
SUSY QCD Corrections to Squark Loops in Higgs Boson Production via Gluon Gluon Fusion

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In coll. with M. Spira
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Outline of the talk

- *Introduction*
- *QCD corrections to squark loops in $gg \rightarrow$ Higgs*
- *Conclusions*

Introduction

Gluon gluon fusion: dominant production process at existing and future hadron colliders.

QCD corrections to top & bottom loops

NLO (SM, MSSM): increase σ by $\sim 10\text{...}100\%$ Spira, Djouadi, Graudenz, Zerwas
Dawson; Kauffman, Schaffer

SM; $\tan\beta \lesssim 5$: limit $M_\Phi \ll m_t$ - approximation $\sim 20\text{-}30\%$ Krämer, Laenen, Spira

NNLO @ $M_\Phi \ll m_t \Rightarrow$ further increase by 20-30% Harlander, Kilgore
Anastasiou, Melnikov
scale dependence: $\Delta \lesssim 10 - 15\%$ Ravindran, Smith, van Neerven

Estimate of NNNLO effects \rightsquigarrow improved perturbative convergence Moch, Vogt
Ravindran

Soft gluon resummation: $\sim 10\%$ Catani, de Florian, Grazzini, Nason

NLO corrections: to squark loops

only in heavy squark limit Dawson, Djouadi, Spira

full SUSY-QCD corrections in heavy mass limit Harlander, Steinhauser
Harlander, Hofmann

$m_{\tilde{Q}} \lesssim 400 \text{ GeV}$: squarks play a significant role \rightsquigarrow

calculation of the full squark mass dependence at NLO.

The MSSM Higgs sector

MSSM Higgs sector – supersymmetry & anomaly free theory \Rightarrow 2 complex Higgs doublets

EW
 $\xrightarrow{\text{SB}}$

neutral, CP-even h, H

neutral, CP-odd A

charged H^+, H^-

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Higgs masses

$$M_h \lesssim 140 \text{ GeV}$$

$$M_{A,H,H^\pm} \sim \mathcal{O}(v) \dots 1 \text{ TeV}$$

Ellis et al; Okada et al; Haber, Hempfling;
Hoang et al; Carena et al; Heinemeyer et al;
Zhang et al; Brignole et al; ...

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Modified couplings with respect to the SM: (decoupling limit) Gunion, Haber

| Φ | $g_{\Phi u\bar{u}}$ | $g_{\Phi d\bar{d}}$ | $g_{\Phi VV}$ |
|--------|---|---|----------------------------------|
| h | $c_\alpha/s_\beta \rightarrow 1$ | $-s_\alpha/c_\beta \rightarrow 1$ | $s_{\beta-\alpha} \rightarrow 1$ |
| H | $s_\alpha/s_\beta \rightarrow 1/\text{tg}\beta$ | $c_\alpha/c_\beta \rightarrow \text{tg}\beta$ | $c_{\beta-\alpha} \rightarrow 0$ |
| A | $1/\text{tg}\beta$ | $\text{tg}\beta$ | 0 |

$$\tan \beta \uparrow \Rightarrow g_{\Phi uu} \downarrow$$

$$g_{\Phi dd} \uparrow$$

$$g_{\Phi VV}^{MSSM} \lesssim g_{\Phi VV}^{SM}$$

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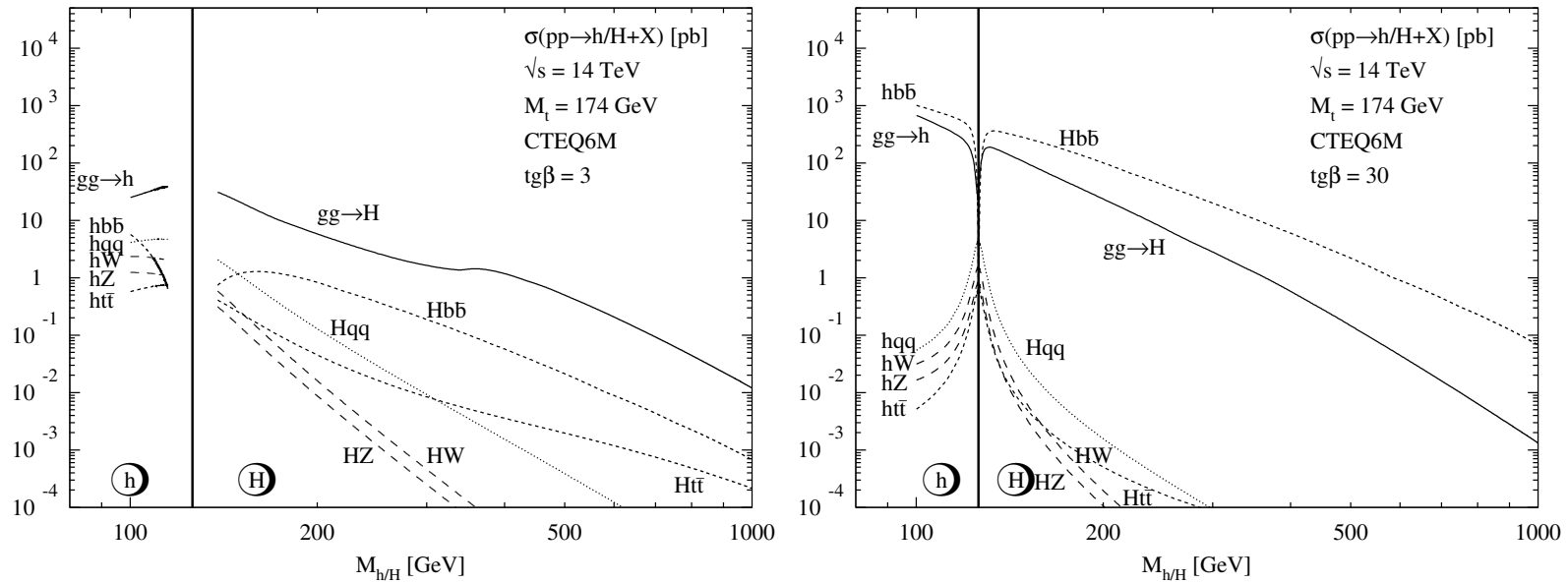
$$\tan \beta \uparrow \Rightarrow g_{\Phi uu} \downarrow$$

$$g_{\Phi dd} \uparrow$$

$$g_{\Phi VV}^{MSSM} \lesssim g_{\Phi VV}^{SM}$$

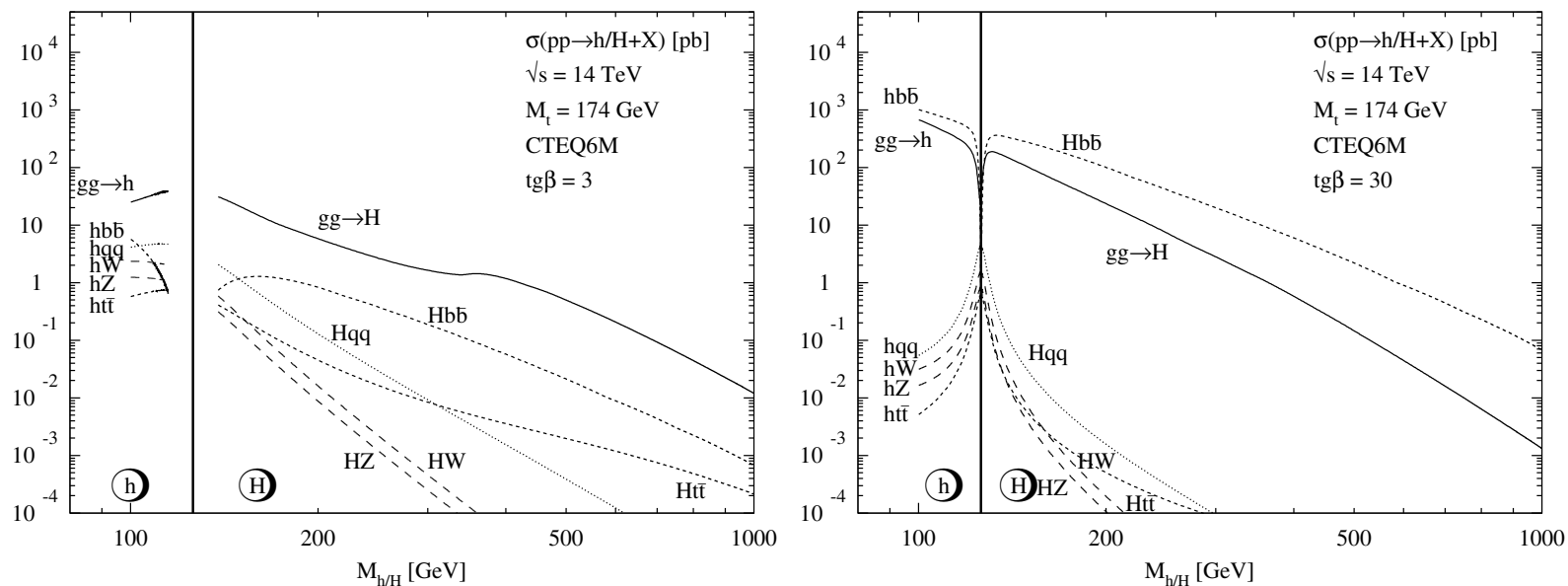
Dominant production mechanism at the LHC: $gg \rightarrow \Phi$
 $\Phi b\bar{b}$ for $\tan \beta$ large

MSSM Higgs Boson Production at the LHC

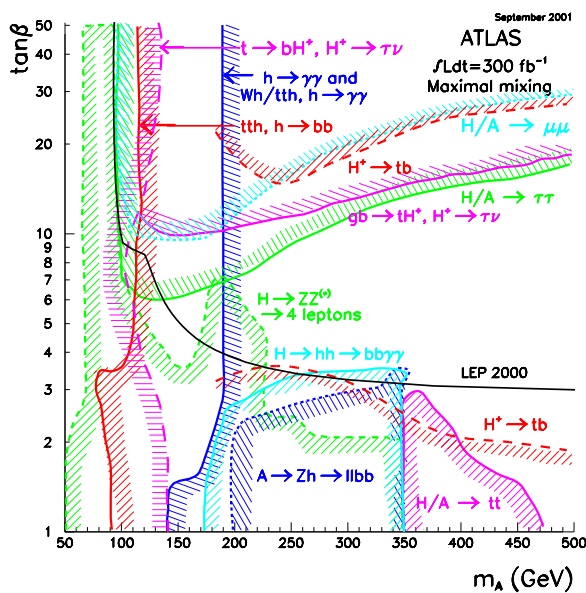


Spira

MSSM Higgs Boson Production at the LHC



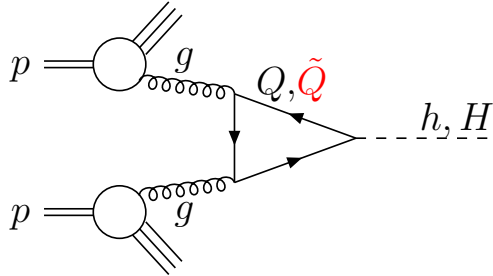
Spira



ATLAS

gg → H, h at leading order

Lowest order - 1 loop



$$\sigma(pp \rightarrow \Phi + X) = \sigma_0 \tau_\Phi \frac{d\mathcal{L}^{gg}}{d\tau_\Phi}$$

$$\sigma_0 = \frac{G_F \alpha_S^2(\mu_R)}{288\sqrt{2}\pi} \left| \sum_Q g_Q^\Phi F(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}}) \right|^2$$

$$\tau_\Phi = \frac{M_\Phi^2}{s}, \quad \tau_{Q,\tilde{Q}} = \frac{4m_{Q,\tilde{Q}}^2}{M_\Phi^2}$$

$$F(\tau_Q) = \frac{3}{2}\tau_Q \left[1 + (1 - \tau_Q)f(\tau_Q) \right]$$

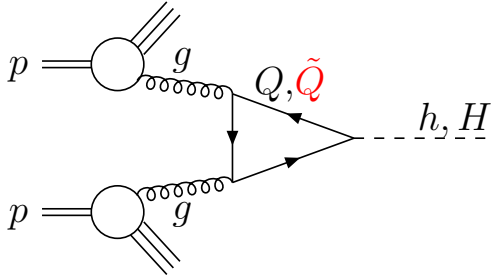
$$\tilde{F}(\tau_{\tilde{Q}}) = -\frac{3}{4}\tau_{\tilde{Q}} \left[1 - \tau_{\tilde{Q}}f(\tau_{\tilde{Q}}) \right]$$

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left[\log \left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} \right) - i\pi \right]^2 & \tau < 1 \end{cases}$$

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} g(x, \mu_F^2) g(\tau/x, \mu_F^2)$$

gg → H, h at leading order

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$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} g(x, \mu_F^2) g(\tau/x, \mu_F^2)$$

Remarks: - MSSM: $\tan \beta \uparrow \Rightarrow b/\tilde{b} \uparrow + t/\tilde{t} \downarrow$

- heavy quarks dominant

$\Phi Q Q \sim m_Q \rightsquigarrow t, b$

- $g_{\tilde{Q}}^\Phi \sim m_Q^2/m_{\tilde{Q}}^2 \rightsquigarrow \tilde{t}, \tilde{b}$

- $gg \rightarrow A$ no \tilde{Q} contribution at LO

The Squark Loops

Calculation of the QCD corrections to squark loops including the full mass dependence

Scenario:

The gluophobic Higgs scenario [$m_t = 174.3$ GeV]

Carena, Heinemeyer, Wagner, Weiglein

$$M_{SUSY} = 350 \text{ GeV}, \mu = M_2 = 300 \text{ GeV}, X_t = -770 \text{ GeV}, A_b = A_t, m_{\tilde{g}} = 500 \text{ GeV}$$

$$\tan \beta = 3$$

$$m_{\tilde{t}_1} = 156 \text{ GeV} \quad m_{\tilde{t}_2} = 517 \text{ GeV}$$

$$m_{\tilde{b}_1} = 346 \text{ GeV} \quad m_{\tilde{b}_2} = 358 \text{ GeV}$$

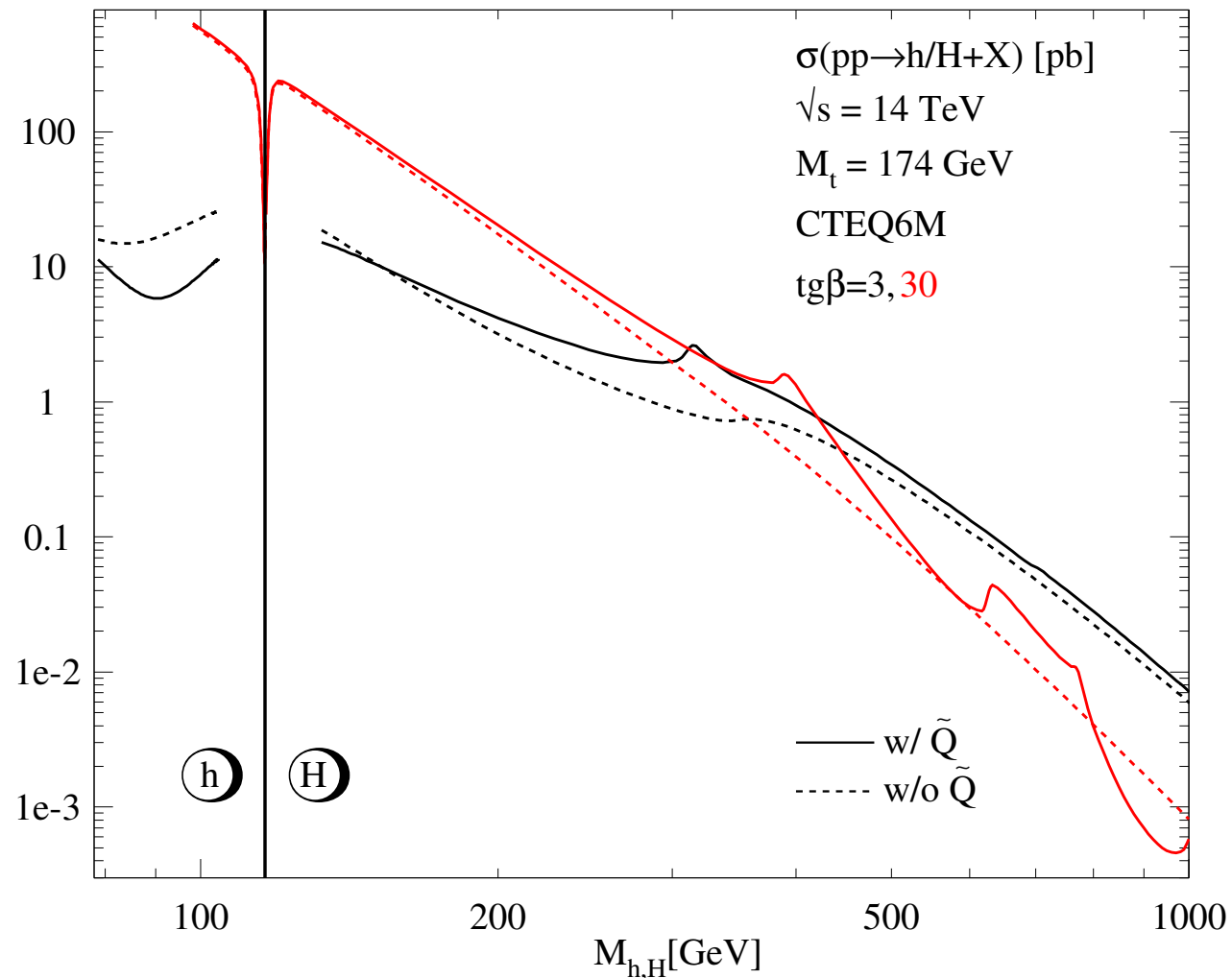
$$\tan \beta = 30$$

$$m_{\tilde{t}_1} = 155 \text{ GeV} \quad m_{\tilde{t}_2} = 516 \text{ GeV}$$

$$m_{\tilde{b}_1} = 314 \text{ GeV} \quad m_{\tilde{b}_2} = 388 \text{ GeV}$$

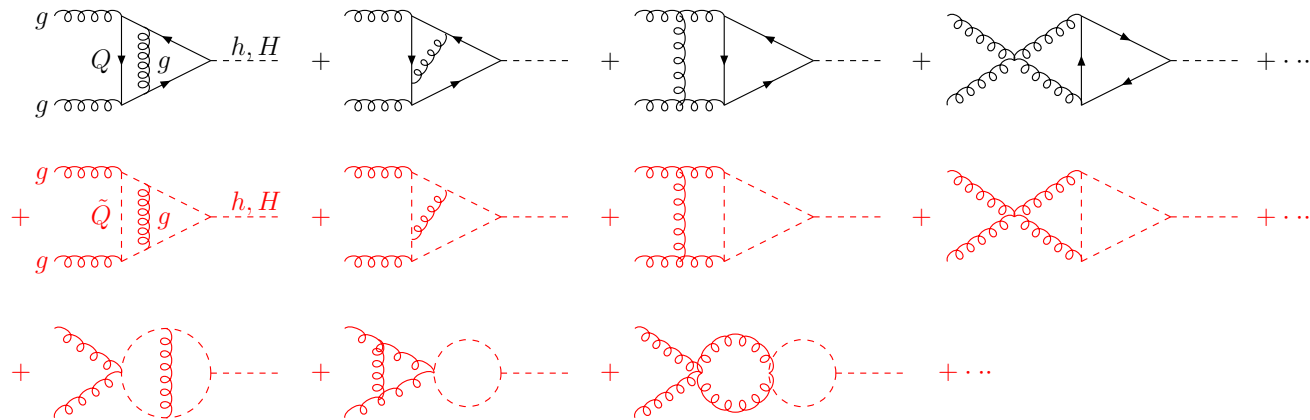
Squark contribution at leading order →

The \mathcal{LO} Cross Section w/ and w/o Squarks



QCD corrections

Virtual corrections [2 loops, no gluino] \rightsquigarrow UV-,IR-,Coll-singularities in $n = 4 - 2\epsilon$ dimensions.



General case (arbitrary $M_\Phi, m_Q, m_{\tilde{Q}}$)

- Interference $b, t, \tilde{b}, \tilde{t}$
- 5-dim. Feynman integrals \rightarrow 1-dimensional [Trilogarithms] + purely numerical solution

(analytically: Harlander, Kant
Anastasiou, Beerli, Bucherer, Daleo, Kunszt
Aglietti, Bonciani, Degrassi, Vicini \rightarrow Comparison: full agreement)

QCD corrections - Suite

Lagrangian separates gluon and gluino exchange contributions in a renormalizable way

$$\mathcal{L} = -\frac{1}{4}G^{a\mu\nu}G_{\mu\nu}^a + -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{Q}(i\not{D} - m_Q)Q + |D_\mu\tilde{Q}|^2 - m_{\tilde{Q}}^2|\tilde{Q}|^2 \\ -g_Q^\Phi\frac{m_Q}{v}\bar{Q}Q\Phi - g_{\tilde{Q}}^\Phi\frac{m_{\tilde{Q}}^2}{v}|\tilde{Q}|^2\Phi + \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{M_\Phi^2}{2}\Phi^2$$

$$iD_\mu = i\partial_\mu - g_S G_\mu^a T^a + ieA_\mu Q$$

Renormalization: - $m_{Q,\tilde{Q}}$: on-shell
- α_s : $\overline{\text{MS}}$ scheme, 5 active flavours

Virtual correction:

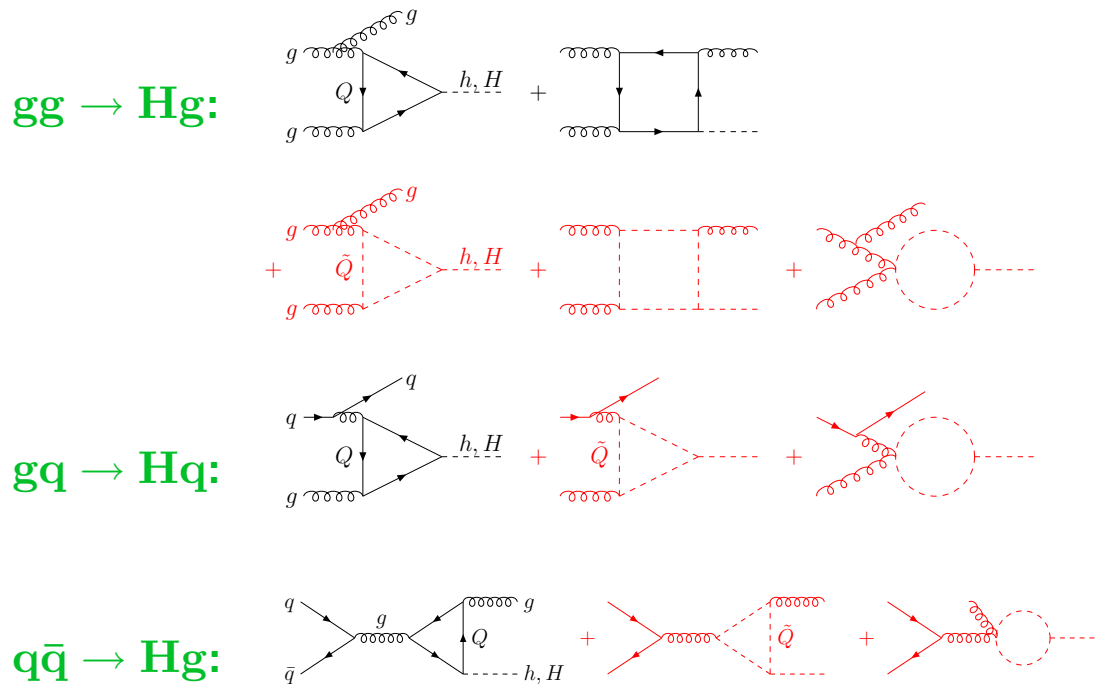
- Recovers heavy squark limit via effective Lagrangian
- After renormalization: IR & coll. singularities

Dawson,Djouadi,Spira

↷ Real corrections have to be added.

Real Corrections

Real corrections - 3 incoherent processes:



Phase space integration in $n = 4 - 2\epsilon$ dimensions \rightsquigarrow IR, Coll. singularities: poles in ϵ

- IR, Coll. poles in real corrections subtract the corresponding ones of the virtual corrections.
- Remaining coll. poles in the real corrections (Altarelli-Parisi kernels as coefficients)
 \rightsquigarrow absorbed in NLO structure functions.

Result

- μ =Ren. scale, Q =Fact. scale, $\mu^2 = Q^2 \approx M_\Phi^2$

$$\sigma(pp \rightarrow \Phi + X) = \sigma_0 \left[1 + C \frac{\alpha_S}{\pi} \right] \tau_\Phi \frac{d\mathcal{L}_{gg}}{d\tau_\Phi} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$C = \pi^2 + C_1(\tau_Q, \tau_{\tilde{Q}}) + \frac{33-2N_F}{6} \log \frac{\mu^2}{M_\Phi^2}$$

$$\Delta\sigma_{gg} = \int_{\tau_\Phi}^1 d\tau \frac{d\mathcal{L}_{gg}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \left\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log \frac{Q^2}{\hat{s}} + d_{gg}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \right. \\ \left. + 12 \left[\left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - \hat{\tau} [2 - \hat{\tau}(1-\hat{\tau})] \log(1-\hat{\tau}) \right] \right\}$$

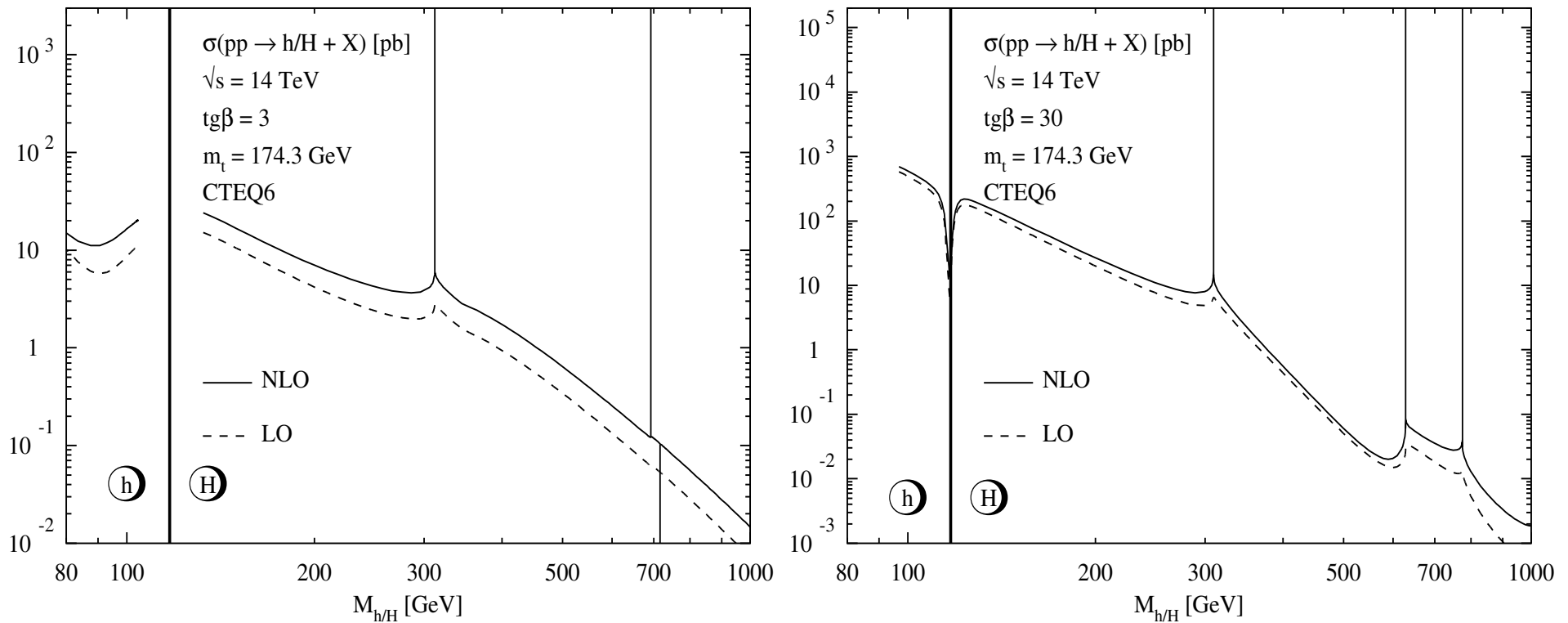
$$\Delta\sigma_{gq} = \int_{\tau_\Phi}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}_{gq}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 \left\{ -\frac{\hat{\tau}}{2} P_{gq}(\hat{\tau}) \left[\log \frac{Q^2}{\hat{s}} - 2 \log(1-\hat{\tau}) \right] \right. \\ \left. + d_{gq}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \right\}$$

$$\Delta\sigma_{q\bar{q}} = \int_{\tau_\Phi}^1 d\tau \sum_q \frac{d\mathcal{L}_{q\bar{q}}}{d\tau} \frac{\alpha_S}{\pi} \sigma_0 d_{q\bar{q}}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}})$$

$$C_1(\tau_Q, \tau_{\tilde{Q}}) \rightarrow \frac{11}{2} + \frac{7}{2} \text{Re} \left\{ \frac{\sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}})}{\sum_Q g_Q^\Phi F(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^\Phi \tilde{F}(\tau_{\tilde{Q}})} \right\} \quad d_{gg}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \rightarrow -\frac{11}{2} (1-\hat{\tau})^3$$

$$d_{gq}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \rightarrow -1 + 2\hat{\tau} - \frac{\hat{\tau}^2}{3} \quad d_{q\bar{q}}(\hat{\tau}, \tau_Q, \tau_{\tilde{Q}}) \rightarrow \frac{32}{27} (1-\hat{\tau})^3$$

The LO and NLO cross section w/ Squarks



$\Delta \sim 20 - 100\%$

Kinks, bumps, spikes: $\tilde{t}_1\tilde{t}_1, \tilde{b}_1\tilde{b}_1, \tilde{b}_2\tilde{b}_2$ thresholds in consecutive order with rising Higgs mass.

Coulomb singularities

$\tilde{Q}\bar{Q}$ thresholds: Formation of 0^{++} states \rightsquigarrow Coulomb singularities

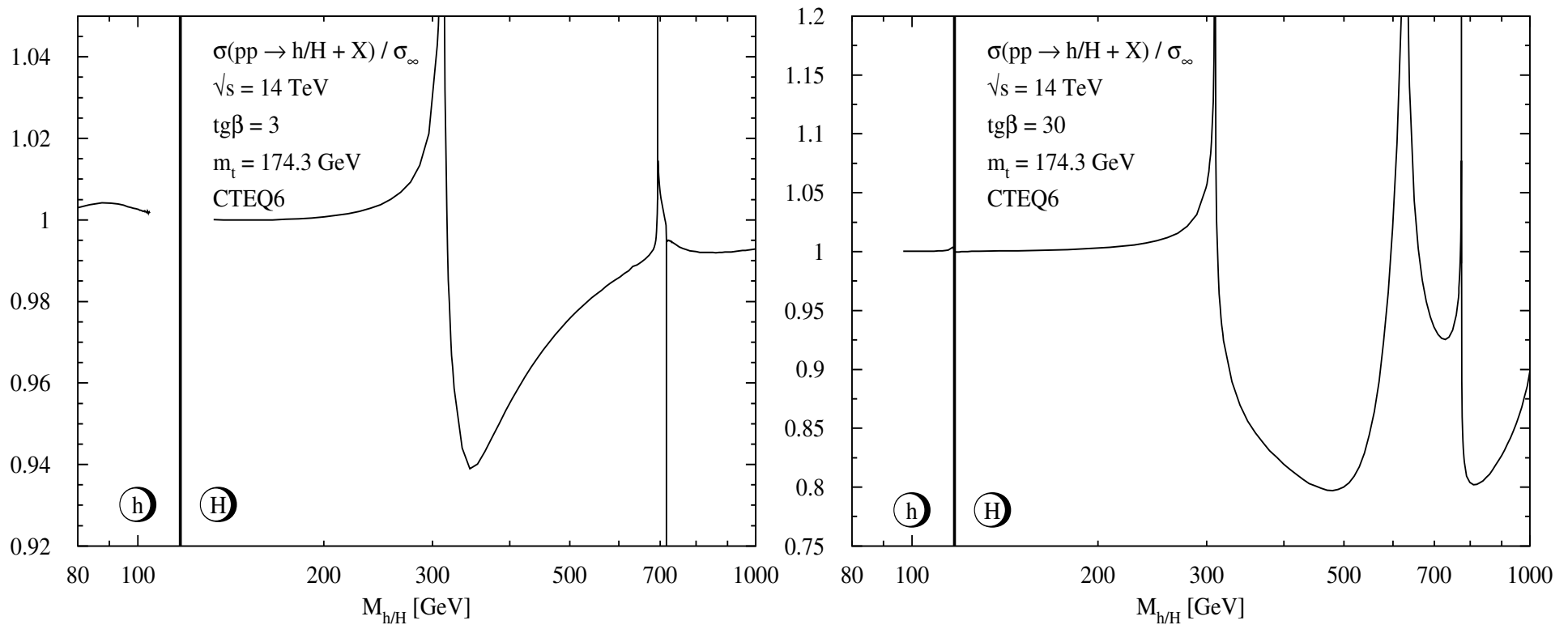
Singular behaviour can be derived from the Sommerfeld rescattering corrections \rightsquigarrow

At each specific $\tilde{Q}_0\bar{Q}_0$ threshold:

$$C_1(\tau_Q, \tau_{\tilde{Q}}) \rightarrow \text{Re} \left\{ \frac{g_{\tilde{Q}_0}^{\Phi} \tilde{F}(\tilde{Q}_0) \frac{16\pi^2}{3(\pi^2-4)} \left[-\ln(\tau_{\tilde{Q}_0}^{-1}-1) + i\pi + \text{const} \right]}{\sum_Q g_Q^{\Phi} F(\tau_Q) + \sum_{\tilde{Q}} g_{\tilde{Q}}^{\Phi} \tilde{F}(\tau_{\tilde{Q}})} \right\}$$

Agrees quantitatively with numerical results.

σ_{NLO} w/ full squark mass dependence / σ_{NLO} in the heavy squark limit



$\sigma(pp \rightarrow h/H + X) / \sigma_\infty$ up to 20%

Kinks, bumps, spikes: $\tilde{t}_1\tilde{t}_1, \tilde{b}_1\tilde{b}_1, \tilde{b}_2\tilde{b}_2$ thresholds in consecutive order with rising Higgs mass.

Scalar Higgs couplings to photons

LHC: $M_H \lesssim 140$ GeV – $H \rightarrow \gamma\gamma$ final state plays an important role for the Higgs boson search

Photon Collider: $\gamma\gamma \rightarrow h/H$ important role for the MSSM Higgs search at a PLC at high energies.

$$\langle \sigma(\gamma\gamma \rightarrow h/H) \rangle = \frac{8\pi^2}{M_{h/H}^3} \Gamma(h/H \rightarrow \gamma\gamma) \frac{d\mathcal{L}^{\gamma\gamma}}{d\tau_{h/H}}$$

Relation $\langle \sigma(\gamma\gamma \rightarrow h/H) \rangle \leftrightarrow \Gamma(h/H \rightarrow \gamma\gamma)$ holds also in NLO QCD:

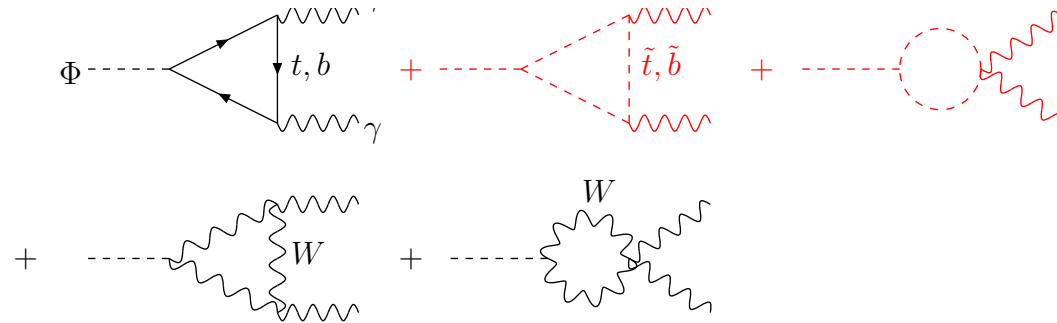
Single gluon radiation vanishes due to color charge conservation as well as due to the Furry theorem.

Photon fusion cross section measurable with an accuracy of a few percent. Melles, Stirling, Khoze
Krawczyk, Niezurawski, Zarnecki
Jikia, Söldner-Rembold

Precise knowledge of QCD corrections to $H \rightarrow \gamma\gamma$ important

H, h $\rightarrow \gamma\gamma$ at leading order

Lowest order - 1 loop



$$\Gamma_{LO}(\Phi \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_\Phi^3}{36\sqrt{2}\pi^3} \left| g_W^\Phi A_W^\Phi(\tau_W) + \sum_f N_{cf} e_f^2 g_f^\Phi A_f^\Phi(\tau_f) + \sum_{\tilde{f}} N_{c\tilde{f}} e_{\tilde{f}}^2 g_{\tilde{f}}^\Phi A_{\tilde{f}}^\Phi(\tau_{\tilde{f}}) \right|^2$$

$$A_W^\Phi(\tau) = -[2 + 3\tau + 3\tau(2 - \tau)f(\tau)]$$

$$A_f^\Phi(\tau) = 2\tau[1 + (1 - \tau)f(\tau)]$$

$$A_{\tilde{f}}^\Phi(\tau) = -\tau[1 - \tau f(\tau)]$$

$$\tau_i = \frac{4m_i^2}{M_\Phi^2}$$

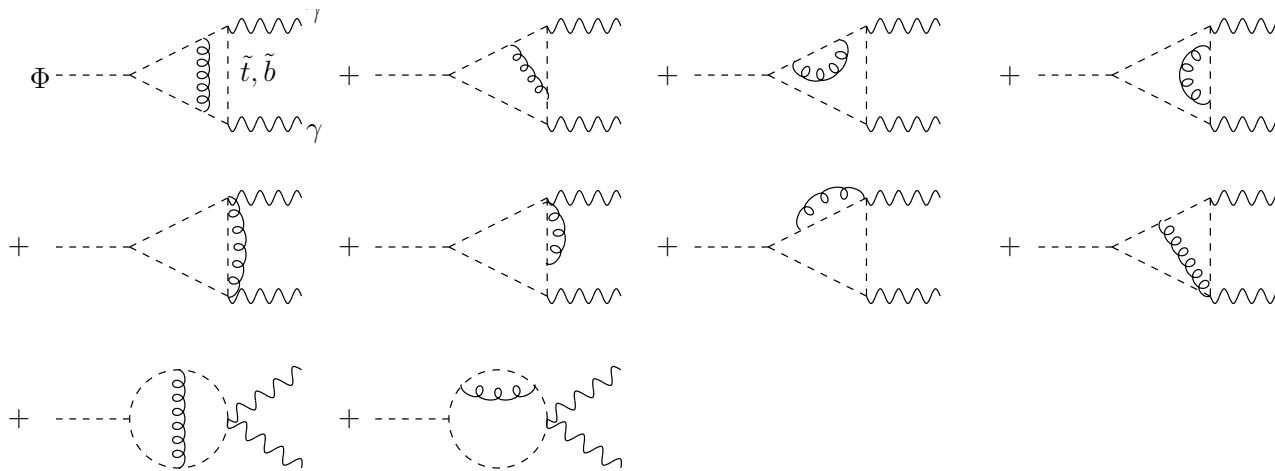
Remark: Chargino, H^\pm , charged \tilde{l} contributions neglected here, however, included in our work/results.

QCD corrections

$$A_Q^\Phi(\tau_Q) \rightarrow A_Q^\Phi(\tau_Q) \left[1 + C_Q^\Phi(\tau_Q) \frac{\alpha_S}{\pi} \right]$$

$$A_{\tilde{Q}}^\Phi(\tau_{\tilde{Q}}) \rightarrow A_{\tilde{Q}}^\Phi(\tau_{\tilde{Q}}) \left[1 + C_{\tilde{Q}}^\Phi(\tau_{\tilde{Q}}) \frac{\alpha_S}{\pi} \right]$$

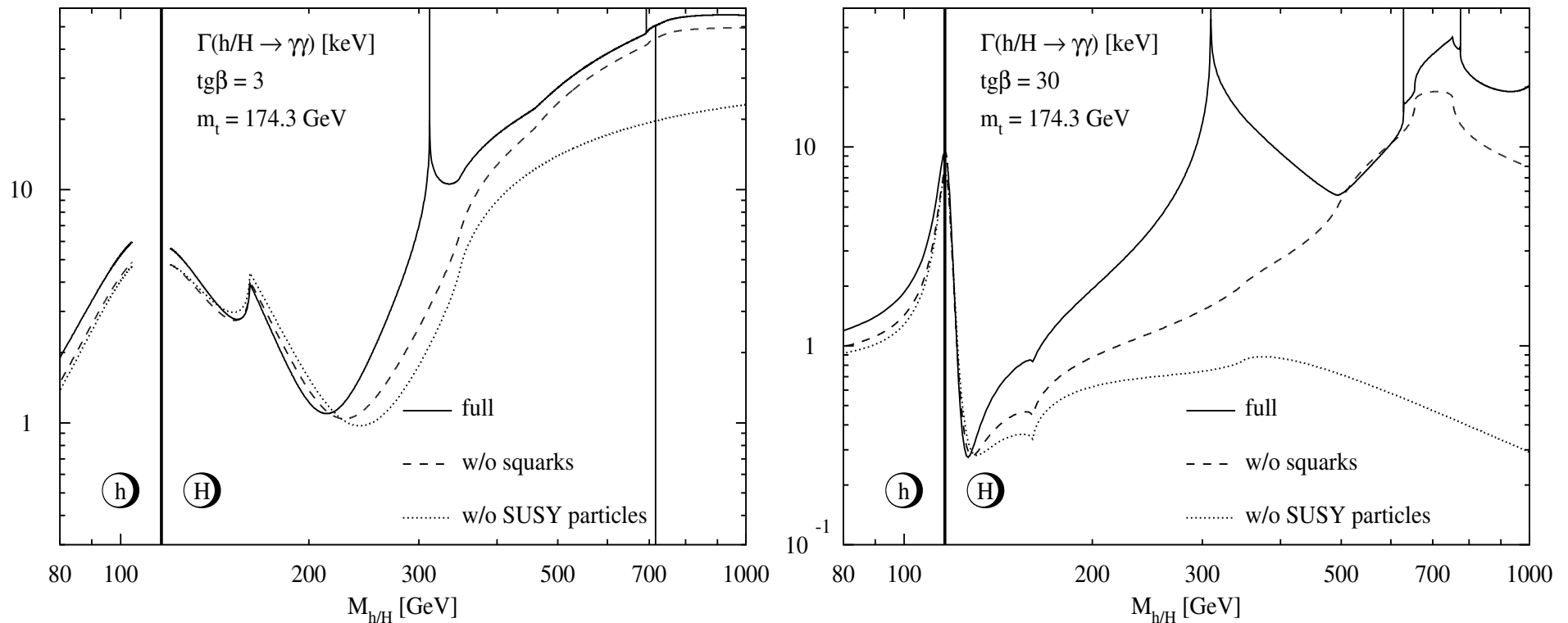
Virtual corrections [2 loops, no gluino contributions]



Remarks:

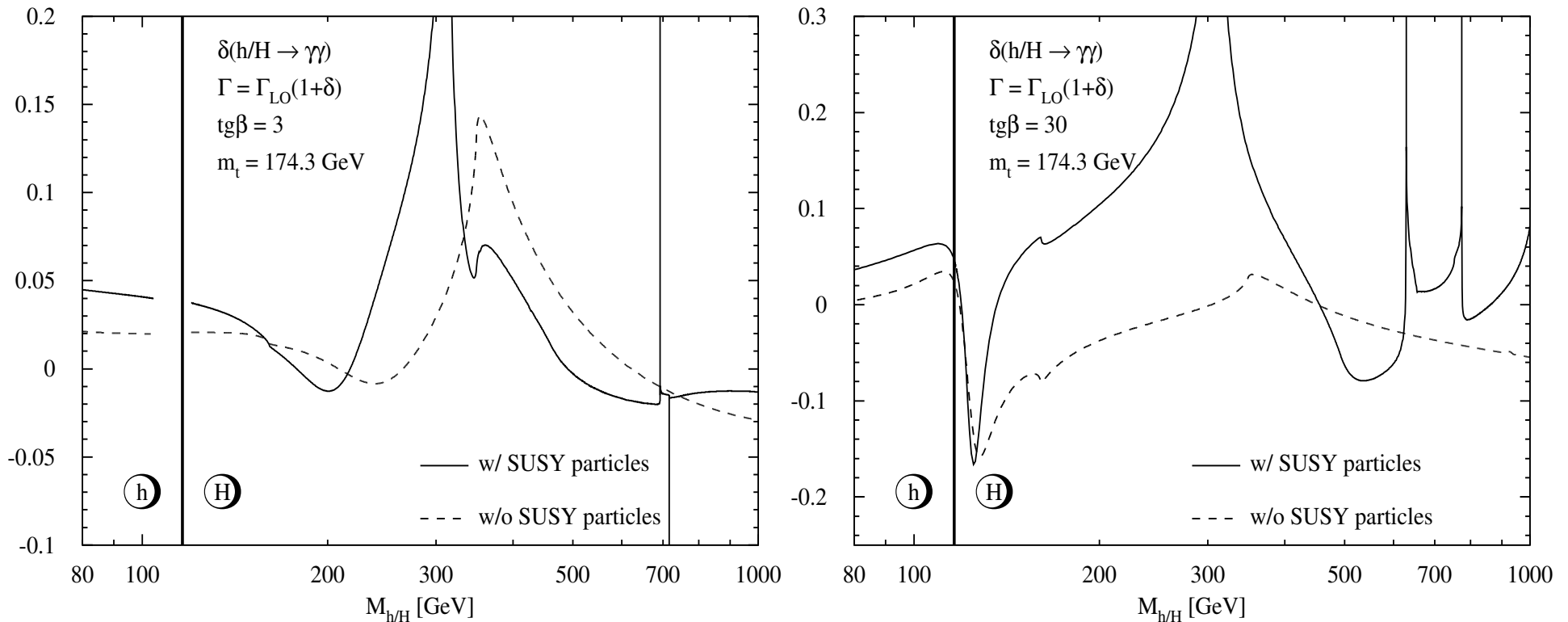
- Improve perturbative behaviour: squark loop contributions in terms of **running squark masses** $m_{\tilde{Q}}(\mu = M_\Phi/2)$, strong coupling $\alpha_S(\mu = M_\Phi)$.
- Massive QCD corrections to quark/squark loops **implemented in HDECAY**. Djouadi, Kalinowski, MMM, Spira

QCD corrected partial decay widths



Kinks, bumps, spikes: $WW, \tilde{t}_1\tilde{t}_1, t\bar{t}, \tilde{b}_1\tilde{b}_1, \tilde{\tau}_1\tilde{\tau}_1, \tilde{\tau}_2\tilde{\tau}_2, \tilde{b}_2\tilde{b}_2$ thresholds in consecutive order with rising Higgs mass.

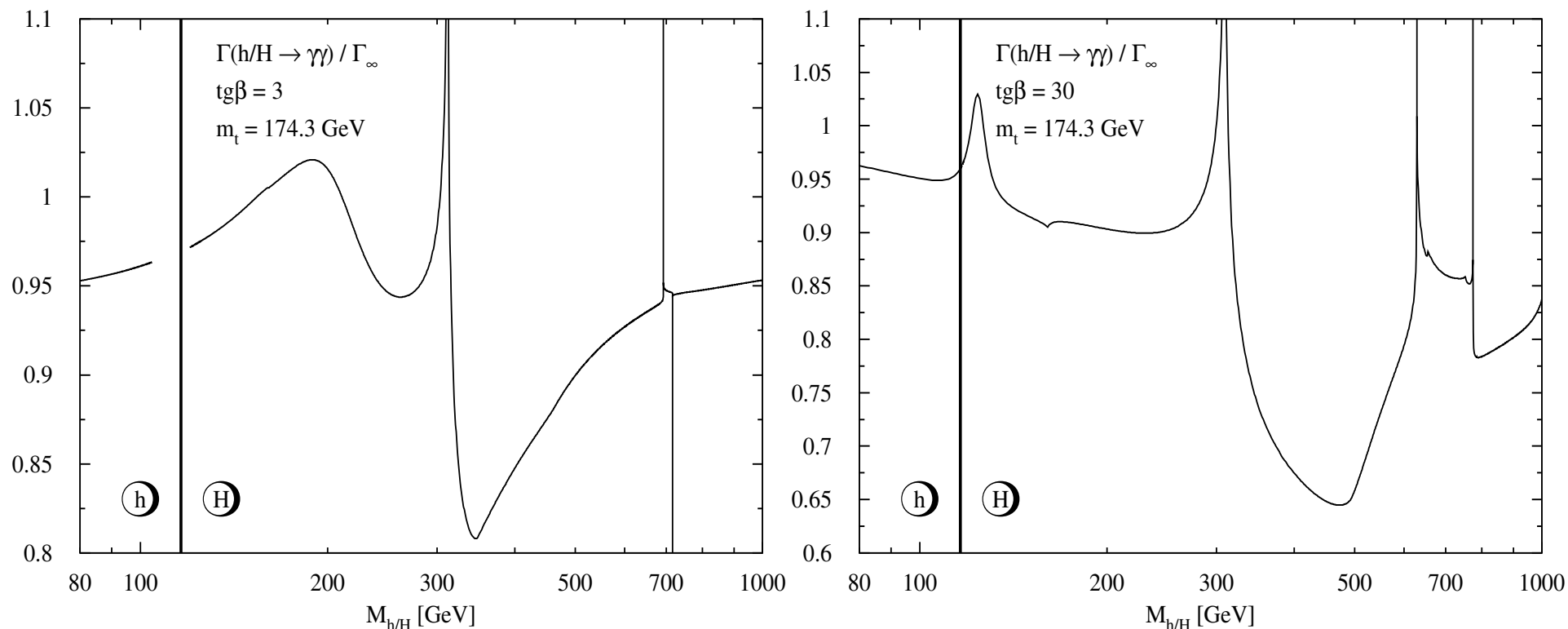
Relative QCD corrections to the partial decay widths



$\delta(h/H \rightarrow \gamma\gamma) \sim 10 - 20\%$

Kinks, bumps, spikes: $WW, \tilde{t}_1\tilde{t}_1, t\bar{t}, \tilde{b}_1\tilde{b}_1, \tilde{\tau}_1\tilde{\tau}_1, \tilde{\tau}_2\tilde{\tau}_2, \tilde{b}_2\tilde{b}_2$ thresholds in consecutive order with $M_{h/H} \uparrow$.

Γ_{NLO} w/ full squark mass dependence / Γ_{NLO} in the heavy squark limit



$\Gamma(h/H \rightarrow \gamma\gamma) / \Gamma_\infty$ up to $\sim 30\%$

Kinks, bumps, spikes: $WW, \tilde{t}_1\tilde{t}_1, t\bar{t}, \tilde{b}_1\tilde{b}_1, \tilde{\tau}_1\tilde{\tau}_1, \tilde{\tau}_2\tilde{\tau}_2, \tilde{b}_2\tilde{b}_2$ thresholds in consecutive order with $M_{h/H} \uparrow$.

Conclusions

- Calculated NLO corrections to $gg \rightarrow h/H$, $h/H \rightarrow \gamma\gamma$ including the full squark mass dependence. [Implemented in HIGLU.]
- K-factor with squarks included is large.
- K-factor very similar to the case of quark loops alone \rightsquigarrow large corrections to squark loops, too.
- Inclusion of full squark mass dependence has significant effects on the K-factor compared to the heavy squark mass limit. The deviation can be as large as $\mathcal{O}(20\%)$ for $gg \rightarrow h/H$,
 $\mathcal{O}(30\%)$ for $h/H \rightarrow \gamma\gamma$.

Note: 2 independent papers by Anastasiou, Beerli, Bucherer, Daleo, Kunszt :
Aglietti, Bonciani, Degrossi, Vicini

Virtual corrections to quark & squark loops in gg fusion derived analytically.

However, no full numerical analysis of the gluon fusion processes at NLO.

Comparison with second group: full agreement.

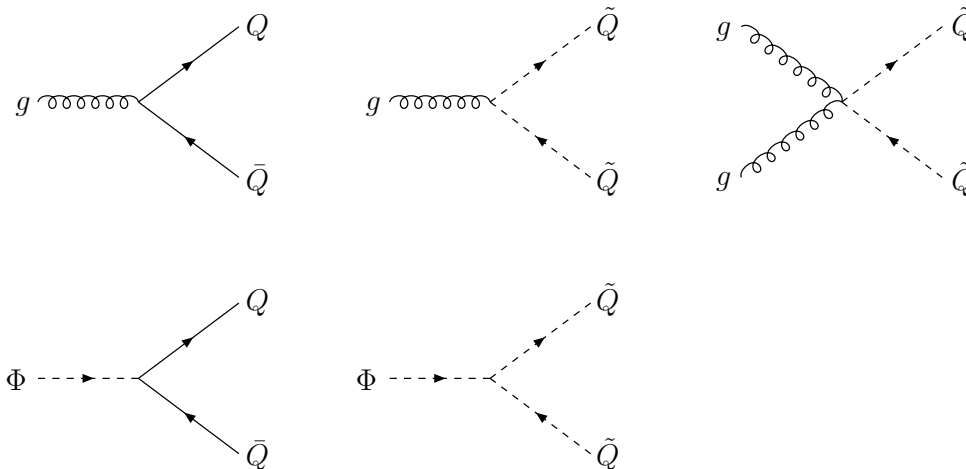
Renormalization

Lagrangian separates gluon and gluino exchange contributions in a renormalizable way

$$\mathcal{L} = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu}^a + -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{Q}(i\not{D} - m_Q)Q + |D_\mu\tilde{Q}|^2 - m_{\tilde{Q}}^2|\tilde{Q}|^2 - g_Q^\Phi \frac{m_Q}{v} \bar{Q}Q\Phi - g_{\tilde{Q}}^\Phi \frac{m_{\tilde{Q}}^2}{v} |\tilde{Q}|^2\Phi + \frac{1}{2}(\partial_\mu\Phi)^2 - \frac{M_\Phi^2}{2}\Phi^2$$

$$iD_\mu = i\partial_\mu - g_S G_\mu^a T^a + ieA_\mu Q$$

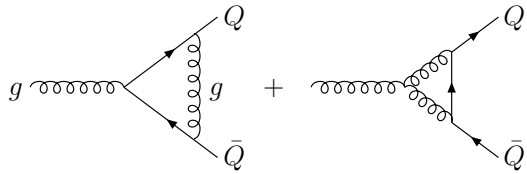
Gluon, $\Phi = \text{H/h}$ interaction vertices:



Renormalization - Suite

- Quark/Squark mass $m_{Q,\tilde{Q}}$: on-shell

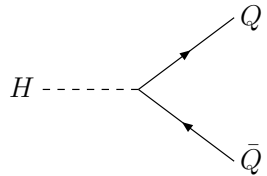
- $gQ\bar{Q}$ vertex:



$$Z_1 = Z_{\alpha_S}^{1/2} Z_3^{1/2} Z_2$$

[Slavnov-Taylor identity]

- $HQ\bar{Q}$ vertex:

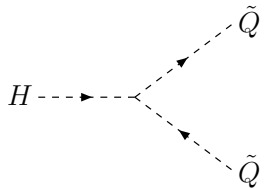


$$\mathcal{L}_{\text{int}} = -g_Q^H \frac{m_{Q_0}}{v} \bar{\Psi}_0 \Psi_0 H = -g_Q^H \frac{m_Q}{v} \bar{\Psi} \Psi H \underbrace{\left[Z_2 - \frac{\delta m_Q}{m_Q} \right]}_{Z_{HQQ}} + \mathcal{O}(\alpha_S^2)$$

$$\Gamma_{H\bar{Q}Q}(q^2 = 0) \neq Z_{HQQ}$$

Braaten, Leveille

- $H\tilde{Q}\tilde{Q}$ vertex:



$$\mathcal{L}_{\text{int}} = -g_{\tilde{Q}}^H \frac{m_{\tilde{Q}_0}^2}{v} \tilde{Q}_0^* \tilde{Q}_0 H = -g_{\tilde{Q}}^H \frac{m_{\tilde{Q}}^2}{v} \tilde{Q}^* \tilde{Q} H \underbrace{\left[Z_2^{\tilde{Q}} - \frac{\delta m_{\tilde{Q}}^2}{m_{\tilde{Q}}^2} \right]}_{Z_{H\tilde{Q}\tilde{Q}}} + \mathcal{O}(\alpha_S^2)$$

$$\Gamma_{H\tilde{Q}\tilde{Q}}(q^2 = 0) \neq Z_{H\tilde{Q}\tilde{Q}}$$

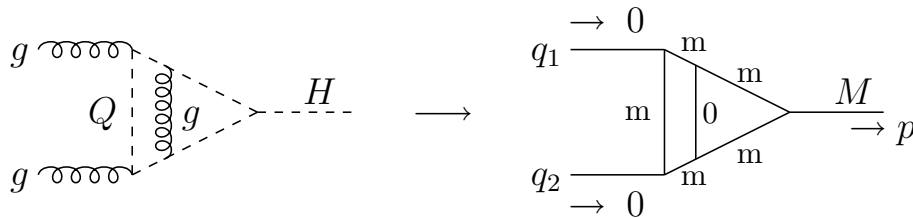
disregard renorm. of $g_{\tilde{Q}}^H$!

General case (arbitrary $M_\Phi, m_Q, m_{\tilde{Q}}$)

- Interference $b, t, \tilde{b}, \tilde{t}$
- 5-dim. Feynman integrals \rightarrow 1-dimensional [Trilogarithms] + purely numerical solution

(analytically: Harlander, Kant
 Anastasiou, Beerli, Bucherer, Daleo, Kunszt
 Aglietti, Bonciani, Degrassi, Vicini \rightarrow Comparison: full agreement)

Example:



$$S = \int \frac{d^n k d^n q}{(2\pi)^{2n}} \frac{1}{(k^2 - m^2)[(k - q_1)^2 - m^2][(k + q_2)^2 - m^2][(k + q - q_1)^2 - m^2][(k + q + q_2)^2 - m^2]q^2}$$

$$= -\frac{\Gamma(2 + 2\epsilon)}{(4\pi)^4 m^4} \left(\frac{4\pi\mu^2}{m^2}\right)^{2\epsilon} \times I$$

$$I = \int_0^1 dx dy dz dr ds \frac{xz}{N^2} \qquad \rho = \frac{M_\Phi^2}{m_{\tilde{Q}}^2} (1 + i0)$$

$$N = 1 + \rho \{ rx(1-x)(1-y-z)(1-y-zs) - [y + (1-y-z)x][1-y-x(1-y-zs)] \}$$