$\Downarrow$ CMB: spatially isotropic & spatial homogeneity

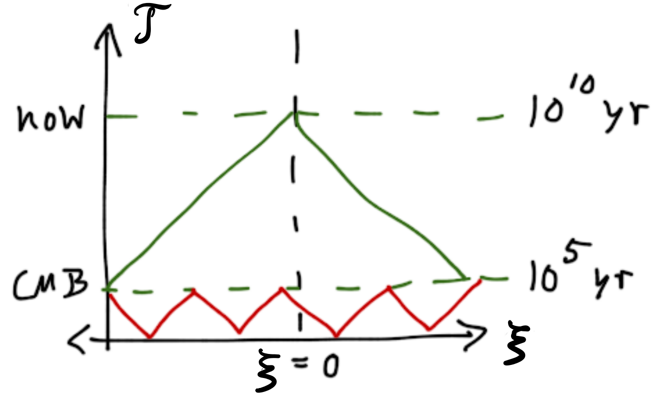
FRW space-time:

$$ds^2 = dt^2 - a^2(t) \cdot \left[d\xi^2 + f^2(\xi) \cdot d\Omega_2^2 \right]$$

$$\left.\frac{S^3,\ H^3,\ \mathbb{R}^3}{k = +1, -1, 0}\right\}$$ maximally symm. 3-space

$d\xi = \frac{dr}{a}$ 'comoving distance'

use conformal time: $dt \rightarrow d\tau = \frac{dt}{a}$



<u>horizon problem</u>

Einstein's equations

$$\Rightarrow \quad H^2 = \frac{1}{3M_p^2}\rho - \frac{k}{a^2}$$ Friedmann equation

energy density: matter, radiation, ...

spatial curvature

ρ drives expansion ...

flatness problem: $\Omega_i \equiv \frac{\rho_i}{\rho_{flat}}$

$$\frac{\rho_k}{\rho_{rad.}}\Big|_{now} = \frac{\Omega_k}{\Omega_{rad.}}\Big|_{now} \sim e^{120}\frac{\rho_k}{\rho_{rad.}}\Big|_{t_{GUT}} \lesssim 0.01$$

$$\Rightarrow \text{need } \frac{\rho_k}{\rho_{rad}}\Big|_{t_{GUT}} \simeq \Omega_k\big|_{t_{GUT}} < e^{-120}\,!$$

solution? $\longrightarrow$ $\ddot a > 0$ acceleration early on

comoving particle horizon:

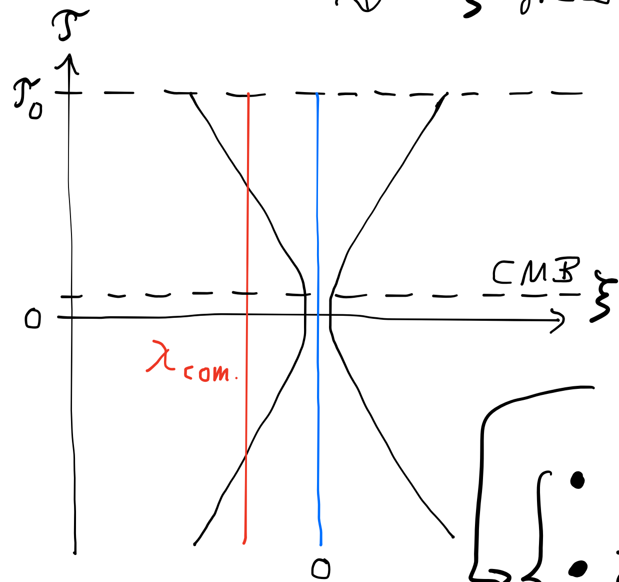
$$\xi = \int_0^{\tau_0} d\tau = \int_0^{t_0}\frac{dt}{a} = \int_0^{a_0}\frac{d\ln a}{aH}$$

comoving spatial distance

and $\frac{1}{aH} \sim a^{\frac{1}{2}(1+3w)}$

$\Rightarrow$ for $w > -\frac{1}{3}$: $\frac{1}{aH}$ dominated at large a

$\rightsquigarrow$ ξ growing with a



$\frac{d}{dt}\left(\frac{1}{aH}\right) > 0$

$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0$

- $\dot a, H < 0$: bounce

or

- $\ddot a, H > 0$: inflation

$\epsilon_H = -\frac{\dot H}{H^2} < 1$

if ρ mostly:	c.c.	matter	radiation
$a(t)$	e^{Ht}	$t^{2/3}$	$\sqrt{t}$
$H(t)$	const.	$\sim \frac{1}{t}$	$\sim \frac{1}{t}$
p	$-\rho$	0	$\frac{1}{3}\rho$

inflation!

$\frac{\rho_k}{\rho_{c.c.}} \sim a^{-2} \Rightarrow$ need $e^{-N_e} \equiv e^{-Ht} \sim e^{-60}$

I slow-roll inflation (Linde/Albrecht & Steinhardt '82)

1 scalar field ϕ, action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

scalar potential

$\delta S / \delta g^{\mu\nu}$

$$T_{\mu\nu} : \rho = \frac{1}{2}(\partial\phi)^2 + V, \; p = \frac{1}{2}(\partial\phi)^2 - V$$

consider: $\vec{\nabla}\phi = 0$, only $\dot{\phi}$

redshifts fast, if $a \sim e^{H \cdot t}$

then if: $\boxed{\dot{\phi}^2 << V} \Rightarrow p = -\rho$

$\Downarrow$

motion dominated by potential energy

$$\begin{cases} a \sim e^{H \cdot t} \\ H \simeq \text{const.} \end{cases}$$

can ensure this, if slow-roll:

e.o.m. for ϕ:

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \equiv -V'$$

slow-roll: $|\ddot{\phi}| << |3H\dot{\phi}|, |V'|$

$\Rightarrow 3H\dot{\phi} = -V'$ slow-roll e.o.m. (*)

then i): $p \simeq -\rho$

$$\Rightarrow 1 \gg \frac{\dot{\phi}^2}{V} = \frac{V'^2}{9VH^2}, \quad H^2 \simeq \frac{V}{3}$$

$$= \frac{1}{3}\left(\frac{V'}{V}\right)^2 \equiv \frac{2}{3}\epsilon, \quad \boxed{\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2}$$

$$\Rightarrow \boxed{\epsilon \ll 1 \text{ ensures } p \simeq -\rho.}$$

1st slow-roll condition

ii) ensure slow-roll for long time:

↳ maintain: $\ddot{\varphi} \ll 3H\dot{\varphi}$

from (*) $\Rightarrow \dot{\varphi}^2 = \frac{V'^2}{3V}$

and using (*) compute:

$$\frac{\ddot{\varphi}}{3H\dot{\varphi}} = 2\epsilon - \frac{1}{3}\eta, \quad \boxed{\eta \equiv \frac{V''}{V}}$$

$$\Rightarrow \frac{\ddot{\varphi}}{3H\dot{\varphi}} \ll 1 \text{ implies } \boxed{\eta \ll 1} \text{ if } \epsilon \ll 1$$

2nd slow-roll condition

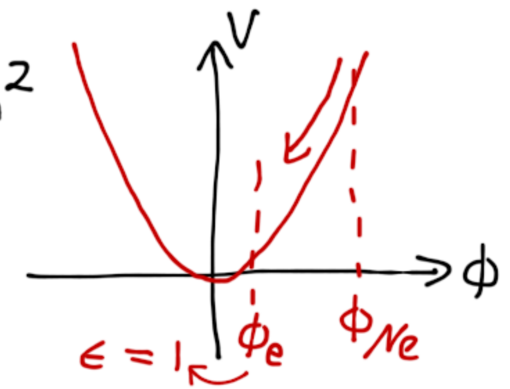
2 classes of $V(\phi)$

i) large-field models

examples: $V(\phi) \sim \phi^p$, $p \geq 2$

e.g. $V(\phi) = \frac{m^2}{2}\phi^2$

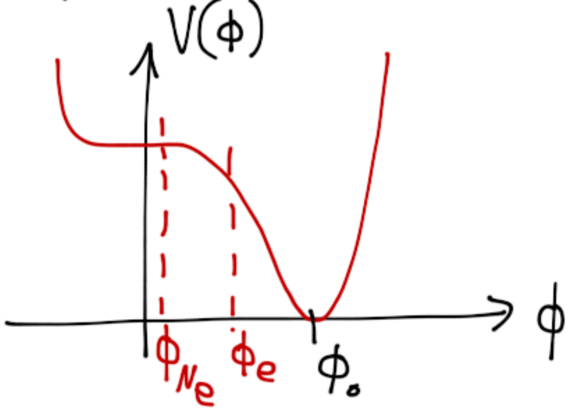
ϕ_e: $\epsilon(\phi_e) = 1$



$$\epsilon = \frac{p^2}{2\phi^2} \Rightarrow \boxed{\phi_e \sim M_P}$$

$$N_e = \int_{t_e}^{t_{N_e}} H\,dt = \int_{\phi_e}^{\phi_{N_e}} \frac{H}{\dot{\phi}}\,d\phi = \int_{\phi_e}^{\phi_{N_e}} \frac{d\phi}{\sqrt{2\epsilon}} \simeq \frac{\phi_{N_e}^2}{2p} \Rightarrow \boxed{\phi_{N_e} = \sqrt{2p N_e} \gg M_P}$$

ii) small-field models



$$V(\phi) = V_0\left(1 - \sqrt{2\epsilon_0}\cdot\phi - \frac{4}{\phi_0^3}\phi^3 + \frac{3}{\phi_0^4}\phi^4\right)$$

$$V'(\phi_0) = 0 \,, \quad V(\phi_0) = 0$$

$$\Rightarrow \sqrt{2\epsilon} = \left|\frac{V'}{V}\right| = \frac{\sqrt{2\epsilon_0} - \frac{12}{\phi_0^3}\phi^2}{1 - \sqrt{2\epsilon_0}\,\phi}$$

for $\phi \ll 1$, $\epsilon = 1$: $\quad \frac{\phi_e}{\phi_0} = \frac{2^{1/4}}{2\sqrt{3}}\sqrt{\frac{\phi_0}{M_P}}$

$\Rightarrow \phi_e \ll 1$ if $\phi_0 \ll 1$.

Why $\phi \sim M_P$ as discriminator?

$\rightarrow$ consider higher-dim. operators correcting V...

$\Rightarrow$ generically we get among them:

$$\Delta V_d \sim V \cdot \left(\frac{\phi}{M_P}\right)^{d-4}$$

$$\Rightarrow \Delta\eta = \frac{\Delta V_d''}{V} \sim (\phi/M_p)^{d-6}$$

$$\Rightarrow \Delta\epsilon \sim \left(\frac{\phi}{M_p}\right)^{2(d-5)} \to 1 \text{ at } \phi \sim M_p$$

→ destroys slow-roll: "eta-problem"

2 ways out:

- keep $\phi \ll M_p$ "small-field" and look for UV-theory to enumerate finite # of dim-6 corrections – and tune
- find a symmetry, that forbids all higher terms → then $\phi \gg M_p$ possible...

∿ example: $V(\phi) \sim \phi^p$ + gravity

graviton vertex $\sim T_{\mu\nu} \sim V, V''$

$\Rightarrow \Delta V \sim V^2, V''\cdot V$ not $V \cdot \frac{\phi^2}{M_p^2}$

→ shift symmetry of gravity...

∿ look at scalar corrections as well – 'daisy' diagrams e.g. for $V_0 = \lambda \cdot \phi^4$:

individually catastrophic!

$+ \ldots =$

$$= (-1)^n \lambda \phi^4 \left(\frac{\phi}{M_p}\right)^{2n-4}$$

benign alternating series

$$= V_0(\phi) \cdot \left(1 + c \cdot \ln \frac{\phi}{M_p}\right)$$