Freeze out of heavy particles and decoupling

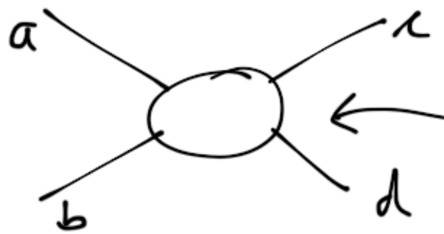
→ 'freeze-out' = decoupling from the thermal bath

↝ need to know reaction rate Γ:

particle speed $\simeq c$

$$\Gamma_a = n_a \cdot \sigma \cdot v$$

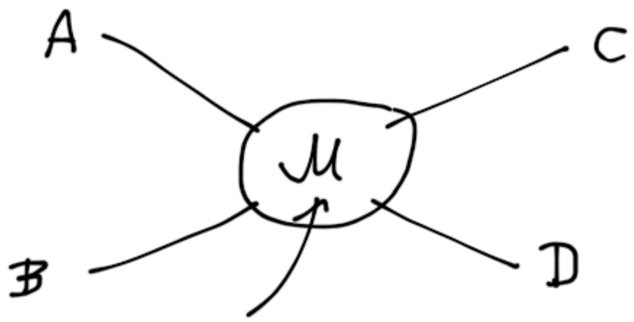
production of a-particle



cross section σ

description of freeze-out

reaction



$A + B \to C + D$

QFT: matrix element $\mathcal{M}$ of reaction

semi-classically:

$$\frac{d\#}{dt} = \Gamma_{\text{production}} - \Gamma_{\text{annihil./decay}}$$

for e.g. particle type A in more detail:

$$\#(A) = N_A = n_A V \sim n_A a^3$$

and QFT

$$\Rightarrow \frac{1}{a^3}\frac{d}{dt}\left(n_A a^3\right) = \int \frac{d^3p_A}{(2\pi)^3 E_A} \int \frac{d^3p_B}{(2\pi)^3 E_B} \int \frac{d^3p_C}{(2\pi)^3 E_C} \int \frac{d^3p_D}{(2\pi)^3 E_D} \qquad (B)$$

$$\times (2\pi)^4 \cdot \delta^{(3)}\left(\vec{p}_A + \vec{p}_B - \vec{p}_C - \vec{p}_D\right) \cdot \delta\left(E_A + E_B - E_C - E_D\right)$$

$$\times |\mathcal{M}|^2 \cdot \Big\{ \underbrace{f_C f_D (1 \pm f_A)(1 \pm f_B)}_{\text{production of } A} - \underbrace{f_A f_B (1 \pm f_C)(1 \pm f_D)}_{\text{annihilation/decay of } A} \Big\}$$

Matrix element

$+$: boson
$-$: fermion

$f_i(E)$, $i = A, B, C, D$
partition function of particle species i

(B) is the Boltzmann equation

$\downarrow$

look at two examples ...

i) freeze-out of thermally produced weakly interacting dark matter

Why dark matter (DM)?

- galaxy rotation curves:

$$\frac{m v^2}{r} \sim G \frac{Mm}{r^2} \Rightarrow v^2(r) \sim \frac{1}{r}$$

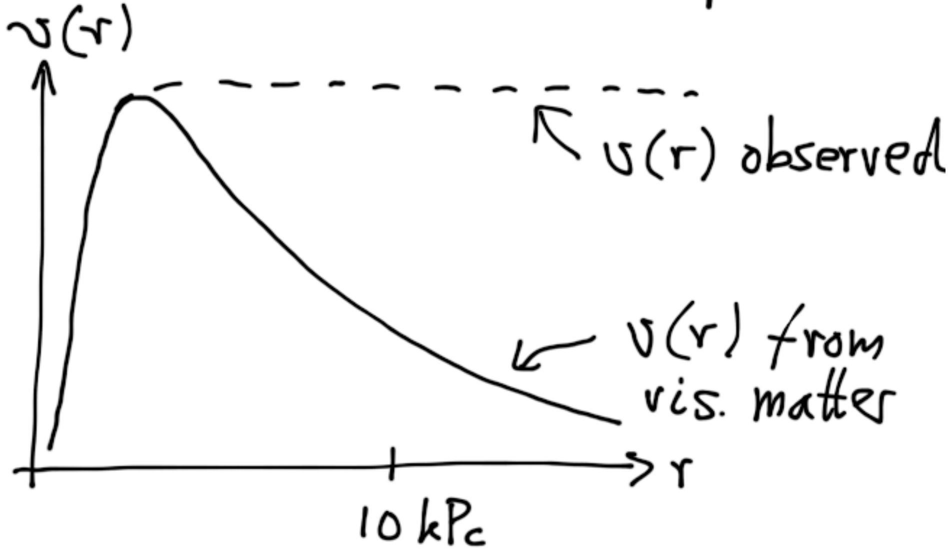
$\leadsto$ observation:

$$v(r) \simeq \text{const.}$$

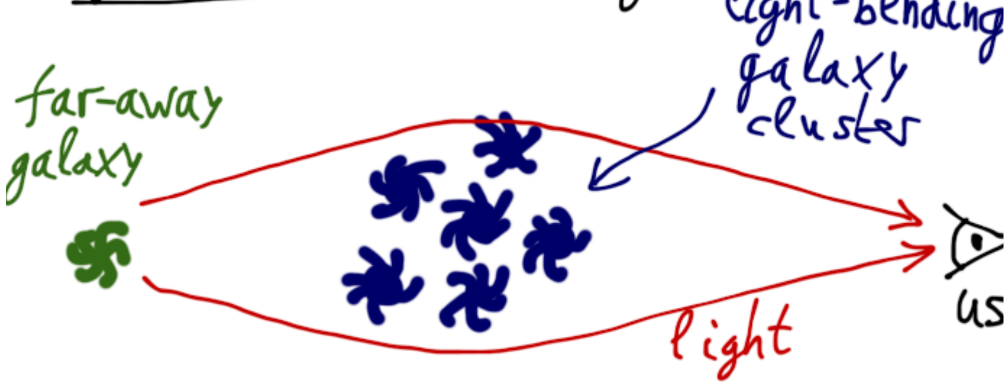
$\leadsto$ need $M(r) \sim r$ to compensate: invisible/dark ...

galaxy rotation curves:

visible matter : $M(r) \sim \frac{1}{r^\#}$, $\# > 0$



gravitational lensing:



- X-ray observations of gas bound in galaxies:

  requires some extra dark gravitating stuff to keep gas inside the galaxy

- WMAP – CMB + BBN:

  BBN fixes $\eta_B \Rightarrow \Omega_B \simeq 0.04$

  WMAP : $\Omega_\Lambda \simeq 0.7$

  $\Omega_0 \simeq 1$

  $\Rightarrow \Omega_{DM} \simeq 0.3$

- gravitational lensing

  $\rightsquigarrow \Omega_m \simeq 0.3$

- structure formation requires some 'extra' gravitational wells : $\Omega_m \simeq 0.3$

$\Rightarrow$ There are many possibilities for DM.

Will focus on WIMP:

new "Weakly Interacting Massive Particle" beyond the SM

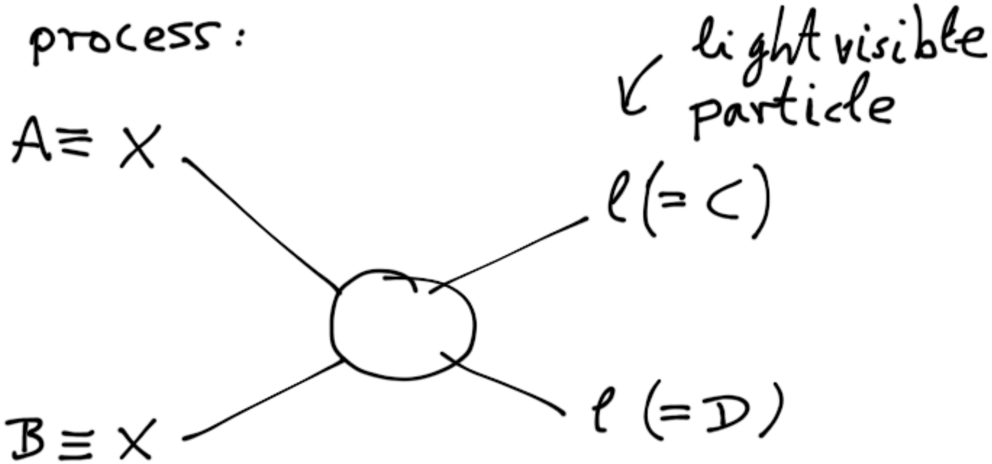
$\rightarrow$ has weak-scale interactions

$\rightarrow$ heavy

$\rightsquigarrow$ well-motivated e.g. from SUSY.

single constituent DM:

process:



X : heavy DM WIMP

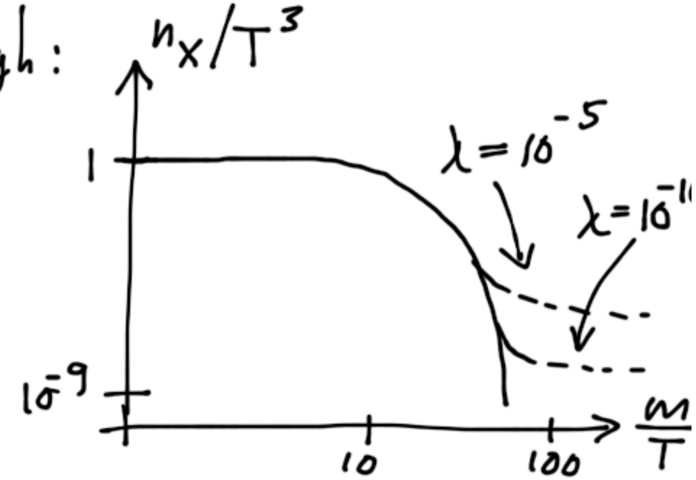
the Boltzmann eq. is again (B) .

big picture:

DM particles X start at high T in equilibrium. If they stayed so always, then:

$$n_X \sim e^{-\frac{m_X}{T}} \quad \text{for } T < m_X$$

<u>but</u>, if $\Gamma_X < H$, DM density freezes out & survives, if it lives long enough:



here: $\lambda = \frac{m^2 \langle \sigma v \rangle}{H(m)}$

$\rightsquigarrow$ will turn out that WIMPS with weak scale masses give $\Omega_{DM} \simeq 0.3$

note: at high T the reaction is in kinetic equilibrium, but not always in chemical equilibrium ...

This implies:

$$f(E) \sim e^{-\frac{E-\mu}{T}}$$

for $T \ll E - \mu$

$\rightsquigarrow$ put differently, μ need not be at its equilibrium value.

$\Rightarrow$ Can rewrite the bracket $\{\ldots\}$ of eq. (B) as:

$$\{\ldots\} = e^{-\frac{E_A+E_B}{T}} \cdot \left(e^{\frac{\mu_C+\mu_D}{T}} - e^{\frac{\mu_A+\mu_B}{T}} \right)$$

$$(E_A + E_B = E_C + E_D)$$

use the n_i for $T \ll m_i$:

$$\Rightarrow \{\ldots\} = e^{-\frac{E_A+E_B}{T}} \cdot \left[\frac{n_C n_D}{n_C^{(0)} n_D^{(0)}} - \frac{n_A n_B}{n_A^{(0)} n_B^{(0)}} \right]$$

with: $\frac{n_i}{n_i^{(0)}} = e^{\mu_i/T}$

equil. n_i at $\mu_i = 0$

now define the thermally averaged cross section:

$$\langle \sigma v \rangle = \frac{1}{n_A^{(0)} n_B^{(0)}} \prod_i \int \frac{d^3 p_i}{(2\pi)^3 E_i} \, e^{-\frac{E_A + E_B}{T}} \cdot |\mathcal{M}|^2 \cdot (2\pi)^4$$

$$\times \, \delta^{(4)}(p)$$

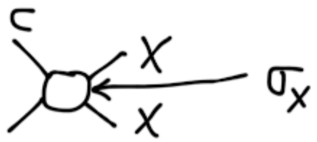
Simplifies the Boltzmann eq. (B):

$$\frac{1}{a^3} \frac{d(n_A a^3)}{dt} = n_A^{(0)} n_B^{(0)} \langle \sigma v \rangle \cdot \left[\frac{n_C n_D}{n_C^{(0)} n_D^{(0)}} - \frac{n_A n_B}{n_A^{(0)} n_B^{(0)}} \right]$$

$$\Downarrow$$

$$\frac{1}{a^3} \frac{d(n_X a^3)}{dt} = \left(n_X^{(0)}\right)^2 \langle \sigma_X v \rangle \cdot \left[1 - \left(\frac{n_X}{n_X^{(0)}} \right)^2 \right] \qquad (B')$$

for X-production:
($\mu_c = 0$ early on)



to obtain freeze-out value of n_X, go to limit:

$$\frac{dn_X}{dt} \to 0 \text{ for } t \to \infty$$

We have:

$$\frac{1}{a^3} \frac{d(n_X a^3)}{dt} = \frac{dn_X}{dt} + 3 \frac{\dot a}{a} \cdot n_X =$$

$$= \frac{dn_X}{dt} + 3H \cdot n_X$$

thus we demand:

$$\frac{dn_X}{dt} = 0 \text{ at late times}$$

furthermore, at late times:

$$\frac{m_X}{T} \to \infty$$

$$\Rightarrow \quad n_X^{(0)} \sim e^{-\frac{m_X}{T}} \to 0$$

exponentially fast at late times

$\Rightarrow$ (B') becomes:

$$3H \cdot n_X \simeq \langle \sigma_X v \rangle \cdot n_X^2 \quad (B'')$$

if we realize:

$$\langle \sigma_X v \rangle n_X = \langle \Gamma_X \rangle$$

thermally averaged reaction rate

this is nothing else than:

$$H \simeq \langle \Gamma_X \rangle$$

our old freeze-out criterion!

We get:

$$(B'') \Leftrightarrow n_X \sim \frac{H}{\langle \sigma_X v \rangle}$$

thus we get for ρ_X:

$$\rho_X = m_X n_X \sim \frac{m_X H}{\langle \sigma_X v \rangle} \quad (**)$$

and at freeze-out:

$$H \sim \frac{T_{f.o.}^2}{M_P}, \quad T_{f.o.} \sim m_X$$

$$(**) \Rightarrow \rho_X \sim \frac{m_X T_{f.o.}^2}{M_P \langle \sigma_X v \rangle} \sim \frac{T_{f.o.}^3}{M_P \langle \sigma_X v \rangle}$$

now, ρ_X is ρ_X at freeze-out, dilutes until now → non-relat. matter:

$$\rho_{X,0} = \rho_X \left(\frac{a_{f.o.}}{a_0} \right)^3 \sim \frac{T_0^3}{M_P \langle \sigma_X v \rangle} \left(\frac{T_{f.o.}}{T_0} \cdot \frac{a_{f.o.}}{a_0} \right)^3$$

now, because:

$$S = \text{const.} = s \cdot a^3 \Rightarrow T \sim \frac{1}{a}$$

$$\Rightarrow \frac{T_{f.o.}}{T_0} \cdot \frac{a_{f.o.}}{a_0} \sim \mathcal{O}(1)$$

$$\Rightarrow \rho_{X,0} \sim \frac{T_0^3}{M_P \langle \sigma_X v \rangle}$$

we can now calculate to-day's density parameter $\Omega_{X,0}$ of the X particles:

$$\Omega_{X,0} = \frac{\rho_{X,0}}{\rho_{cr,0}} \sim \frac{T_0^3}{M_P^3} \cdot \frac{1}{H_0^2 \langle \sigma_X v \rangle}$$

now we have:

$$T_0 \sim 10^{-4}\,\mathrm{eV}\,, \quad M_P \sim 2\cdot 10^{18}\,\mathrm{GeV}$$

$$\frac{H_0}{c} \simeq 3\cdot 10^{-9}\,\frac{\mathrm{s}}{\mathrm{m}}\cdot\frac{72\cdot 10^{3}\,\mathrm{m}}{\mathrm{s}\cdot\mathrm{Mpc}}$$

$$\simeq \frac{2\cdot 10^{-4}}{\mathrm{Mpc}} \simeq \frac{2\cdot 10^{-4}}{3\cdot 10^{6}\cdot 10\cdot 10^{12}\,\mathrm{km}}$$

$$\simeq 6\cdot 10^{-29}\,\frac{1}{\mathrm{cm}}$$

plug into $\Omega_{X,0}$:

$$\Omega_{X,0} \sim 10^{-94}\cdot\frac{10^{58}}{36}\cdot\frac{\mathrm{cm}^2}{\langle\sigma_X v\rangle}$$

$$\Rightarrow \Omega_{X,0} \sim \frac{3\cdot 10^{-38}\,\mathrm{cm}^2}{\langle\sigma_X v\rangle} \quad (\Omega)$$

for $\langle\sigma_X v\rangle$ we get:

σ_X :

$\alpha \sim 0.01$

(EW interaction)

$$\Rightarrow \mathcal{M} \sim \frac{\alpha}{M_W^2} \sim \alpha G_F$$

$$\Rightarrow \langle\sigma_X v\rangle \sim \alpha^2 G_F^2 m_X^2$$

$$= 10^{-4}\cdot\left(\frac{1}{(300\,\mathrm{GeV})^2}\right)^2\cdot(100\,\mathrm{GeV})^2\cdot\left(\frac{m_X}{100\,\mathrm{GeV}}\right)^2$$

$$\simeq 10^{-10}\,\mathrm{GeV}^{-2}\cdot\left(\frac{m_X}{100\,\mathrm{GeV}}\right)^2$$

little conversion help:

$$\mathrm{GeV}^{-1} \sim 0.2\,\mathrm{fm} = 2\cdot 10^{-14}\,\mathrm{cm}$$

$$\Rightarrow \langle \sigma_\chi v \rangle \sim \left(\frac{m_\chi}{100\,\mathrm{GeV}}\right)^2 \cdot 10^{-37}\,\mathrm{cm}^2$$

plug this into (Ω):

$$\Rightarrow \Omega_{\chi,0} \sim 0.3 \cdot \left(\frac{100\,\mathrm{GeV}}{m_\chi}\right)^2 \quad (83)$$

$\rightsquigarrow$ A WIMP with mass of ~ 100 GeV, natural in EW-scale SUSY, is a perfect DM candidate!