

Theory prediction for $t\bar{t}$ threshold enhancement at hadron colliders

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mostly on the basis of

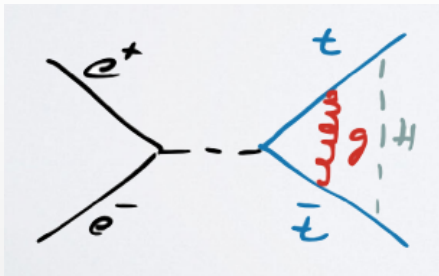
M.V.G., G. Limatola, S.-O. Moch, M. Steinhauser, O. Zenaiev,
[arXiv:2412.16685]

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FH Particle Physics Discussion: a spotlight on the $t\bar{t}$ threshold at the LHC
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$e^+e^- \rightarrow t\bar{t}$ close to threshold at lepton colliders



- * Top quark non relativistic close to threshold

($v_t/c = \beta_t = \sqrt{1 - 4m_t^2/M_{t\bar{t}}^2} \ll 1$) in $t\bar{t}$ rest frame. Regime for NRQCD. Plenty of studies, due to sensitivity to m_t .

- * Simultaneous sensitivity to Γ_t , α_s , $y_{t\bar{t}h}$, to be considered when the process is used for m_t extraction via top-threshold scan in short-distance m_t renormalization scheme ($\Delta m_t \sim \text{some tens MeV}$).

- * Even $e^+e^- \rightarrow W^+bW^-\bar{b}$ predictions are available (non-resonant contributions to m_t).

$t\bar{t} + X$ production close to threshold at hadron colliders

- * Far less studies at hadron colliders w.r.t. lepton colliders
- * Interest suddenly enhanced by the threshold excesses seen in the results of recent CMS analyses searching for BSM $H \rightarrow t\bar{t}$ decays (to be cross-checked by ATLAS).
- * NRQCD predictions, including singlet and octet contributions for the $m_{t\bar{t}}$ distribution $\Rightarrow$ *Revival of the theory chain in Kiyo, Kühn, Moch, Steinhauser and Uwer, [arXiv:0812.0919]*
- * bound-state effects related to resummation of terms depending on α_s^n/β_t^m , associated to the exchanges of Coulomb-like virtual gluons between the t and $\bar{t}$ quarks
- * plus resummation of large logarithms due to real soft-gluon emissions close to threshold at NLL.

$M_{t\bar{t}}$ differential distribution

$$M_{t\bar{t}} \frac{d\sigma_{P_1 P_2 \rightarrow T}}{dM_{t\bar{t}}}(S, M_{t\bar{t}}^2) = \sum_{i,j} \int_{\rho}^1 d\tau \left[\frac{d\mathcal{L}_{ij}}{d\tau} \right](\tau, \mu_f^2) M_{t\bar{t}} \frac{d\hat{\sigma}_{ij \rightarrow T}}{dM_{t\bar{t}}}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2)$$

with $\tau = \hat{s}/S$, $\rho = M_{t\bar{t}}^2/S$,

$$\left[\frac{d\mathcal{L}_{ij}}{d\tau} \right](\tau, \mu_f^2) = \int_0^1 dx_1 \int_0^1 dx_2 f_{i/P_1}(x_1, \mu_f^2) f_{j/P_2}(x_2, \mu_f^2) \delta(\tau - x_1 x_2)$$

and

$$M_{t\bar{t}} \frac{d\hat{\sigma}_{ij \rightarrow T}}{dM_{t\bar{t}}}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2) = F_{ij \rightarrow T}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2) \frac{1}{m_t^2} \text{Im} G^{[1,8]}(M_{t\bar{t}} + i\Gamma_t)$$

valid in NRQCD for the considered distribution at NLO:

- still to be generalized at higher-orders
- still to be generalized for other distributions

Factorization of the partonic cross-section

$$M_{t\bar{t}} \frac{d\hat{\sigma}_{ij \rightarrow T}}{dM_{t\bar{t}}}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2) = F_{ij \rightarrow T}(\hat{s}, M_{t\bar{t}}^2, \mu_f^2) \frac{1}{m_t^2} \text{Im} G^{[1,8]}(M_{t\bar{t}} + i\Gamma_t)$$

valid in NRQCD for the considered distribution at NLO

- $F_{ij \rightarrow T}$: hard-scattering function for producing the $T = {}^{2S+1}L_J^{[1,8]}$ state, containing threshold logarithms becoming large close to the partonic threshold (i.e. for τ reaching its minimum value ρ , equivalent to $z \rightarrow 1$).
- $G^{[1,8]}$: attractive and repulsive non-relativistic Green's functions, solution of Schroedinger eq. accounting for exchange of potential gluons between t and $\bar{t}$. They depend on Γ_t and m_t .
 - $G^{[1]}$ from attractive Coulomb-like potential, responsible for “toponium” quasi-bound states
 $\Gamma_t \gg \Lambda_{QCD}$: the top quark decays before a proper bound state can be formed.
 - $G^{[8]}$ from repulsive Coulomb-like potential.

Resummation of threshold logarithms

- * We resum threshold logarithms up to NLL for the most relevant T channels ($gg \rightarrow^1 S_0^{[1]}, ^1 S_0^{[8]}$ and $q\bar{q} \rightarrow^3 S_1^{[8]}$), working in N -space (N conjugate to z).

- * We consider the Mellin transform of the hard function

$$F_{ij \rightarrow T}^N(M_{\tilde{t}\tilde{t}^*}^2, \mu_f^2) = \int_0^1 dz z^{N-1} F_{ij \rightarrow T}(\hat{s}, M_{\tilde{t}\tilde{t}^*}^2, \mu_f^2)$$

and the Mellin transform $\mathcal{L}_{ij}^N$ of the luminosity function $\mathcal{L}_{ij}(\tau)$.

- * We then recover predictions in z -space from the inverse Mellin transform of $(\mathcal{L} \otimes \mathcal{F})^N = \mathcal{L}^N \mathcal{F}^N$, performed numerically, using the minimal prescription.

- * This includes matching of the resummed results to the NLO ones, avoiding double-counting, leading to NLO+NLL predictions.

- * Uncertainties related to different prescriptions for avoiding the Landau pole in the inverse Mellin transform and matching not yet included.

Theory predictions: input

* Present setup:

- $\text{NNPDF3.1 NNLO PDFs} + \alpha_s(M_Z)$,
- $m_t = 172.5 \text{ GeV}$,
- $\Gamma_t = 1.36 \text{ GeV}$,
- $\mu_R = \mu_F = [m_t, 4m_t]$.

* Ongoing study of parametric uncertainties.

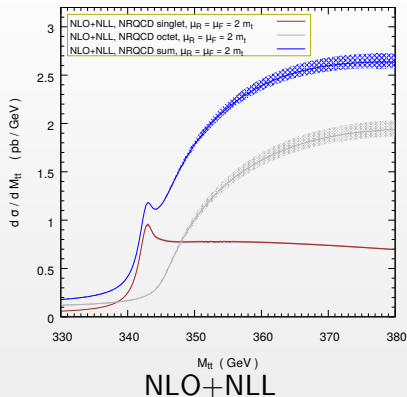
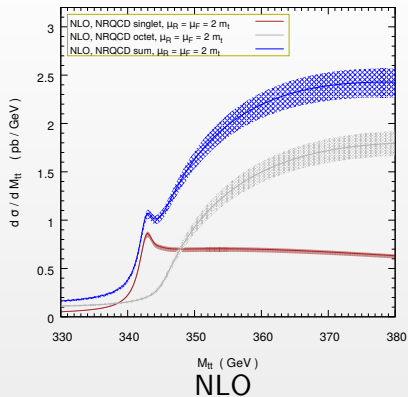
* Possibility to provide predictions with different inputs:
please feel free to ask.

Predictions for the convolution $\mathcal{L} \otimes \mathcal{F}$

	NLO			resummed		
$gg \rightarrow {}^1S_0^{[1]}$	18.2	18.7	18.3	19.4	20.5	21.1
$gg \rightarrow {}^1S_0^{[8]}$	55.8	55.2	52.8	60.0	61.5	62.0
$q\bar{q} \rightarrow {}^3S_1^{[8]}$	21.7	22.3	22.0	22.4	22.4	22.0

TABLE I. Comparison of the NLO and NLO+NLL resummed result of the convolution $\mathcal{L} \otimes F$ (in units 10^{-6} GeV^{-2}) for the LHC configuration $\sqrt{s} = 13 \text{ TeV}$ with NNPDF3.1 PDFs at the reference point $M_{t\bar{t}} = 2m_t$. The three columns correspond to the scale choices $\mu_r = \mu_f = \{m_t, 2m_t, 4m_t\}$.

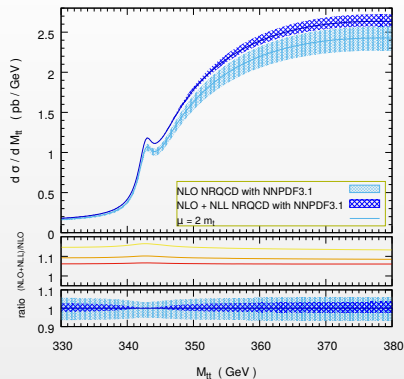
$M_{t\bar{t}}$ distribution for $pp \rightarrow t\bar{t} + X$ close to threshold



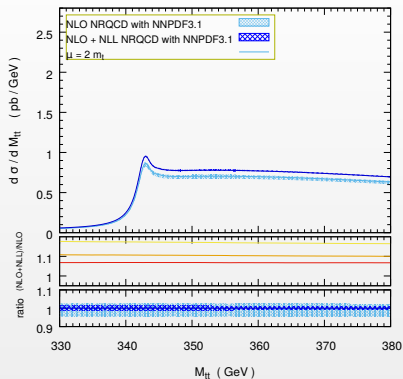
* Multiple singlet and octet contributions included at fixed-order: all S -wave channels, i.e. the $gg, gq, q\bar{q} \rightarrow {}^1S_0^{[1,8]}$ channels, the $gg, gq, q\bar{q} \rightarrow {}^3S_1^{[8]}$ channels, as well as the $gg \rightarrow {}^3S_1^{[1]}$ one.

* NLL effects enhance predictions and reduce considerably uncertainty bands w.r.t. NLO.

Comparison of NLO and NLO+NLL predictions



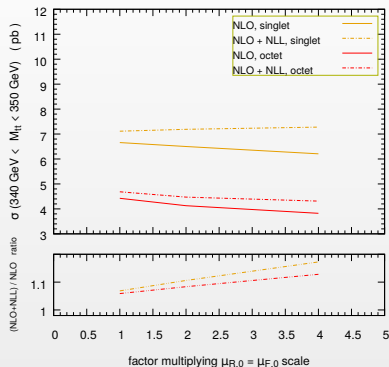
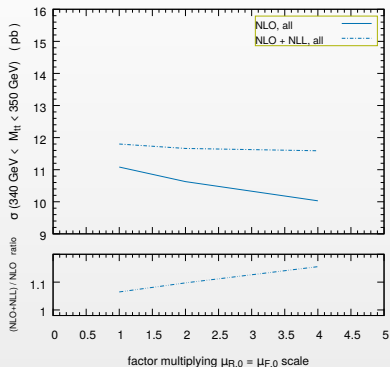
including octets



only singlet

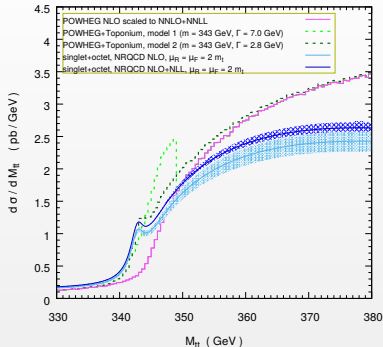
- * K -factors $\sim 10\%$, depending on the scale
- * Uncertainties reduced in NLO+NLL w.r.t. NLO, especially in the peak region.

Integrated cross-section in the region of validity of NRQCD vs. scale



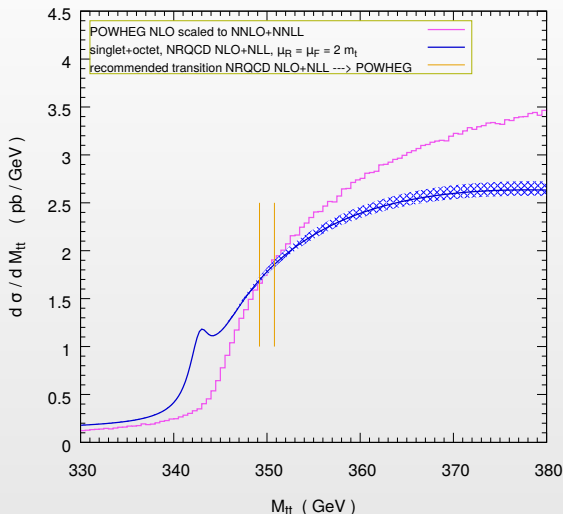
- * K -factors $\sim 10\%$ for both singlet and octet
- * NLL effects increase with the value of the scale $\mu_{R,0} = \mu_{F,0}$
- * other scale choices under investigation

Comparison with predictions of Fuks model 2021

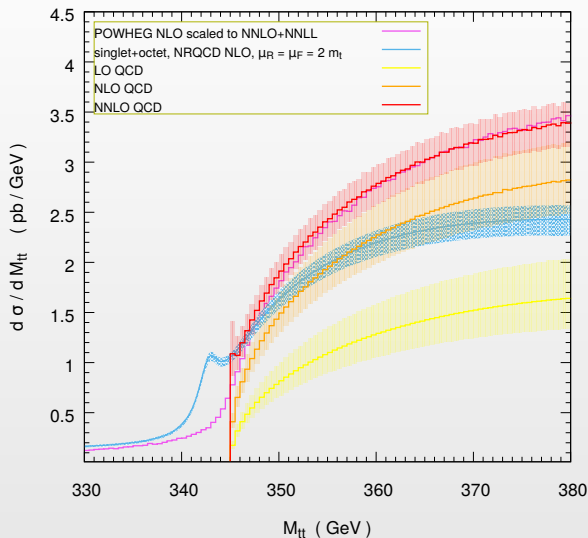


- * POWHEG and Fuks et al. model input from Laurids et al.
- * Height, position and width of Fuks-modelled singlet do not coincide with our NLO (+ NLL) QCD singlet + octet predictions.
- * Big uncertainties on the Green's functions used in our NLO QCD predictions, not shown in our plots. Missing higher orders would enhance these functions.

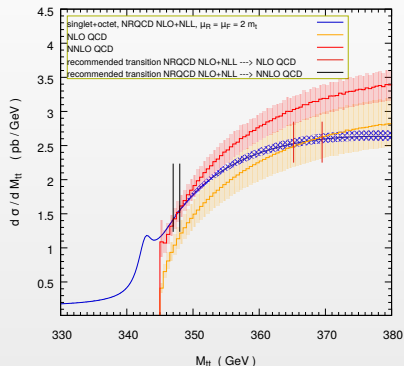
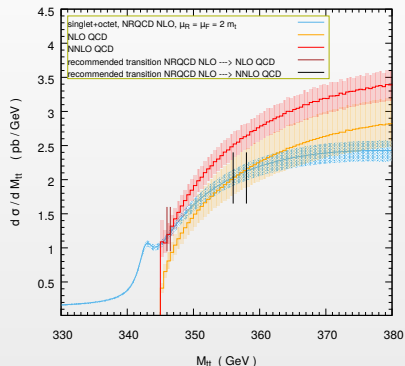
Recommended transitions from NRQCD to QCD POWHEG predictions



Reweightd Powheg vs. fixed-order predictions vs. NRQCD



Recommended transitions from NRQCD to QCD fixed-order predictions



* fixed-order predictions with MATRIX at LO, NLO, NNLO, with their scale uncertainty bands.

* A proper matching between NRQCD and QCD predictions still to be devised

Conclusions

- * Further studies ongoing, especially focused on uncertainties.
- * So far, we are limited to inclusive predictions, cuts still to be applied.
- * Investigation of how to best use our predictions:
bin-by-bin reweighting ? Alternatives ?
- * How safe/robust is extrapolating to other distributions (i.e. the distributions currently measured by the experimentalists)?
- * Future interesting studies: compare shape of enhancements at threshold from models with BSM Higgs decaying into a $t\bar{t}$ pair to our results, and understand up to which extent it is possible to discriminate between the effects of BSM Higgs and toponium.
- * Please note that our threshold enhancement, corresponding to the color-singlet config., occurs mostly **below** threshold, in contrast with many BSM Higgs models.
- * Our current predictions: lower limit to the true ones.