The $\overline{B} \to X_s\gamma$ decay rate: Finalizing the NNLO program

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in collaboration with
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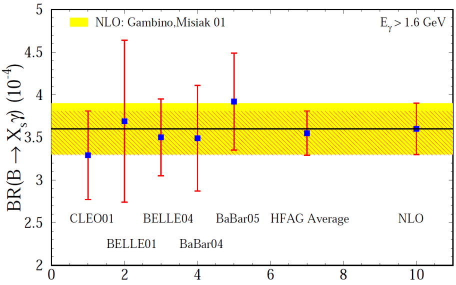
Loops and Legs 2008, Sondershausen
24. April 2008

Motivation

- $\overline{B} \to X_s\gamma$ decay most precise $\Delta B = 1$ FCNC measurement:

$$\mathcal{B}(\overline{B} \to X_s\gamma)_{\text{exp}}^{E_\gamma > 1.6\text{GeV}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$$ [HFAG 2006]



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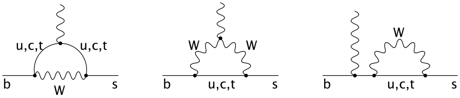
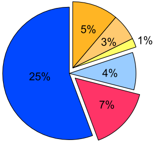


- branching ratio: strong constraints on new physics
- photon spectrum: determination of HQET parameters

$$\mathcal{B}(\overline{B} \to X_s\gamma)\big|_{E_\gamma > 1.6\text{GeV}} = \mathcal{B}(\overline{B} \to X_c e\overline{\nu}) \left[\frac{\Gamma(\bar{b} \to s\gamma)}{\Gamma(b \to ce\overline{\nu})}\right]_{LO} f\left(\frac{\alpha_s(M_w)}{\alpha_s(m_b)}\right)$$

$$\times \left[1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_{em}) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\frac{\Lambda^2}{m_b^2}) + \mathcal{O}(\frac{\Lambda^2}{m_c^2}) + \mathcal{O}(\alpha_s \frac{\Lambda}{m_b})\right]$$



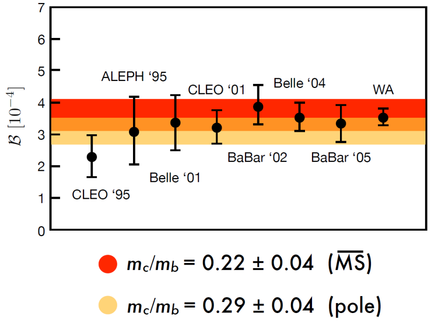
SM expectation for $\overline{B} \to X_s\gamma$ at NLO

$$\mathcal{B}(\overline{B} \to X_s\gamma)^{E_\gamma > 1.6\text{GeV}}_{\text{th, NLO}} = (3.57 \pm 0.30) \times 10^{-4}$$

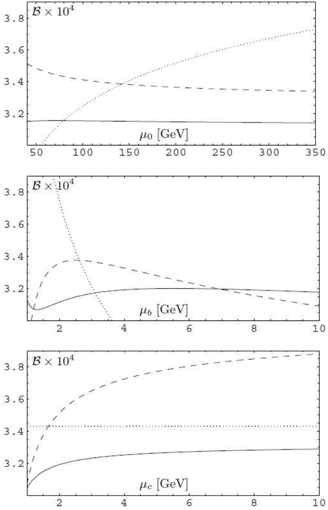
[Misiak et al. 2001, Buras et al. 2002]

- exp. error < theoretical error
- large scale dependence
- large m_c scheme dependence

$\Rightarrow$ need for NNLO corrections



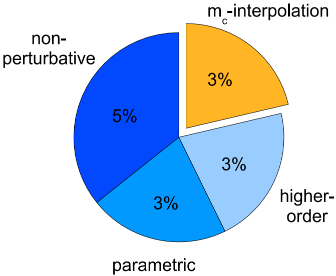
The first NNLO estimate of $\overline{B} \to X_s\gamma$



$$\mathcal{B}^{E_\gamma > 1.6\text{GeV}}_{\text{th, NNLO}} = (3.15 \pm 0.23) \times 10^{-4}$$

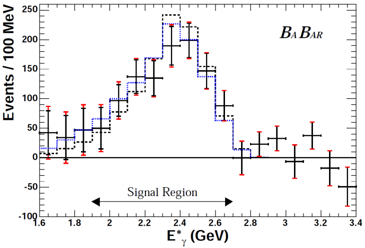
[Misiak et al., 2006]

- nice reduction of μ-dependence in comparison to NLO
- most pronounced effect for μ_c



'Inclusiveness' and Photon Energy Cut

- measurement of total rate not possible due to large background
- experimental cut on photon energy $E_\gamma \geq E_{\text{cut}} = 1.8\text{GeV}$
- perturbation theory:
  - branching ratio prediction for $E_{\text{cut}} \approx 0$ to avoid dependence on heavy quark distribution function
  - assumption: fixed-order evaluation of photon spectrum with dominant contribution $\langle s\gamma|Q_7|b\rangle$ valid up to $E_0 = 1.6\text{GeV}$ [Melnikov, Mitov 2005]
- measured branching ratio extrapolation to photon energy cut E_0 [Buchmüller, Flächer 2006]



Theoretical Framework

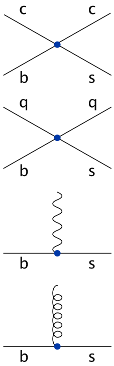
effective theory without top- and W-fields

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QEDxQCD}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i(\mu) + R$$

- operator basis

$$\begin{aligned}
Q_{1,2} &= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b) & |C_i(m_b)| &\approx 1 \\
Q_{3,4,5,6} &= (\bar{s}\Gamma_i c)\sum_q (\bar{q}\Gamma_i' q), & |C_i(m_b)| &< 0.07 \\
Q_7 &= \frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & C_7(m_b) &\approx -0.3 \\
Q_8 &= \frac{g m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_T G^a_{\mu\nu}, & C_8(m_b) &\approx -0.15
\end{aligned}$$

- large logarithms $\log \frac{m_b^2}{m_W^2} \approx -6$ resummed with renormalization group techniques



Major Steps of Calculation

- Matching
  match effective theory on full theory calculating vacuum graphs
  $\Rightarrow$ Wilson Coefficients at large scale $C(\mu \approx m_W)$
- Mixing
  renormalization group running of Wilson coefficients through anomalous dimensions of effective operators
  $\Rightarrow$ log resummed Wilson coefficients at small scale $C(\mu \approx m_b)$
- Matrix Elements
  $\overline{B} \to X_s \gamma$ inclusive
  $\Rightarrow$ calculate matrix elements in perturbation theory

Contributions at NNLO

Matching

- $(Q_1, \ldots, Q_6)$ at 2-loops
  [Bobeth, Misiak, Urban 00]
- Q_7 and Q_8 at 3-loops
  [Misiak, Steinhauser 04]

Matrix Elements

- Q_7, Q_8
  [Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov 05] [Asatrian, Hovhannisyan, Poghosyan, Ewerth, Greub, Hurth 06]
- Q_1, Q_2, Q_7, Q_8 in large-β_0 approx.
  [Bieri, Greub, Steinhauser 03]

Mixing

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  [Gorbahn, Haisch 05]
- (Q_7, Q_8) at 3-loops
  [Gorbahn, Haisch, Misiak 05]
- $(Q_1, \ldots, Q_6)$ to (Q_7, Q_8) at 4-loops
  [Czakon, Haisch, Misiak 06]

- Q_1, Q_2 as large-m_c expansion
  [Misiak, Steinhauser 06]
- Q_1, Q_2, fermionic corrections
  [Boughezal, Czakon, T.S. 07]

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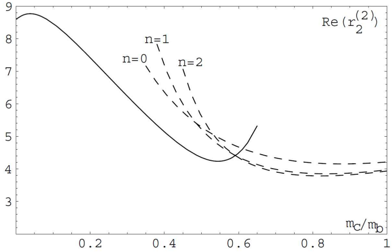
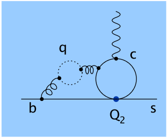
$\Rightarrow$ $Q_{1,2}$ matrix elements not fully available

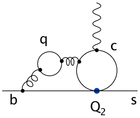
$Q_{1,2}$ Matrix Elements and Interpolation in m_c

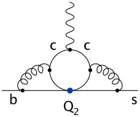
[Misiak, Steinhauser 2006]

- large-β_0 approximation: expansions in the limits $m_c/m_b \to 0, \infty$ match nicely, already for the first term





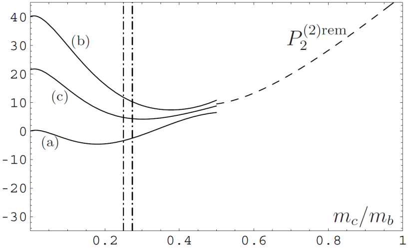


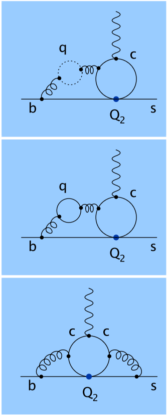


$Q_{1,2}$ Matrix Elements and Interpolation in m_c

[Misiak, Steinhauser 2006]

- large-β_0 approximation: expansions in the limits $m_c/m_b \to 0, \infty$ match nicely, already for the first term
- beyond β_0 parts: calculation of first term in the large m_c limit and extrapolation to yet unknown region $m_c = 0$
  $\Rightarrow$ uncertainty of 3% in branching ratio



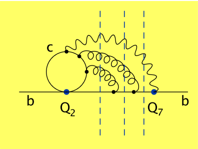


$m_c > m_b$

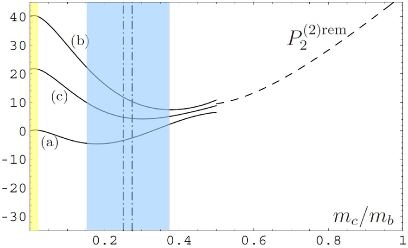
$Q_{1,2}$ Matrix Elements and Interpolation in m_c

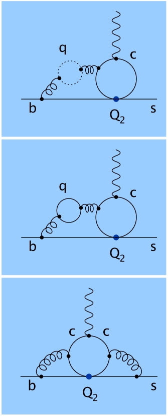
[Misiak, Steinhauser 2006]

- large-β_0 approximation: expansions in the limits $m_c/m_b \to 0, \infty$ match nicely, already for the first term
- beyond β_0 parts: calculation of first term in the large m_c limit and extrapolation to yet unknown region $m_c = 0$
  $\Rightarrow$ uncertainty of 3% in branching ratio
- improvement of m_c interpolation



$m_c = 0$





$m_c \neq 0$

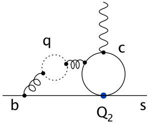
$m_b \neq 0$

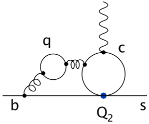
Calculation: $Q_{1,2}$ Matrix Elements at $\mathcal{O}(\alpha_s^2)$

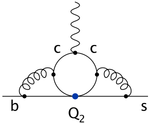
NNLO $\langle s\gamma|O_{1,2}|b\rangle$ at physical m_c

$\Rightarrow$ virtual corrections

- approx. 400 3-loop on-shell vertex diagrams involving two scales (m_b, m_c)
- approx. 500 master integrals
- determination of master integrals: differential equations







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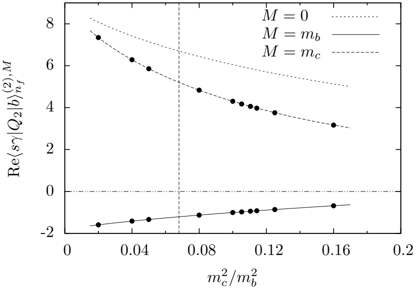
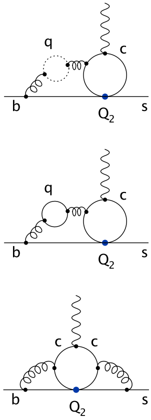
⇒ virtual corrections

→ fermionic parts completed [Boughezal, Czakon, T.S. 2007]





→ confirmation of large-β_0 parts [Bieri et al. 03]

→ decoupling-like effect for b-quark loop insertions

⇒ estimated impact on branching ratio: +1-2%[a]

[a] thanks to M. Misiak

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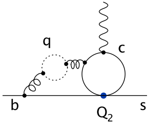
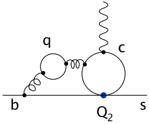
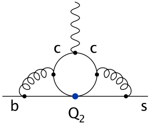
⇒ working apparatus available





⇒ IBP reduction still in progress (but close . . .)

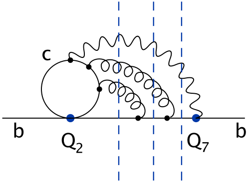


Calculation: $Q_{1,2}$ Matrix Elements at $\mathcal{O}(\alpha_s^2)$

NNLO $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c = 0$

$\Rightarrow$ virtual + real corrections

- 4-loop on-shell propagators with selective cuts on interference (Q_2, Q_7)
- possible cuts: $s\gamma$, $s\gamma g$, $s\gamma gg$, $s\gamma q\bar{q}$, $s\gamma gq\bar{q}$
- IBP reduction completed



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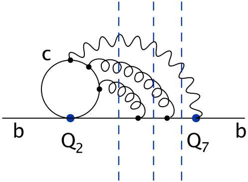
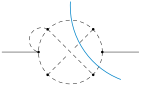


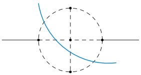
Determination of masters:

- massless master integrals: Mellin-Barnes based method

  $\rightarrow$ completed up to 4-particle cuts



$$= -\frac{3.142}{\epsilon^5} + \frac{i\,19.74 - 12.567}{\epsilon^4} + \frac{i\,78.957 + 86.326}{\epsilon^3} + \ldots$$



$$= \frac{15.105}{\epsilon^2} + \frac{i\,47.46 + 211.15}{\epsilon} + (i\,663.3 + 1448.5) + \ldots$$

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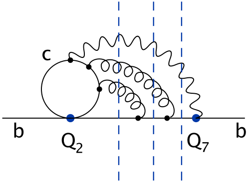

Determination of masters:

- massless master integrals: Mellin-Barnes based method
  $\rightarrow$ completed up to 4-particle cuts

- master integrals with internal b-quark: differential equations applied to off-shell integrals

Determination of massive Master Integrals

- evaluation of off-shell master integrals $V_i(z,\epsilon)$ with help of numerical differential equations (deqns) [Caffo, , Czyż, Remiddi 2002]

$$\frac{d}{dz}V_i(z,\epsilon) = A_{ij}(z,\epsilon)V_j(z,\epsilon), \quad z = p^2/m_b^2$$

- Idea:
  - calculate integrals at some "simple" point (e.g. large-mass limit $p^2 \ll m_b^2$)
  - Integrate system of deqns starting at this limit up to the on-shell condition $z = 1$

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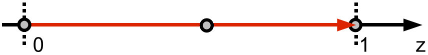

$\rightarrow$ but: deqns singular in both endpoints! (and on naive contour $z \in \mathbb{R}$)

$\Rightarrow$ solution: combine expansions with numerical integration in complex plane

Determination of massive Master Integrals

Solution of the system of differential equations:

- expand in ϵ and z in the limit $z \to 0$ with the power logarithmic ansatz:

$$V_i(z,\epsilon) = \sum_{nmk} c^0_{inmk} \epsilon^n z^m \log^k z$$

- solve recursively for c^0_{inmk} up to high powers in z
- boundary conditions:
  - automatized diagrammatic large-mass expansions for $p^2 \ll m_b^2$
  - Mellin Barnes based method

$\Rightarrow$ high precision values for $z \approx 0$

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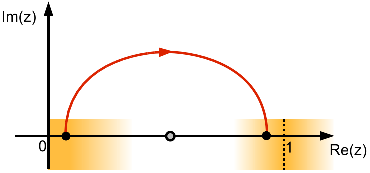
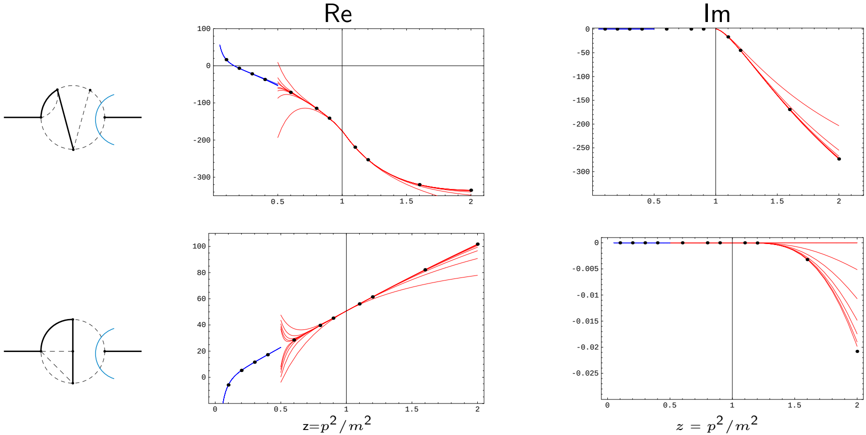
$\Rightarrow$ high precision values for $z \approx 0$

- use these values as starting point for numerical integration (in complex plane) up to $z \approx 1$ (zvode)
- perform another power logarithmic expansion around $z \to 1$ and solve coefficients c^1_{inmk} recursively
- use numerical integration to fix the remaining c^1_{inmk}



Massive Masters for $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c = 0$

First Results: 2-particle Cuts

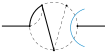


- Expansions:
  - $z \to 0$: up to z^{20}
  - $z \to 1$: up to $(z-1)^{12}$
- Numerics: starting at $z = 0.05$
- Matching: at $z = 0.9$

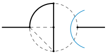
Massive Masters for $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c = 0$

First Results: 2-particle cuts

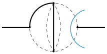
- Numerics at $z = 1$:



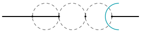
$$= -\frac{2.09440}{\epsilon^3} - \frac{12.7029}{\epsilon^2} - \frac{52.6065}{\epsilon} - 175.318 + \ldots$$



$$= \frac{1.04720}{\epsilon^3} + \frac{3.18387}{\epsilon^2} + \frac{10.9594}{\epsilon} + 35.8017 + \ldots$$



$$= \frac{2.09440}{\epsilon^3} + \frac{15.1844}{\epsilon^2} + \frac{65.1880}{\epsilon} + 240.5444 + \ldots$$



$$= \frac{3.14159}{\epsilon^3} - \frac{20.4204 - i\,9.86964}{\epsilon^2} - \frac{77.7544 - i\,64.1524}{\epsilon} + \ldots$$

- 2-particle cuts: almost there
- 3-particle cuts: boundaries, expansion for $z \to 0$ and numerics available
- 4-particle cuts: in preparation
- Outlook: Complete error analysis, cross checks (two completely independent computations), improvement of precision to at least 16 digits ...

Conclusions

- NNLO corrections to the $\overline{B} \to X_s\gamma$ decay rate needed to match the current and future experimental precision
- Most parts of the NNLO program are available
- Reducing/Removing the remaining interpolation uncertainty requires the full evaluation of $\langle s\gamma|O_{1,2}|b\rangle$ at NNLO

$\Rightarrow$ First step: Computation of fermionic corrections at physical m_c

$\longrightarrow$ independent confirmation of the findings of [Bieri et al. 2003] in the massless case

$\longrightarrow$ keeping full mass dependence: impact on branching ratio $\approx +1$-2%

$\Rightarrow$ Work in progress:

Differential equations (numerics + expansions) provide a powerful tool for the determination of

$\rightarrow$ full virtual corrections at physical m_c

$\rightarrow$ full virtual + real corrections at $m_c = 0$

$\Rightarrow$ Finalizing the $\overline{B} \to X_s\gamma$ program at NNLO comes into reach ...