Top quark pair + 1-jet production at next-to-leading order QCD

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*) Funded through Heisenberg fellowship and SFB-TR09

Deutsche Forschungsgemeinschaft DFG

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Why is top quark physics interesting ?

Top quark is the heaviest elementary particle discovered so far

- Top mass close to the scale of electroweak symmetry breaking, special role in EWSB?
  Is the unnatural natural Yukawa coupling natural ? *)

- Is the top quark still pointlike ?

→ Top quark plays special role in many extensions of the SM

In the SM top quark couplings highly constrained by gauge structure

Are the quantum numbers as predicted by the SM ?

*) Giudice, Naturally Speaking: The naturalness Criterion and Physics at the LHC, arXiv:0801.2562

Unique feature of the top quark

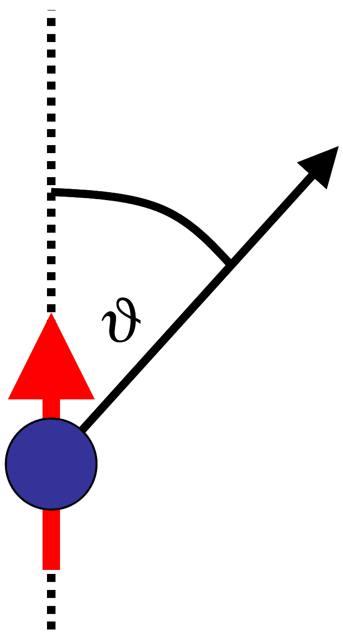
Top quark extremely short lived:

$$\Gamma_t = 1.48 GeV \rightarrow \tau_t = 0.44 \times 10^{-24} s < \tau_{QCD} \approx 3 \times 10^{-24} s$$

Top quark decays essentially as a free quark

[Bigi, Dokshitzer, Khoze, Kühn, Zerwas 86]

The polarisation is transferred to the angular distribution of the decay products via the parity violating decay



$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\vartheta} = \frac{1}{2}\left(1 + \kappa_f \cos\vartheta\right)$$

	$\ell^+, \bar{d}$	ν_ℓ^+, u	b	W^+	least energetic jet from $q\bar{q}'$
κ_f	1	-0.31	-0.41	0.41	0.51

QCD corrections also known

[Czarnecki, Jezabek, Kühn 91, Brandenburg, Si, P.U. '02]

Top quark spin correlation

Quantum mechanics:

$$^{(2s+1)}l_j$$

$$^3S_1 \qquad\qquad ^1S_0$$

close to threshold:

$$q\bar{q} \to t\bar{t} \qquad\qquad gg \to t\bar{t}$$

→ Spins are parallel or anti-parallel close to threshold

$$C = \frac{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow)}$$

Spinasymmetry can be observed double diff. distributions

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\vartheta_\ell \, d\cos\vartheta_{\bar{\ell}}} = \frac{1}{4}(1 - C\cos\vartheta_\ell\cos\vartheta_{\bar{\ell}})$$

Task for Tevatron and the LHC

Measure top quark properties as precise as possible

At the Tevatron analysis are limited by the statistics

LHC is a top quark factory

L ~ 10 / fb / year → ~8 millionen top quark pairs
~3 millionen single top quarks

Expectations:

$$\Delta m_t \approx 1\,\text{GeV}$$

$$\frac{\Delta\sigma_{tt}}{\sigma_{tt}} \approx 10\%$$

Important measurements

- ✓ tt cross section — Precise determination of top mass, consistency checks with theo. predictions, search for new physics in the tt invariant mass spectrum
- ✓ W-Polarization in top decay — Test of the V-A structure in top decay
- ✓ ttH cross section — Measurement of the Yukawa coupling
- ✓ ttZ cross section — Measurement of the Z couplings
- ✓ Single top production — Direct measurement of the CKM matrix element Vtb, top polarization, search for anomalous Wtb couplings
- ✓ Spin correlations — Weak decay of a `free' quark, bound on the top width and V_{tb}, search for anomalous couplings
- ✓ tt+Jet(s) production — Search for anomalous couplings, important background
- ttγ cross section — Measurement of the electric charge

A lot of progress recently

Why is $t\bar{t}$ + 1 Jet important ?

1. Phenomenological importance:

- Important signal process
  - Large fraction of inclusive tt are due to tt+jet
  - Search for anomalous couplings
  - New physics ?
  - Forward-backward charge asymmetry (Tevatron)
  - Top quark pair production at NNLO ? *)
- Important background process
  - Dominant background for Higgs production via WBF and many new physics searches
  - ...

*) Recent progress due to [Czakon, Mitov, Moch]

Motivation: Why is $t\bar{t}$ + 1 Jet important ?

2. "Technical importance":

Important benchmark process for one-loop calculations for the LHC

Significant complexity due to:

- All partons are coloured
- Additional mass scale m_t
- Infrared structure complicated
- Many diagrams, large expressions

Ideal test ground for developing and testing of new methods for one-loop calculations

Technical details

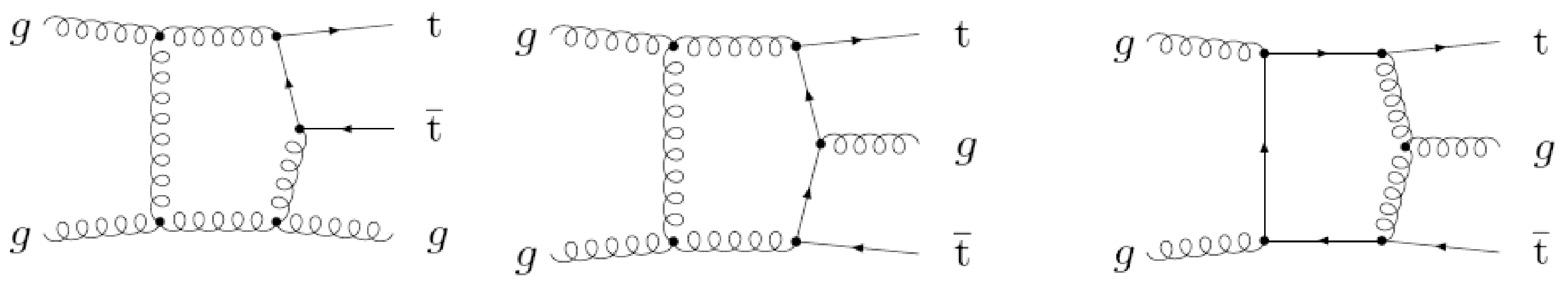
Virtual corrections

Partonic processes: $gg \to t\bar{t}g,\ \underbrace{q\bar{q} \to t\bar{t}g,\ qg \to t\bar{t}q,\ g\bar{q} \to t\bar{t}\bar{q}}$

related by crossing

Number of 1-loop diagrams ~ 350 (100) for $gg\ (q\bar{q})$

Most complicated 1-loop diagrams pentagons of the type:



Algebraic decomposition of amplitudes:

color, i.e. $C_1 = (T_{a_1} T_{a_2} T_{a_3})_{\bar{t}t}$

$$\mathcal{A}(gg \to t\bar{t}g) = \sum_{k,l} f_{kl}(\{(p_i \cdot, p_j\}) \times S_k \times C_l$$

standard matrix elements, i.e. $S_1 = \langle k_{\bar{t}} | \varepsilon_1 | k_t \rangle (\varepsilon_2 \cdot \varepsilon_3),$

Reduction of tensor integrals — what we did…

Four and lower-point tensor integrals:

Reduction à la Passarino-Veltman,
with special reduction formulae in singular regions,
→ two complete independent implementations !

Five-point tensor integrals:

- Apply 4-dimensional reduction scheme, 5-point tensor integrals are reduced to 4-point tensor integrals

  → No dangerous Gram determinants!

  [Denner, Dittmaier 02]

  Based on the fact that in 4 dimension 5-point integrals can be reduced to 4 point integrals

  [Melrose ´65, v. Neerven, Vermaseren 84]

- Reduction à la Giele and Glover [Duplancic, Nizic 03, Giele, Glover 04]

  Use integration-by-parts identities to reduce loop-integrals

Real corrections

Numerical evaluation of the amplitude in the helicity bases

Methods:
- Feynman diagramatic approach
- Berends-Giele recurrence relations
- Madgraph

Treatment of soft and collinear singularities à la Catani and Seymour

[Frixione,Kunszt,Signer ´95, Catani,Seymour ´96, Nason,Oleari 98, Phaf, Weinzierl 02, Catani,Dittmaier,Seymour, Trocsanyi ´0

$$\sigma_{\mathrm{NLO}} = \underbrace{\int_{m+1} [\sigma_{\mathrm{real}} - \sigma_{\mathrm{sub}}]}_{\mathrm{finite}} + \underbrace{\int_{m} [\sigma_{\mathrm{virt.}} + \bar{\sigma}^{1}_{\mathrm{sub}}]}_{\mathrm{finite}} + \underbrace{\int dx \int_{m} [\sigma_{\mathrm{fact.}}(x) + \bar{\sigma}_{\mathrm{sub}}(x)]}_{\mathrm{finite}}$$

With:

$$0 = -\int_{m+1} \sigma_{\mathrm{sub}} + \int_{m} \bar{\sigma}^{1}_{\mathrm{sub}} + \int dx \int_{m} \bar{\sigma}_{\mathrm{sub}}(x)$$

$\sigma_{\mathrm{sub}} \longrightarrow \sigma_{\mathrm{real}}$ in all single-unresolved regions

Subtraction

$$\sigma_{\text{sub}} = \sum_{\text{dipoles}} \mathcal{D}_{ij,k}(p_i, p_j, p_k)$$

$$\mathcal{D}_{ij;k} = -\frac{1}{(p_i + p_j) - m_{ij}^2} \langle \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \left| \frac{\mathbf{T}_a \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}} V_{ij,k} \right| \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \rangle$$

Note: there are many of them (i.e. 36 for gg→ttgg)

Two independent libraries to calculate the dipoles

Significant amount of computing power goes into dipoles!

Main issue: **Speed !**

Dipole subtraction method — implementation

emacs@pcth188.cern.ch

File Edit Options Buffers Tools C++ Help

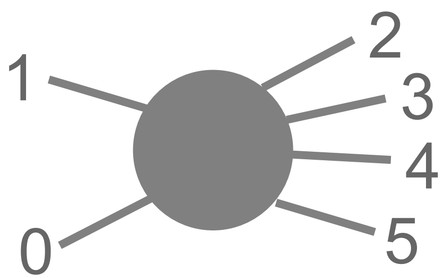
```cpp
double ggttgg_counterterm(const vector<FourMomentum>& momenta) {

  static ggttg ggttgamplitude;
  static Correlator correlator(ggttgamplitude);

  static Dipole d[36] = {
    // FinalFinal:
    Dipole(2,4,3), Dipole(2,4,5), Dipole(2,5,3), Dipole(2,5,4),
    Dipole(3,4,2), Dipole(3,4,5), Dipole(3,5,2), Dipole(3,5,4),
    Dipole(4,5,2), Dipole(4,5,3),
    //FinalInitial:
    Dipole(2,4,0), Dipole(2,4,1), Dipole(2,5,0), Dipole(2,5,1),
    Dipole(3,4,0), Dipole(3,4,1), Dipole(3,5,0), Dipole(3,5,1),
    Dipole(4,5,0), Dipole(4,5,1),
    // InitialFinal:
    Dipole(0,4,2), Dipole(0,4,3), Dipole(0,4,5), Dipole(0,5,2),
    Dipole(0,5,3), Dipole(0,5,4), Dipole(1,4,2), Dipole(1,4,3),
    Dipole(1,4,5), Dipole(1,5,2), Dipole(1,5,3), Dipole(1,5,4),
    // IntialIntial:
    Dipole(0,4,1), Dipole(0,5,1), Dipole(1,4,0), Dipole(1,5,0)
  };


  SplittingKernels splittings(momenta,particles);


  double sum = 0.;

  for (int i = 0; i<36; i++) {
    splittings.Kernel(d[i]);
    correlator.EvalfDipole(d[i]);
    sum += d[i].value;
  }

  return( sum );

}
```

Dipole(1,5,0) $\longrightarrow \mathcal{D}_{15,0}$

--:-- Dipoles.cpp 3:44PM 0.02 (

LO – amplitude, with colour information, i.e. correlations

List of dipoles we want to calculate



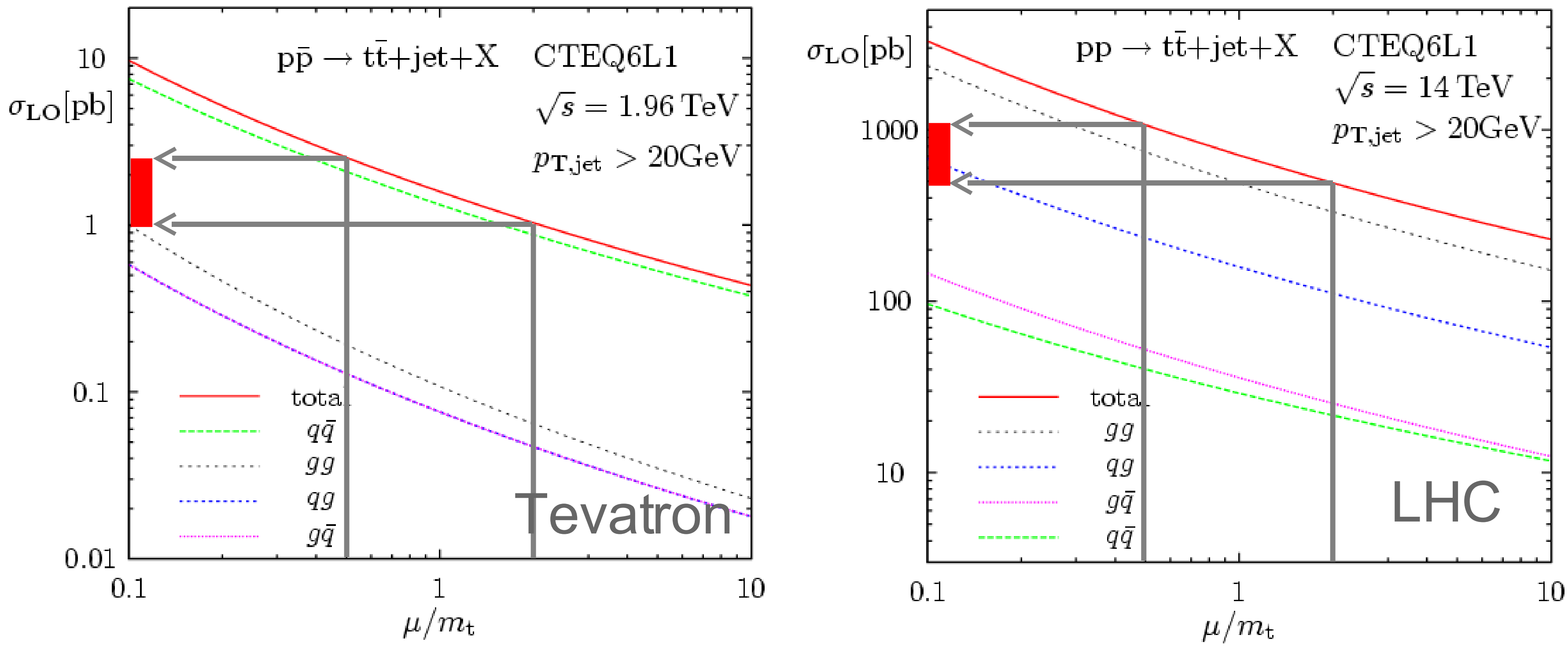
reduced kinematics, “tilde momenta”

Dipole d_i

Indeed: no rocket science…

Results

Leading-order results — some features



Observable:

- Assume top quarks as always tagged
- To resolve additional jet demand minimum k_t of 20 GeV

Note:

- Strong scale dependence of LO result
- No dependence on jet algorithm
- Cross section is **NOT** small

Checks of the NLO calculation

- Leading-order amplitudes checked with Madgraph
- Subtractions checked in singular regions
- Structure of UV singularities checked
- Structure of IR singularities checked

CHECKED !

Most important:

- Two complete independent programs using a complete different tool chain and different algorithms, complete numerics done twice !

For example:

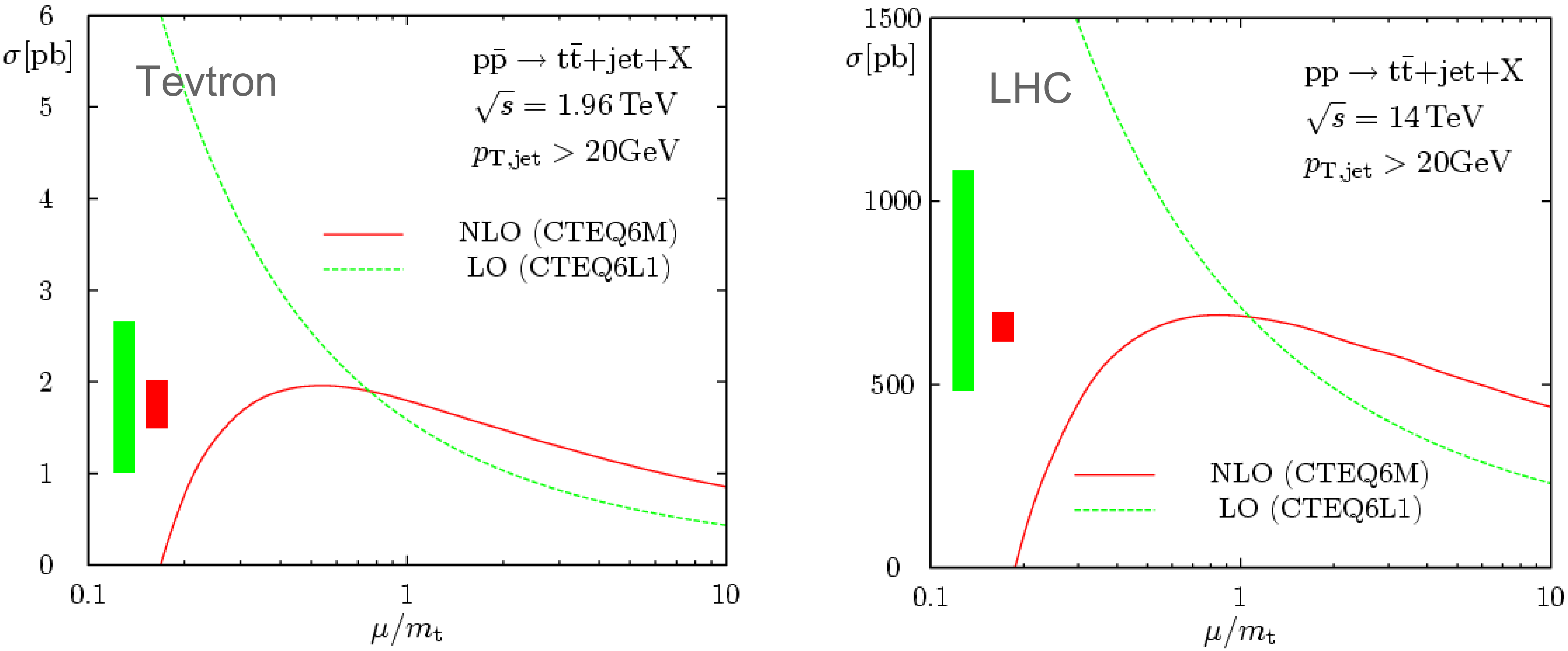
Virtual corrections:

→ Feynarts 1.0 — Mathematica — Fortran77

→ QGraf — Form3 — C,C++

Top-quark pair + 1 Jet Production at NLO

[Dittmaier, P.U., Weinzierl PRL 98:262002, '07]



- Scale dependence is improved
- Sensitivity to the jet algorithm
- Corrections are moderate in size
- Arbitrary (IR-safe) observables calculable

→ work in progress

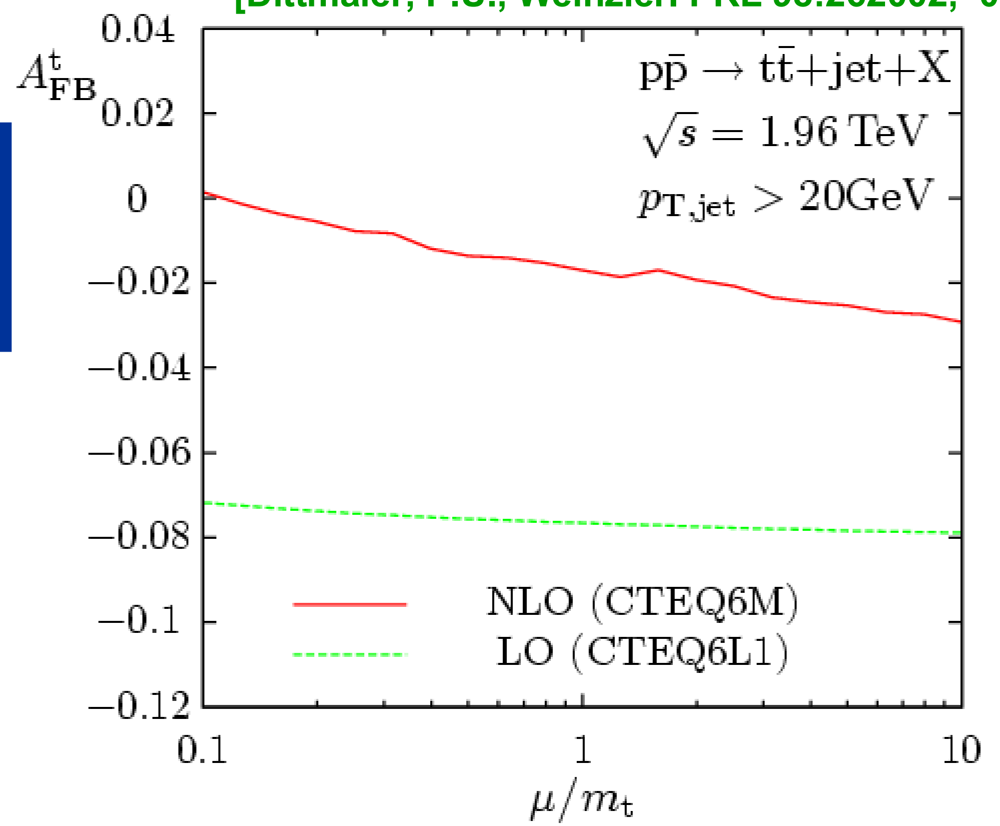
Forward-backward charge asymmetry (Tevatron)

$$A_{\mathrm{FB}}^{t} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)}$$

Effect appears already in top quark pair production

[Kühn, Rodrigo]

[Dittmaier, P.U., Weinzierl PRL 98:262002, '07]



- Numerics more involved due to cancellations
- Large corrections, LO asymmetry almost washed out
- Refined definition (larger cut, different jet algorithm…) ?

Forward-backward charge asymmetry

Tevatron

p_T^{cut} [GeV]	cross section [pb] LO	cross section [pb] NLO	charge asymmetry [%] LO	charge asymmetry [%] NLO
20	$1.583(2)^{+0.96}_{-0.55}$	$1.791(1)^{+0.16}_{-0.31}$	$-7.69(4)^{+0.10}_{-0.085}$	$-1.77(5)^{+0.58}_{-0.30}$
30	$0.984(1)^{+0.60}_{-0.34}$	$1.1194(8)^{+0.11}_{-0.20}$	$-8.29(5)^{+0.12}_{-0.085}$	$-2.27(4)^{+0.31}_{-0.51}$
40	$0.6632(8)^{+0.41}_{-0.23}$	$0.7504(5)^{+0.072}_{-0.14}$	$-8.72(5)^{+0.13}_{-0.10}$	$-2.73(4)^{+0.35}_{-0.49}$
50	$0.4670(6)^{+0.29}_{-0.17}$	$0.5244(4)^{+0.049}_{-0.096}$	$-8.96(5)^{+0.14}_{-0.11}$	$-3.05(4)^{+0.49}_{-0.39}$

central value

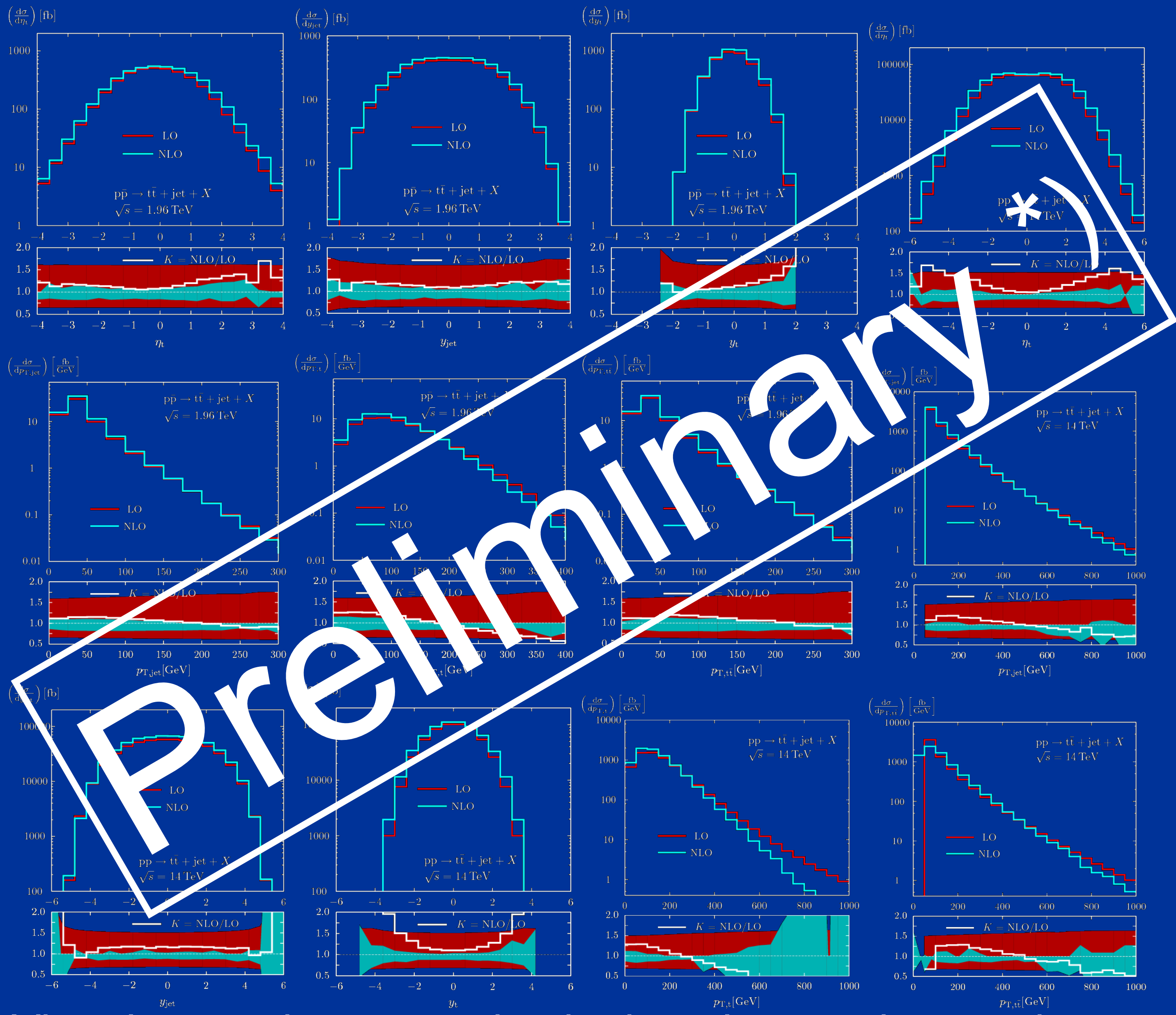
uncertainty
num. intgration

shift towards μ=2m,m/2

- cross section receives moderate corrections
- scale dependence largely reduced
- large corrections to the asymmetry

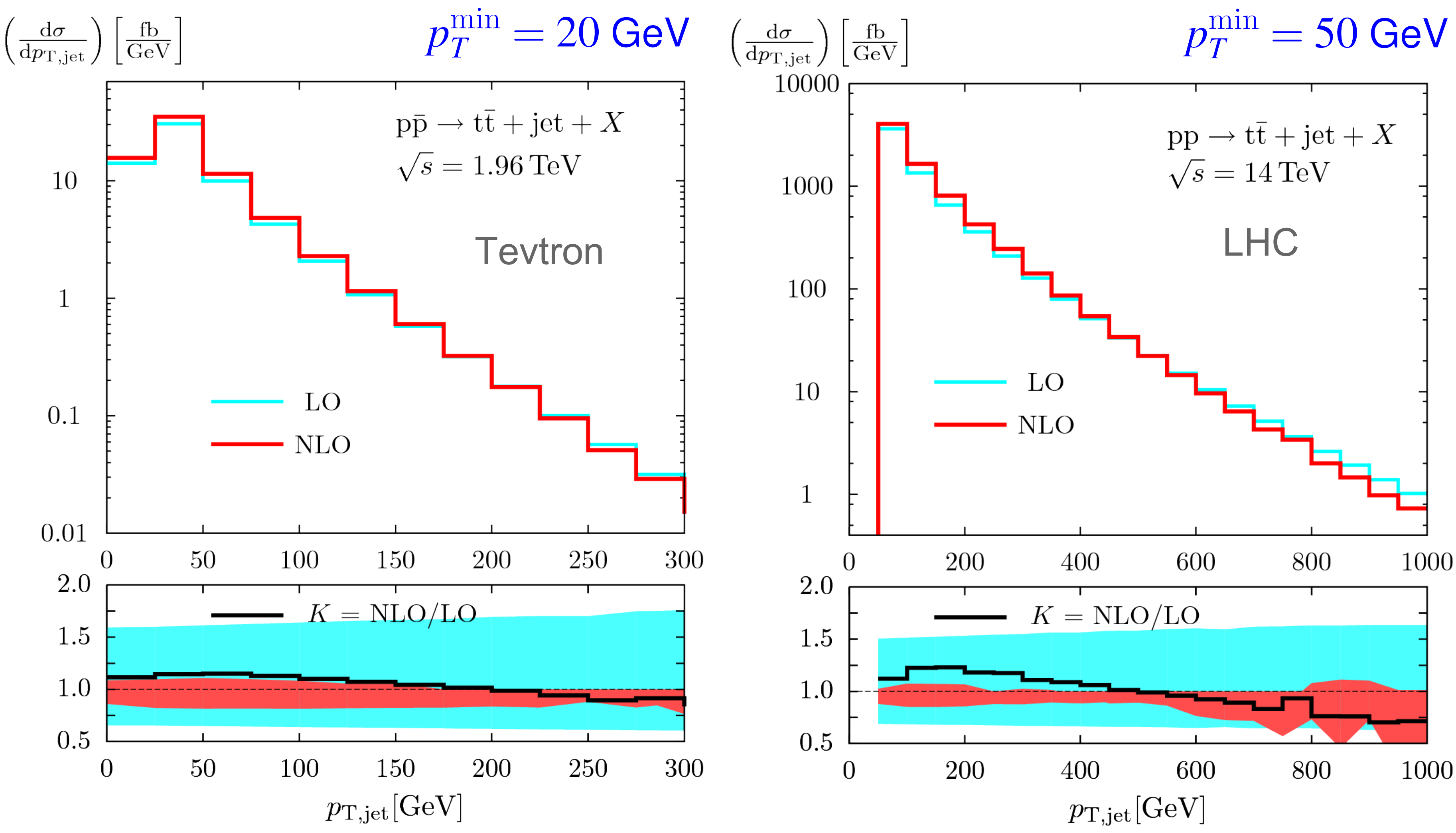
→ no conclusive picture yet

Differential distributions



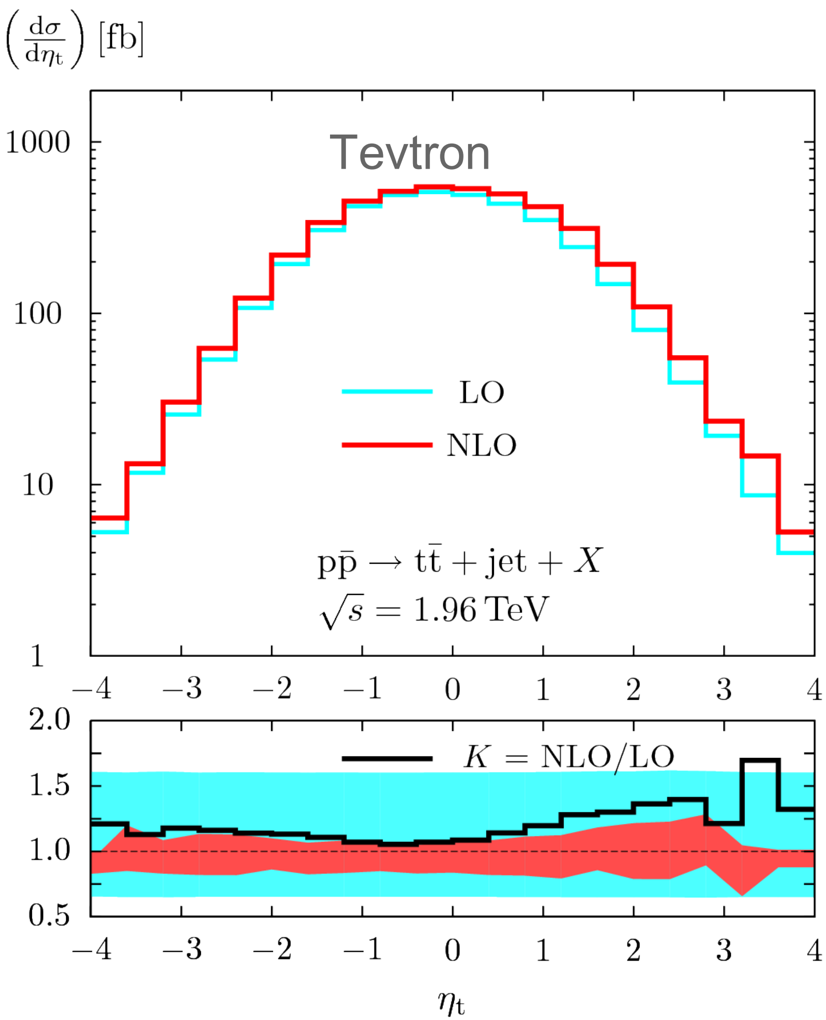
*) Virtual correction cross checked, real corrections underway

p_T distribution of the additional jet



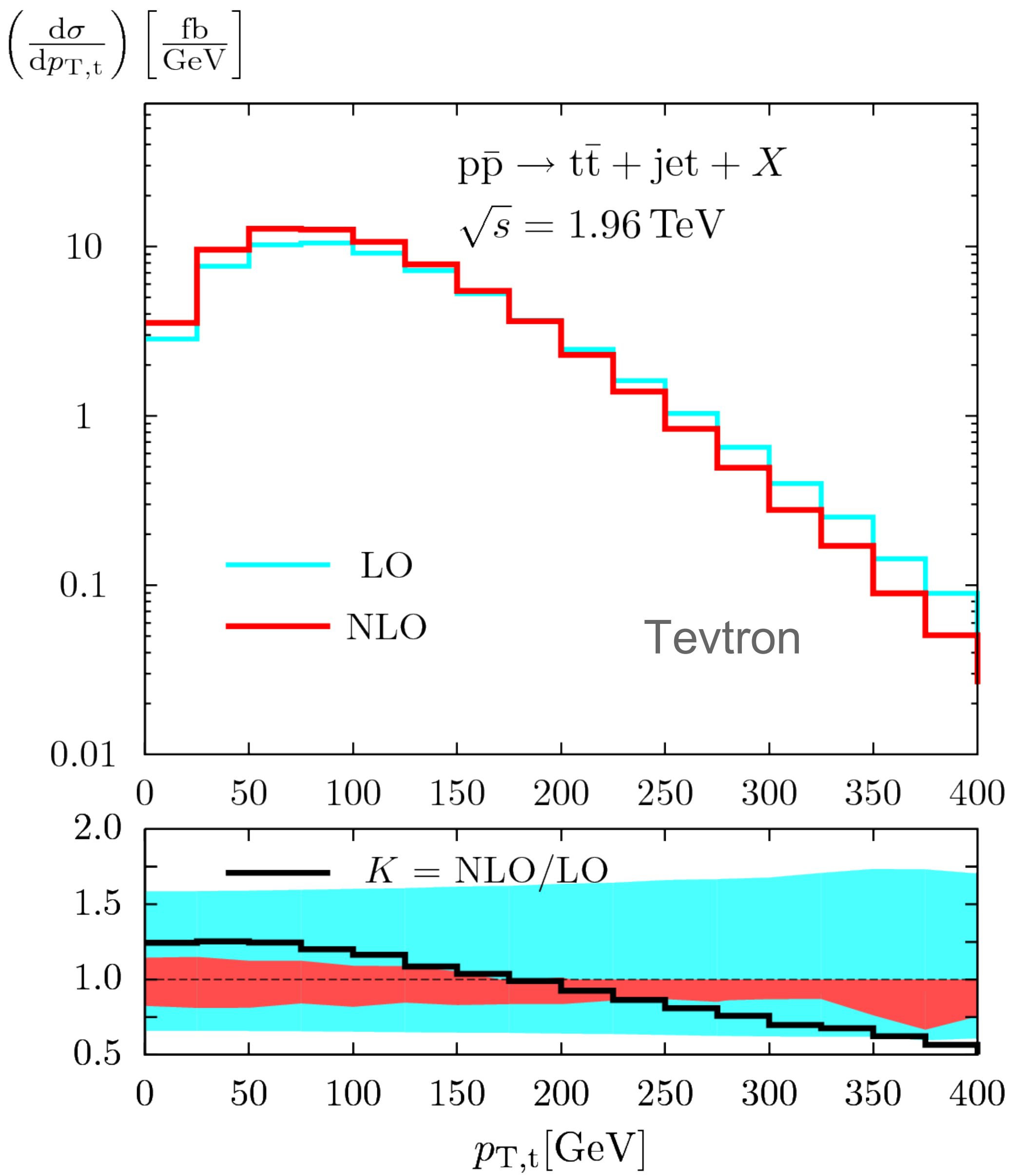
Corrections of the oder of 10-20 %,
again scale dependence is improved

Pseudo-Rapidity distribution



again: charge asymmetry is washed out by the corrections

Top quark p_t distribution



The K-factor is not a constant!

→ Phase space dependence, dependence on the observable

Conclusions

Top quark physics:

- Many interesting measurements possible at LHC and Tevatron
- A lot of progress as far as theory is concerned

Top quark pair + 1-Jet production at NLO:

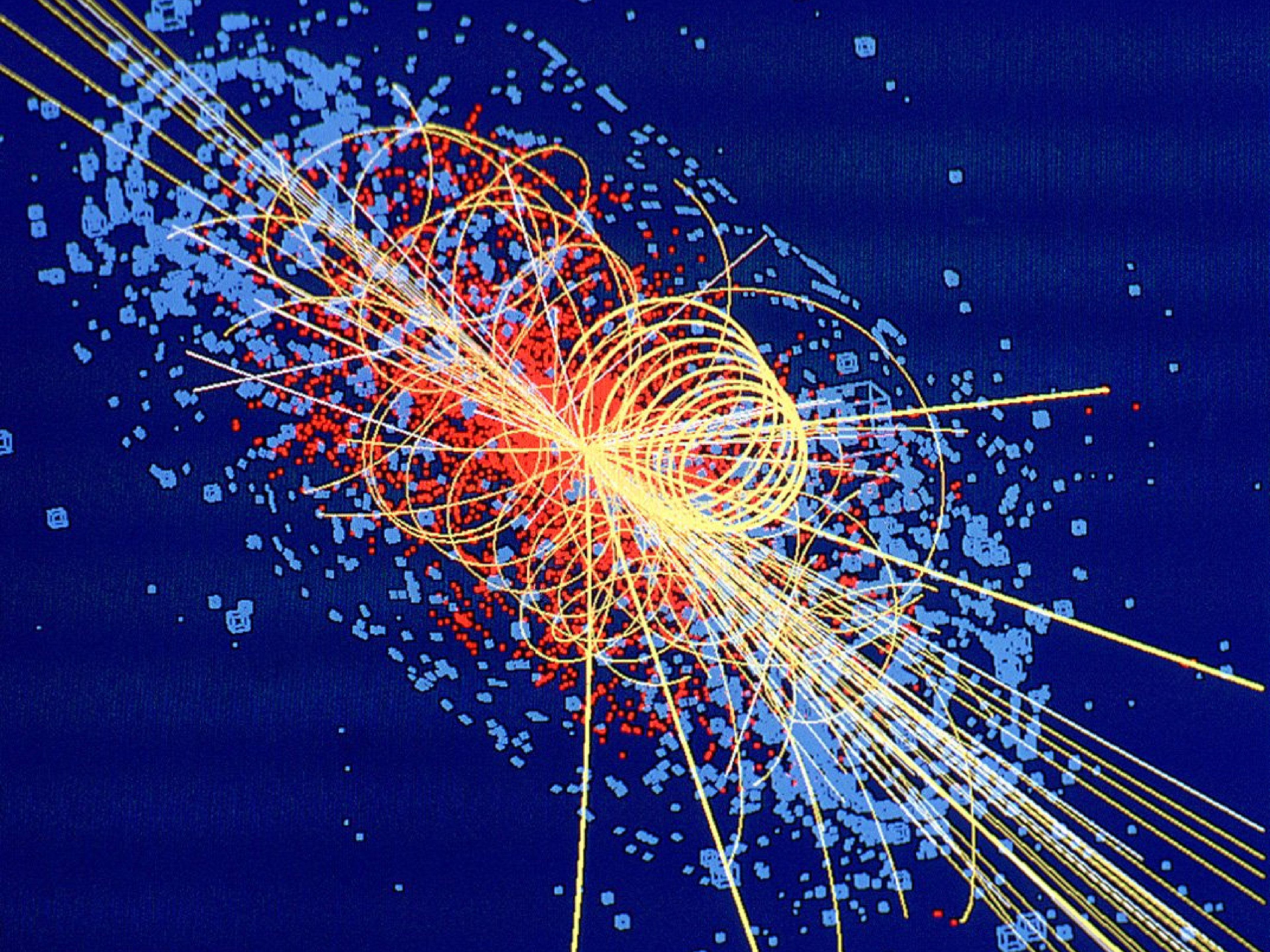
- Non-trivial calculation
- Two complete independent calculations
- Methods used work very well
- Cross section corrections are under control
- Further investigations for the FB-charge asymmetry necessary (Tevatron)
- Preliminary results for distributions

Outlook

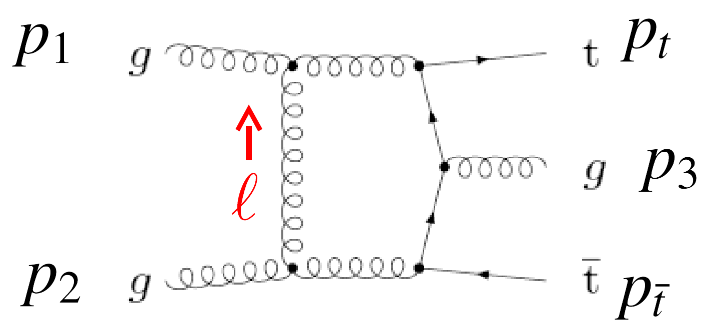
- Proper definition of FB-charge asymmetry
- Top decay
- Further improvements possible
  (remove redundancy, further tuning, except. momenta,…)
- Apply tools to other processes, i.e. WWj@NLO

[Dittmaier, Kallweit, PU]

see also Stefan Kallweit´s talk



Tensor integrals



$$\int d^d\ell \frac{\ell_\mu \ell_\nu \ell_\rho \ldots}{(\ell^2 + i\varepsilon)((\ell + p_1)^2 + i\varepsilon)((\ell + p_1 - p_t)^2 - m_t^2 + i\varepsilon)}$$
$$\times \frac{1}{((\ell - p_1 + p_{\bar{t}})^2 - m_t^2 + i\varepsilon)} \frac{1}{((\ell - p_2)^2 + i\varepsilon)}$$

Issues: Numerical stable and fast reduction to scalar integrals

Motivation: One loop calculations for LHC

„State of the art“:

2→3 reactions at the border of what is feasible with current techniques*)

High demand for one-loop calculations for the LHC:

Les Houches 05: NLO wishlist for LHC

Les Houches 07 wishlist

process ($V \in \{Z, W, \gamma\}$)	# groups working on
1. $pp \to V\,V$ jet	2
2. $pp \to t\bar{t}\,b\bar{b}$	1
3. $pp \to t\bar{t} + 2$ jets	
4. $pp \to W\,W\,W$	1
5. $pp \to V\,V\,b\bar{b}$	
6. $pp \to V\,V + 2$ jets	
7. $pp \to V + 3$ jets	
8. $b\bar{b}b\bar{b}$	1
9. $gg \to W^*W^*$ (NLO, 2 loops)	?
10. EW corrections to VBF	1
11. NNLO to VBF, $t\bar{t}$, Z/γ+jet, W+jet	

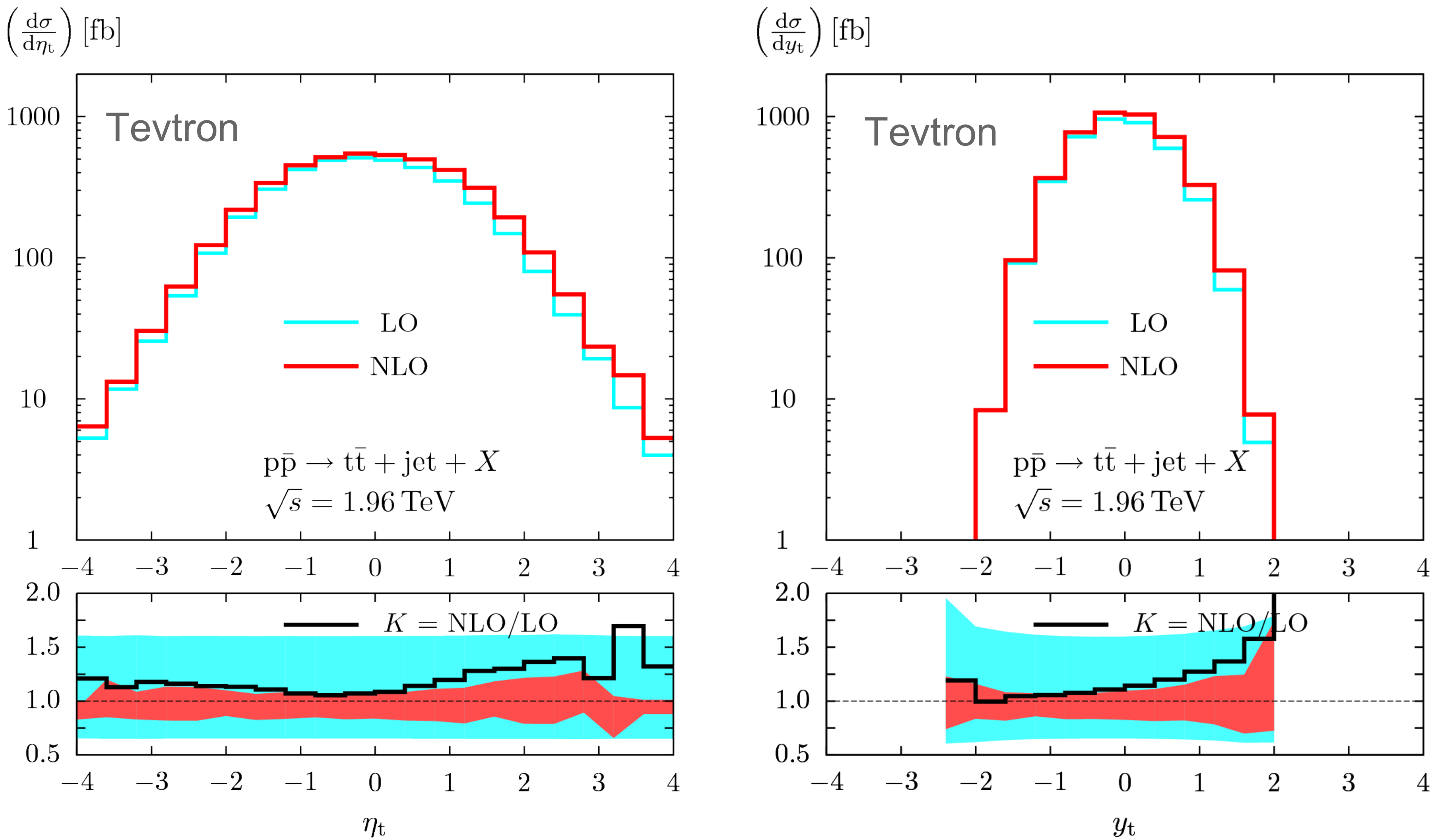
Summary of activities in NLO multi-leg working group

Nice overview of current Status in Gudruns opening talk Les Houches ´07

[Gudrun Heinrich]

*) Only one 2→4 calculation available so far [Denner, Dittmaier, Roth, Wieders 05], many uncalculated 2→3 processes...

Rapidity versus Pseudo-Rapidity



Performance

Accuracy:

Both methods for tensor reduction agree to high accuracy

→ 10 Digits agreement for individual phase space points

After integration: complete agreement within stat. error

Runtime: (3GHz P4)

~ 30 ms for the evaluation of gg→ttg@1-loop

some improvements possible: remove redundancy