Two and Three Loop Heavy Flavor Corrections in DIS

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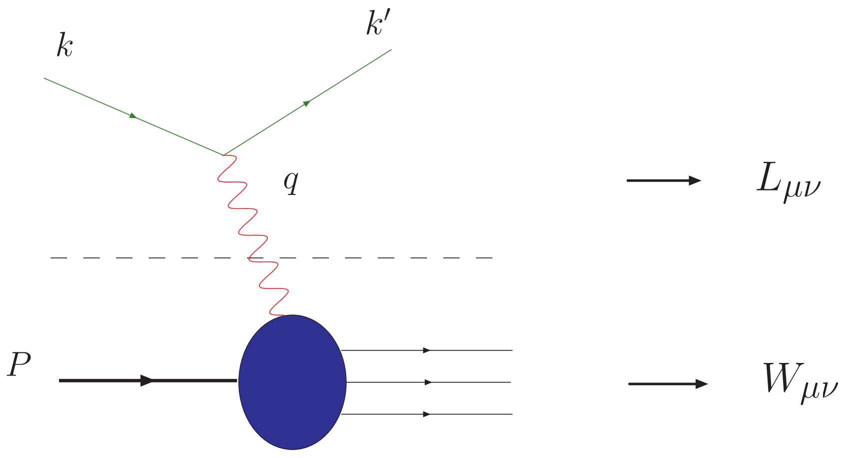
- Introduction
- Renormalization of the OME's to 3 Loops
- O(ϵ) terms at 2 Loops
- Towards Fixed Moments of $A_{Qg}^{(3)}$
- Conclusions

based on:

- I. Bierenbaum, J. Blümlein, S. K., and C. Schneider
  `arXiv:0707.4759 [math-ph];`
  `arXiv:0803.0273 [hep-ph].`
- I. Bierenbaum, J. Blümlein, and S. K.,
  Phys. Lett. **B648** (2007) 195;
  Nucl. Phys. **B780** (2007) 40;
  arXiv:0706.2738 [hep-ph];
  Acta Phys. Polon. B **38** (2007) 3543;
- J. Blümlein, A. De Freitas, W.L. van Neerven, and S. K.,
  Nucl. Phys. **B755** (2006) 272.

1. Introduction

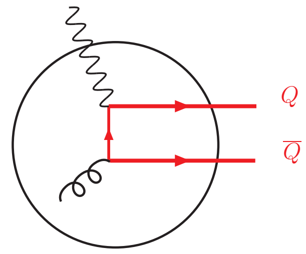
Deep–Inelastic Scattering (DIS):



$$Q^2 := -q^2, \quad x := \frac{Q^2}{2pq} \quad \text{Bjorken–x}$$

$$\frac{d\sigma}{dQ^2\, dx} \sim W_{\mu\nu} L^{\mu\nu}$$

LO:



Hadronic tensor for heavy quark production via single photon exchange:

$$W_{\mu\nu}^{Q\bar{Q}}(q,P,s) = \frac{1}{4\pi}\int d^4\xi \exp(iq\xi)\langle P,s \mid [J_\mu^{em}(\xi), J_\nu^{em}(0)] \mid P,s\rangle_{Q\bar{Q}}$$

$$\text{unpol.}\left\{ \quad = \frac{1}{2x}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_L^{Q\bar{Q}}(x,Q^2) + \frac{2x}{Q^2}\left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu}\right) F_2^{Q\bar{Q}}(x,Q^2)\right.$$

$$\text{pol.}\left\{ \quad -\frac{M}{2Pq}\varepsilon_{\mu\nu\alpha\beta}q^\alpha\left[s^\beta g_1^{Q\bar{Q}}(x,Q^2) + \left(s^\beta - \frac{sq}{Pq}p^\beta\right) g_2^{Q\bar{Q}}(x,Q^2)\right]\right. .$$

- Light quarks: In Bjorken limit, $\{Q^2,\ \nu\} \rightarrow \infty$, x fixed , at twist $\tau = 2$–level:

$$\underbrace{F_i(x,Q^2)}_{\text{structure functions}} = \sum_j \underbrace{C_i^j\left(x, \frac{Q^2}{\mu^2}\right)}_{\text{Wilson coefficients, perturbative}} \otimes \underbrace{f_j(x,\mu^2)}_{\text{parton densities, non-perturbative}},$$

Different flavor contributions to $F_2(x,Q^2)$:

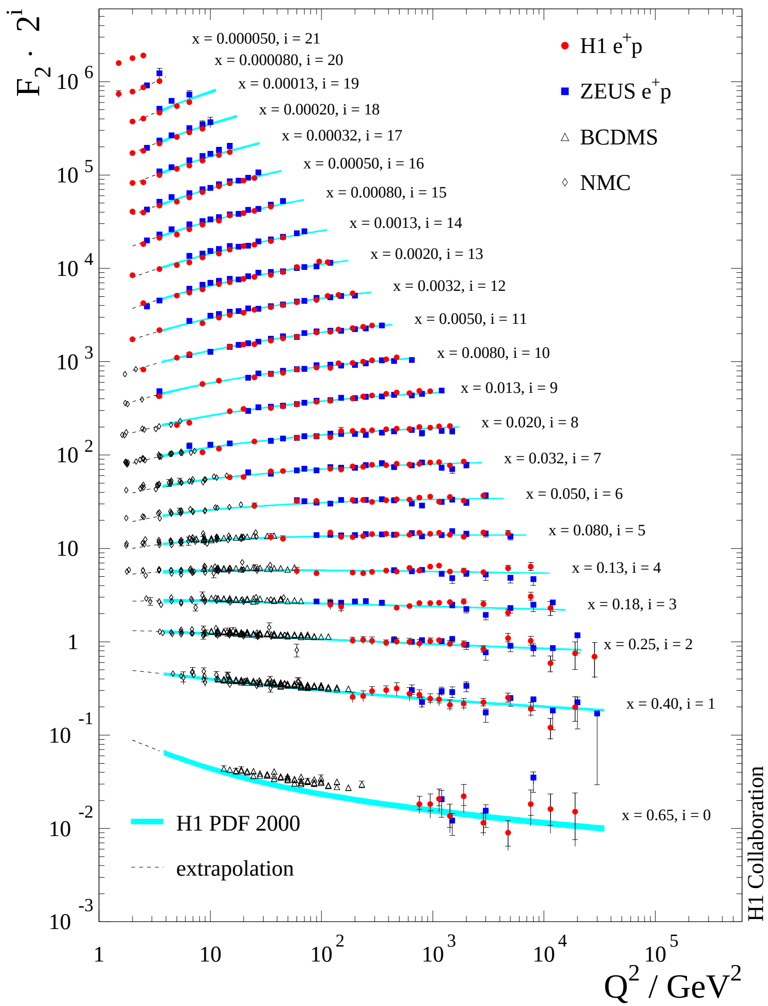
	light flavor	heavy flavor
LO:	$x[f_q + f_{\bar{q}}]\vert_{q=u,d,s}$	$a_s \left[C_g^{(1),H} \otimes xg\right]$
NLO:	$a_s \left[C_q^{(1)} \otimes x[f_q + f_{\bar{q}}] + C_g^{(1)} \otimes xg\right]$	$a_s^2 \left[C_{q,NS}^{(2),H} \otimes x[f_q + f_{\bar{q}}] + C_{q,PS}^{(2),H} \otimes x\Sigma + C_g^{(2),H} \otimes xg\right]$
$\mathrm{N}^2\mathrm{LO}$:	$\vdots$	$\vdots$

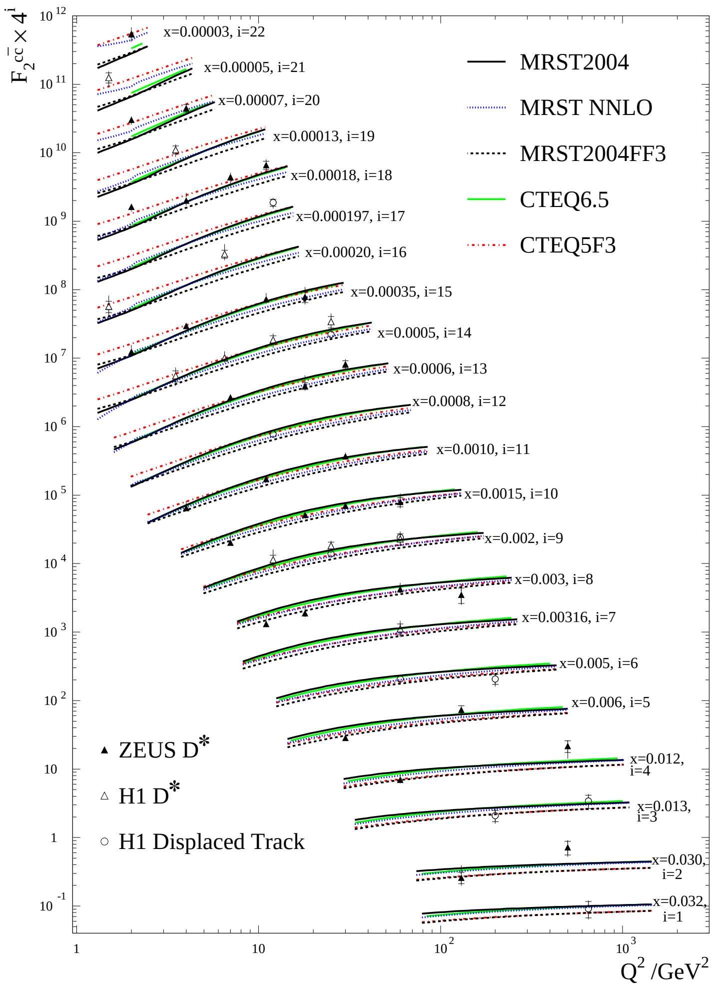
3-loop analysis: $\mathrm{O}(a_s^3)$ heavy flavor Wilson coefficients needed.

Need for the calculation:

- Heavy flavor (charm) contributions to DIS structure functions are rather large [20–40 % at lower values of x] .
- Increase in accuracy of the perturbative description of DIS structure functions.
- $\Longleftrightarrow$ QCD analysis and determination of $\Lambda_{\rm QCD}$, resp. $\alpha_s(M_Z^2)$, from DIS data: $\delta\alpha_s/\alpha_s < 1$ %.
- $\Longleftrightarrow$ Precise determination of the gluon and sea quark distributions.
- $\Longleftrightarrow$ Derivation of variable flavor number scheme for heavy quark production to $O(a_s^3)$.

Goal: Calculation of the heavy flavor Wilson coefficients to higher orders for $Q^2 \geq 25\,{\rm GeV}^2$ [sufficient in many applications] .





[Thompson, 2007]

High statistics in both cases

$F_2^{c\bar{c}}(x, Q^2) \sim 20 - 40\,\%\; F_2(x, Q^2)$ for small values of x, but different scaling violations

Previous calculations:

Unpolarized DIS :

- LO : [Witten, 1976; Babcock, Sivers, 1978; Shifman, Vainshtein, Zakharov 1978; Leveille, Weiler, 1979]
- NLO : [Laenen, Riemersma, Smith, van Neerven, 1993, 1995]
  asymptotic: [Buza, Matiounine, Smith, Migneron, van Neerven, 1996; Bierenbaum, Blümlein, Klein, 2007]
- Observation: $F_2^{c\bar{c}}(x, Q^2)$ is very well described by $F_2^{c\bar{c}}(x, Q^2)|_{Q^2 \gg m^2}$ for $Q^2 \gtrsim 10\, m_c^2$.

Polarized DIS :

- LO : [Watson, 1982; Glück, Reya, Vogelsang, 1991; Vogelsang, 1991]
- NLO :
  asymptotic: [Buza, Matiounine, Smith, van Neerven, 1997; Bierenbaum, Blümlein, Klein, 2008, to appear]

Mellin–space expressions: [Alekhin, Blümlein, 2003].

Variable flavor number scheme at $O(a_s^2)$: [Buza, Matiounine, Smith, van Neerven, 1998]

In the following, we report on results for unpolarized and polarized Heavy Flavor Wilson coefficients beyond NLO.

2. The Method

- massless RGE and light–cone expansion in Bjorken–limit $\{Q^2, \ \nu\} \ \rightarrow \ \infty, \ x$ fixed:

$$\lim_{\xi^2 \to 0} \left[J(\xi), J(0) \right] \propto \sum_{i,N,\tau} c_{i,\tau}^N(\xi^2, \mu^2) \xi_{\mu_1} ... \xi_{\mu_N} O_{i,\tau}^{\mu_1 \cdots \mu_m}(0, \mu^2) \ .$$

- Operators: flavor non-singlet (≤ 3), pure–singlet and gluon; consider leading twist.

- RGE for collinear singularities
  $\Longrightarrow$ mass factorization of the structure functions into Wilson coefficients and parton densities:

$$F_i(x, Q^2) = \sum_j \underbrace{C_i^j\left(x, \frac{Q^2}{\mu^2}\right)}_{\text{perturbative}} \otimes \underbrace{f_j(x, \mu^2)}_{\text{non-perturbative}}$$

- Light-flavor Wilson coefficients: process dependent

$$C_{(2,L);i}^{\text{fl}}\left(\frac{Q^2}{\mu^2}\right) = \delta_{i,q} + \sum_{l=1}^{\infty} a_s^l C_{(2,L),i}^{\text{fl},(l)}, \qquad i = q, g$$

$\Longrightarrow$ Known up to $O(a_s^3)$ [Moch, Vermaseren, Vogt, 2005.]

- Heavy quark contributions given by heavy quark Wilson coefficient, $H_{(2,L),i}^{\rm S,NS}\left(\frac{Q^2}{\mu^2},\frac{m^2}{Q^2}\right)$.
- In the limit $Q^2 \gg m_Q^2$ [$Q^2 \approx 10\ m_Q^2$ for F_2]:
  massive RGE, derivative $m^2\partial/\partial m^2$ acts on Wilson coefficients only: all terms but power corrections calculable through partonic operator matrix elements, $\langle i|A_l|j\rangle$, which are process independent objects!

$$H_{(2,L),i}^{\rm S,NS}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = \underbrace{A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right)}_{\text{massive OMEs}} \otimes \underbrace{C_{(2,L),k}^{\rm S,NS}\left(\frac{Q^2}{\mu^2}\right)}_{\text{light–parton–Wilson coefficients}}.$$

- holds for polarized and unpolarized case. OMEs obey expansion

$$A_{k,i}^{\rm S,NS}\left(\frac{m^2}{\mu^2}\right) = \langle i|O_k^{\rm S,NS}|i\rangle = \delta_{k,i} + \sum_{l=1}^{\infty} a_s^l A_{k,i}^{{\rm S,NS},(l)}\left(\frac{m^2}{\mu^2}\right), \qquad i = q, g$$

[Buza, Matiounine, Migneron, Smith, van Neerven, 1996; Buza, Matiounine, Smith, van Neerven, 1997.]

3. Renormalization

Unrenormalized massive operator matrix elements:

$$\hat{\hat{A}}_{ij} = \delta_{ij} + \sum_{k=0}^{\infty} \hat{a}_s^k \hat{\hat{A}}_{ij}^{(k)}$$

need for:

- Mass renormalization
- Charge renormalization
- Renormalization of ultraviolet singularities
- Factorization of collinear singularities

$\longrightarrow$ use $\overline{\mathrm{MS}}$ scheme and decoupling formalism [Ovrut, Schnitzer 1981; Bernreuther, Wetzel 1982].

Since the light-cone expansion is used, external legs obtain self-energy insertions due to heavy quarks.

Mass renormalization:

on–mass–shell scheme for quarks

[Tarrach 1981; Nachtmann, Wetzel 1981; Gray, Broadhurst, Graefe, Schilcher 1990]

$$\hat{m} \quad = \quad m + \hat{a}_s \, \delta m_1 + \hat{a}_s^2 \, \delta m_2 + O(\hat{a}_s^3)$$

$$\delta m_1 \quad = \quad C_F S_\varepsilon m \left(\frac{m^2}{\mu^2}\right)^{\varepsilon/2} \left[\frac{6}{\varepsilon} - 4 + \left(4 + \frac{3}{4}\zeta_2\right) \varepsilon\right]$$

$$\delta m_2 \quad = \quad C_F S_\varepsilon^2 m \left(\frac{m^2}{\mu^2}\right)^{\varepsilon} \Bigg[\frac{1}{\varepsilon^2}\left(18 C_F + 22 C_A - 8 T_F (N_l + N_h)\right) + \frac{1}{\varepsilon}\left(-\frac{45}{2} C_F + \frac{91}{2} C_A - 14 T_F (N_l + N_h)\right)$$

$$+ C_F \left(\frac{199}{8} - \frac{51}{2}\zeta_2 + 48\ln(2)\zeta_2 - 12\zeta_3\right) + C_A \left(-\frac{605}{8} + \frac{5}{2}\zeta_2 - 24\ln(2)\zeta_2 + 6\zeta_3\right) + T_F \left[N_l \left(\frac{45}{2} + 10\zeta_2\right) + N_h \left(\frac{69}{2} - 14\zeta_2\right)\right]\Bigg]$$

Charge renormalization: $\overline{\text{MS}}$-scheme

$$\hat{a}_s(\varepsilon) \quad = \quad Z_g^2(\varepsilon, \mu^2) a_s(\mu^2) = a_s(\mu^2)\left[1 + \delta a_{s,1} a_s(\mu^2) + \delta a_{s,2} a_s^2(\mu^2)\right] + O(a_s^4)$$

$$\delta a_{s,1} \quad = \quad S_\varepsilon \frac{2\beta_0}{\varepsilon}, \qquad \delta a_{s,2} = S_\varepsilon^2 \left[\frac{4\beta_0^2}{\varepsilon^2} + \frac{\beta_1}{\varepsilon}\right]$$

Operator renormalization:

ultraviolet divergences, renormalized by Z–factors.

Generic formula in terms of anomalous dimensions $\gamma_{ij,k}$. Has to be adapted e.g. for the various cases – three-loop non-singlet, pure-singlet, etc. $(i,j,m,n \in \{q,g\})$:

$$
\begin{aligned}
Z_{ij}(N,a_s,\varepsilon) \quad = \quad & \delta_{i,j} + a_s S_\varepsilon \frac{\gamma_{ij,0}}{\varepsilon} + a_s^2 S_\varepsilon^2 \left\{ \frac{1}{\varepsilon^2}\left[\frac{1}{2}\gamma_{im,0}\gamma_{mj,0} + \beta_0\gamma_{ij,0}\right] + \frac{1}{2\varepsilon}\gamma_{ij,1}\right\} \\
& + a_s^3 S_\varepsilon^3 \left\{ \frac{1}{\varepsilon^3}\left[\frac{1}{6}\gamma_{in,0}\gamma_{nm,0}\gamma_{mj,0} + \beta_0\gamma_{im,0}\gamma_{mj,0} + \frac{4}{3}\beta_0^2\gamma_{ij,0}\right] \right. \\
& + \frac{1}{\varepsilon^2}\left[\frac{1}{6}\left(\gamma_{im,1}\gamma_{mj,0} + 2\gamma_{im,0}\gamma_{mj,1}\right) + \frac{2}{3}\left(\beta_0\gamma_{ij,1} + \beta_1\gamma_{ij,0}\right)\right] \\
& \left. + \frac{\gamma_{ij,2}}{3\varepsilon}\right\}
\end{aligned}
$$

$$
Z_{qq}^{PS} = Z_{qq} - Z_{NS}.
$$

The anomalous dimensions $\gamma_{ij,k}(N)$ are related to the splitting functions by

$$
\gamma_{ij,k}(N) = -\int_0^1 dz z^{N-1} P_{ij}^{(k)}(z) \ .
$$

Mass factorization: Collinear singularities are factored into Γ_{NS}, $\Gamma_{ij,S}$ and $\Gamma_{qq,PS}$.

For massless quarks: $\Gamma_{\rm NS} = Z^{-1}_{\rm NS}$, $\Gamma_{ij,{\rm S}} = Z^{-1}_{ij,{\rm S}}$, $\Gamma_{qq,{\rm PS}} = Z^{-1}_{qq,{\rm PS}}$

$$\begin{aligned}
\Gamma_{\rm NS}(N,a_s,\varepsilon) &= 1 - a_s S_\varepsilon \frac{\gamma_{\rm NS,0}}{\varepsilon} + a_s^2 S_\varepsilon^2 \left[\frac{1}{\varepsilon^2}\left(\frac{1}{2}\gamma_{NS,0}^2 - \beta_0 \gamma_{\rm NS,0}\right) - \frac{1}{2\varepsilon}\gamma_{\rm NS,1}\right] \\
\Gamma_{ij,{\rm S}}(N,a_s,\varepsilon) &= \delta_{ij} - a_s S_\varepsilon \frac{\gamma_{ij,0}}{\varepsilon} + a_s^2 S_\varepsilon^2 \left[\frac{1}{\varepsilon^2}\left(\frac{1}{2}\gamma_{ik,0}\gamma_{kj,0} - \beta_0 \gamma_{ij,0}\right) - \frac{1}{2\varepsilon}\gamma_{ij,1}\right] \\
\Gamma_{qq,{\rm PS}}(N,a_s,\varepsilon) &= -a_s^2 S_\varepsilon^2 \left[\frac{1}{2\varepsilon^2}\gamma_{qg,0}\gamma_{gq,0} + \frac{1}{2\varepsilon}\gamma_{qq,{\rm PS},1}\right] .
\end{aligned}$$

Here: in each diagram at least one quark line is massive

$\Rightarrow$ Γ–matrices apply to parts of the diagrams with massless lines only

$\leftrightarrow$ at most 2–loop sub-graphs

$\leftrightarrow$ mass factorization is different in various sub–classes of contributing Feynman diagrams

$\rightarrow$ singularities contained in $\Gamma_{\rm NS}, \Gamma_{ij,{\rm S}}$, and $\Gamma_{qq,{\rm PS}}$ are absorbed into the bare parton densities, which become scale–dependent

The renormalized operator matrix elements are obtained removing the ultraviolet singularities and collinear singularities of the operator matrix elements,

$$A_{ij} = Z^{-1}_{ik}\hat{A}_{kl}\Gamma^{-1}_{lj} = \delta_{ij} + a_s A^{(1)}_{ij} + a_s^2 A^{(2)}_{ij} + a_s^3 A^{(3)}_{ij} \ .$$

For example:

$$A_{Qg} = Z^{-1}_{qq}\hat{A}^{PS}_{Qq}\Gamma^{-1}_{qg} + Z^{-1}_{qq}\hat{A}_{Qg}\Gamma^{-1}_{gg} + Z^{-1}_{qg}\hat{A}_{gq,Q}\Gamma^{-1}_{qg} + Z^{-1}_{qg}\hat{A}_{gg,Q}\Gamma^{-1}_{gg} \ .$$

$$\Longrightarrow A^{(2)}_{Qg} = \hat{A}^{(2)}_{Qg} + Z^{-1,(1)}_{qq}\hat{A}^{(1)}_{Qg} + Z^{-1,(1)}_{qg}\hat{A}^{(1)}_{gg,Q} + Z^{-1,(2)}_{qg} + \Big(\hat{A}^{(1)}_{Qg} + Z^{-1,(1)}_{qg}\Big)\Gamma^{-1,(1)}_{gg} \ .$$

$\rightarrow$ mixing with $\hat{A}^{PS}_{Qq}$ and $\hat{A}_{gg,Q}$ at $O(a_s^3)$.

$\rightarrow$ $\hat{A}_{gq,Q}$ starts contributing from $\mathrm{O}(a_s^2) \Rightarrow$ at $\mathrm{O}(a_s^4)$ to $\hat{A}_{Qg}$

For example: Renormalized gluonic massive operator matrix elements up to $O(a_s^2)$:

Unrenormalized $\hat{A}_{Qg}$ to $O(a_s^2)$:

$$\hat{A}_{Qg}^{(1)} = S_\varepsilon \left(\frac{m^2}{\mu^2}\right)^{\varepsilon/2} \left\{-\frac{1}{\varepsilon}\hat{P}_{qg}^{(0)} + a_{Qg}^{(1)} + \varepsilon\, \bar{a}_{Qg}^{(1)} + \varepsilon^2\, \bar{\bar{a}}_{Qg}^{(1)}\right\}$$

$$\begin{aligned}\hat{A}_{Qg}^{(2)} = & S_\varepsilon^2 \left(\frac{m^2}{\mu^2}\right)^{\varepsilon} \Bigg[\frac{1}{\varepsilon^2}\left\{\frac{1}{2}\hat{P}_{qg}^{(0)} \otimes (P_{qq}^{(0)} - P_{gg}^{(0)}) + \beta_0 \hat{P}_{qg}^{(0)}\right\} \\ & + \frac{1}{\varepsilon}\left\{-\frac{1}{2}\hat{P}_{qg}^{(1)} - 2\beta_0 a_{Qg}^{(1)} - a_{Qg}^{(1)} \otimes (P_{qq}^{(0)} - P_{gg}^{(0)})\right\} + a_{Qg}^{(2)} + \varepsilon\, \bar{a}_{Qg}^{(2)}\Bigg] \\ & - \frac{2}{\varepsilon} S_\varepsilon \sum_{H=Q}^{t} \beta_{0,H} \left(\frac{m_H^2}{\mu^2}\right)^{\varepsilon/2} \left(1 + \frac{1}{8}\varepsilon^2 \zeta_2 + \frac{\varepsilon^3}{24}\zeta_3\right)\end{aligned}$$

Renormalized A_{Qg} to $O(a_s^2)$:

$$A_{Qg}^{(1)} = -\frac{1}{2}\widehat{P}_{qg}^{(0)} \ln\left(\frac{m^2}{\mu^2}\right)$$

$$\begin{aligned}A_{Qg}^{(2)} = & \frac{1}{8}\left\{\widehat{P}_{qg}^{(0)} \otimes \left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right]\right\} \ln^2\left(\frac{m^2}{\mu^2}\right) - \frac{1}{2}\widehat{P}_{qg}^{(1)} \ln\left(\frac{m^2}{\mu^2}\right) \\ & + \overline{a}_{Qg}^{(1)}\left[P_{qq}^{(0)} - P_{gg}^{(0)} + 2\beta_0\right] + a_{Qg}^{(2)} \quad ; \qquad a_{Qg}^{(1)} \equiv 0\end{aligned}$$

- $\overline{a}_{Qg}^{(1)}$: $O(\varepsilon)$ of the OME $A_{Qg}^{(1)}$ in the $\overline{\text{MS}}$–scheme. Enters $A_{Qg}^{(2)}$ through renormalization.
- For the renormalization of $A_{Qg}^{(3)}$: $\overline{a}_{Qg}^{(2)}$, $\overline{a}_{gg,Q}^{(2)}$, $\overline{a}_{Qq}^{(2),\mathbf{PS}}$ are needed.

4. Calculation Techniques

- Calculation in Mellin-space for space–like q^2, $Q^2 = -q^2$: $\quad 0 \le x \le 1$
- use of generalized hypergeometric functions for general analytic results
- use of Mellin-Barnes integrals for numerical checks (MB, [Czakon, 2006]) and some analytic results
- Summation of lots of new infinite one-parameter sums into harmonic sums. E.g.:

$$N \sum_{i,j=1}^{\infty} \frac{S_1(i) S_1(i+j+N)}{i(i+j)(j+N)} = 4S_{2,1,1} - 2S_{3,1} + S_1\left(-3S_{2,1} + \frac{4S_3}{3}\right) - \frac{S_4}{2}$$
$$- S_2^2 + S_1^2 S_2 + \frac{S_1^4}{6} + 6S_1\zeta_3 + \zeta_2\left(2S_1^2 + S_2\right) .$$

  use of integral techniques and the Mathematica package SIGMA [C. Schneider, 2007], [I. Bierenbaum, J. Blümlein, S. K., C. Schneider, arXiv:0707.4659 [math-ph]; arXiv:0803.0273 [hep-ph]]

- Partial checks for fixed values of N using SUMMER, [Vermaseren, Int. J. Mod. Phys. A **14** (1999)].
- Algebraic and structural simplification of the harmonic sums [J. Blümlein, 2003, 2007].

5. Results

Unpolarized case, Singlet, O(ε)

$$
\begin{aligned}
\overline{a}_{Qg}^{(2)} &= T_F C_F \Bigg\{ \frac{2}{3}\frac{(N^2+N+2)(3N^2+3N+2)}{N^2(N+1)^2(N+2)}\zeta_3 + \frac{P_1}{N^3(N+1)^3(N+2)} S_2 + \frac{N^4-5N^3-32N^2-18N-4}{N^2(N+1)^2(N+2)} S_1^2 \\
&+ \frac{N^2+N+2}{N(N+1)(N+2)}\Big(16S_{2,1,1} - 8S_{3,1} - 8S_{2,1}S_1 + 3S_4 - \frac{4}{3}S_3S_1 - \frac{1}{2}S_2^2 - S_2S_1^2 - \frac{1}{6}S_1^4 + 2\zeta_2S_2 - 2\zeta_2S_1^2 - \frac{8}{3}\zeta_3S_1\Big) \\
&- 8\frac{N^2-3N-2}{N^2(N+1)(N+2)}S_{2,1} + \frac{2}{3}\frac{3N+2}{N^2(N+2)}S_1^3 + \frac{2}{3}\frac{3N^4+48N^3+43N^2-22N-8}{N^2(N+1)^2(N+2)}S_3 + 2\frac{3N+2}{N^2(N+2)}S_2S_1 + 4\frac{S_1}{N^2}\zeta_2 \\
&+ \frac{N^5+N^4-8N^3-5N^2-3N-2}{N^3(N+1)^3}\zeta_2 - 2\frac{2N^5-2N^4-11N^3-19N^2-44N-12}{N^2(N+1)^3(N+2)}S_1 + \frac{P_2}{N^5(N+1)^5(N+2)}\Bigg\} \\
&+ T_F C_A \Bigg\{ \frac{N^2+N+2}{N(N+1)(N+2)}\Big(16S_{-2,1,1} - 4S_{2,1,1} - 8S_{-3,1} - 8S_{-2,2} - 4S_{3,1} - \frac{2}{3}\beta''' + 9S_4 - 16S_{-2,1}S_1 \\
&+ \frac{40}{3}S_1S_3 + 4\beta''S_1 - 8\beta'S_2 + \frac{1}{2}S_2^2 - 8\beta'S_1^2 + 5S_1^2S_2 + \frac{1}{6}S_1^4 - \frac{10}{3}S_1\zeta_3 - 2S_2\zeta_2 - 2S_1^2\zeta_2 - 4\beta'\zeta_2 - \frac{17}{5}\zeta_2^2\Big) \\
&+ \frac{4(N^2-N-4)}{(N+1)^2(N+2)^2}\Big(-4S_{-2,1} + \beta'' - 4\beta'S_1\Big) - \frac{2}{3}\frac{N^3+8N^2+11N+2}{N(N+1)^2(N+2)^2}S_1^3 + 8\frac{N^4+2N^3+7N^2+22N+20}{(N+1)^3(N+2)^3}\beta' \\
&+ 2\frac{3N^3-12N^2-27N-2}{N(N+1)^2(N+2)^2}S_2S_1 - \frac{16}{3}\frac{N^5+10N^4+9N^3+3N^2+7N+6}{(N-1)N^2(N+1)^2(N+2)^2}S_3 - 8\frac{N^2+N-1}{(N+1)^2(N+2)^2}\zeta_2S_1 \\
&- \frac{2}{3}\frac{9N^5-10N^4-11N^3+68N^2+24N+16}{(N-1)N^2(N+1)^2(N+2)^2}\zeta_3 - \frac{P_3}{(N-1)N^3(N+1)^3(N+2)^3}S_2 - \frac{2P_4}{(N-1)N^3(N+1)^3(N+2)^2}\zeta_2 \\
&- \frac{P_5}{N(N+1)^3(N+2)^3}S_1^2 + \frac{2P_6}{N(N+1)^4(N+2)^4}S_1 - \frac{2P_7}{(N-1)N^5(N+1)^5(N+2)^5}\Bigg\}.
\end{aligned}
$$

Unpolarized case, Pure–Singlet and Non–singlet, O(ε)

$$\overline{a}_{Qq}^{\rm PS,(2)} = C_F T_F \Biggl\{ -2 \frac{(5N^3+7N^2+4N+4)(N^2+5N+2)}{(N-1)N^3(N+1)^3(N+2)^2} \Bigl(2S_2+\zeta_2\Bigr) - \frac{4}{3} \frac{(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)} \Bigl(3S_3+\zeta_3\Bigr) + 2 \frac{P_9}{(N-1)N^5(N+1)^5(N+2)^4} \Biggr\} .$$

$$\overline{a}_{qq,Q}^{\rm NS,(2)} = C_F T_F \Biggl\{ \frac{4}{3} S_4 + \frac{4}{3} S_2 \zeta_2 - \frac{8}{9} S_1 \zeta_3 - \frac{20}{9} S_3 - \frac{20}{9} S_1 \zeta_2 + 2 \frac{3N^2+3N+2}{9N(N+1)} \zeta_3 + \frac{112}{27} S_2 + \frac{3N^4+6N^3+47N^2+20N-12}{18N^2(N+1)^2} \zeta_2 - \frac{656}{81} S_1 + \frac{P_8}{648N^4(N+1)^4} \Biggr\} .$$

Polarized case, Singlet O(ε)

$$
\begin{aligned}
\Delta\overline{a}_{Qg}^{(2)} = & \, T_F C_F \Biggl\{ \frac{N-1}{N(N+1)} \Bigl(16 S_{2,1,1} - 8 S_{3,1} - 8 S_{2,1} S_1 + 3 S_4 - \frac{4}{3} S_3 S_1 - \frac{1}{2} S_2^2 - \frac{1}{6} S_1^4 - \frac{8}{3} S_1 \zeta_3 - S_2 S_1^2 + 2 S_2 \zeta_2 - 2 S_1^2 \zeta_2 \Bigr) \\
& - 8 \frac{S_{2,1}}{N^2} + \frac{3N^2+3N-2}{N^2(N+1)(N+2)} \Bigl(2 S_2 S_1 + \frac{2}{3} S_1^3 \Bigr) + \frac{3N^4+48N^3+123N^2+98N+8}{N^2(N+1)^2(2+N)} S_3 + 4 \frac{N-1}{N^2(N+1)} S_1 \zeta_2 \\
& + \frac{2}{3} \frac{(N-1)(3N^2+3N+2)}{N^2(N+1)^2} \zeta_3 + \frac{P_1}{N^3(N+1)^3(N+2)} S_2 + \frac{N^3-6N^2-22N-36}{N^2(N+1)(N+2)} S_1^2 + \frac{P_2}{N^3(N+1)^3} \zeta_2 \\
& - 2 \frac{2N^4-4N^3-3N^2+20N+12}{N^2(N+1)^2(N+2)} S_1 + \frac{P_3}{N^5(N+1)^5(N+2)} \Biggr\} \\
& + T_F C_A \Biggl\{ \frac{N-1}{N(N+1)} \Bigl(16 S_{-2,1,1} - 4 S_{2,1,1} - 8 S_{-3,1} - 8 S_{-2,2} - 4 S_{3,1} + \frac{2}{3} \beta''' - 16 S_{-2,1} S_1 - 4 \beta'' S_1 + 8 \beta' S_2 + 8 \beta' S_1^2 + 9 S_4 \\
& + \frac{40}{3} S_3 S_1 + \frac{1}{2} S_2^2 + 5 S_2 S_1^2 + \frac{1}{6} S_1^4 + 4 \zeta_2 \beta' - 2 \zeta_2 S_2 - 2 \zeta_2 S_1^2 - \frac{10}{3} S_1 \zeta_3 - \frac{17}{5} {\zeta_2}^2 \Bigr) + \frac{N-1}{N(N+1)^2} \Bigl(-16 S_{-2,1} - 4 \beta'' + 16 \beta' S_1 \Bigr) \\
& - \frac{16}{3} \frac{N^3+7N^2+8N-6}{N^2(N+1)^2(N+2)} S_3 + 2 \frac{3N^2-13}{N(N+1)^2(N+2)} S_2 S_1 - \frac{2}{3} \frac{N^2+4N+5}{N(N+1)^2(N+2)} S_1^3 - 8 \frac{\zeta_2 S_1}{(N+1)^2} - \frac{2}{3} \frac{(N-1)(9N+8)}{N^2(N+1)^2} \zeta_3 \\
& - 8 \frac{N^2+3}{N(N+1)^3} \beta' - \frac{P_4}{N^3(N+1)^3(N+2)} S_2 - \frac{N^4+2N^3-5N^2-12N+2}{N(N+1)^3(N+2)} S_1^2 - 2 \frac{P_5}{N^3(N+1)^3} \zeta_2 \\
& + 2 \frac{P_6}{N(N+1)^4(N+2)} S_1 - 2 \frac{P_7}{N^5(N+1)^5(N+2)} \Biggr\} .
\end{aligned}
$$

[I. Bierenbaum, J. Blümlein and S. K., 2008, to appear]

Polarized case, Pure–Singlet and Non–singlet, O(ε)

$$
\begin{aligned}
\Delta\overline{a}_{Qq}^{\mathrm{PS},(2)} &= -\frac{4(N-1)(N+2)}{3N^2(N+1)^2}(3S_3+\zeta_3)+2\frac{(N+2)(N^3+2N+1)}{N^3(N+1)^3}(2S_2+\zeta_2) \\
&\quad +2\frac{(2+N)(N^5-7N^4+6N^3+7N^2+4N+1)}{(N+1)^5N^5}\,, \\
\Delta\overline{a}_{qq,Q}^{\mathrm{NS},(2)} &= \overline{a}_{qq,Q}^{\mathrm{NS},(2)}\,.
\end{aligned}
$$

- Use of 't Hooft–Veltman–scheme for γ_5. A finite renormalization has to be applied to maintain the Ward–identities.
- The first moment of $\Delta\overline{a}_{Qg}^{(2)}$ vanishes. $\Longrightarrow$ sum rule of $\Delta H_g(x)$.

$\hat{A}_{gg,Q}^{(2)}$ unpolarized

$$\hat{A}_{gg,Q}^{(2)} =$$

$$T_F C_A \Bigg\{ \frac{1}{\varepsilon^2}\left(-\frac{32}{3}S_1 + \frac{64(N^2+N+1)}{3(N-1)N(N+1)(N+2)}\right) + \frac{1}{\varepsilon}\left(-\frac{80}{9}S_1 + \frac{16P_1}{9(N-1)N^2(N+1)^2(N+2)}\right)$$

$$+\left(-\frac{8}{3}\zeta_2 S_1 + \frac{16(N^2+N+1)\zeta_2}{3(N-1)N(N+1)(N+2)} - 4\frac{56N+47}{27(N+1)}S_1 + \frac{2P_3}{27(N-1)N^3(N+1)^3(N+2)}\right)$$

$$+\varepsilon\Bigg(-\frac{8}{9}\zeta_3 S_1 - \frac{20}{9}\zeta_2 S_1 + \frac{16(N^2+N+1)}{9(N-1)N(N+1)(N+2)}\zeta_3 + \frac{2N+1}{3(N+1)}S_2 - \frac{S_1^2}{3(N+1)}$$

$$+\frac{4P_1\zeta_2}{9(N-1)N^2(N+1)^2(N+2)} - 2\frac{328N^4+256N^3-247N^2-175N+54}{81(N-1)N(N+1)^2}S_1 + \frac{P_5}{81(N-1)N^4(N+1)^4(N+2)}\Bigg)\Bigg\}$$

$$+T_F C_F \Bigg\{ \frac{1}{\varepsilon^2}\left(\frac{16(N^2+N+2)^2}{(N-1)N^2(N+1)^2(N+2)}\right) + \frac{1}{\varepsilon}\left(\frac{4P_2}{(N-1)N^3(N+1)^3(N+2)}\right)$$

$$+\left(\frac{4(N^2+N+2)^2\zeta_2}{(N-1)N^2(N+1)^2(N+2)} - \frac{P_4}{(N-1)N^4(N+1)^4(N+2)}\right)$$

$$+\varepsilon\left(\frac{4(N^2+N+2)^2\zeta_3}{3(N-1)N^2(N+1)^2(N+2)} + \frac{P_2\zeta_2}{(N-1)N^3(N+1)^3(N+2)} + \frac{P_6}{4(N-1)N^5(N+1)^5(N+2)}\right)\Bigg\}$$

$\longrightarrow$ Result obtained in terms of Γ and Ψ functions to all orders in ε

$\longrightarrow$ to $O(\varepsilon^0)$ agreement with van Neerven et al.

$\longrightarrow$ $O(\varepsilon)$ new: term needed too to derive variable flavor number scheme to $O(\alpha_s^3)$.

A remark on the mathematical structure of the O(1) and O(ε) terms:

van Neerven et al. to O(1): unpolarized: 48 basic functions; polarized: 24 basic functions.

O(1): $\{S_1,\ S_2,\ S_3,\ S_{-2}, S_{-3}\}$, $S_{-2,1} \Longrightarrow$ 2 basic objects.

O(ε): $\{S_1,\ S_2,\ S_3,\ S_4,\ S_{-2}, S_{-3}, S_{-4}\}$, $S_{2,1}$, $S_{-2,1}$, $S_{-3,1}$, $S_{2,1,1}$, $S_{-2,1,1}$
$\Longrightarrow$ 6 basic objects

These objects are in common to all single scale higher order processes.

Str. Functions, DIS HQ, Fragm. Functions, DY, Hadr. Higgs-Prod., s+v contr. to Bhabha scatt., ...

harmonic sums with index $\{-1\}$ cancel (holds even for each diagram)

[J. Blümlein, 2004; J. Blümlein, V. Ravindran, 2005,2006; J. Blümlein, S. Klein, `arXiv: 0706.2426 [hep-ph]`; J. Blümlein and S. Moch in preparation.]

Expectation for 3–loops: weight 5 (6) harmonic sums

6. Fixed moments of $A_{ij,Q}^{(3)}$

Contributing OMEs:

Singlet	A_{Qg}	$A_{gg,Q}$	$A_{gq,Q}$	mixing
Pure–Singlet		$A_{Qq}^{\rm PS}$		mixing
Non–Singlet	$A_{qq,Q}^{\rm NS,+}$	$A_{qq,Q}^{\rm NS,-}$	$A_{qq,Q}^{\rm NS,v}$	

- All 2–loop $O(\varepsilon)$–terms in the unpolarized case are known:

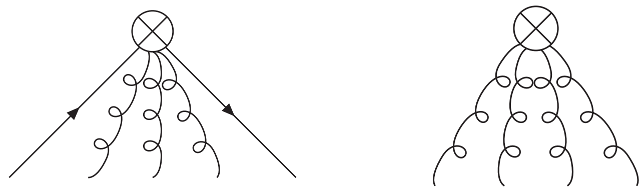
$$\overline{a}_{Qg}^{(2)}, \quad \overline{a}_{Qq}^{(2),\mathbf{PS}}, \quad \overline{a}_{gg,Q}^{(2)}, \quad \overline{a}_{gq,Q}^{(2)}, \quad \overline{a}_{qq,Q}^{(2),\mathbf{NS}}.$$

- Unpolarized anomalous dimensions are known up to $O(a_s^3)$ [Moch, Vermaseren, Vogt, 2004.]
  $\Longrightarrow$ All terms needed for the renormalization of unpolarized 3–Loop heavy OMEs are present.
  $\Longrightarrow$ Calculation will provide first independent checks on $\gamma_{qg}^{(3)}$, $\gamma_{qq}^{(3),\rm PS}$ and on respective color projections of $\gamma_{qq}^{(3),\rm NS\pm,v}$, $\gamma_{gg}^{(3)}$ and $\gamma_{gq}^{(3)}$.

- Calculation proceeds in the same way in the polarized case. Known so far :

$$\Delta\overline{a}_{Qg}^{(2)}, \quad \Delta\overline{a}_{Qq}^{(2),\mathbf{PS}}, \quad \Delta\overline{a}_{qq,Q}^{(2),\mathbf{NS}} = \overline{a}_{qq,Q}^{(2),\mathbf{NS}} \; .$$

- 3–loop OMEs are generated with QGRAF [Nogueira 1993]
- New operator insertions emerge:



We consider first the terms with 2 quark loops (at least one heavy): # of diagrams for $A_{Qg}^{(3)}$

- 489 diagrams with two quark loops
- 1478 diagrams with one quark loop

$\Longrightarrow$ number of diagrams can be reduced by using symmetry arguments.

First step: Calculation of fixed moments of $A_{Qg}^{(3)}(N)$, $N = 2, 4, 6, ...$

- three–loop "self-energy" type diagrams with an operator insertion
- Extension: additional scale compared to massive propagators: Mellin variable N
- Genuine tensor integrals due to

$$\Delta^{\mu_1}...\Delta^{\mu_n} \langle p|O_{\mu_1...\mu_n}|p\rangle = \Delta^{\mu_1}...\Delta^{\mu_n} \langle p|S\, \bar{\Psi}\gamma_{\mu_1} D_{\mu_2}...D_{\mu_n}\Psi|p\rangle = A(N)\cdot(\Delta p)^N$$

$$D_\mu = \partial_\mu - igt_a A_\mu^a \quad , \qquad \Delta^2 = 0.$$

- Construction of a projector to obtain the desired moment in N [undo Δ-contraction] $\surd$
- Color factors are calculated with [Ritbergen, Schellekens, Vermaseren 1998]
- Translation to suitable input for MATAD [Steinhauser, 2001] $\surd$

Tests performed: Various 2–loop calculations for $N = 2, 4, 6, ...$ were repeated
$\longrightarrow$ agreement with our previous calculation; checked against result of $A_{gg,Q}^{(2)}$

Status:

- automated chain to evaluate the OMEs $\surd$
- Terms $\propto$ O$(d_{abc}d^{abc})$ vanish
- currently investigated: terms $\propto$ $T_F^2 C_F$, $T_F^2 C_A$–terms

7. Conclusions

- The heavy flavor contributions to F_2 are rather large in the region of lower values of x.
- QCD precision analyses therefore require the description of the heavy quark contributions to 3–loop order.
- We calculate the heavy flavor DIS Wilson Coefficients in the asymptotic regime $[Q^2 \geq 10m^2]$ using massive operator matrix elements.
- We newly presented first contributions to these corrections:
  - $\bar{a}_{Qg}^{(2)}$, $\bar{a}_{Qq}^{PS,(2)}$, $\bar{a}_{gg,Q}^{(2)}$, $\bar{a}_{qq}^{NS,(2)} = \Delta\bar{a}_{qq,Q}^{NS,(2)}$
  - $\Delta\bar{a}_{Qg}^{(2)}$, $\Delta\bar{a}_{Qg}^{PS,(2)}$

  in the unpolarized and polarized case for general values of the Mellin variable. These terms contribute to $H_{ij}^{(3)}$, $\Delta H_{ij}^{(3)}$ respectively, through renormalization.

- The calculation is performed in Mellin space, which allows to obtain compact results.
  - The analytic results were obtained using representations in terms of generalized hypergeometric functions.
  - Numerical checks were performed applying Mellin–Barnes integrals.
- Integral techniques and the summation package SIGMA have been used for summation. The results are given in the form of nested harmonic sums.
- We developed a programme–chain to calculate the massive operator matrix elements $A_{ij}^{(3)}$ for fixed Mellin moments based on QGRAF and MATAD
- We will report on first 3–loop results in the near future.