

Including Coulomb corrections $t\bar{t}$ production

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Plan

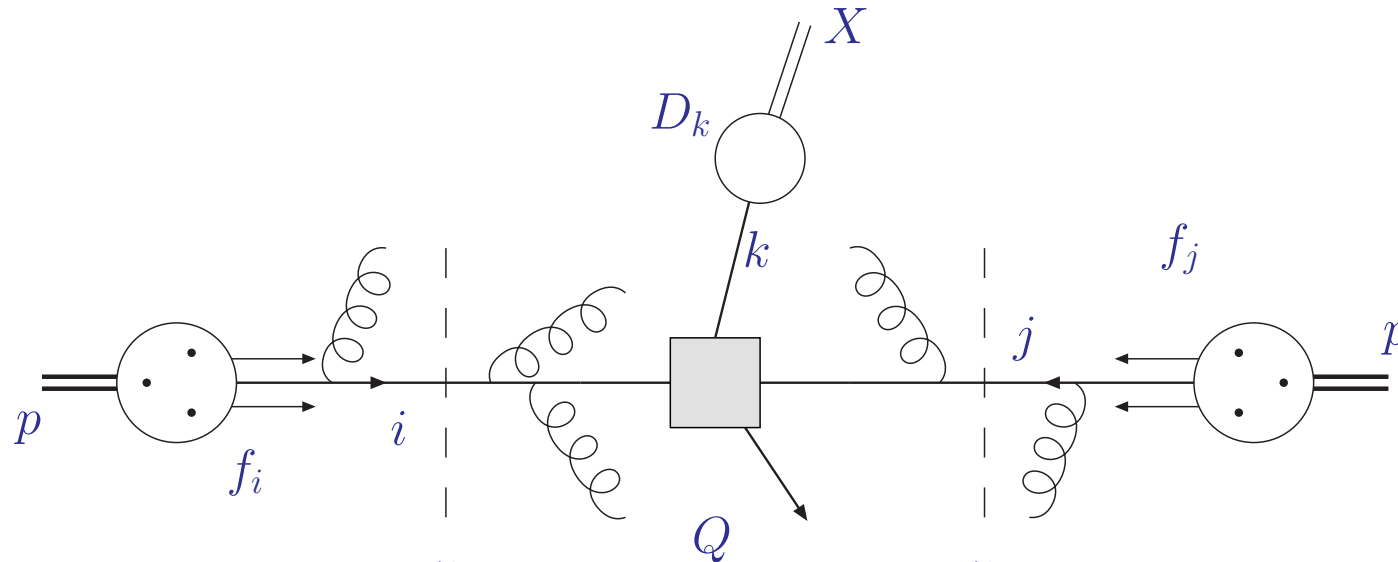
Coulomb corrections

- *Top-quark pair production near threshold at LHC*
Y. Kiyo, J. H. Kühn, S. M., M. Steinhauser and P. Uwer [arXiv:0812.0919](#)

Phenomenology update

- *Updated predictions for toponium production at the LHC*
M.V. Garzelli, G. Limatola, S. M., M. Steinhauser and S. Zenaiev
[arXiv:2412.16685](#)

QCD factorization



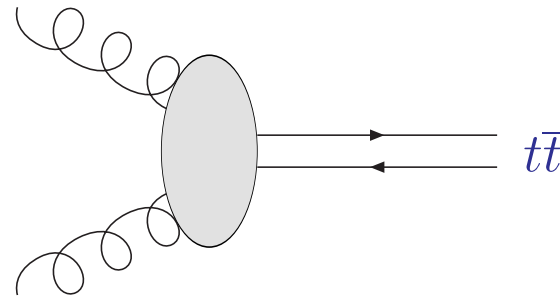
$$\sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2)$$

- Factorization at scale μ
 - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij \rightarrow X}$ calculable in perturbation theory
 - cross section $\hat{\sigma}_{ij \rightarrow k}$ for parton types i, j and hadronic final state X
- Non-perturbative parameters: parton distribution functions f_i , strong coupling α_s , particle masses m_X
 - known from global fits to exp. data, lattice computations, ...

Heavy quark production

Quantum numbers

- Quantum numbers of heavy quark pair $t\bar{t} \left({}^{2S+1}L_J^{[1,8]} \right)$
- Notation from spectroscopy
 - orbital angular momentum L :
→ S -wave, P -wave, ...
 - spin $2S + 1$: singlet, triplet
 - total angular momentum J
 - color $[1, 8]$: singlet, octet



Kinematics

- Close to production threshold
 - velocity $\beta_{t\bar{t}} = \sqrt{1 - 4m^2/s}$ of heavy quarks
- Born cross section near threshold
 - S -wave: $\sigma_{S\text{-wave}}^{(0)} \sim \beta$
 - P -wave: $\sigma_{P\text{-wave}}^{(0)} \sim \beta^3$

Cross section (I)

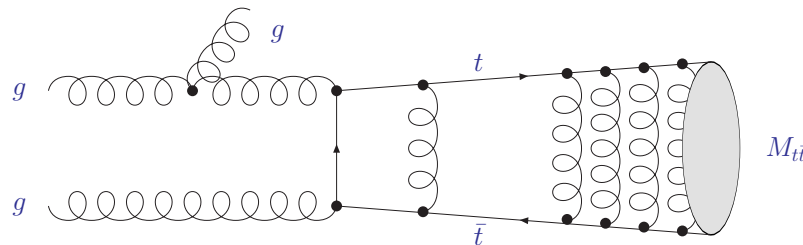
- Parametric form of cross section in $\beta_{t\bar{t}} = \sqrt{1 - 4m^2/s}$

Threshold logarithms

- Sudakov logarithms from emission of soft and collinear gluons
 - all orders in perturbation theory $\alpha_s^n \ln^k(\beta_{t\bar{t}})$ with $2n \geq k \geq 1$
 - resummation of large logarithms (renormalization group equation)
Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; ...

Coulomb corrections

- Singular contributions from gluon exchanges among $t\bar{t}$ final state



- all orders in perturbation theory $\alpha_s^n / \beta_{t\bar{t}}^k$ with $n \geq k \geq 1$
- resummation with non-relativistic QCD (NRQCD)

Bodwin, Braaten, Lepage '95

Cross section (II)

Perturbation theory

- Partonic cross section

$$\hat{\sigma}_{t\bar{t}} = \hat{\sigma}_{t\bar{t}}^{(0)} \times \sum_{k=0} \left(\frac{\alpha_s}{\beta_{t\bar{t}}} \right)^k \times \exp \left\{ \underbrace{\ln(\beta_{t\bar{t}}) g_0(\alpha_s \ln(\beta_{t\bar{t}}))}_{\text{LL}} + \underbrace{g_1(\alpha_s \ln(\beta_{t\bar{t}}))}_{\text{NLL}} + \dots \right\}$$

- Hadronic cross section

- convolution with PDFs $\mathcal{L}_{ij} = f_i \otimes f_j$
- parametric form

$$\frac{d\sigma_{t\bar{t}}}{d\beta_{t\bar{t}}} \simeq \beta_{t\bar{t}} \mathcal{L}_{ij}(\beta_{t\bar{t}}) \hat{\sigma}_{ij \rightarrow t\bar{t}}(\beta_{t\bar{t}})$$

- Power counting

- one power $\beta_{t\bar{t}}$ from Born cross section $\sigma_{S\text{-wave}}^{(0)}$
- one power $\beta_{t\bar{t}}$ from phase space in convolution with $\mathcal{L}_{ij}$

- Breakdown of fixed order QCD perturbation theory

$$\bullet \quad d\sigma_{t\bar{t}}^{\text{LO}} \sim \beta_{t\bar{t}}^2, \dots, d\sigma_{t\bar{t}}^{\text{N}^3\text{LO}} \sim \frac{1}{\beta_{t\bar{t}}}, \quad d\sigma_{t\bar{t}}^{\text{N}^4\text{LO}} \sim \frac{1}{\beta_{t\bar{t}}^2}, \dots$$

Phenomenology

- **FCC-ee**: much work (theory and phenomenology) [many people]
 - fixed center-of-mass energy $\sqrt{S}$ allows $t\bar{t}$ threshold scan at $\sqrt{S} \sim 2m_t$
 - dominant color-singlet production $\rightarrow t\bar{t} \left({}^3S_1^{[1]} \right)$
- **FCC-ee**: bound state corrections also for other charged final states
 - W^+W^- -pairs with corrections from Coulomb photons
 $\Delta\sigma/\sigma_{\text{Born}} \simeq \mathcal{O}(\text{few}) \%$ Actis, Beneke, Falgari, Schwinn '08
- **LHC**: effects on top-mass measurement Hagiwara, Sumino, Yokoya '08
 - complete NLO NRQCD result Petrelli, Cacciari, Greco, Maltoni, Mangano '97 (corrections by Hagiwara, Sumino, Yokoya '08)
 - detailed study in NRQCD assembling existing knowledge at NLO/NLL Kiyo, Kühn, S.M., Steinhauser, Uwer '08
 - bound-state effects on kinematical distributions Sumino, Yokoya '10
 - more studies since 2020 . . .

Theory framework for hadro-production

- Recall master equation $\sigma_{pp \rightarrow t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}$
 - top-quark pairs produced as color-singlet/octet $\rightarrow t\bar{t} \left({}^{2s+1}S_J^{[1,8]} \right)$
 - threshold at $M_{t\bar{t}} \sim 2m_t$ with $M_{t\bar{t}} = (p_t + p_{\bar{t}})^2$

NRQCD factorization

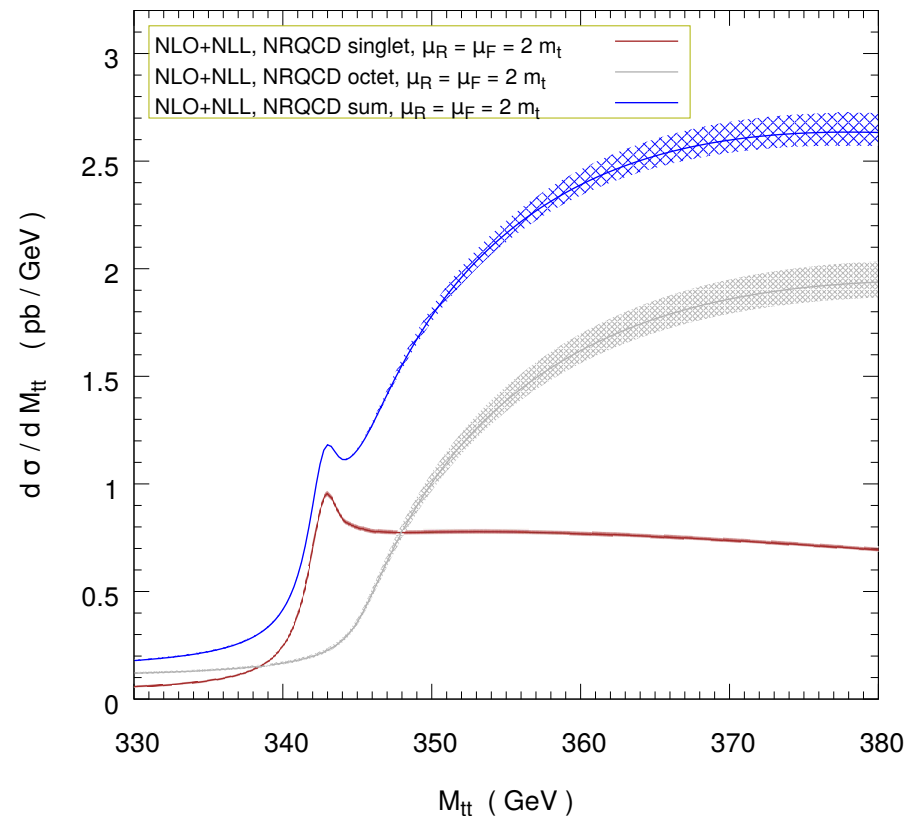
- Partonic cross section $\hat{\sigma}_{ij \rightarrow t\bar{t}} = F_{ij \rightarrow T} \times \Im G(M_{t\bar{t}})$
 - free $t\bar{t}$ production rate $F_{ij \rightarrow T}$
 - “bound state” Green’s function $\Im G$ (imaginary part)
- Coulomb resummation from solution of Schrödinger equation in NRQCD with QCD heavy quark potential $V_{\text{QCD}}(\vec{x})$
 - small energies $E = E_t + E_{\bar{t}} - 2m_t$ in toponium rest frame

$$\left\{ -\frac{\vec{\nabla}^2}{m_t} + V_{\text{QCD}}(\vec{x}) - (E + i\Gamma_t) \right\} G(E, \vec{x}) = \delta^{(3)}(\vec{x})$$

- Differential kinematics $\frac{d\hat{\sigma}_{ij \rightarrow t\bar{t}}}{dM_{t\bar{t}}^2} = F_{ij \rightarrow T} \times \Im G^{[1,8]}(M_{t\bar{t}})$
 - soft-collinear resummation in $F_{ij \rightarrow T}$ at NLL and matching at NLO

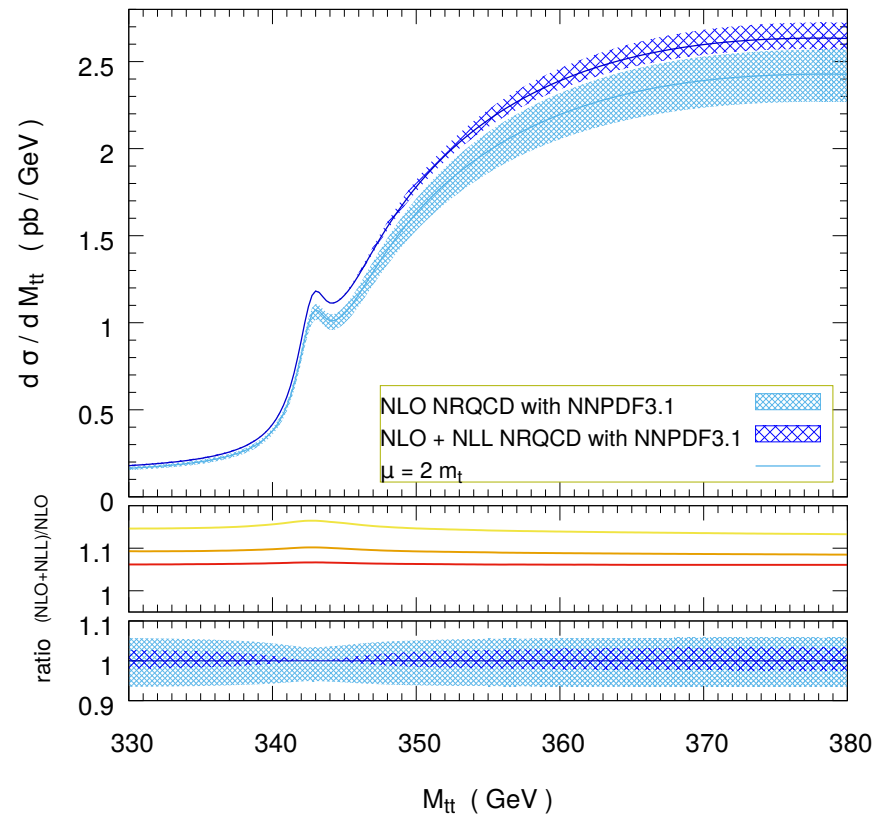
Invariant mass distribution (I)

- $d\sigma/dM_{t\bar{t}}$ at LHC ($\sqrt{S} = 13$ TeV) Garzelli, Limatola, S.M., Steinhauser, Zenaiev '24
- Parton channels with large threshold logarithms
 - $gg \rightarrow t\bar{t} \left({}^1S_0^{[1]} \right)$ attractive interaction $\rightarrow$ Coulomb resonance
 - $gg \rightarrow t\bar{t} \left({}^1S_0^{[8]} \right), q\bar{q} \rightarrow t\bar{t} \left({}^3S_1^{[8]} \right)$ repulsive interaction



Invariant mass distribution (II)

- $d\sigma/dM_{t\bar{t}}$ at LHC ($\sqrt{S} = 13$ TeV) for top quark pole mass $m_t = 172.5$ GeV
 - fixed order NLO vs. NLO+NLL resummed NRQCD predictions
 - scale variation $\mu_r = \mu_f \in \{m_t, 2m_t, 4m_t\}$

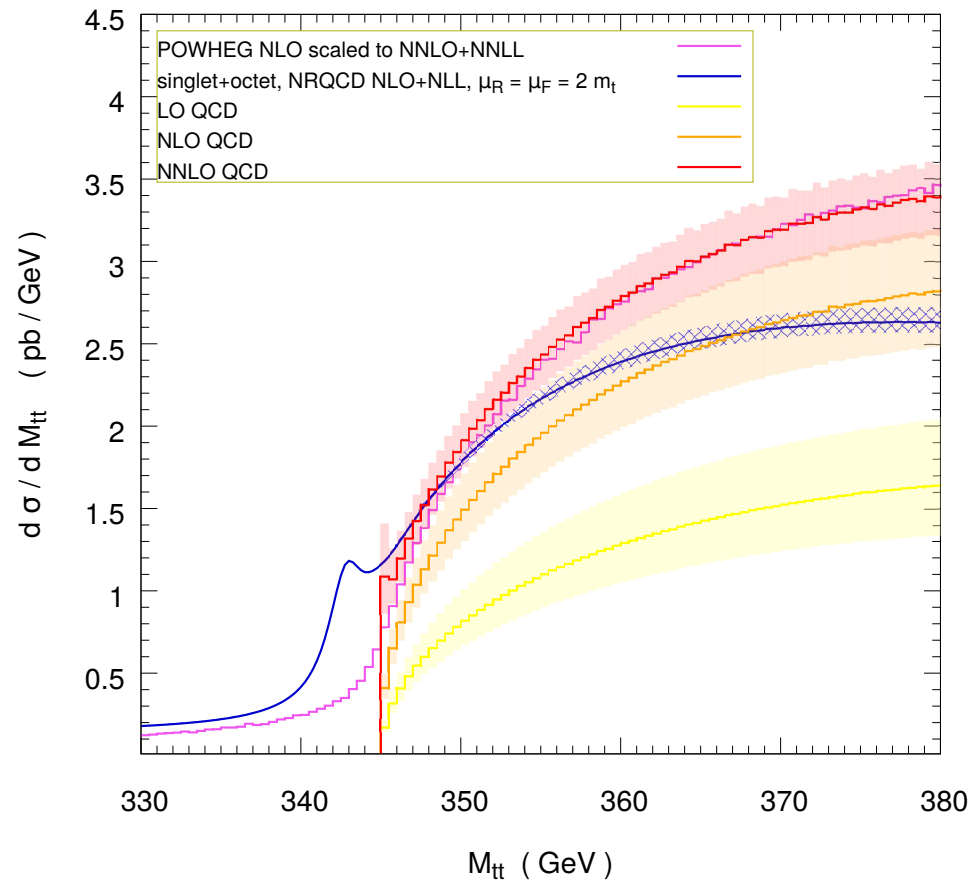


- S -wave channels without large threshold logarithms omitted
 - $gg \rightarrow {}^3S_1^{[1,8]}, q\bar{q} \rightarrow {}^1S_0^{[1,8]}, gq \rightarrow {}^1S_0^{[1,8]}, gq \rightarrow {}^3S_1^{[8]}$ sum up to $\mathcal{O}(5\%)$

Matching to fixed order (I)

- $d\sigma/dM_{t\bar{t}}$ with NNPDF3.1 PDFs; threshold at $dM_{t\bar{t}} = 345$ GeV
 - NLO+NLL resummed NRQCD predictions
 - fixed order QCD at LO, NLO, NNLO with adapted version of MATRIX

Garzelli, Mazzitelli, S.M., Zenaiev '23
 - NLO+POWHEG (rescaled to NNLO+NNLL total cross section)
- Anuar, Grohsjean, Jeppe, Schwanenberger [private work]

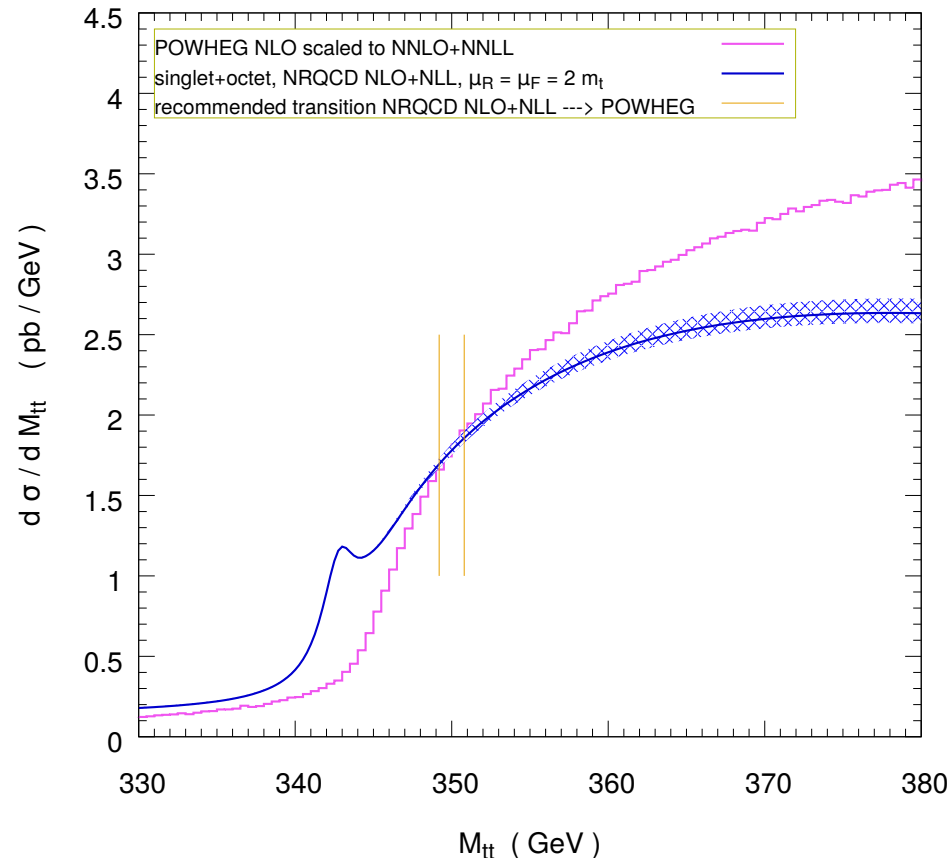


Matching to fixed order (II)

- $d\sigma/dM_{t\bar{t}}$ with matching of NRQCD to fixed order perturbation theory
 - NLO+NLL resummed NRQCD predictions
 - NLO+POWHEG (rescaled to NNLO+NNLL total cross section)

Anuar, Grohsjean, Jeppe, Schwanenberger [private work]

- Validity of NRQCD theory framework in range $M_{t\bar{t}} \in [340, 350]$ GeV



Recent phenomenology

- Resummation (small $\beta_{t\bar{t}}$) in SCET

Wan-Li Ju, Guoxing Wang, Xing Wang, Xiaofeng Xu, Yongqi Xu, Li Lin Yang '20

- double-differential distribution in $M_{t\bar{t}}$ and $y_{t\bar{t}}$

- Signatures of toponium formation

Fuks, Hagiwara, Ma, Zheng '21

- color-singlet spin-0 toponium bound state η_t of $t\bar{t}$
 - di-lepton decay $gg \rightarrow \eta_t \rightarrow l\bar{l}b\bar{b}\nu\bar{\nu}$

- Re-weighting of $t\bar{t}$ matrix elements with a NRQCD Green's function

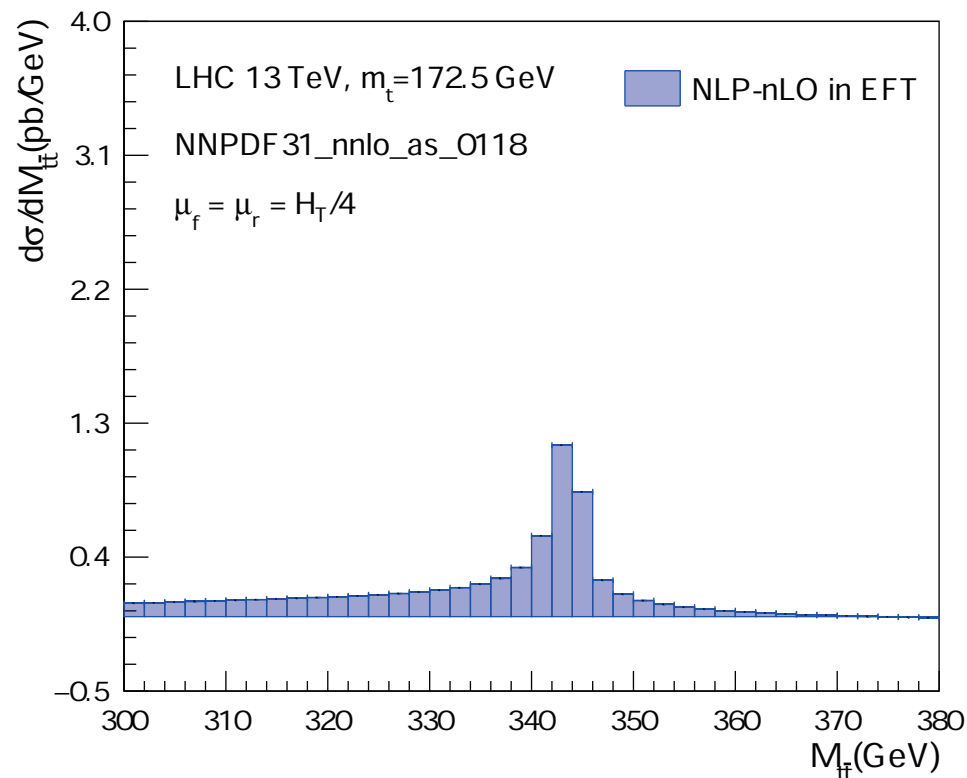
Fuks, Hagiwara, Ma, Zheng '24

Comment on 2004.03088

- Resummation effects in top quark mass determination with claimed shift of $m_t \sim 1.4$ GeV

Wan-Li Ju, Guoxing Wang, Xing Wang, Xiaofeng Xu, Yongqi Xu, Li Lin Yang '20

- Contributions from resummation in region $300 \text{ GeV} \leq M_{t\bar{t}} \leq 380 \text{ GeV}$ at the 13 TeV LHC



Comment on 2102.11281

- Signatures of spin-0 toponium bound state η_t decay
Fuks, Hagiwara, Ma, Zheng '21
 - theory estimate based on Sumino, Yokoya '10
 - di-lepton decay $gg \rightarrow \eta_t \rightarrow l\bar{l}b\bar{b}\nu\bar{\nu}$
- Short-comings
 - toponium cross section essentially as integral over $M_{t\bar{t}}$
(way too large range in $M_{t\bar{t}}$ for application of NRQCD)
 - neglect color-octet configurations (no enhancement)
- Published ratios for toponium cross sections σ_{η_t} vs. $\sigma_{t\bar{t}}$ not suitable for re-weighting of MC event samples in experimental analysis

Comment on 2411.18962

- Re-weighting matrix elements for $t\bar{t}$ production, combined with parton showering Fuks, Hagiwara, Ma, Zheng '24
- Short-comings
 - no higher order effects
 - no distinction of S -wave, P -wave, ...
 - no distinction of color-singlet/octet configurations
- Very large suggested range for re-weighting of matrix elements
 - $|M_{t\bar{t}} - 2m_t| \leq 20 \text{ GeV}$ too large?

Summary

- Coulomb corrections and their resummation for $t\bar{t} + X$ production
 - update of resummation studies at NLL for CMS analyses
- Combination with QCD perturbation theory to NNLO for $t\bar{t} + X$ production
 - bin-wise matching