

The Role of Applications in the History of Quantum Mechanics

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Göttingen, 31 March 2025

UNIVERSITY OF COPENHAGEN



INTERNATIONAL YEAR OF
Quantum Science
and Technology

2025
Quantum2025

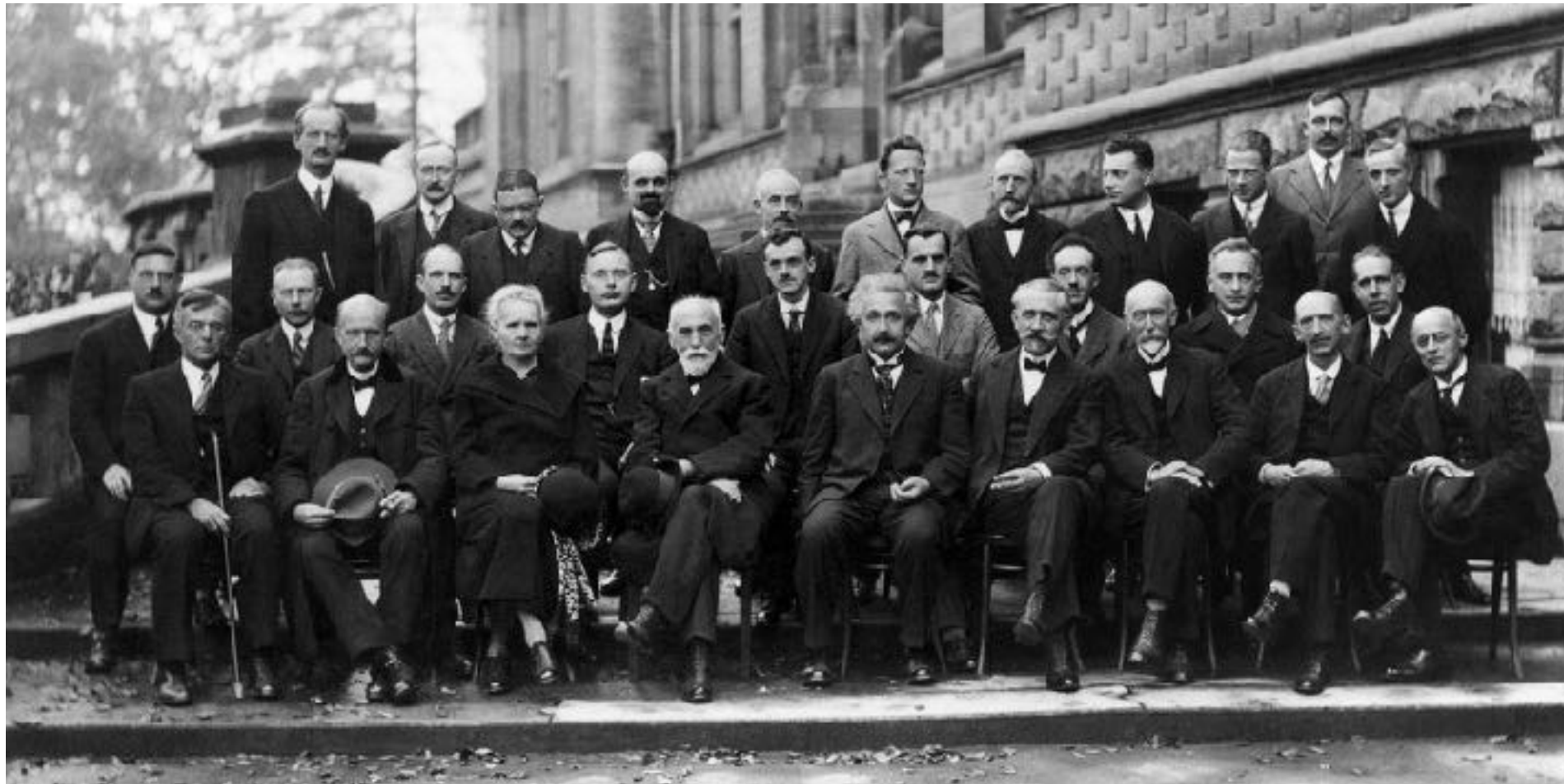
The genesis of quantum theory and quantum mechanics

- **Quantum theory** emerged from 1900 within a network of only a handful of institutions.
- **Quantum mechanics** developed in 1925–1927 in close exchange between (mainly) Göttingen, Copenhagen, and Munich.
- Since the early 1920s: **exchange of young students**, especially between Max Born, Niels Bohr, and Arnold Sommerfeld (“*Knabenphysik*”).
- Unlike the **relativity revolution** that centered on Einstein, the **quantum revolution** was a collaborative and multicentric endeavor that involved **several dozen actors**.
- This presents a **challenge** to traditional models of **linear story-telling historiography**.



Quantum historiography

- Not just quantum theory, but also the **writing of its history**, is therefore an **interdisciplinary and highly collaborative enterprise** involving professional historians and philosophers of physics as well as physicists all over the world.
- We have come a long way from **actors' histories** (Werner Heisenberg, Friedrich Hund) to **early professional accounts** (Thomas S. Kuhn, Max Jammer) to an **international community of quantum historians** who regularly meet (SYHQ at this conference; HQ-5 in Brazil this summer) and who back in 2020 were among the **initiators of this year's quantum century celebrations**.



Fifth Solvay Conference in
Bruxelles, October 1927

Outline of my talk

- 1 Backdrop: Key Milestones in Quantum Theory**
- 2 Main argument: The Role of Applications**
- 3 Three Examples of Early Applications**
- 4 Conclusion and Outlook: Why apply?**

1 Backdrop: Key Milestones in Quantum Theory

Key milestones in quantum theory



Max Planck (1858–1947)

$$\rho(\nu, T)d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} d\nu$$

black-body radiation law

1900 Planck (quantum of action)



Key milestones in quantum theory



First Solvay Conference, 1911

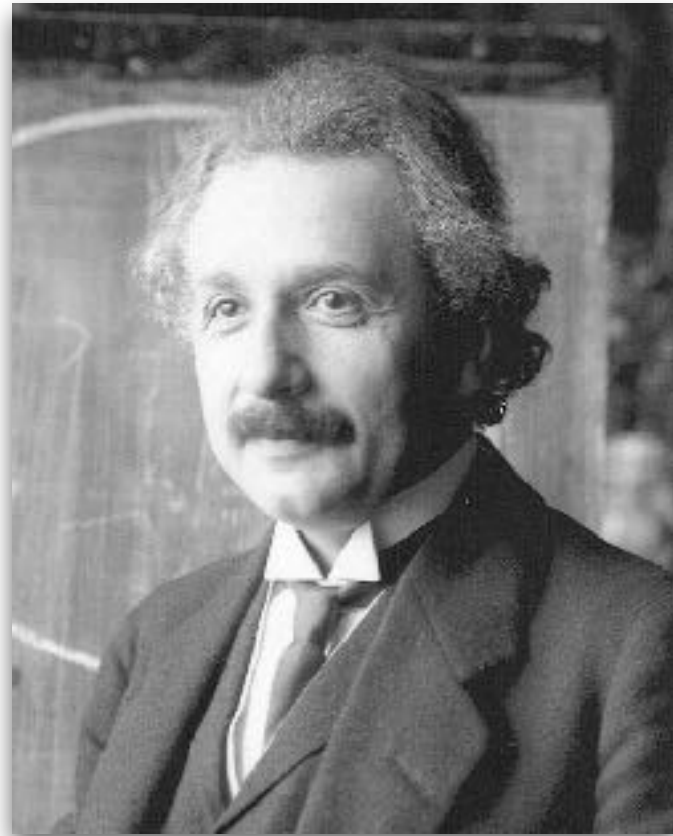
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black-body radiation law



Max Planck
(1858–1947)

Key milestones in quantum theory



Albert Einstein (1879–1955)

$$E = h\nu$$

energy of a light quantum

1900 Planck (quantum of action)

1905 Einstein (light quantum)



Key milestones in quantum theory

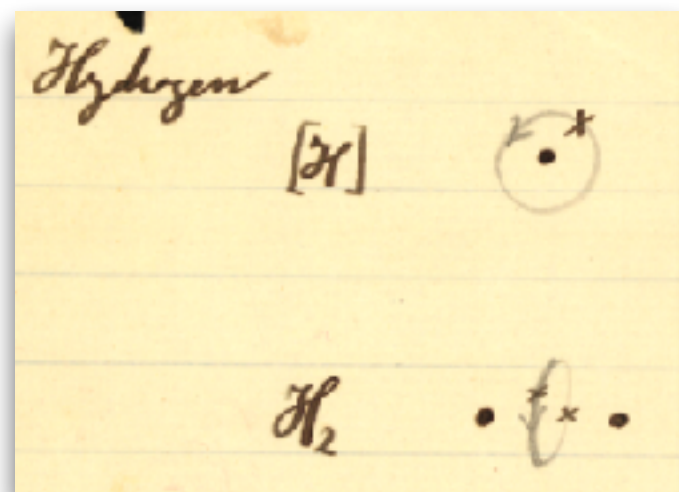


Niels Bohr (1885–1962)

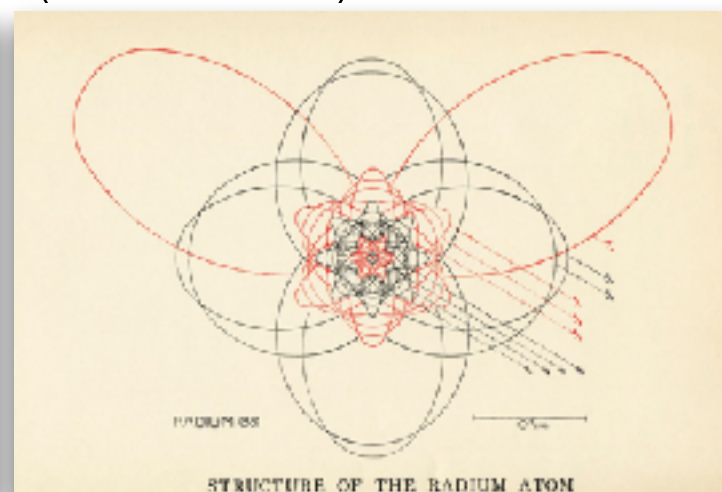
1900 Planck (quantum of action)

1905 Einstein (light quantum)

1913 Bohr (Bohr atomic model)



1912 Manchester memorandum



structure of the Radium atom according to Bohr's model

atomic model

Key milestones in quantum theory



Arnold Sommerfeld (1868–1951) and
Niels Bohr in Lund, Sweden

**Bohr-Sommerfeld (phase-integral)
quantization condition**

$$\oint p dq = nh$$

1900 Planck (quantum of action)

1905 Einstein (light quantum)

1913 Bohr (Bohr atomic model)

1916 Sommerfeld (quantization condition)

Key milestones in quantum theory



Paul S. Epstein
(1883–1966)

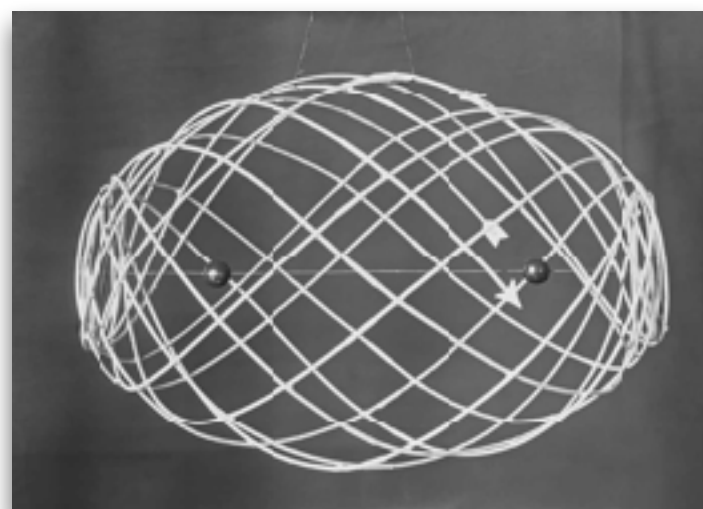


Karl Schwarzschild
(1873–1916)

$$\left. \begin{aligned} W &= W(q_1, \dots, q_f; u_1, \dots, u_f), \\ p_i &= \frac{\partial W}{\partial q_i}, \quad u_i = \frac{\partial W}{\partial u_i}. \end{aligned} \right\}$$

action and angle variables
(Hamilton-Jacobi theory)

model of the hydrogen
molecule ion
according to Pauli and
the old quantum
theory (Deutsches
Museum Munich)



1900 Planck (quantum of action)

1905 Einstein (light quantum)

1913 Bohr (Bohr atomic model)

1916 Sommerfeld (quantization condition)

1916 Epstein and Schwarzschild
(action and angle variables)

Key milestones in quantum theory



Copenhagen conference (1936). Front row, from left: Pauli, Jordan, Heisenberg, Born

“sharpening” of Bohr’s correspondence principle

$$pq - qp = \frac{h}{2\pi i} \mathbf{1}$$

“Umdeutung” ➤ matrix mechanics

1900 Planck (quantum of action)

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1916 Sommerfeld (quantization condition)
1916 Epstein and Schwarzschild
(action and angle variables)

1925 Heisenberg, Born, Jordan
(matrix mechanics)

Key milestones in quantum theory



Erwin Schrödinger (1887–1961)

$$\Delta\psi + \frac{2m}{K^2} \left(E + \frac{e^2}{r} \right) \psi = 0$$

wave equation

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1905 Einstein (light quantum)

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1916 Epstein and Schwarzschild
(action and angle variables)

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1926 Schrödinger (wave mechanics)

Key milestones in quantum theory



Erwin Schrödinger (1887–1961)

$$\Delta\psi + \frac{2m}{K^2} \left(E + \frac{e^2}{r} \right) \psi = 0$$

wave equation

equivalence!

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Key milestones in quantum theory



Werner Heisenberg, 1927 Niels Bohr at Como, 1927

$$\Delta q \cdot \Delta p \sim h$$

**uncertainty, complementarity ►
“Copenhagen interpretation”**



Niels Bohr's Copenhagen Institute in 1921

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1913 Bohr (Bohr atomic model)

1916 Sommerfeld (quantization condition)

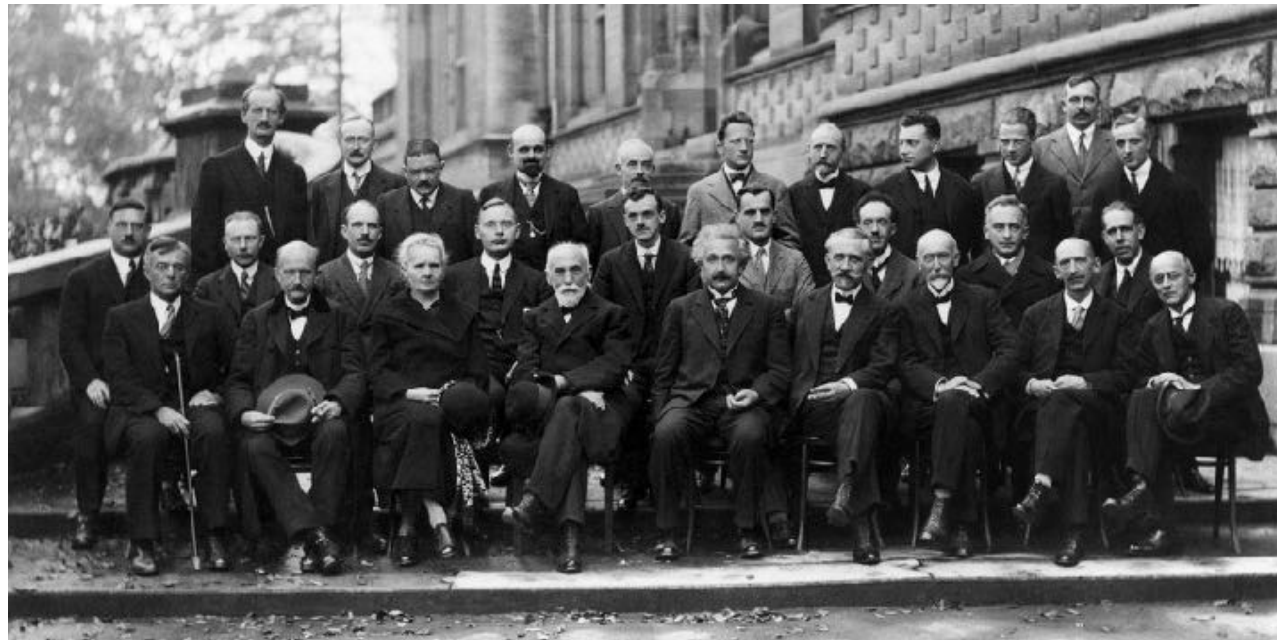
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(action and angle variables)

1925 Heisenberg, Born, Jordan
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1926 Schrödinger (wave mechanics)

1927 Bohr (“Copenhagen” interpretation)

Key milestones in quantum theory



Fifth Solvay Conference in Bruxelles, October 1927

We consider quantum mechanics to be a **closed theory** [geschlossene Theorie], whose fundamental physical and mathematical assumptions are **no longer susceptible of any modification**.

Max Born and Werner Heisenberg in their report at the Fifth Solvay Conference, October 1927, quoted after Bacciagaluppi and Valentini, *Quantum Theory at the Crossroads* (Cambridge: CUP, 2013).

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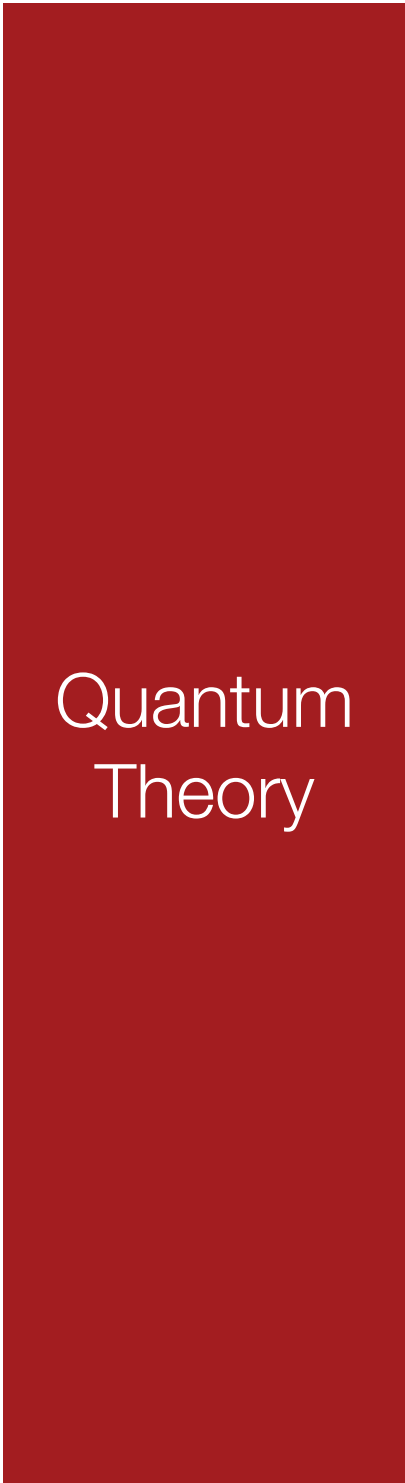
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1926 Schrödinger (wave mechanics)

1927 Bohr ("Copenhagen" interpretation)

Some historiographical terminology



Quantum
Theory



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Some historiographical terminology



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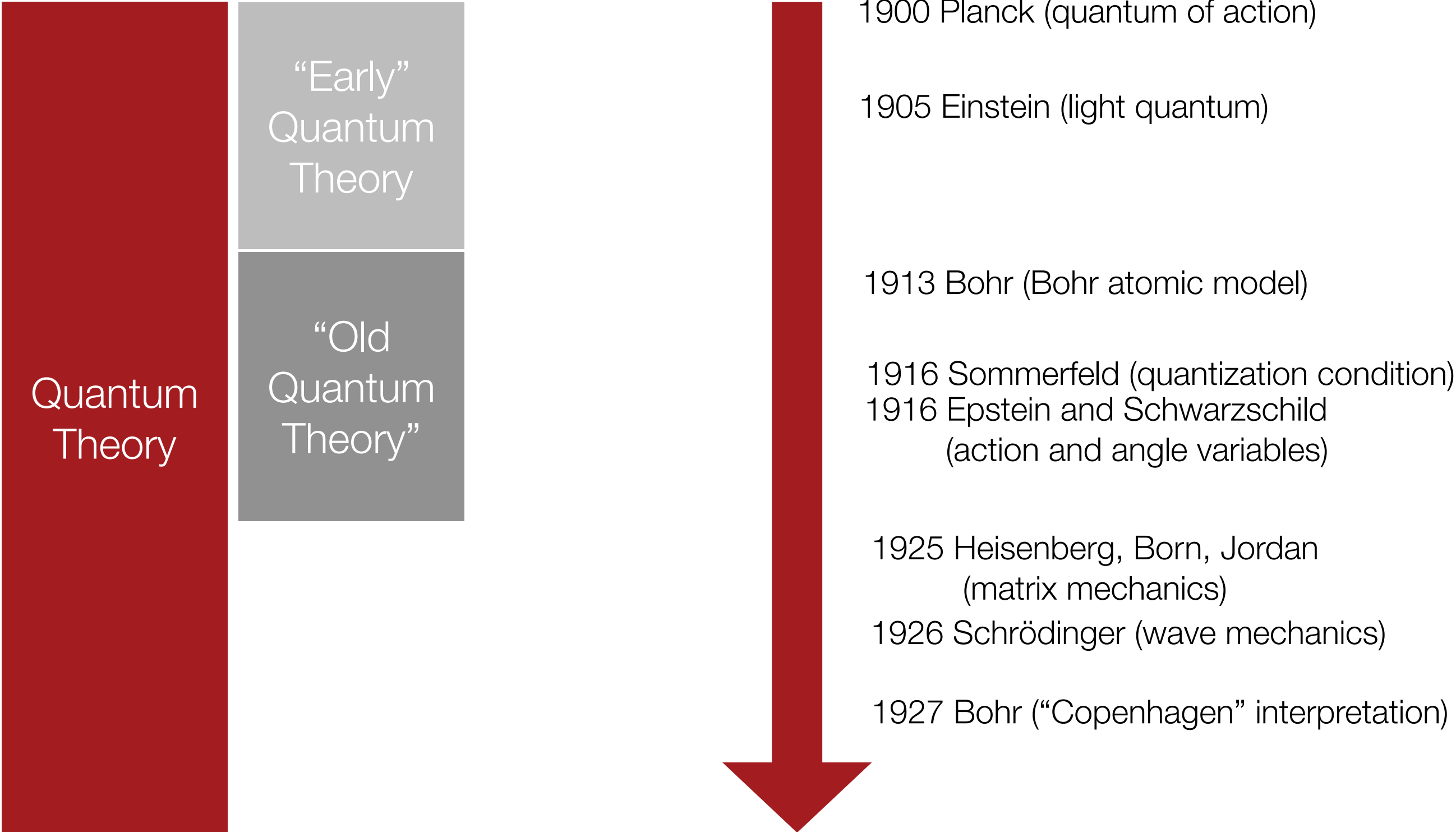
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Some historiographical terminology



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“Old
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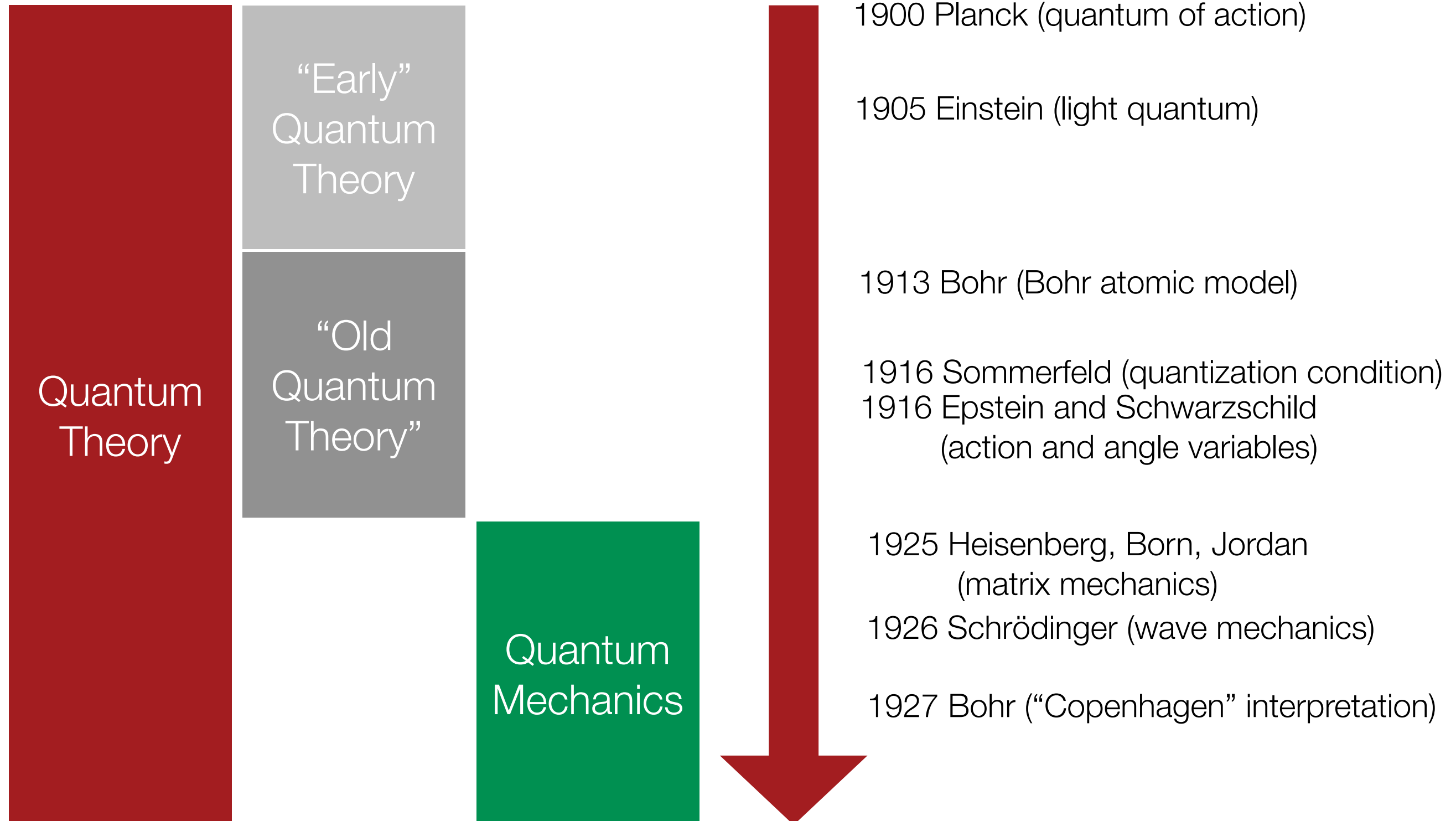
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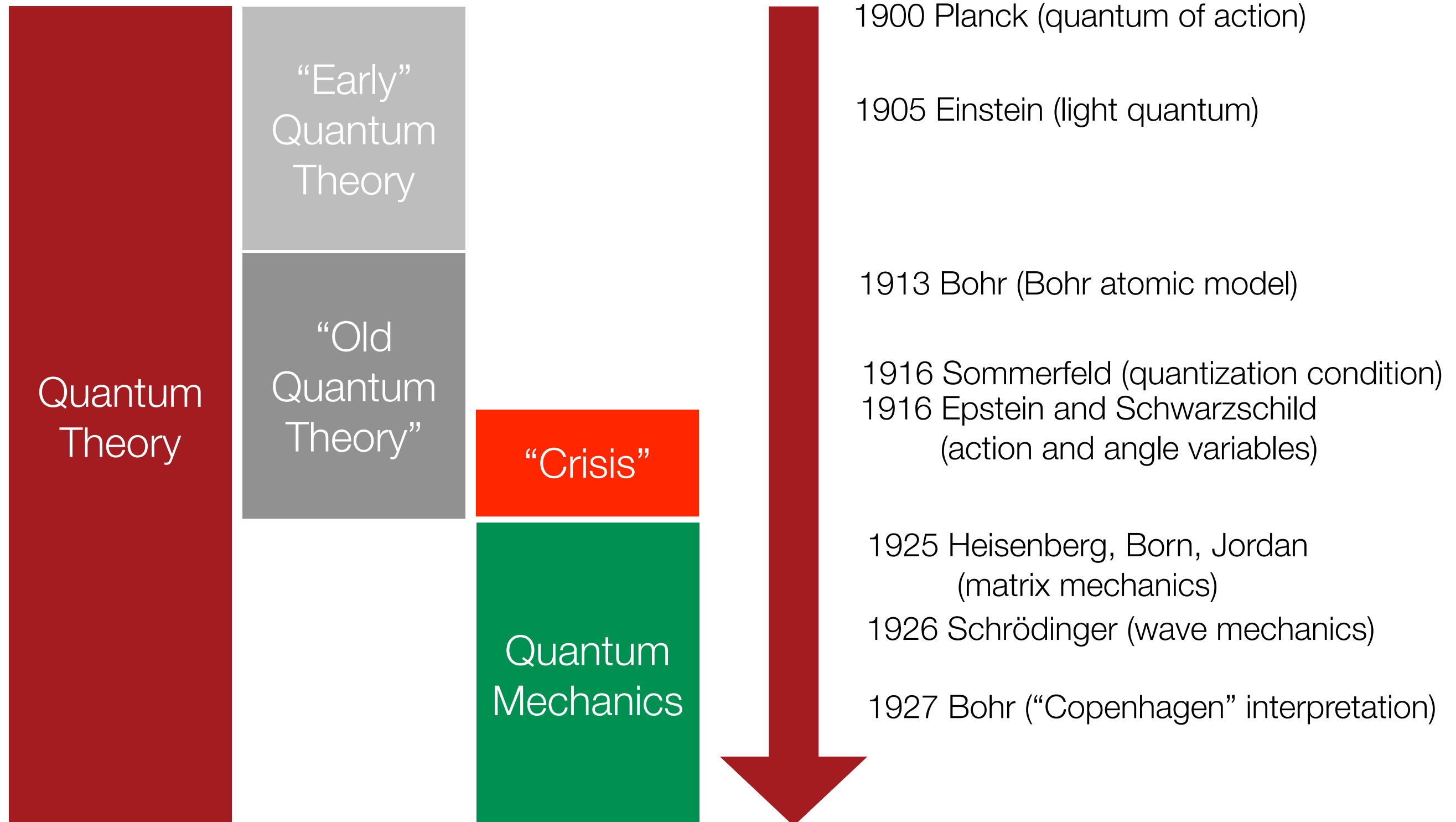
1926 Schrödinger (wave mechanics)

1927 Bohr (“Copenhagen” interpretation)

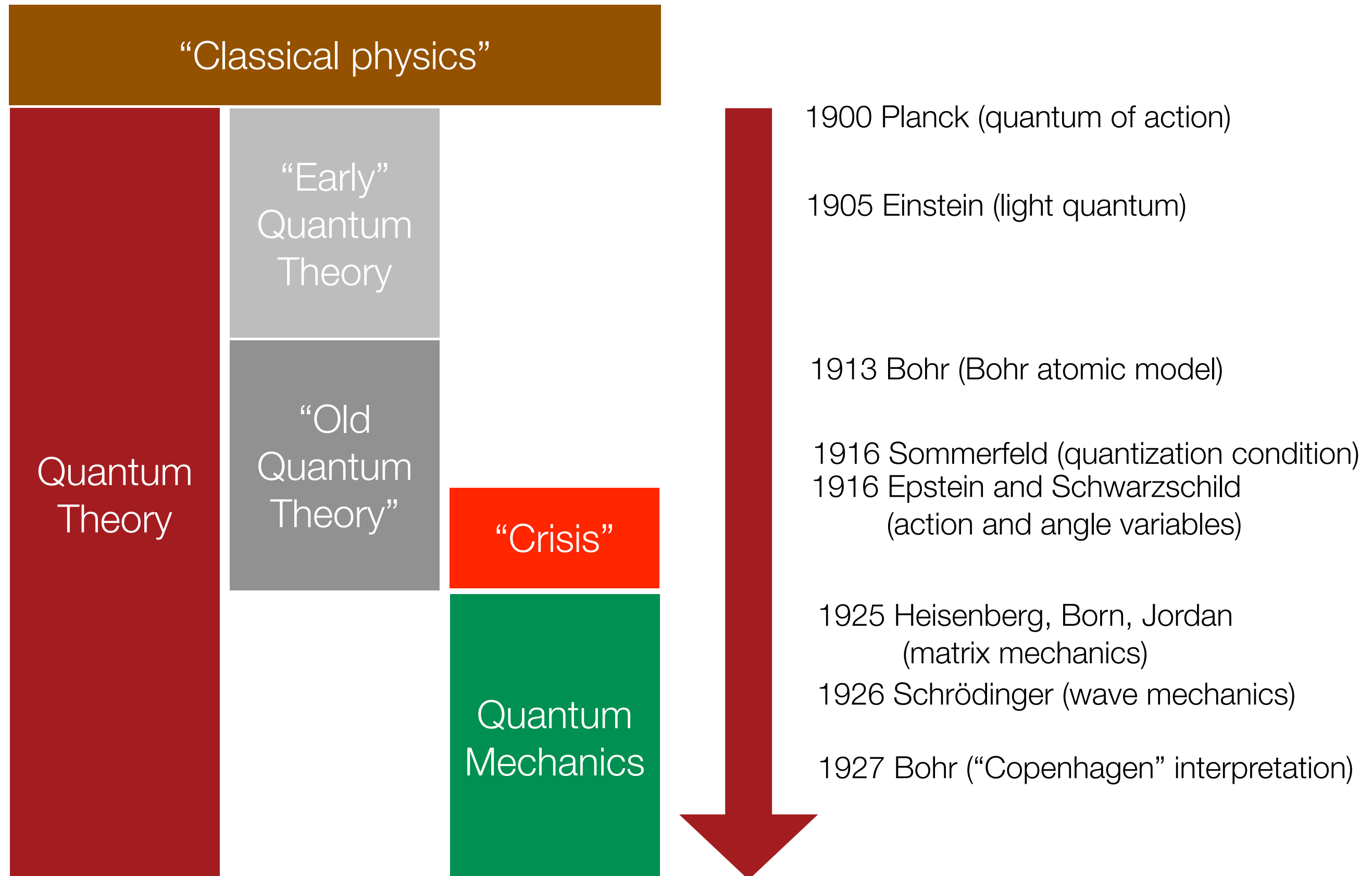
Some historiographical terminology



Some historiographical terminology



Some historiographical terminology



2 Main argument: The Role of Applications

Original papers from ca. 1925–1935



Original papers from ca. 1925–1935



**quantum mechanical
formalism**

(Heisenberg 1925,
Born&Jordan 1925,
Schrödinger
1926a,b,d,e, ...)

Original papers from ca. 1925–1935

interpretation

(Born 1926a,b;
Heisenberg 1927;
Bohr 1928a,b, EPR
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applications of quantum mechanics

Original papers from ca. 1925–1935

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**applications of
quantum
mechanics**

**Q: What role (if any?) did applications
of quantum mechanics play for the
foundations of the theory?**

Dimensions of applying a theory

1. Validation of the theory through problem solving

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Simultaneously with the progressive elucidation of the general laws of quantum mechanics, **new evidence for the empirical correctness** of this theory has been provided by a large number of applications by various authors.

Jordan, P. (1927). Die Entwicklung der neuen Quantenmechanik. *Die Naturwissenschaften*, 15(3), 614–23, on 616.



Pascual Jordan
(1902–1980)

Dimensions of applying a theory

1. Validation of the theory through problem solving
2. **Extending range of validity of the theory into new empirical domains**

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Friedrich Hund
(1896–1997)

With the probability interpretation and transformation theory, the principles of quantum mechanics were by and large known at the end of 1926. Since spring 1926, the Schrödinger equation had been a convenient method for solving the simpler problems, adapted to the mathematical knowledge of physicists at that time. Around 1927, these circumstances led to a **flood of applications** and the development of practical methods of calculation

Hund, F. (1967). *Geschichte der Quantentheorie* (Mannheim: BI), on 167.

Dimensions of applying a theory

1. Validation of the theory through problem solving
2. **Extending range of validity of the theory into new empirical domains**

Satisfied that the theory “works,” since it provided unambiguous answers whenever invoked, physicists engaged themselves rather in **solving problems** which so far had defied all previous attempts or which promised to open up new avenues of research. The year 1927 thus not only became the year in which the quantum-mechanical formalism, in all its essential points, received a formal completion and a consistent interpretation; 1927 also witnessed a veritable **avalanche of elaborations and applications** of the new conceptions and led to new insights in atomic physics to an unprecedented extent.

Max Jammer, *The Conceptual Development of Quantum Mechanics* (New York: McGraw Hill, 1966), on 362 (my emphasis).



Max Jammer
(1915–2010)

Dimensions of applying a theory

1. Validation of the theory through problem solving
2. **Extending range of validity of the theory into new empirical domains**

Were applications thus just a “flood” or “avalanche” of **normal science** and **problem solving** in which an otherwise completed formalism was put to practical use?

Dimensions of applying a theory

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2. **Extending range of validity of the theory into new empirical domains**

Were applications thus just a “flood” or “avalanche” of **normal science** and **problem solving** in which an otherwise completed formalism was put to practical use?

No—there is a third dimension of applying a theory that played an important and constitutive role in the history of quantum mechanics.

Dimensions of applying a theory

1. Validation of the theory through problem solving
2. Extending range of validity of the theory into new empirical domains
3. **Further articulating, modifying—or potentially even overthrowing—the theory and elucidating its meaning and interpretation**

The open-endedness of early quantum mechanics

- Many actors did not see quantum mechanics as a finished formalism, but **expected another imminent theoretical innovation.**



Erwin Schrödinger and Fritz London
(1900–1954) in Berlin, 1928

The open-endedness of early quantum mechanics

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- In the old quantum theory, it had proven a **viable and successful research strategy** to extend and clarify the theory through applications.



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An hypothesis proves its worth not before being **applicable to other areas** than the one for which it was formulated originally. From this point of view, the quantum hypothesis has stood the test **brilliantly**... Recently, many other phenomena have been identified, in areas such as magnetism or the conduction of electricity or heat, which certainly can only be explained through quantum theory.

Peter Debye, Inaugural Lecture, Utrecht, 1913



Peter Debye
(1884–1966)
in 1912

The open-endedness of early quantum mechanics

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- In the old quantum theory, it had proven a **viable and successful research strategy** to extend and clarify the theory through applications.
- Obvious places to look for an extension of quantum mechanics were its **relativistic extension**, the study of the **atomic nucleus**, but also **aperiodic phenomena**, **many-body systems** in atomic and molecular physics, **spin**, and **quantum statistics**.



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- The **cut** we sometimes make between “foundations” and (mere) “applications” is **artificial** and **anachronistic**: Applications played important functional roles in the development of quantum mechanics into a finished, canonized theory as it can be found in today’s **textbooks**.



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- Realizing this **creative tension between foundations and applications** in the practice of the actors also makes us broaden our outlook on where to look for actors’ statements on the foundations and interpretation of quantum mechanics.



Erwin Schrödinger and Fritz London
(1900–1954) in Berlin, 1928

3 Three Examples of Early Applications

Three Examples of Early Applications

- Scattering and Born's probability interpretation
- Complex spectra, quantum statistics, and resonant exchange
- Tunneling in molecules and nuclei

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Scattering and Born's probability interpretation

- In June-July 1926, Max Born, submits two papers on “quantum mechanics of scattering processes” that are famous for his introduction of the **probability interpretation** of Schrödinger's wave function.

Max Born
(1882–1970)



Zur Quantenmechanik der Stoßvorgänge.

[Vorläufige Mitteilung.]

Von Max Born, Göttingen.

(Eingereicht am 25. Juli 1926.)

Durch die Untersuchung der Stoßvorgänge wird die Auffassung entwickelt, die Quantenmechanik in der Schrödingerschen Form nicht nur die stationären Zustände, sondern auch die Übergangswahrscheinlichkeiten zu beschreiben gestattet.

Die von Heisenberg begründete Quantenmechanik ist bisher ausschließlich angewandt worden zur Berechnung f und der den Übergängen zugeordneten Schwingungszahlen. Aber diese Fragestellung betrifft nur theoretische Probleme; daneben erhebt sich als physikalisches Problem die Frage nach dem Wesen der „Übergänge“ selbst. Hinsichtlich der Meinung geteilt zu sein; viele sehen den Übergang von der Quantenmechanik in der klassischen Mechanik, andere darin, daß hier neue Begriffe eingeführt werden, die mit dem Gedanken der durch den Aufbau der Quantenmechanik zu der Vermutung vollständig sein und das Übergangsproblem mitgelöst wird, wenn man nur zeigen kann, daß dies auch der Fall ist. Schon Bohr hat die Aufmerksamkeit auf die prinzipiellen Schwierigkeiten der Quantenmechanik bei Emission und Absorption von Licht durch Atome und Moleküle hingewiesen. Bei diesen hat man es mit den Wellenfeldern ausschließlich mit Systemen zu tun, die dem Formalismus der Quantenmechanik unterworfen sind. In der Tat ist das Problem in Angriff genommen, die Teilchen (z. B. Elektronen oder Photonen) und ihre Wechselwirkungen zu untersuchen und festzustellen, ob nicht der vorliegenden Theorie eine Beschreibung des Übergangs hinzugefügt werden muß.

¹⁾ Diese Mitteilung war ursprünglich für die „Naturwissenschaften“ bestimmt, wurde aber wegen Raummangels nicht aufgenommen. Sie ist in der Zeitschrift für Physik, Bd. XXXVI, 1926, Nr. 1, S. 1, veröffentlicht.

Quantenmechanik der Stoßvorgänge¹⁾.

Von Max Born in Göttingen.

(Eingereicht am 31. Juli 1926.)

Die Schrödingersche Form der Quantenmechanik erlaubt in natürlicher Weise die Herleitung eines Satzes, der die Wahrscheinlichkeit der Übergänge zwischen stationären Zuständen in einer Theorie der Stoßvorgänge, bei der die Übergangswahrscheinlichkeiten durch die asymptotischen Verhältnisse periodischer Lösungen bestimmt werden.

Einführung. Die Stoßvorgänge haben nicht nur die überzeugendsten experimentellen Beweise für die Grundannahmen der Quantentheorie geliefert, sondern scheinen auch geeignet, Aufklärung zu geben über die physikalische Bedeutung der formalen Gesetze der sogenannten „Quantenmechanik“. Diese liefert zwar, wie es scheint, stets die richtigen Termwerte der stationären Zustände und die richtigen Amplituden der bei den Übergängen ausgestrahlten Schwingungen, aber über die physikalische Interpretation der Formeln sind die Meinungen geteilt. Die von Heisenberg begründete, von ihm gemeinsam mit Jordan und dem Verfasser dieser Mitteilung entwickelte Matrizenform der Quantenmechanik²⁾ geht von dem Gedanken aus, daß eine exakte Darstellung der Vorgänge in Raum und Zeit überhaupt unmöglich ist, und begnügt sich daher mit der Aufstellung von Relationen zwischen beobachtbaren Größen, die nur im klassischen Grenzfall als Eigenschaften von Bewegungen gedeutet werden können. Schrödinger³⁾ auf der anderen Seite sieht den Wellen, die er nach de Broglies Vorgang als die Träger der atomaren Prozesse ansieht, eine Realität von derselben Art zuschreiben, wie die Lichtwellen besitzen; er versucht, „Wellengruppen“ aufzubauen, welche in allen Richtungen relativ kleine Abmessungen⁴⁾ haben und die offenbar die bewegte Korpuskel direkt darstellen sollen.

Keine dieser beiden Auffassungen scheint mir befriedigend. Ich möchte versuchen, hier eine dritte Interpretation zu geben und ihre Brauchbarkeit an den Stoßvorgängen zu erproben. Dabei knüpfe ich an

¹⁾ Hierin eine vorläufige Mitteilung, ZS. f. Phys. 37, 463, 1926.

²⁾ W. Heisenberg, ZS. f. Phys. 33, 879, 1925; M. Born und P. Jordan, ebenda 34, 858, 1925; M. Born, W. Heisenberg und P. Jordan, ebenda 35, 202, 1926. Siehe auch P. A. M. Dirac, Proc. Roy. Soc. 109, 642, 1925; 110, 261, 1926.

³⁾ E. Schrödinger, Ann. d. Phys. 78, 361, 489, 734, 1925. Vgl. besonders die zweite Mitteilung, S. 489. Ferner Naturw. 14, 664, 1925.

⁴⁾ Zeitschrift für Physik. Bd. XXXVI.

Scattering and Born's probability interpretation

- In June-July 1926, Max Born, submits two papers on “quantum mechanics of scattering processes” that are famous for his introduction of the **probability interpretation** of Schrödinger's wave function.

“Born's aim in his first collision paper was not to contribute to the clarification of **interpretational issues**, as his later recollections suggest, but to **solve** a particular (yet crucial) scientific **problem**.

...
The aim of [the] collision papers was not to argue the reality of particles and indispensability of indeterminism, but rather to describe and theoretically to substantiate Bohr's concepts of ‘quantum jumps’—the discrete discontinuous energy changes within an atom. Born saw direct evidence for the existence of discrete energy levels in the **Franck-Hertz experiments**, which he sought to explicate theoretically.”

Mara Beller, “Born's Probabilistic Interpretation: A Case Study of ‘Concepts in Flux,’” *SHPS* 21, no. 4 (1990): 563–588, on 564.

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Ich selbst kam durch den Gedanke der durch den Aufbau der Quantenmechanik zu der Vermutung, daß die Übergänge vollständig sein und das Übergangsproblem mit der Quantenmechanik selbst gelöst werden kann. Ich glaube, daß es mir jetzt gelungen ist, dies zu zeigen.

Schon Bohr hat die Aufmerksamkeit auf die prinzipiellen Schwierigkeiten der Quantenmechanik bei Emission und Absorption von Licht durch Atomwechselwirkung von Atomen auf kurze Ketten der Stoßvorgänge. Bei diesen hat man es mit diskontinuierlichen Wellenfeldern ausschließlich mit Systemen zu tun, die dem Formalismus der Quantenmechanik unterworfen sind. Daher ist das Problem in Angriff genommen, die freien Teilchen (z. B. Strahlung oder Elektronen) und ihre Wechselwirkung mit Atomen zu untersuchen und festzustellen, ob nicht der vorliegenden Theorie eine Beschreibung des Übergangs hinzugefügt werden kann.

¹⁾ Diese Mitteilung war ursprünglich für die „Naturwissenschaften“ bestimmt, wurde aber wegen Raummangel nicht aufgenommen. Siehe auch die zweite Mitteilung, die dem „Zentralblatt für Physik“ 34, XXXVII, 1926, 1927, 1928, 1929, 1930, 1931, 1932, 1933, 1934, 1935, 1936, 1937, 1938, 1939, 1940, 1941, 1942, 1943, 1944, 1945, 1946, 1947, 1948, 1949, 1950, 1951, 1952, 1953, 1954, 1955, 1956, 1957, 1958, 1959, 1960, 1961, 1962, 1963, 1964, 1965, 1966, 1967, 1968, 1969, 1970, 1971, 1972, 1973, 1974, 1975, 1976, 1977, 1978, 1979, 1980, 1981, 1982, 1983, 1984, 1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 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Three Examples of Early Applications

- Scattering and Born's probability interpretation
- Complex spectra, quantum statistics, and resonant exchange
- Tunneling in molecules and nuclei

Complex spectra: Pauli's exclusion principle

- Already before Quantum Mechanics in January 1925, Wolfgang Pauli postulates a novel rule for complex spectra: the **Pauli exclusion principle** (“housing office for equivalent electrons”).
- No derivation or physical interpretation available.



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(1900–1958)



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“The problem of a further justification of the general rule over the occurrence of equivalent electrons in the atom ... likely can only be successfully tackled after a **future deepening** of the fundamental principles of quantum theory.”

Wolfgang Pauli, “Über den Zusammenhang des Abschusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren,” *Zeitschrift Für Physik* 31 (1925): 765–783, on 783.



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- However, even the deepening of fundamental principles brought along by quantum mechanics at first **did not** provide a lead on how to tackle the physical interpretation of the exclusion principle.
- This only occurred in the **context of applications**.



Wolfgang Pauli
(1900–1958)



Complex spectra: Fermi statistics

- In March 1926, Enrico Fermi submits a paper on the quantization of the ideal monatomic gas and realizes that in order to account for degeneracy, he needs **something in addition** to the quantization rules of the old quantum theory.



Enrico Fermi
(1901–1954)
in 1927



“We therefore conjecture that the quantization of ideal gases necessitates **an additional rule** [Pauli’s exclusion principle] to complement Sommerfeld’s quantization conditions.”

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- No mention or use of **quantum mechanics** (in March 1926!) .
- Along the way, Fermi shows that a gas of particles obeying Pauli’s exclusion principle satisfies **a new statistics**: Fermi statistics.

Complex spectra: Dirac many-particle wavefunctions

- In August 1926, Paul Dirac independently derives **similar results** in a more general way from the formalism of quantum mechanics.

“Thus the symmetrical eigenfunctions alone or the antisymmetrical eigenfunctions alone give a complete solution of the problem. The theory at present is **incapable of deciding** which solution is the correct one.”

Paul Adrien Maurice Dirac, “On the Theory of Quantum Mechanics,” Proceedings of the Royal Society of London. Series A 112, no. 762 (1926): 661–77, on 669.



Paul Dirac
(1902–1983)

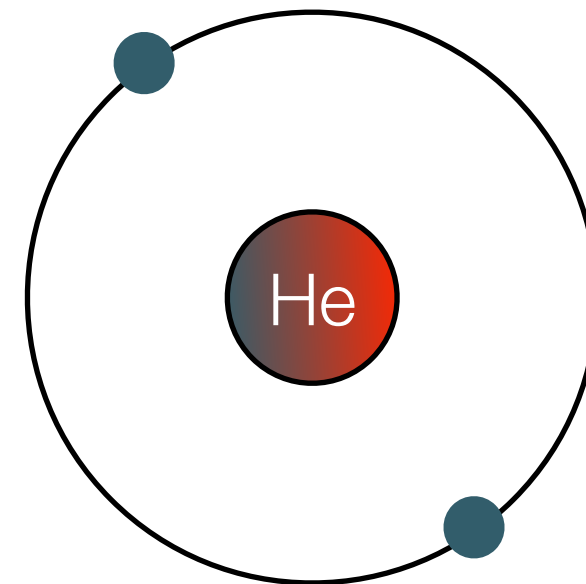
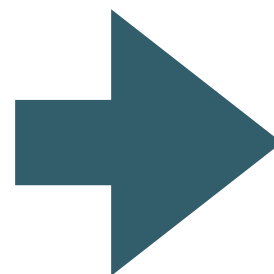
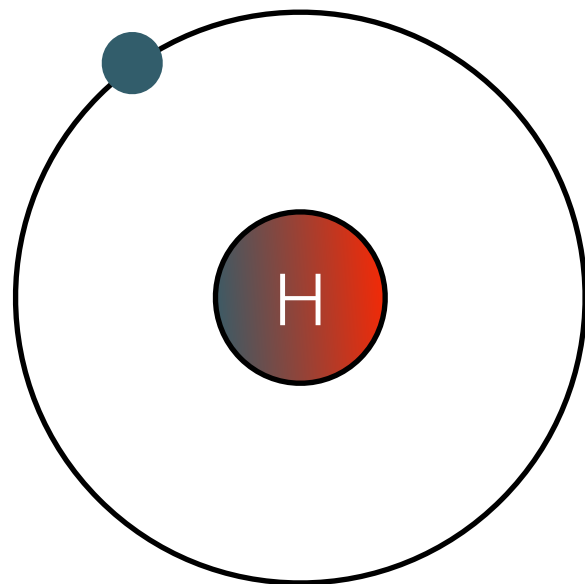
- Symmetrical** eigenfunctions: Bose-Einstein, correct for light quanta.
- Antisymmetrical** eigenfunctions: Pauli exclusion principle trivial consequence, lead to “different statistical mechanics,” “probably the correct one for gas molecules.”

$$\sum_{\alpha_1, \dots, \alpha_r} \psi_{n_1}(\alpha_1) \psi_{n_2}(\alpha_2) \dots \psi_{n_r}(\alpha_r),$$

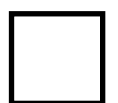
$$\begin{vmatrix} \psi_{n_1}(1), & \psi_{n_1}(2) & \dots & \psi_{n_1}(r) \\ \psi_{n_2}(1), & \psi_{n_2}(2) & \dots & \psi_{n_2}(r) \\ \cdot & \cdot & \cdot & \cdot \\ \psi_{n_r}(1), & \psi_{n_r}(2) & \dots & \psi_{n_r}(r) \end{vmatrix}.$$

Complex spectra: atoms with more than one electron

- In January 1926, Pauli had succeeded in applying matrix mechanics to the **hydrogen atom**, just weeks before Schrödinger explained the hydrogen spectrum in the context of wave mechanics.
- How to extend quantum mechanics to atoms with more than one electron (or molecules, or gases of many atoms) was **absolutely nontrivial**.



???

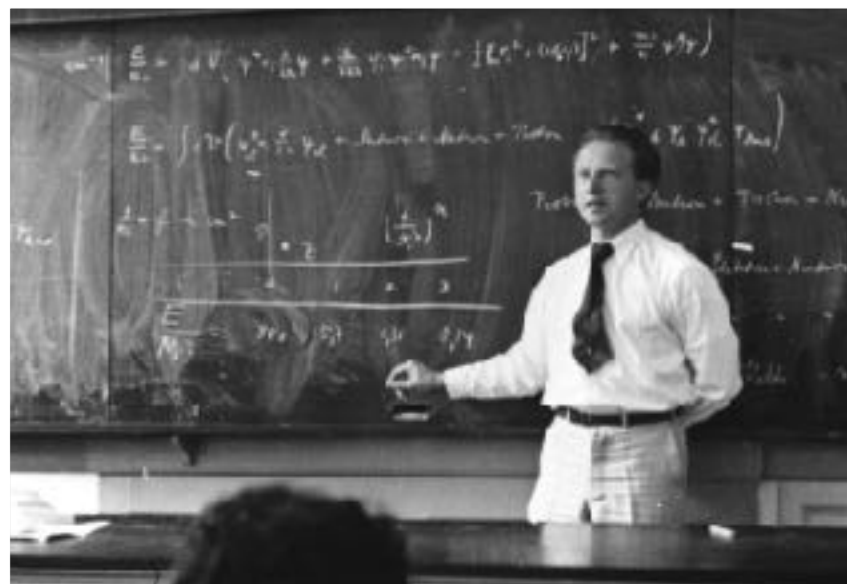


explained by early quantum mechanics

not explained by quantum
mechanics until mid-to-late 1926

Complex spectra: Heisenberg's resonant exchange

- On 5 May 1926, Heisenberg sends Pauli a hand-drawn postcard and notes on its back:



Werner Heisenberg (1901–1976)
lecturing in Copenhagen, 1936

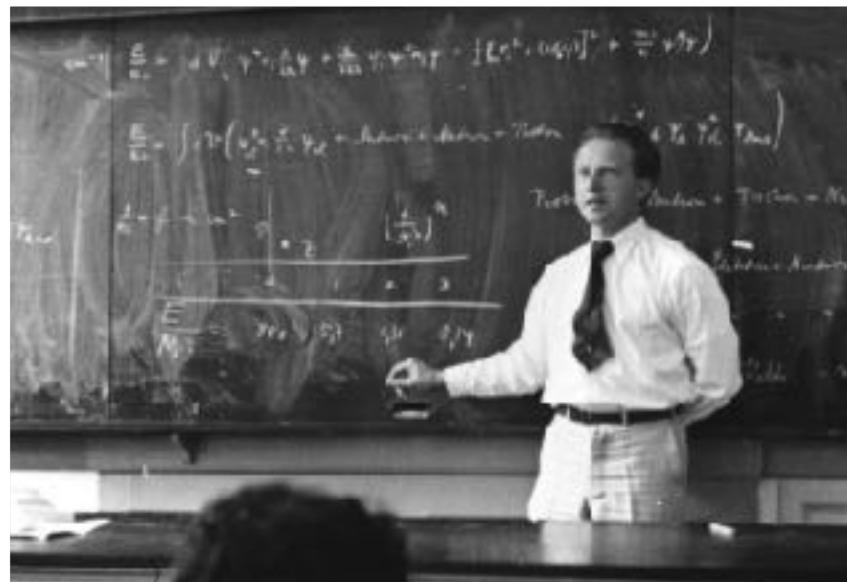
“We have found a rather decisive argument that your exclusion of equivalent orbits is **connected** to the distance between singlet and triplet [terms in neutral Helium]. ... Thus, para- and ortho[helium] do *indeed* have different energies, independent of the interaction between the magnets [i.e., the magnetic moments associated with potentially “spinning” electrons].”

Heisenberg to Pauli, 5 May 1926



Recto of the postcard. According to Friedrich Hund, the drawing refers to the imminent call to an *Extraordinariat* in Leipzig that Heisenberg (H) believed would go to Pauli (P) and threaten to end the latter's career as a researcher. Note that it is not the ordinary professorship in Leipzig that Heisenberg would be called to in 1927.

Complex spectra: Heisenberg's resonant exchange



Werner Heisenberg (1901–1976)
lecturing in Copenhagen, 1936

- Heisenberg prepares a manuscript titled “Many-body problem and resonance” in which he introduces the concept of **resonant exchange** to account for the Helium spectrum.
- Shortly before sending the manuscript off from Copenhagen, he writes to Max Born in Göttingen

“But I think it is truly a step forward that one sees that **Pauli’s exclusion [principle] and Bose’s rule are the same**, that **they do not contradict quantum mechanics**, and that one can calculate the energy values and so forth quantum-mechanically.”

Heisenberg to Born, 26 May 1926, AHQP, M/f No. 18,
Sect. 002–011

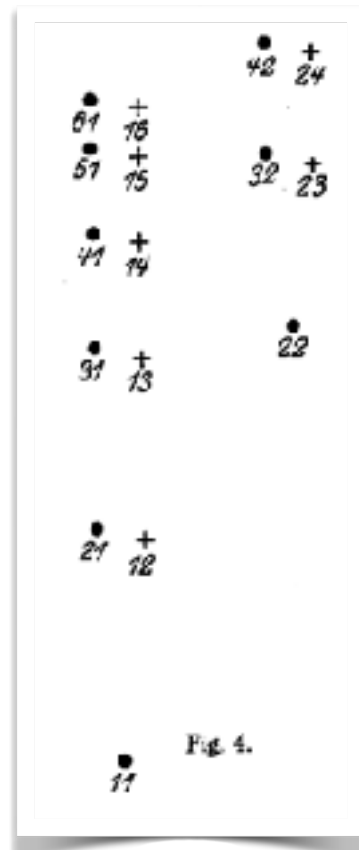
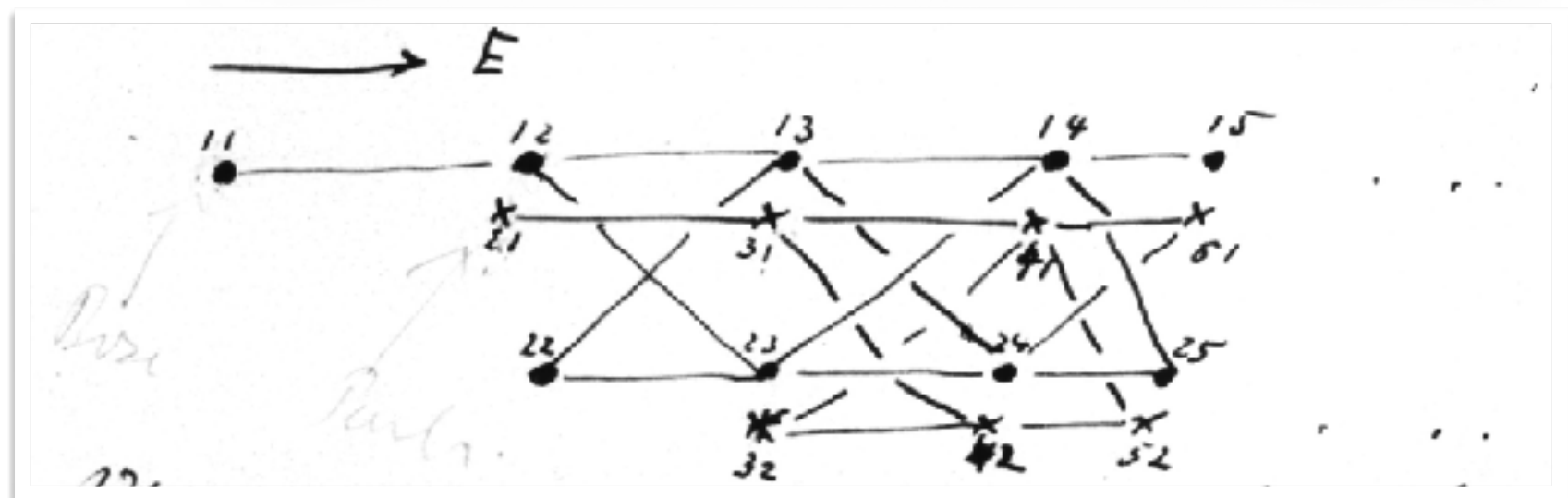
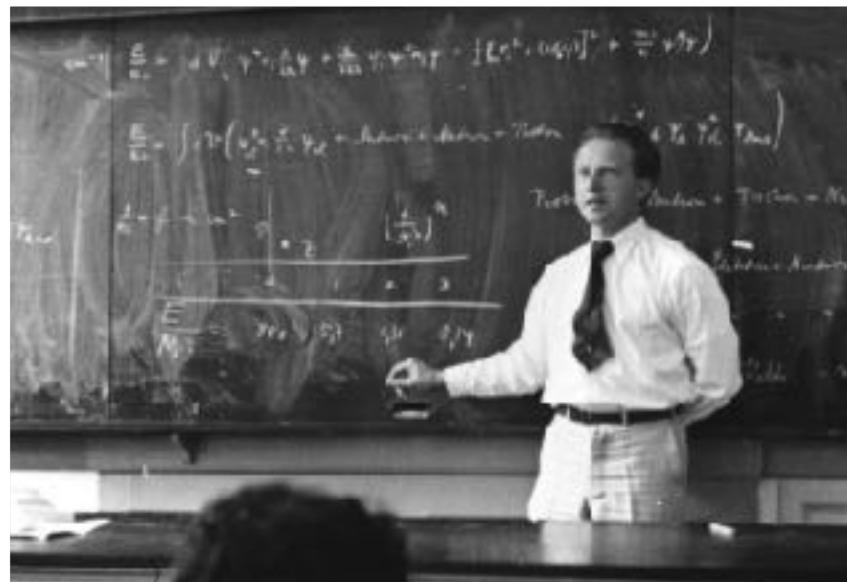


Fig. 4.



Complex spectra: Heisenberg's resonant exchange



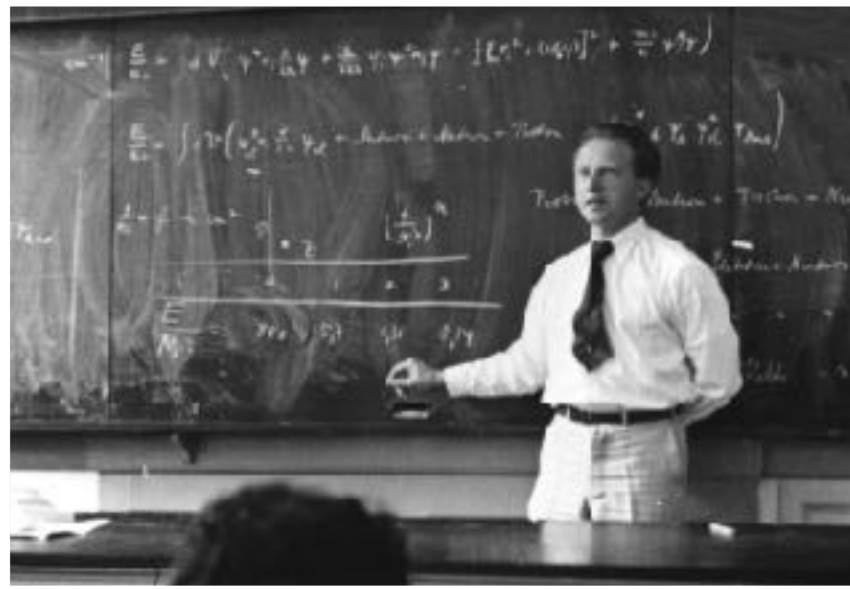
Werner Heisenberg (1901–1976)
lecturing in Copenhagen, 1936

- Many-body problem paper: **Analogy** with classical picture of coupled oscillators: Helium spectrum splits up into **two non-combining subsystems**.
- Heisenberg chooses subsystem that does not contain equivalent orbits (i.e., satisfies **Pauli's rule**), without further physical justification.
- In a subsequent paper that Heisenberg described as “**steamrolling**” due to the amount of perturbation theoretical computations it contained, he concluded

“The calculations carried out here were intended to show that **quantum mechanics also enables a qualitative description of the spectrum for atoms with two electrons, down to the finest details** [...]. For the selection of the one term system that does not contain states with equivalent orbits of the electrons, a clear justification is still missing. It is to be hoped that when attempting to eliminate this deficiency, one will reveal **deeper-lying connections**.”

Werner Heisenberg, “Über Die Spektra von Atomsystemen mit zwei Elektronen,” *Zeitschrift für Physik* 39(1926): 499–518, on 518.

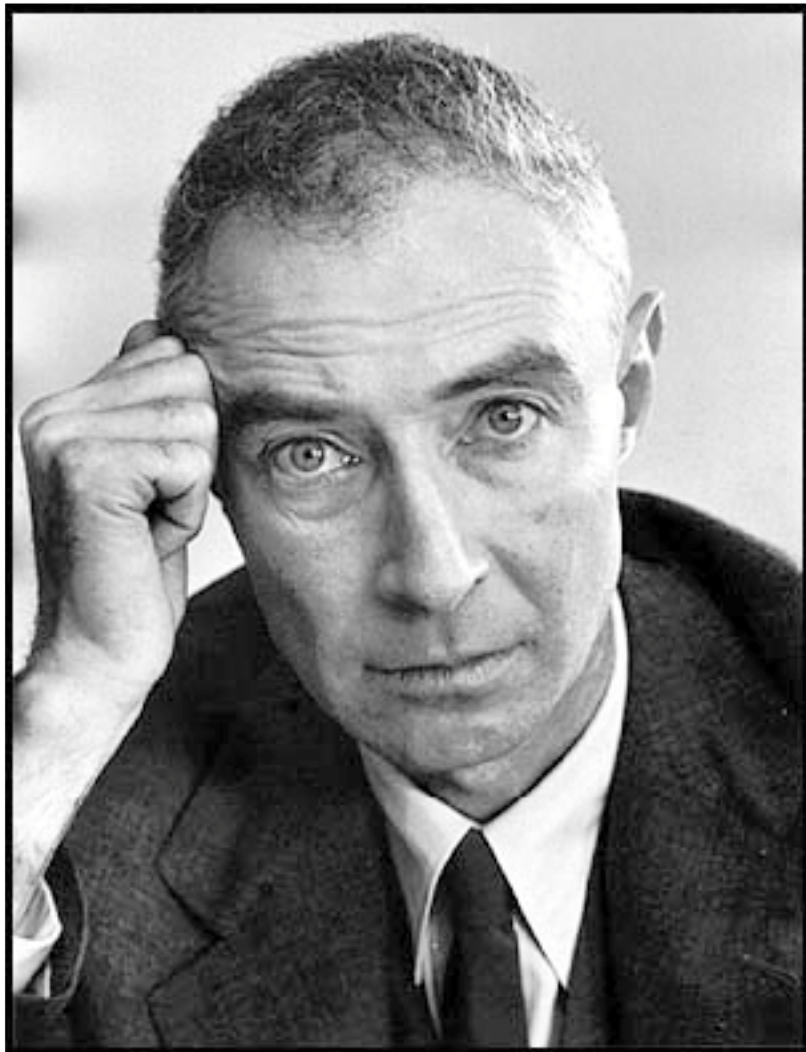
Complex spectra: Heisenberg's resonant exchange



Werner Heisenberg (1901–1976)
lecturing in Copenhagen, 1936

- Those deeper-lying connections were later uncovered by Pauli and Heisenberg: **Fermi-Dirac statistics** (not Bose-Einstein statistics) was the one consistent with Pauli's exclusion principle.
- This finally led to a first **consistent integration** of the quantum statistics of the old quantum theory into quantum mechanics.
- Heisenberg's application-borne concept of resonant exchange would prove **immensely fruitful** in many different areas of physics, e.g., for the Heitler-London theory of the covalent bond, for Heisenberg's own theory of ferromagnetism in solids, for the theory of nuclear structure, and for the very **notion of force** itself.
- Likewise, the study of complex spectra continued to be a **source of new quantum-mechanical ideas**, among them Eugene Wigner's application of **group theory** to quantum mechanics, **Slater determinants**, and **self-consistent field methods**.

Complex spectra: Oppenheimer on Heisenberg's resonant exchange



J. Robert Oppenheimer
(1904–1967)

“I regarded it as a kind of **discovery of the meaning of quantum theory**. ... I think that if Heisenberg had found that there wasn't anything new but just that the integrals of wave functions happened to give the helium spectrum right, it would have been **problem solving**.

It was the fact that there was an element of novelty [resonant exchange] and something which had never been described before which turned it from solving a problem into **exploring the content and meaning** [of quantum mechanics].”

J. Robert Oppenheimer interviewed by Thomas S. Kuhn, 20 November 1963, AHQP

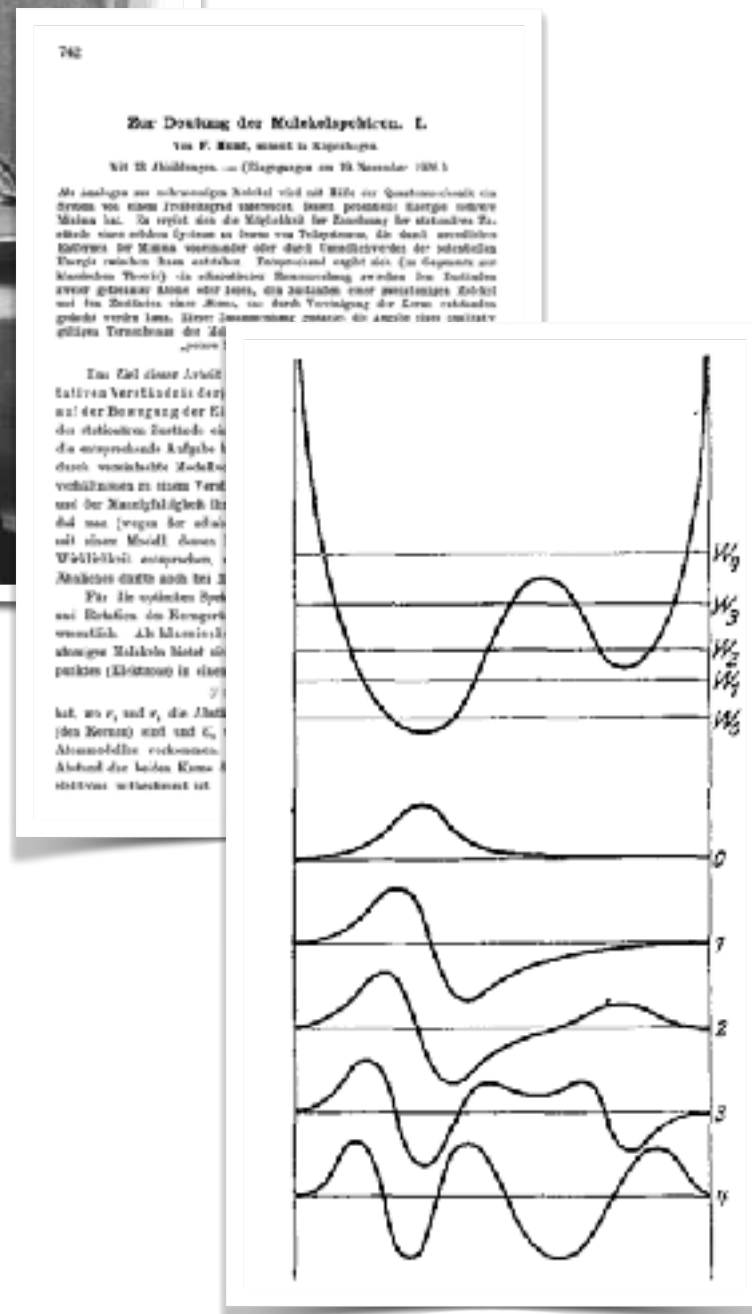
Three Examples of Early Applications

- Scattering and Born's probability interpretation
- Complex spectra, quantum statistics, and resonant exchange
- Tunneling in molecules and nuclei

Quantum Tunneling



Friedrich Hund
(1896–1997)



- In a series of papers written in Copenhagen and Göttingen in 1926–1930, Friedrich Hund explores **molecular spectra** and attempts to systematize their interpretation.
- Already in his first paper, submitted in November 1926 from Copenhagen, Hund considers **wavefunctions in double potential wells** and shows that there is a **continuous transition** from wavefunctions localized in one well to wavefunctions spanning both wells.
- This is the first step towards the concept of **quantum tunneling** (the term itself was first coined in the 1930s).

Quantum Tunneling

- In Cambridge in 1927–1928, Lothar Nordheim, later joined by Ralph Fowler, studies the **behavior of electrons at metal surfaces** and finds a “remarkable new phenomenon” that now is part of virtually every quantum mechanics textbook.

Here a **remarkable new phenomenon** occurs which can significantly alter the situation. If the potential in the intermediate layer is greater than outside the metal, the result that total reflection takes place when the available kinetic energy is not sufficient to overcome the potential threshold is no longer valid. This is because the electron density does not disappear immediately after the jump point, but only decreases exponentially. [...] Thus, **according to quantum mechanics, some electrons will pass this threshold**, whereas according to classical theory they would be held back.

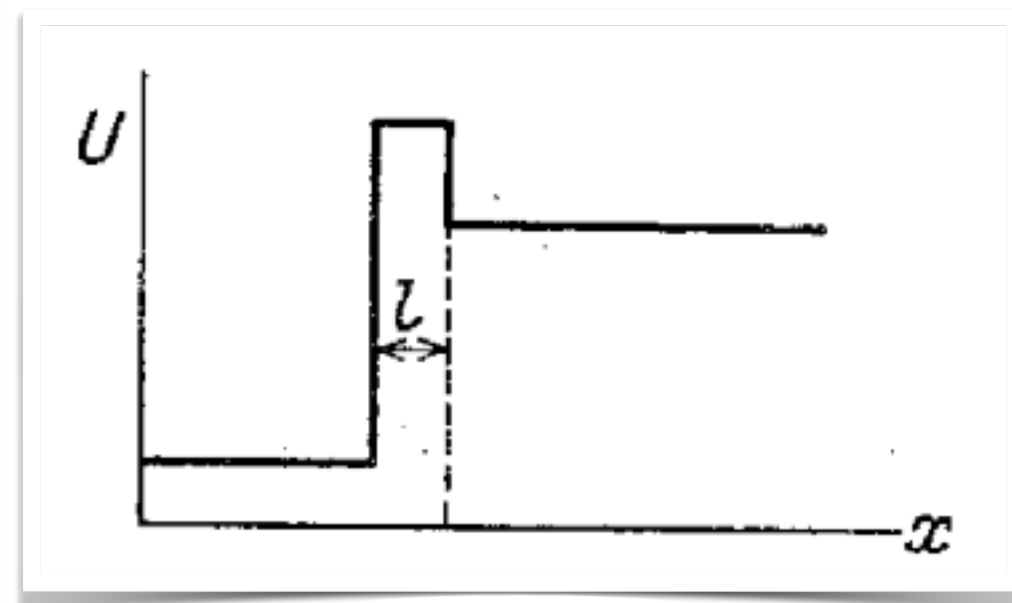
Lothar Nordheim, “Zur Theorie der thermischen Emission und der Reflexion von Elektronen an Metallen,” *Zeitschrift für Physik* 46, no. 1 (1928): 833–855, on 849.



Lothar Nordheim
(1899–1985)



Ralph H. Fowler
(1889–1944)

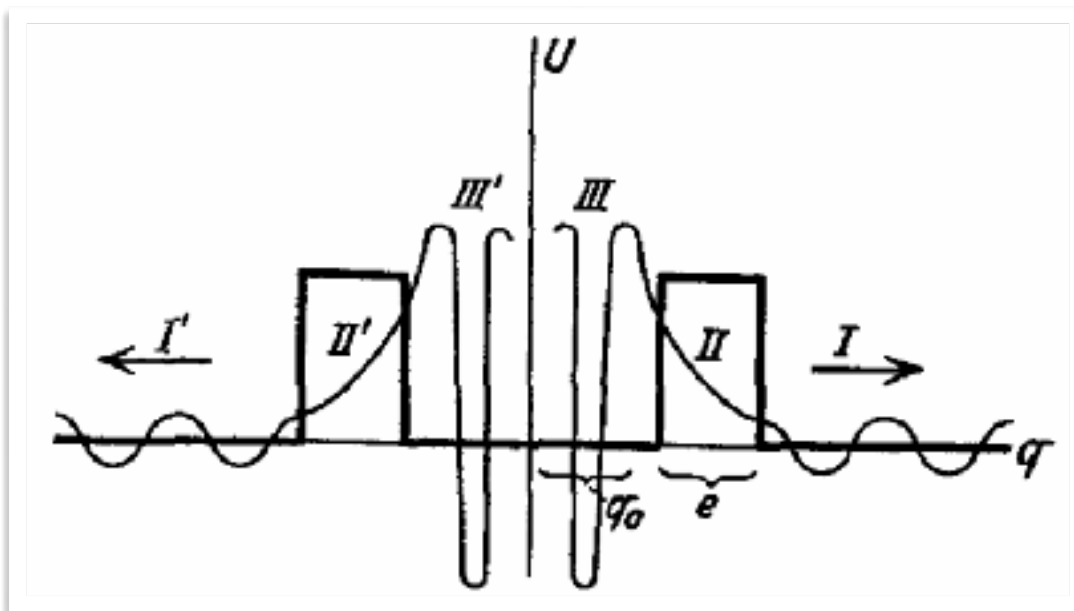


Quantum Tunneling

George Gamow burst upon the European community of physicists like a meteor from outer space. The origin of his trajectory was distant Leningrad; his point of impact was Göttingen; the time was mid-June 1928.

Roger H. Stuewer, “Gamow’s Theory of Alpha-Decay” in *The Kaleidoscope of Science*, vol. 1, ed. E. Ullmann-Margalit (Dordrecht: Reidel, 1986), pp.147–186, on 147.

- Soon after his arrival in Göttingen, Gamow explains **alpha decay in nuclei** quantum mechanically using **tunneling**. Almost simultaneously, Ronald Gurney and Edward Condon in Princeton arrive at similar results.



George Gamow (1904–1968) with Pauli in Switzerland, 1930

Quantum Tunneling

- Important **evidence** that quantum mechanics was **valid in the nuclear domain**.
- Gamow's work marks the starting point of **nuclear physics as a subdiscipline of physics**.

In my experience **nuclear physics starts** with the sudden appearance, one morning in the library of the Göttingen Institute, of a fair-haired giant, with shortsighted, half-shut eyes behind his spectacles, who introduced himself, with a broad smile, by declaring: "I am Gamow."

Léon Rosenfeld, "Nuclear Reminiscences," in *Cosmology, Fusion & Other Matters: George Gamow Memorial Volume*, ed. Frederick Reines (Boulder, CO: Associated University Press, 1972), pp. 289–99, on 289 (quoted after Stuewer 2018).



George Gamow
(1904–1968) with Pauli
in Switzerland, 1930

Many further examples:

- **Atoms and Molecules:**

- Diatomic molecules (1926 Schrödinger: **entanglement**).
- Molecule formation (1926 Hund: **adiabatic transitions** between free and bound atoms)
- Molecular structure (1927 **Born-Oppenheimer approximation**).
- Covalent bond (1927 Heitler-London: **exchange forces**).
- Many-electron atoms and ions (1927 Hartree, 1930 Fock: **self-consistent field methods**)
- ...

- **Solids:**

- Paramagnetism (1926 Pauli: **Fermi surface** in solids).
- Electron theory of metals (1927 Sommerfeld: **Fermi-Dirac electron gas in metals**).
- Ferromagnetism (1928 Heisenberg: **exchange forces**).
- Anomalous Hall effect (1928 Peierls: **hole conduction**)
- Insulators and thermal conductivities (1929 Peierls: **Umklapp processes**)
- Long mean-free path of electrons in metals (1929 **Bloch waves**).
- Valence electrons in crystals (1931 Kronig and Penney: **band structure and band gaps**)
- ... (**Brillouin zone, Wigner-Seitz cell, quasiparticles...**)

- **Nuclei:**

- Nuclear structure (1932–3 Heisenberg: **nuclear exchange forces**)
- ...

4 Conclusion and Outlook: Why apply?

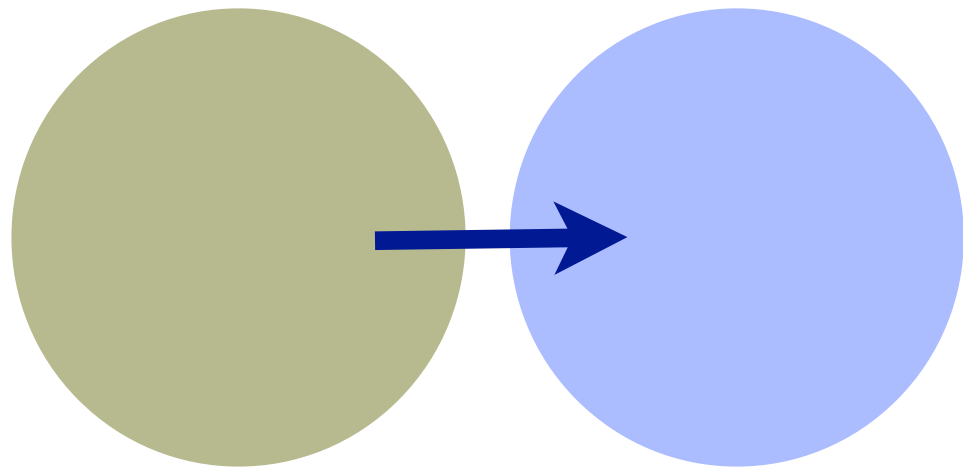
Conclusion and Outlook: Why apply?

- Applications uncovered avenues to a potential **extension or even revision** of quantum mechanics.
- Applications **recovered successes of the old quantum theory** by integrating them into the new formalism (e.g., quantum statistics).
- Applications helped **extend the domain of applicability** of quantum mechanics far beyond its initially narrow range of application.
- Applications contributed to a **deeper understanding** of what the new quantum-mechanical formalism really meant.
- In that vein, applications gave rise to **novel (often intermediate-level) concepts** (resonant exchange, tunneling, Bloch wave, spin waves, Brillouin zone, Fermi surface) and techniques of computation/approximation (Rayleigh-Schrödinger perturbation theory, Slater determinants, Pauli spin matrices, Born-Oppenheimer approximation).
- Applications thus **established the generality of quantum mechanics in practice and not just in theory** and helped bridge the gap between fundamental laws and empirical observations.



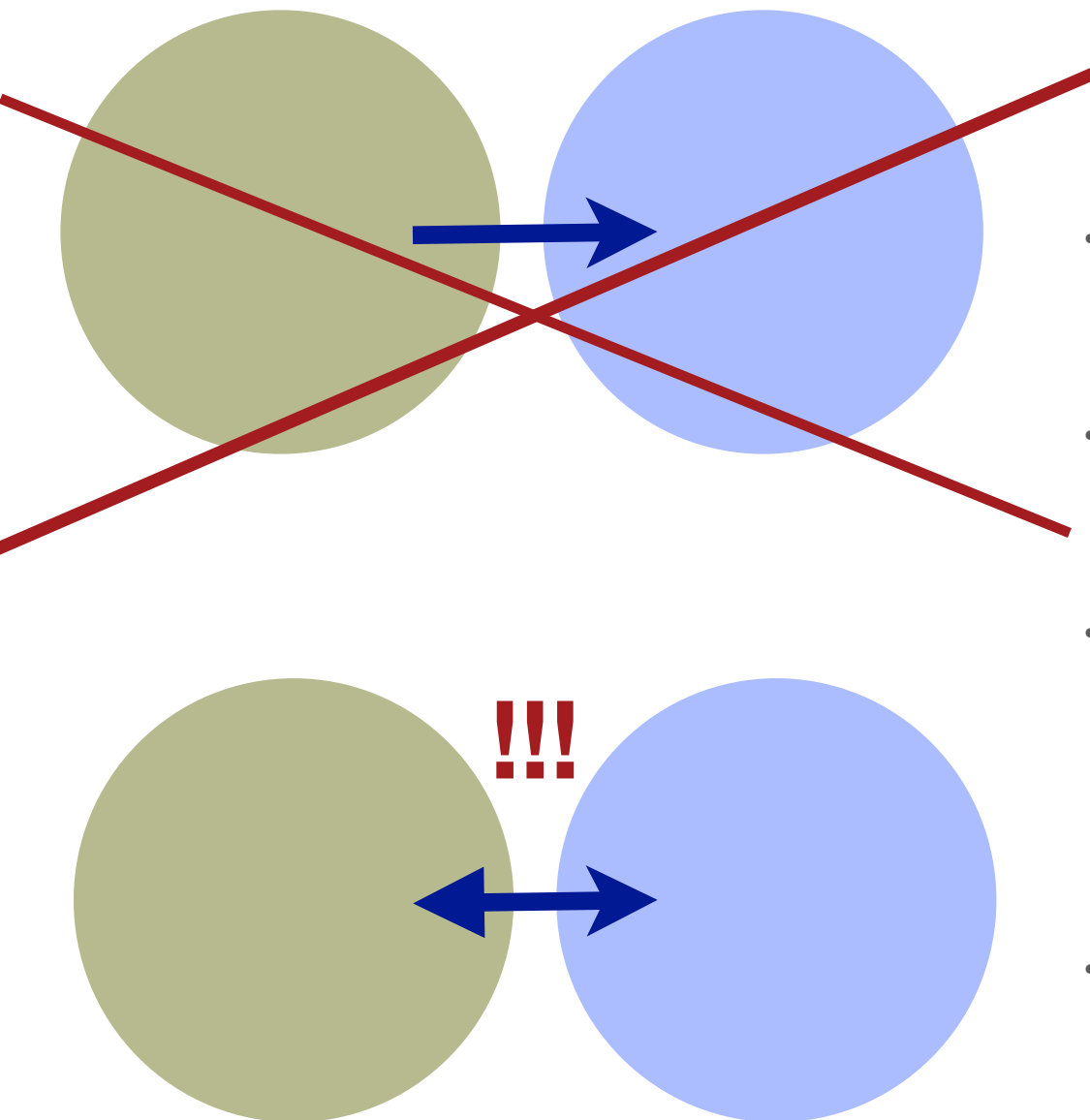
Wolfgang Pauli (1900–1958) and George Gamow (1904–1968) in Copenhagen (unknown date)

Conclusion and Outlook: Why apply?



- Applications of quantum mechanics were also at the outset of **new, fruitful research fields and subdisciplines** that emerged during the twentieth century: quantum chemistry, solid state physics, nuclear physics.
- When a theoretical formalism gets applied to new domains, it is **never a one-way street**.

Conclusion and Outlook: Why apply?



- Applications of quantum mechanics were also at the outset of **new, fruitful research fields and subdisciplines** that emerged during the twentieth century: quantum chemistry, solid state physics, nuclear physics.
- When a theoretical formalism gets applied to new domains, it is **never a one-way street**.
- Rather, the new fields contributed **new concepts and techniques** to quantum mechanics as we know it today.
- Quite generally, “applying” a theory to a different empirical phenomenon likely affects not only how we explain that phenomenon, but at least potentially also has a **back-effect** on the theory itself.
- There are **prominent examples** of this between, e.g., condensed matter and high energy physics during the course of the twentieth century.
- Applications are thus not always “mere problem solving” but sometimes a **promising strategy to better understand a theory’s foundations**.

Conclusion and Outlook: Why apply?

- Therefore, instead of reading a handful of papers over and over again, researchers interested in **the foundations and interpretation of quantum mechanics** should study more papers, by a broader range of authors, and also on more mundane and applied topics such as complex atoms, molecules, solids, and nuclei.
- Sadly, most of this literature is **not available in English** (van der Waerden never published Volume 2 of his *Sources of Quantum Mechanics...*).
- Such a broadened purview casts doubt on the dominant narrative of an “**interpretational dark age**” in the history of quantum physics from the mid-1930s until the early 1960s.
- In light of claims of an ongoing “**second quantum revolution**” that centers on practical applications in quantum optics, quantum information, and quantum computing, we should expect that **these applications are affecting our views on the foundations of quantum physics**.



Bohr, Pauli, Nordheim, Fues,
Rosenfeld in Auditorium A,
Copenhagen, 1929

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THANK YOU!



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