Feebly Interacting Particles in the Early Universe

based on work in collaboration with

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Enrico Morgante, Pedro Schwaller,
Cristina Puchades Ibañez, and Carlos Tamarit

Based on 2312.17246, 2502.01729, and upcoming work.

Mathias Becker

What is a Feebly Interacting Particle (FIP)?

For the context of this talk: A FIP does not thermalize in the early universe.

Simplified DM (SUSY-like)

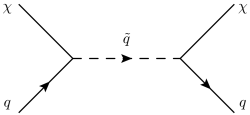

Fig. from 1903.05650

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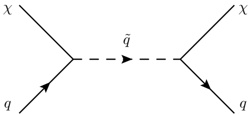

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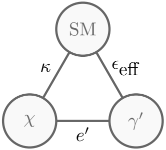
Vector Portal



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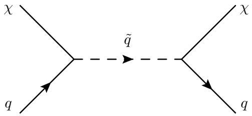

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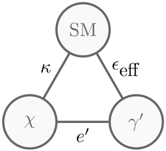

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Axion-Like Particle

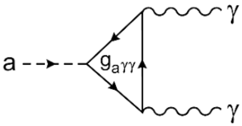

Fig. from 2106.03424

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LLP searches

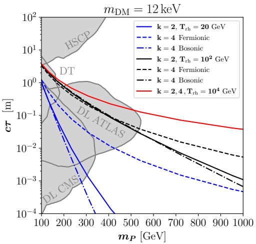
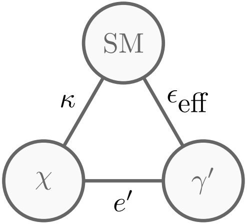
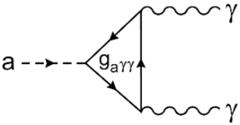


Fig. from 2306.17238





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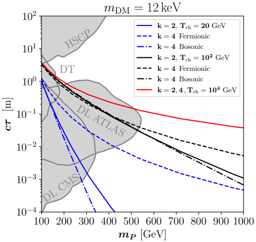


Fig. from 2306.17238

Direct detection

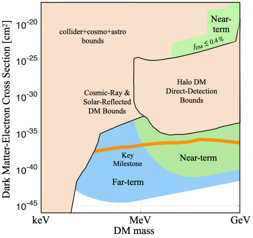
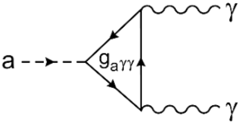


Fig. from 2203.08297



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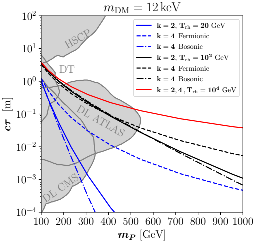

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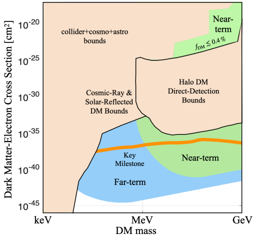


Fig. from 2203.08297

Constraints on ALP-$\gamma\gamma$

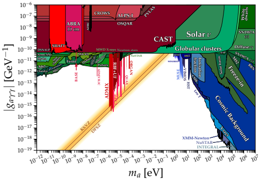
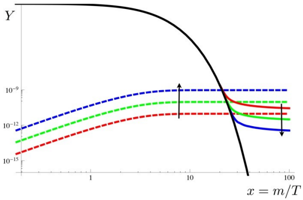


Fig. from O'Hare



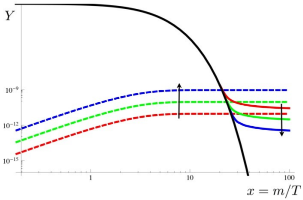
<u>Freeze-In</u>

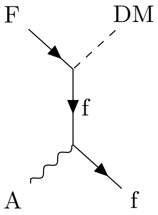
$$T_{f.i} \sim \frac{M}{2} - \frac{M}{5}$$

<u>Freeze-Out</u>

$$T_{f.o} \sim \frac{M}{25} - \frac{M}{30}$$

→ Finite Temperature Corrections relevant for Freeze-In





Freeze-In

$$T_{f.i} \sim \frac{M}{2} - \frac{M}{5}$$

Freeze-Out

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→ Finite Temperature Corrections relevant for Freeze-In

$$\sigma \sim \int dt |\mathcal{M}|^2 \sim \int \frac{dt}{t} \sim \ln\left(\frac{m_f}{T}\right)$$
$$\Rightarrow \text{divergent for } m_f \ll T$$

Common Treatment: Thermal masses

$$m_f \to m_f(T) \sim T$$

see for instance [Belanger et. al(2020)], [No et. al(2020)], [Calibbi et. al(2021)]

What do we do?

→ Calculate the DM production rate in the real time formalism of thermal QFT

→ Compare out results to:

- Thermal QFT calculations in Hard Thermal Loop approximation
- Boltzmann approach employing scattering rates regulated with thermal masses

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Models: s (scalar DM), F (gauge charged Parent), f (SM Fermion)

$$\mathcal{L}_{\mathrm{int}} = \mathrm{y}_{\mathrm{DM}}\, \bar{\mathrm{f}}_{\mathrm{SM}}\, \mathrm{F}\, \mathrm{s} + \mathrm{h.c.}$$

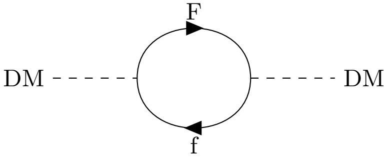
Relevant Parameters

$$\mathrm{G} = \sum_{\mathrm{i}} \mathrm{C}_2(\mathcal{R}_{\mathrm{i}})\mathrm{g}_{\mathrm{i}}^2(\mathrm{m}_{\mathrm{F}}) \ , \qquad \delta = \frac{\mathrm{m}_{\mathrm{F}} - \mathrm{m}_{\mathrm{DM}}}{\mathrm{m}_{\mathrm{DM}}}$$

DM Time Evolution

$$\dot{n}_{\mathrm{DM}} + 3Hn_{\mathrm{DM}} = \gamma_{\mathrm{DM}} \sim \int d^3p \frac{\Pi^{\mathcal{A}}_{\mathrm{DM}}}{E_{\mathrm{DM}}} f_{\mathrm{DM}}\left(E_{\mathrm{DM}}\right)$$

Spectral Self-Energy $\Pi^{\mathcal{A}}_{\mathrm{DM}} = -\mathrm{Im}\Pi^{R}_{\mathrm{DM}} =$



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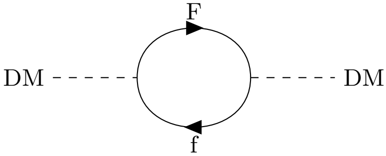
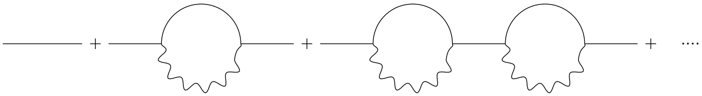


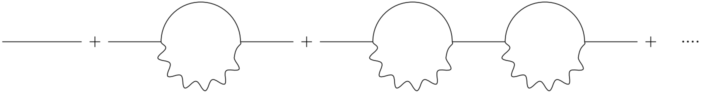
DM Self-Energy

$$\Pi^{\mathcal{A}}_{DM}\left(P\right) \sim \int dK\, S^{\mathcal{A}}_{F}\left(K\right) S^{\mathcal{A}}_{f}\left(K-P\right) + \text{higher order contributions}$$

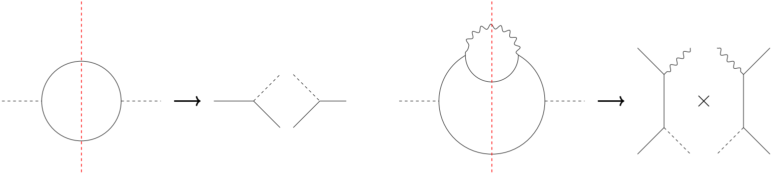
Resummed Propagators



Resummed Propagators



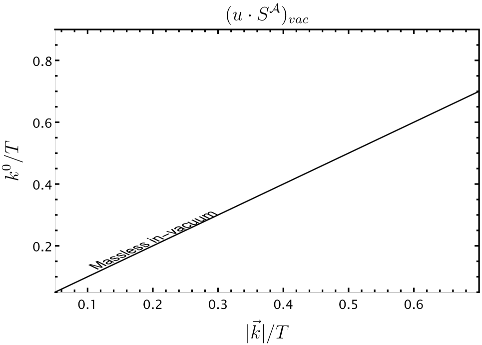
→ scattering contributions also arise at leading order of the effective action (DM self-energy) expansion



Spectral Propagator

Free Propagator

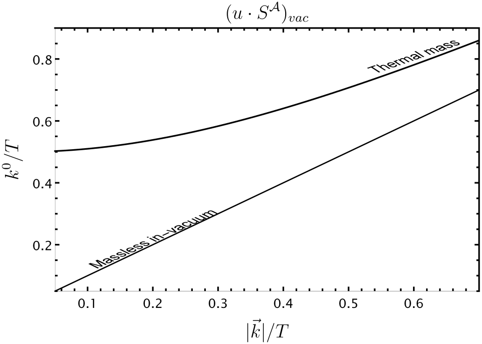
$$\not{S}^{\mathcal{A}}(\mathrm{k}) \sim \delta\left(\mathrm{k}^2\right)$$



Spectral Propagator

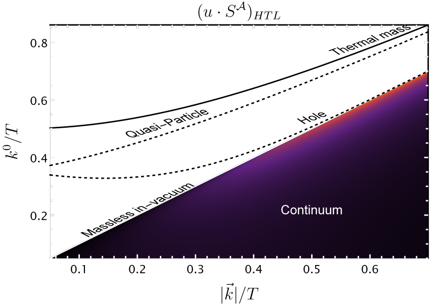
Thermal Mass

$$\not{S}^{\mathcal{A}}(\mathrm{k}) \sim \delta\left(\mathrm{k}^2 - \mathrm{m}_{\mathrm{th}}^2\right)\,, \quad \mathrm{m}_{\mathrm{th}}^2 = \frac{\mathrm{G}}{2}\mathrm{T}^2$$



HTL Resummed

$$\not{S}^{\mathcal{A}}(\mathrm{k}) \sim \delta\left(\mathrm{k}^2 - \Sigma^{\mathcal{H}}(\mathrm{k})\right) + \not{S}^{\mathcal{A}}_{\mathrm{cont.}}$$



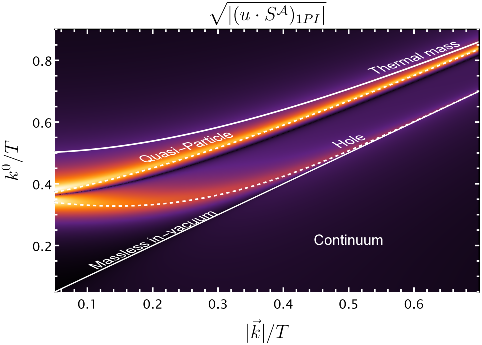
The HTL approximation:

- assumes a large temperature limit $\mathrm{T} \gg \mathrm{M}$
- is an expansion in G/π

Spectral Propagator

1PI-Resummed

$$\not{S}^{\mathcal{A}}(k) \sim (\not{k} - \not{\Sigma}^{\mathcal{H}}) \frac{\Gamma(k)}{\Omega^2(k) + \Gamma^2(k)} - \not{\Sigma}^{\mathcal{A}}(k) \frac{\Omega(k)}{\Omega^2(k) + \Gamma^2(k)}$$



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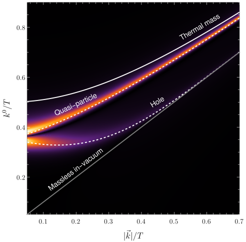

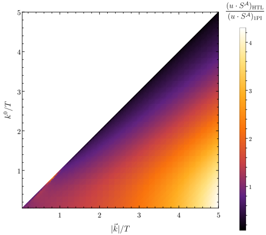

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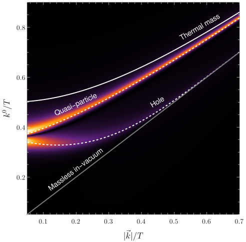
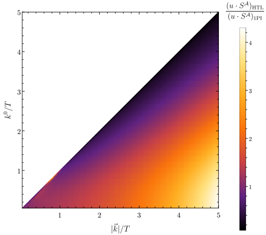
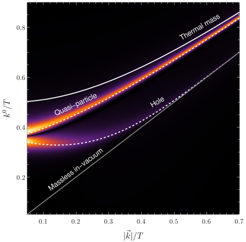


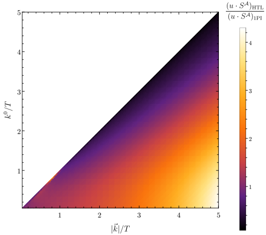


HTL approximation simplifies numerics but only reliable for k $\lesssim$ gT!

Freeze-In occurs around T $\sim$ M $\rightarrow$ breakdown of HTL.

Spectral Propagator





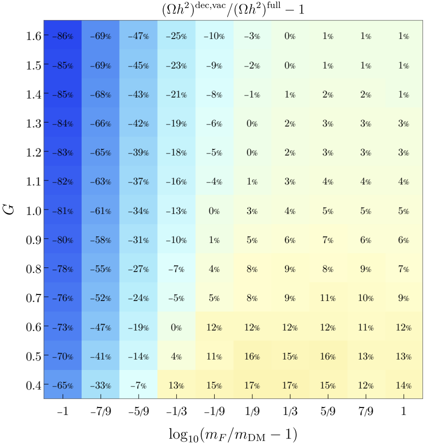
HTL approximation simplifies numerics but only reliable for k $\lesssim$ gT!

Freeze-In occurs around T $\sim$ M $\rightarrow$ breakdown of HTL.

<u>Method we use:</u>
Full form of resummed propagator
[Rychkov, Strumia(2007)])
[Drewes, Kang (2013)])
[Hamaguchi, Moroi, Mukaida(2011)])

<u>Alternative method:</u>
Match at scale gT
[Biondini, Ghiglieri(2020)])
[Ghiglieri, Laine(2020)])
[Bolz, Brandenburg, Buchmüller(2001)])

Results: Vacuum Decays

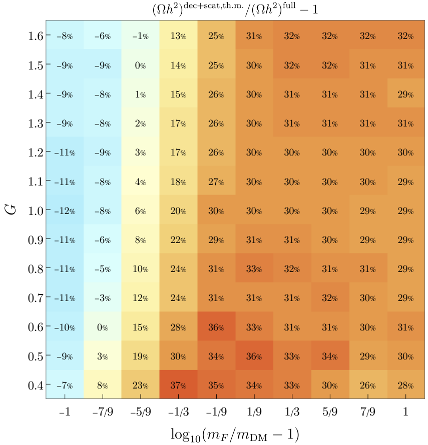


→ Strongly underestimates Ω_{DM} for small mass splittings.

→ (Accidentally) correct for certain parameters.

→ Including thermal masses only for decays worsens the result!

Results: Thermal Masses for Decays and Scatterings

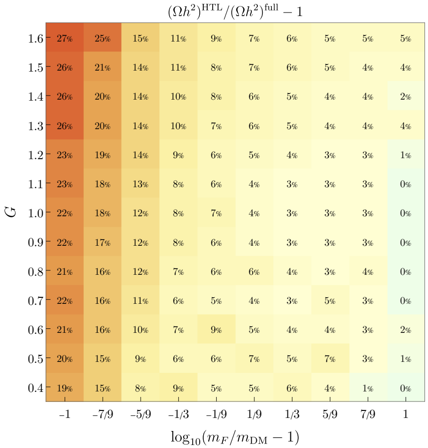


→ Underestimates Ω_{DM} for small mass splittings.

→ Overestimates Ω_{DM} for large mass splittings.

→ Including Fermi-Dirac/Bose-Einstein statistics reduces deviation in $\sim$ half.

Results: Hard Thermal Loop Approximation



→ Overestimates Ω_{DM} for small mass splittings.

→ Percent level accurate Ω_{DM} for large mass splittings.

→ Better results for smaller effective gauge couplings G.

Takeaways:

→ IR freeze-in dynamics most relevant at T $\sim$ M: Complicates calculation of the interaction rate.

→ The accuracy of the analyzed methods varies with model parameters, ranging from percent-level precision to nearly missing complete the contribution, with typical deviations of $\mathcal{O}(10\%)$.

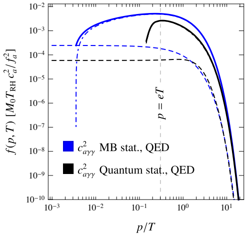
Necessary extention:

→ Inclusion of the LPM effect arising at NLO of the effective action: Will be included in a follow paper soon!

Part II: Thermal Axions

Photophilic ALP

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{2} m_a^2 a^2 - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{c_i \alpha_i}{8\pi f_a} a B_{\mu\nu} \tilde{B}^{\mu\nu}$$



from [Baumholzer et. al(2021)]

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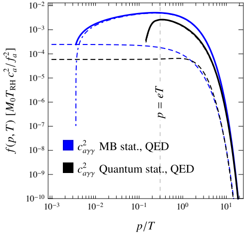

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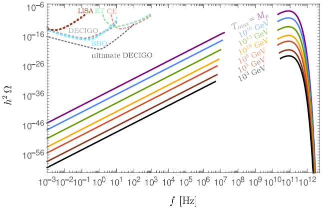

from [Ringwald et. al(2020)], see also [Ghiglieri et. al(2015)]

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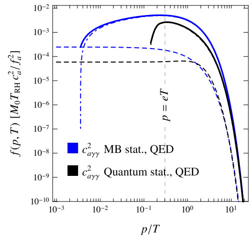

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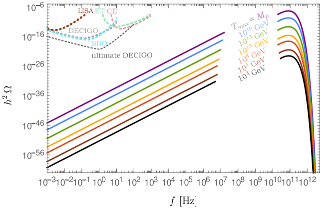
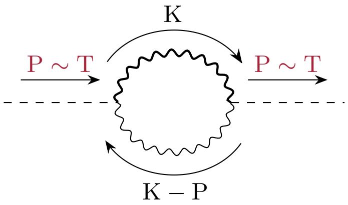
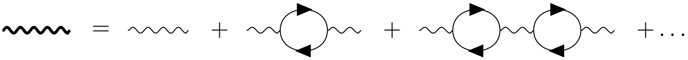


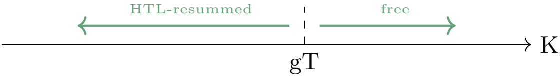
from [Ringwald et. al(2020)], see also [Ghiglieri et. al(2015)]

Goal: Obtain production rate at $g^4 T \leq p \leq gT$

Braaten/Buchmüller thermal axions

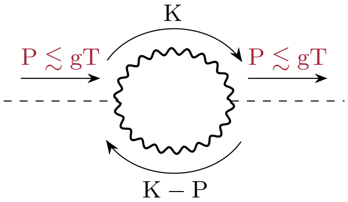


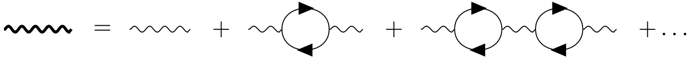


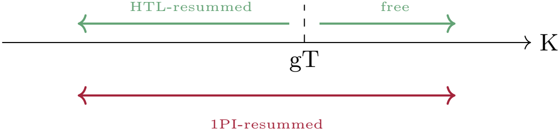


Improved matching/substraction and alternative scheme discussed in [Bouzoud,Ghiglieri (2024)]

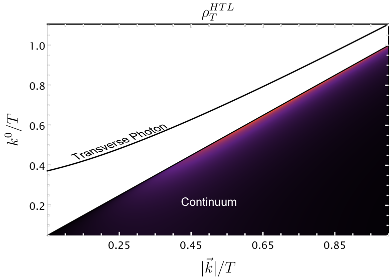
Soft thermal axions

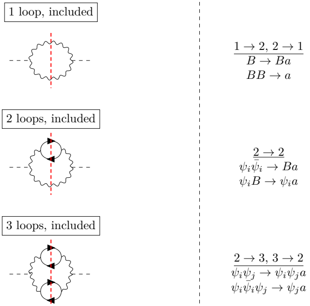






Reminder: $\rho^{\mathrm{HTL}} \sim \delta(\mathrm{K}^2 - \mathrm{m}^2) + \theta(-\mathrm{K}^2)\rho_{\mathrm{cont.}}$



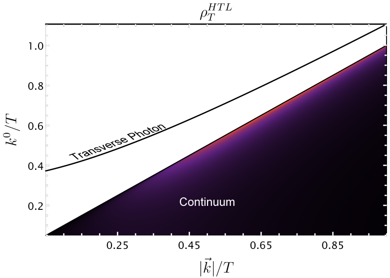


Double Timelike: (Inverse) Decays

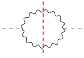
Time-Spacelike: $2 \leftrightarrow 2$ scatterings

Double Spacelike: $2 \leftrightarrow 3$ scatterings

Reminder: $\rho^{\mathrm{HTL}} \sim \delta(\mathrm{K}^2 - \mathrm{m}^2) + \theta(-\mathrm{K}^2)\rho_{\mathrm{cont.}}$

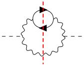


1 loop, included



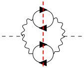
$\underline{1 \to 2,\ 2 \to 1}$
$B \to Ba$
$BB \to a$

2 loops, included



$\underline{2 \to 2}$
$\psi_i \bar{\psi}_i \to Ba$
$\psi_i B \to \psi_i a$

3 loops, included



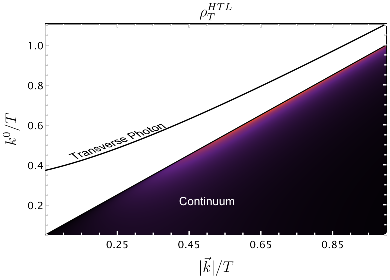
$\underline{2 \to 3,\ 3 \to 2}$
$\psi_i \psi_j \to \psi_i \psi_j a$
$\psi_i \bar{\psi}_i \psi_j \to \psi_j a$

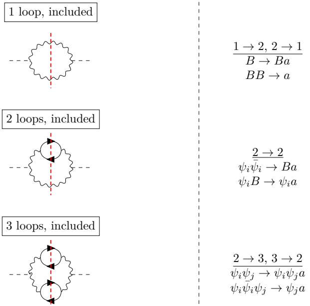
Double Timelike: (Inverse) Decays $\to$ kinematically suppressed

Time-Spacelike: $2 \leftrightarrow 2$ scatterings

Double Spacelike: $2 \leftrightarrow 3$ scatterings

Reminder: $\rho^{\mathrm{HTL}} \sim \delta(\mathrm{K}^2 - \mathrm{m}^2) + \theta(-\mathrm{K}^2)\rho_{\mathrm{cont}}.$



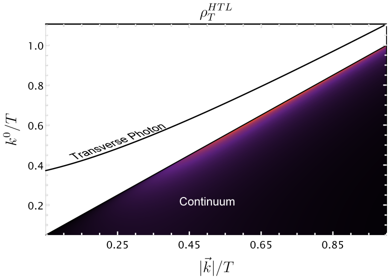


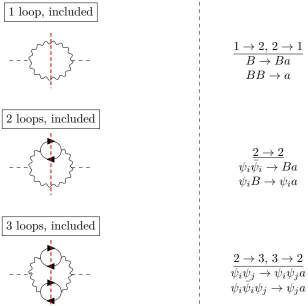
Double Timelike: (Inverse) Decays $\to$ kinematically suppressed

Time-Spacelike: $2 \leftrightarrow 2$ scatterings $\to$ dominates at hard ALP momentum

Double Spacelike: $2 \leftrightarrow 3$ scatterings

Reminder: $\rho^{\mathrm{HTL}} \sim \delta(\mathrm{K}^2 - \mathrm{m}^2) + \theta(-\mathrm{K}^2)\rho_{\mathrm{cont.}}$



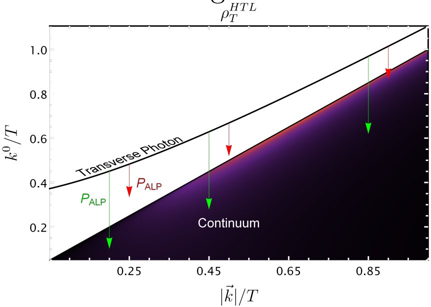


Double Timelike: (Inverse) Decays $\to$ kinematically suppressed

Time-Spacelike: $2 \leftrightarrow 2$ scatterings $\to$ dominates at hard ALP momentum

Double Spacelike: $2 \leftrightarrow 3$ scatterings $\to$ dominates at soft ALP momentum

Soft momentum scaling



$$K^2 > 0 \land (K-P)^2 < 0$$
$$\downarrow$$
$$k \gtrsim \frac{g^2T^2}{p}$$

Time-Spacelike ($2 \leftrightarrow 2$)

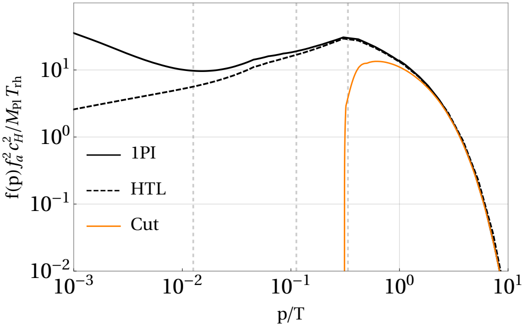
- $\Pi_{TS} \sim \exp(-g^2T/p)$
- Exponential suppression for $p < g^2T$

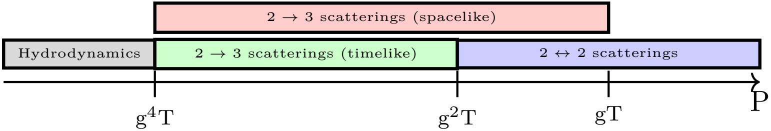
Double Spacelike ($2 \leftrightarrow 3$)

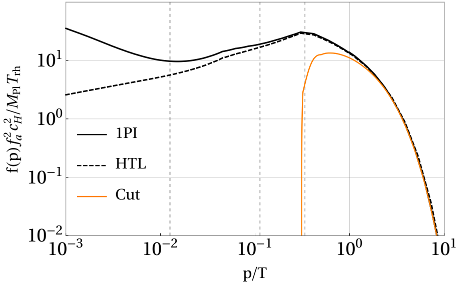
- $\Pi_{SS} \sim p^{4/3}$
- Only power-law suppression

Conclusion: $2 \to 3$ scatterings dominate the interaction rate.

Results


f(p) $f_a^2 c_H^2 / M_{\rm Pl}\, T_{\rm rh}$
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
p/T
1PI
HTL
Cut

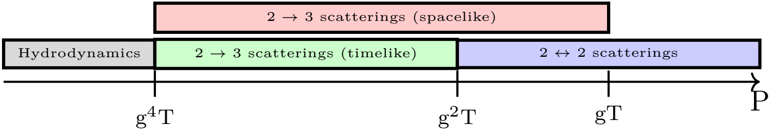

$2 \to 3$ scatterings (spacelike)
Hydrodynamics
$2 \to 3$ scatterings (timelike)
$2 \leftrightarrow 2$ scatterings
g^4T
g^2T
gT
P



Integrated quantities such as:

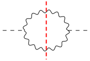
$$n \sim \int d^3p\, f(p)$$
$$\langle p \rangle \sim \int d^3p\, p\, f(p)$$

are only moderately affected ($\mathcal{O}(1-10\%)$).



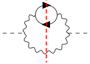
Full leading order result

1 loop, included



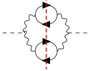
$\underline{1 \to 2,\ 2 \to 1}$
$B \to Ba$
$BB \to a$

2 loops, included



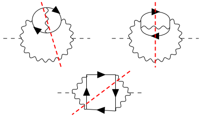
$\underline{2 \to 2}$
$\psi_i \bar{\psi}_i \to Ba$
$\psi_i B \to \psi_i a$

3 loops, included



$\underline{2 \to 3,\ 3 \to 2}$
$\psi_i \psi_j \to \psi_i \psi_j a$
$\psi_i \bar{\psi}_i \psi_j \to \psi_j a$

3 loops, not included



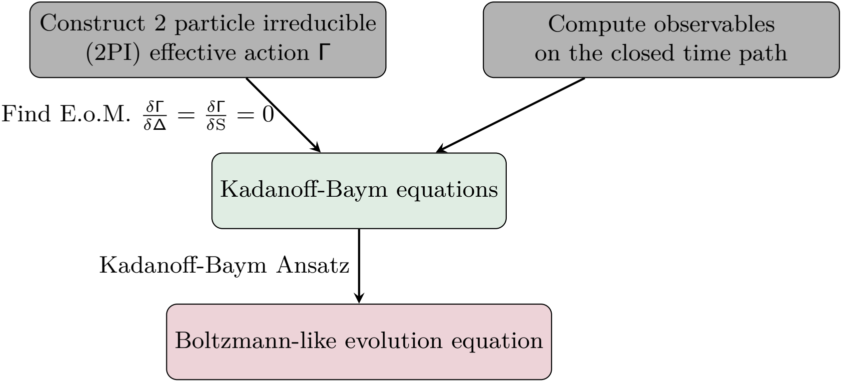
$\underline{2 \to 3,\ 3 \to 2}$
$\psi_i \bar{\psi}_i \to BBa$
$\psi_i B \to \psi_i Ba$
$BB \to \psi_i \bar{\psi}_i a$
$\psi_i \bar{\psi}_i B \to Ba$
$\psi_i BB \to \psi_i a$

- Must include all self-energies implying $2 \to 3$ scatterings.
- Requires extension beyond simple 1-loop 1PI-resummation.
- We include soft ALP radiation from virtual photons
- We do not include soft ALP emission from real photons yet.

Conclusion

- A gap in the interaction rate exists for $g^4T \leq p \leq gT$, relevant e.g. ALPs, gravitons.
- Extrapolating the leading-order rate from hard ($p \gtrsim gT$) to soft momenta yields unphysical (negative) rates.
- For $p \lesssim g^2T$, $2 \to 3$ scatterings dominate, but full leading-order accuracy is still in progress.
- Impact on integrated quantities (e.g., number density, $\langle p \rangle$) is relatively small ($\sim 10\%$) but crucial for full momentum dependence, affecting e.g. the gravitational wave spectrum and possibly N_{eff}.

Closed Time Path (CTP/Keldyish-Schwinger/real time) formalism



for details see [Garbrecht(2018)],[Berges(2004)],[Prokopec et. al(2003)]

Results

We compare

- Complete 1PI-resummed results (our work)
- Hard Thermal Loop resummed results
- Boltzmann Equations with scatterings regulated by thermal masses
- Leading Order Vacuum results

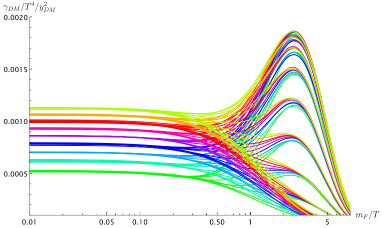
in terms of two relevant parameters

- the effective gauge coupling $\mathrm{G} = \sum_{\mathrm{i}} \mathrm{C}_2\left(\mathcal{R}_{\mathrm{i}}\right) \mathrm{g}_{\mathrm{i}}^2 \in [0.4, 1.6]$
- the mass splitting between the parent F and DM $\delta = 1 - \mathrm{m}_{\mathrm{DM}}/\mathrm{m}_{\mathrm{F}} \in [0.1, 10]$

1PI-resummed Interacaction Rates from the CTP

Fit of the Interaction Rate

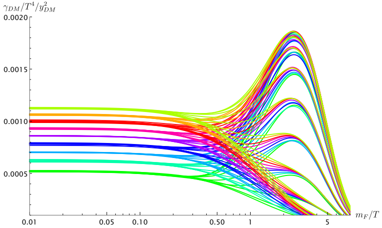
$$\frac{\gamma_{\mathrm{DM}}}{y_{\mathrm{DM}}^2 T^4} = \underbrace{A z^3 K_1(az)}_{\text{Decays}} + \underbrace{B z K_1(bz)}_{\text{Scatterings}}$$



1PI-resummed Interacaction Rates from the CTP

Fit of the Interaction Rate

$$\frac{\gamma_{\mathrm{DM}}}{y_{\mathrm{DM}}^2 T^4} = \underbrace{A z^3 K_1(a z)}_{\text{Decays}} + \underbrace{B z K_1(b z)}_{\text{Scatterings}}$$



Ultra-relativistic limit

$$B b^{-1} = 5 \cdot 10^{-4} G - 1.66 \cdot 10^{-4} G \ln G$$

Non-relativistic limit

$$A a^{-0.5} = 0.033 \frac{\delta^2(\delta+2)^2}{16\pi(1+\delta)^4}$$

Goal: Eventually provide all 4 parameters of this fit in terms of G and δ.

Quality of the Approximation Schemes

Boltzmann/Thermal Mass Approach:

- Underestimates Scattering Contribution
- Overestimates Decay Contribution

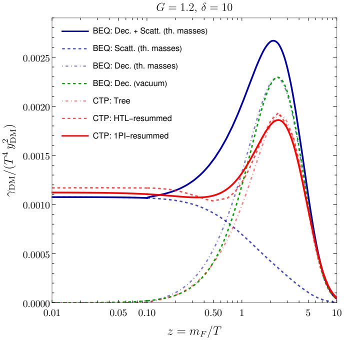
Hard Thermal Loop Approximation:

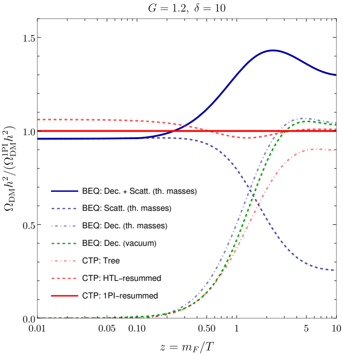
- Overestimates Scattering Contribution
- (Somewhat) accurately captures Decay Contribution

Models

	Y	SU(2)	SU(3)	G	$\mu = M_Z$	10^4 GeV	10^7 GeV	10^{10} GeV
e_L	$-1/2$	2	1	$\frac{g_1^2}{4} + \frac{3g_2^2}{4}$	0.38	0.4	0.46	0.52
u_L	$+1/6$	2	3	$\frac{g_1^2}{36} + \frac{3g_2^2}{4} + \frac{4g_3^2}{3}$	2.3	1.6	1.2	1.0
d_L	$+1/6$	2	3	$\frac{g_1^2}{36} + \frac{3g_2^2}{4} + \frac{4g_3^2}{3}$	2.3	1.6	1.2	1.0
e_R	-1	1	1	g_1^2	0.21	0.22	0.24	0.26
u_R	$+2/3$	1	3	$\frac{4g_1^2}{9} + \frac{4g_3^2}{3}$	2.1	1.3	0.9	0.7
d_R	$-1/3$	1	3	$\frac{g_1^2}{9} + \frac{4g_3^2}{3}$	2.0	1.2	0.8	0.6

Comparison of Interaction Rates/ Relic Density





Boltzmann/Thermal Mass Approach:

- underestimates scatterings
- overestimates decays

Hard Thermal Loop Approximation:

- overestimates scatterings
- accurately captures decays

Estimate of LPM corrections