Introduction to String Theory

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I.) Motivation: The quest for a fundamental theory

The Principles of Modern Physics

Modern physics rests upon 2 mighty pillars:

- **General Relativity** (GR)
- **Yang-Mills Theory as a Quantum Field Theory** (YM)

GR describes gravitational physics at astronomical length scales:

- Carrier of the interactions is spacetime curvature itself:
  $R_{\mu\nu} - \frac{1}{2} R G_{\mu\nu} + \Lambda G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
- excellent experimental confirmation,
  e.g. gravitational lenses, perihelium of Mercury, gravitational waves...

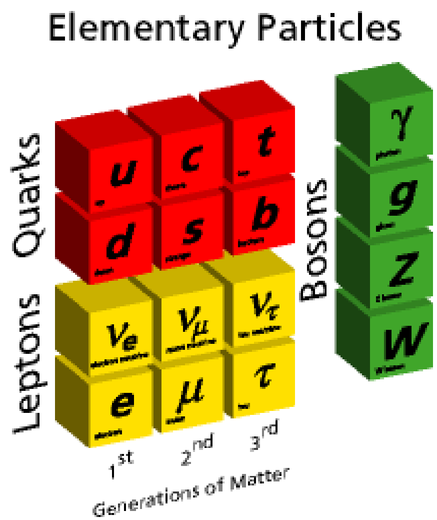
YM describes particle physics at (sub)atomic distances

- based on principle of gauge interaction
- Carrier of interactions are gauge bosons: spin 1 fields A_μ

Standard Model of Particle Physics

A description of Nature requires 3 different YM theories:

- electromagnetism $\leftrightarrow$ gauge group U(1): γ_μ photon
- strong interaction $\leftrightarrow$ gauge group SU(3) : g^a_μ: gluons a=1,. . . , 8
- weak interaction $\leftrightarrow$ gauge group SU(2): $W^\pm_\mu, Z_\mu$



- matter $\leftrightarrow$ spin $\frac{1}{2}$ fermions
  $\Rightarrow$ 3 families of matter
- mass via Higgs boson

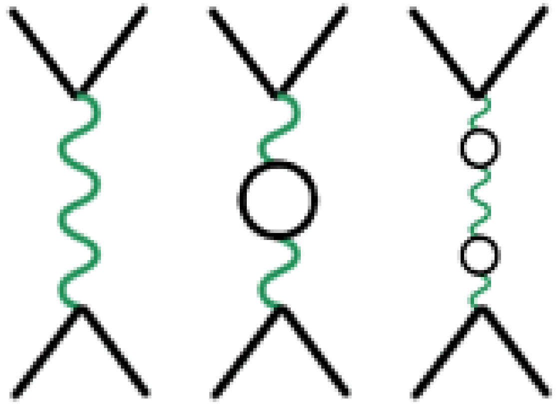
$\Rightarrow$ **Standard Model of elementary particles**

- fundamental objects are point particles
- 19 empirical constants: masses, mixing angles...
- spectacular precision measurements at particle colliders

Shortcomings of the SM

Despite this phenomenological success at large and small scales, the theory suffers from severe fundamental shortcomings:

YM theory leads to ultraviolet divergences in the computation of elementary scattering processes



- From pragmatic perspective:
  no problem thanks to regularisation and renormalisation.
- From theoretical perspective:
  theory cannot be valid at high energies = small distances.
- Renormalisation introduces dimensionful and dimensionless coupling constants, masses etc. which cannot be computed from first principles.

YM theory/QFT is a low-energy effective theory

Criticism of GR

GR is well-defined only as a classical theory.

One can try to formulate GR as a perturbative gauge theory:

- Carrier of the interaction: spin 2 bosons $\leftrightarrow$ gravitons $h_{\mu\nu}$
- Unlike YM theory GR is not perturbatively renormalisable:
  One needs infinitely many counter-terms.

$\rightarrow$ Not a fundamental quantum theory of gravitation!

Another hint for incompleteness of GR: Black Holes

- At the centre of the black hole there is a curvature singularity
- Interpretation similar to YM divergences:
  beyond validity of effective theory

Quest for a fundamental theory

If QFT and GR are really effective theories valid at low energies, what would we call a fundamental theory?

Minimal requirements:

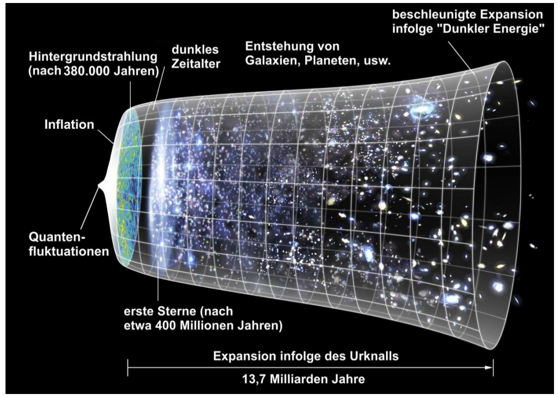
1. No ultraviolet divergences
   $\leftrightarrow$ describe microscopic degrees of freedom correctly at all energies
2. No free dimensionless parameters
   $\leftrightarrow$ no 'hiding of ignorance' in tunable parameters

Search for unification

Possible Objection: All these issues are not a problem in practice.

However:

1) At least understanding early time cosmology likely to require a fundamental theory



2) Many conceptual questions:

- Many energy scales in the SM are not technically natural.
  Most important example: The Higgs mass lies in the TeV region, but naive application of QFT yields $\mathcal{O}(10^{19}\,\mathrm{GeV})$ quantum corrections $\leftrightarrow$ **hierarchy problem**
- Dark Matter? Dark Energy?
- Why are gravitation and the 3 gauge interactions so different?

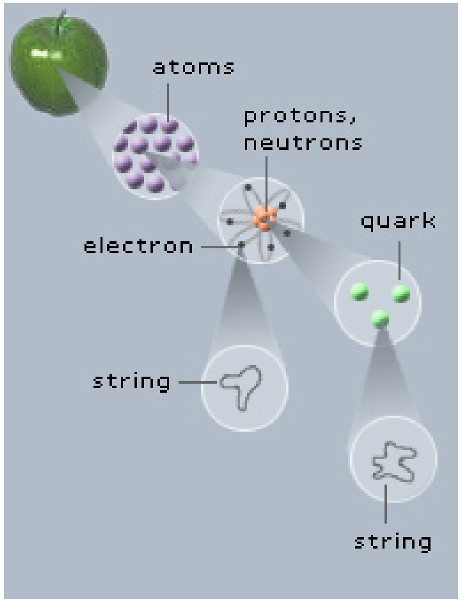
Basic ideas of string theory

String theory ...

✓ solves the problem of UV divergences

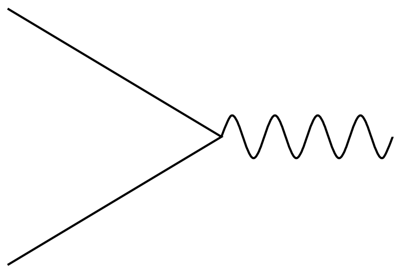
✓ in a unified description of YM and GR

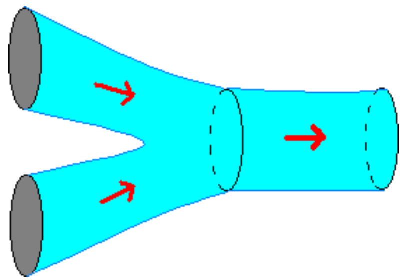
- **Dynamical input**: The fundamental objects in Nature are not pointlike, but 1-dimensional strings
- **Kinematic input**: Describe these strings via the familiar rules of quantum theory and general covariance



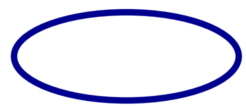
Consequences of string theory - I

- Interactions without UV divergences due to smoothening of interaction vertex





There is only one kind of strings,



- but 2 possible topologies:

closed $\leftrightarrow$ open

open strings: spin 1 object $\leftrightarrow$ gauge boson

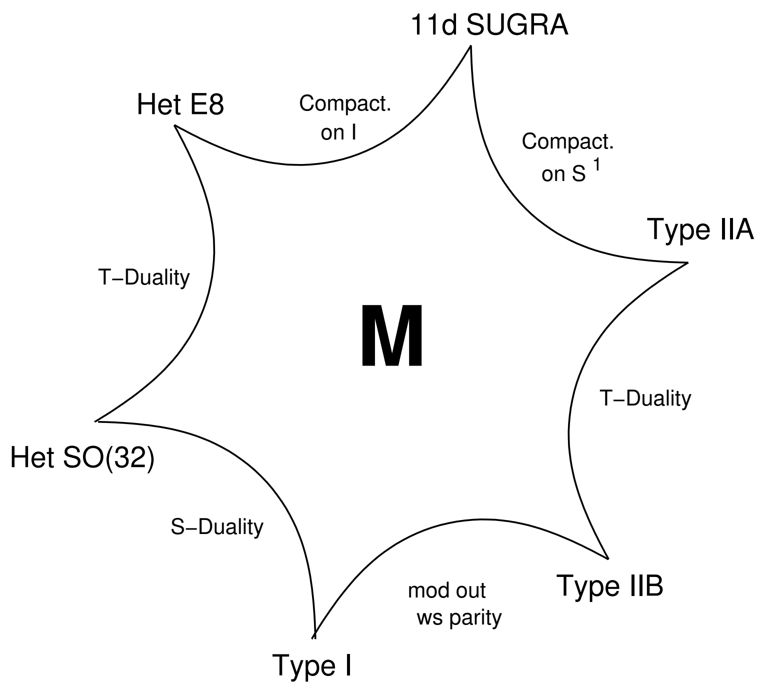
closed strings: spin 2 object $\leftrightarrow$ graviton

$\rightsquigarrow$ predicts YM and GR as 2 fundamental interactions

Consequences of string theory - II

The theory's internal consistency conditions bear further consequences:

- Spacetime is not 4-dimensional, but 10-dimensional.
- 10-dim. theory is supersymmetric:
  Each boson has a fermionic superpartner.
- In 10 Dimensions there is only one unified string theory.
  It has various formulations, all related by dualities.



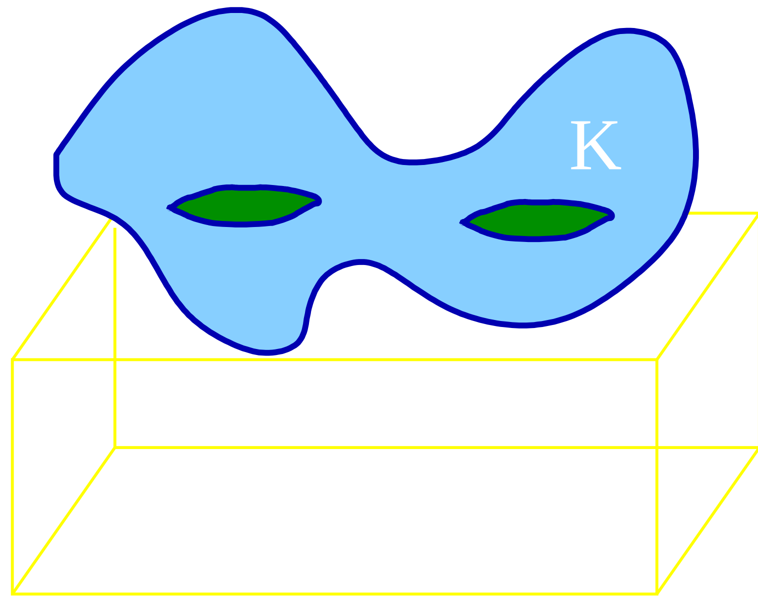
String compactification

String theory is well-defined only if spacetime is 10 dimensional.

But we only observe 4 large spacetime dimensions!

However: Extra compact dimensions of size $\leq 10^{-5} cm$ allowed by experiment

- **Compactify** string theory on a compact six dimensional space with 4 large dimensions remaining $\mathcal{M}^{1,9} = \mathcal{M}^{1,3} \times K$



$\Longrightarrow$ Realm of string compactifications and model building (see later)

2.) Classical strings

A symphony from 1 string

✓ A string can vibrate just like the string of a violin does.

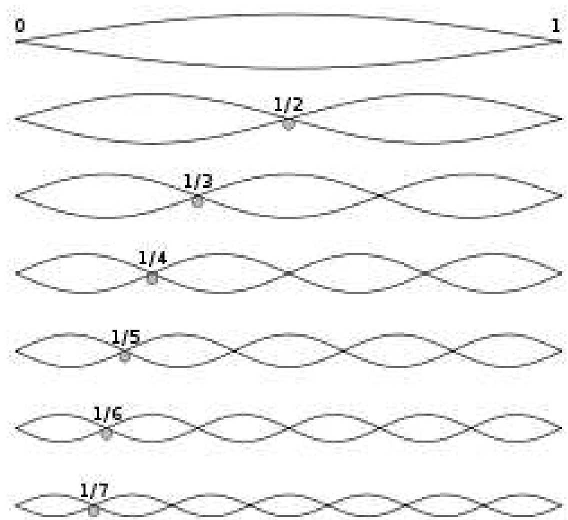
✓ The different oscillation modes (tones) correspond to different particles.

⇒ **Maximal unification:**

- only one kind of "stuff" - the string
- all physics is buried in its excitations

Analogy:

There is only one violin string, but many different oscillations imply a full symphony of different tones.



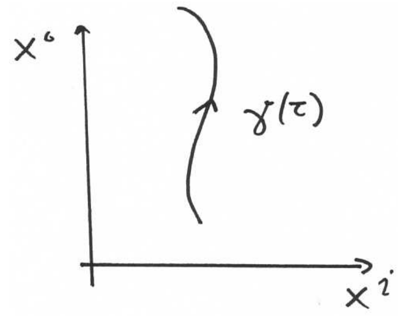
Program:

- Describe classical string oscillations as harmonic oscillator
- Quantise the system by standard techniques

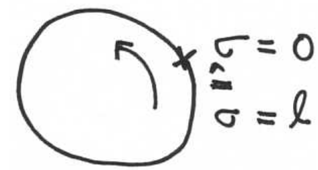
Classical Strings - Kinematics

Kinematics:

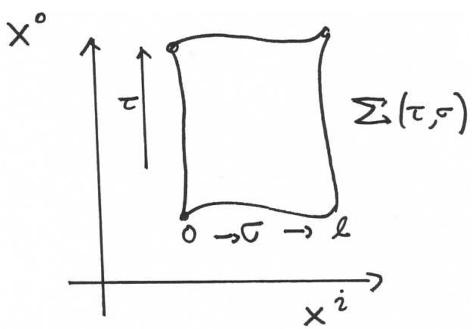
- Point particle traces out worldline $\gamma(\tau)$

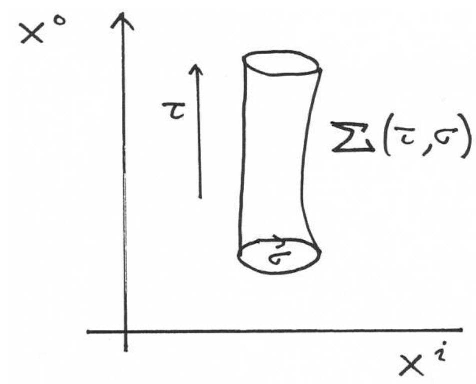


- String: Parametrise the position along string by $0 \leq \sigma < \ell$



- Together with time τ this gives the worldsheet coordinates (τ, σ)
- Worldsheet $\Sigma(\tau, \sigma)$: tracetory of string in ambient spacetime





Classical Strings - Dynamics I

Equations of motion

- free point particle: $(\frac{\partial}{\partial\tau})^2 X^\mu(\tau) = 0$
- free string:

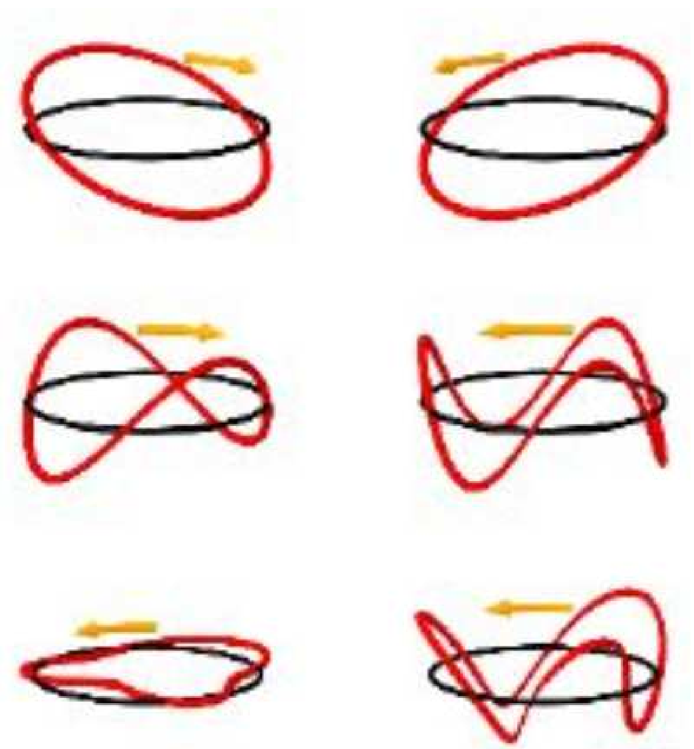
$$\Big((\frac{\partial}{\partial\tau})^{\mathbf{2}} - (\frac{\partial}{\partial\sigma})^{\mathbf{2}}\Big)\mathbf{X}^\mu(\tau,\sigma) = \mathbf{0} \leftrightarrow \textbf{wave equation in 2D}$$

Strings carry energy

- c.o.m. momentum
- oscillations along string

Strings carry spin

$\leftrightarrow$ polarisation of oscillation



Energy scale set by **string length**

$$\ell_s \equiv 2\pi\sqrt{\alpha'}$$

Classical Strings - Dynamics II

- 2D wave equation: $\left(\left(\frac{\partial}{\partial\tau}\right)^2 - \left(\frac{\partial}{\partial\sigma}\right)^2\right) X^\mu(\tau,\sigma) = 0$

- Ansatz: $X^\mu(\tau,\sigma) = \underbrace{X_R^\mu(\tau-\sigma)}_{\text{right-moving wave}} + \underbrace{X_L^\mu(\tau+\sigma)}_{\text{left-moving wave}}$

- Boundary conditions for **closed string**

$$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\ell) \qquad \ell : \text{circumference of string}$$

Most general solution: Fourier expansion

$$X_R^\mu = \frac{1}{2}x^\mu + \frac{\pi\alpha'}{\ell}p^\mu(\tau-\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}\neq 0}\frac{1}{n}\alpha_n^\mu e^{-i\frac{2\pi}{\ell}n(\tau-\sigma)}$$

$$X_L^\mu = \frac{1}{2}x^\mu + \frac{\pi\alpha'}{\ell}p^\mu(\tau+\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}\neq 0}\frac{1}{n}\tilde{\alpha}_n^\mu e^{-\frac{2\pi}{\ell}in(\tau+\sigma)}$$

- Frequencies: $\frac{2\pi}{\ell}n$ Amplitudes: α_n^μ/n (Right) $\tilde{\alpha}_n^\mu/n$ (Left)
- c.o.m momentum p^μ and position x^μ

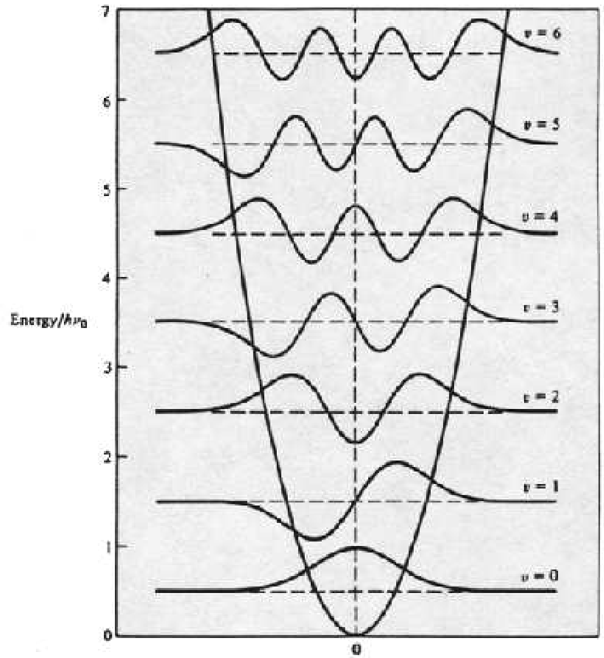
3.) Quantum strings

String Quantisation - I

= quantisation of waves along the string

Each excitation mode $\alpha_m^\mu, \tilde{\alpha}_m^\mu$ represents a **harmonic oscillator:**

$$[\alpha_m^\mu, \alpha_n^\nu] = m\, \delta_{m+n,0}\, \eta^{\mu\nu}$$



States:

- c.o.m. momentum p: $|0, p\rangle$
- Excite each left/right oscillation frequency $\frac{2\pi}{\ell} n$ arbitrarily often:

$$\prod_{m>0,\mu} (\alpha_{-m}^\mu)^{n_{m,\mu}} \prod_{m>0,\mu} (\tilde{\alpha}_{-m}^\mu)^{\tilde{n}_{m,\mu}} |0; p\rangle$$

(Special technicality here: equal number of left/rightmoving quanta)

String Quantisation - II

Tower of string excitiations - characterized by oscillation number $N_L = \tilde{N}_R$

- $N_L = 0 = \tilde{N}_R$: $|0, p\rangle$: momentum eigenstate with zero oscillations
- $N_L = 1 = \tilde{N}_R : \zeta_{\mu\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0; p\rangle$: first mode excited
- $\ldots$

Mass of string excitations: (for bosonic string)

$$M^2 = \frac{4}{\alpha'}(N - a) \qquad a = 1 \qquad N = N_L = \tilde{N}_R$$

$\alpha' \simeq \ell_s^2$ $\qquad$ ℓ_s: string length $\leftrightarrow$ sets scale of oscillations

$N_L = 0 = N_R$: tachyon - removed in superstring theory

$N_L = 1 = N_R$: massless excitations

$N = 2, 3, \ldots$: massive states of mass-squared set by $\frac{1}{\alpha'}$

Each oscillation appears as object with mass and spin = particle.

Gravitons from closed strings

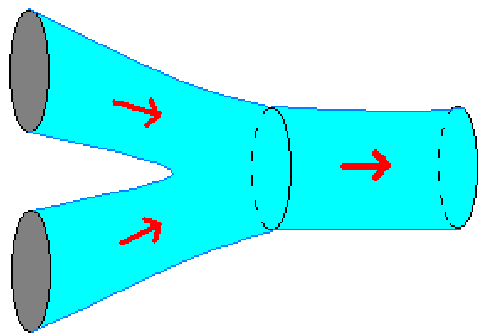
Low-energy regime ($E << \ell_s^{-1}$): only massless modes relevant

$$\text{Closed massless} : \zeta_{\mu\nu}\alpha^{\mu}_{-1}\tilde{\alpha}^{\nu}_{-1}|0;p\rangle, \quad \zeta_{\mu\nu} : \text{polarisation tensor}$$

- This object contains a spin-2 mode = 2-index symmetric tensor.
- **This must be the graviton $h_{\mu\nu}$.**
  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$: fluctuation around background

Direct check:

- Compute interactions in string perturbation theory
- Find same interactions as for perturbative graviton



High energy regime ($E \geq \ell_s^{-1}$):

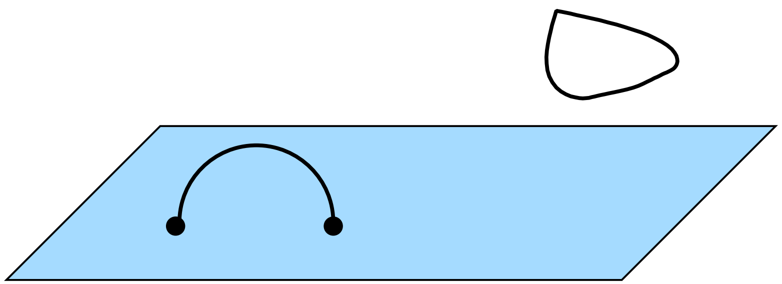
Characteristic tower of massive, higher spin excitations visible

$$M^2 \simeq N/\ell_s^2 \qquad J \simeq \ell_s^2 M^2$$

Photons from open strings

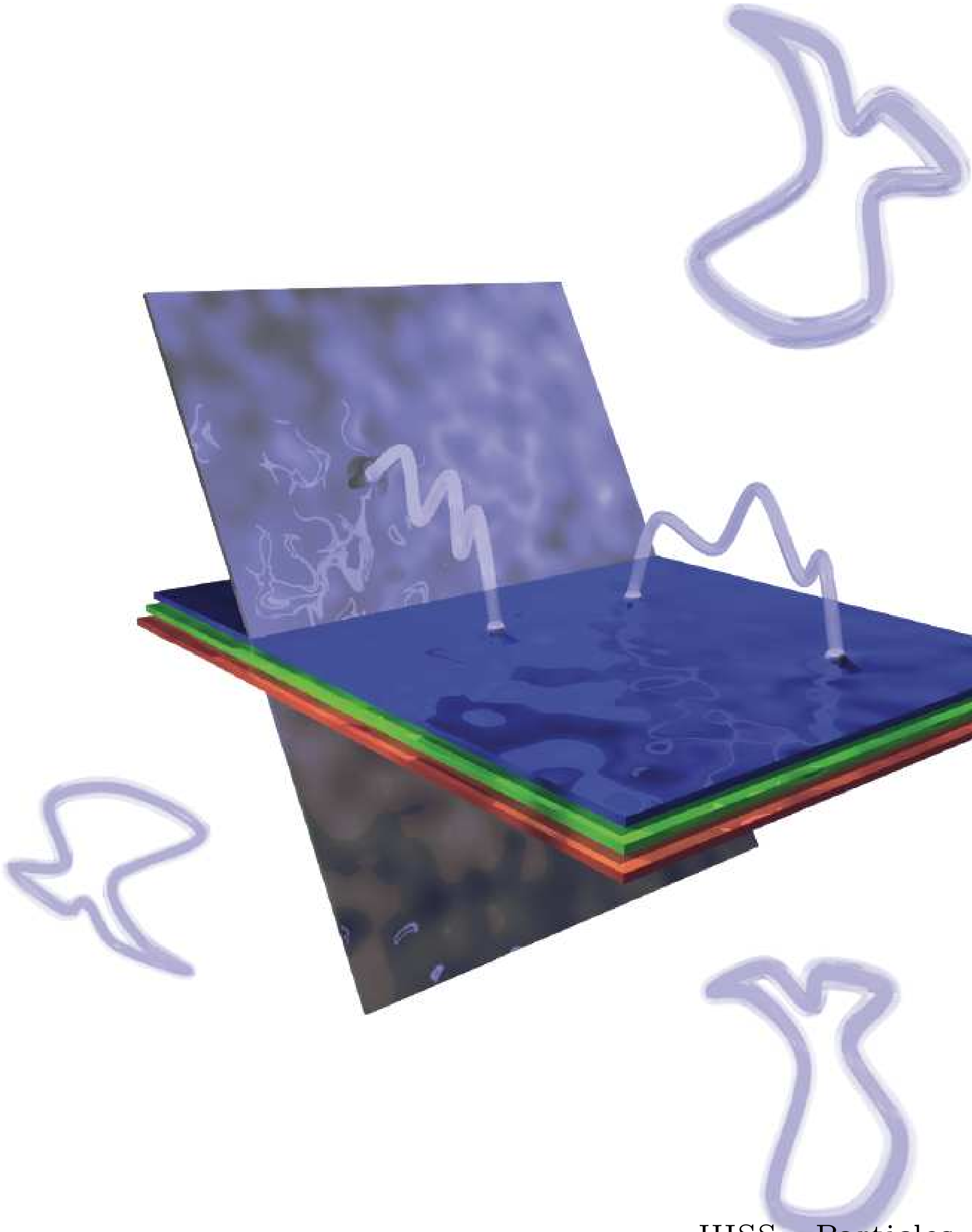
- An open string has two endpoints at $\sigma = 0$ and $\sigma = \ell$
- Repeat program of classical solutions and quantisation with suitable boundary conditions
- Result: String endpoints can move freely along an object called a **Dp-brane** = (p+1)-dimensional hypersurface of spacetime

Polchinski 1996



- Boundary conditions relate left/rightmoving waves
- Massless level: $\zeta_\mu \alpha^\mu_{-1}|0;p\rangle$: spin-1 particle
- Interpretation as vector boson responsible for a U(1) gauge theory

Gravity in bulk - EM on brane



String Quantisation - III

Technical complications:

- For $\mu = 0$ we get the wrong sign in the commutation relations:

$$[\alpha_m^\mu, \alpha_n^\nu] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}$$

$\Longrightarrow$ negative norm states

(for experts: same as in QED cf. Gupta-Bleuler quantisation)

- These ghosts can be removed precisely if the number d of spacetime dimensions takes a special value.

$\rightsquigarrow$ **Superstring Theory: d= 9+1**

(precursor theory: Bosonic String: d = 25+1)

Prediction of number of spacetime dimensions from straightforward consistency condition of quantum theory!

Brane worlds

Summary:

- String theory is well-defined within 10D spacetime.
- Within the full 10D bulk a graviton propagates.
- In 10D spacetime there are lower dimensional D-branes along which a gauge boson propagates.

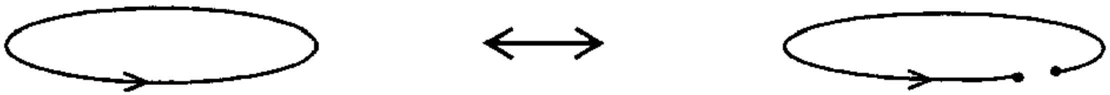
$\Longrightarrow$ **Braneworld idea**:

Gauge interactions confined to branes - gravity propagates within bulk

A crucial consistency check

In string theory, gauge theory implies gravity.

- Strings interact by joining and splitting.



- Open string endpoints can join to form a stable closed string.

  (The converse is not always true)

✓ Behaviour consistent with universality of gravity:

photons $\Longrightarrow$ energy $\Longrightarrow$ gravity

✓ In string theory, gauge interactions and gravity are not independent.

They are linked by the internal consistency of the theory.

String theory is the only known theory with this property.

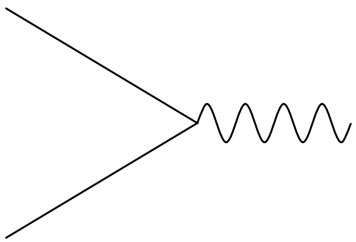
String theory goes beyond Einstein gravity:

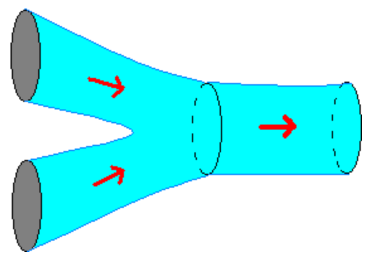
Systematically compute higher order corrections to Einstein action.

UV finiteness

General picture: **String as intrinsic UV regulator**

- High energy scattering probes string length $\leftrightarrow$ non-local behaviour
- Point-like interaction vertex is smoothened out.

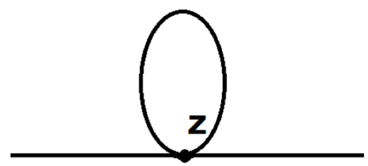




Loop diagrams

1) Point particle

- Feynman diagram = circle S^1

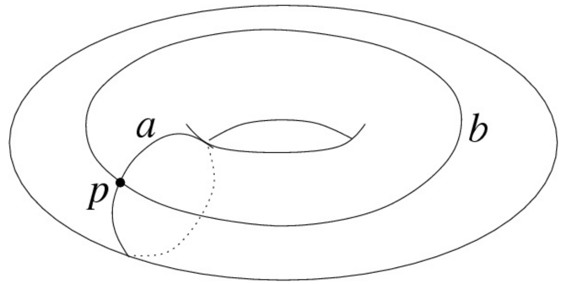


- circle parameter: radius R
- UV region: $R \to 0$

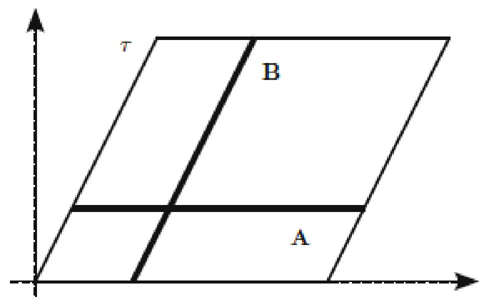
UV finiteness

2) String:

- Feynman diagram = Torus T^2



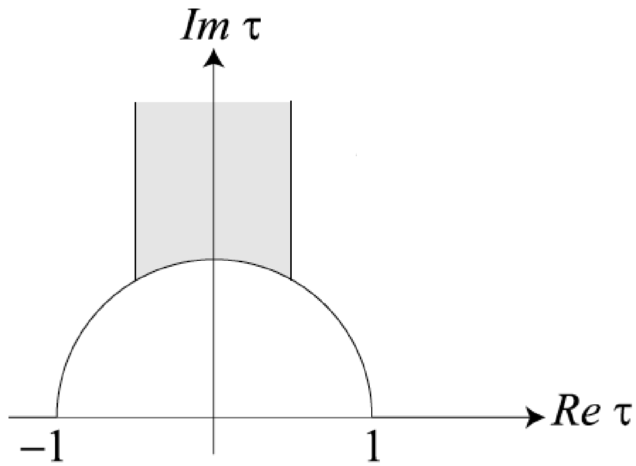
- Torus parameter:
  $\tau = \tau_1 + i\tau_2$ (shape of T^2)



- UV region: $\tau_2 \to 0$

- T^2 has symmetry $\tau \to -\frac{1}{\tau}$

  $\Rightarrow \tau$ takes values in fundamental domain

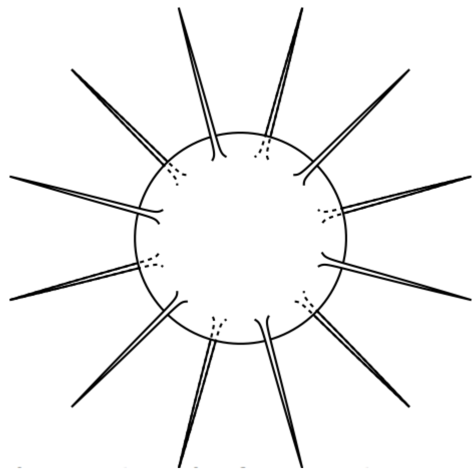


UV divergent region is absent.

Why strings are special

Can a particle have even higher-dimensional substructure?

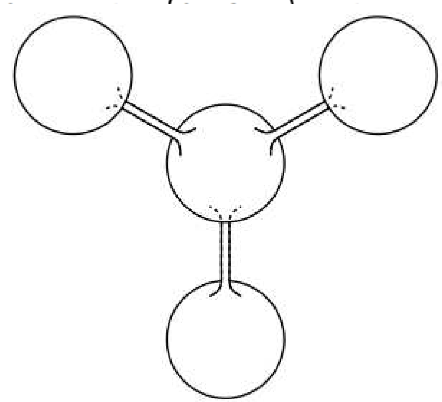
Model particle as a **membrane** -
2 spatial dimensions



Tubes of length L and radius R have spatial volume $\simeq L \times R$.

Quantum fluctuations:

- Long, thin tubes can form without energy cost.
- Membranes automatically describe multi-particle states.



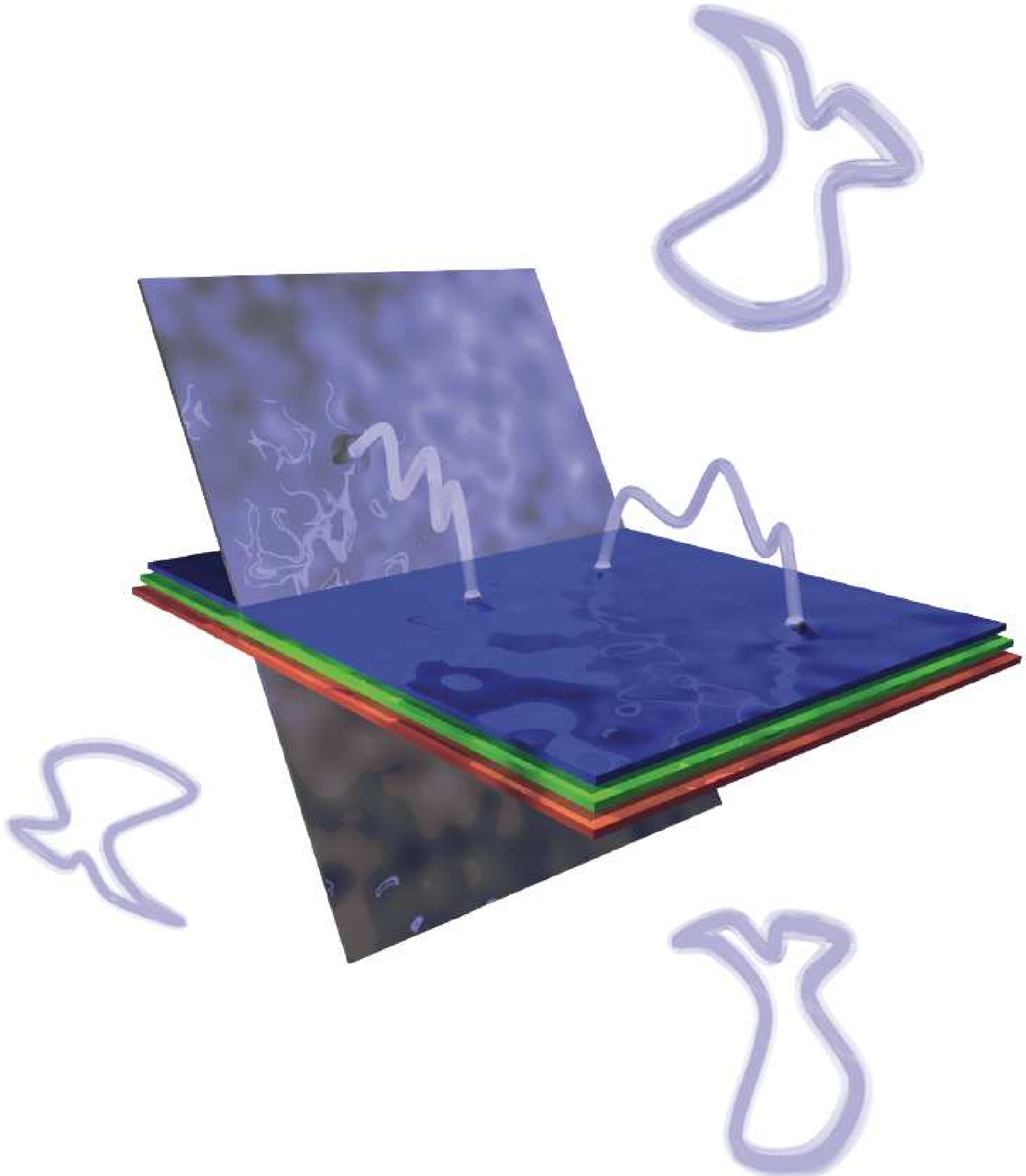
No first quantisation of higher-branes à la strings possible.

[DeWit et al 1988; Banks et al. 1997]

Take away

- String theory is a maximally unifying theory:
  All physics descends from 1 type of stuff - the string.
- Its oscillation modes give different particles.
- Closed strings: massless graviton $\rightarrow$ Einstein gravity $+$ corrections
  Open strings: massless vector boson $\rightarrow$ Yang-Mills interactions
- Open strings end on D-branes.
- Theory consistent in 10 dimensions.
- In 10 dimensions the theory is unique up to dualities and makes definite predictions.

Part II



Previously on this show

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  Open strings: massless vector boson $\rightarrow$ Yang-Mills interactions
- Open strings end on D-branes.
- Theory consistent in 10 dimensions.
- In 10 dimensions the theory is unique up to dualities and makes definite predictions.

4.) KK compactification and T-duality

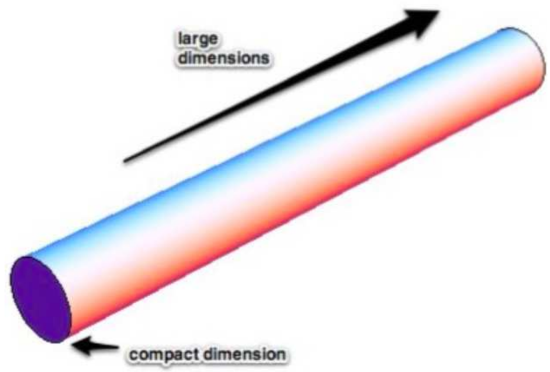
T-duality - I

String Theory is an example of a theory of extra dimensions.

Such theories are considered also in context of point particle framework.

- Extra dimensions are compact and very small.
- Characteristic feature: tower of Kaluza-Klein excitations

Toy example:



- Consider theory in 5 dimensions: $x^M, M = \underbrace{0, 1, \ldots 3}_{\mu}, 4$
- Compactify direction x^4 along circle S^1 of radius R
- Consider massless particle in 5D: $p^M p_M = 0$
  Momentum p^4 is now quantised: $p^4 = \frac{n}{R}$, $n \in \mathbb{N}$
  (cf. quantum mechanics in a box)

T-duality - II

Suppose radius R of S^1 along direction x^4 is very "small"

- Expect an effectively 4 dimensional theory in x^μ, $\mu = 0, 1, \ldots, 3$
- Compute effective mass in 4D M^2_{eff}:

$$0 = -p^M p_M = -p^\mu p_\mu - (p^4)^2$$

$$\Longrightarrow M^2_{\text{eff}} = -p^\mu p_\mu = -p^M p_M + (p^4)^2 = \frac{n^2}{R^2}$$

Experimental signatures of higher-dimensional point particle theory:

- One would find KK tower with equidistant mass spacing in $\frac{1}{R}$
- As $R \to 0$: first KK excitation disappears from low-energy spectrum

T-duality - III

In string theory, new stringy features appear related to extended/non-local nature of string.

Compactify dimension D on a circle S^1 of radius R

2 consequences:

- Momentum p^D is quantised: $p^D = \frac{n}{R}, n \in \mathbb{N}$
  $\leftrightarrow$ typical point particle behaviour
- Can have winding strings looping w times around S^1

$$\sigma \to \sigma + \ell : \quad X^D \to X^D + 2\pi\omega R$$

Important:

A string has tension.

Winding a string costs extra energy - the string wants to contract.

$\Rightarrow$ extra contribution to effective mass from winding

T-duality - IV

From string quantisation:

$$M_{\text{eff}}^2 = \frac{n^2}{R^2} + \frac{\omega^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2a)$$

- $n = \omega = 0 \rightarrow$ familiar states present also for $R \rightarrow \infty$
- $\omega = 0, n \neq 0$: Kaluza-Klein tower of massive excitations characteristic for extra dimensions
  present also for point particle theory
- $\omega \neq 0$: **winding states: truly stringy effect**

T-duality - V

$$M_{\text{eff}}^2 = \frac{n^2}{R^2} + \frac{\omega^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2a)$$

Consider limit $R \to 0$:

- KK tower $m_{KK}^2 = \frac{n^2}{R^2}$ disappears from low-energy spectrum. If this were the only effect, we would say the theory becomes effectively a theory in only $1 + (D-1)$ dimensions.
- But the winding states become lighter $m_{\text{wind}}^2 = \frac{1}{\alpha'}\omega^2 R^2$.

There remains a memory of the Dth dimension in the low-energy spectrum.
Theory remains effectively $1 + D$ dimensional.
Oberve: The spectrum is invariant under the T-duality transformation

$$n \leftrightarrow \omega, \qquad R \leftrightarrow \alpha'/R$$

exchanging momentum and winding states
Physics at $R < \sqrt{\alpha'}$ "dual to" Physics at $R > \sqrt{\alpha'}$
$R = \sqrt{\alpha'}$ is the minimal length of a compact dimension

String compactification

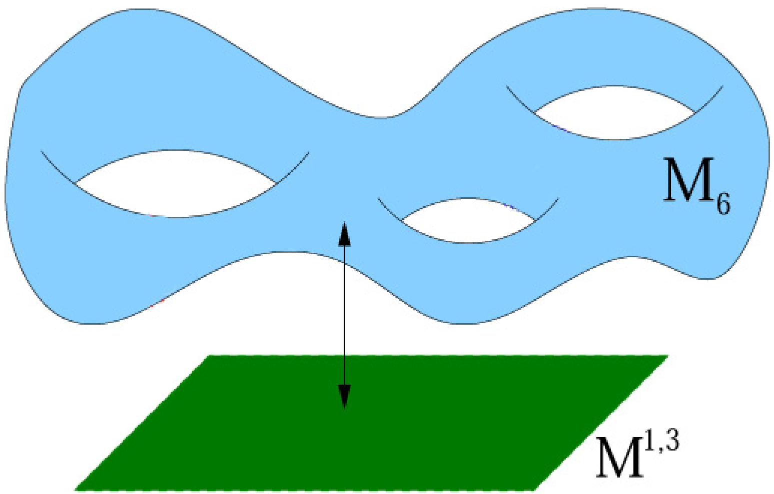
String theory well-defined in d = 10 dimensions

To arrive at 4 large extra dimensions we need to compactify 6 dimensions.

- Simplest solution:
  Each dimension is a circle S^1 internal space is a six-dimensional torus
  $T^6 = S^1 \times \ldots \times S^1$
  x_0, x_1, x_2, x_3: macroscopic $x_4, x_5, \ldots, x_9$: rolled up on T^6

- More generally can think of other six-dimensional manifolds.

- Each different compactification manifold leads to different physics in 4 dimensions.



- But not every choice is admissable due to consistency conditions

Remaining road map

Remaining tasks for us today

1. Understand which types of compactifications are interesting from a particle physics perspective:
   Intersecting Brane Worlds

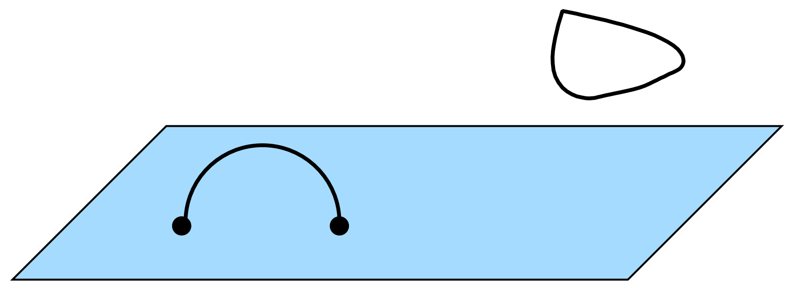
2. Get an intuitive idea of what we mean by consistency conditions
3. Which is the right vacuum? The Landscape of String Vacua

1.) The Standard Model from Intersecting Branes

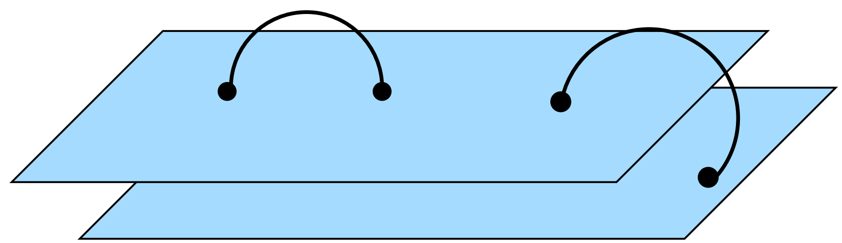
Intersecting Brane Worlds - I

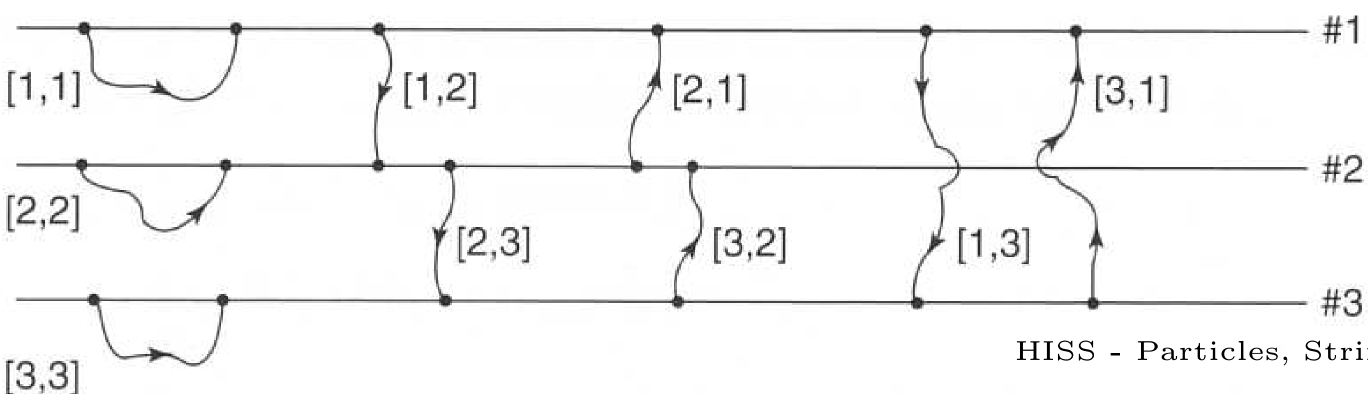
An interesting class of compactifications relies on Dp-branes:
(p+1)-dim. hypersurfaces on which open strings end

Recall:
String excitations with 2 endpoints along same single Dp-brane
$\rightarrow U(1)$ gauge boson $A^i, i = 0, \ldots p$



N coincident Dp-branes
$\rightarrow U(N)$ gauge symmetry





Intersecting Brane Worlds - II

Compare:

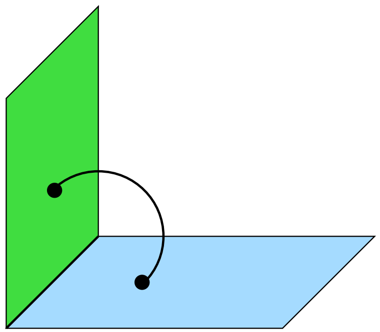
- Standard Model gauge group is $SU(3) \times SU(2) \times U(1)_Y$
- Ignoring for now difference between U(N) and SU(N) D-branes are just right to give the SM gauge groups!

Matter

Claim:

At intersection of Dp-branes:

Chiral fermions in bifundamental resprentation $(\overline{N}_a, N_b)$



Tasks:

1) Understand/Recall what this means

2) Provide more details on how it arises

Intermezzo I: Chirality

Some facts from particle physics:

1) Massless fermions have definite helicity $h = \vec{S} \cdot \hat{\vec{p}}$:

$h = +1/2$ 'left-handed' $\qquad$ $h = -1/2$ 'right-handed'

- left-handed particles $f_L \Longleftrightarrow$ right-handed anti-particles $\overline{f}_R$
- right-handed particles $f_R \Longleftrightarrow$ left-handed anti-particles $\overline{f}_L$

2) Charge of f_L = - Charge of $\overline{f}_R$

Consider electromagnetism: $U(1)_{e.m.}$ and neglect masses

left-handed electron: $q_{e_L} = -1$ $\qquad$ right-handed positron: $q_{\overline{e}_R} = +1$

$\rightarrow$ suffices to specify charges of f_L and f_R

3) A theory is chiral if f_L and f_R do not have the same charges.

Example $U(1)_{e.m.}$:

- left-handed electron: $q_{e_L} = -1$ $\qquad$ right-handed electron: $q_{e_R} = -1$

  $\Longrightarrow U(1)_{e.m.}$ is non-chiral

Intermezzo II: Bifundamentals

Consider $SU(3)$ - e.g. think of QCD

- Quarks are charged under $SU(3)$ - they carry "colour"
- Each quark comes in 3 colours - it forms a triplet under SU(3)
- Represent quark as vector with 3 entries on which SU(3) matrices act

$$\text{quark} \simeq \mathbf{3} \text{ - fundamental under SU(3)}$$

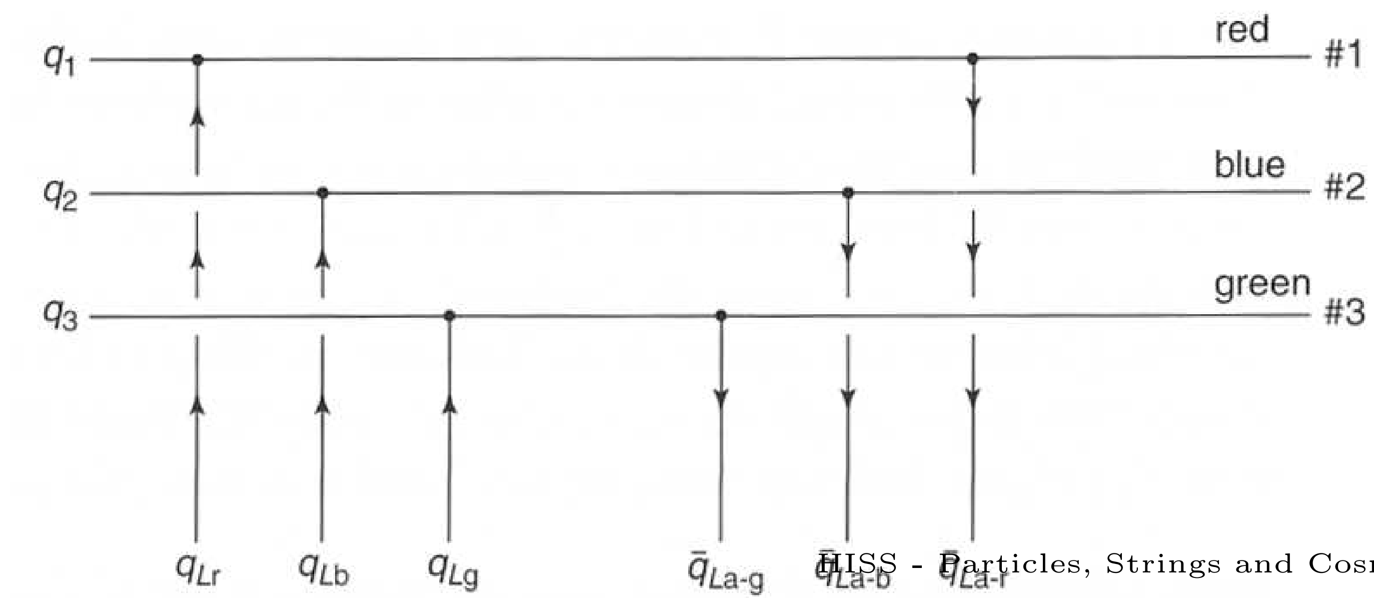
- Left-handed quarks are also charged under $SU(2)_w$:
  $Q_L \simeq (\mathbf{3}, \mathbf{2})$ - bifundamental
- $SU(2)_w$ is a chiral theory:
  Only lefthanded fermions interact weakly
  e.g. $Q_L \simeq (\mathbf{3}, \mathbf{2})$ but $u_R \simeq (\mathbf{3}, 1)$, $d_R \simeq (\mathbf{3}, 1)$

Matter from branes - I

Fundamental charge of fermions associated with endpoints of open strings on D-branes

Consider 3 coincident branes $\mathcal{D}_c \to$ Gauge group $U(3)_c$
Recall: Open string is oriented $\leftrightarrow$ 2 endpoints $\sigma = 0$ and $\sigma = \pi$:

- Open string ending on $\mathcal{D}_c$ gives left-handed quark Q_L in $\mathbf{3}$ (plus right-handed anti-particle)
- Open string starting on $\mathcal{D}_c$ gives left-handed quark Q_L in $\overline{\mathbf{3}}$ (plus right-handed anti-particle)



Matter from branes - II

✓ Strings between 2 different stacks of branes give bifundamental matter

✓ Massless bifundamental matter localised at brane intersections

Simple example: Type IIA theory with intersecting D6-branes

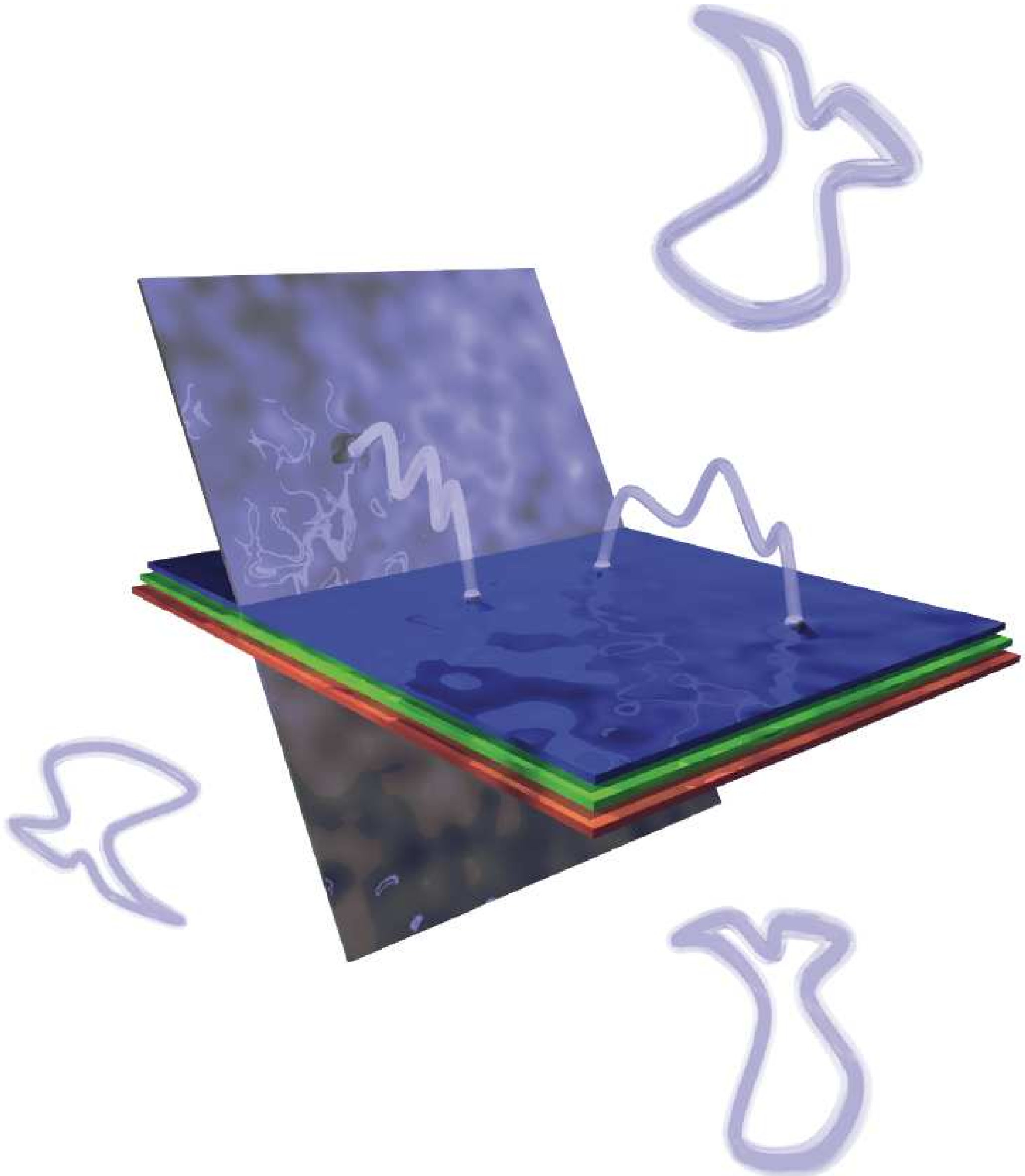
- D6-brane fills $1+6$ dimensions
- Consider 2 intersecting D6-branes: N_a copies of $\mathcal{D}_a$, N_b copies of $\mathcal{D}_b$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$\mathcal{D}_a$	✓	✓	✓	✓	✓	-	✓	-	✓	-
$\mathcal{D}_b$	✓	✓	✓	✓	-	✓	-	✓	-	✓

Common locus: x_0, x_1, x_2, x_3 - intersecting at $x_4 = 0 = \ldots = x_9$

- $U(N_i) = SU(N_i) \times U(1)_i$ gauge bosons along $\mathcal{D}_i$, $i = a, b$
- massless fermions charged as $(\mathbf{N_a}, \mathbf{N_b})$ under $U(N_a) \times U(N_b)$ at intersection locus, i.e. in x_0, x_1, x_2, x_3

Intersecting branes



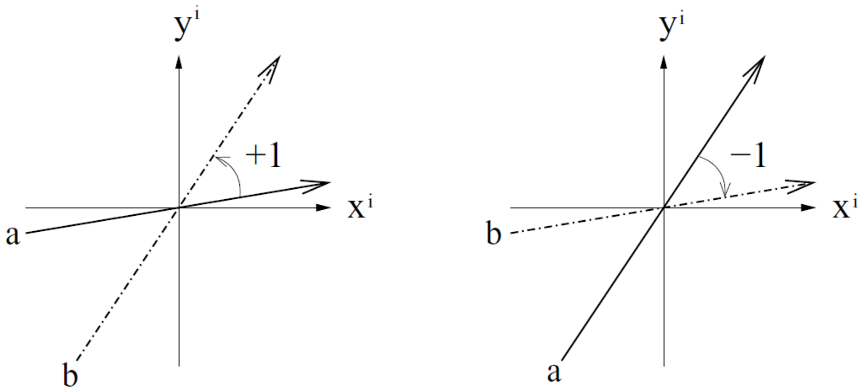
Matter from branes - III

Depending on relative orientation of branes at each intersection point:

- **either** a left-handed f_L in $(N_1, \overline{N}_2)$ + antiparticle $\overline{f}_R$ in $(\overline{N}_1, N_2)$
- **or** a left-handed f_L in $(\overline{N}_1, N_2)$ + antiparticle $\overline{f}_R$ in $(N_1, \overline{N}_2)$

View $\mathbb{R}^6 = \mathbb{R}^2_1 \times \mathbb{R}^2_2 \times \mathbb{R}^2_3$

- Define topological intersection number I^i_{ab} in each factor of $\mathbb{R}^2_i$:



- Total intersection number is the product of three different:
  $I_{ab} = I^1_{ab} \times I^2_{ab} \times I^3_{ab}$
- $I_{ab} > 0$: f_L in $(N_1, \overline{N}_2)$ + anti-particle
  $I_{ab} < 0$: f_L in $(\overline{N}_1, N_2)$ + anti-particle

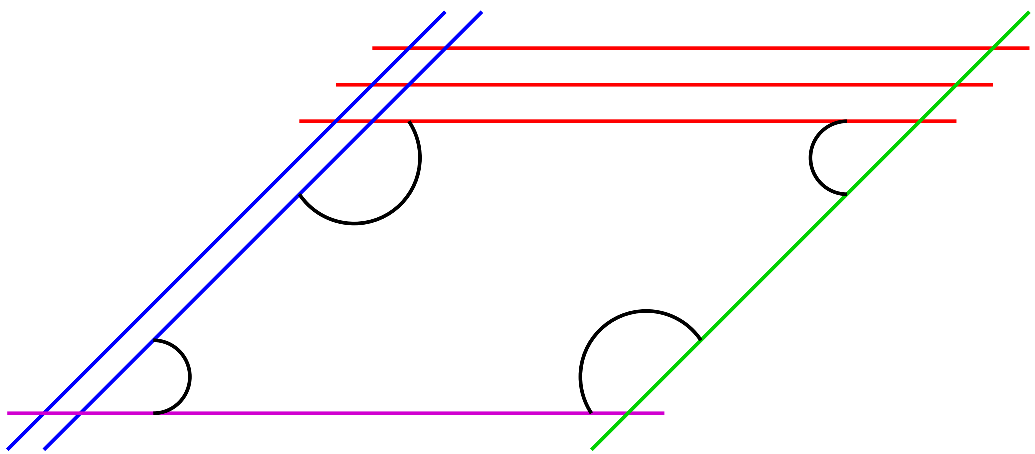
Intersecting Brane Models

→ Simple realisation of gauge groups of the type $\prod_i U(N_i)$ with chiral matter in bifundamental representations

→ Basic ingredients of the Standard Model

$$SU(3) \times SU(2) \times U(1)_Y$$

Direct implementations of Standard Model gauge interactions and matter via "Intersecting Brane Worlds"



Intersecting Brane Worlds

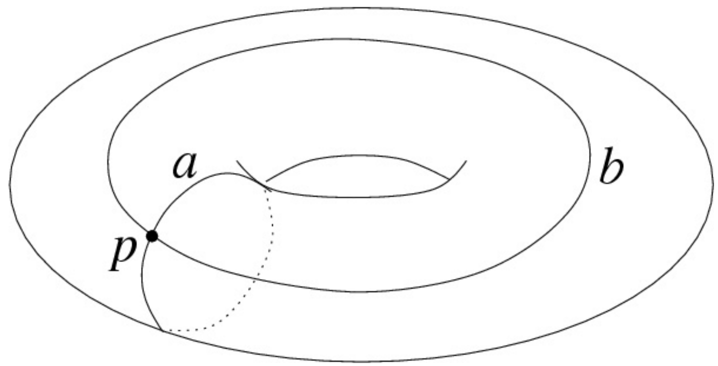
Compactify $\mathcal{R}^{1,9} = \mathcal{M}^{1,3} \times M_6$ M_6: Calabi-Yau manifold

- Our 4 dimensions $\mathcal{M}^{1,3}$ are filled by all the branes.
- The remaining 3 dimensions of the $D6$-branes are wrapped along a 3-cycle Π_a.

What is a cycle?

Toy example:

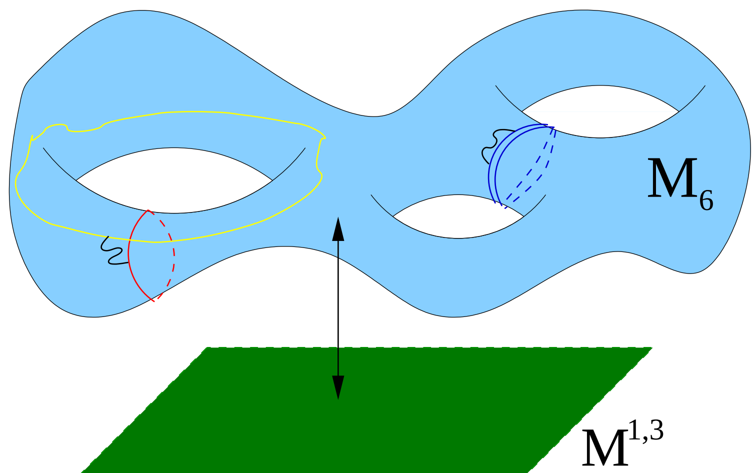
torus $T^2 \rightarrow$ 1-cycles a,b



Generalisation to M_6:

3-cycle = 3-dimensional subspace with no boundary and that is not a boundary itself

$\rightarrow$ brane cannot slip off



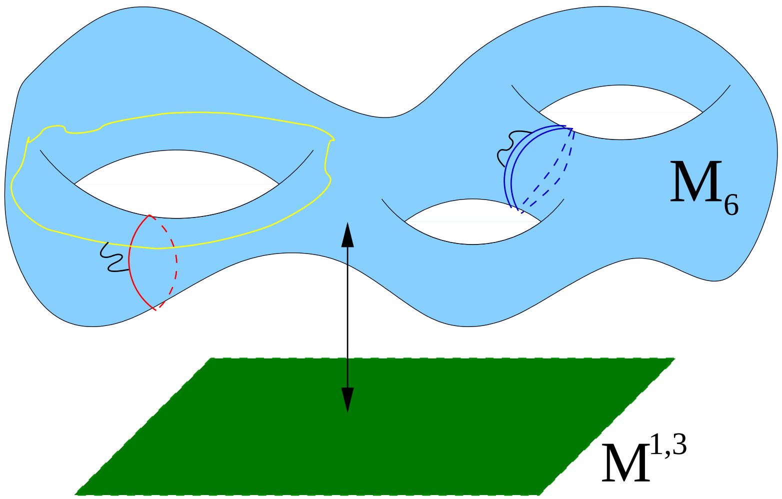
Intersecting Brane Models

Two 3-cycles Π_a and Π_b intersect in points

Each intersection point hosts a chiral fermion in $(\overline{N}_a, N_b)$

$\#$ of generations $=$ $\#$ of intersections

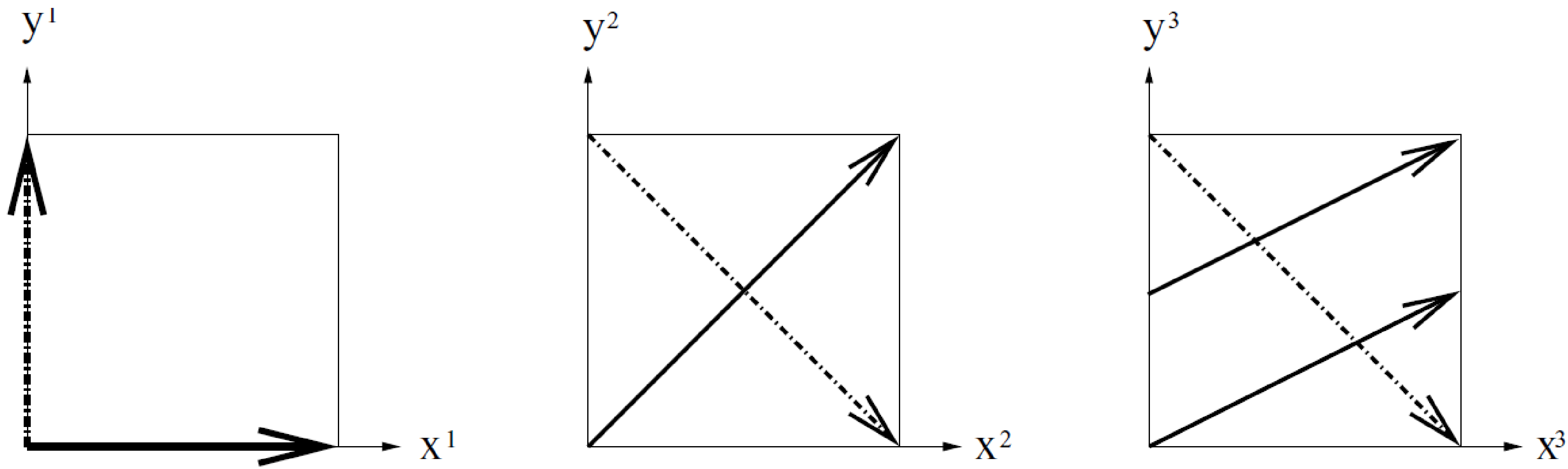
$\Longrightarrow$ Geometric rationale for family replication



Toroidal models

Simplest example (again): $M_6 = T^2 \times T^2 \times T^2$

- Special class of 3-cycles wrap 1-cycle on each T^2
- Specified by wrapping numbers $(n_1, m_1), (n_2, m_2), (n_3, m_3)$
- Intersection number $I^i_{ab} = n^a_i m^b_i - n^b_i m^a_i$



2.) Consistency conditions

D-branes are dynamical

Things are not quite so simple:

Severe consistency conditions, partly related to the fact that a D-brane is a dynamical object: Polchinski 1996

- gravitates and
- acts as charge for certain (generalised) gauge potentials

Some more background information:

Closed string sector contains massless higher form potentials

$C_p = \frac{1}{p!} C_{\mu_1 \dots \mu_p} \, dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}$

- natural generalisation of Yang-Mills 1-form gauge potential
  $A = A_\mu dx^\mu$
- field strength: $F_{p+1} = dC_p$ propagates in entire 10 dim space
- kinetic terms: $S = -\frac{1}{2\kappa^2} \int_{\mathcal{M}_{10}} F_{p+1} \wedge * F_{p+1}$ $\qquad \kappa^2 = \frac{1}{2}(2\pi)^7 \alpha'^4$

IBM: Consistency Conditions

1) Gauss' law:

- D6-branes charged under antisymmetric tensor field

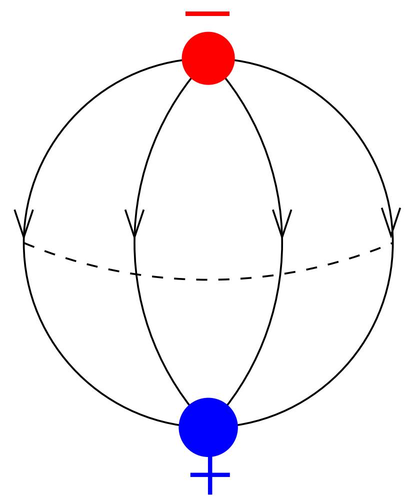
QED:

- $A = A_\mu dx^\mu$
- $S_{coup} = q \int A_\mu dx^\mu$

String Theory:

- $C_7 = C_{[a_1 \dots a_7]} \, dx^{a_1} \wedge \dots \wedge dx^{a_7}$
- $S = \mu_6 \int_{\mathcal{M}^{1,3} \times \Pi_a} C_7$

- total charge under C_7 has to vanish on compact internal space $\leftrightarrow$ Analogy: no single point charge on S^2!



$\rightarrow$ Need to introduce objects of negative charge

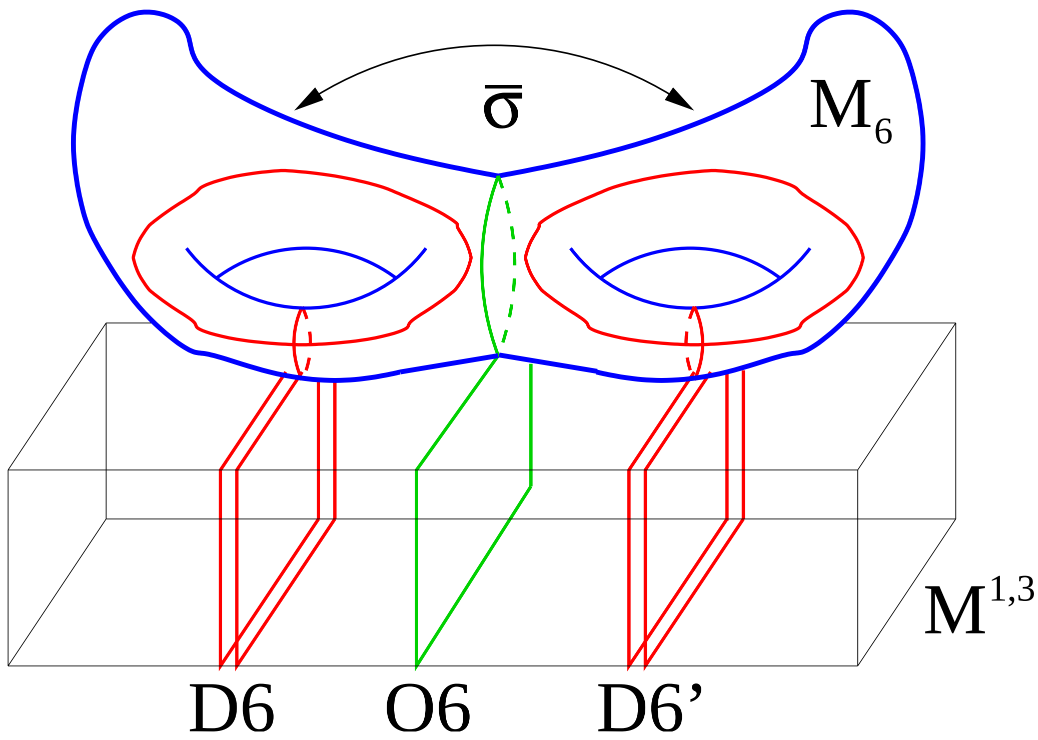
Simplest option: orientifold 6-planes $O6$ on cycle Π_{O6}
arise as fix-poined set of discrete Z_2 symmetry $\overline{\sigma}$ of M_6

IBM: Consistency Conditions

Type IIA/$\Omega\overline{\sigma}$ orientifold:

- mod out by Ω: symmetry of the string worldsheet
- include image branes on $\Pi_{a'}$ subject to Gauss' law:

$$\sum_a N_a([\Pi]_a + [\Pi]_{a'}) = 4[\Pi_{O6}]$$



IBM: Consistency Conditions

2) Supersymmetry at string scale:

guarantees stability and is phenomenologically attractive

- Compactification on Calabi-Yau $\Rightarrow \mathcal{N} = 2$ SUSY
- D-brane on Π_a preserves at best $\mathcal{N} = 1$ subalgebra

$\rightarrow$ cycles Π_a must be volume minimizing = special Lagrangian

$\rightarrow$ all Π_a must preserve the same $\mathcal{N} = 1$ supersymmetry $\leftrightarrow$ D-term

IBM: Consistency Conditions

Tadpole condition: $\sum_a N_a([\Pi]_a + [\Pi]_{a'}) = 4[\Pi_{O6}]$

N_a: # of D-branes along 3-cycle $[\Pi_a]$ $\leftrightarrow$ rank of gauge group $U(N_a)$

- It implies anomaly cancellation in 4D, but it is much stronger.
- Given a specific geometric background, not every gauge group can be constructed on it!

Comparison with bottom-up QFT:

- In 4D we can write down every anomaly-free gauge theory.
- String theory is more restrictive due to consistent coupling to gravity.

Insight:

A consistent compactification manifold with a consistent set of D-branes is a solution to the string equations of motion.

Terminology:

Such a solution is called a **string vacuum.**

IBM: Effective action

✓ Given specific string vacuum, can compute details of effective action

✓ Turns out: All couplings/interactions depend on details of the geometry

Example 1: Planck scale vs. string scale

- 10D effective action: $S_{10D} = \frac{2\pi}{\ell_s^8} \int_{\mathbb{R}^{1,9}} \sqrt{-g} e^{-2\phi} R + \ldots$

  ϕ: dilaton (= massless scalar field from $t_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu} + \phi$)

- Planck scale in 10d: $M^8_{\mathrm{Pl},10} = \frac{2\pi}{\ell_s^8} e^{-2\phi}$: depends on scalar field VEV!

- Compactification ansatz for metric $g^{(10)}_{\mu\nu} = \begin{pmatrix} g^{(4)}_{\mu\nu} & 0 \\ 0 & g^{(6)}_{ij} \end{pmatrix}$

- Result: $S_{\mathrm{4D,eff}} = \frac{2\pi}{\ell_s^8} \int_{\mathbb{R}^{1,3}} \sqrt{-g^{(4)}} R^{(4)} \times \int_{M_6} \sqrt{-g^{(6)}} e^{-2\phi} + \ldots$

  $\Longrightarrow M^2_{\mathrm{Pl},4} = \frac{4\pi}{\ell_s^8} \int_{M_6} \sqrt{-g^{(6)}} e^{-2\phi} = 4\pi M_s^2 e^{-2\phi} \mathrm{Vol}(M_6)$

  $M_s = \ell_s^{-1}$: string scale $\qquad$ $\mathrm{Vol}(M_6)$: volume in units of ℓ_s

IBM: Effective action

Example 1: Planck scale vs. string scale

$M_{\mathrm{Pl},4}^2 = 4\pi M_s^2 e^{-2\phi}\mathrm{Vol}(M_6)$

$M_s = \ell_s^{-1}$: string scale $\qquad$ $\mathrm{Vol}(M_6)$: volume in units of ℓ_s

Perturbatively controlled setups:

For $\mathrm{Vol}(M_6) \geq 1$, $\qquad$ at weak coupling $e^{-2\phi} = \frac{1}{g_s^2} \geq 1$:

$$M_s \leq M_{\mathrm{Pl}} = 10^{19}\mathrm{GeV}$$

Large extra dimensions:

$\mathrm{Vol}(M_6) \gg 1$ gives hierarchically small $M_s \ll M_{\mathrm{Pl}}$

IBM: Effective action

Example 2: Gauge coupling

- Know from fundamental theory:
  $S_{D6} = -T_6 \int_{D6} e^{-\phi} \sqrt{det(g_{\mu\nu} + F_{\mu\nu})}, \qquad T_6 = \frac{2\pi}{\ell_s^7}$: brane tension
- Evaluate for metric $g^{(10)}_{\mu\nu} = \begin{pmatrix} g^{(4)}_{\mu\nu} & 0 \\ 0 & g^{(6)}_{ij} \end{pmatrix}$, $D6 = \mathbb{R}^{1,3} \times \Pi$
  and expand square root
- Result: $\int_{\mathbb{R}^{1,3}} \frac{1}{4g^2_{YM}} F_{\mu\nu} F^{\mu\nu}$ with $\frac{1}{g^2_{YM}} = e^{-\phi}\mathrm{Vol}(\Pi)$
  $\rightarrow$ volume of Π sets strength of gauge interactions

Similarly can show that strength of Yukawa interactions depend on certain cycle volumes.

IBM: Effective action

Important implication: All 4D physical couplings are dynamical!

- Gauge coupling $\leftrightarrow$ volume of cycle Γ: $\int_\Gamma \sqrt{\det(g_{ij}^{(6)})}$
- Metric is itself dynamical and can be viewed as a string field.

  ✓ In GR, fluctuations around flat metric $=$ graviton

  ✓ Graviton $=$ massless spin 2 excitation of closed string

  ✓ curved metric $=$ coherent state of such excitations

  Analogy from QED:

  ✓ photon γ $=$ excitation of electrodynamic vacuum

  ✓ laser field $=$ coherent state of such photons

- Suggested interpretation:

  4D coupling $\leftrightarrow$ cycle volume $\quad = \quad$ expectation value of string states

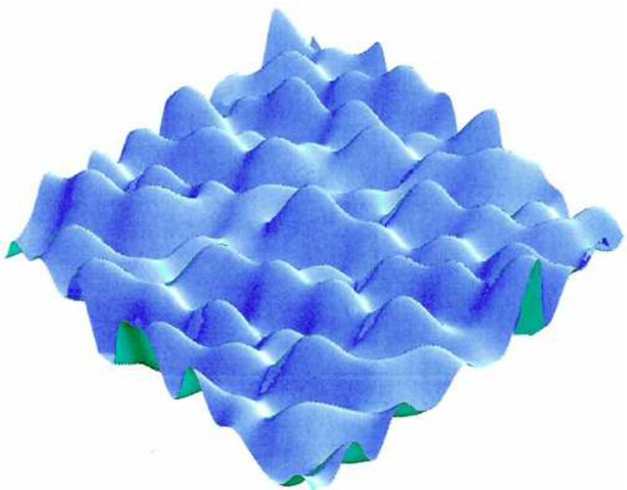
Moduli stabilisation

- Each 2-cycle volume is a dynamical 4D scalar field ϕ_i.
- In presence of certain gauge fluxes and/or by non-perturbative effects, these receive a potential $V(\phi_i)$
- The string vacuum corresponds to a minimum of $V(\phi_i)$

This is what one means by dynamical generation of couplings.

The study of such string solutions is an active area of research.

Preliminary state of art:
There is not just a single, but a multitude of consistent 4D string vacua = the Landscape of String Vacua.



Is there a well-controlled vacuum with or quasi-vacuum with positive cosmological constant?

3.) The landscape of string vacua

The landscape of string vacua

The existence of a multitude of solutions is a common phenomenon:

Example: **Einstein gravity** - One theory with many solutions.

- The theory does not tell us a priori the distance of the earth - sun or the number of planets.

- In fact, a multitude of solar systems exists as consistent solutions (many of them even realized).
- To make predictions we must first specify the relevant solution.
- Lucky case for astronomy:

  Telescopes probe exactly the length scales at which Einstein gravity operates.

  $\leftrightarrow$ Measurements available to fix the boundary conditions of solution.

One theory with many solutions

- A plethora of different 4d string vacua exists.
- Each solution makes definite predictions for physics all the way up to the Planck scale.

Practical difficulty:

- String theory becomes directly testable at energies $E \simeq M_s = \ell_s^{-1}$.
- String scale M_s is the only parameter of the theory.
- Current LHC constraints $M_s \geq 7 \times 10^3 \mathrm{GeV}$.

If M_s is much higher direct probes of string theory in colliders hard (never say never, but at least not next year or so)

This is a problem of every theory of quantum gravity - it really kicks in at the scale of quantum gravity and this can be as high as $10^{19} \mathrm{GeV}$

Lessons from the Landscape

At least in two instances scales of observed physics appears "fine-tuned".

- Cosmological Constant
- Higgs mass

Concerning Higgs mass:

- Many dynamical solutions have been suggested.
- The vast majority involves new physics at TeV scale (SUSY, Large Extra Dimensions, Technicolour,...)
- If LHC finds no new physics beyond the Higgs, then the Higgs mass **might** just be fine-tuned. (Controversial)

Then the string landscape offers a huge set of solutions - each with a different Higgs mass.

Fine-tuning is ok as long as Higgs mass scans in the landscape.

In string theory such considerations can be made within a theoretically sound framework.

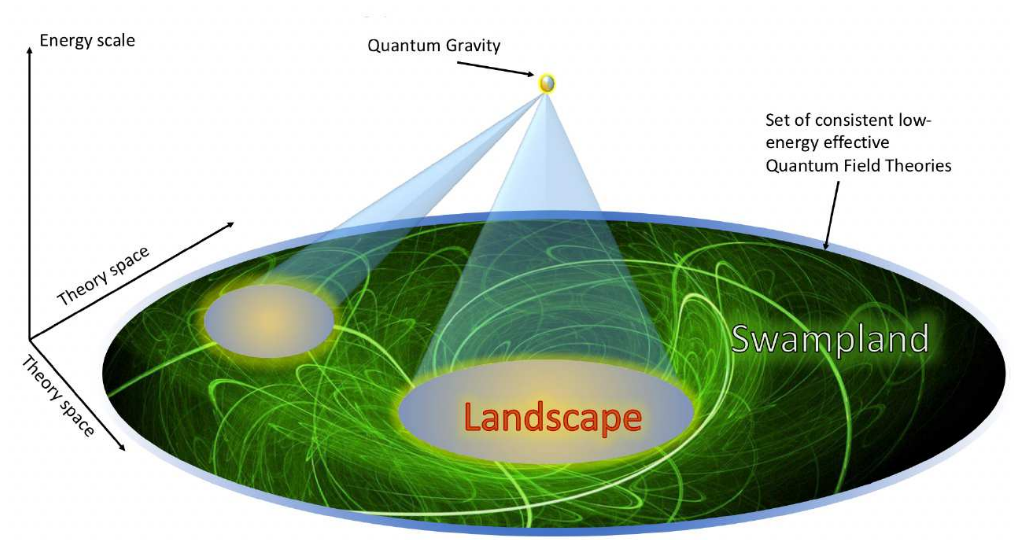
4.) The Swampland Idea

The Swampland idea

Which constraints arise for an EFT from the fact that it is the low-energy approximation of a Quantum Gravity (QG) theory?

Terminology: [Vafa 2005]

Swampland		**Landscape**
EFT consistent as a local Quantum Field Theory but not as a Quantum Gravity	$\Longleftrightarrow$	EFT fully consistent as a Quantum Gravity



Pic from review by [Palti'19]

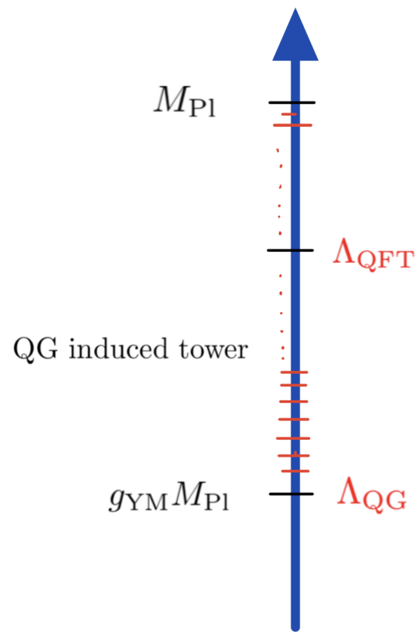
Swampland Surprises

Surprise 1 (Conjecture):

The true cutoff of the EFT in QG may lie *parameterically* below $\Lambda_{\text{naive}} \sim M_{\text{Pl}}$.

Example: U(1) gauge theory coupled to gravity as EFT from a QG

$$F_{\text{U}(1)} = -\frac{g^2_{\text{U}(1)}}{4\pi}\frac{q_1 q_2}{r^2} \qquad\qquad F_{\text{grav}} = G_{\text{N}}\frac{M_1 M_2}{r^2}$$

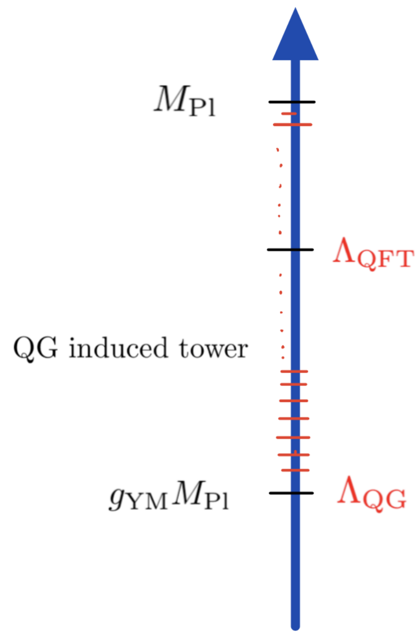


Conjecture:

QG imposes cutoff for EFT parametrically below naive cutoff M_{Pl}:

$$\Lambda = g_{\text{U}(1)} M_{\text{Pl}} \ll M_{\text{Pl}} \quad \text{if} \quad g_{\text{U}(1)} \to 0$$

Swampland Surprises - Teaser



Conjecture:

QG imposes cutoff for EFT parametrically below naive cutoff $M_{\rm Pl}$*:*

$$\Lambda = g_{U(1)} M_{\rm Pl} \ll M_{\rm Pl} \quad \text{if} \quad g_{U(1)} \to 0$$

Consequences:

- If the EFT has states above $g_{U(1)} M_{\rm Pl}$, then it is inconsistent as an EFT and hence in the Swampland.
- In the limit $g_{U(1)} \to 0$, the EFT breaks down since $\Lambda_{\rm QG} \to 0$.

Swampland Surprises - Teaser

Surprise 2 (Conjecture):

Not all types of matter and interactions consistent in absence of gravity can be coupled to Quantum Gravity.

More precisely:

Not all anomaly free gauge groups and matter are consistent in presence of gravity.

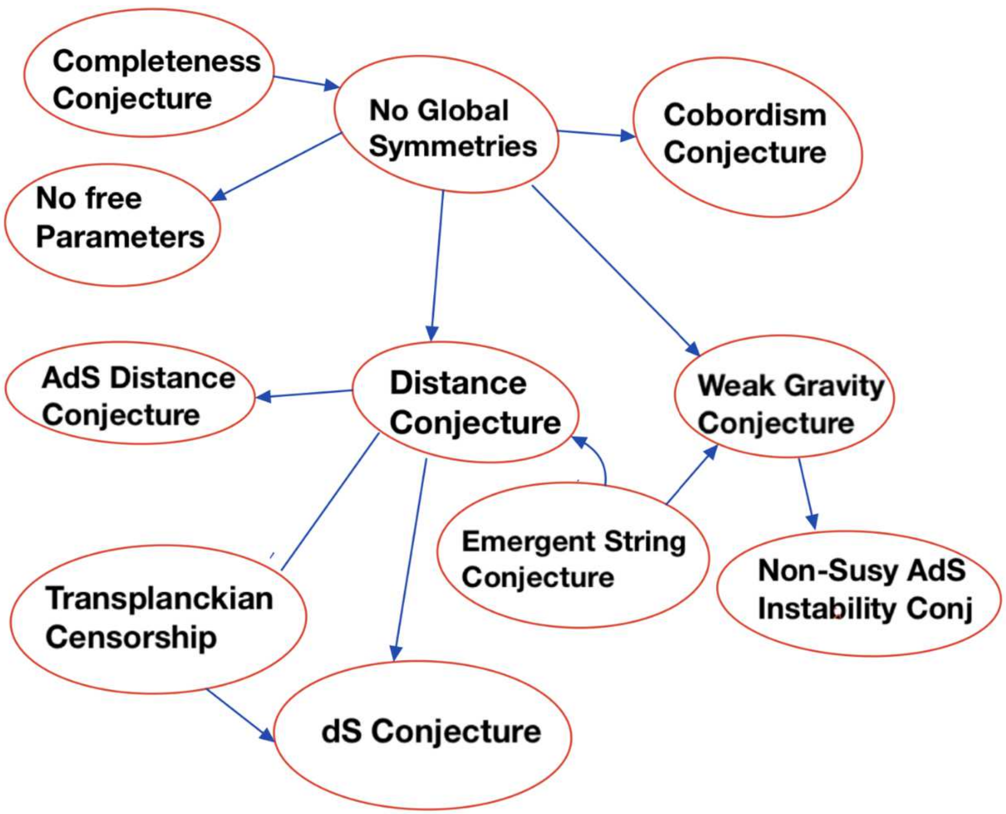
Example:

Matter content in many higher-dimensional theories with gravity indeed bounded.

Quantum Gravity Conjectures

Try to find general principles which every hypothetical QG should encompass.

Broad in scope, but speculative and oftentimes only heuristic:
$\Longrightarrow$ **Web of Conjectures with many logical connections**



Quantum Gravity Conjectures

Try to find general principles which every hypothetical QG should encompass.

Broad in scope, but speculative and oftentimes only heuristic:
$\Longrightarrow$ Web of Conjectures with many logical connections

Within string theory:
Conjectures can be tested rigorously and refined.

Long-term goal:
Find generic properties of quantum gravity.