Unitarity Techniques for QCD

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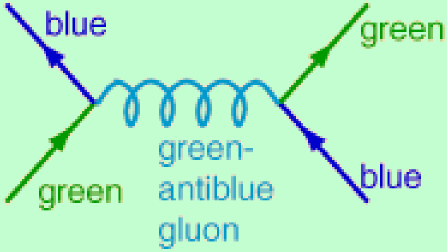
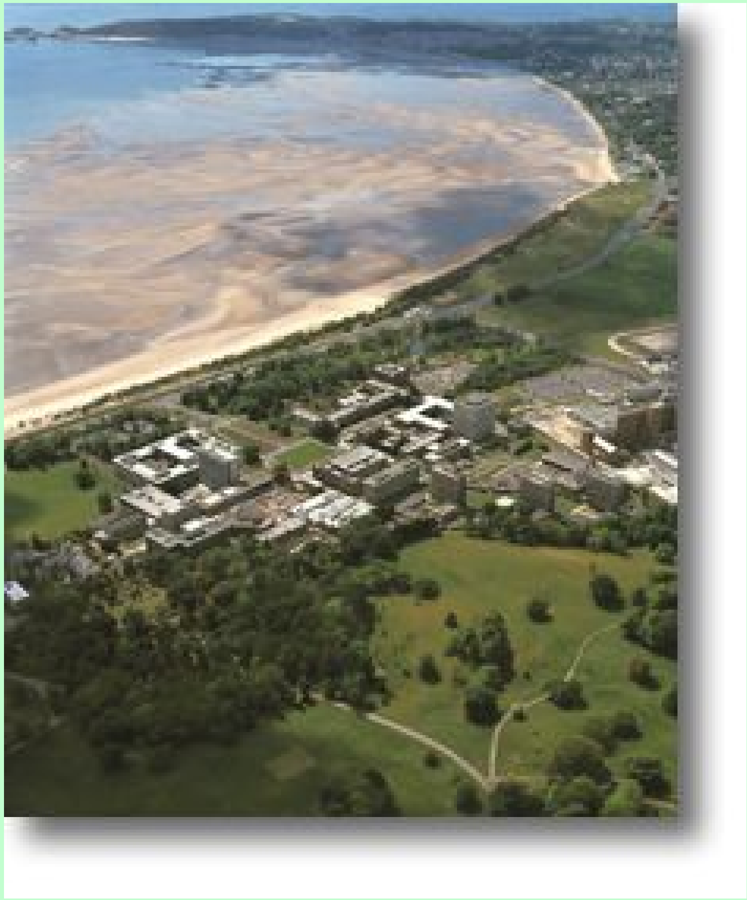
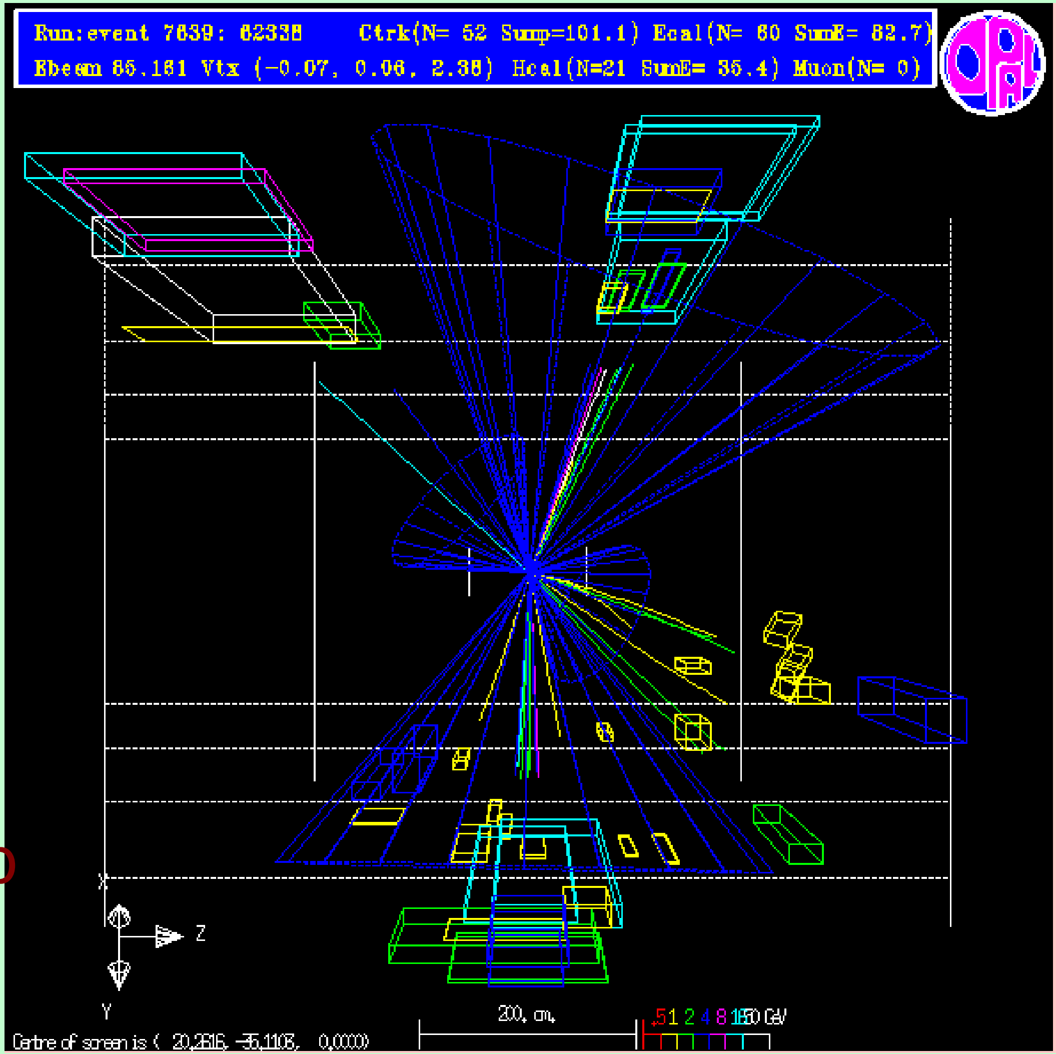




Loops and Legs 2008

QCD Matrix Elements



- -QCD matrix elements are an important part of calculating the QCD background for processes at LHC

- -NLO calculations (at least!) are needed for precision

  - - apply to 2g → (n-2)g

-this talk about one-loop n-gluon scattering n>5

-using analytic unitarity methods

Passarino-Veltman reduction of 1-loop

$$I_n[F^m(l)] = \sum c_i I^i_{n-1}[F^{m-1}(l)]$$

Decomposes a n-point integral into a sum of (n-1) integral functions obtained by collapsing a propagator

$$A_n^{1-loop} = \sum c_i I_4^i + \sum_i d_i I_3^i + \sum_i e_i I_2^i + R$$

-coefficients are rational functions of |ki§ using spinor helicity

Organisation: Supersymmetric Decomposition

Supersymmetric gluon scattering amplitudes are the linear combination of QCD ones+scalar loop

$$
\begin{aligned}
A_n^{\mathcal{N}=4} &\equiv A_n^{[1]} + 4A_n^{[1/2]} + 3A_n^{[0]} \\
A_n^{\mathcal{N}=1\text{ vector}} &\equiv A_n^{[1]} + A_n^{[1/2]} \\
A_n^{\mathcal{N}=1\text{ chiral}} &\equiv A_n^{[1/2]} + A_n^{[0]}
\end{aligned}
$$

-this can be inverted

$$
\begin{aligned}
A_n^{[1]} &= A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1\text{ chiral}} + A_n^{[0]} \\
A_n^{[1/2]} &= A_n^{\mathcal{N}=1\text{ chiral}} - A_n^{[0]}
\end{aligned}
$$

$$
A_n^{1-loop} = \sum c_i I_4^i + \sum_i d_i I_3^i + \sum_i e_i I_2^i + R
$$

N=4

N=1

One-Loop QCD Amplitudes

One Loop Gluon Scattering Amplitudes in QCD

-Four Point : Ellis+Sexton, Feynman Diagram methods

-Five Point : Bern, Dixon,Kosower, String based rules

-Six-Point and beyond--- present problem

The Six Gluon one-loop amplitude

	$N=4$	$N=1$	scalar C	scalar R
$A(++++++)$	-	-	-	93
$A(-+++++)$	-	-	-	93
$A(--++++)$	94	94	94	06
$A(-+-+++)$	94	94	05	06
$A(-++-++)$	94	94	05	06
$A(---+++)$	94	05	05	06
$A(--+-++)$	94	05	06	06
$A(-+-+-+)$	94	05	06	06

~14 papers

81% `B'

Berger, Bern, Dixon, Forde, Kosower

Bern, Dixon, Dunbar, Kosower

Britto, Buchbinder, Cachazo, Feng

Bidder, Bjerrum-Bohr, Dixon, Dunbar

Bern, Chalmers, Dixon, Kosower

Bedford, Brandhuber, Travaglini, Spence

Forde, Kosower

Xiao,Yang, Zhu

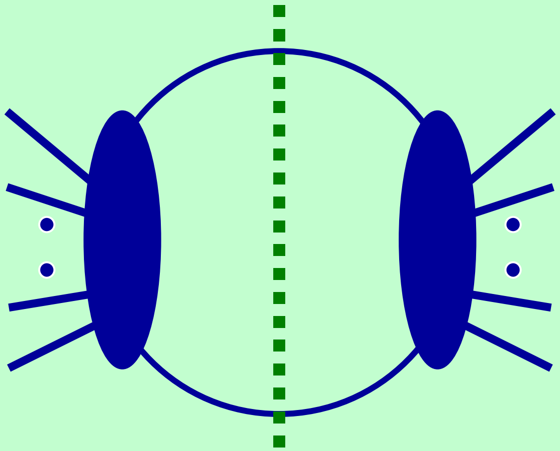
Bern, Bjerrum-Bohr, Dunbar, Ita

Britto, Feng, Mastriolia

Mahlon

Unitarity Methods

$$C_2 = \int dLIPS(l_1, l_2) M(l_1, \cdots, l_2) \times M(-l_2, ..., -l_1)$$

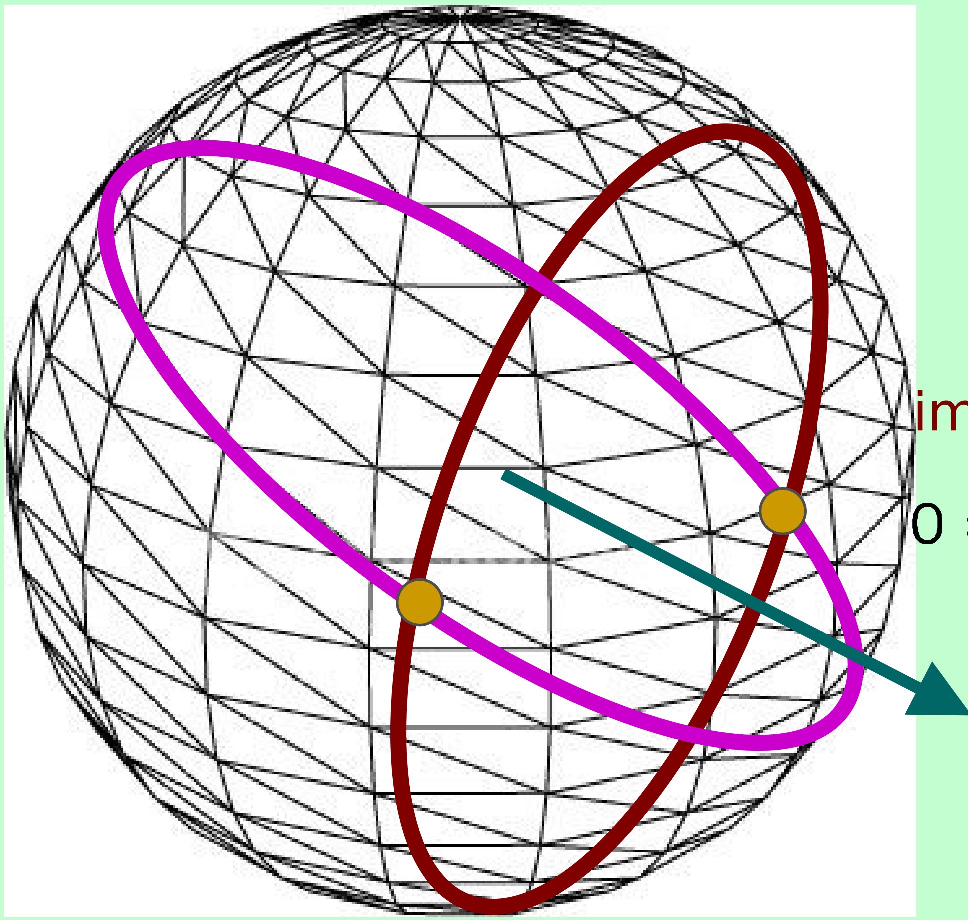


-look at the two-particle cuts

$$= \sum c_i I_4^i|_{disc} + \sum_i d_i I_3^i|_{disc} + \sum_i e_i I_2^i|_{disc}$$

-use this technique to identify the coefficient

Topology of Cuts



-look when K is timelike, in frame where $K=(K_0,0,0,0)$

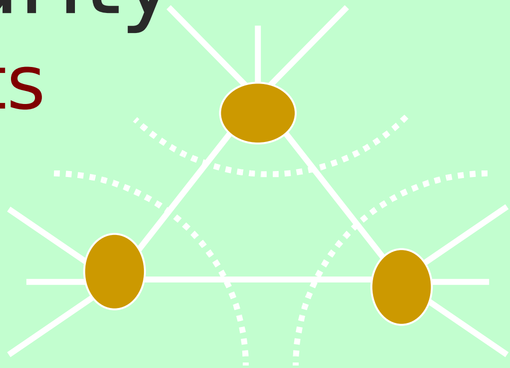
l_1 and l_2 are back to back on surface of sphere

imposing an extra condition

$$0 = (l-Q)^2 \longrightarrow l\cdot Q = Q^2$$

Generalised Unitarity

-use info beyond two-particle cuts

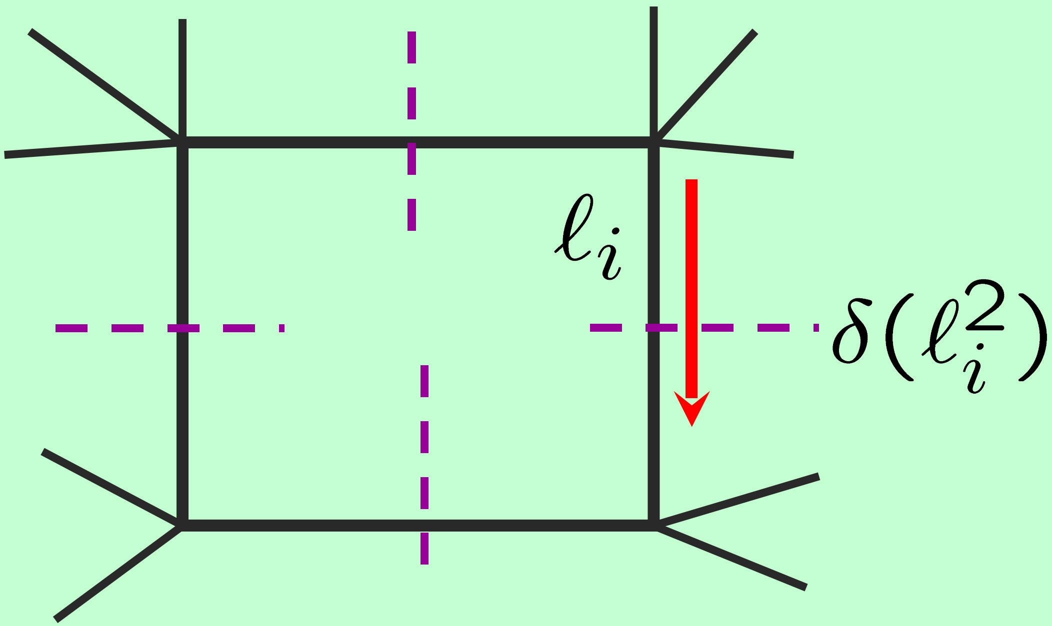


$$C_3 = \int dLIPS \quad M(l_1, \cdots, l_2) \times M(-l_2, \cdots, l_3) \times M(-l_3, \cdots, -l_1)$$

$$= \sum c_i I_4^i|_{disc} + d_i I_3^i|_{disc}$$

Box-Coefficients Britto,Cachazo,Feng

$$\hat{c} = \frac{1}{2}\sum_{\mathcal{S}} (A^{\text{tree}}(\ell_1, i_1, \ldots, i_2, \ell_2) \times A^{\text{tree}}(\ell_2, i_3, \ldots, i_4, \ell_3)$$
$$\times A^{\text{tree}}(\ell_3, i_5, \ldots, i_6, \ell_4) \times A^{\text{tree}}(\ell_4, i_7, \ldots, i_8, \ell_1))$$



-works for massless corners (complex momenta)

Canonical Basis of Terms in Cut

-expand cut as a sum of simple forms whose effect is

$$A^{tree} \times A^{tree} = \sum_i c_i F_i$$

-look at quad cuts for boxes,triple cut for triangles, double cut for bubbles

$$A^{tree} \times A^{tree} \times A^{tree} = \sum_i c_i F_i$$

-relate to four dimensional covariant integrals

-can also use two-dimensional fermionic integration

Britto, Buchbinder Cachazo Feng

-can relate to real 1+2 dimensional integration

Britto, Feng Mastrolia

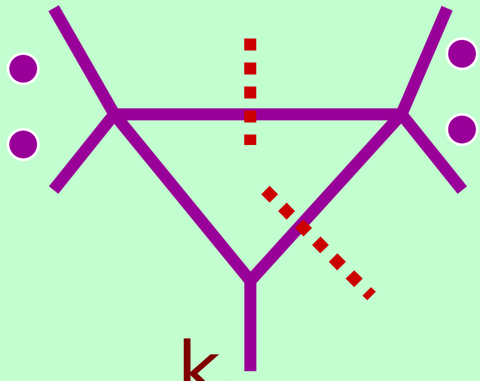
Forde

-see also talk by Papadopoulos

Canonical Basis of Terms for Bubble Coefficients

$$H_1 = \frac{\langle a l \rangle}{\langle b l \rangle}$$ in the two-particle cut

$$H_1 = \frac{[b|l|a\rangle}{(l+k_b)^2}$$



-linear triangle

$$l \longrightarrow l + k_b a_1 + P a_2$$

$$H \longrightarrow [b|P|a\rangle I_3[a_2] \longrightarrow \frac{[b|P|a\rangle}{[b|P|b\rangle} I_2$$

Extend the canonical form

$$H_n = \frac{\langle a_1 l\rangle \cdots \langle a_n l\rangle}{\langle b_1 l\rangle \cdots \langle b_n l\rangle}$$

$$= \sum_i c_i \frac{\langle a_1 l\rangle}{\langle b_i l\rangle} \longrightarrow \sum_i c_i H_1(b_i, a_1)$$

$$c_i = \frac{\prod_{j<n} \left\langle B_j\, A_i \right\rangle}{\prod_{j\neq i} \left\langle A_j\, A_i \right\rangle} \qquad \sum_i \frac{\prod_{j<n} \left\langle B_j\, A_i \right\rangle}{\prod_{j\neq i} \left\langle A_j\, A_i \right\rangle} \frac{\langle B_n|P|A_i]}{\langle A_i|P|A_i]}$$

Fermionic Unitarity

$$\int dLIPS F = \int \langle \lambda, d\lambda \rangle [\bar{\lambda}, d\bar{\lambda}] F(\lambda, \bar{\lambda})$$

Cachazo, Svecek, Witten

Britto,Feng,Mastrolia,Yang

-try to use analytic structure to identify terms within two-particle cuts

$$\int \langle \lambda, d\lambda \rangle [\bar{\lambda}, d\bar{\lambda}] \frac{H(\lambda)}{[\lambda|P|\lambda\rangle^2} \longrightarrow$$ bubbles

-gives same result although we are trying to work using loop momenta

more canonical forms

$$G_0 = \frac{[A|l|a\rangle}{(l+Q)^2}$$

Use identity,

$$\frac{[Al]}{(l+Q)^2} = \frac{1}{(l+Q)^2}\frac{[A|P(P+Q)Q|l\rangle}{\langle l|PQ|l\rangle} - \frac{[A|P|l\rangle}{\langle l|PQ|l\rangle}$$

$$G_0 \equiv \frac{\langle l|a\rangle\langle l|P|A]}{\langle l|PQ|l\rangle} \sim H_2(\hat{P}, \hat{Q}, a, P|A])$$

$$\hat{P} = P + xQ, \hat{P}^2 = 0 \quad P^2 x = -P\cdot Q + \sqrt{\Delta_3}$$

-better to recombine and rationalise

$$\frac{\langle a\, l_1\rangle\,\langle b\, l_1\rangle}{\langle l_1|PQ|l_1\rangle} \longrightarrow \frac{\langle a|(PQ-QP)|b\rangle}{2\Delta_3}$$

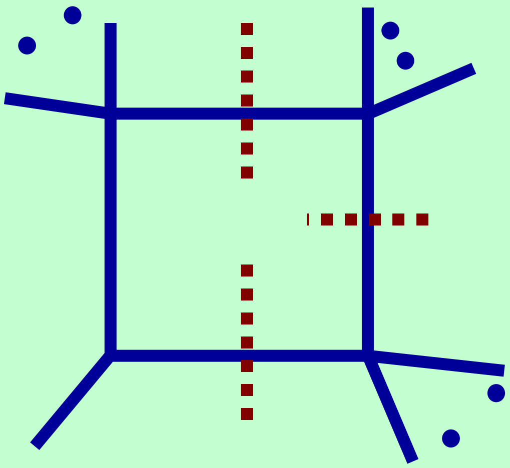
$$G_1(A;b,c;d;Q;l_1) = \frac{1}{(l_1+Q)^2}\frac{[A|l_1|b\rangle\,\langle l_1\, c\rangle}{\langle l_1\, d\rangle}$$

$$\frac{[A|P[P,Q]|D\rangle\langle B|[P,Q]|C\rangle}{4\Delta_3\langle D|PQ|D\rangle}+\frac{[A|P|D\rangle}{2\langle D|PQ|D\rangle}\left(\frac{(\langle B\, D\rangle\,\langle C|P|D]+B\leftrightarrow C}{\langle D|P|D]}\right)$$

Canonical Basis of Terms for Triple Cut

$$H_1 = \frac{\langle a l \rangle}{\langle b l \rangle}$$

$$= \frac{[b|l|a\rangle}{(l+k_b)^2}$$



-Linear box

-need formulae to generate triangle coefficient

Canonical Basis of Terms: Triple Cut

$$\frac{\langle a\, l\rangle}{\langle b\, l\rangle} \longrightarrow_{triangle} \frac{\langle a|(K_1K_3 - K_3K_1)|b\rangle}{2\langle b|K_1K_3|b\rangle} \qquad \langle b\, K_i\rangle \neq 0$$

$$\frac{\langle a\, l\rangle}{\langle K_3\, l\rangle} \longrightarrow_{triangle} \frac{\langle a|K_1|K_3]}{\langle K_3|K_1|K_3]}$$

$$\frac{\langle l\, a\rangle\, \langle l\, b\rangle}{\langle l|K_3K_1|l\rangle} \longrightarrow_{triangle} \frac{\langle a|(K_1K_3 - K_3K_1)|b\rangle}{2\Delta_3}$$

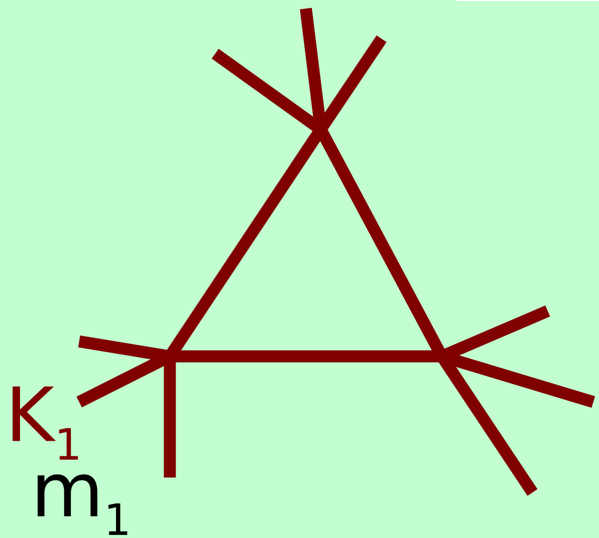
$$\frac{\langle l\, a\rangle\, \langle l\, b\rangle\, \langle l\, c\rangle}{\langle l|K_3K_1|l\rangle\, \langle l\, d\rangle} \longrightarrow_{triangle} \frac{\langle b|(K_1K_3 - K_3K_1)|d\rangle\langle c|(K_1K_3 - K_3K_1)|a\rangle + \Delta_3\, \langle b\, d\rangle\, \langle c\, a\rangle}{2\Delta_3\langle d|K_3K_1|d\rangle} + \frac{\langle d\, b\rangle\, \langle d\, c\rangle\, \langle a|(K_1K_3 - K_3K_1)|d\rangle}{\langle d|K_3K_1|d\rangle^2}$$

Example:use this to obtain three Mass triangle for n-point NMHV, N=1

$$\frac{-i}{K_3^2 \prod_{i \neq r,s,t} \langle i\, i+1 \rangle} \sum_{y \in Y_6} \frac{\langle m_1^{32}\, y\rangle \langle m_2^{32}\, y\rangle \langle m_2^{31}\, y\rangle \langle m_3\, y\rangle \langle m_3^{31}\, y\rangle}{\prod_{z \in Y_6, z \neq y} \langle z\, y\rangle} \times \left(\frac{\langle m_1|[K_1, K_2]|y\rangle \langle X|[K_1,K_2]|X\rangle}{2\Delta_3 \langle y|K_1 K_2|y\rangle} + \frac{\langle y\, X\rangle \langle y\, m_1\rangle \langle X|[K_1,K_2]|y\rangle}{2\langle y|K_1 K_2|y\rangle^2} \right).$$

$$Y_6 = \{r, r+1, s^{32}, (s+1)^{32}, t^{31}, (t+1)^{31}\}$$

$$|a^{ij}\rangle \equiv K_j K_i |a\rangle.$$



Six Point Example, N=1, NMHV

$A(---+++)$ $A(--+-++)$ $\boxed{A(-+-+-+)}$

Britto,BuchbinderCachazo,Feng

$$A_n^{1-loop} = \sum c_i I_4^i + \sum_i d_i I_3^i + \sum_i e_i I_2^i$$

quadruple cuts

massless:IR

-two-particle cut

massive: triple cuts

$$A = \sum_i c_i I_4^{1m,i} + c_i I_4^{2m:h} + d_i I_3^{1m} + d_i I_3^{2m} + d_j I_3^{3m} + e_i I_2^i$$

$$e_3 = e_1(3,4,5,6,1,2) \quad e_2 = \bar{e}_1(2,3,4,5,6,1)$$

$$\sum_h A(l_1^{-h}, 1^-, 2^+, l_2^h) \times A(l_2^{-h}, 3^-, 4^+, 5^-, 6^+, l_1^h)$$

$$\begin{aligned}
A(l_2^{-h}, 3^-, 4^+, 5^-, 6^+, l_1^h) =& \frac{[l_2|P_{456}|5\rangle^2 [l_1|P_{456}|5\rangle^2}{t_{456}\,[l_1\,l_2]\,[l_2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,[l_1|P_{456}|4\rangle [3|P_{456}|6\rangle} \left(\frac{[l_2|P_{456}|5\rangle}{[l_1|P_{456}|5\rangle}\right)^h \\
&+ \frac{[6|P_{l_2 34}|3\rangle^2 (\langle l_2\,3\rangle\,[6\,l_1])^2}{t_{l_2 34}\,\langle l_2\,3\rangle\,\langle 3\,4\rangle\,[5\,6]\,[6\,l_1]\,[5|P_{34}|l_2\rangle [l_1|P_{56}|4\rangle} \left(\frac{[6|P_{234}|3\rangle}{\langle l_2\,3\rangle\,[6\,l_1]}\right)^h \\
&+ \frac{[4|P_{345}|l_1\rangle^2 [4|P_{345}|l_2\rangle^2}{t_{345}\,\langle 6\,l_1\rangle\,\langle l_1\,l_2\rangle\,[3\,4]\,[4\,5]\,[3|P_{345}|6\rangle [5|P_{345}|l_2\rangle} \left(\frac{[4|P_{345}|l_1\rangle}{[4|P_{345}|l_2\rangle}\right)^h
\end{aligned}$$

$$\begin{aligned}
cut =& \frac{\langle 1\,l_2\rangle}{\langle 1\,2\rangle\,\langle 2\,l_2\rangle\,\langle l_2\,l_1\rangle} \times \frac{[l_2|P_{456}|5\rangle [l_1|P_{456}|5\rangle\,\langle 1\,2\rangle^2\,[2|P_{456}|5\rangle^2}{t_{456}\,[l_1\,l_2]\,[l_2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,[l_1|P_{456}|4\rangle [3|P_{456}|6\rangle} \\
&+ \frac{\langle 1\,l_2\rangle}{\langle 1\,2\rangle\,\langle 2\,l_2\rangle\,\langle l_2\,l_1\rangle} \times \frac{[6|P_{l_2 34}|3\rangle\,[6\,l_1]\,\langle X_{2a}\,l_2\rangle^2}{t_{l_2 34}\,\langle 3\,4\rangle\,[5\,6]\,[6\,l_1]\,[5|P_{34}|l_2\rangle [l_1|P_{56}|4\rangle} \\
&+ \frac{\langle 1\,l_2\rangle}{\langle 1\,2\rangle\,\langle 2\,l_2\rangle\,\langle l_2\,l_1\rangle} \times \frac{[4|P_{345}|l_1\rangle [4|P_{345}|l_2\rangle [4|P_{345}|2\rangle^2\,\langle l_1\,l_2\rangle^2}{t_{345}\,\langle 6\,l_1\rangle\,\langle l_1\,l_2\rangle\,[3\,4]\,[4\,5]\,[3|P_{345}|6\rangle\,[5|P_{345}|l_2\rangle}
\end{aligned}$$

$$\begin{aligned}
c_{12} =& \frac{[2|P_{456}|5\rangle^2}{[1\,2]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,[3|P_{456}|6\rangle t_{456}} \times \overline{H}_3(1, 3, P_{56}|4\rangle; 2, P_{46}|5\rangle, P_{46}|5\rangle) \\
&+ \frac{1}{\langle 1\,2\rangle\,\langle 3\,4\rangle\,[5\,6]} \times G_3(2, P_{34}|5], P_{12}P_{56}|4\rangle; 6; 1, 3, X_{2a}, X_{2a}; Q_{34}) \\
&+ \frac{[4|P_{345}|1\rangle^2}{\langle 1\,2\rangle\,[3\,4]\,[4\,5]\,[3|P_{345}|6\rangle t_{345}} \times H_3(2, 6, P_{34}|5]; 1, P_{345}|4], P_{345}|4])
\end{aligned}$$

-compact rational form

Seven-Point NMHV amplitudes, N=1

$\boxed{A(--+-+++)}$

$A(---++++)$

$A(--++-++)$

$A(-+-+-++)$

$$A(1^-2^-3^+4^-5^+6^+7^+) = c_2 I_2(s_{23}) + c_3 I_2(s_{34} + c_4 I_2(s_{45}) + c_7 I_2(s_{71})$$

$$+ \sum_{i \neq 5} e_i I_2(t_{ii+1i+2})$$

$$+ boxes + triangles$$

$$A(7^+, 1^-, l_2, l_1) \times A_7(l_1, l_2, 2^-, 3^+, 4^-, 5^+, 6^+)$$

-need seven point tree…$A(l_1,l_2,3^-,4^+,5^-,6^+,7^+)$..

$$T^B_{1a} = \frac{[4|P_{234}|5\rangle^2\,[2\,4]^2\,\langle 5\,1\rangle^2}{\langle 5\,6\rangle\,\langle 6\,7\rangle\,\langle 7\,1\rangle\,[2\,3]\,[3\,4]\,t_{234}[2|P_{234}|5\rangle\,[4|P_{234}|1\rangle} \times \left(\frac{[2\,4]\,\langle 5\,1\rangle}{[4|P_{234}|5\rangle}\right)^{2h}$$

$$T^B_{1b} = \frac{-\,\langle 2\,3\rangle\,\langle 13\rangle^2[4|P_{67}|1\rangle^4}{\langle 6\,7\rangle\,\langle 7\,1\rangle\,\langle 12\rangle\,[4\,5]\,[5|P_{67}|1\rangle[4|P_{23}|1\rangle\langle 1|P_{67}P_{45}|3\rangle\langle 6|P_{45}P_{23}|1\rangle} \times \left(\frac{\langle 13\rangle}{\langle 2\,3\rangle}\right)^{2h}$$

$$T^B_{1c} = \frac{t_{671}[2|P_{671}|1\rangle^2\,\langle 3\,5\rangle^4}{\langle 6\,7\rangle\,\langle 7\,1\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,[2|P_{71}|6\rangle\,[2|P_{345}|5\rangle\langle 1|P_{67}P_{45}|3\rangle t_{345}} \left(\frac{-[2|P_{671}|1\rangle}{t_{671}}\right)^{2h}$$

$$T^B_2 = \frac{-\,[2\,7]^2\,[1\,7]\,\langle 3\,5\rangle^4}{[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,[2|P_{712}|6\rangle[7|P_{712}|3\rangle t_{712}} \times \left(-\frac{[2\,7]}{[1\,7]}\right)^{2h}$$

$$T^B_3 = \frac{\langle 1\,3\rangle^2\,\langle 2\,3\rangle^2\,[7|P_{123}|5\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,[7|P_{123}|3\rangle\,[7|P_{123}|4\rangle\langle 1|P_{23}P_{45}|6\rangle t_{123}t_{456}} \times \left(\frac{\langle 1\,3\rangle}{\langle 2\,3\rangle}\right)^{2h}$$

$$T^B_4 = \frac{-\,\langle 1\,3\rangle^2\,\langle 2\,3\rangle^2\,\langle 5\,6\rangle^2\,[6\,7]^3}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 5\,6\rangle\,[7|P_{56}|4\rangle\,[5|P_{567}|1\rangle t_{567}s_{56}} \times \left(\frac{\langle 1\,3\rangle}{\langle 2\,3\rangle}\right)^{2h}$$

-this gives us the coefficient

$$
\begin{aligned}
&= \frac{1}{\langle 5\,6\rangle \langle 6\,7\rangle \langle 1\,2\rangle [3\,4]} G_5(1,7,P_{12}P_{34}|5\rangle, P_{12}|3], P_{567}|4]; 4; 2,5,5,X_{1a},X_{1a},P_{12}|4]; Q_{567}) \\
&+ \frac{\langle 2\,3\rangle^2}{\langle 1\,2\rangle \langle 6\,7\rangle [4\,5]} \times H_6(1,7,P_{67}|5], P_{567}|4], P_{67}P_{45}|3\rangle, P_{4567}P_{45}|6\rangle; 2,3,P_{67}|4], P_{67}|4], P_{67}|4], P_{67}|4],) \\
&+ \frac{\langle 3\,5\rangle^4}{\langle 1\,2\rangle \langle 3\,4\rangle \langle 4\,5\rangle \langle 6\,7\rangle\, t_{345}} \sum_{\substack{a\in\{1,2\}\\ b\in\{6,7\}}} [a\,b]\, H_5(2,a,b,P_{67}P_{345}|2\rangle, P_{67}P_{345}|2\rangle; 1,7,P_{12}P_{345}|6\rangle, P_{12}P_{34}|5\rangle, P_{67}P_{45}|3\rangle)) \\
&+ \frac{\langle 3\,5\rangle^4 [7\,1]^2 \langle 1\,2\rangle^2}{\langle 3\,4\rangle \langle 4\,5\rangle \langle 5\,6\rangle \langle 1\,2\rangle\, s_{12}[7|P_{712}|3\rangle t_{712}} \times H_2(2, P_{12}P_{712}|6\rangle; 1, P_{12}|7]) \\
&+ \frac{[7|P_{123}|5\rangle^4 \langle 2\,3\rangle^2}{\langle 1\,2\rangle \langle 4\,5\rangle \langle 5\,6\rangle\, [7|P_{123}|3\rangle [7|P_{123}|4\rangle t_{123} t_{456}} \times H_2(2, P_{123}P_{45}|6\rangle; 1, 3) \\
&+ \frac{[6\,7]^3 \langle 2\,3\rangle^2}{\langle 1\,2\rangle \langle 3\,4\rangle [5\,6]\, t_{567} [7|P_{56}|4\rangle} \times H_2(1, P_{67}|5]; 2, 3)
\end{aligned}
$$

-the other coefficients follow analogously giving the complete N=1 amplitude

Spurious Singularities

-spurious singularities are singularities which occur in

Coefficients but not in full amplitude

-need to understand these to do recursion

-link coefficients together

Example of Spurious singularities

$$A_6^{N=1 \text{ chiral}}(1^-,2^-,3^-,4^+,5^+,6^+) = a_1 K_0[s_{61}] + a_2 K_0[s_{34}]$$
$$-\frac{i}{2}\Big[b_1 \frac{L_0[s_{345}/s_{61}]}{s_{61}} + b_2 \frac{L_0[s_{234}/s_{34}]}{s_{34}} + b_3 \frac{L_0[s_{234}/s_{61}]}{s_{61}} + b_4 \frac{L_0[s_{345}/s_{34}]}{s_{34}}\Big]$$

$$L_0(r) = \frac{\ln(r)}{1-r}, \;\; K_0(r) = \frac{1}{\epsilon} + \ln(r)$$

$$b_1 = \frac{[6|\not P|3\rangle^2 [6|(\not 2 \not P \not P - \not P \not 2 \not P)|3^+\rangle}{[2|\not P|5\rangle\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,P^2}, \quad P = P_{345}$$
$$b_4 = \frac{[6|\not P|3\rangle^2 [6|(\not 5 \not P \not P - \not P \not 5 \not P)|3^+\rangle}{[2|\not P|5\rangle\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,P^2}, \quad P = P_{345}$$

Co-planar singularity

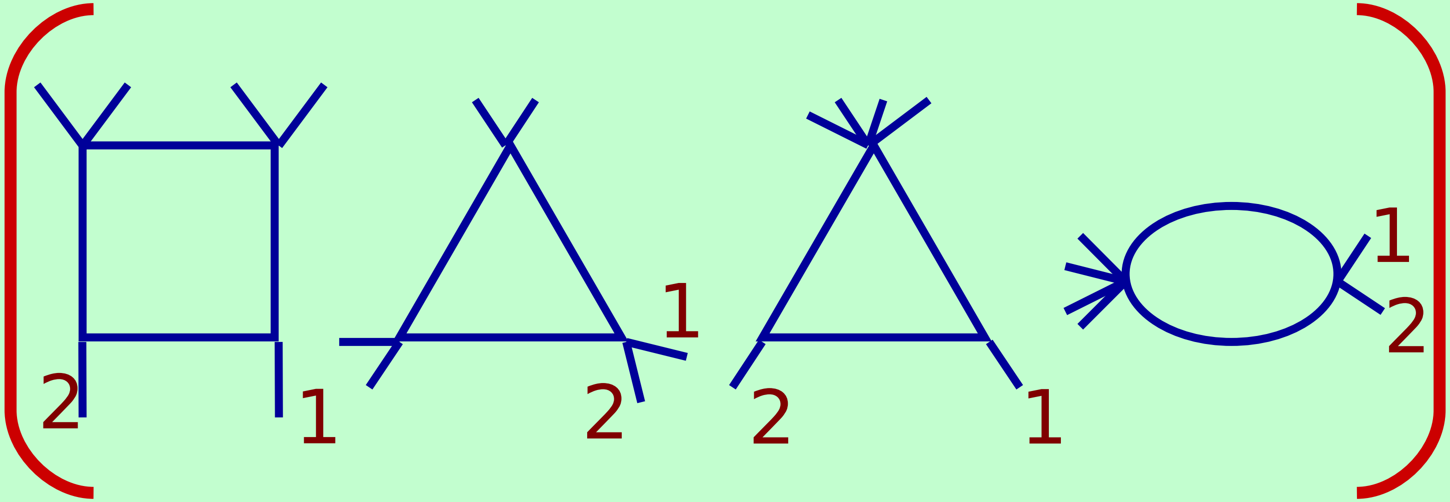
Collinear Singularity

$$P_{34} = \alpha k_2 + \beta k_5$$

Multi-particle pole

Spurious singularities link different coefficients

$$\frac{1}{[2|K_{234}|5\rangle^2}$$



=0 at singularity

-these singularities link different coefficients togeth

-use a basis where these cancel automatically

Campbell Glover, Miller

Bern, Dixon Kosower

Conclusions

-Unitarity looking competitive for NLO processes, 7pt N=1 example

-for pure gluon, massless QCD works very well

-important extensions to rational term , Anastasiou, Britto,Feng,Kunszt, Mastrolia

massive particles, Britto, Feng, Yang; Badger

-are there better basis of functions?

-automation?, already much progresss Bern,Berger,Dixon,Kosower, Febres Cordero,Forde, Ita, Mai

Ellis,Giele, Kunszt; Maitre Mastrolia

-can use a variety of techniques : possible simultaneously

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