Narrow-width approximation for BSM physics: limitations and improvements

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Narrow-width approximation importance

- accurate LHC/ILC predictions: need N^{n}LO calculations ($n \geq 1$)
- (s)particle decay chains $\longrightarrow$ many-particle final states
- already 4,5,6-leg one-/two-loop calculations tech. very demanding

NWA to the rescue

sub- and nonresonant/nonfactorizable contributions can be neglected in a theoretically consistent way (gauge inv., ...)

- huge calculational simplifications (already at tree level)
- employed in nearly all SM precision calculations & BSM studies
- good NWA uncertainty control crucial

Branching ratio measurement implies NWA (& spin averaging):

$$\text{BR}_X \equiv \frac{\Gamma_X}{\Gamma} = \frac{\sigma_{\text{NWA}}}{\sigma_p} \approx \frac{\sigma_X}{\sum_X \sigma_X}$$

Narrow-width approximation (NWA) recap

$$D(q^2) \equiv \frac{1}{(q^2 - M^2)^2 + (M\Gamma)^2} \quad \sim \quad 2\pi\, K \cdot \delta(q^2 - M^2) \quad \text{for } \Gamma \to 0$$

$$\text{with} \quad K = \frac{1}{2M\Gamma} = \int_{-\infty}^{\infty} \frac{dq^2}{2\pi} D(q^2)$$

$\longrightarrow$ σ approx. decouples into **on-shell** production σ_p and decay Γ_X
(multiple resonances and spin/polarization straightforward)

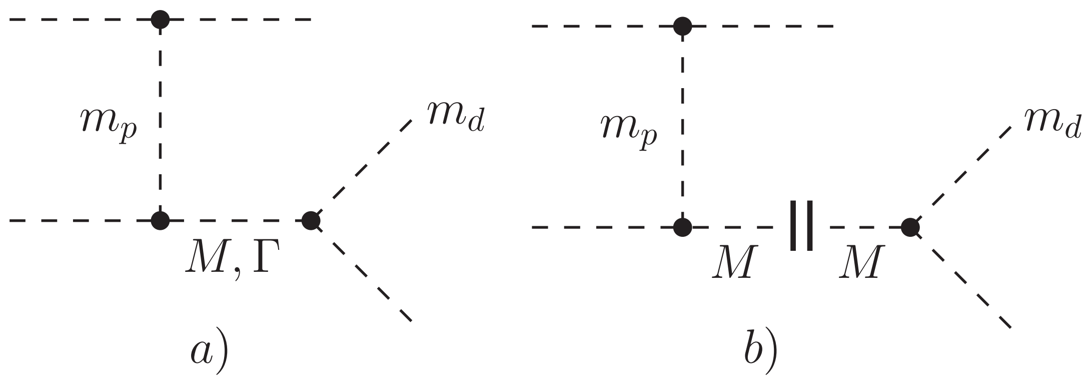
$$\sigma = \frac{1}{2s} \int_{q^2_{\text{min}}}^{q^2_{\text{max}}} \frac{dq^2}{2\pi} \int d\phi_p |\mathcal{M}_p(q^2)|^2 D(q^2) \int d\phi_d |\mathcal{M}_d(q^2)|^2$$

$$\approx \sigma_{\text{NWA}} \equiv \frac{1}{2s} \int d\phi_p |\mathcal{M}_p(M^2)|^2 K \int d\phi_d |\mathcal{M}_d(M^2)|^2$$

Scales occurring in $D(q^2) \longrightarrow$ error estimate $\mathcal{O}(\Gamma/M)$

Limitations of the $\mathcal{O}(\Gamma/M)$ uncertainty estimate

Relative deviation $R \equiv \sigma/\sigma_{\text{NWA}} - 1 = R^{(1)} + \mathcal{O}(\Gamma^2)$ for scalar process:

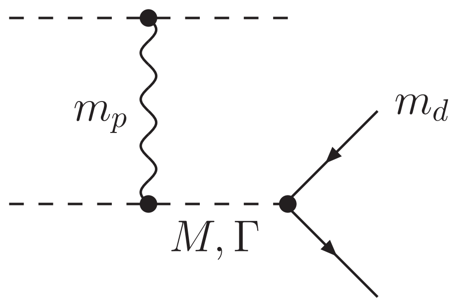


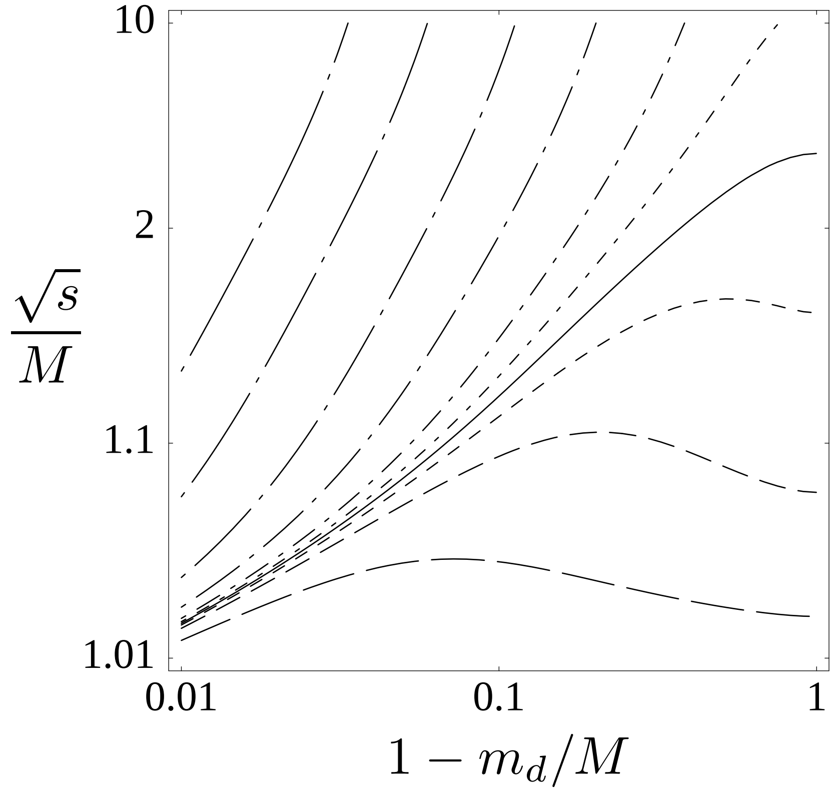
$$R^{(1)} = \Bigg\{ \frac{M\left(s-m_d^2\right)}{\pi\left(m_d^2-M^2\right)\left(s-M^2\right)} + \left[\pi M\left(m_d^2-M^2\right)\left(s-M^2\right)\left(s+m_p^2\right)\left(s-M^2+m_p^2\right)\right]^{-1} \Bigg[m_d^2\, s\left(s-M^2+m_p^2\right)^2 \ln\frac{s}{m_d^2}$$

$$+\left(s+m_p^2\right)\left(m_d^2\left(s\left(s+m_p^2\right)-2M^2 s+M^4\right)-M^4 m_p^2\right) \ln\frac{M^2-m_d^2}{s-M^2} + M^4 m_p^2\left(s-m_d^2+m_p^2\right) \ln\frac{s-m_d^2+m_p^2}{m_p^2} \Bigg] \Bigg\} \cdot \mathbf{\Gamma}$$

additional scales: $\sqrt{s}$, particle masses in production or decay, selection cuts, ...

$\{\ldots\} \overset{?}{\approx} \mathbf{1}/\boldsymbol{M}$:   Not when $\sqrt{s} \to M$ (prod. threshold ✓) or $\boldsymbol{m_d \to M}$





Contour lines for

$$\boldsymbol{R/(\Gamma/M) \in \{-10, -3, -1, 0, 1, 3, 10, 30, 100, 300\}}$$

with $\Gamma/M = 0.01$ and $m_p = 0.02\,M$.

Dash length increases with magnitude.

Origin of amplified deviations $\longrightarrow$ improvements

Off-shell vs. NWA cross section:

$$\sigma = \frac{1}{2s}\int \frac{dq^2}{2\pi}\, D(q^2)\int d\phi_p(q^2)\int d\phi_d(q^2)\, |\mathcal{M}_r(q^2)|^2$$

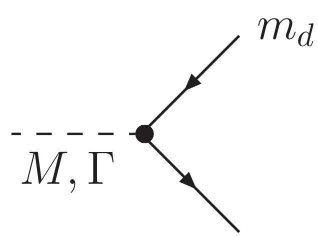
$$\sigma_{\text{NWA}} = \frac{1}{2s} K_{\text{NWA}} \int d\phi_p(M^2)\int d\phi_d(M^2)\, |\mathcal{M}_r(M^2)|^2$$

with $K_{\text{NWA}} = \frac{1}{2M\Gamma} = \int_{-\infty}^{\infty} \frac{dq^2}{2\pi}\, D(q^2)$

Deforming factors in phase space element and $|\mathcal{M}_d|^2$:

$$\frac{d\phi_{2d}}{d\Omega^*} = \frac{1}{32\pi^2}\sqrt{\left[1-\frac{(m_1+m_2)^2}{q^2}\right]\left[1-\frac{(m_1-m_2)^2}{q^2}\right]}$$

$$= \frac{1}{32\pi^2}\,\frac{q^2-m_d^2}{q^2} \quad \text{for } m_d \equiv m_1, m_2 = 0$$

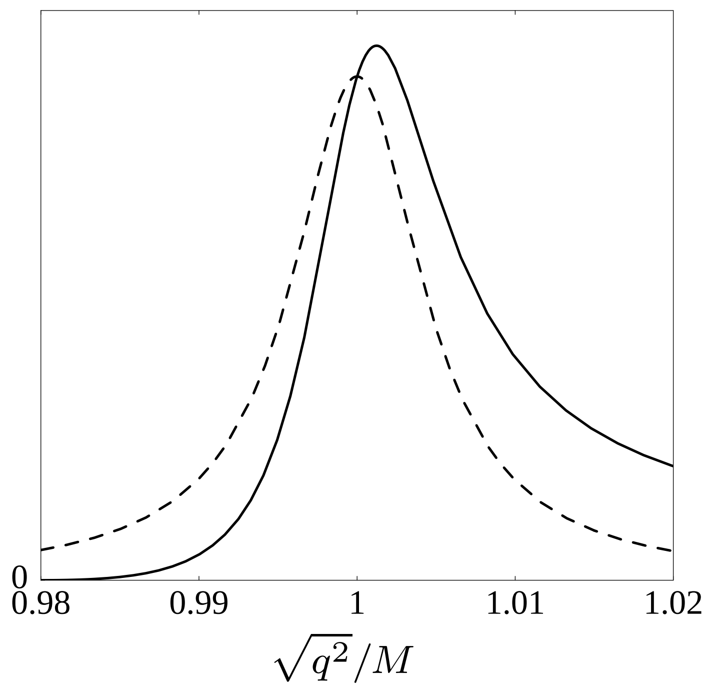


Strategy for improvements: absorb amplifying factors into K

Breit-Wigner shape deformation by threshold factors

$$R_2' \equiv K_{\text{INWA}}/K_{\text{NWA}} =$$

$$\left(\int_{m^2}^{q^2_{\text{max}}} \frac{dq^2}{2\pi} \frac{1}{(q^2-M^2)^2+(M\,\Gamma)^2}\, \frac{(q^2-m^2)^2/q^2}{(M^2-m^2)^2/M^2}\right) \Big/ \left(\int_{-\infty}^{\infty} \frac{dq^2}{2\pi} \frac{1}{(q^2-M^2)^2+(M\,\Gamma)^2}\right)$$



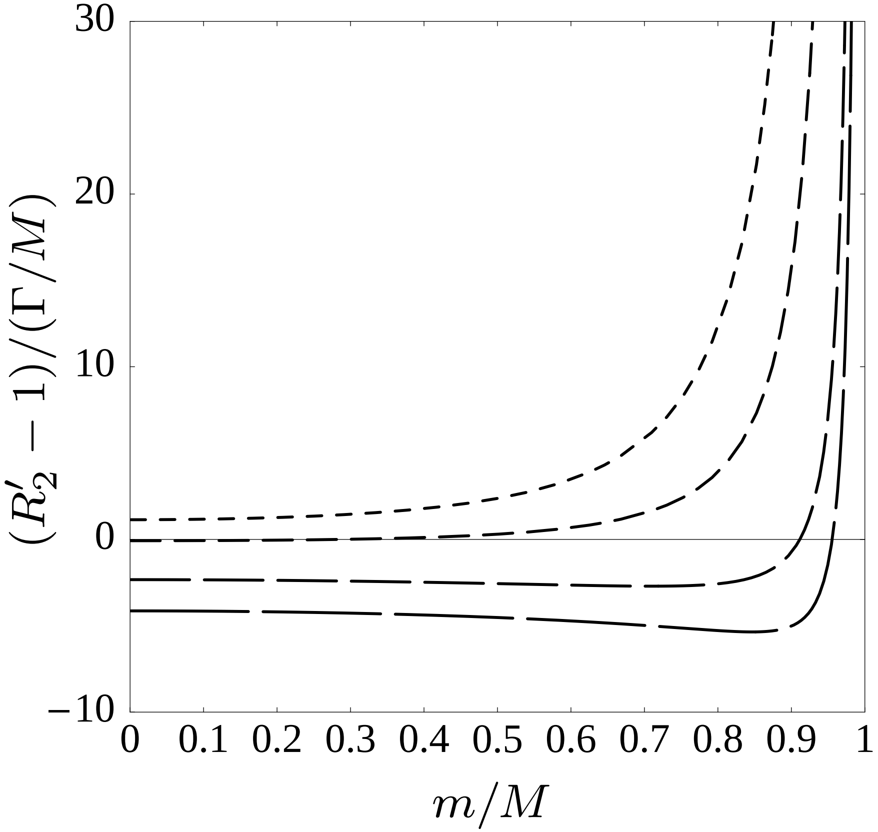
Integrand of numerator (solid) and denominator (dashed)

for $m = M - 2\Gamma$ and $\Gamma/M = 1\%$

Explicitly

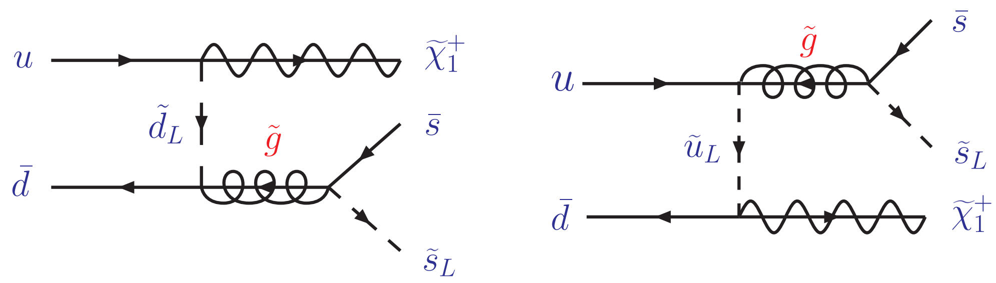
$$R_2' = \frac{1}{\pi}\left[\tan^{-1}\frac{\beta^2}{\gamma} + \tan^{-1}\frac{\lambda}{\gamma}\right] + \frac{\gamma}{\pi}\left[\left(\frac{2}{\beta^2}-1\right)\ln\frac{\lambda}{\beta^2} + \left(\frac{1}{\beta^2}-1\right)^2 \ln\frac{q_{\text{max}}^2}{m^2}\right]$$

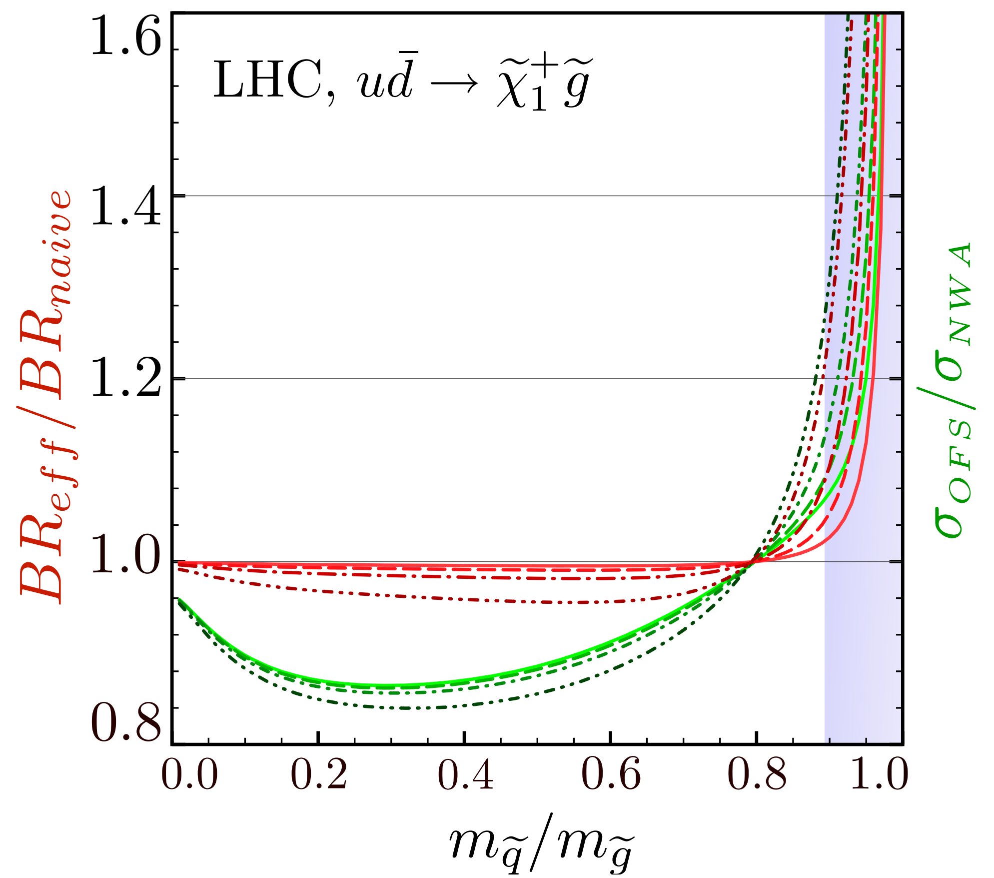
with $\gamma \equiv \Gamma/M$, $\beta = \sqrt{1-m^2/M^2}$ and $\lambda \equiv q_{\text{max}}^2/M^2 - 1$



$\sqrt{q_{\text{max}}^2}/M \in \{1.05, 1.1, 2, 10\}$, dash length decreases with increasing q_{max}^2, $\Gamma/M = 1\%$

Example in Minimal Supersymmetric Standard Model

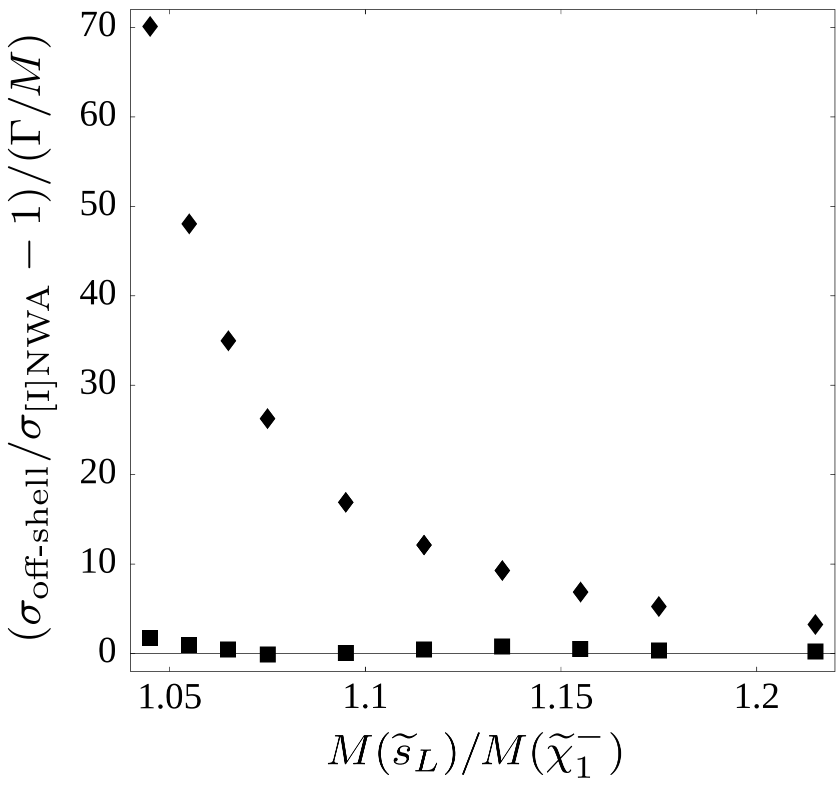


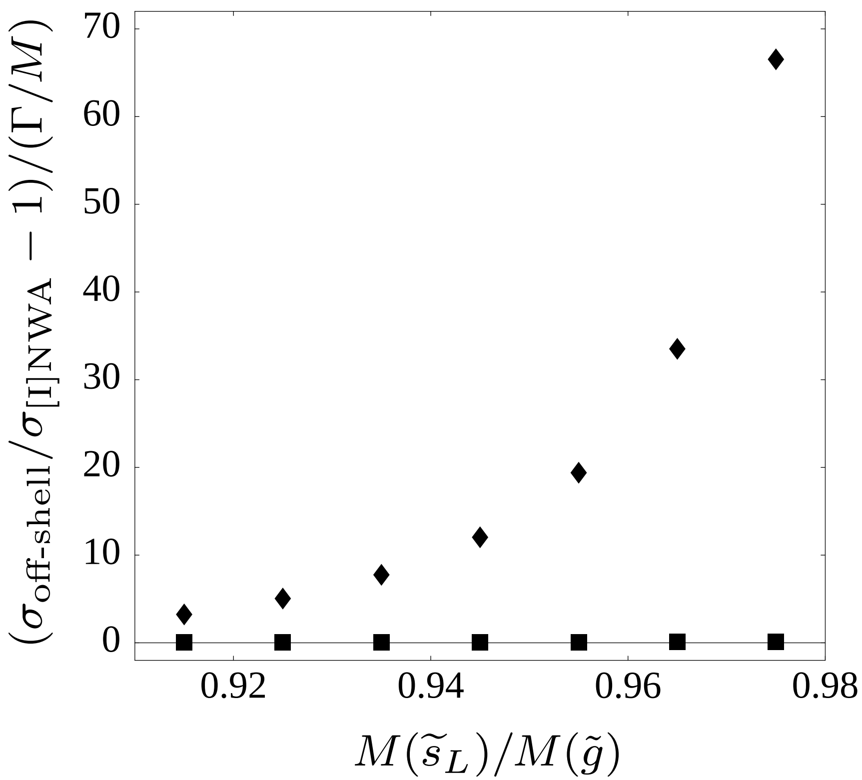


$\Gamma(\widetilde{g})/M(\widetilde{g})$ $\approx$ 6 GeV/600 GeV = 1%, (SPS1a)

Application to MSSM cascade decay

$pp \to \tilde{g}\,\widetilde{u}_L$ followed by the cascade decay $\tilde{g} \to (\widetilde{s}_L \to \widetilde{\chi}_1^- c)\bar{s}$





Variable strange squark mass approaching the chargino (left), gluino mass (right)

Deviations for the standard NWA (diamonds) and the improved NWA (boxes)

LHC, (SPS1a)

Conclusions and Outlook

Standard narrow-width approximation not reliable

- when production close to threshold dominates
- for decay chains with similar intermediate masses

General approach exists to improve NWA in these cases

Interesting questions:

- How much better is *leading-pole approximation*?
- Can mass determination be affected by peak shift?