Hamburg Summer School 2025

Particles, Strings & Cosmology

Theoretical Cosmology

Week 2: July 23-25 , 2025

Lecturer:

Alexander Westphal

Programme

Wednesday: FRW metric, Einstein equation, derivation of Friedmann equation, solutions,

Thursday: Particle decoupling, neutrino decoupling, thermodynamics, relic photons, derivation of g_*(T), entropy density, neutrino temperature, photon decoupling , dark matter thermal production: freeze-out, relic abundance computation

basics of inflation, slow-roll models, stability under quantum corrections

Friday: inflation continued, quantum fluctuations during inflation, basics of CMB, basics of eternal inflation

References

Textbooks:

-Kolb and Turner, The early universe

-Gorbunov and Rubakov, Introduction to the theory of the early universe

-Bailing and Love, Cosmology in gauge field theory and string theory

-Dodelson, Modern cosmology

-Weinberg, Gravitation and cosmology

-Weinberg, Cosmology

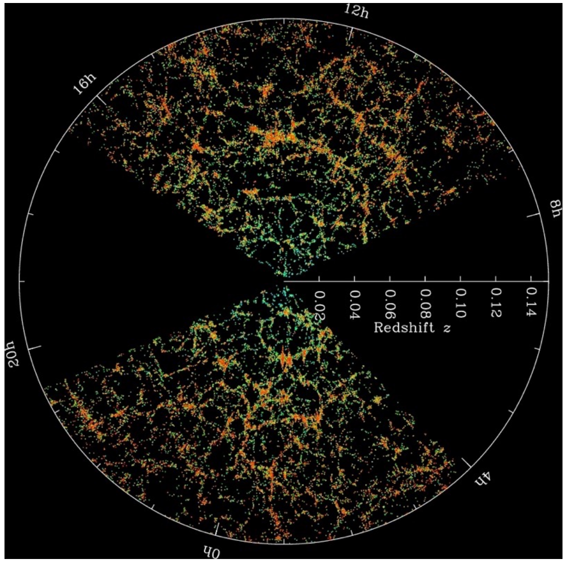
-Baumann: Cosmology

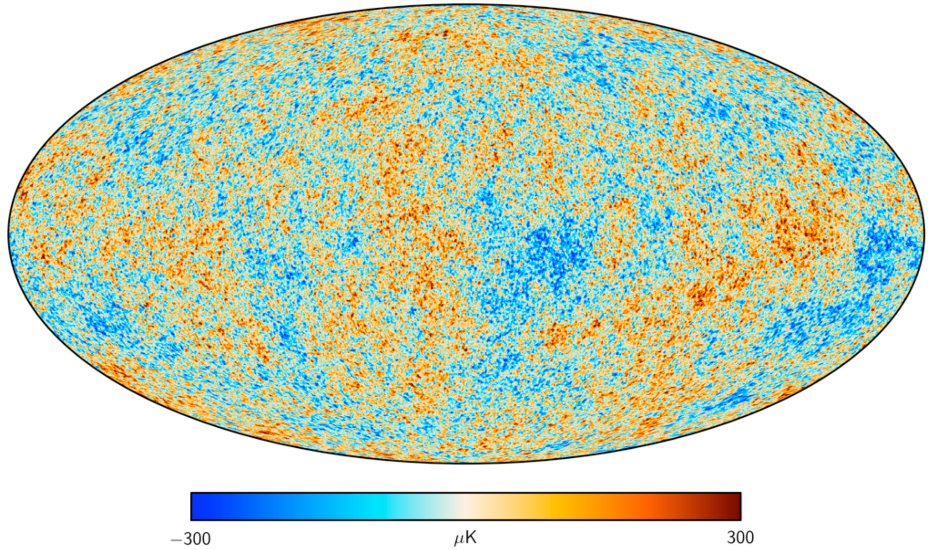
+ many lecture notes available on the arXiv

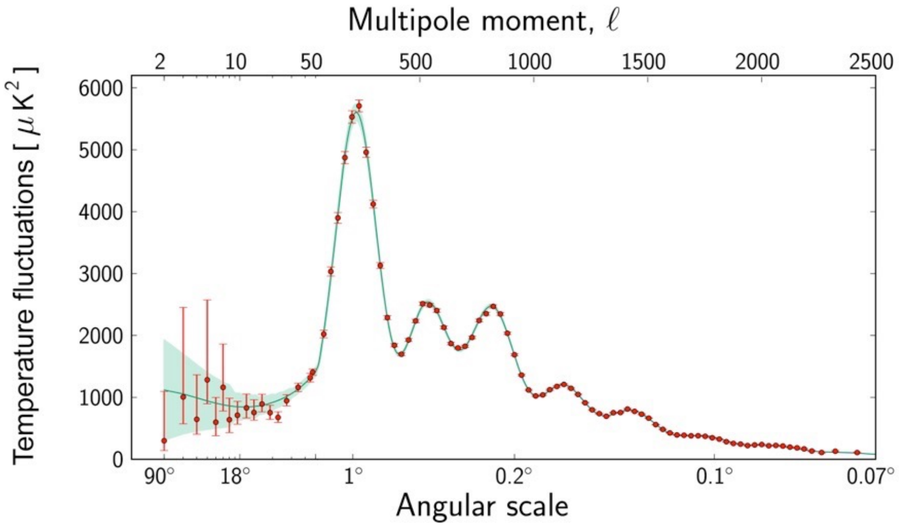
recommended : Daniel Baumann's lecture notes

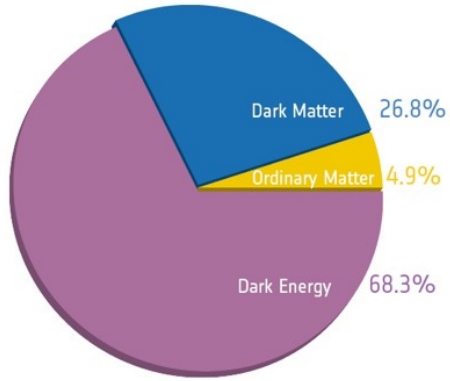
INTRODUCTION

SDSS Galaxy Map

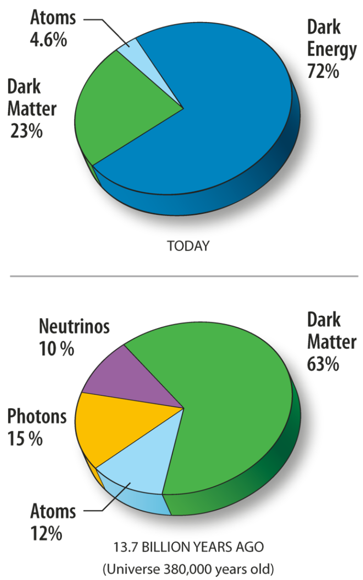


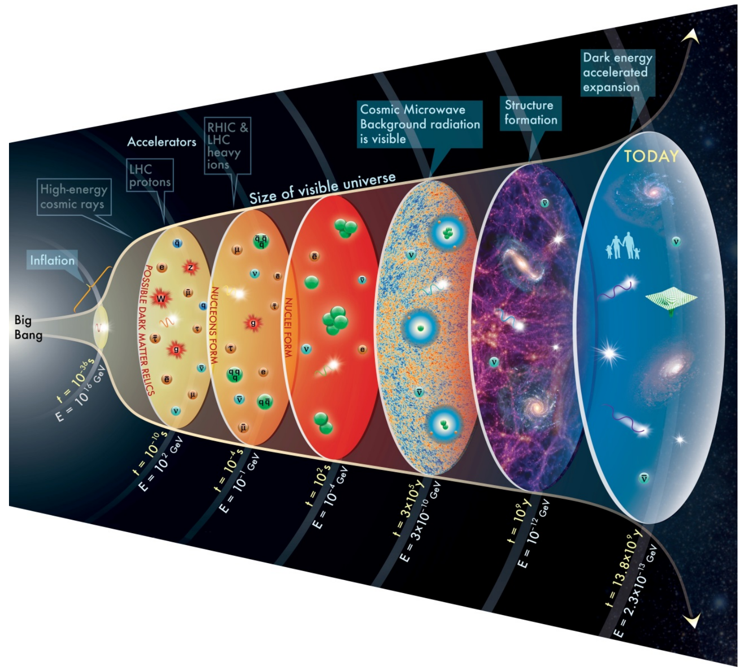


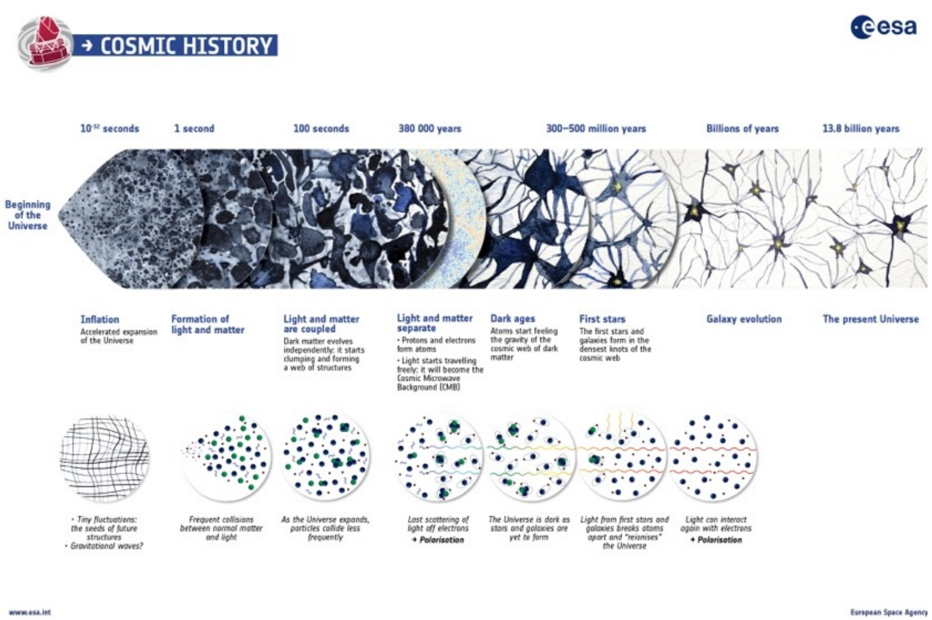




After Planck



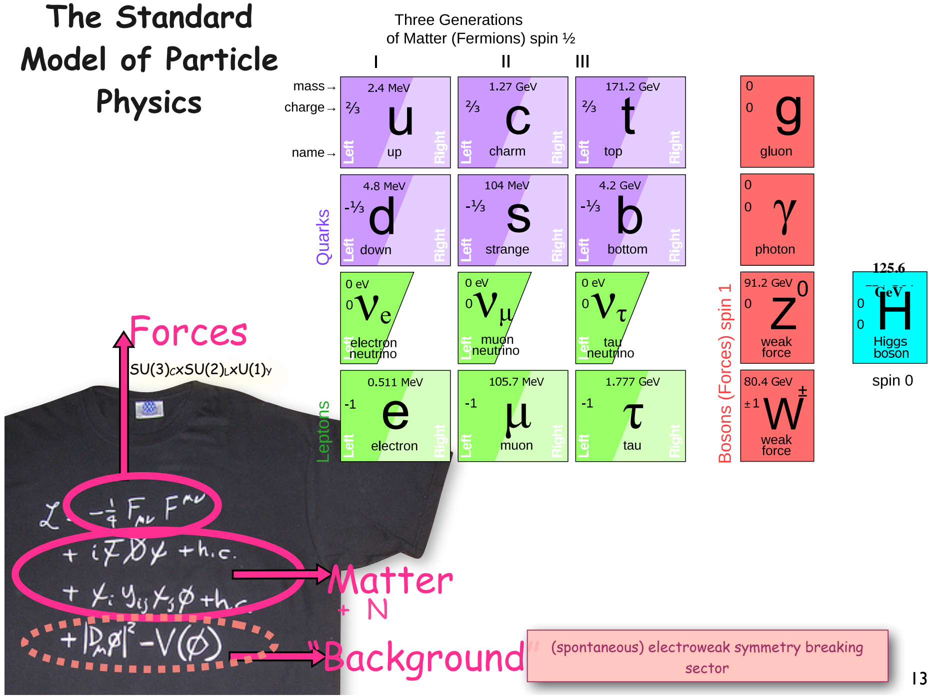


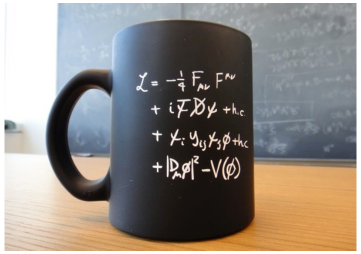


Key events in the thermal history of the universe

Event	time t	redshift z	temperature T	
Inflation	10^{-34} s (?)	–	–	
Baryogenesis	?	?	?	
EW phase transition	20 ps	10^{15}	100 GeV	
QCD phase transition	20 μs	10^{12}	150 MeV	
Dark matter freeze-out	?	?	?	$X+\bar{X} \leftrightarrow \ell+\bar{\ell}$.
Neutrino decoupling	1 s	6×10^9	1 MeV	$\nu_e+\bar{\nu}_e \leftrightarrow e^++e^-$, $e^-+\bar{\nu}_e \leftrightarrow e^-+\bar{\nu}_e$
Electron-positron annihilation	6 s	2×10^9	500 keV	$e^++e^- \leftrightarrow \gamma+\gamma$
Big Bang nucleosynthesis	3 min	4×10^8	100 keV	$n+\nu_e \leftrightarrow p^++e^-$, $n+e^+ \leftrightarrow p^++\bar{\nu}_e$, $n \leftrightarrow p^++\bar{\nu}_e+e^-$, $n+p^+ \leftrightarrow \mathrm{D}+\gamma$, $\mathrm{D}+p^+ \leftrightarrow {}^3\mathrm{He}+\gamma$, $\mathrm{D}+{}^3\mathrm{He} \leftrightarrow {}^4\mathrm{He}+p^+$.
Matter-radiation equality	60 kyr	3400	0.75 eV	
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV	$e^-+p^+ \leftrightarrow \mathrm{H}+\gamma$
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV	Thomson $e^-+\gamma \leftrightarrow e^-+\gamma$
Reionization	100–400 Myr	11–30	2.6–7.0 meV	
Dark energy-matter equality	9 Gyr	0.4	0.33 meV	
Present	13.8 Gyr	0	0.24 meV	

The Standard Model of Particle Physics





The Standard Model of Particle Physics fails to explain:

Matter-antimatter

Dark Matter

Dark Energy

Inflation

Quantum Gravity

All related to physics of the early universe

THEORETICAL COSMOLOGY

Aim: Understanding structure, evolution & origin of the universe

Relies on two "Standard Models"

- **of particle physics**
- **of cosmology (Hubble diagram, BBN, CMB)**

Cosmology

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Inflation & CMB

(A. Westphal, DESY)

literature:

- Inflation and String Theory: <u>Chapt. 2</u>
  (textbook by Baumann & McAllister)
  also on arXiv: 1404.2601
- reviews:

  arXiv: 0907.5424
  (Baumann)

  arXiv: hep-th/0503203
  (textbook by Andrei Linde)
- Modern Cosmology
  (textbook by Scott Dodelson)
- Introduction to the Theory of the Early Universe – Vol. 2
  (textbook by Gorbunov & Rubakov)

we will work with natural units:

$$\hbar = c = k_B = 1$$

(Planck constant, speed of light in vacuo, Boltzmann constant)

$\rightsquigarrow$ quantities have mass dimensions:

$$[\text{energy}] = [\text{mass}] = [\text{temperature}]$$
$$= [\text{length}]^{-1} = [\text{time}]^{-1}$$

in these units the Planck scale, where quantum gravity effects get strong, is:

$$M_P^2 = \frac{1}{8\pi G_N} = 2.4 \cdot 10^{18}\,\text{GeV}$$

(G_N ← gravitational const.)

sometimes we also set: $G_N = 1 \Rightarrow M_P = 1$

mass unit $1\,\text{GeV} \simeq 1.78 \cdot 10^{-24}\,g \simeq m_p$ (proton mass)
time unit $1\,\text{GeV}^{-1} \simeq 6.59 \cdot 10^{-25}\,s$
length unit $1\,\text{GeV}^{-1} \simeq 0.20\,\text{fm}$
temperature $1\,\text{GeV} \simeq 1.16 \cdot 10^{13}\,K$

distance scales in the cosmos

- $R_{\text{Earth}} \sim 6000\,\text{km}$
- solar system $\sim 7 \cdot 10^9\,\text{km}$
- Milky Way (visible part) (our galaxy) $\sim 10^{18}\,\text{km}$ $\sim 10^5\,\text{ly}$ (light years)
- $1\,\text{AU} \equiv$ mean Earth-Sun distance $\simeq 1.5 \cdot 10^8\,\text{km}$
- $1\,\text{ly} \simeq 10^{18}\,\text{cm}$
- $1\,\text{pc} \equiv$ distance, from where $1\,\text{AU}$ appears with angular diameter $1\,\text{arcsec} = 1'' = \frac{1\,\text{AU}}{1''} = 3.26\,\text{ly}$
- diameter of Milky Way $\simeq 30\,\text{kpc}$
  width of its disk $\simeq 10\,\text{kpc}$
- Sol is at $\simeq 8\,\text{kpc}$ from center of the Milky Way
- closest major galaxy: Andromeda (M31) $\sim 770\,\text{kpc}$ distant
- typical size of galaxy clusters: $10\,\text{Mpc}$

The visible universe today

(1) main feature: large-scale spatial homogeneity & isotropy

$\rightsquigarrow$ "cosmological principle"

observable patch $\simeq$ 5000 Mpc

(2) contains 200...2000 billion galaxies

→ galaxy mass $\sim 10^{12} M_\odot$

(3) distribution of structures:

homogeneous @ large scales (> 100 Mpc)

very inhomogeneous @ small scales (< 100 Mpc)

→ galaxies, clusters, superclusters, filaments, voids

↓

Formed by gravitational instability from small initial quantum fluctuations during inflation (seeds of future structures).

Initial quantum fluctuations

→ CMB temperature variations

FRW metric

evidence:

→ good for spatial isotropy (count # of galaxies in all directions)

→ harder for spatial homogeneity (measure distances between galaxies & reconstruct 3D map)

Spatial isotropy & homogeneity:

mathematical description with space-time metric where spatial slices (hypersurfaces of constant cosmic time) are maximally symmetric ...

in 3D polar spatial coord.s

[see sect. 1 of Baumann]

$$ds^2 = dt^2 - a^2(t) \cdot \left(\frac{dr^2}{1 - k \cdot r^2} + r^2 \cdot d\Omega_2^2 \right)$$

$a^2(t)$ ← scale factor

$k = $ +1 closed universe, 0 flat universe, −1 open universe

"Friedmann-Robertson-Walker (FRW) metric"