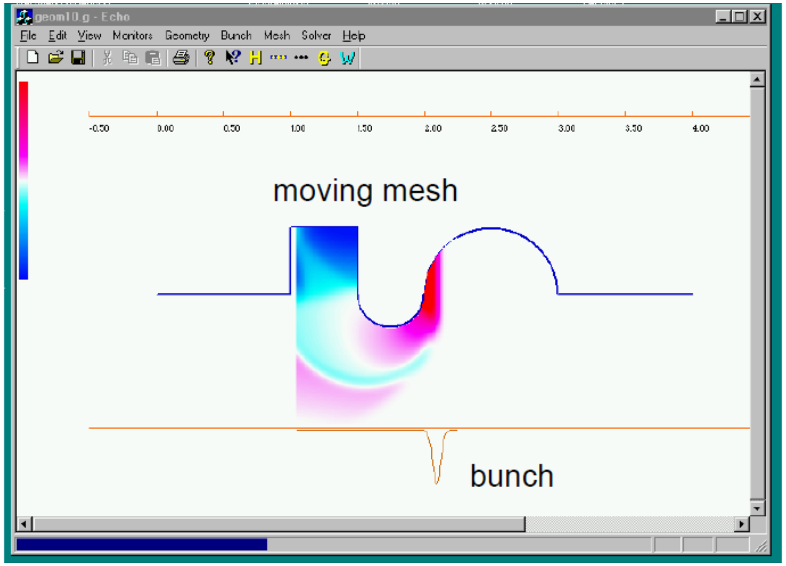
Status of the ECHO Code

Algorithms and Applications



Igor Zagorodnov

HOMSC2025 Workshop

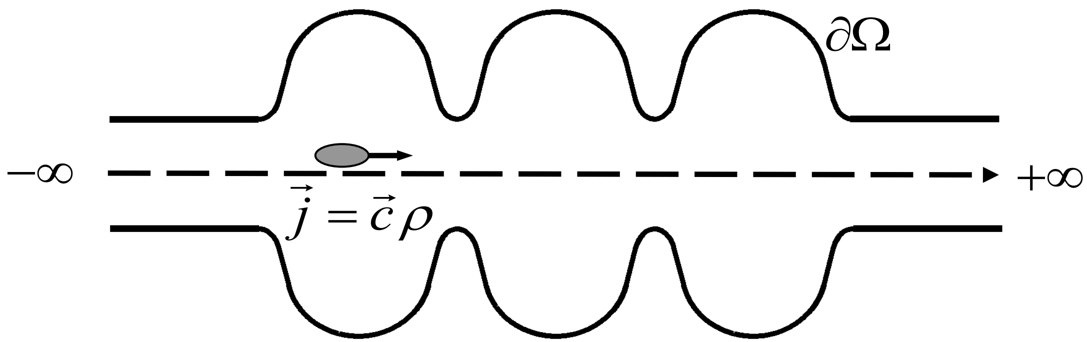
Hamburg
7 October 2025

HELMHOLTZ GEMEINSCHAFT

DESY

Motivation

Wake field calculation – estimation of the effect of the geometry variations on the bunch



Wake potential

$$W_{\parallel}(\mathbf{r}_0, \mathbf{r}, s) = \frac{1}{Q} \int_{-\infty}^{\infty} E_z\left(\mathbf{r}_0, \mathbf{r}, z, \frac{z-s}{c}\right) dz$$

$$\frac{\partial}{\partial s} \mathbf{W}_{\perp}(\mathbf{r}_0, \mathbf{r}, s) = \nabla_{\perp} W_{\parallel}(\mathbf{r}_0, \mathbf{r}, s)$$

First codes in **time domain** ~ 1980

A. Novokhatski (BINP),

T. Weiland (CERN)

Motivation (in year 2000)

MAFIA*

- rotationally symmetric and 3D
- longitudinal and transverse wakes
- triangular geometry approximation;
- arbitrary materials
- dispersion error

NOVO**

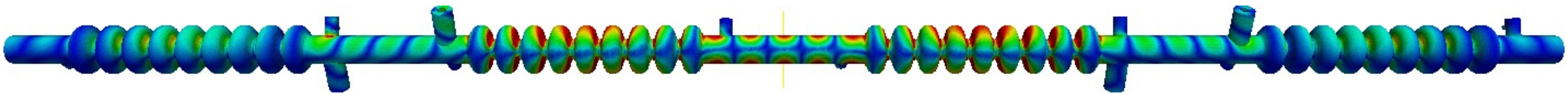
- rotationally symmetric
- only longitudinal wake
- "staircase" geometry approximation;
- only PEC
- low dispersion error

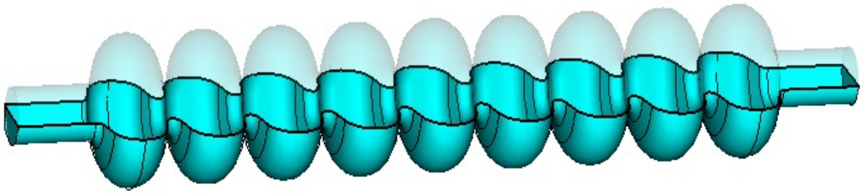
* MAFIA Collaboration, MAFIA Manual, CST GmbH, Darmstadt, 1997.

** A. Novokhatski, M. Timm, T. Weiland, *Transition Dynamics of the Wake Fields of Ultra Short Bunches*. Proc. of the ICAP 1998, Monterey, California, USA.

Motivation

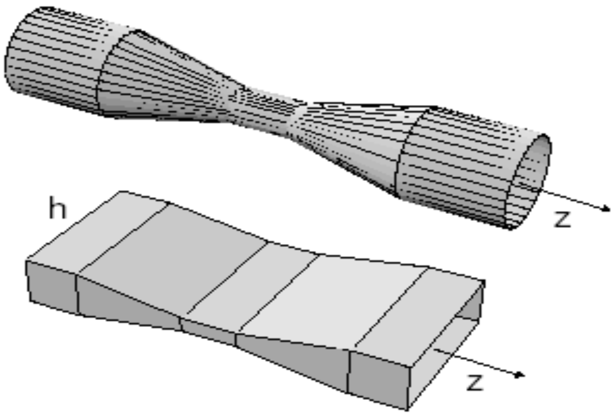
(Picture from W.Ackermann, TU Darmstadt, 2012)





New projects with

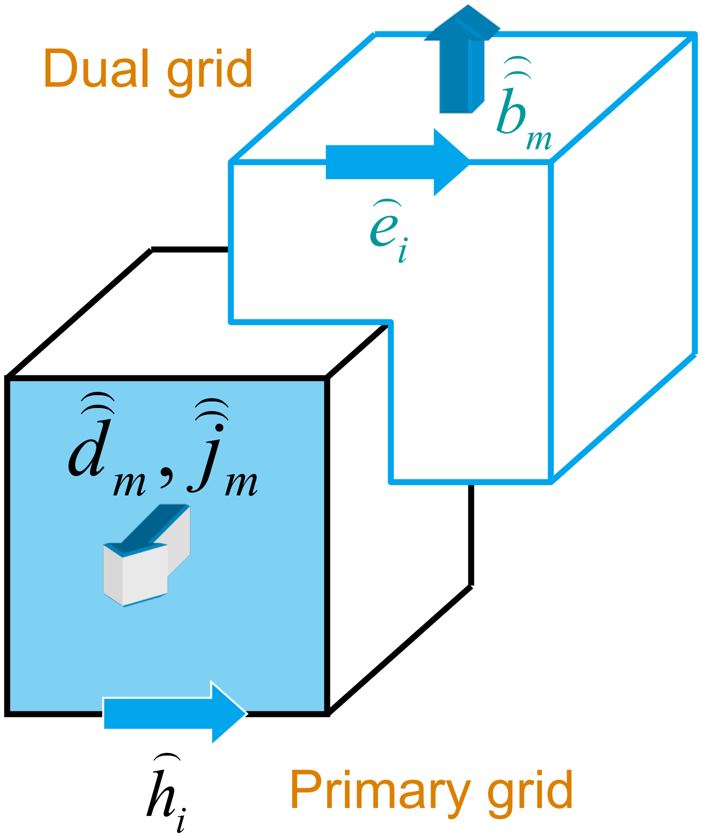
- short bunches;
- long structures;
- tapered collimators



Solutions

- zero dispersion in longitudinal direction;
- “conformal” meshing;
- moving mesh and “explicit” or “split” methods

Low-dispersive schemes



Maxwell Grid Equations*

$$\mathbf{C}\widehat{\mathbf{e}} = -\frac{d}{dt}\widehat{\widehat{\mathbf{b}}}\ , \qquad \tilde{\mathbf{C}}\widehat{\mathbf{h}} = \frac{d}{dt}\widehat{\widehat{\mathbf{d}}} + \widehat{\widehat{\mathbf{j}}},$$

$$\mathbf{S}\widehat{\widehat{\mathbf{b}}} = \mathbf{0}, \qquad \tilde{\mathbf{S}}\widehat{\widehat{\mathbf{d}}} = \mathbf{q},$$

$$\widehat{\mathbf{e}} = \mathbf{M}_{\varepsilon^{-1}}\widehat{\widehat{\mathbf{d}}}\ , \qquad \widehat{\mathbf{h}} = \mathbf{M}_{\mu^{-1}}\widehat{\widehat{\mathbf{b}}}$$

$$curl \sim \mathbf{C} = \begin{pmatrix} \mathbf{0} & -\mathbf{P}_z & \mathbf{P}_y \\ \mathbf{P}_z & \mathbf{0} & -\mathbf{P}_x \\ -\mathbf{P}_y & \mathbf{P}_x & \mathbf{0} \end{pmatrix}$$

* T. Weiland, *A discretization method for the solution of Maxwell's equations for six-component fields*, Electronics and Communication (AEÜ) **31**, p. 116 (1977)

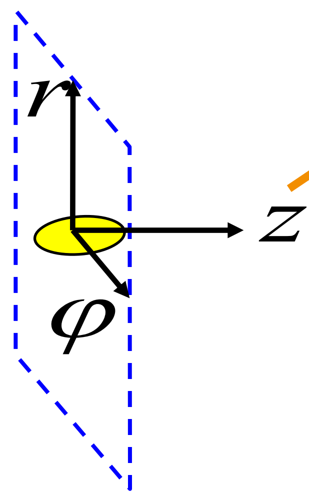
Low-dispersive schemes

$$\mathbf{C} = \begin{pmatrix} 0 & -\mathbf{P}_z & \mathbf{P}_\varphi \\ \mathbf{P}_z & 0 & -\mathbf{P}_r \\ -\mathbf{P}_\varphi & \mathbf{P}_r & 0 \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} 0 & 0 & \mathbf{P}_\varphi \\ 0 & 0 & -\mathbf{P}_r \\ -\mathbf{P}_\varphi & \mathbf{P}_r & 0 \end{pmatrix}$$

$$\mathbf{L} = \begin{pmatrix} 0 & -\mathbf{P}_z & 0 \\ \mathbf{P}_z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

transversal plane



longitudinal direction

Low-dispersive schemes

FDTD (1966)

$$\frac{\mathbf{e}^{n+1/2}-\mathbf{e}^{n-1/2}}{\Delta t}=\mathbf{M}_{\varepsilon^{-1}}(\mathbf{C}^{*}\mathbf{h}^{n}-\mathbf{j}^{n})$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^{n}}{\Delta t}\mathbf{h}^{n}=\mathbf{M}_{\mu^{-1}}\mathbf{C}\mathbf{e}^{n+1/2}$$

$$\mathbf{h}^{n}=\begin{pmatrix}\mathbf{h}_x^n\\ \mathbf{h}_y^n\\ \mathbf{h}_z^n\end{pmatrix}\qquad \mathbf{e}^{n+0.5}=\begin{pmatrix}\mathbf{e}_x^{n+0.5}\\ \mathbf{e}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

Implicit Scheme* (2002)

$$\frac{\mathbf{e}^{n+1/2}-\mathbf{e}^{n-1/2}}{\Delta t}=\mathbf{M}_{\varepsilon^{-1}}(\mathbf{T}^{*}\overline{\mathbf{h}}^{n}+\mathbf{L}^{*}\mathbf{h}^{n}-\mathbf{j}^{n})$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^{n}}{\Delta t}\mathbf{h}^{n}=\mathbf{M}_{\mu^{-1}}\mathbf{C}\mathbf{e}^{n+1/2}$$

$$\overline{\mathbf{h}}^{n}\equiv\theta\mathbf{h}^{n+1}+(1-2\theta)\mathbf{h}^{n}+\theta\mathbf{h}^{n-1}$$

* I. Zagorodnov, R. Schuhmann, T. Weiland, *Long-Time Numerical Computation of Electromagnetic Fields in the Vicinity of a Relativistic Source*, Journal of Computational Physics **191**, No.2 , pp. 525-541 (2003)

Low-dispersive schemes

FDTD (1966)

$$\frac{\mathbf{e}^{n+0.5}-\mathbf{e}^{n-0.5}}{\Delta\tau}=\mathbf{C}_0{}^*\mathbf{h}^n+\mathbf{j}^n,$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^n}{\Delta\tau}=-\mathbf{C}_0\mathbf{e}^{n+0.5}$$

$$\mathbf{h}^n=\begin{pmatrix}\mathbf{h}_x^n\\ \mathbf{h}_y^n\\ \mathbf{h}_z^n\end{pmatrix}\qquad \mathbf{e}^{n+0.5}=\begin{pmatrix}\mathbf{e}_x^{n+0.5}\\ \mathbf{e}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

dispersion error

TE/TM splitting (2004)

$$\frac{\mathbf{u}^{n+0.5}-\mathbf{u}^{n-0.5}}{\Delta\tau}=\mathbf{T}_0\frac{\mathbf{u}^{n+0.5}+\mathbf{u}^{n-0.5}}{2}+\mathbf{L}\mathbf{v}^n+\mathbf{j}_u^n,$$

$$\frac{\mathbf{v}^{n+1}-\mathbf{v}^n}{\Delta\tau}=\mathbf{T}_1\frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2}-\mathbf{L}^*\mathbf{u}^{n+0.5}+\mathbf{j}_v^{n+0.5}$$

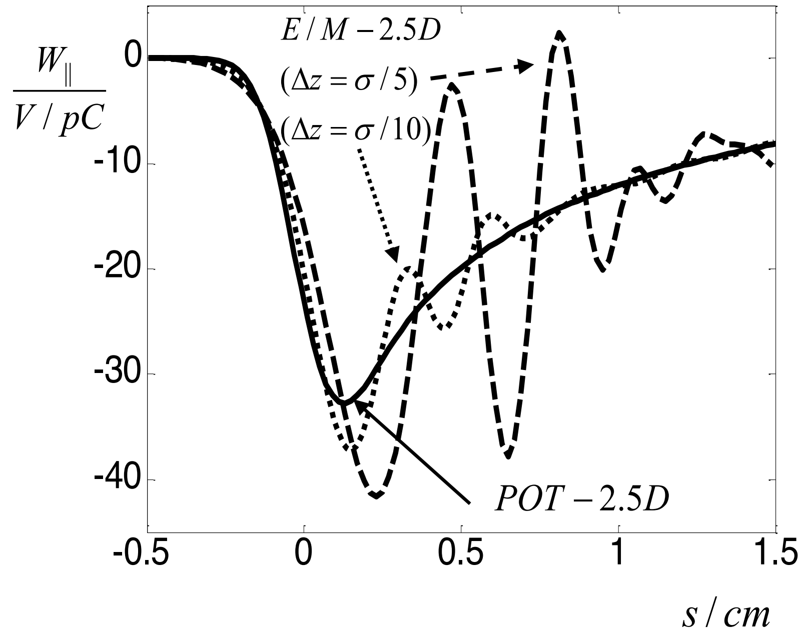
$$\mathbf{v}^n=\begin{pmatrix}\mathbf{e}_x^n\\ \mathbf{e}_y^n\\ \mathbf{h}_z^n\end{pmatrix}\qquad \mathbf{u}^{n+0.5}=\begin{pmatrix}\mathbf{h}_x^{n+0.5}\\ \mathbf{h}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

dispersion error suppressed for $\Delta\tau=\Delta z$

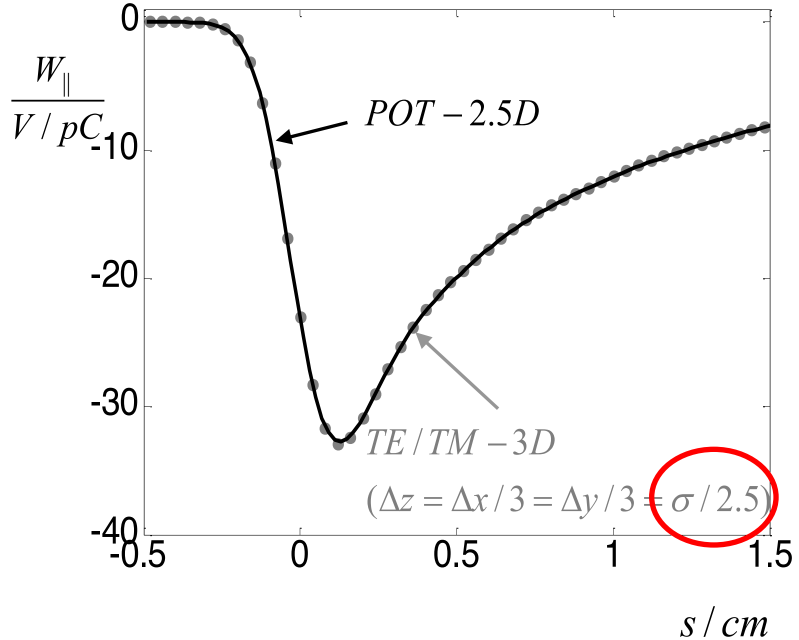
* Zagorodnov I.A, Weiland T., *TE/TM Field Solver for Particle Beam Simulations without Numerical Cherenkov Radiation,* Phys. Rev. ST Accel. Beams **8**, 042001 (2005)

Low-dispersive schemes

FDTD



ECHO-3D (TE/TM)



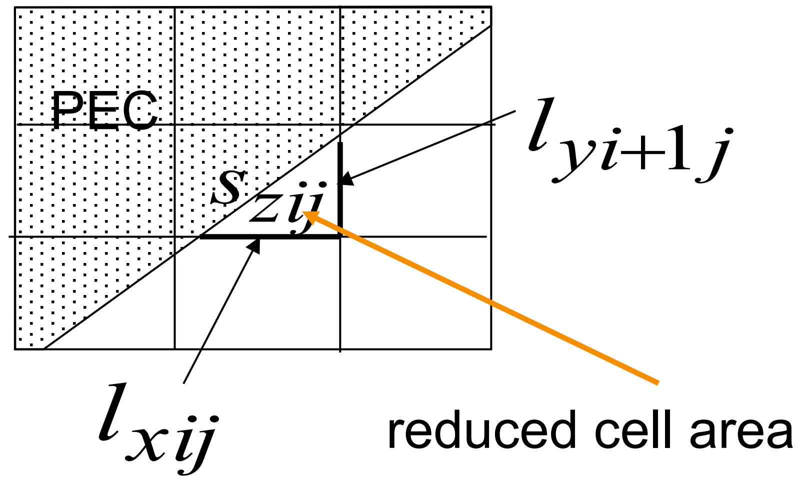
Comparison of the wake potentials for structure consisting of 20 TESLA cells excited by Gaussian bunch with RMS length $\sigma = 1$ mm

E/M splitting $\Delta z \sim \sqrt{\frac{\sigma^3}{L}}$

TE/TM splitting $\Delta z \sim \sigma$

Boundary approximation

Standard Conformal Scheme



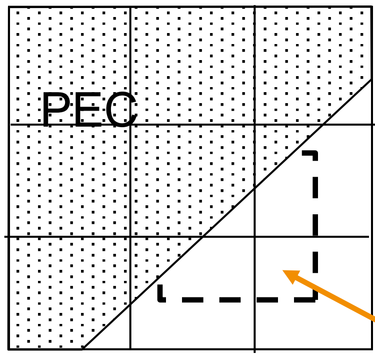
time step must be reduced

Dey S, Mittra R. A locally conformal finite-difference time-domain (FDTD) algorithm for modeling three-dimensional perfectly conducting objects. IEEE Microwave and Guided Wave Letters 7(9):273–275 (1997)

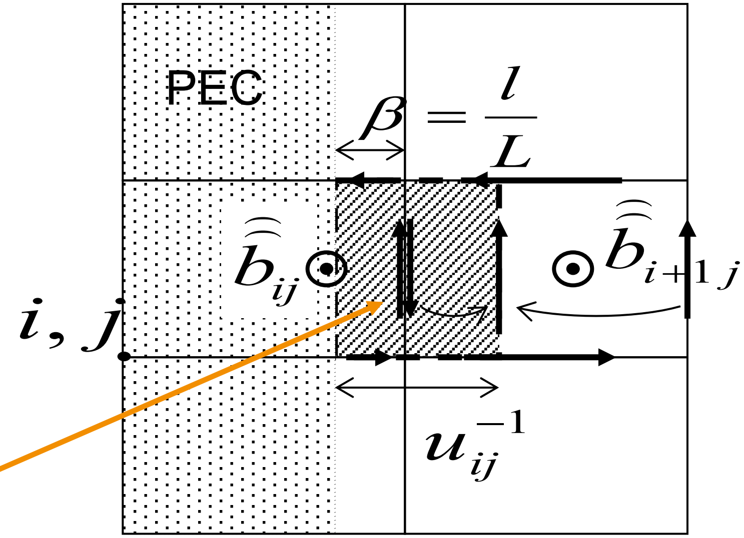
Thoma P. Zur numerischen Lösung der Maxwellschen Gleichungen im Zeitbereich. Dissertation DI7: TH Darmstadt,1997.

Boundary approximation

New Conformal Scheme* (**2002**)



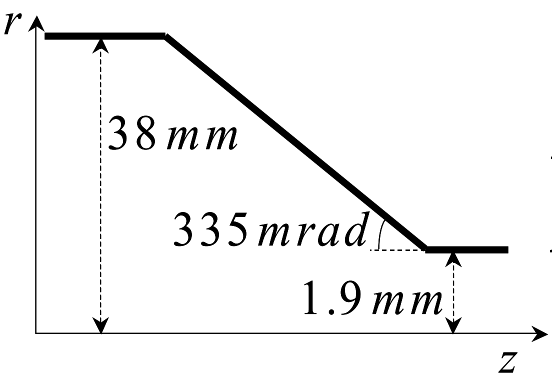
virtual cell

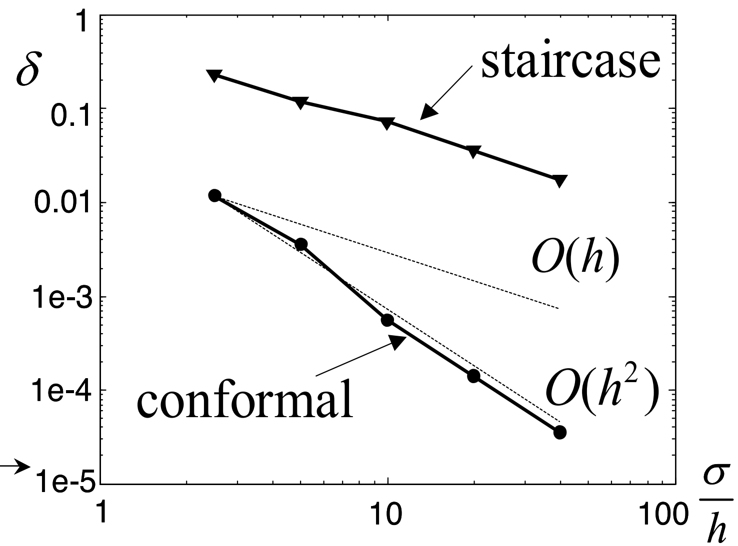


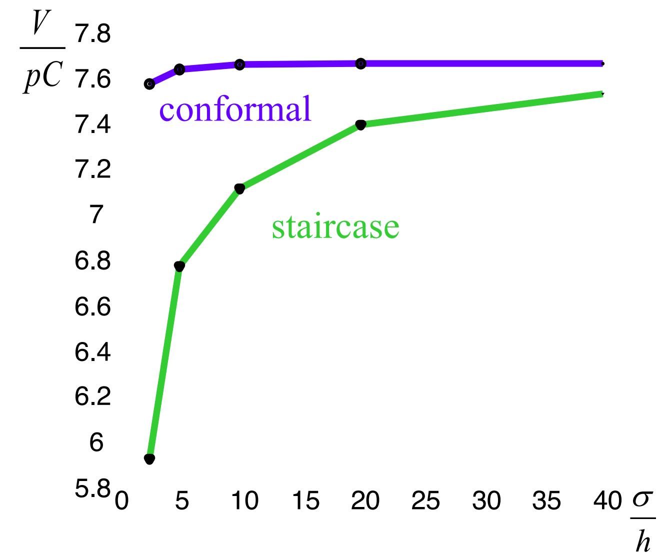
Time step is not reduced!

* Zagorodnov I., Schuhmann R.,Weiland T., *A Uniformly Stable Conformal FDTD-Method on Cartesian Grids*, International Journal on Numerical Modeling, vol. 16, No.2, pp. 127-141 (2003)

Boundary approximation





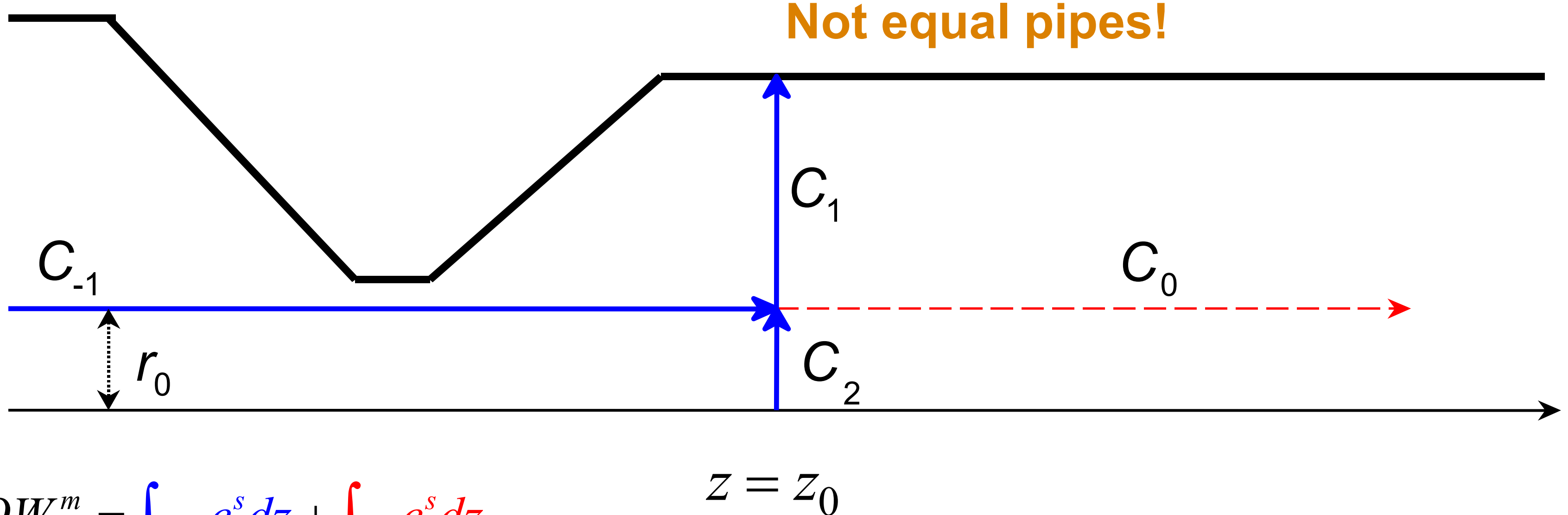


Error in loss factor for a taper $\delta = \left| L_{calc} - L \right| L^{-1}$

The error δ relative to the extrapolated loss factor L=-7.63777 Vp/C for bunch with σ=1 mm is shown.

Indirect Integration Algorithm

Not equal pipes!



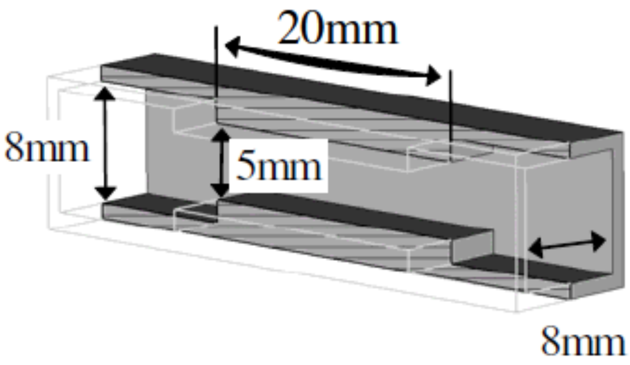
$$QW_{\parallel}^{m} = \int_{C_{-1}} e_z^s dz + \int_{C_0} e_z^s dz$$

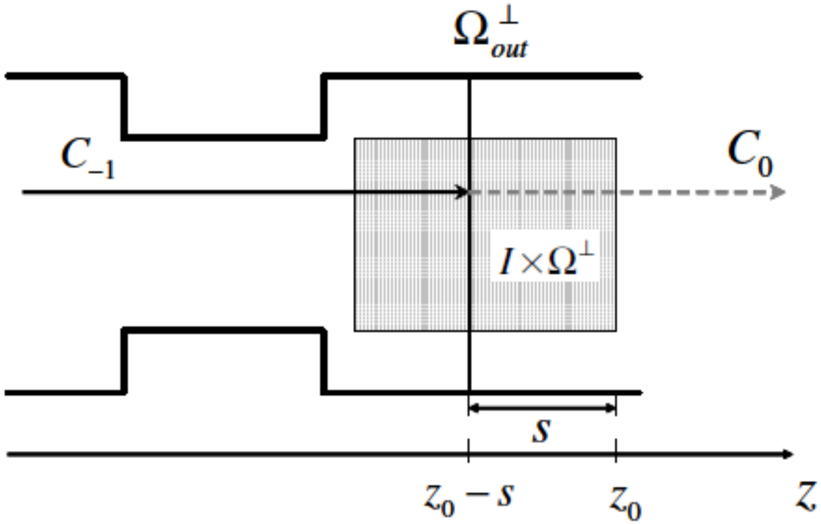
$$\int_{C_0} e_z^s dz = -\frac{1}{2}\left(\int_{C_1} \left(r_0^m \omega_D + r_0^{-m} \omega_S - \frac{\beta}{a^m} \omega_S \right) - \frac{\beta}{a^m} \int_{C_2} \omega_S \right)$$

I. Zagorodnov, R. Schuhmann, T. Weiland, Journal of Computational Physics **191**, No.2 , pp. 525-541 (2003)

O. Napoly, Y. Chin, and B. Zotter, Nucl. Instrum. Methods Phys. Res., Sect. A **334**, 255 (1993)

Indirect Integration Algorithm





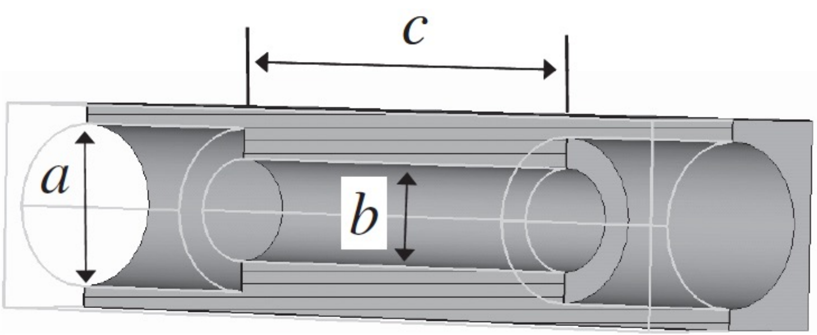
$$QW_{\parallel}(\vec{r}_0, s) = -\int_{C_{-1}(\vec{r}_0, z_0 - s)} E_z^{sc}[\vec{r}_0, z, t(z, s)]dz - u(\vec{r}_0, s), \qquad t(z, s) = \frac{z + s}{c}$$

$$\Delta u(\vec{r}, s) = -\left[\frac{\partial}{\partial s} + \frac{\partial}{c\,\partial t}\right] E_z^{sc}(\vec{r}, z_0 - s, t_0), \qquad \vec{r} \in \Omega^{\perp}_{\text{out}}, \qquad u(\vec{r}, s) = 0, \qquad \vec{r} \in \partial\Omega^{\perp}_{\text{out}}.$$

Zagorodnov I., *Indirect Methods for Wake Potential Integration,* Phys. Rev. STAB **9**, 102002 (2006)

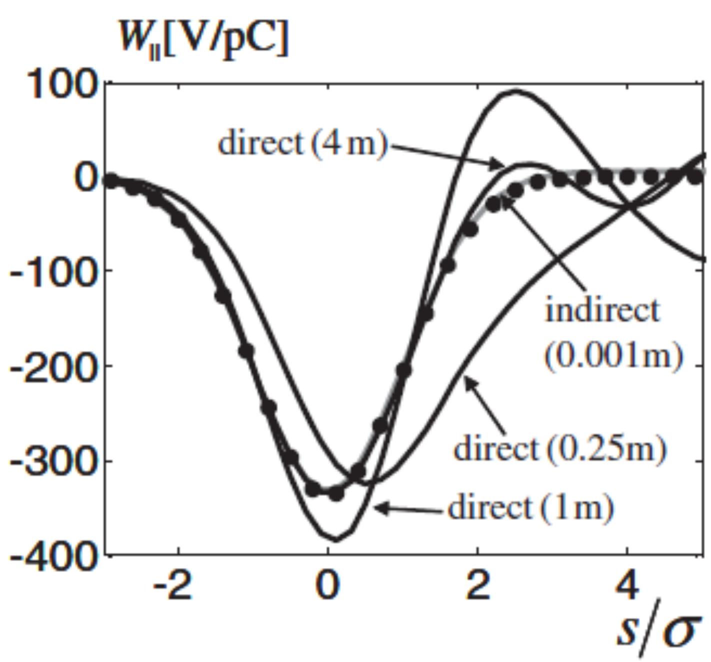
H. Henke and W. Bruns, in Proceedings of EPAC 2006, Edinburgh, Scotland (WEPCH110, 2006)

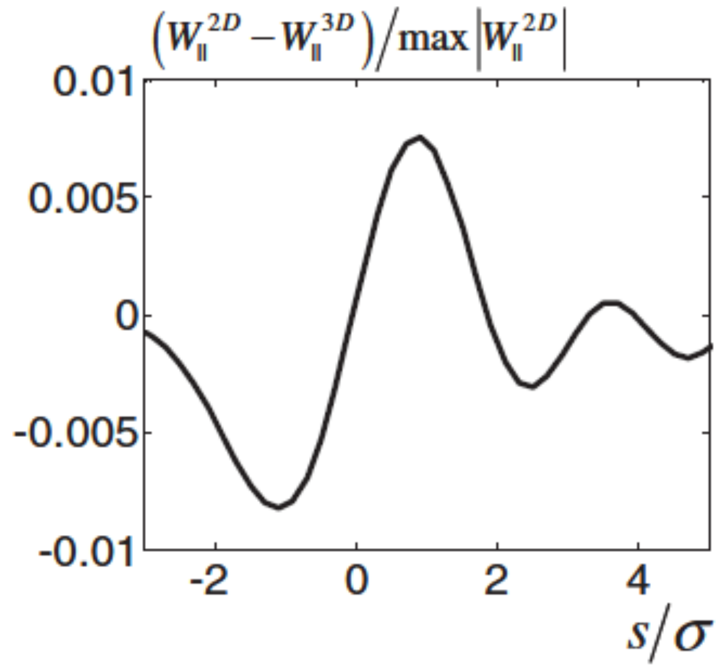
Indirect Integration Algorithm



$a = 8$ mm, $b = 5$ mm, and $c = 20$ mm

Gaussian bunch with rms length 25 µm





Modelling of Conductive Walls

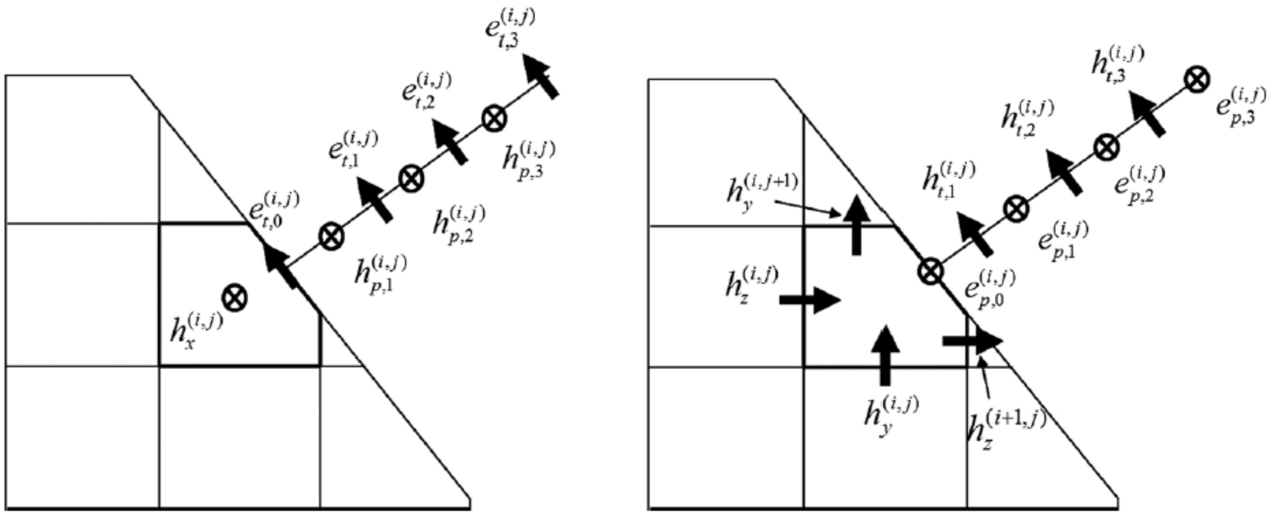


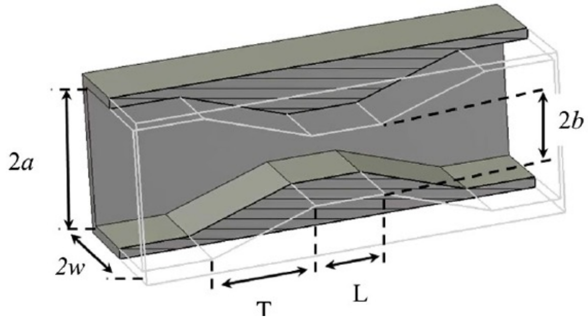
FIG. 6. A boundary cell in the vacuum and 1D conductive line in the metal.

Thoma P. Zur numerischen Lösung der Maxwellschen Gleichungen im Zeitbereich. Dissertation DI7: TH Darmstadt,1997.

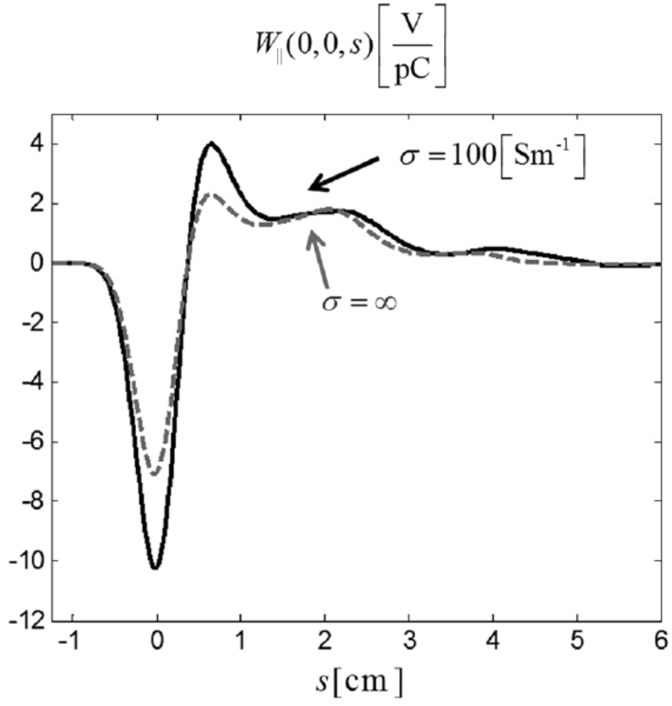
Tsakanian A., Dohlus M., Zagorodnov I., *Hybrid TE-TM scheme for time domain numerical calculations of wakefields in structures with walls of finite conductivity,* Phys. Rev. STAB **15**, 054401 (2012)

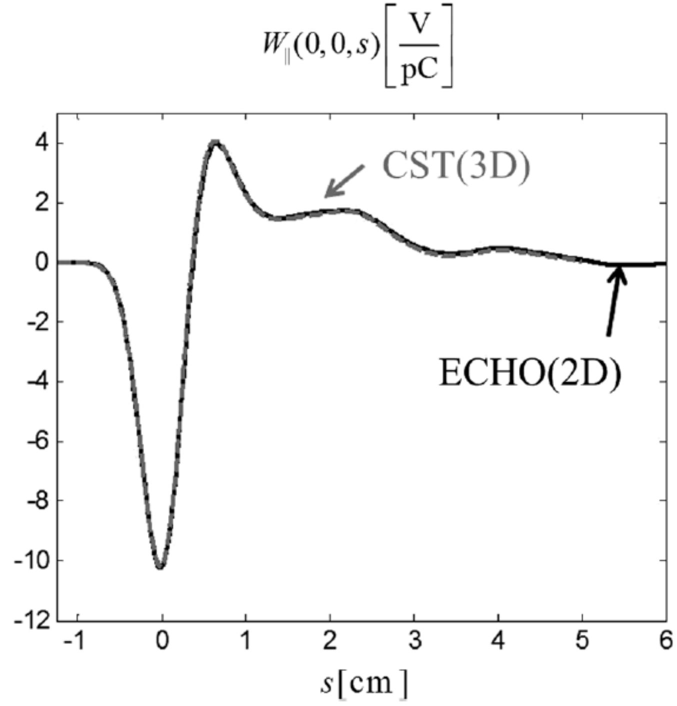
Zagorodnov I., Bane K., Stupakov G. Calculations of wakefields in 2D rectangular structures, Phys. Rev. STAB **18**, 104401 (2015)

Modelling of Conductive Walls



2*w*= 10 cm, *T*=5 cm, 2*b*=2cm, *L*=12 cm.
Gaussian bunch with rms length 25mm





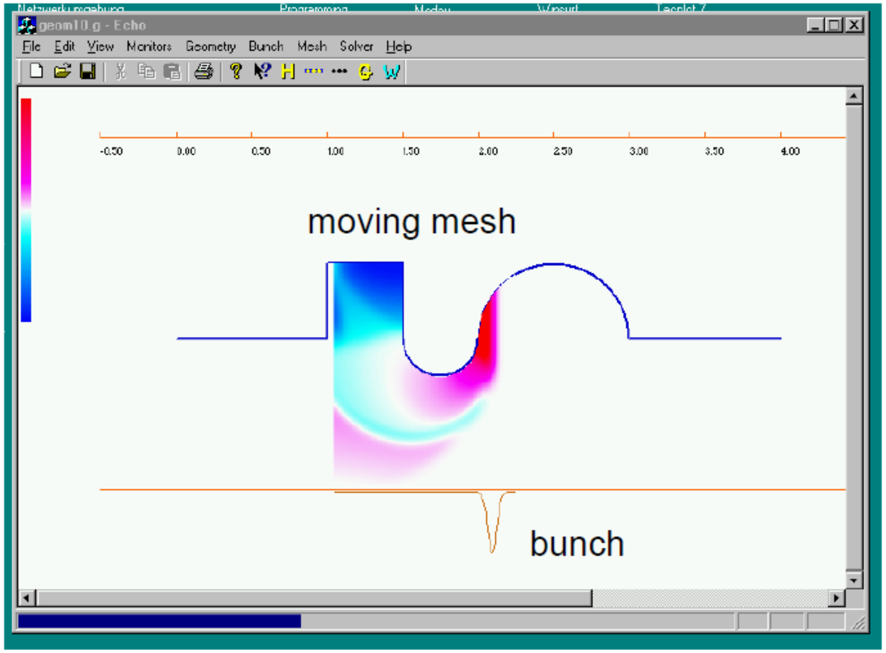
Longitudinal wake potential of tapered collimator

Code Status

- ECHOz1, ECHOz2 (rotationally symmetric)
- ECHO2D (rectangular and rotationally symmetric)
- ECHO3D (fully 3D)
- ECHO1D (anisotropic round and rectangular waveguides)
- iECHO (arbitrary 2D waveguides with surface impedance)

https://www.echo4d.de

ECHOz1



ECHO

Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

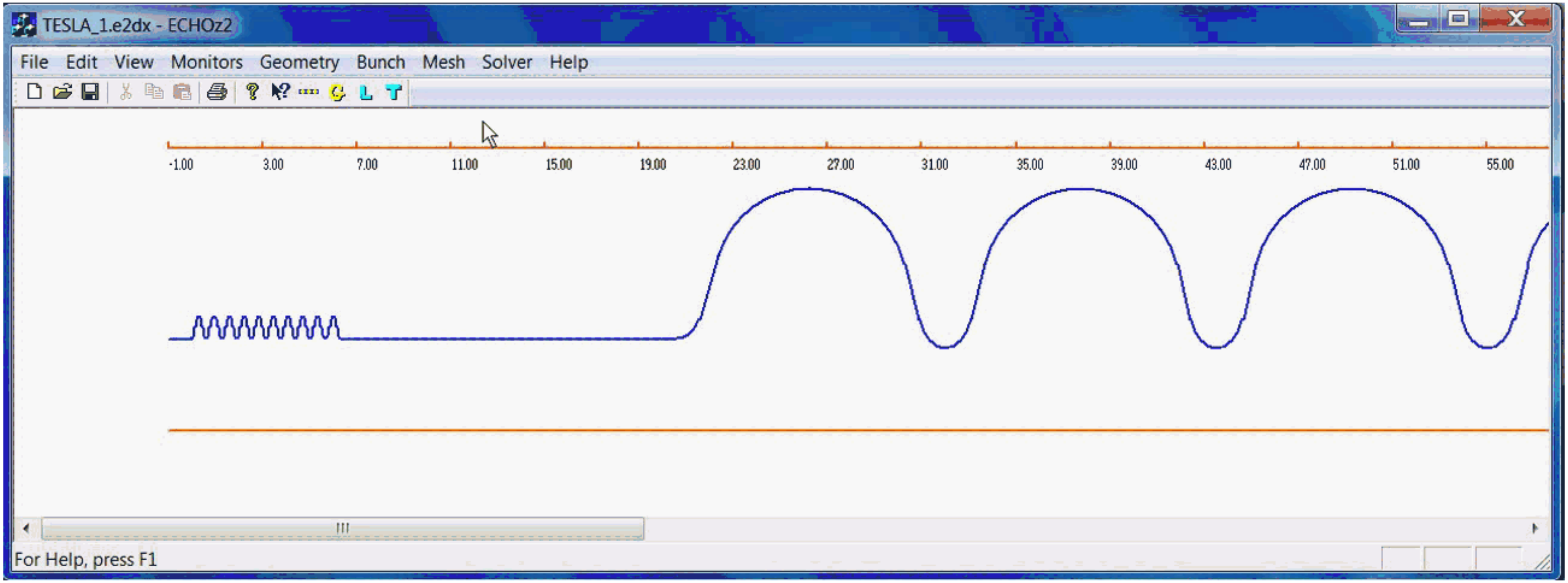
Only fully rotationally symmetric problems (m=0 mode)

- only PEC
- stand-alone Windows GUI application
- parallelized (OpenMP)

ECHOz2

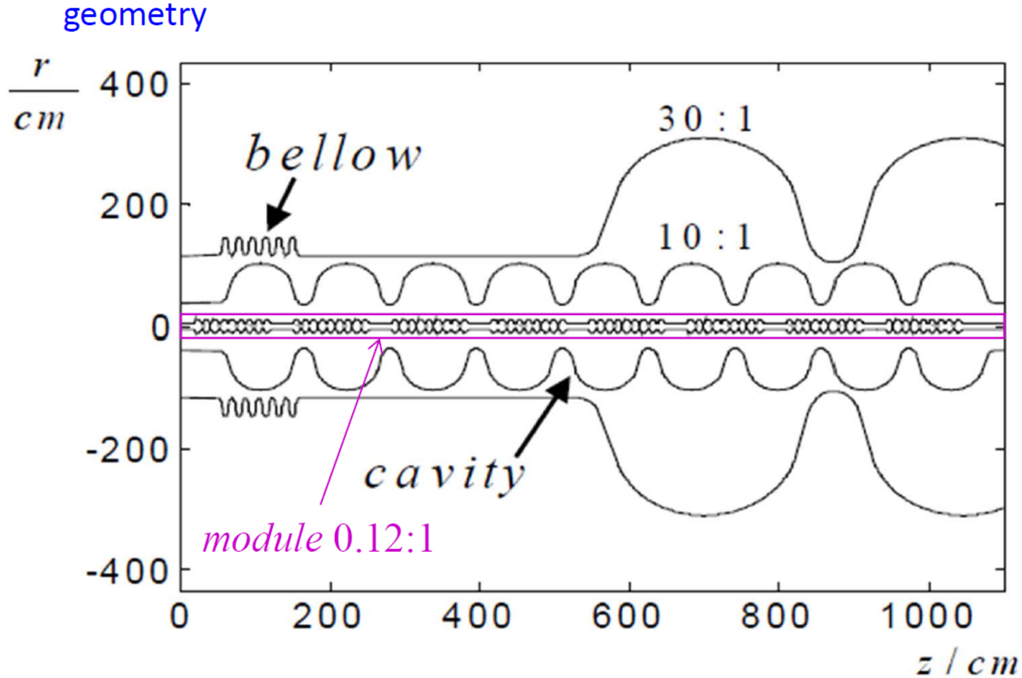
Rotationally symmetric problems (all modes)

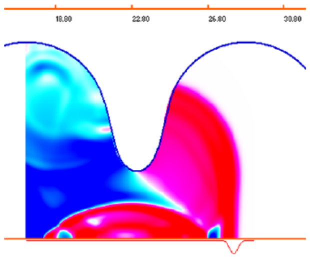
- surface conductivity
- stand-alone Windows GUI application
- parallelized (OpenMP)



ECHOz2

Short-Range Wake Functions for TESLA Cryomodule



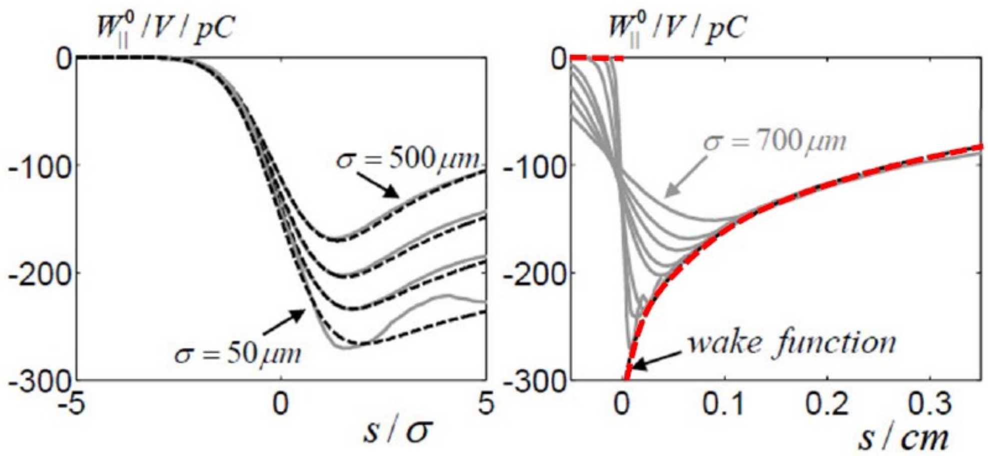


σ = 700 ... **50 μm**

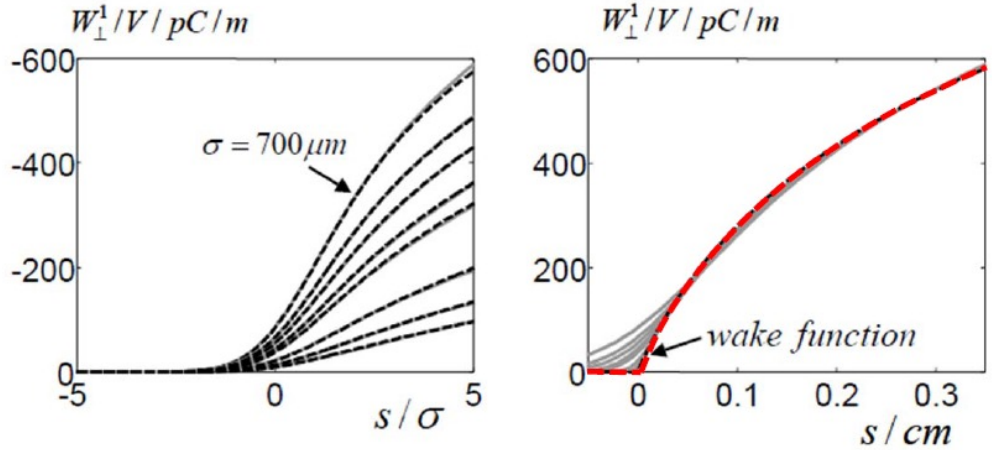
Longitudinal wake functions was found earlier with code NOVO
Novokhatski A., Timm M., Weiland T., Single Bunch Enetgy Spread in the TESLA Cryomodule, DESY TESLA-99-16, 1999
Transverse wake functions is found with code ECHO
T. Weiland, I. Zagorodnov, The short-range transverse wake function for TESLA accelerating structure, TESLA Report 2003-19

ECHOz2

longitudinal wake (monopole)



transverse wake (dipole)



Novokhatski A., Timm M., Weiland T. (1999, NOVO)

$$W_{\parallel}(s) = A\left[\left(1+\beta_0\right)e^{-\sqrt{\frac{s}{s_0}}} - \beta_0\right]$$

Weiland T., Zagorodnov I. (2003, ECHO)*

$$W_{\parallel}(s) = Ae^{-\sqrt{\frac{s}{s_0}}}$$

$$W_{\perp}(s) = A_1\left[1-\left(1+\sqrt{\frac{s}{s_1}}\right)e^{-\sqrt{\frac{s}{s_1}}}\right]$$

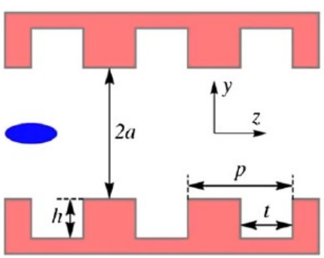
Bane K.L.F., *Short-Range Dipole Wakefields in Accelerating Structures for the NLC*, SLAC-PUB-9663, LCC-0116, 2003

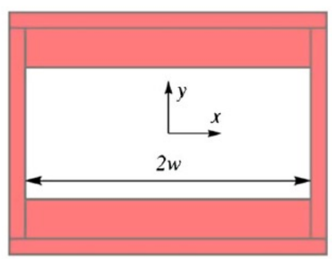
ECHO2D

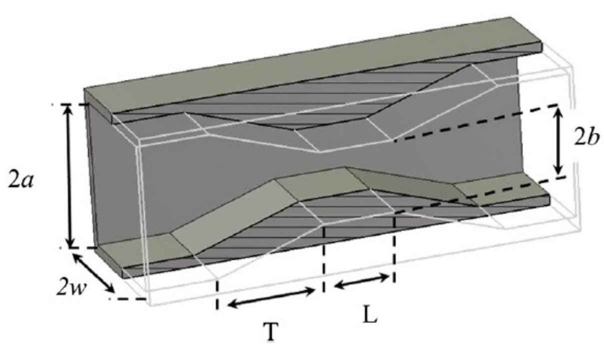
Rectangular Geometries with Constant Width

Rectangular and rotationally symmetric problems (all modes)

- different materials (epsilon and mue)
- surface and volume conductivity
- stand-alone console application (Windows, Mac OS, Linux)
- parallelized (MPI and OpenMP)



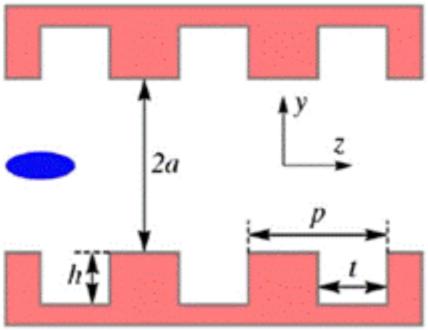


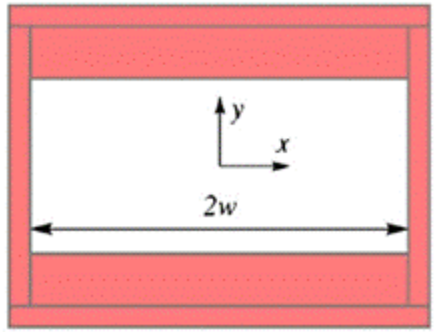


Zagorodnov I., Bane K., Stupakov G. Calculations of wakefields in 2D rectangular structures, Phys. Rev. STAB **18**, 104401 (2015)

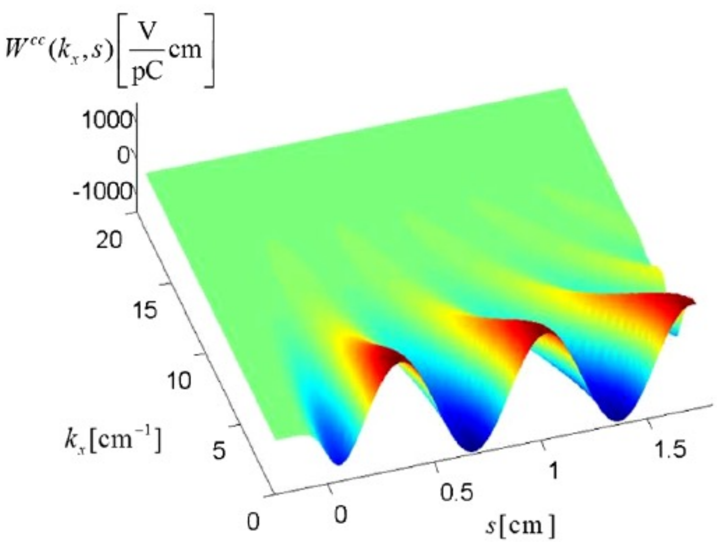
ECHO2D

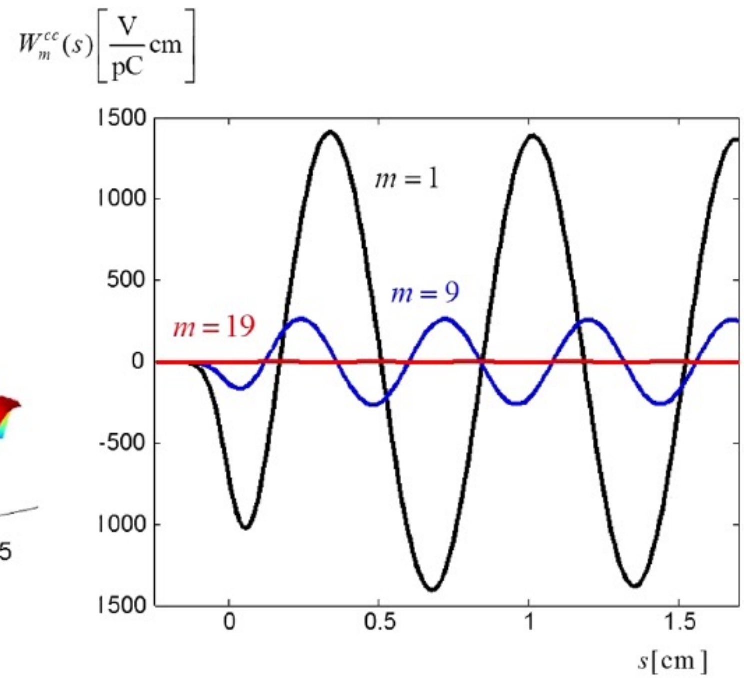
Pohang Dechirper Experiment





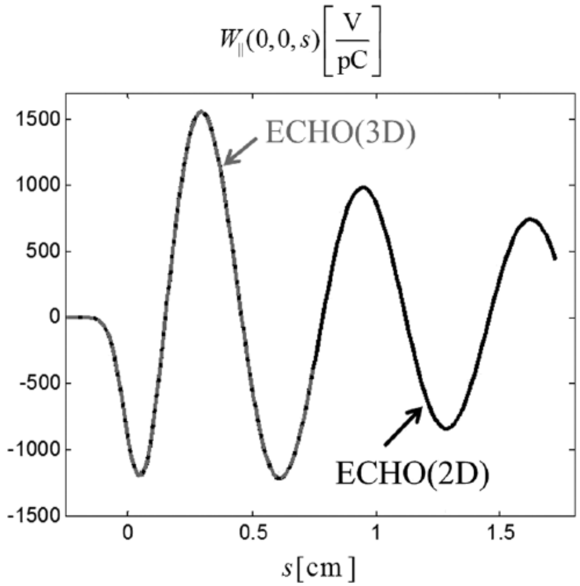
$2w$= 5 cm, p=0.5 mm,
h=0.6mm,
t=p/2, L=1 m, 2a= 6mm.
Gaussian bunch with rms length 0.5mm

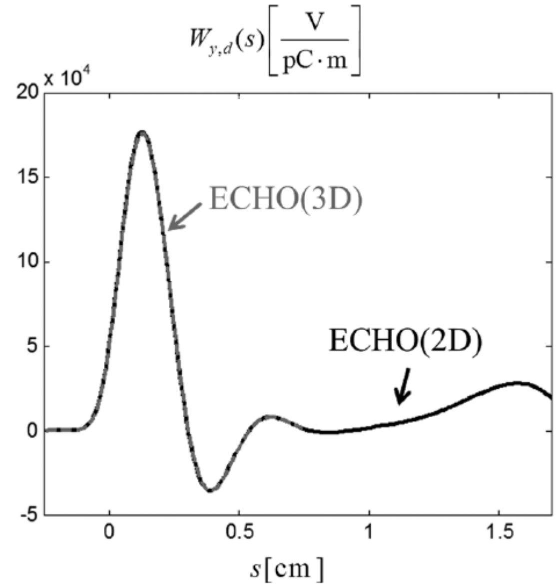




ECHO2D

Pohang Dechirper Experiment*



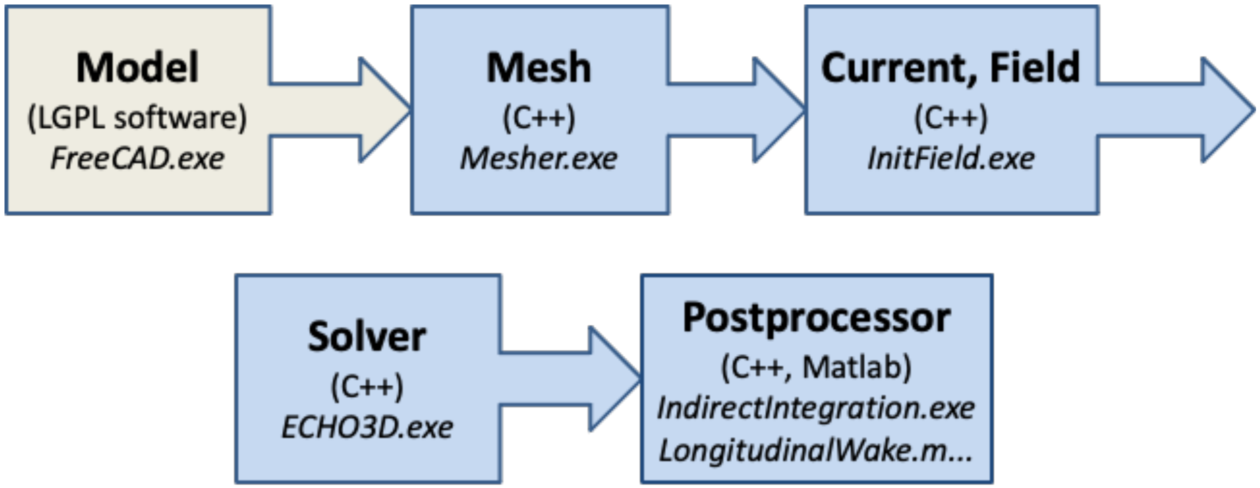


Wake	r[T3P]	r[ECHO]
Longitudinal, loss factor	0.84	0.83
Dipole, kick factor	1.08	0.79
Quad, kick factor	0.73	0.73

$$r = \frac{\text{measured}}{\text{analytical}} = 0.75$$

*P. Emma et al., Phys. Rev. Lett. **112**, 034801 (2014)

ECHO3D

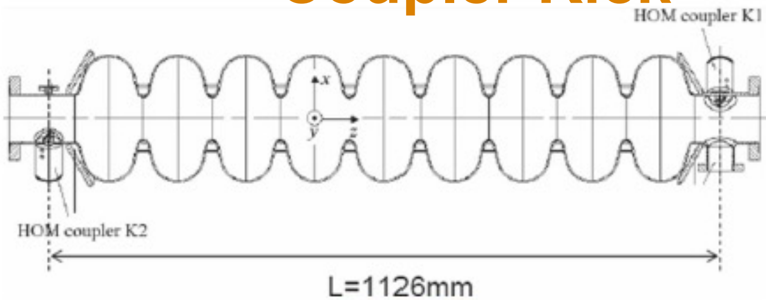


Full 3D geometry

- different materials (epsilon and mue)
- no conductivity
- **Perfectly Matched Layer**
- console application (Windows, Mac OS, Linux)
- parallelized (OpenMP)

ECHO3D

Coupler Kick



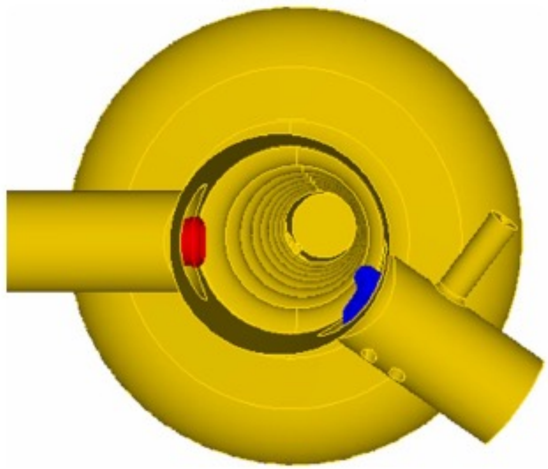
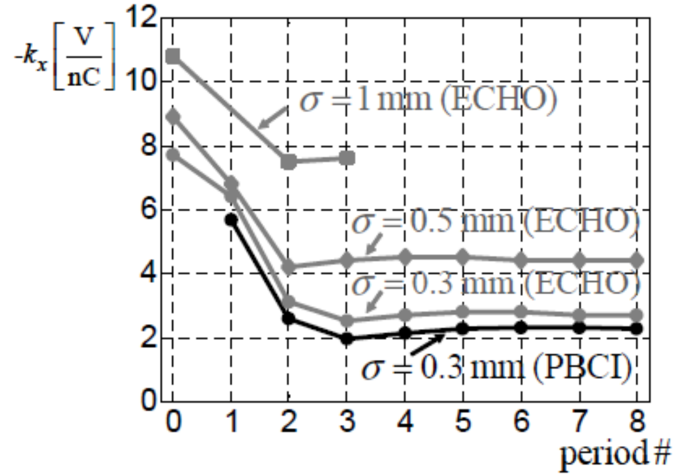


Figure 1: Sketch of an ILC cavity with the HM couplers (top). View from the downstream end of a cavity showing the FM coupler (red) and one HM coupler (blue).

M. Dohlus, I. Zagorodnov, E. Gjonaj, T. Weiland, Coupler kick for very short bunches and its compensation, EPAC08 (2008)



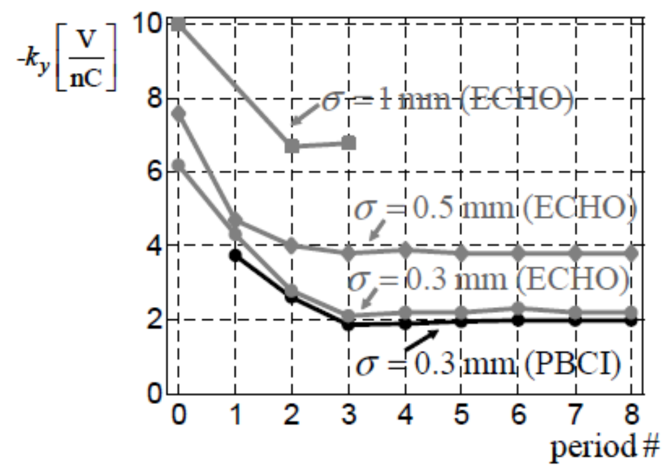
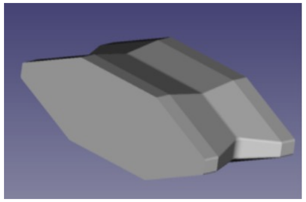


Figure 3: Horizontal and vertical kick vs. number of periods.

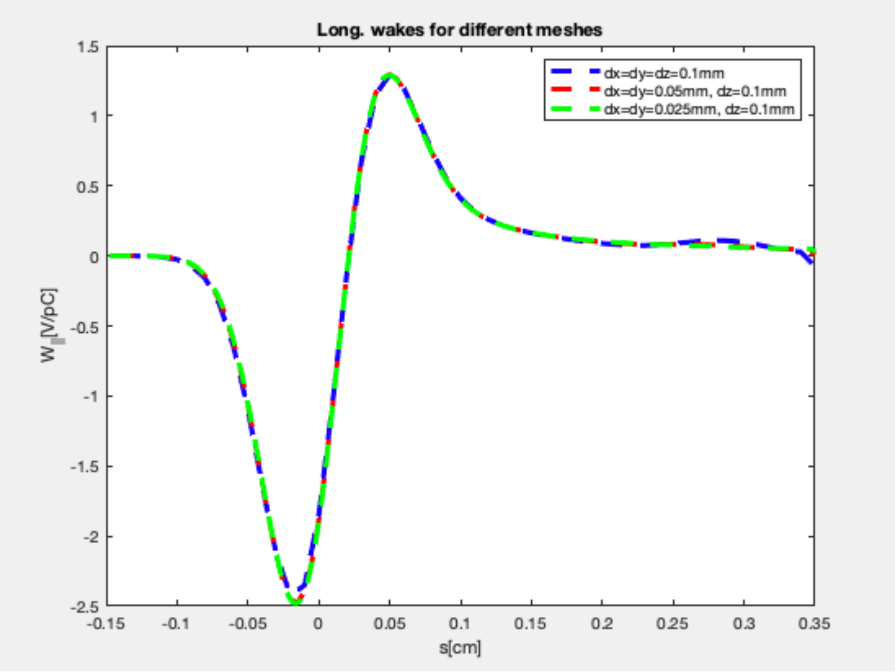
ECHO3D

NSLS-II flange absorber

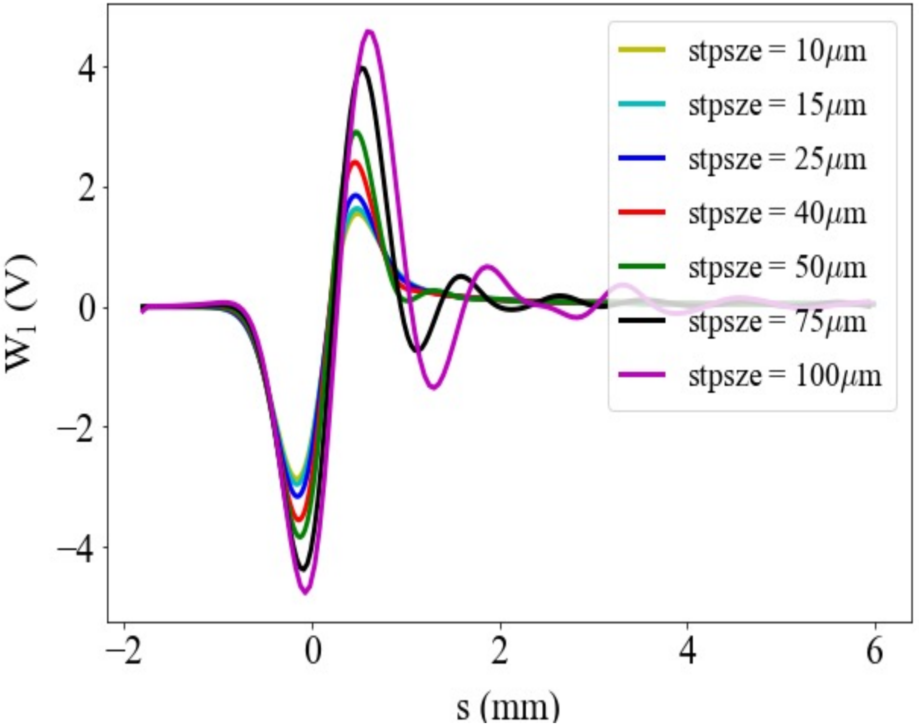


Real geometry. Sigma= 0.3 mm. Longitudinal wake.

ECHO3D



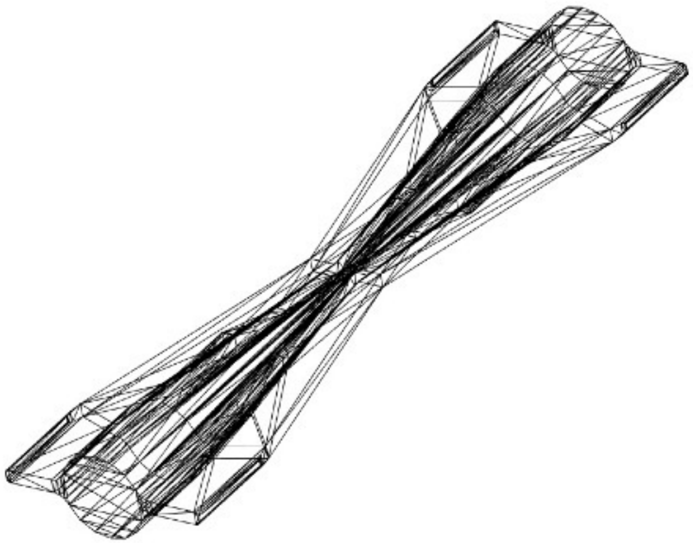
GdfidL (Picture from Aamna Khan, BNL)



Factor 10^4 in mesh points

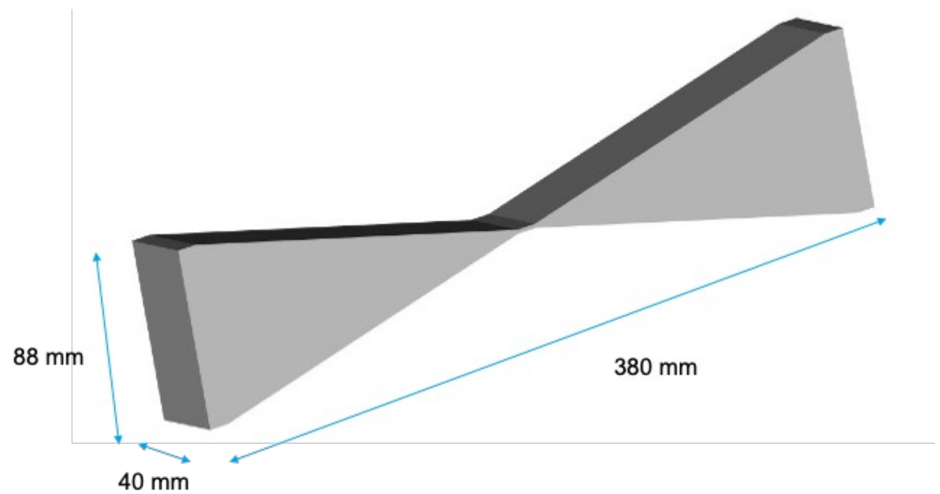
ECHO3D

KEK collimator



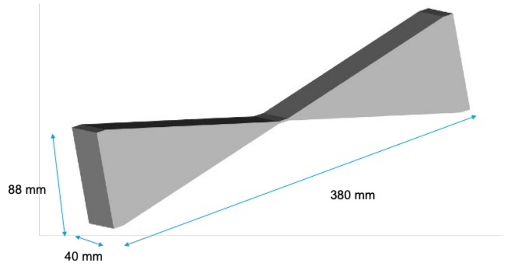
(Picture from Takuya Ishibashi, KEK)

Model geometry can be computed with high accuracy by ECHO2D code.

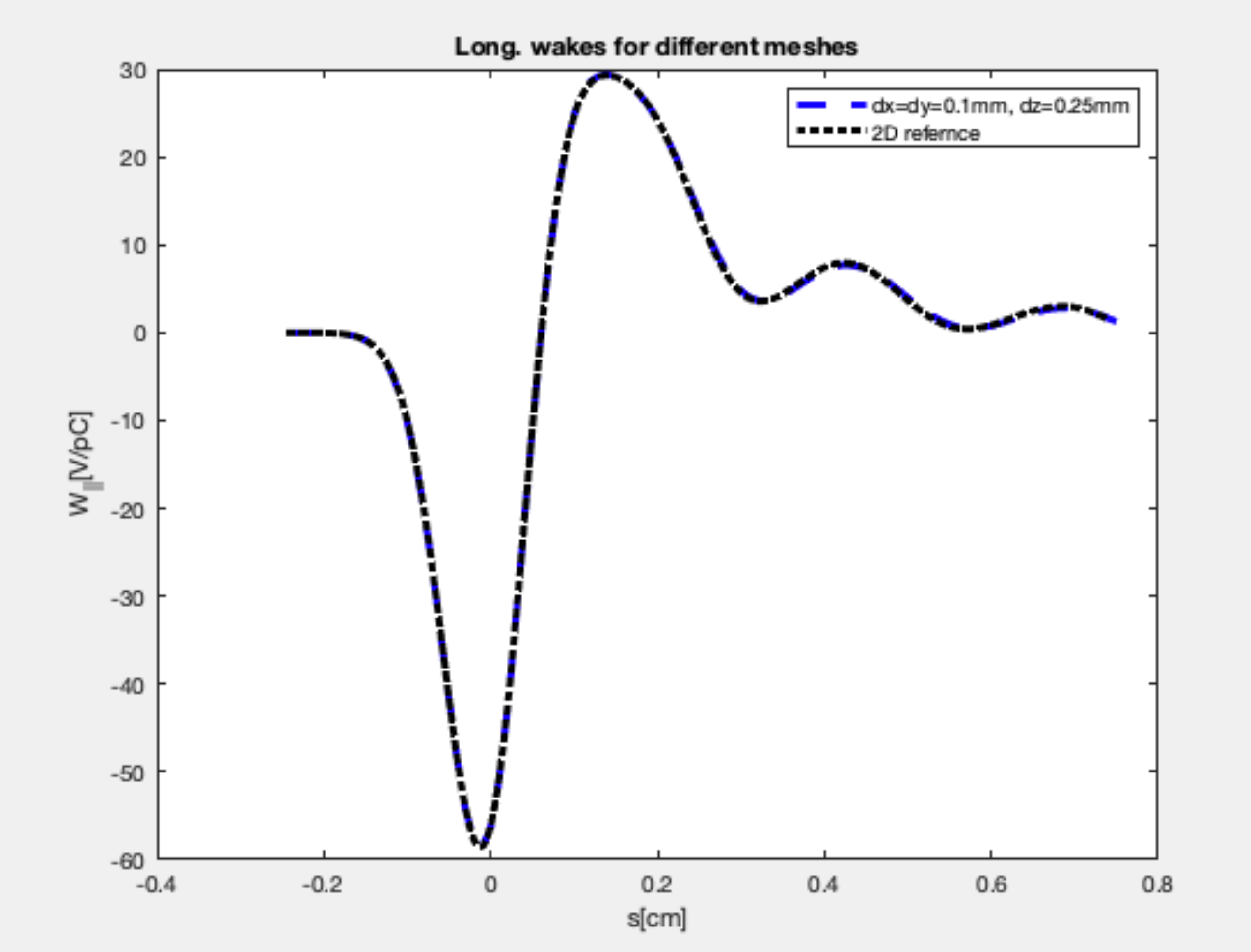


ECHO3D

KEK collimator

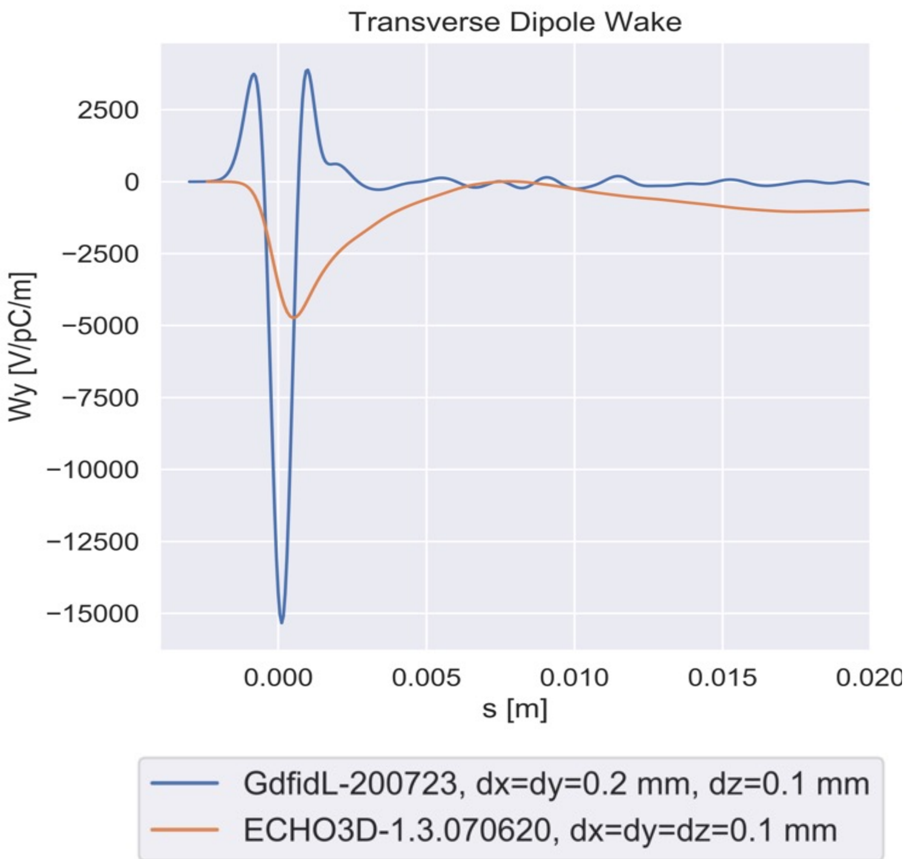


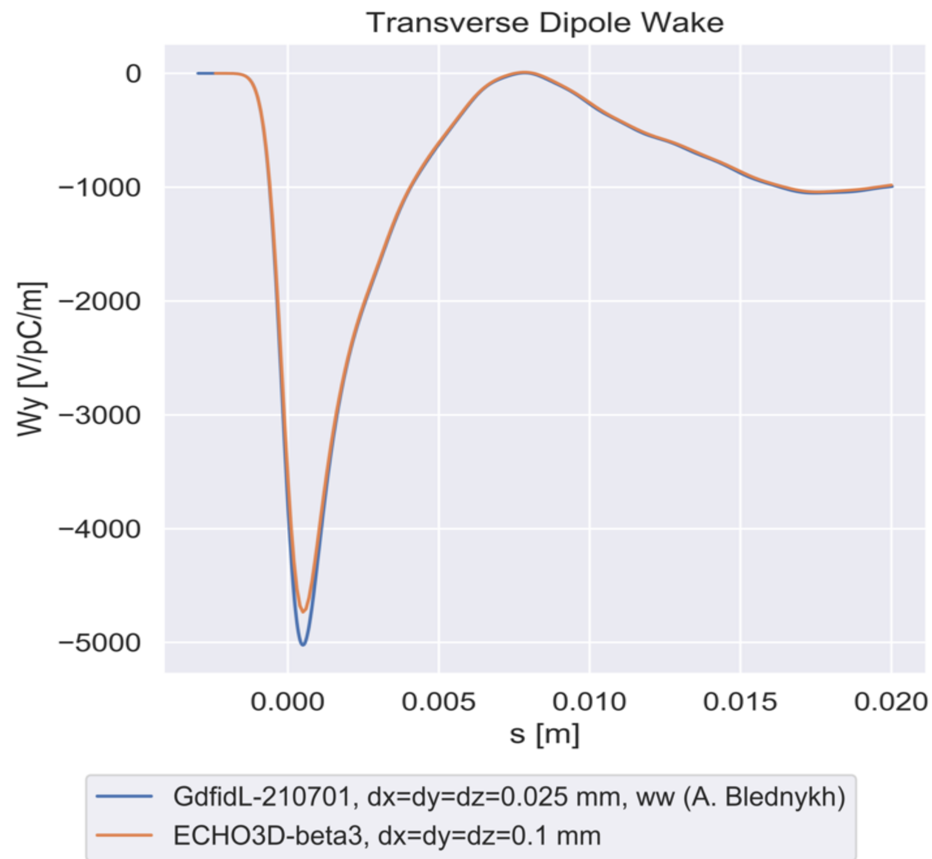
Model problem. Sigma= 0.5 mm. Longitudinal wake.



ECHO3D

KEK collimator (Pictures from Takuya Ishibashi, KEK)





Real geometry. Sigma= 0.5 mm. Dipole wake.

Factor $4^4 = 256$ in mesh points

ECHO3D

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ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-TUPMF071

STATUS OF IMPEDANCE MODELING FOR THE PETRA IV

Yong-Chul Chae† and Rainer Wanzenburg, DESY, Hamburg, Germany

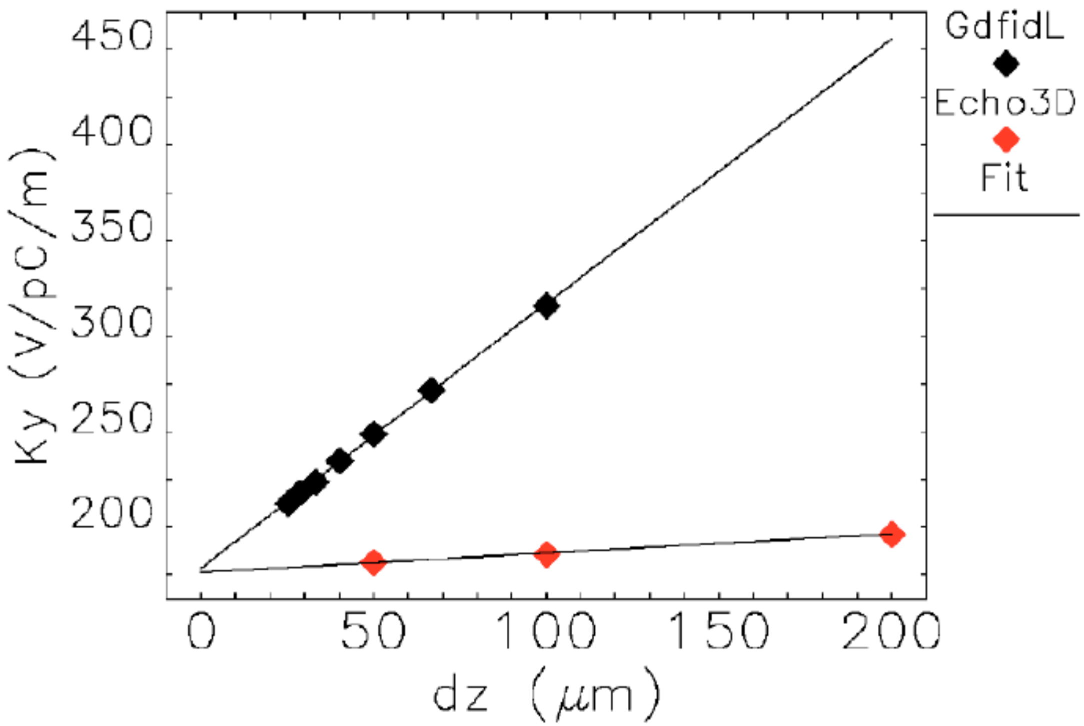
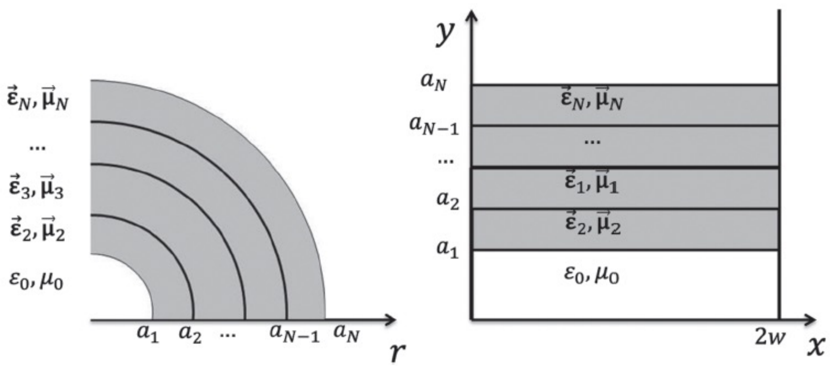
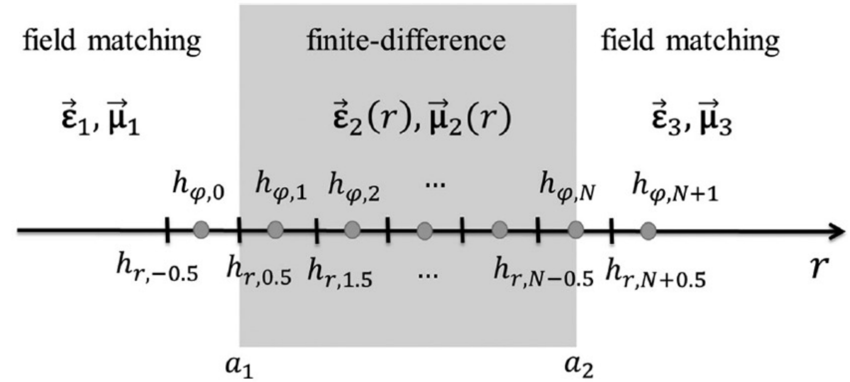


Figure 5: The convergence test of 6 mm gap ID chamber by two programs GdfidL and Echo3D.

ECHO1D

Anisotropic 1D waveguides.





$$\vec{D} = \begin{pmatrix} \varepsilon_r(r,k) & 0 & 0 \\ 0 & \varepsilon_\varphi(r,k) & 0 \\ 0 & 0 & \varepsilon_z(r,k) \end{pmatrix} \vec{E}, \quad \vec{B} = \begin{pmatrix} \mu_r(r,k) & 0 & 0 \\ 0 & \mu_\varphi(r,k) & 0 \\ 0 & 0 & \mu_z(r,k) \end{pmatrix} \vec{H}.$$

I. Zagorodnov, Impedances of anisotropic round and rectangular chambers, Phys. Rev. AB **21**, 064601 (2018)

I. Zagorodnov, M. Dohlus, *Beam wakes for canonical chambers with anisotropic surface impedance,* Phys. Rev. AB *26, 094401 (2023)*

iECHO

A numerical method based **on integral equation in the frequency domain** was developed for arbitrarily shaped 2D waveguides with surface impedance boundary conditions.

The approach was originally proposed by Martin Dohlus in a formulation using **auxiliary sources** (two contours).

In a subsequent work, we reformulated it as a **boundary element method** (one contour).

I. Zagorodnov, M. Dohlus, T. Wohlenberg, Short-range longitudinal wake function of undulator lines at the European X-ray free electron laser, NIM A, 1043 (2022) 167490

W. Qin, M. Dohlus, I. Zagorodnov, Short-range wakefields in an L-shaped corrugated structure, Phys. Rev. AB 26, 064402 (2023)

Different materials can be specified along the boundary.