4. CMB & curvature perturbation

Cosmic Microwave Background (CMB)

→ Planck spectrum with:

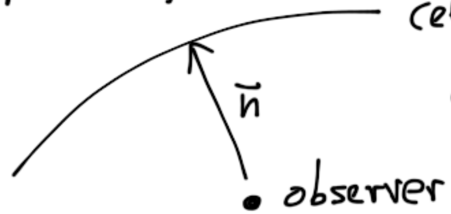
$$T = (2.726 \pm 0.001)\,K$$

anisotropies: $\frac{\Delta T}{T} \sim 10^{-3}$ 'dipole'

$\frac{\Delta T}{T} \sim 10^{-5}$ 'primordial'

needed for galaxy formation

power spectrum:



celestial sphere

$$\delta T(\vec{n}) = T(\vec{n}) - T_0$$

↑ average (T_0)

↑ varying intensity of photon flux

$\delta T/T$ from the curvature perturbation ζ

∿ recall: $\zeta = \delta N_e$

during inflation: $a = e^{N_e}$

$$\Rightarrow \delta N_e = \delta \ln a = \frac{\delta a}{a} = -\frac{\delta T}{T}$$

because: $T \sim \frac{1}{a}$

$$\Rightarrow \zeta = -\frac{\delta T}{T} \equiv \theta \text{ initial}$$

temperature perturbation at the end of inflation ≃ reheating at T_{rh}.

thereafter: radiation/matter domination

$$\Rightarrow a \sim t^p \Rightarrow \theta^{(0)}_{\mathcal{I}} = -\mathcal{O}(1)\cdot\zeta$$

initial temperature perturbation at conformal time $\mathcal{T} > \mathcal{T}_{rh}$ after the end of inflation.

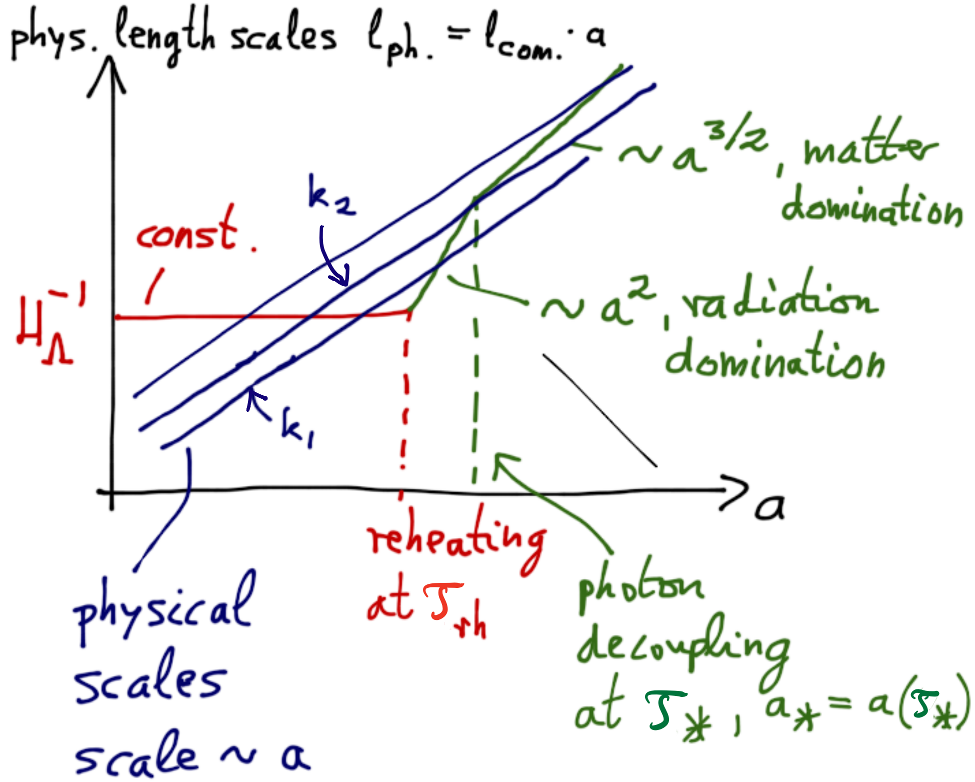
perturbation $\theta_k^{(0)}$ of comoving wavelength $\lambda = \frac{2\pi}{k}$ causes sound wave temperature fluctuation in the plasma after reheating at $\mathcal{T}_{rh}$:

$$\theta_k(\mathcal{T}) = \theta_k^{(0)} \cdot \cos(k \cdot s) = -\mathcal{O}(1) \cdot \zeta_k \cdot \cos(k \cdot s)$$

$s = \int c_s \, d\mathcal{T} \simeq \mathcal{T}/\sqrt{3}$ $\quad c_s = \frac{1}{\sqrt{3}}$: speed of sound in plasma

for evolution to $\mathcal{T} \gg \mathcal{T}_{rh}$

note that ζ_k was frozen-in until horizon re-entry



k_1: horizon re-entry $\tau_{H^{-1}} \ll \tau_*$ before decoupling

$\rightarrow \theta_k^{(0)}$ causes sound wave with comoving wavelength

$\lambda_1 = \frac{2\pi}{k_1}$ and frequency

$\omega_1 = c_s k_1 = \frac{k_1}{\sqrt{3}}$ $\rightarrow$ oscillates until recombination to phase

$k_1 \cdot s_* \simeq \omega_1 \cdot \tau_* \gtrsim 1$

k_2: horizon re-entry $\tau_{H^{-1}} \lesssim \tau_*$

$\rightarrow \theta_k^{(0)}$ causes sound wave with comoving wavelength

$\lambda_2 = \frac{2\pi}{k_2}$ and frequency

$\omega_2 = c_s k_2 = \frac{k_2}{\sqrt{3}}$ $\rightarrow$ oscillates until recombination to phase

$k_2 \cdot s_* \simeq \omega_2 \cdot \tau_* << 1$

$\Rightarrow$ sound wave stays at $\theta_k^{(0)}$, large-scale temperature fluctuations measure directly the primordial initial temperature and curvature perturbation, since

$\theta_k(\tau_*) = \theta_k^{(0)} \sim -\zeta_k$

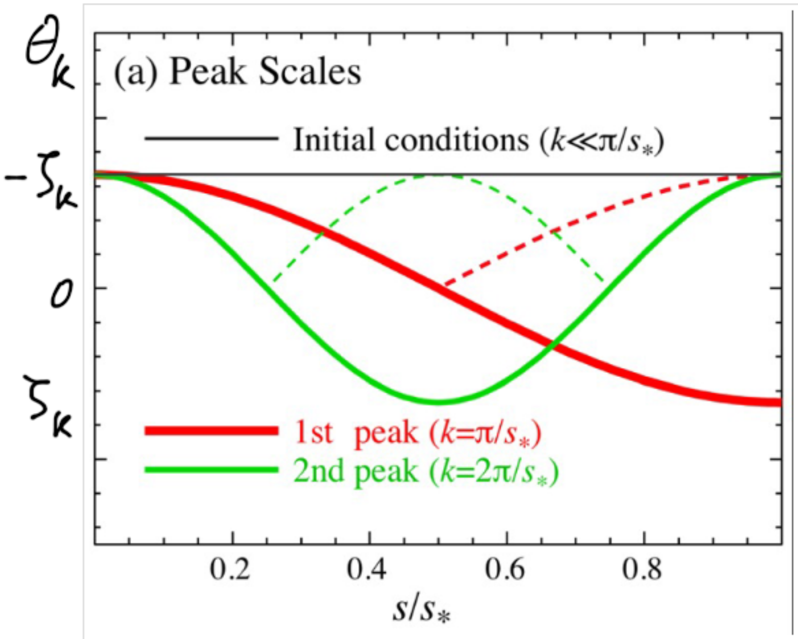
for $k \cdot s_* \simeq \omega \cdot \tau_* << 1$.

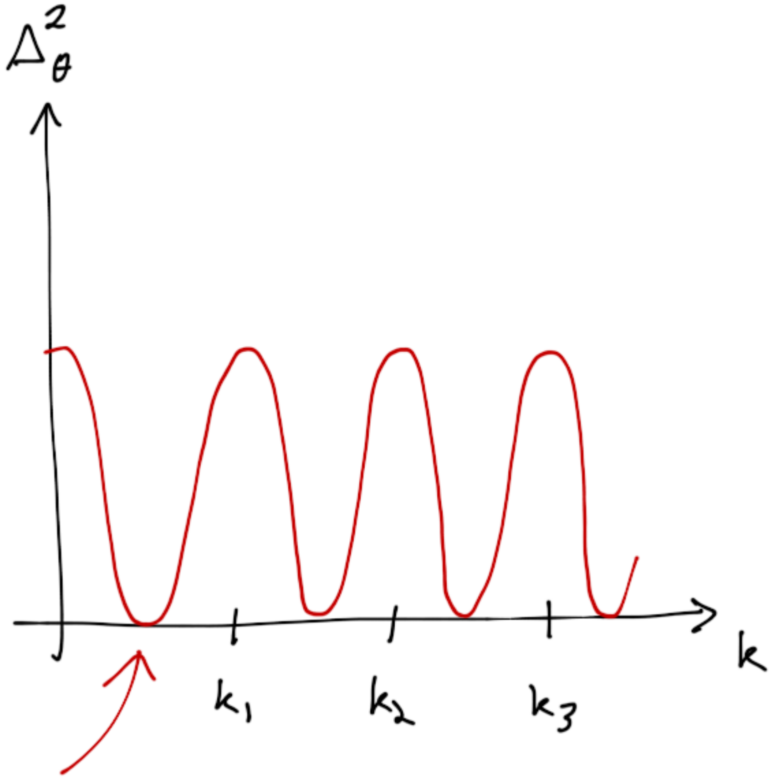
$\Rightarrow$ time evolution of ζ_k perturbation starts with ζ_k given, and $\frac{\partial \zeta_k}{\partial \tau} = 0$ everywhere, as $\cos(k \int c_s d\tau)$ with zero phase everywhere, for all k!

$\rightsquigarrow$ inflationary perturbations define coherent initial phase conditions for sound waves in plasma produced!

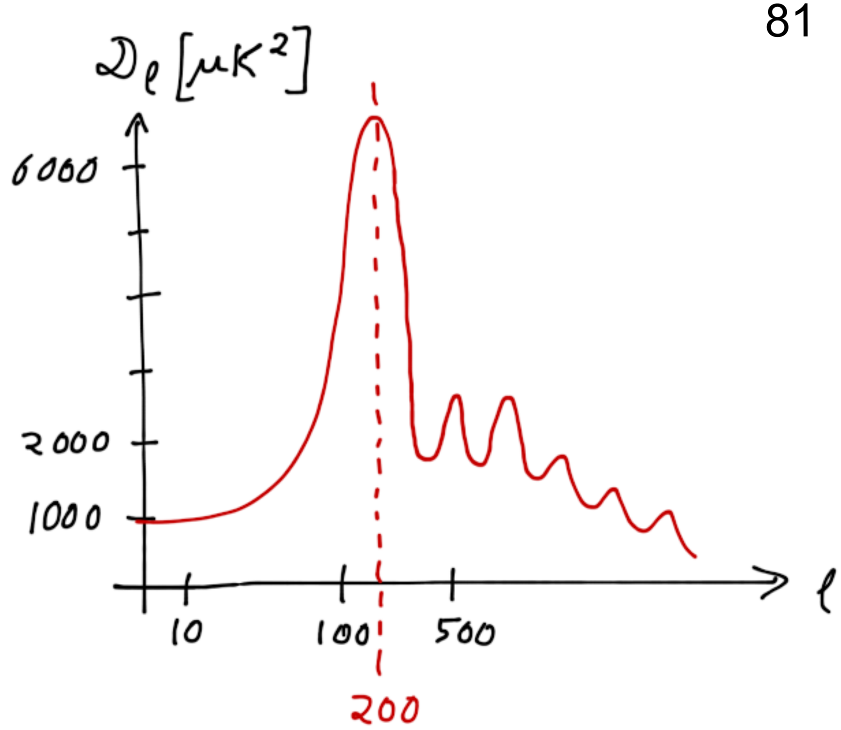
$\Rightarrow$ set of coherent peaks at : $\lambda_n = \frac{2\tau_*}{n\sqrt{3}}$

in the temperature power spectrum Δ_θ^2 !





set of coherent acoustic peaks



$$\sim \frac{dT}{T} \sim 3 \cdot 10^{-5}$$

$$\mathcal{D}_\ell \sim T_0^2 \cdot \ell(\ell+1) \cdot \int dk \cdot k^2 \cdot \Delta_\zeta^2(k) \cdot \left(T_{T\ell}(k)\right)^2$$

decoupling physics: transfer function

How do these peaks appear today?

∿ project to sphere in sky:

comoving distance D since decoupling:

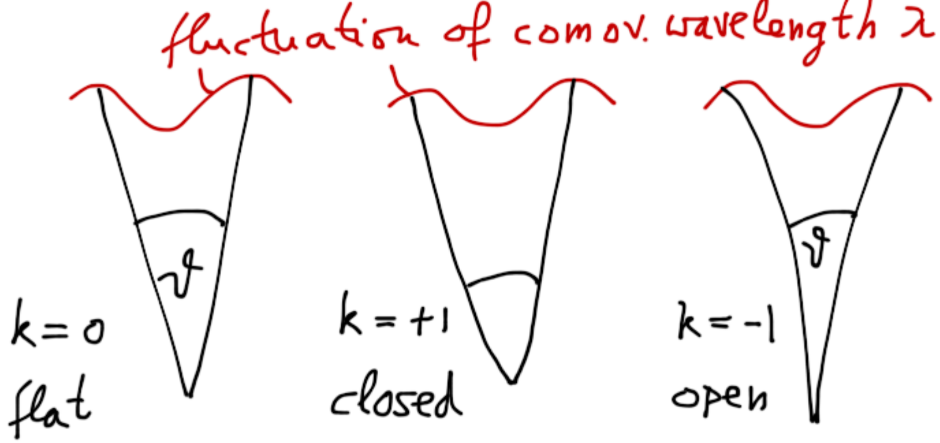
$$D_* = \tau_0 - \tau_* \simeq \tau_0$$

⇒ temperature fluctuation from sound wave with comoving wavelength λ_n of the n^{th} peak appears under angle:

$$\vartheta_n = \frac{\lambda_n}{D_*} = \frac{2}{n\sqrt{3}} \cdot \frac{\tau_*}{\tau_0}$$

in a spatially flat universe.

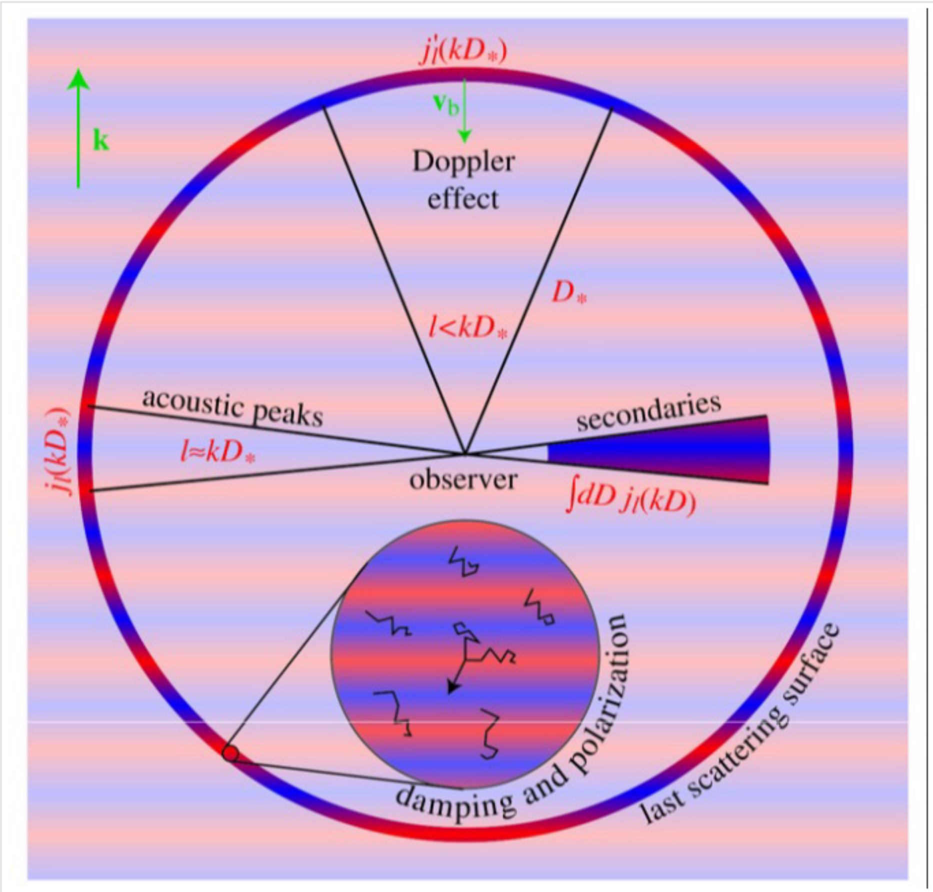
More generally:



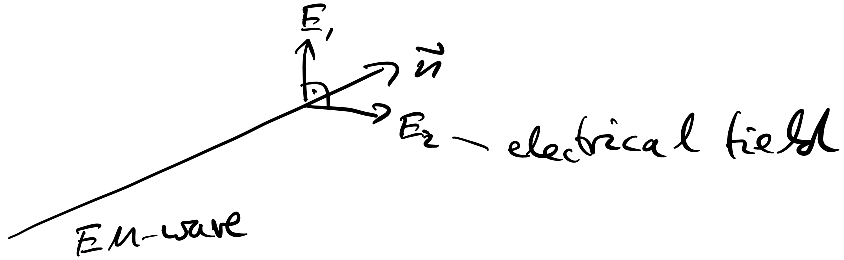
$$\Rightarrow \vartheta_{k=-1} < \vartheta = \vartheta_{flat} < \vartheta_{k=+1}$$

↳ measuring e.g. ϑ_1 accurately in the CMB can determine spatial geometry of the visible universe!

but : $\vartheta_1 : \vartheta_2 : \vartheta_3 : \dots$ peak position ratios do not depend on spatial geometry.



Polarization of CMB:



intensity tensor $I_{ij} = \langle E_i(\vec n) E_j(\vec n) \rangle$

linear polarization described by Stokes parameters:

$$Q = \frac{1}{2}(I_{11} - I_{22}), \quad U = \frac{1}{2} I_{12}$$

decompose with spherical harmonics:

$$(Q+iU)(\vec n) = \sum_{l,m} a_{\pm 2, lm} Y_{lm}(\vec n)$$

split into E- and B-mode polariz.:

$$a_{E/B,lm} = -\frac{1}{2}\left(a_{2,lm} \pm a_{-2,lm}\right)$$

B modes: only generated by GW from inflation!

define:

$$\boxed{r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon \simeq 16\epsilon_V} \qquad (165)$$

"tensor-to-scalar ratio"

Planck 2018 + BAO + BICEP2 & Keck Array:

$$A_s \equiv \Delta_{\mathcal{R}}^2\Big|_{k=k_*=aH}$$

$$= (2.196 \mp 0.06) \cdot 10^{-9}$$

$$n_s = 0.965 \pm 0.004$$

$$r < 0.06 \quad (95\%)$$