Particles, Strings and Cosmology 2025

Strong interactions

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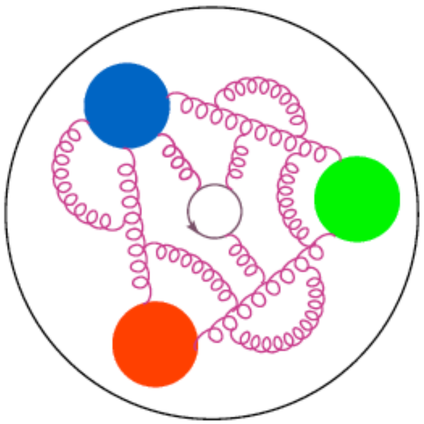
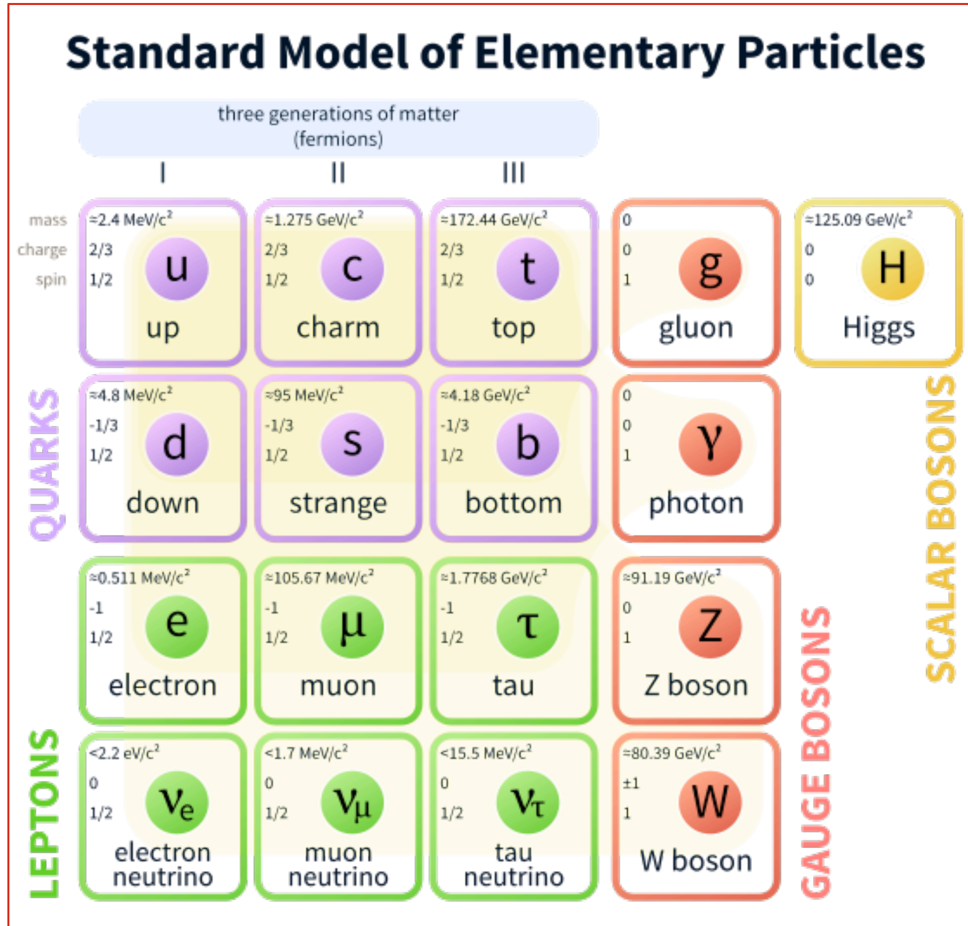
CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

DESY.

Strong interactions



https://videos.cern.ch/record/1989447



The color symmetry

Font for this QCD part: Halzen-Martin

Quantum Chromodynamics

- The theory of strong interactions is based on the SU(3) group*.

- The discovery of states like the Δ^{++} (=uuu) with spin=3/2 posed the problem of having all identical quarks on the same ground state

$$\mathbf{u}\uparrow\ \mathbf{u}\uparrow\ \mathbf{u}\uparrow$$

so that an additional quantum number, the „colour" was introduced. Mesons and baryons are colourless, i.e. the coulour wavefunction of a baryon:

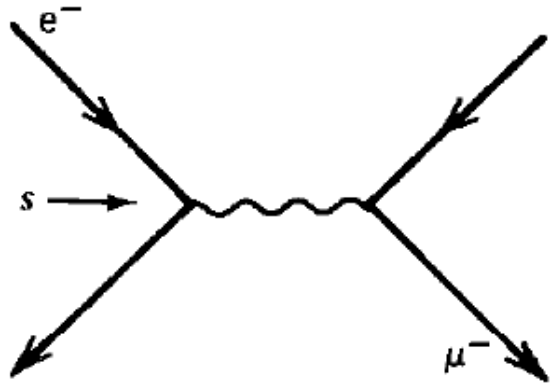
$$(\mathrm{qqq})_{\mathrm{col.\ singlet}} = \sqrt{\tfrac{1}{6}}\,(\mathrm{RGB} - \mathrm{RBG} + \mathrm{BRG} - \mathrm{BGR} + \mathrm{GBR} - \mathrm{GRB})$$

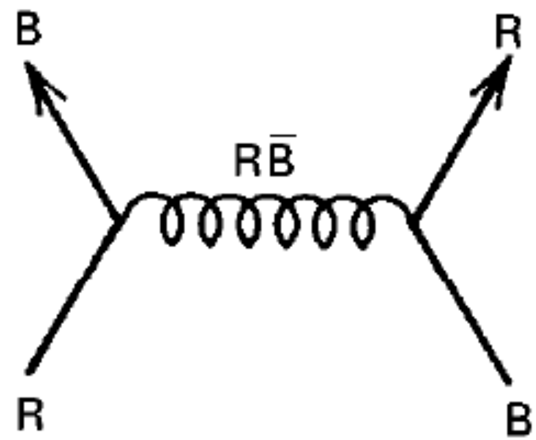
Gluons are coloured and there are 8 generators of the group (3^2-1), as one is a colour singlet

$$\mathbf{R\bar{G}}, \mathbf{R\bar{B}}, \mathbf{G\bar{R}}, \mathbf{G\bar{B}}, \mathbf{B\bar{R}}, \mathbf{B\bar{G}}, \sqrt{\tfrac{1}{2}}\,(\mathbf{R\bar{R}} - \mathbf{G\bar{G}}), \sqrt{\tfrac{1}{6}}\,(\mathbf{R\bar{R}} + \mathbf{G\bar{G}} - 2\mathbf{B\bar{B}})$$

* Three refers to the colours

Analogy to QED





The two diagrams look very similar, however:

- different coupling strength (α=1/128, α_s=0.116 at M(Z))
- QCD is a non-abelian theory, gluon are coloured and there are gluon self-interactions diagrams
- due to confinement, quarks are never observed free, but hadronize in bundles of particles. This process is not perturbative, and this poses limitation to the verification of the theory (<~1% precision at the moment)

Observation of 2 jets

First observed in the electron-positron colliders SPEAR

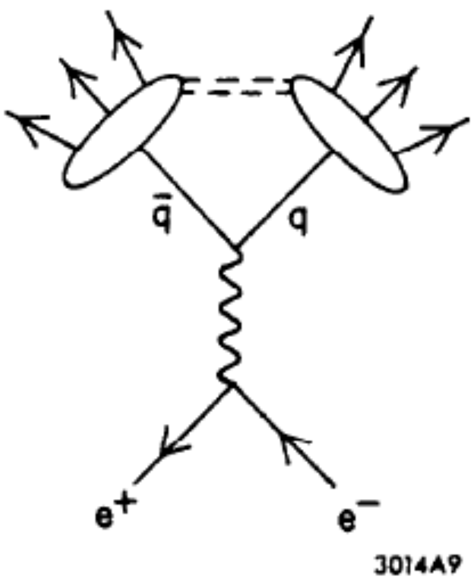


Fig. 9. Quark-parton model picture of production of hadrons in e^+e^- annihilation.

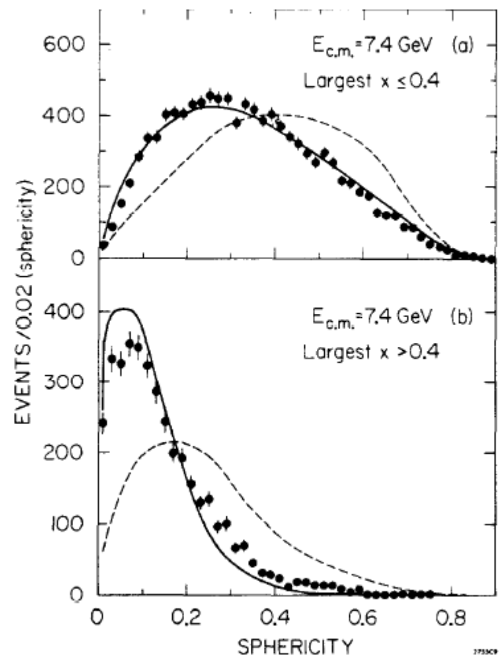
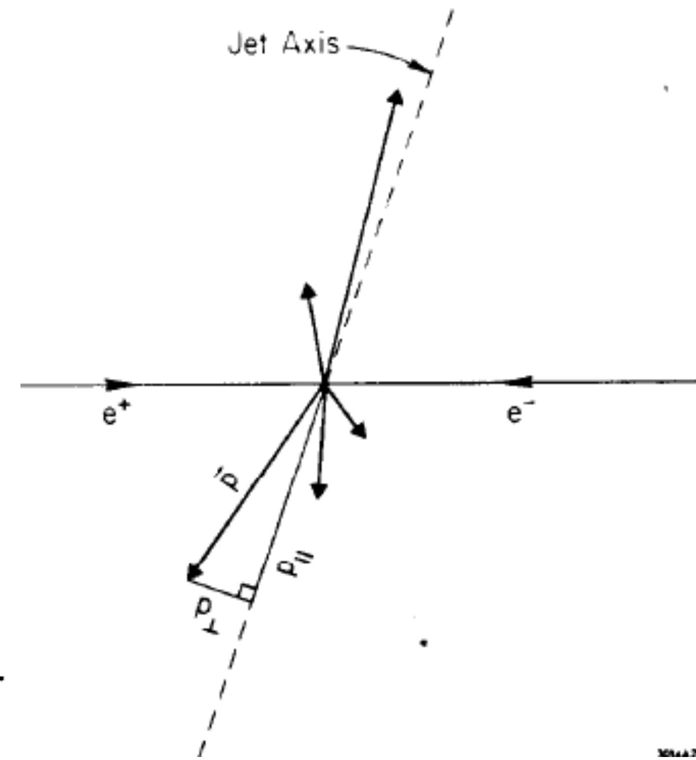
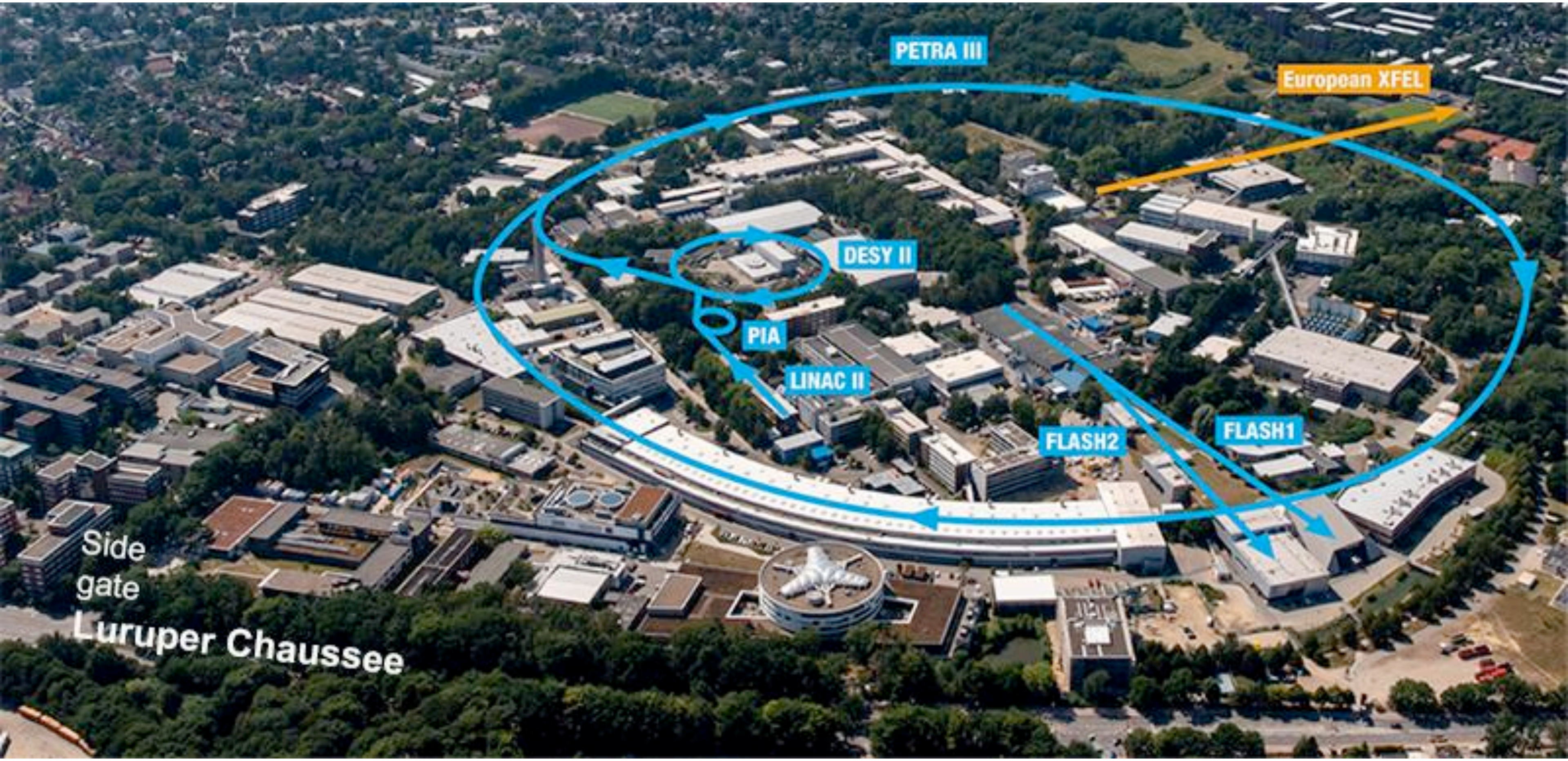


Fig. 16. Observed sphericity distributions at $E_{c.m.} = 7.4$ GeV for data, jet model (solid curves), and phase-space model (dashed curves) for (a) events with largest $x \leq 0.4$ and (b) events with largest $x > 0.4$.



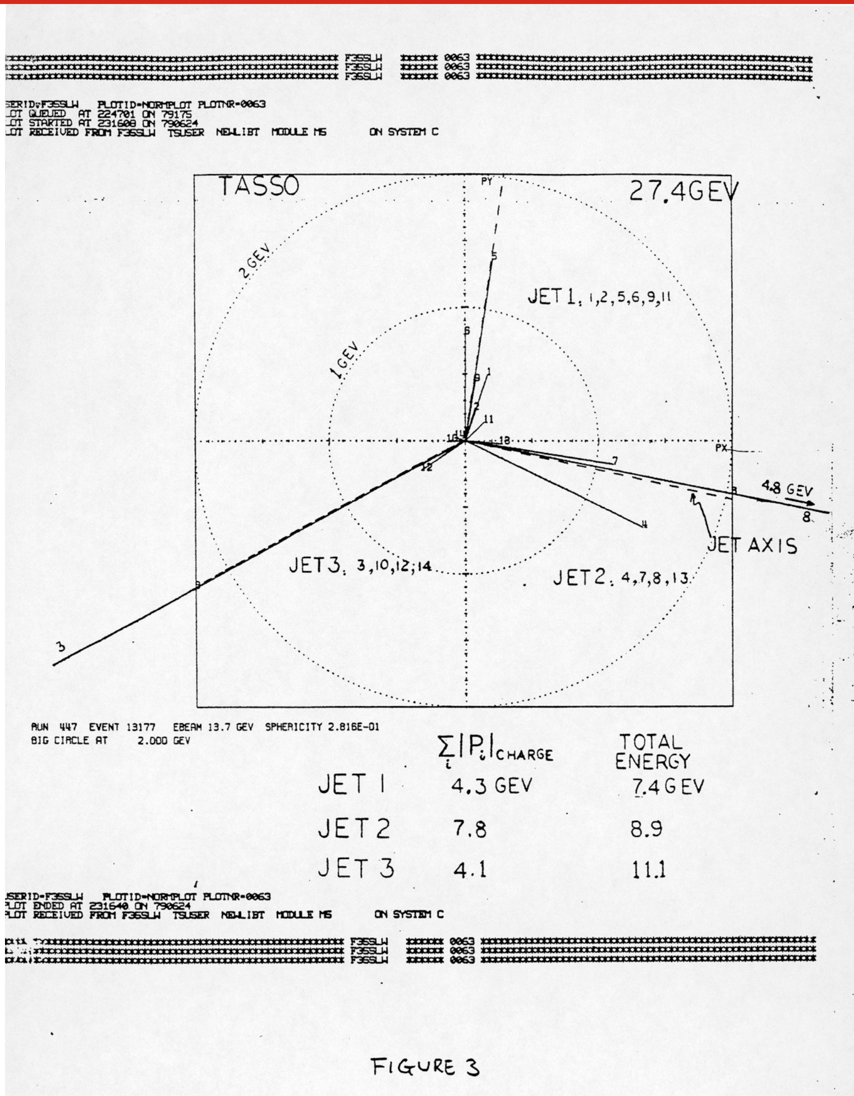
Font: SLAC-PUB-1814, SPEAR observation of 2 jets

DESY today

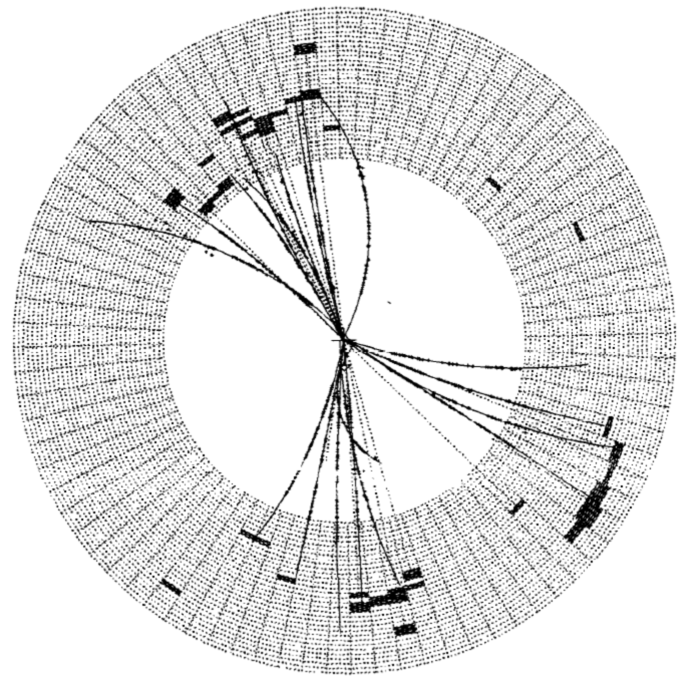


PETRA III
European XFEL
DESY II
PIA
LINAC II
FLASH2
FLASH1
Side
gate
Luruper Chaussee

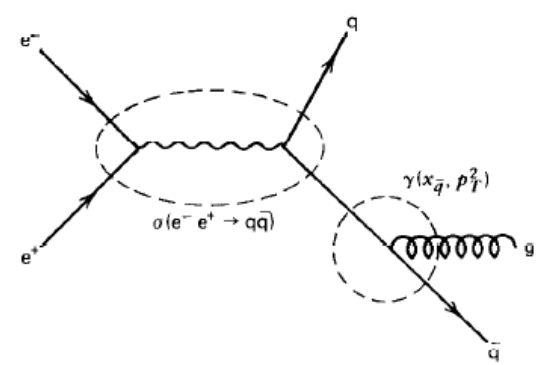
Observation of three jets



First event by TASSO (1979) at PETRA, DESY



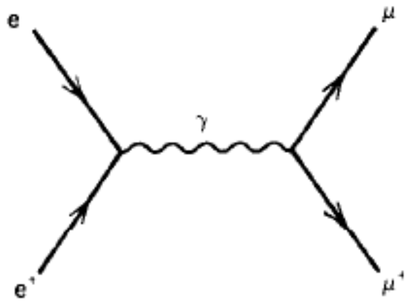
JADE experiment, DESY



Ratio R in e⁺e⁻

In lowest order the cross section for e⁺e⁻ in two muons in QED is given by:

$$\sigma(e^-e^+ \to \mu^-\mu^+) = \frac{4\pi\alpha^2}{3Q^2}$$

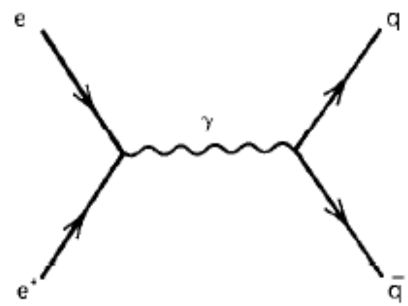


(a)

The diagram for the production of a quark pair is the same, except for:

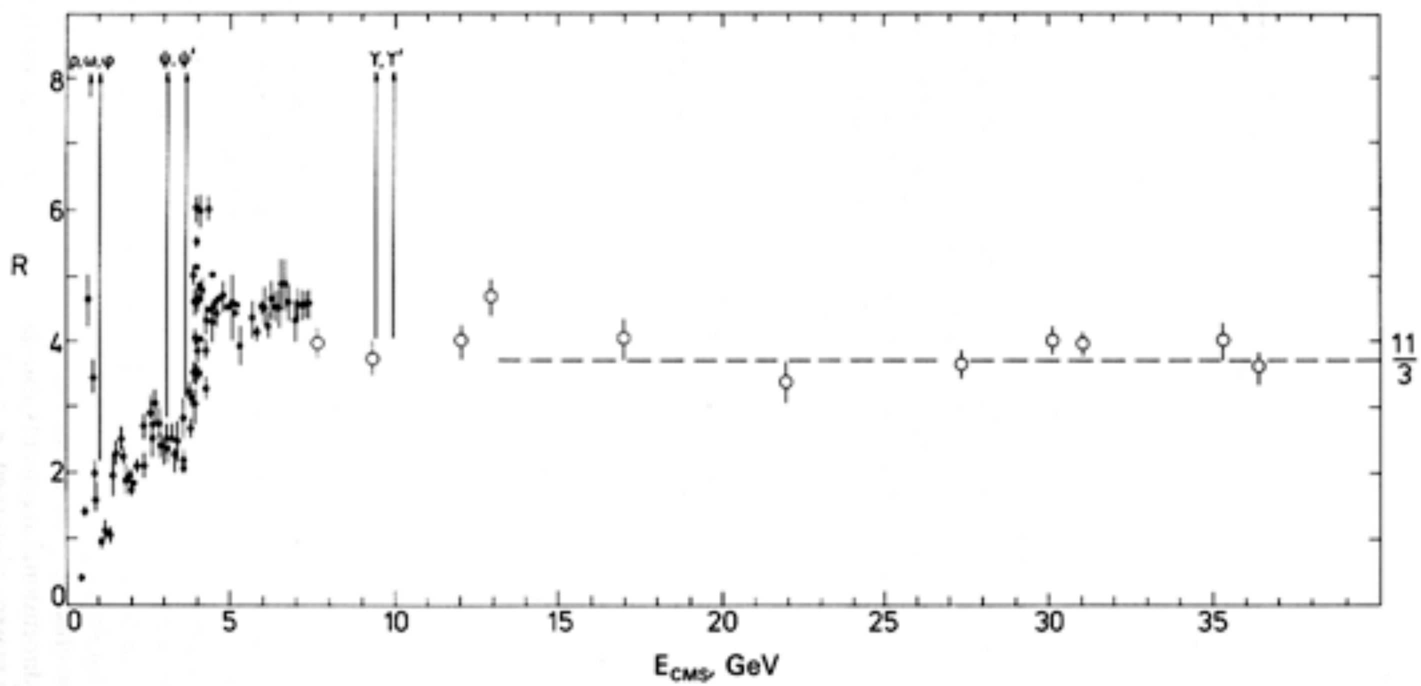
- the quark charge (squared)
- the factor 3 due to the number of coloured state of the quarks

This factor 3 in R was one of the experimental proofs of the 3 „colours" for quarks



$$R \equiv \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^-e^+ \to \mu^-\mu^+)} = 3\sum_q e_q^2.$$

Ratio R in e^+e^-



$$R = 3\left[\left(\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2\right] = 2 \qquad \text{for u, d, s,}$$

$$= 2 + 3\left(\tfrac{2}{3}\right)^2 = \tfrac{10}{3} \qquad \text{for u, d, s, c,}$$

$$= \tfrac{10}{3} + 3\left(\tfrac{1}{3}\right)^2 = \tfrac{11}{3} \qquad \text{for u, d, s, c, b}$$

Lagrangian and α_S

How many of you are familiar with the Lagrangian formalism in quantum field theory?

In QED

We require U(1) local gauge invariance*, where the phase $\alpha(x)$ is dependent on space and time point-to-point (that's what we are interested in)

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

However the Lagrangian of a free particle

$$\mathfrak{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

is not invariant under this trasformation. If we want the Lagrangian to have local gauge invariance, we have to do a trick and introduce the covariant derivative with a field A:

$$D_{\mu} \equiv \partial_{\mu} - ieA_{\mu}$$

$$\mathfrak{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi + e\bar{\psi}\gamma^{\mu}A_{\mu}\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

*invariance under UA(1) implies conservation of current

In QCD

In analogy to what is done in QED, let's start from the Lagrangian of a free particle

$$\mathcal{L}_0 = \bar{q}_j\left(i\gamma^\mu\partial_\mu - m\right)q_j$$

q=3 quark colour fields

And let's require the Lagrangian to be invariant under the local phase transformation SU(3):

$$q(x) \rightarrow Uq(x) \equiv e^{i\alpha_a(x)T_a}q(x)$$

T= 3x3 matrices

In analogy to what is done in QED, one can ask the Lagrangian to be invariant under local color phase transformation to the quark field. This can be achieved by replacing the derivative with the covariant derivative:

$$\partial_\mu q \rightarrow \qquad D_\mu = \partial_\mu + igT_aG^a_\mu$$

G= 8 gluon fields

QCD Lagrangian

The resulting Lagrangian is:

$$\mathfrak{L} = \bar{q}\left(i\gamma^{\mu}\partial_{\mu} - m\right)q - g\left(\bar{q}\gamma^{\mu}T_{a}q\right)G_{\mu}^{a} - \tfrac{1}{4}G_{\mu\nu}^{a}G_{a}^{\mu\nu}$$

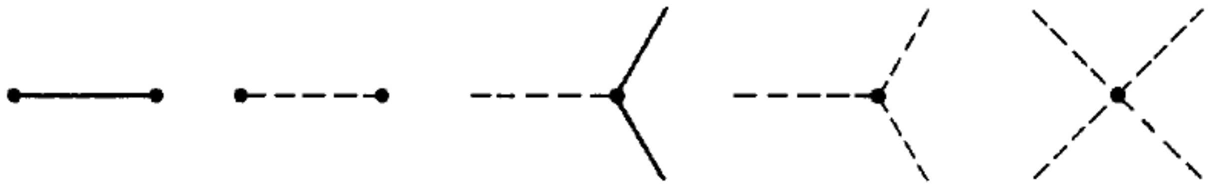
Quark line

Gluon-quark interaction with coupling g

Kinetic term +gluon self-interaction

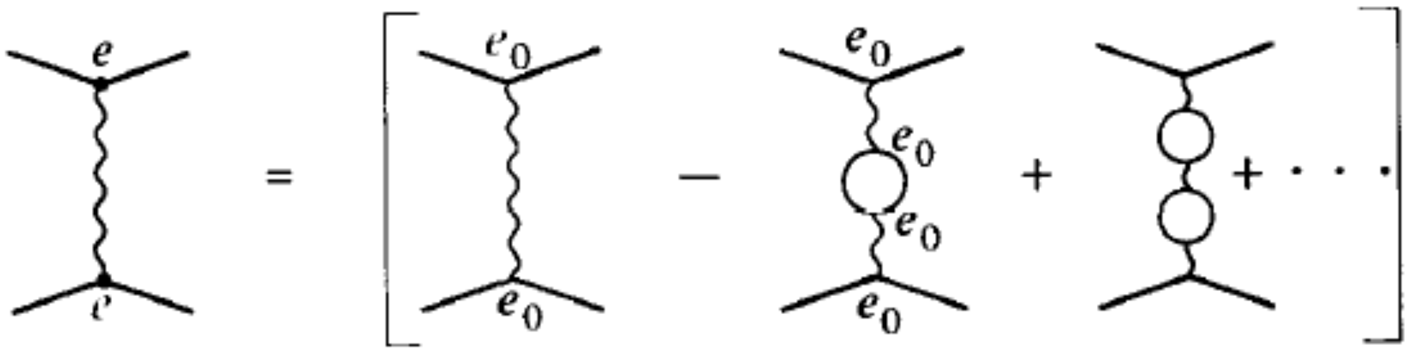
Due to the fact that gluons are „coloured", there are also three-gluon and four-gluon vertices, differently to QED. One can symbolically rewrite the Lagrangian as:

$$\mathfrak{L} = \text{“}\bar{q}q\text{”} + \text{“}G^{2}\text{”} + g\text{“}\bar{q}qG\text{”} + g\text{“}G^{3}\text{”} + g^{2}\text{“}G^{4}\text{”}.$$



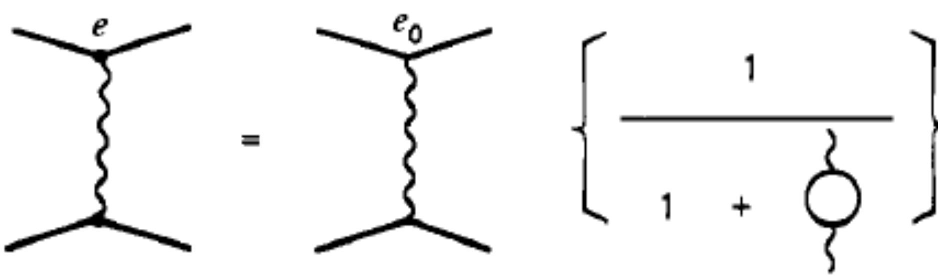
Running of alpha in QED

In QED we have to consider not only the diagrams with the „bare“ charge e_0, but also the loop corrections in the propagators. We can absorb the loop corrections in a redefinition of the charge e_0 -> e, this is the charge that we effectively measure.



at $Q^2 = \mu^2$

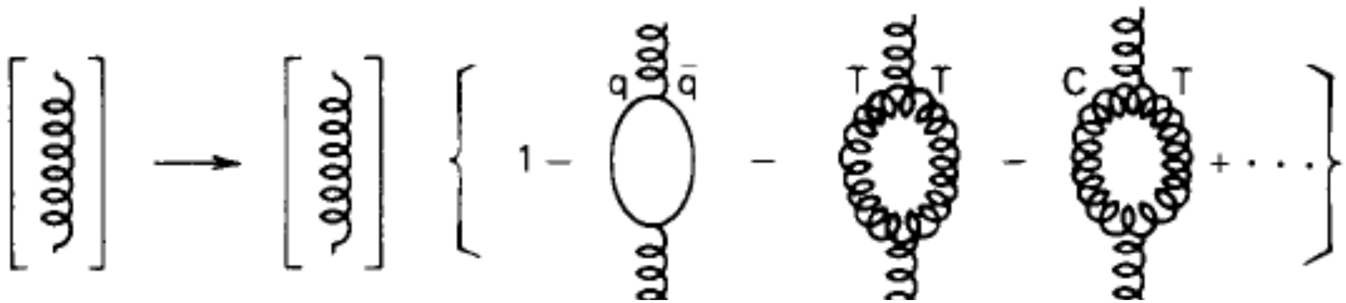
.The relation between the two has to be specified at a particular scale and the charge e (or alpha(QED)) becomes dependent on the scale Q^2 and in particular it increases with energy:



$$\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi}\log\left(\frac{Q^2}{M^2}\right)}$$

Running of α_S in QCD

Renormalization group equation for the strong coupling constant:



$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log(Q^2/\mu^2)}$$

Gluon-gluon term

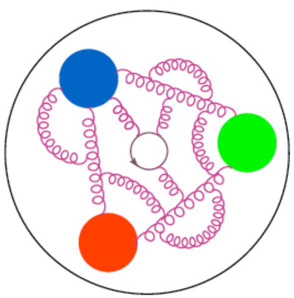
Quark term

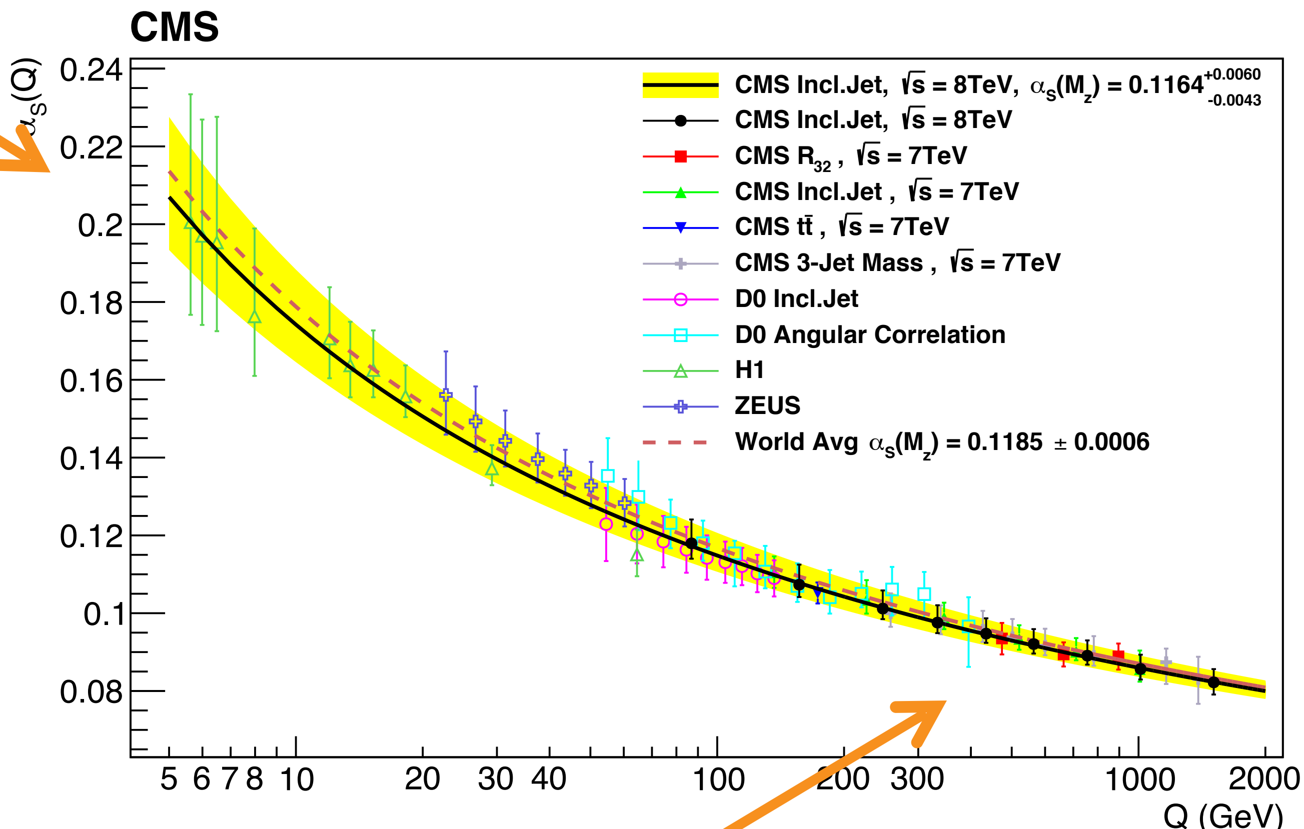
For 5 flavour n_f (true at the scale M(Z)), the dependence of $\boldsymbol{\alpha_S}$ on the energy is opposite to the one of alpha electromagnetic, that is $\boldsymbol{\alpha_S}$ decreases with increasing energy.

Font: Halzen-Martin

Strong coupling constant

α_S large, confinement of quarks inside hadrons



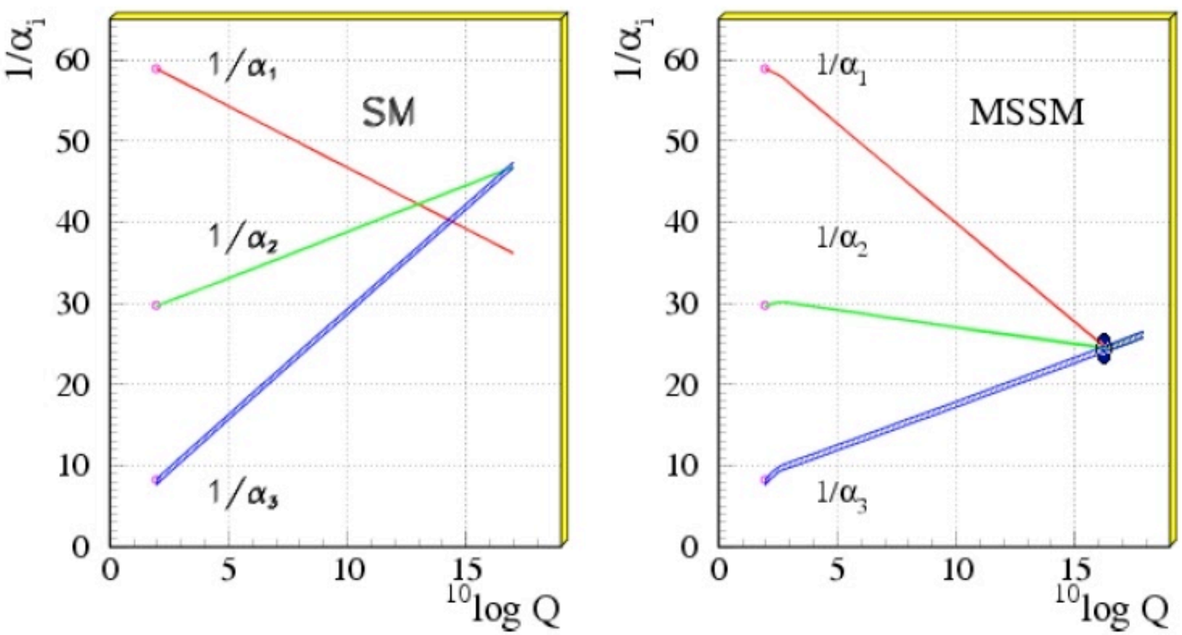


α_S small, can apply perturbative calculations

Asymptotic freedom: quarks are ‚free' at short distances and large energies

Famous plot on running

Running of the three coupling constant (1=e.m., 2=strong, 3=weak) after first precise measurements at LEP.

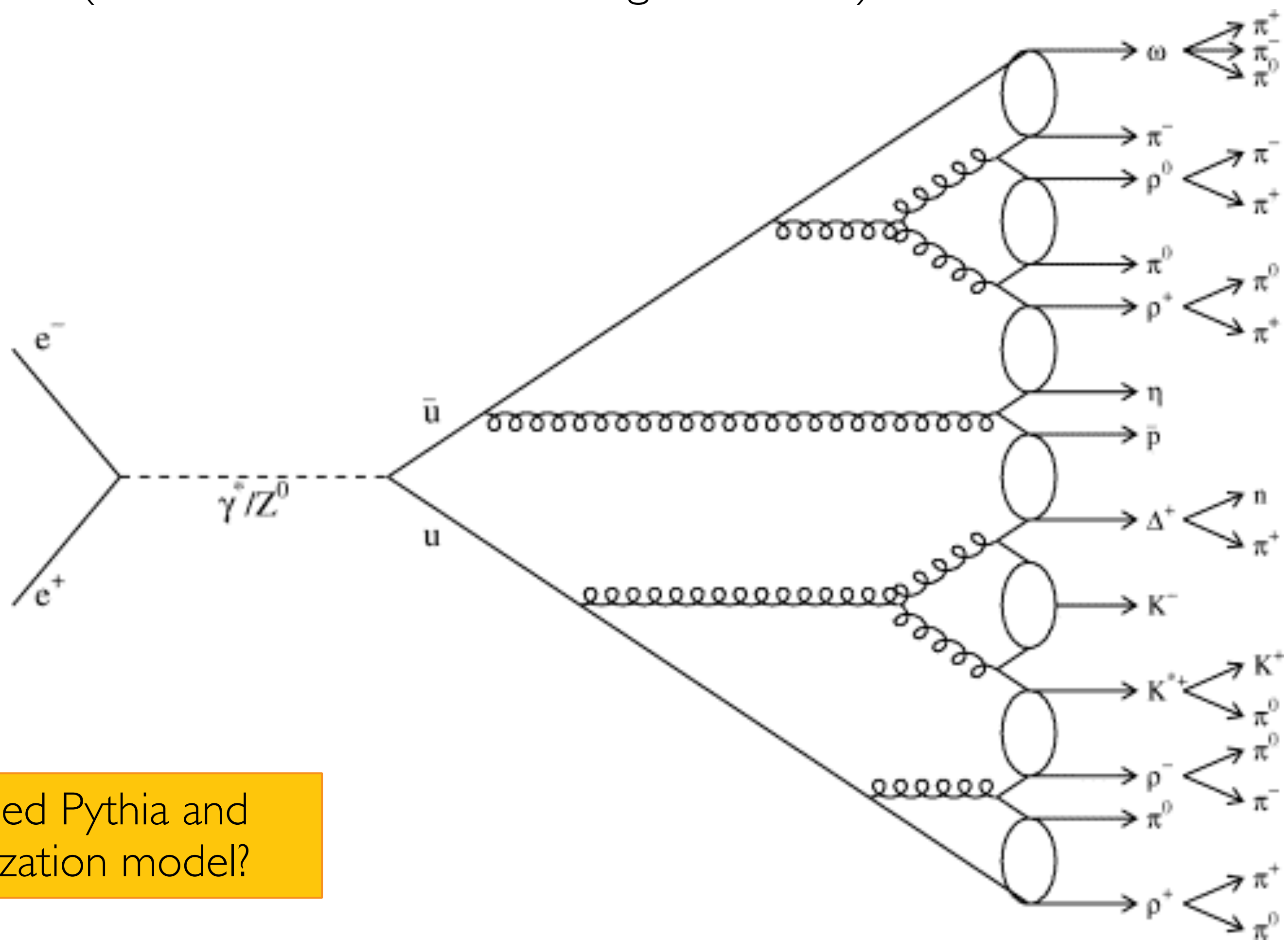


Why is this plot famous?

- A) My professor started to believe in Supersymmetry after seeing this plot
- B) It is not famous at all, and the second plot is a mistake
- C) It shows that the coupling constants will never meet
- D) I do not understand the question

Fragmentation

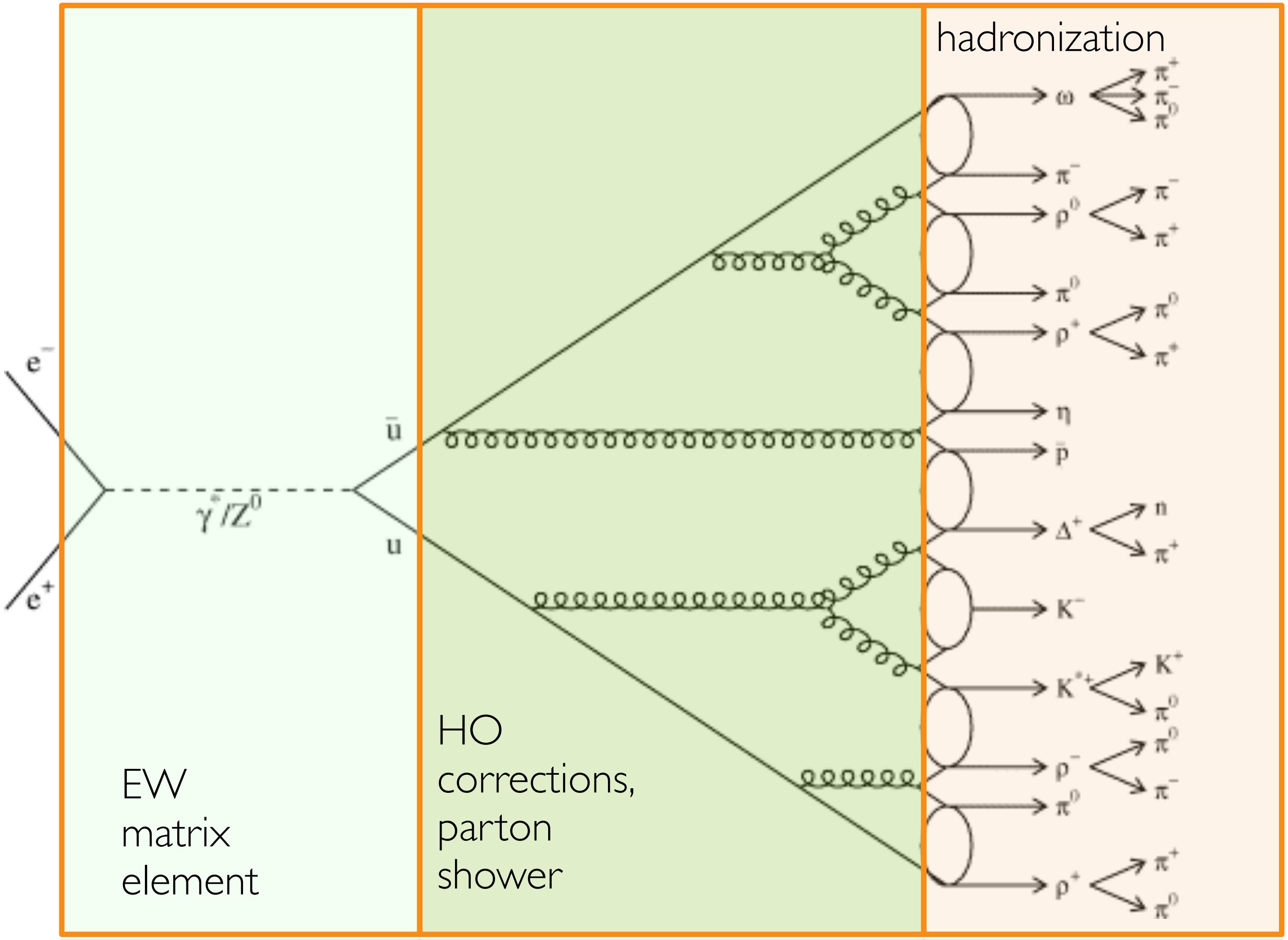
- Quarks are not observed free. What we observe are jets of hadrons in the direction of the quarks.
- The formation of hadrons is a non-perturbative process, described by empirical models (called hadronization or fragmentation)



Have you ever used Pythia and the Lund hadronization model?

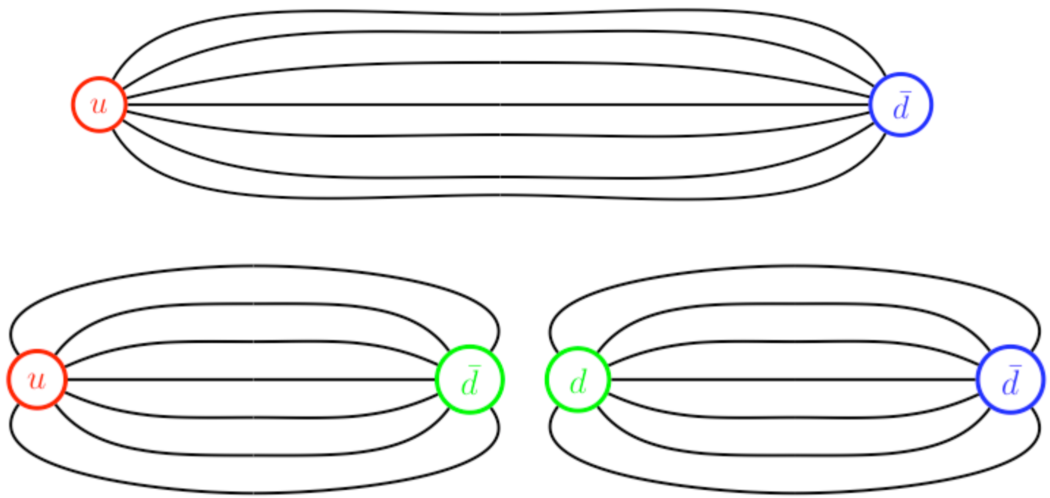
Fragmentation

Three steps in MC models:



Fragmentation models

- Fragmentation is a non perturbative process (small scales, large distances), is treated by phenomenological models, described in Monte Carlo programs with few parameters tuned to data.
- The most popular one is the Lund string fragmentation, called from the town of the developper (Pythia generator, etc.)



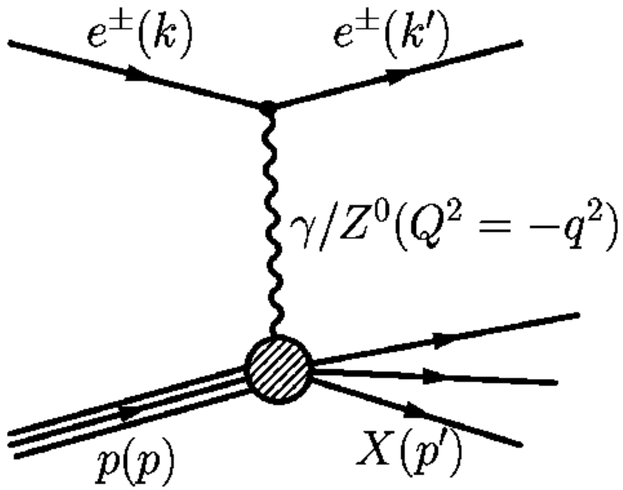
As two quarks try to get apart, a „string" (gluon field lines) is created, which finally breaks and creates another pair of quark-antiquark. At the end the pairs combine into final hadrons

DIS

Are you already familiar with Deep Inelastic Scattering?

Deep Inelastic scattering

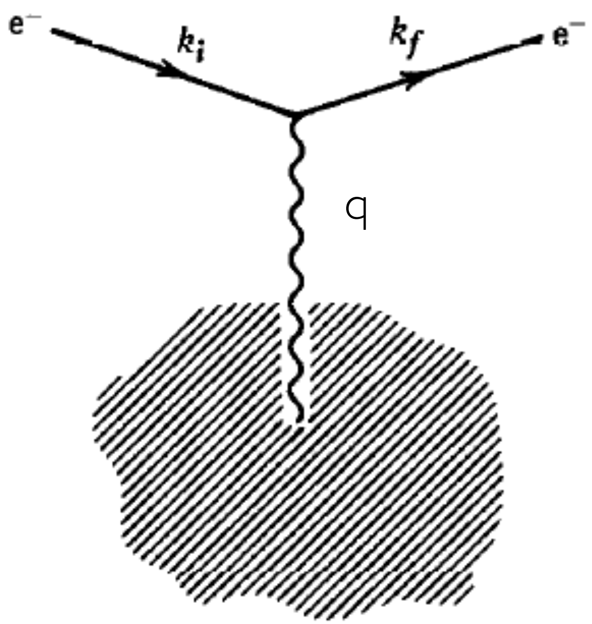
A big confirmation of the quark model first, and later of QCD as theory of strong interactions, came with experiments on Deep Inelastic Scattering:



- A pointlike beam typically probes the proton (electron, muon, neutrino)
- „Deep" : the four-momentum transfer q is „high" ~ few GeV^2
- Elastic: the proton does not break-up
- Inelastic: the proton breaks up in hadrons, like in the figure

Form factors

Let's first consider the scattering of an electron to an extended charged object, for simplicity with spin zero. The cross section is given by:



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(q)|^2,$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \equiv \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 E^2}{4k^4 \sin^4 \frac{\theta}{2}}\left(1 - v^2 \sin^2 \frac{\theta}{2}\right)$$

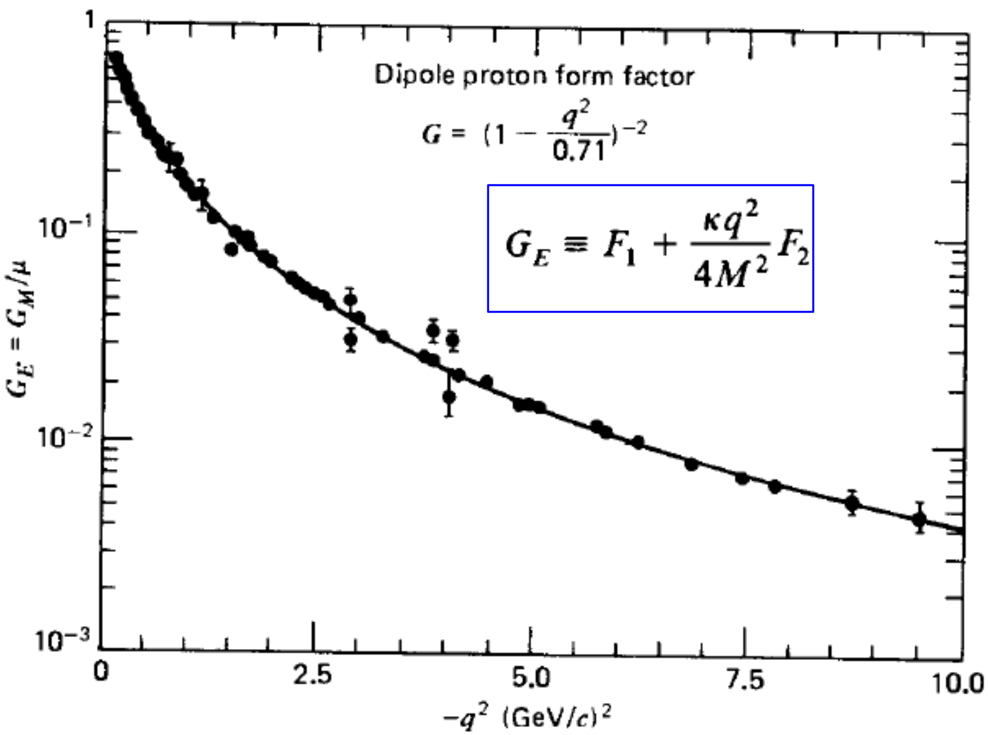
$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3x$$

F(q) is the Fourier trasform of the charge distribution and gives information about the structure of the target, θ is the electron scattering angle

Form factors

For scattering on protons, there is an additional magnetic form factor, due to the fact that the proton has spin ½, and the cross section is given by

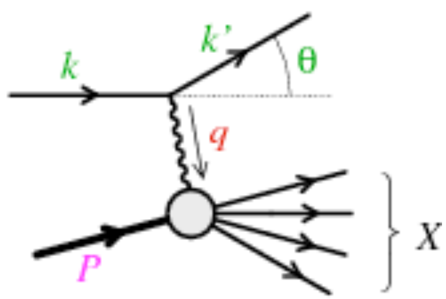
$$\left\{ \quad \right\}_{ep \to ep} = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \delta\left(\nu + \frac{q^2}{2M} \right)$$



Have you ever heard about the Rosenbluth formula?

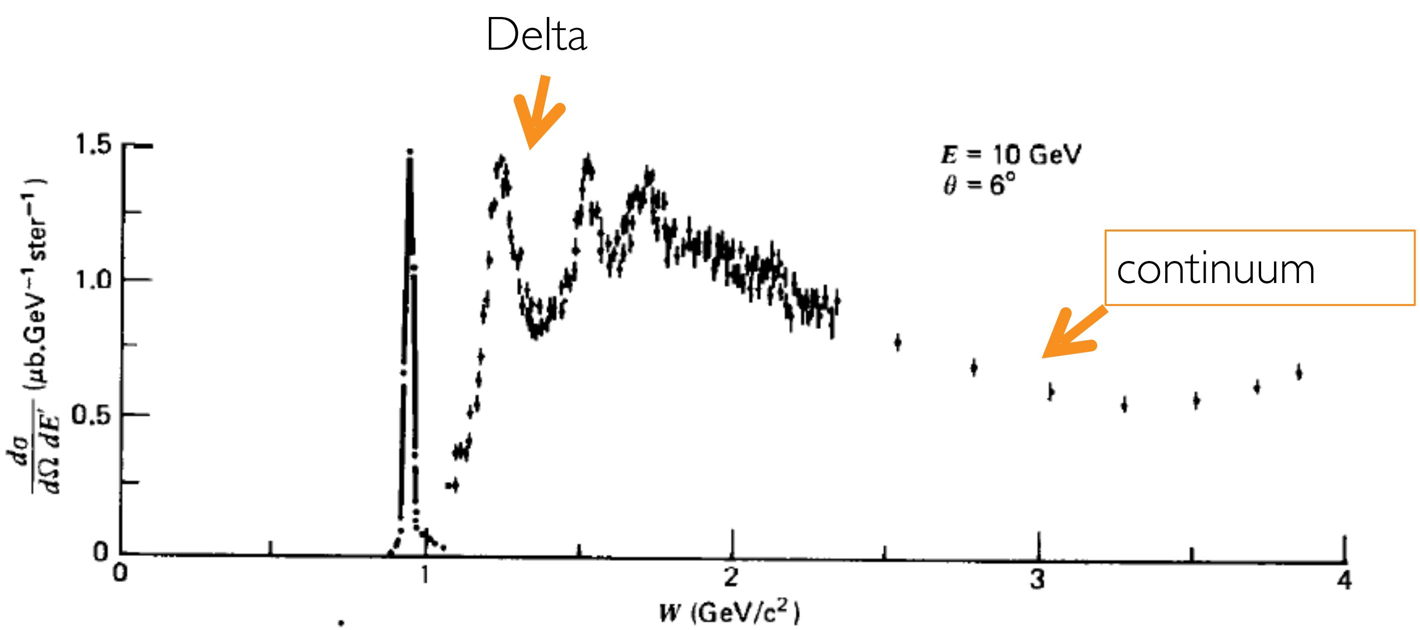
These form factors were determined experimentally measuring the cross section as a function of Q^2, like here for the G_E of the proton. The expected drop with Q^2 is clearly seen

Inelastic ep->eX



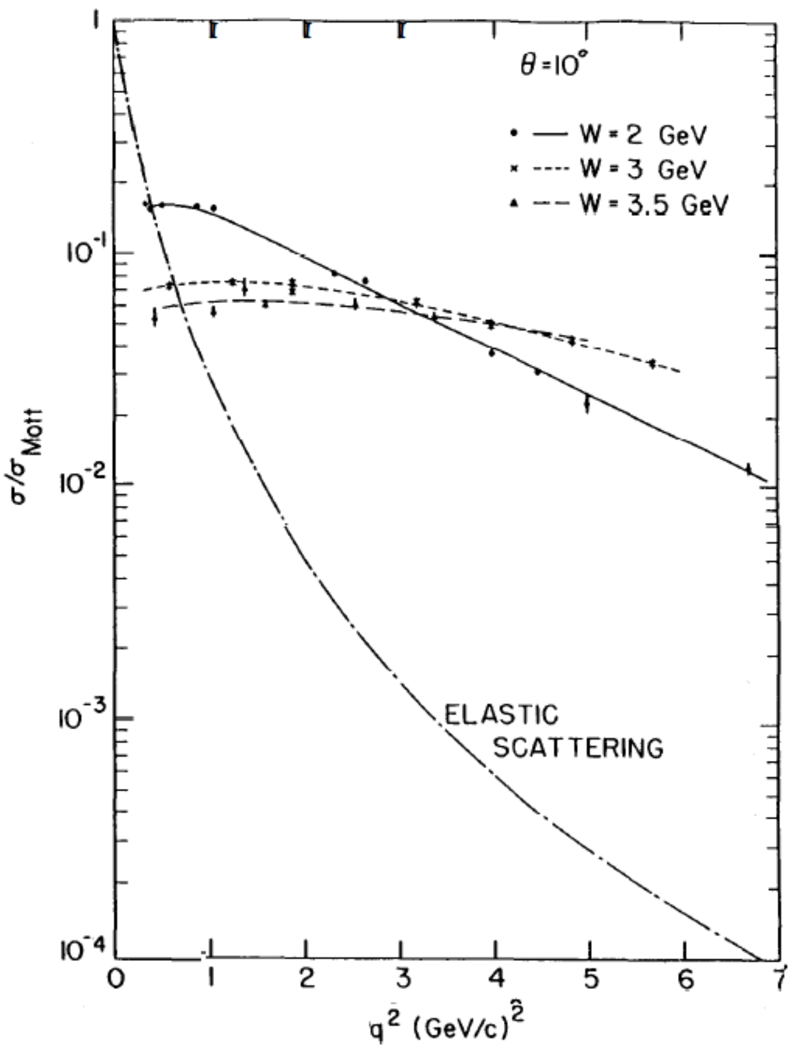
Increasing the Q^2, the proton first gets into an excited state (like the Δ) particle and then loses its identity and breaks up into many hadrons X at high invariant mass W. This is what is called the „continuum" and corresponds to inelastic scattering.

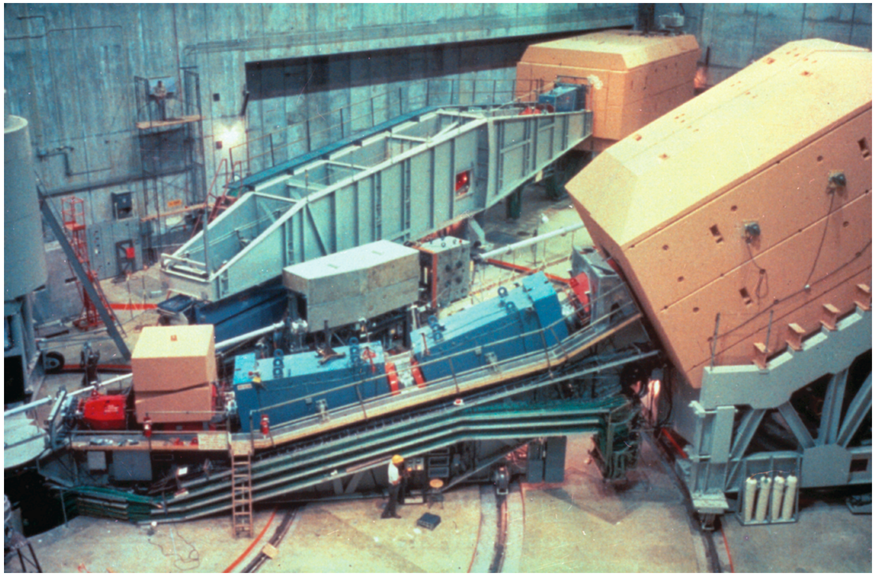
Data at SLAC



Deep Inelastic scattering

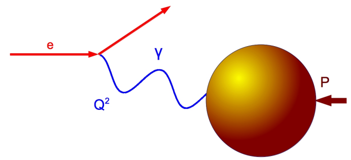
With the further increase of the beam energy at SLAC, a dramatic change in the cross section was observed

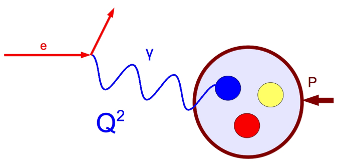


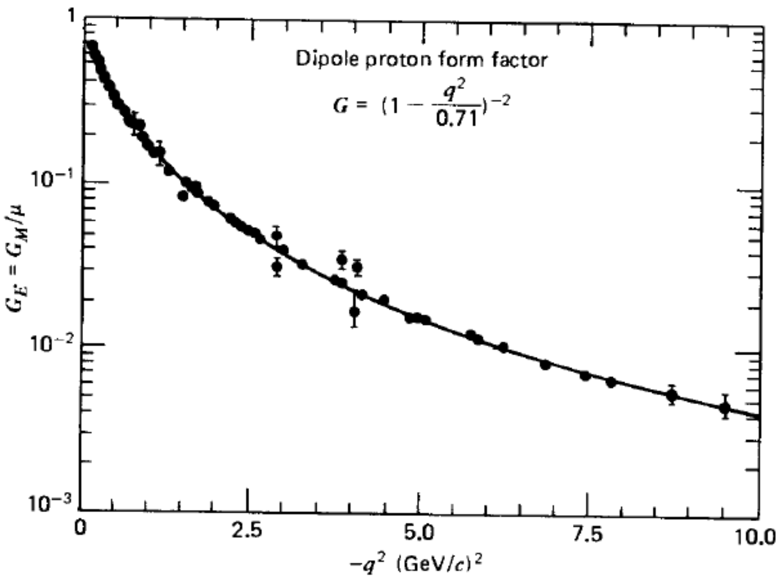


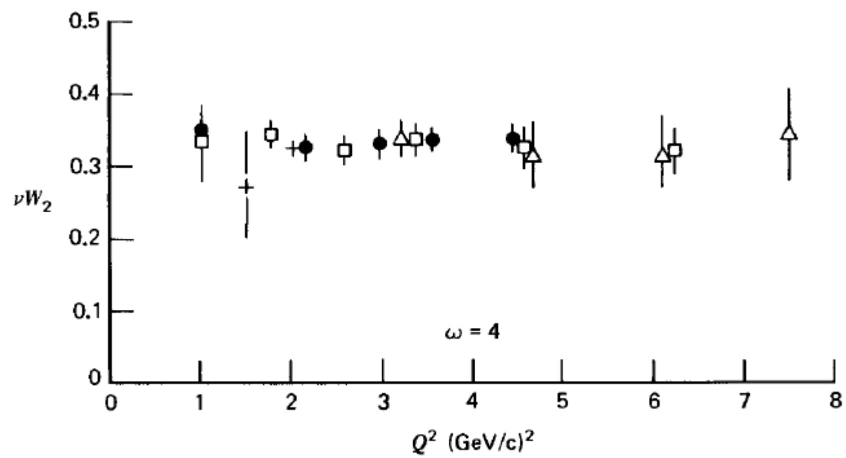
The cross section ratio was found to be constant vs Q^2 and not to have the usual form factor dependence

Bjorken scaling









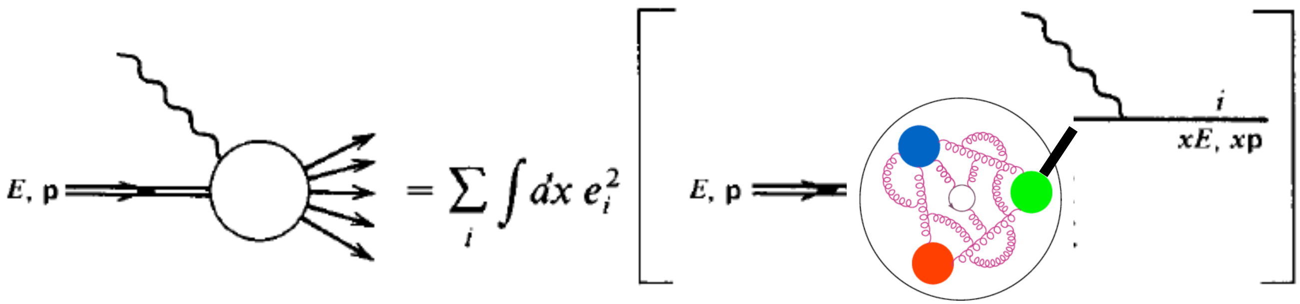
Reminder: d~ h/Q

If an object is pointlike, increasing Q^2, i.e. decreasing the distance, one does not expect to see a change in the structure function.

Interpretation from Feynman: scattering with a poinlike object

Quark Parton Model

Feynman interpreted the scaling observation in the Quark Parton Model



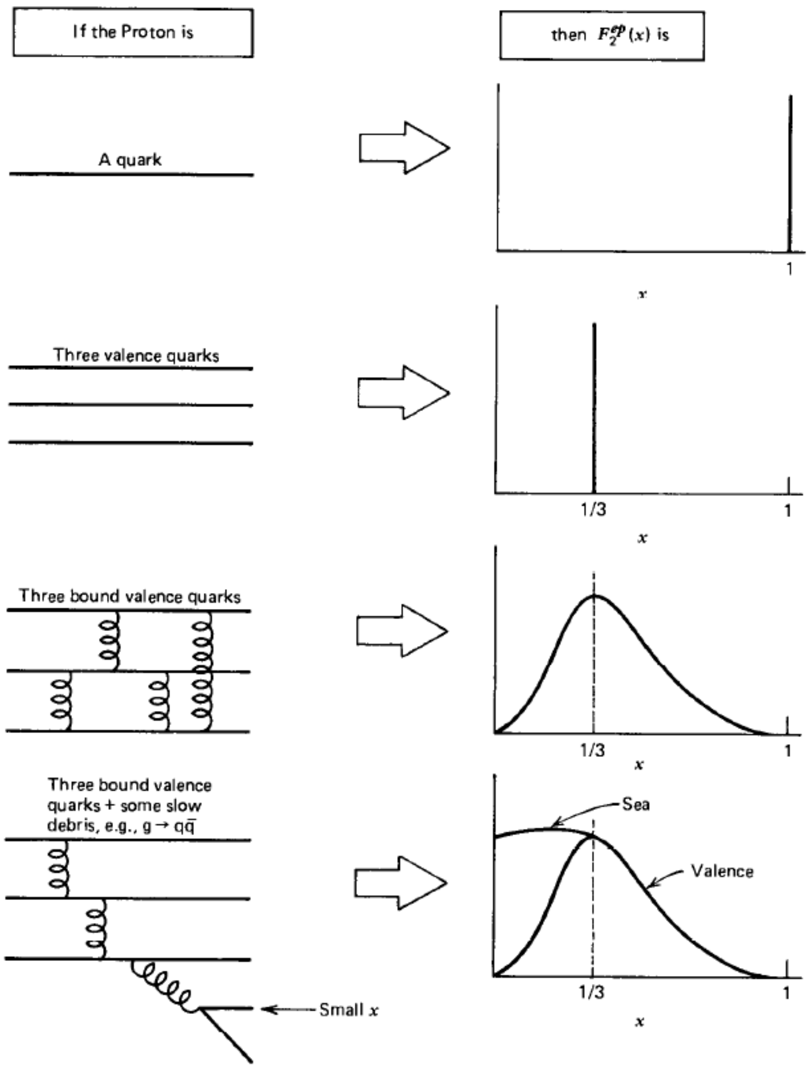
If the Q^2 is very high, the time of interaction of the virtual photon with the partons **(quarks/ gluons)** inside the proton is << than the time of interactions among the partons, which are then see as „quasi-free".
The scattering becomes then as a coherent sum of the individual electron-parton scatterings.

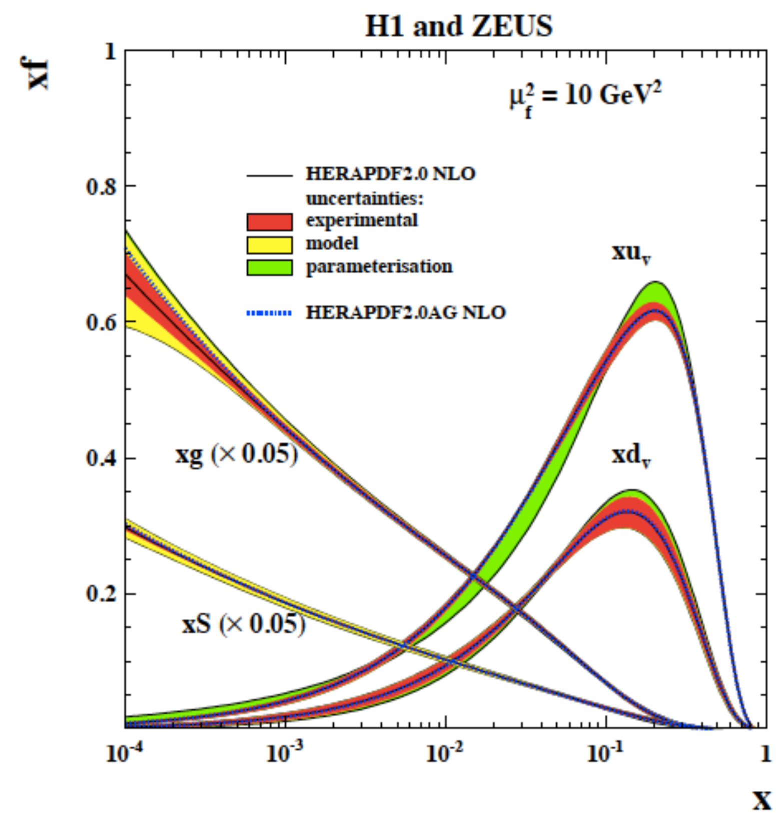
$$F_2(x) = \sum e_q^2 x f(x)$$

- F_2 is called the proton structure function and f(x) the individual parton densities

- The Bjorken scaling variable x is the fraction of proton momentum carried by the struck quark

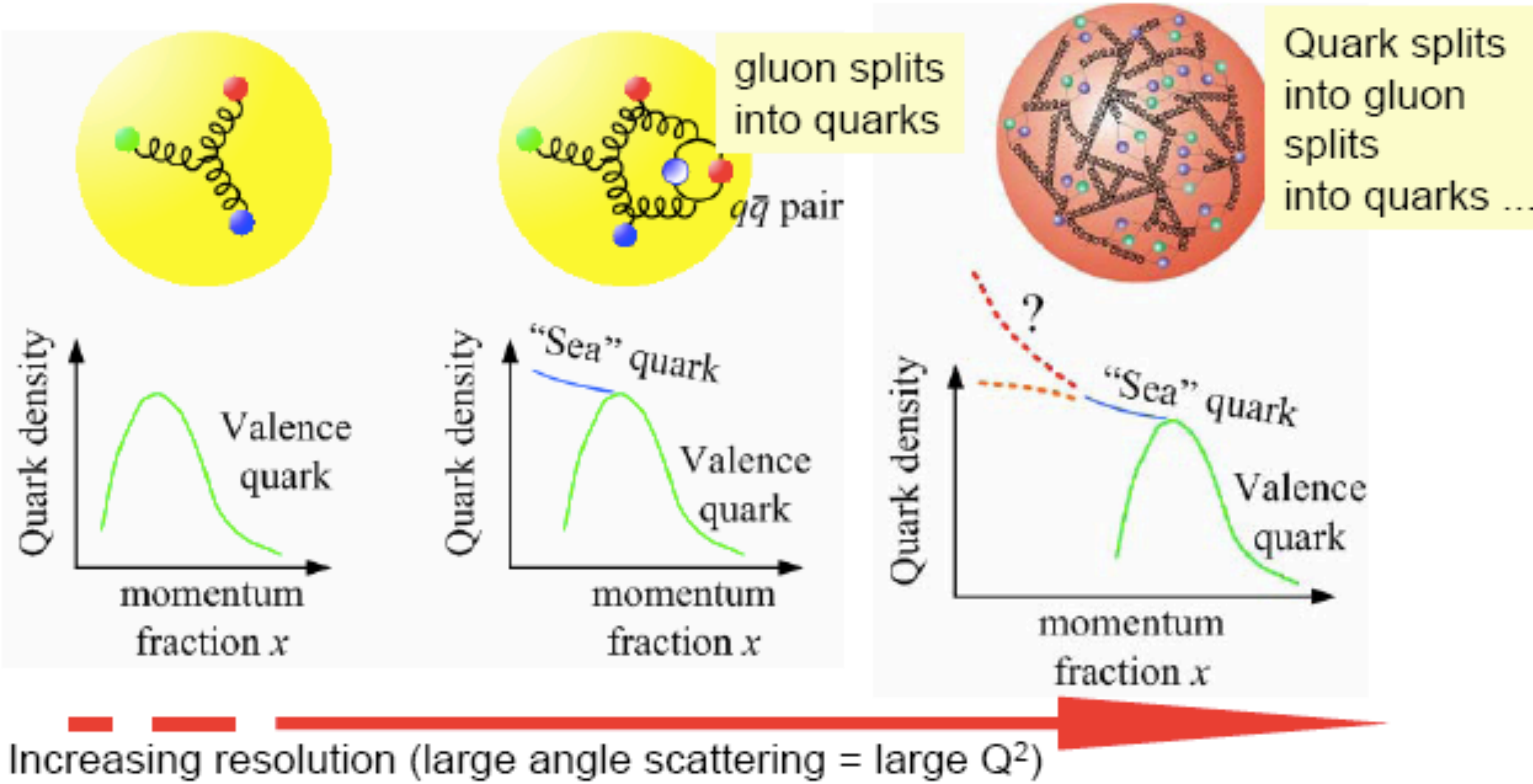
How are the $f_i(x)$?



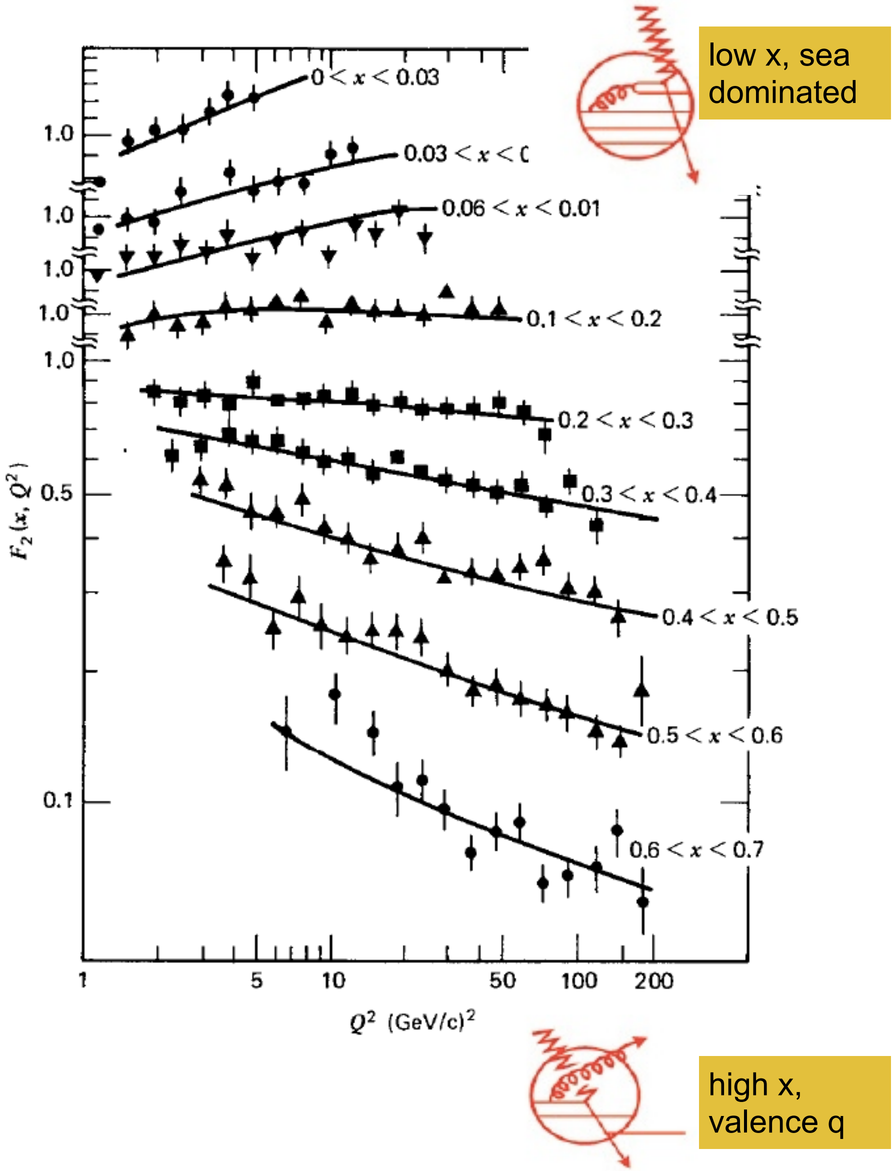


These functions are called PDF (parton density functions)

A proton picture



Scaling violations



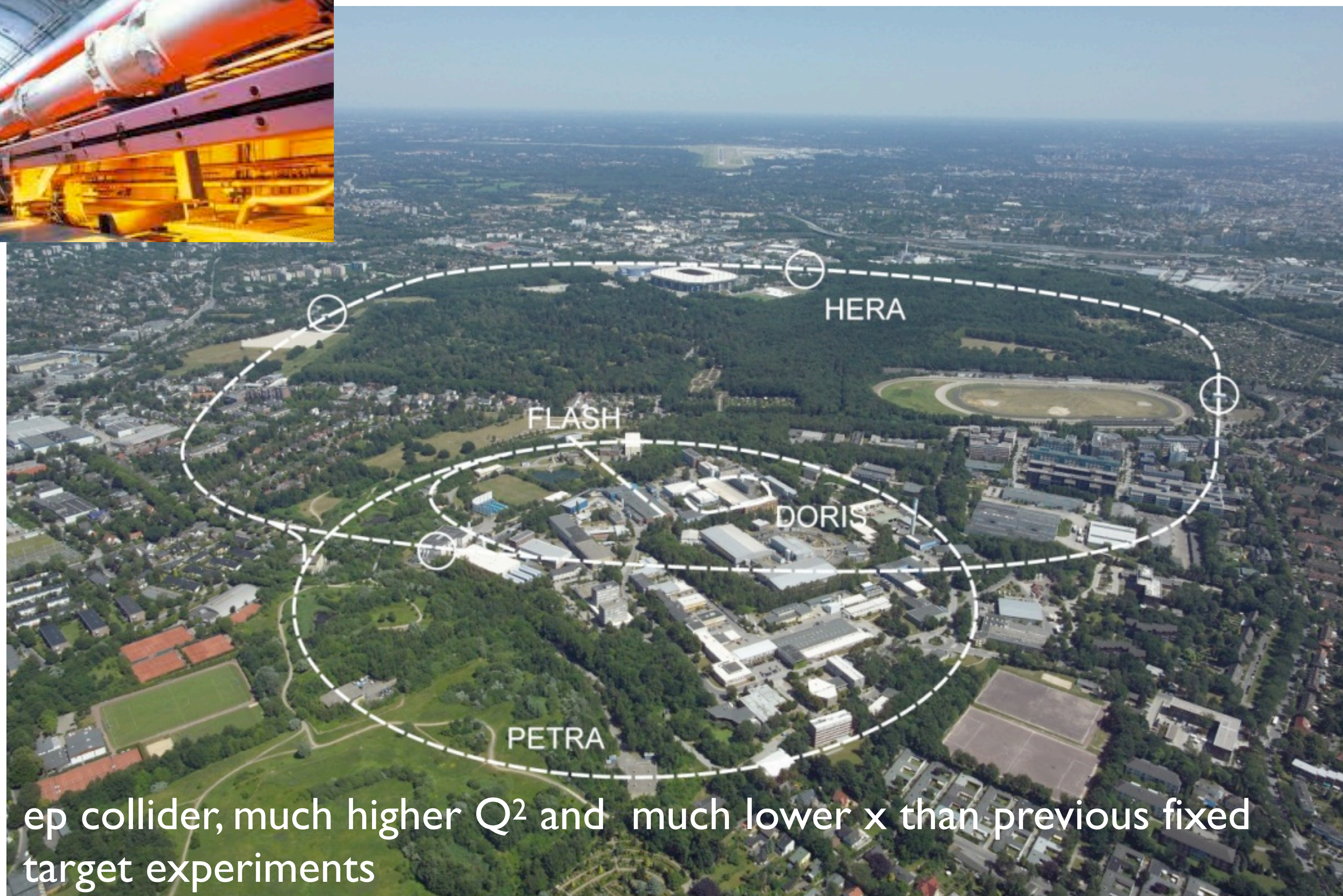
By increasing further the energy, scaling violations were observed:

F_2 increases with Q^2 at low x
And decreases with Q^2 at high x

What happens:

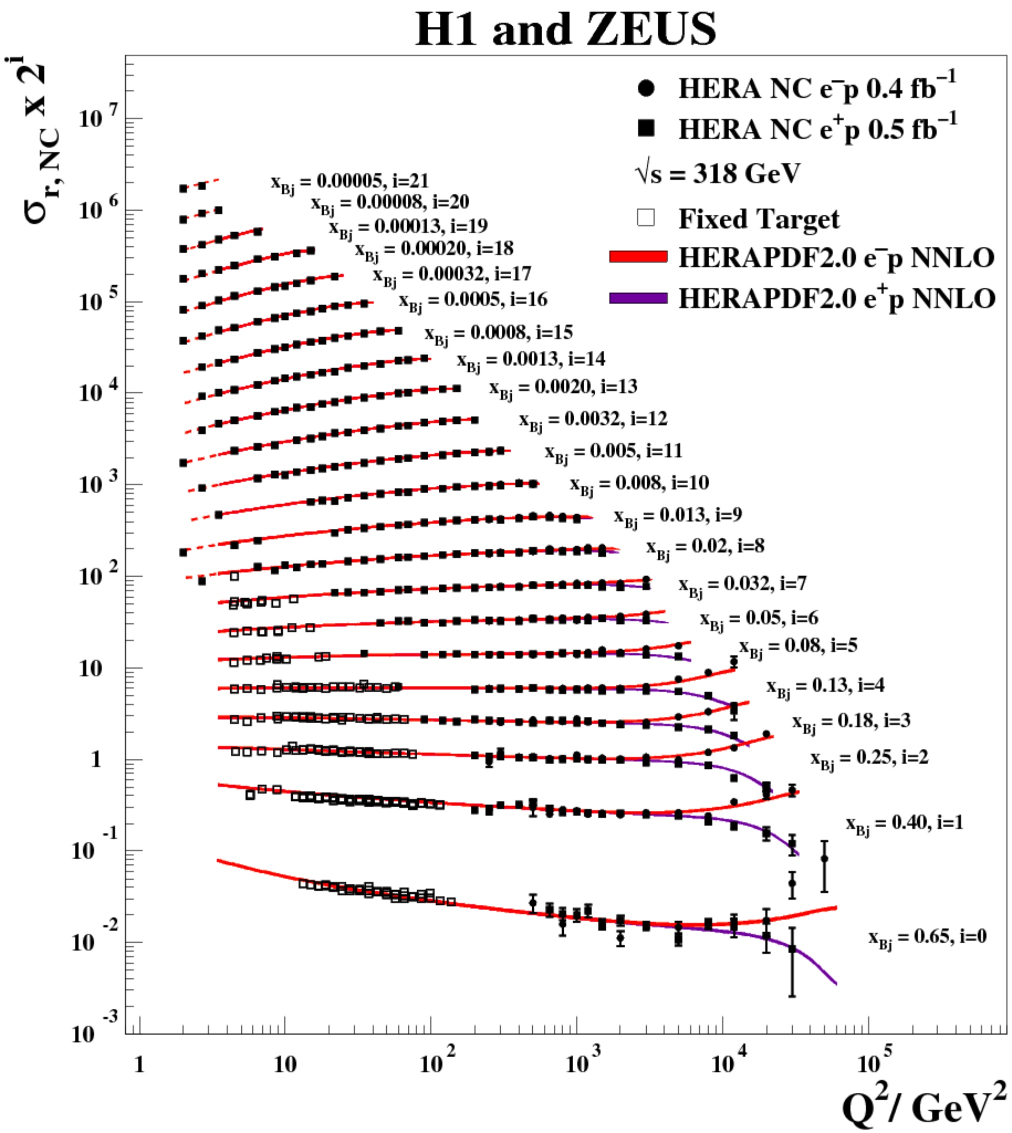
- Quarks emit gluons, at low x more and more sea quarks are created and F_2 increases; at high x the valence quarks loose momentum emitting gluons and F_2 decreases
- The line is pQCD, and it perfectly describes the data, over many orders of magnitude

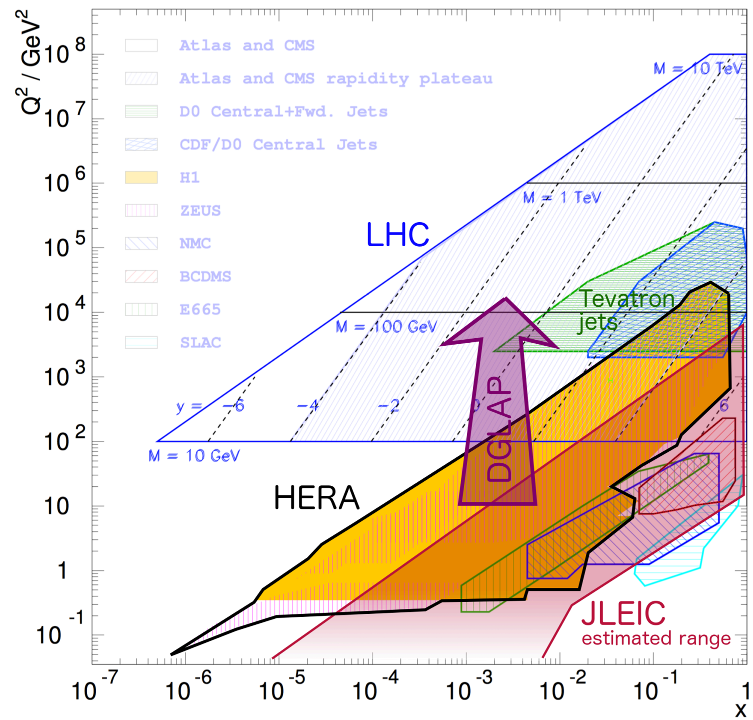
HERA at DESY (1992-2007)



ep collider, much higher Q^2 and much lower x than previous fixed target experiments

HERA at DESY

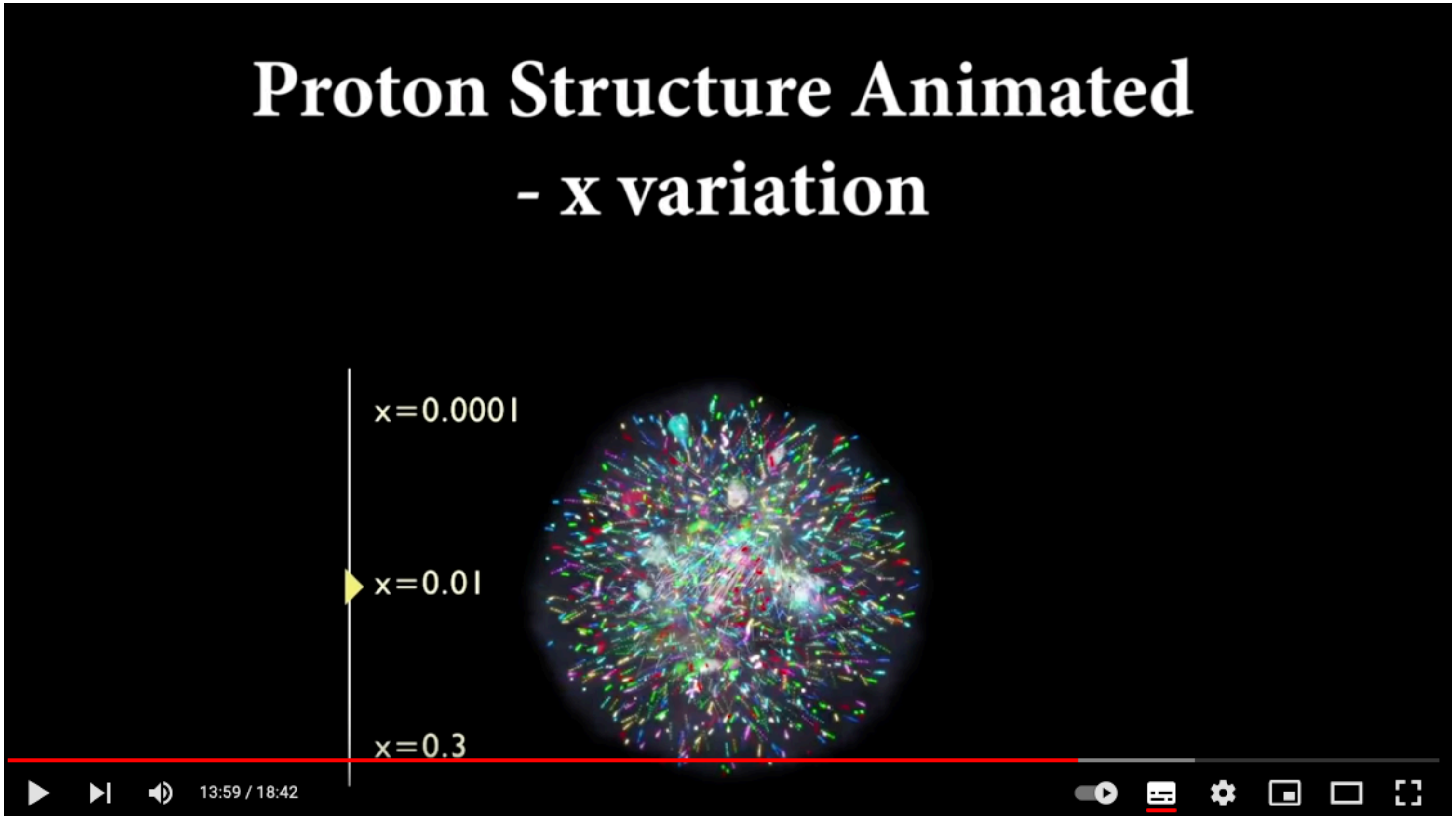




Plot J. Kretzschmar, S. Schmitt

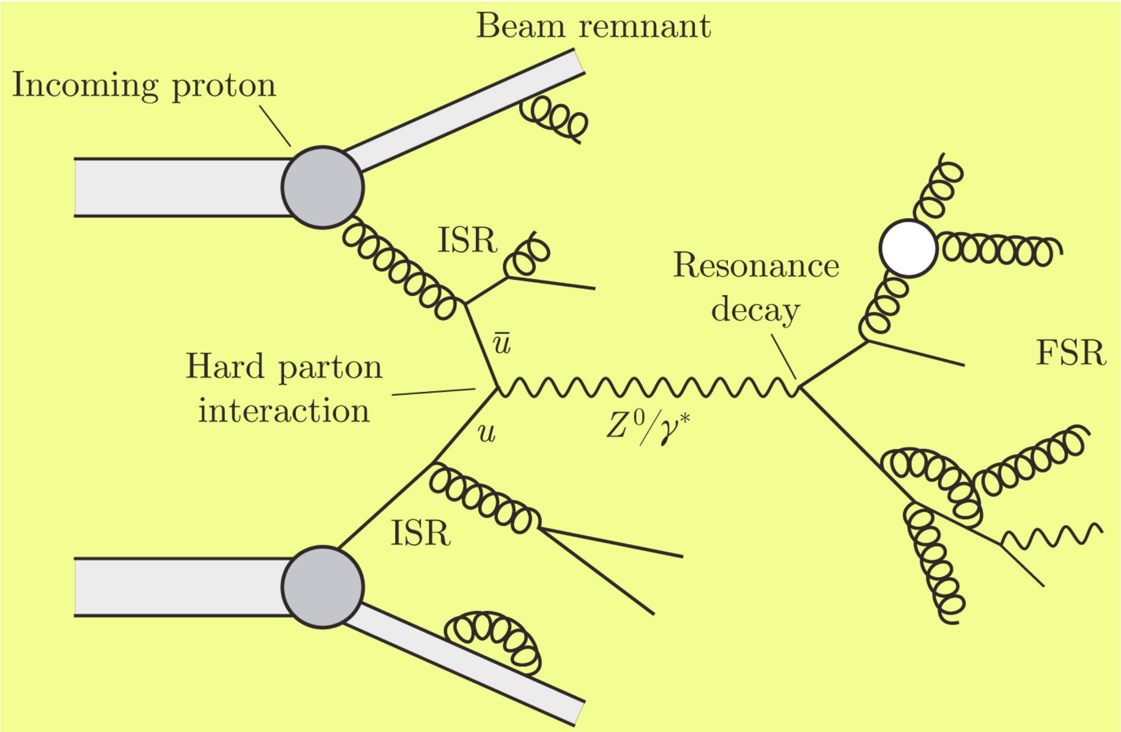
Picture of a proton

https://www.youtube.com/watch?v=G-9I0buDi4s
13:27-15:37



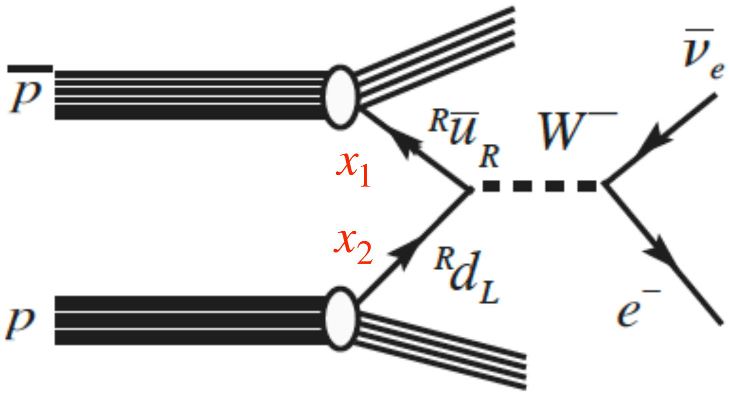
Credit: EIC

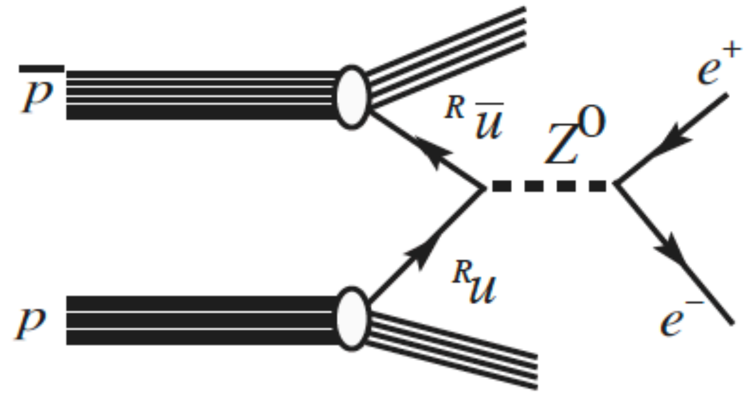
In summary



PDFs enter any of the cross sections in pp collisions. If you will do an LHC analysis, you will hear a lot about it (CTEQ, MSTW, NNPDF, etc..) and you will have to take them into account, at least as a systematic.

Example: W, Z production





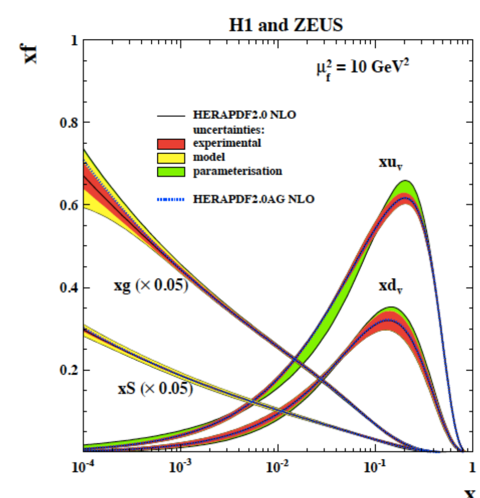
Each quark would have ~x of the beam energy, so the effective c.m. energy sqrt(s) becomes:

$$M_W^2 = 4x_1 E_{\bar{p}} x_2 E_p = x_1 x_2 s$$

$$M_W \simeq x \cdot \sqrt{s} \ \ , M_W = 80\ GeV, \sqrt{s} = 450\ GeV$$

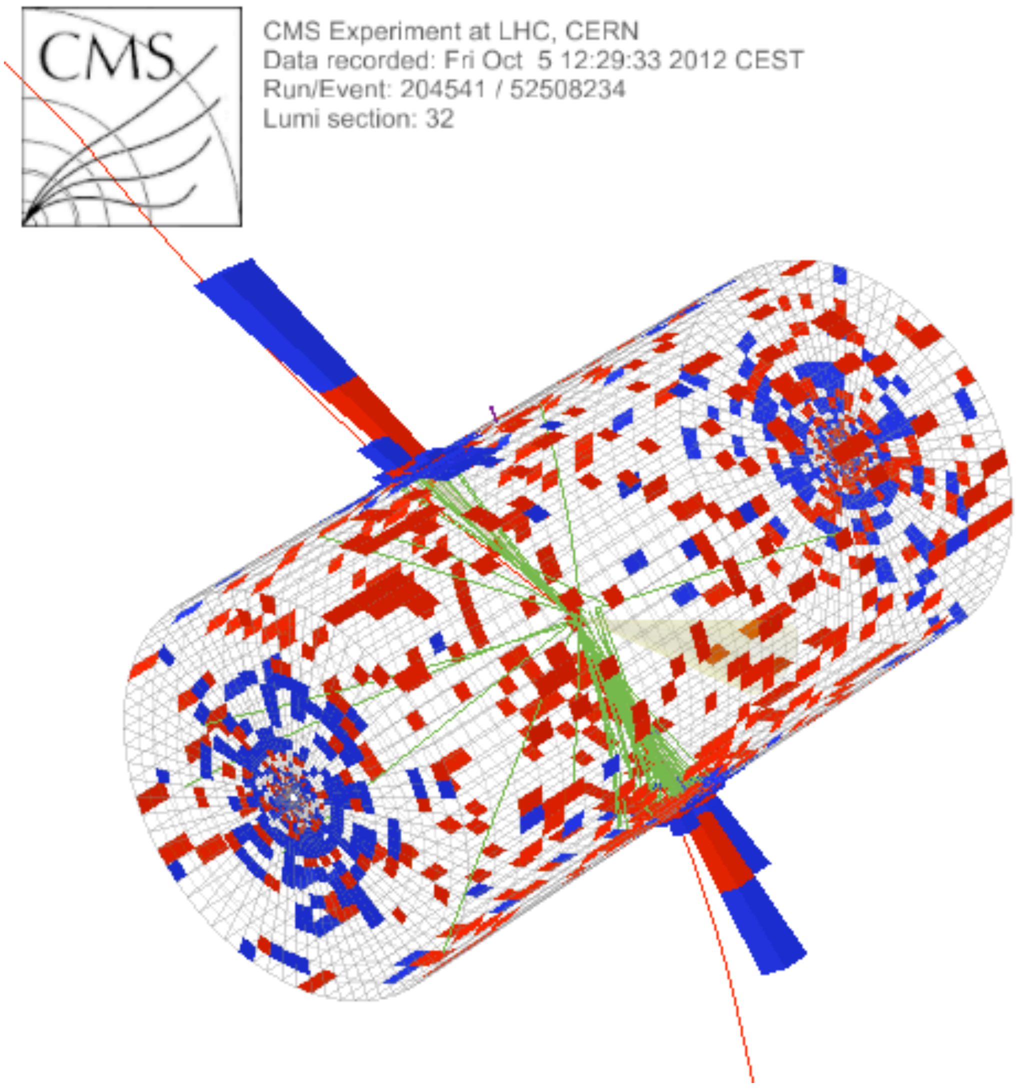
$$x \simeq 0.18$$

In valence region



Font: Bettini

Jets at the LHC



Jets algorithms

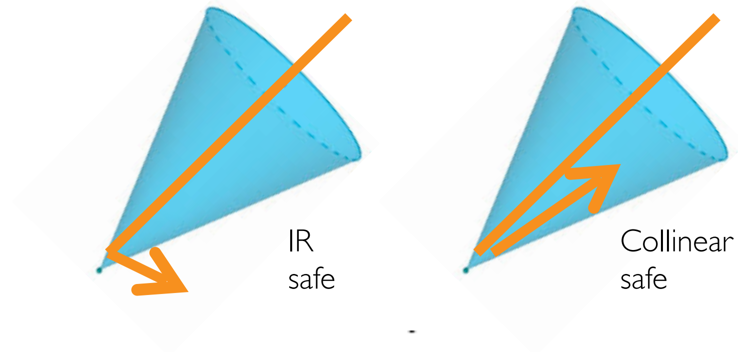
But how do you combine particles and build a jet and compare to theory?
In order to be theoretical sound a jet algorithm:

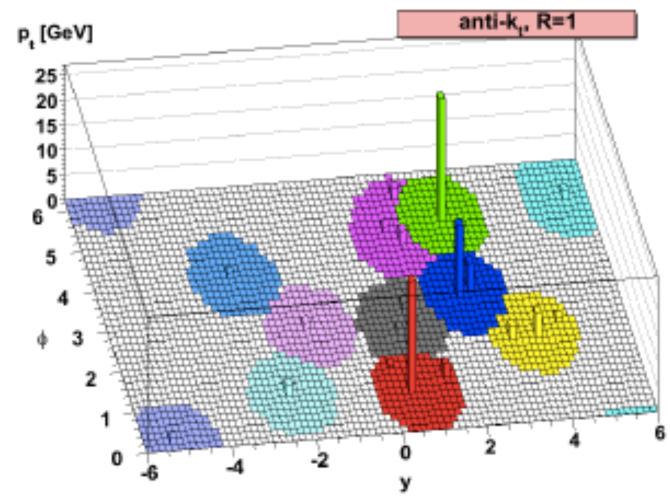
- must be infrared safe -> soft radiation should not change jet
- must be collinear safe -> collinear radiation should not change jet
- must be easy to implement in a computer code, applicable to reconstructed objects
- must be fast, etc.

Typically define a distance:

$$\boxed{\begin{aligned} d_{ij} &= \min(k_{ti}^{2p}, k_{tj}^{2p})\frac{\Delta_{ij}^2}{R^2}, \\ d_{iB} &= k_{ti}^{2p}, \end{aligned}}$$

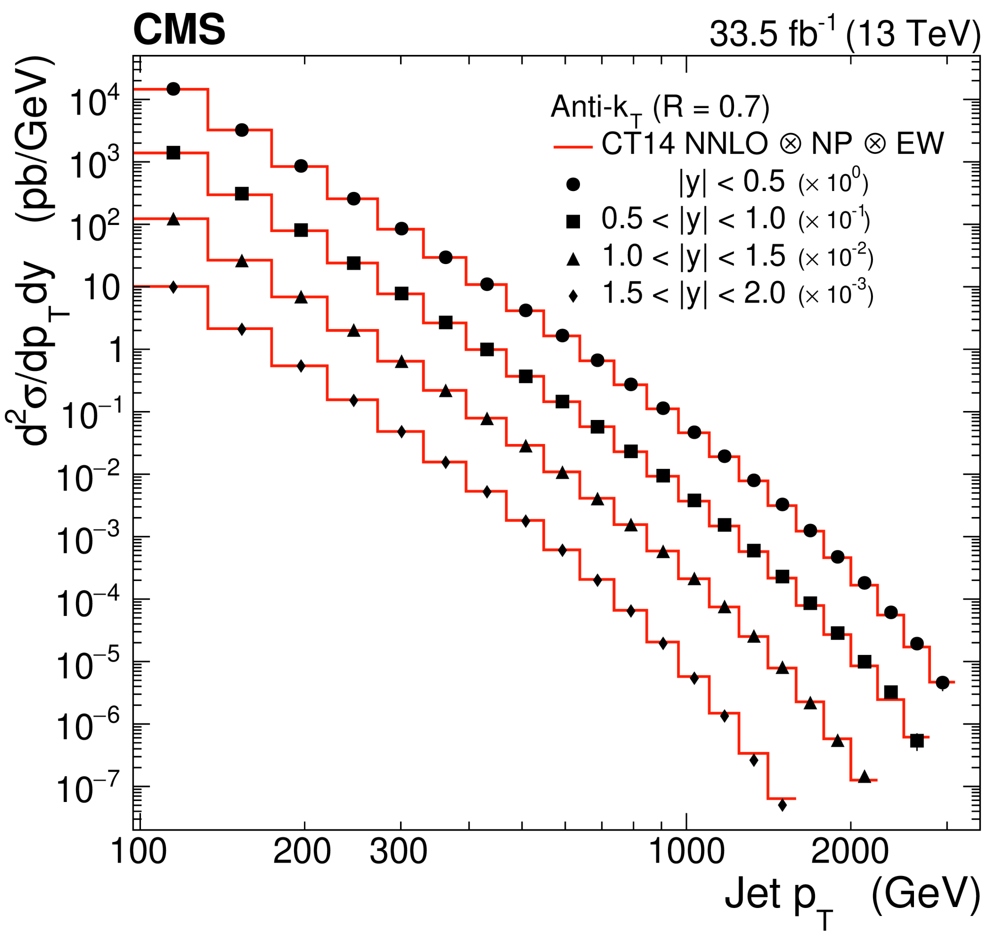
Identifiy the smallest distance in η, ϕ and recombine objects until no entities are left

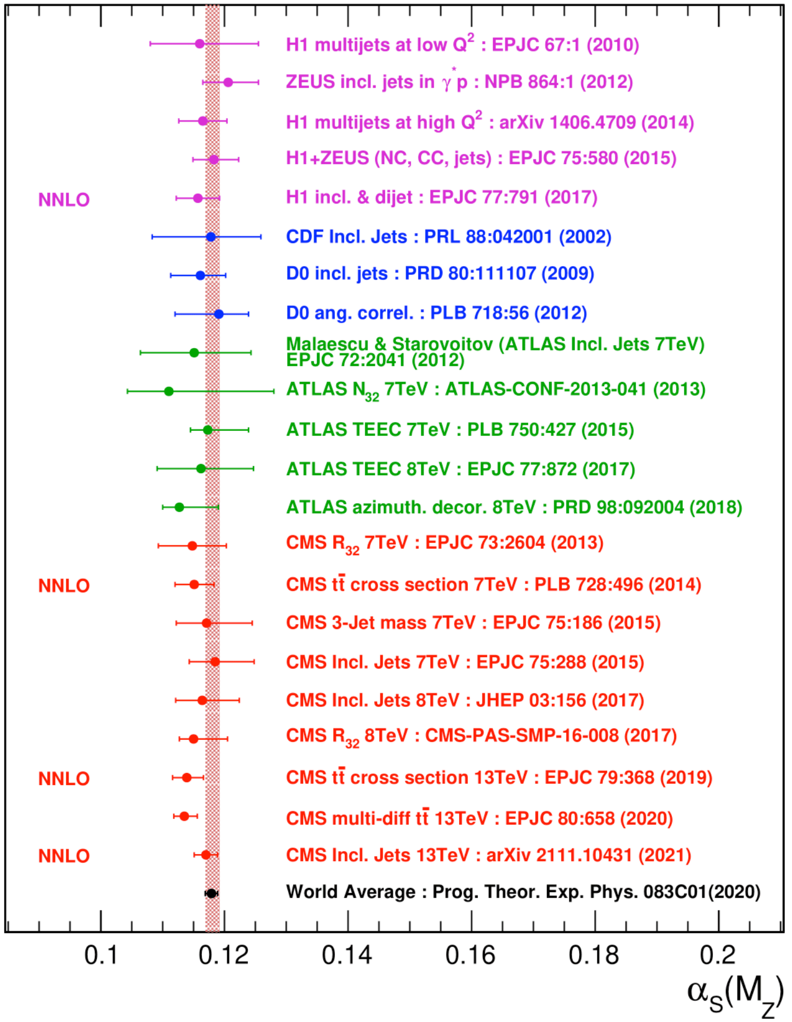




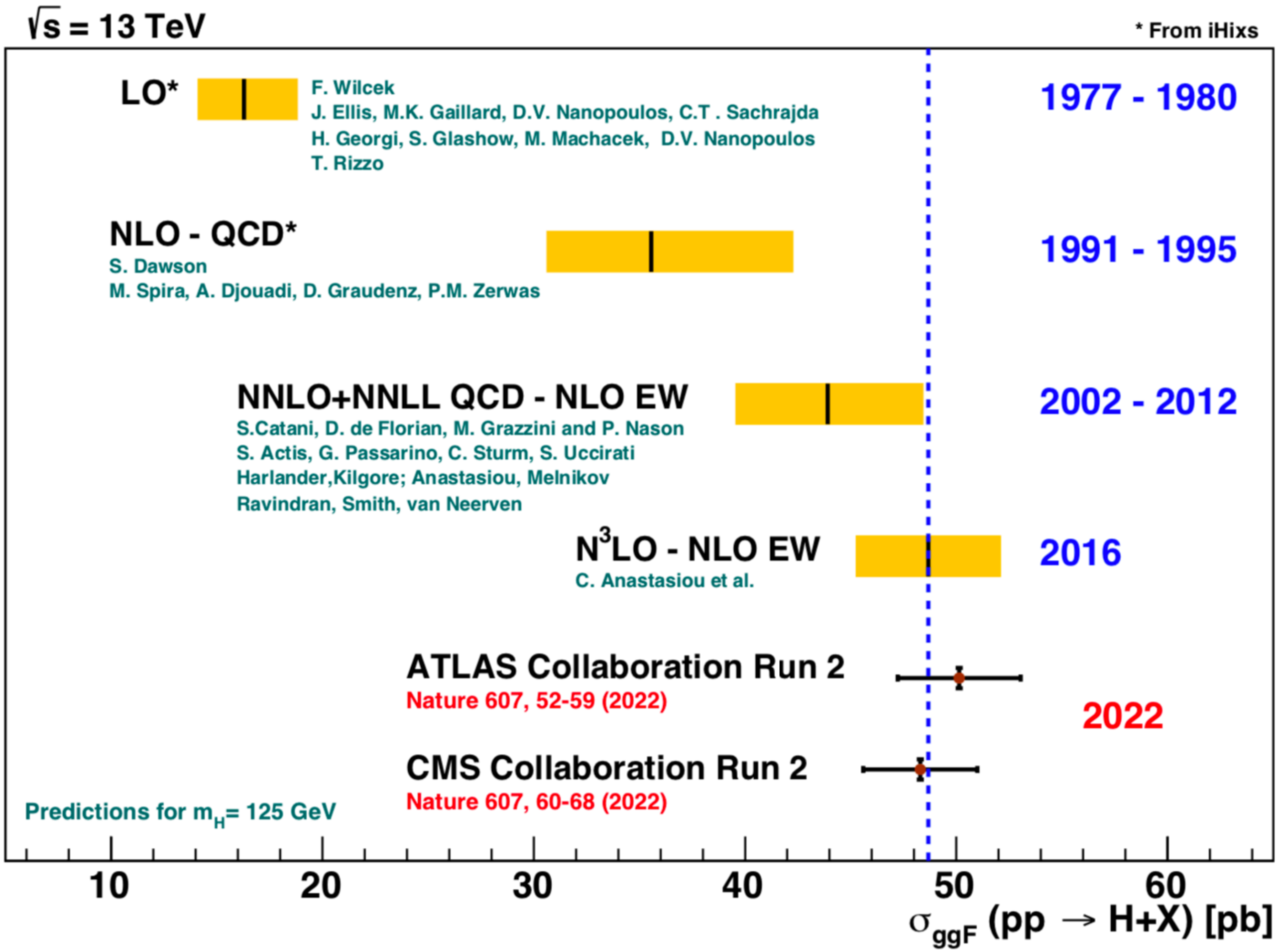
Most popular algorithm now at the LHC: anti-k_T – corresponds to p=-1. It is the one which has more „round" shapes. Usually R=0.4, 0.6 used.

Jet cross sections





QCD and the Higgs



Grazzini, Kado

Backup