Quantum field theory, quantum reference frames and the type of local algebras

CJ Fewster

Department of Mathematics and YCQT, University of York

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Comm. Math. Phys. **406**:19 (2025) `arXiv:2403.11973` with D Janssen, LD Loveridge, K Rejzner, J Waldron

See also a more technical talk in session MP10, 1400 Thursday 3 April

Outline

- A remarkable result in mathematical QFT is that all local observable algebras are isomorphic – they are von Neumann factors of the specific hyperfinite type III_1
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- These differences sound (and are) technical but have physical consequences, casting light on the degrees of freedom available in theories of quantum gravity and bearing on the interaction between measurement and symmetry.
- This talk: describe background, demystify (factors? types? quantum reference frames?) for these results, and briefly present our extension of the above results.

AQFT – the overall vibe

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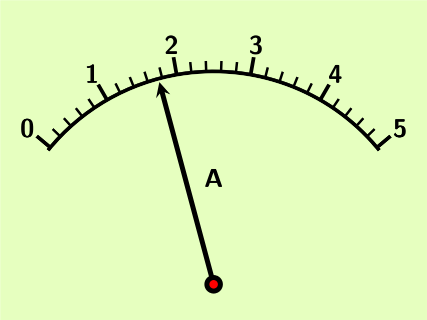
Nonetheless: specific theories can be constructed in AQFT framework,
AQFT has led to new constructive techniques Lechner, Bostelmann, Cadamuro, Tanimoto,...
interacting theories can be constructed in pAQFT Brunetti, Fredenhagen, Hollands, Wald, Duetsch, Rejzner...

The role of algebra



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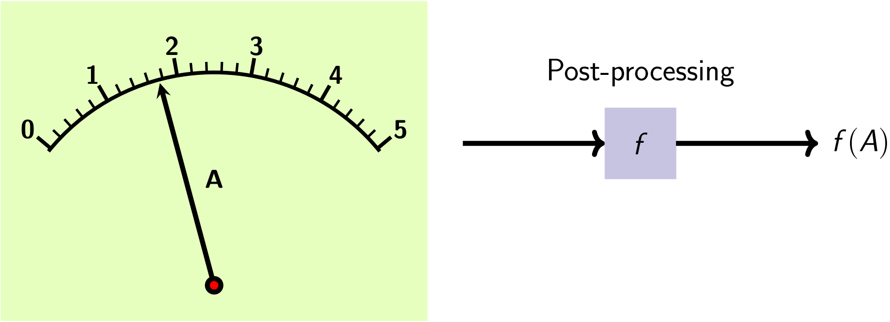

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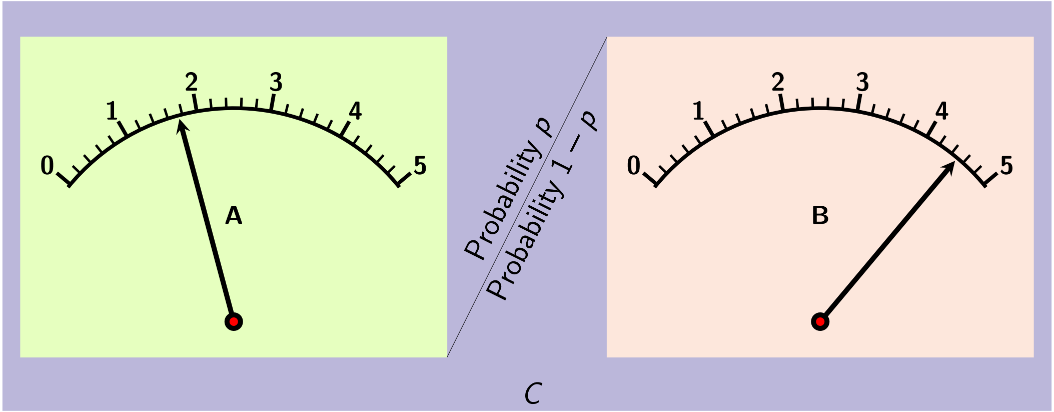

$$\langle C\rangle = p\langle A\rangle + (1-p)\langle B\rangle \qquad \longrightarrow \qquad C = \text{“}pA + (1-p)B\text{”}$$

The role of algebra (continued)

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Given an antisymmetric commutator as well, we define an algebra product

$$AB := A \circ B + \tfrac{1}{2}[A, B].$$

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Conclusion Quantum observables naturally generate a unital $*$-algebra.

$*$-algebras, C^*-algebras, W^*-algebras

Additional structure permits varying strength of functional calculus.

- $*$-algebra A complex vector space, with product (associative & distributive) and an antilinear involution $A \mapsto A^*$, and a unit $\mathbf{1}$.

$$\lambda A, \qquad A + B, \qquad AB, \qquad A^*$$

$$A\mathbf{1} = A, \qquad (A+B)C = AC + BC, \qquad (AB)^* = B^*A^*, \quad \ldots$$

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- $*$-algebra supports polynomial functions
- C^*-algebra A $*$-algebra with a complete norm $\|\cdot\|$ obeying

$$\|AB\| \leqslant \|A\|\,\|B\|, \qquad \|A^*A\| = \|A\|^2$$

Equivalently, a norm-closed self-adjoint subalg. of $\mathcal{B}(\mathcal{H})$ for some Hilbert space $\mathcal{H}$.

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- $*$-algebra supports polynomial functions
- C^*-algebra supports continuous functions
- von Neumann algebra/W^*-algebra A $*$-subalgebra $\mathcal{M} \subset \mathcal{B}(\mathcal{H})$ such that

$$\mathcal{M} = \mathcal{M}'', \qquad \text{where} \quad \mathcal{N}' = \{A \in \mathcal{B}(\mathcal{H}) : [A, B] = 0 \ \forall B \in \mathcal{N}\}$$

Equivalently, a weakly closed unital $*$-subalg. of $\mathcal{B}(\mathcal{H})$.

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Additional structure permits varying strength of functional calculus.

- $*$-algebra supports polynomial functions
- C^*-algebra supports continuous functions
- von Neumann algebra/ W^*-algebra supports bounded functions

An AQFT on spacetime $\boldsymbol{M}$ comprises...

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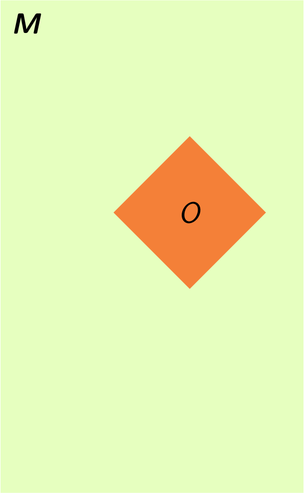

- a C^*-algebra $\mathcal{A}(\boldsymbol{M})$ for the whole spacetime
- to each [suitable] subregion $O \subset \boldsymbol{M}$ a C^*-subalgebra

$$\mathcal{A}(\boldsymbol{M}; O) \subset \mathcal{A}(\boldsymbol{M})$$

Interpretation

Elements of $\mathcal{A}(\boldsymbol{M}; O)$ with $A^* = A$ are observable in O

Prototypical elements are bounded continuous functions of quantum fields averaged within O.

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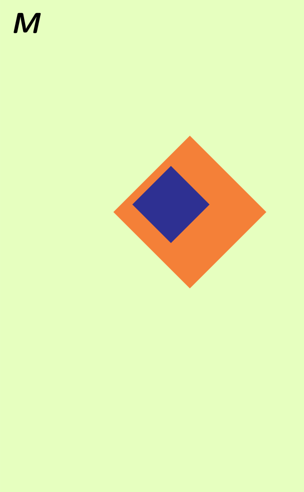

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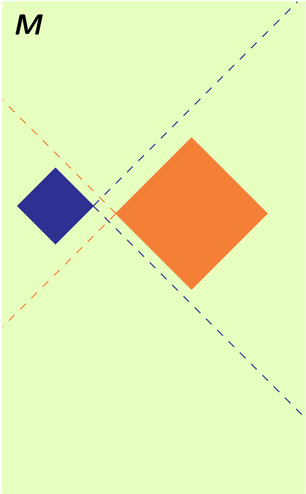

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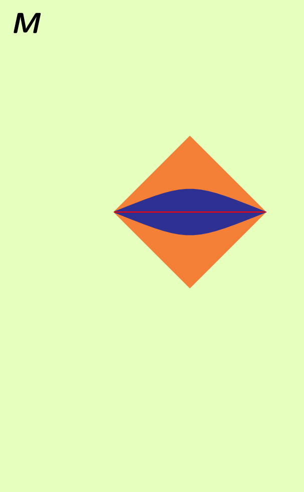

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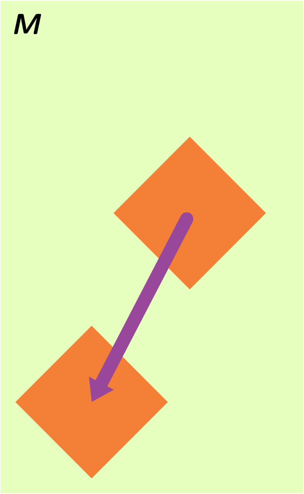

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- a group homomorphism $\alpha : \mathrm{Sym}(\boldsymbol{M}) \to \mathrm{Aut}(\mathcal{A}(\boldsymbol{M}))$ s.t.

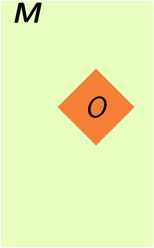
$$\alpha(\tau)(\mathcal{A}(\boldsymbol{M}; O)) = \mathcal{A}(\boldsymbol{M}; \tau(O))$$ Symmetry

When in Hilbert space...

Consider a representation π of $\mathcal{A}(\boldsymbol{M})$ on Hilbert space $\mathcal{H}$ (e.g., a GNS rep.).

For each spacetime region O,

- $\pi(\mathcal{A}(\boldsymbol{M}; O)) \subset \mathcal{B}(\mathcal{H})$ contains at most the same info as $\mathcal{A}(\boldsymbol{M}; O)$



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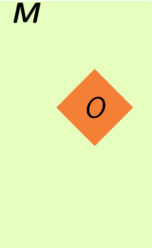

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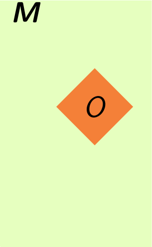

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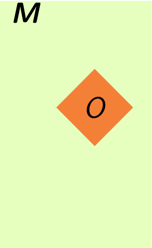

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- Representations that are 'not too different' give isomorphic von Neumann algebras
- The vN algebras can encapsulate the QFT under good physical conditions

Classification of von Neumann algebras Murray & von Neumann... ...Connes

- A vN algebra $\mathcal{M}$ is a factor if $\mathcal{M} \cap \mathcal{M}' = \mathbb{C}\mathbf{1}$

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  - I_n, $n \in \mathbb{N} \cup \{\infty\}$
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- Type I_n: $\mathcal{M} \cong \mathcal{B}(\mathcal{H})$ for Hilbert space $\mathcal{H}$ of dimension n
  - for $n < \infty$ this is matrix theory – every element has a finite trace
  - for $n = \infty$ trace class operators are weakly dense in $\mathcal{M}$ – semifinite trace

Trace = faithful, normal convex map $\tau : \mathcal{M}_+ \to [0, \infty]$, s.t. $\tau(A^*A) = \tau(AA^*)$ for all A.

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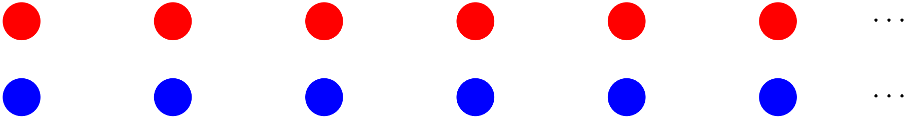
- Type II_∞ factors have a semifinite trace (but are not type I_∞)
- Type III factors do not admit any (semi)finite trace.
  $\tau(A) \in \{0, \infty\}$ for every element $A \in \mathcal{M}_+$.

Trace = faithful, normal convex map $\tau : \mathcal{M}_+ \to [0, \infty]$, s.t. $\tau(A^*A) = \tau(AA^*)$ for all A.

A Type II_1 example

Two spin chains, Hilbert space $\mathcal{H} = \bigotimes_{r=1}^{\infty}(\mathbb{C}^2 \otimes \mathbb{C}^2)$ (suitably defined)



$$\Psi = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \otimes \frac{|00\rangle + |11\rangle}{\sqrt{2}} \otimes \frac{|00\rangle + |11\rangle}{\sqrt{2}} \otimes \frac{|00\rangle + |11\rangle}{\sqrt{2}} \otimes \frac{|00\rangle + |11\rangle}{\sqrt{2}} \otimes \frac{|00\rangle + |11\rangle}{\sqrt{2}} \otimes \cdots$$

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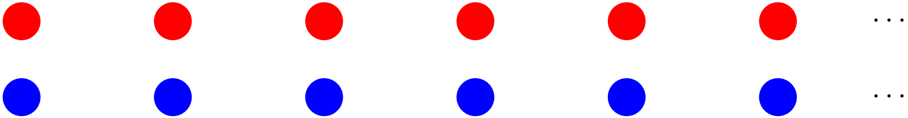

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Generate $\mathcal{M}$ from red chain observables $X = \bigotimes_{r=1}^{\infty} (A_r \otimes \mathbf{1})$ with $A_r = \mathbf{1}$ for suff. large r.

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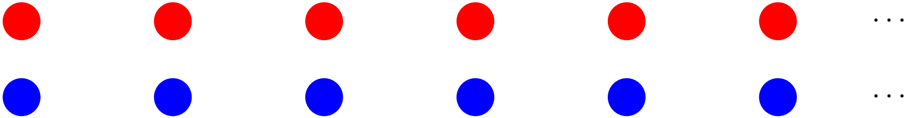

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Generate $\mathcal{M}$ from red chain observables $X = \bigotimes_{r=1}^{\infty} (A_r \otimes \mathbf{1})$ with $A_r = \mathbf{1}$ for suff. large r.

Then $X \mapsto \langle \Psi | X \Psi \rangle = \lim_{R\to\infty} \frac{1}{2^R} \prod_{r=1}^{R} \operatorname{Tr} A_r$ extends to a finite trace on $\mathcal{M}$

so $\mathcal{M}$ is of Type II_1.

$\dim \mathcal{M} = \infty$ + finite trace $\implies$ not Type I_n.

The universal type of local algebras in QFT

Theorem Fredenhagen 1985; Buchholz, D'Antoni, Fredenhagen 1987

For a W^*-AQFT $O \mapsto \mathcal{M}(O)$ obeying reasonable conditions (nuclearity, scaling limits) each $\mathcal{M}(O)$ is isomorphic to the $\underbrace{\text{unique hyperfinite}}_{\text{Haagerup, 1987}}$ factor of type III_1.

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Many consequences – distinguishing QFT from QM – including

Distinction between theories is due to 'relative position' of $\mathcal{M}(O)$'s in $\mathcal{B}(\mathcal{H})$.

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Infinite degeneracy No $\mathcal{M}(O)$ contains a nonzero finite rank projection

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Distinction between theories is due to 'relative position' of $\mathcal{M}(O)$'s in $\mathcal{B}(\mathcal{H})$.

Infinite degeneracy No $\mathcal{M}(O)$ contains a nonzero finite rank projection

Local states are impure No normal state restricts to any $\mathcal{M}(O)$ as a pure state.

See Yngvason *The role of Type* III *factors in quantum field theory* for more.

Recap

Different vN algebra types correspond roughly to different physical settings:

- Type I_n (qubits)
- Type I_∞ (quantum mechanics)
- Type II_1 (some quantum stat. mech. models)
- Type II_∞ (ditto)
- Type III_1 (quantum field theory) Buchholz, D'Antoni, Fredenhagen
- Type ??? (quantum gravity)

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Different vN algebra types correspond roughly to different physical settings:

- Type I_n (qubits) ← finite trace
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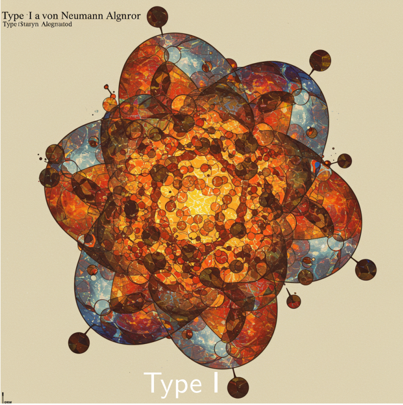
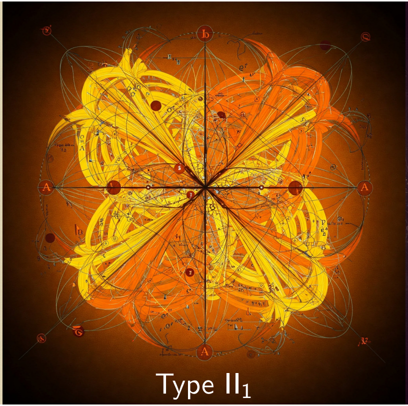
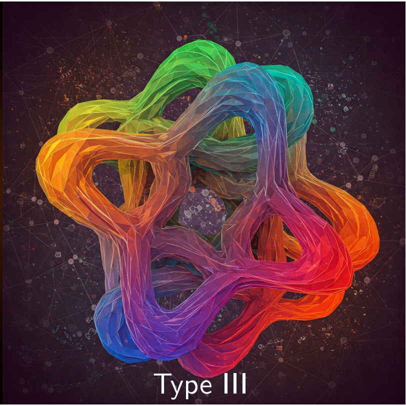
- Type III_1 (quantum field theory) ← no (semi)finite trace
- Type ??? (quantum gravity)

What the AI thinks... Image credits: Google Gemini







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Evidently this is impossible for $\mathcal{M}(O)$ in AQFT.

"I really apologise for that." – Klaus Fredenhagen

QFT vs Gravity

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What is the description of quantum observables allowing for dynamical gravity?

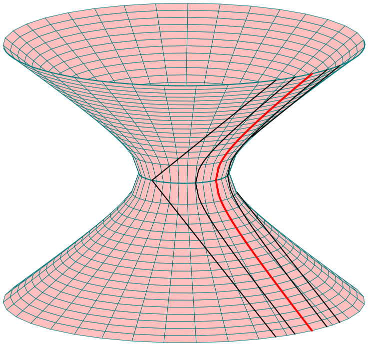
$\longrightarrow$ Investigate in de Sitter spacetime.

QFT in de Sitter spacetime

n-dimensional de Sitter spacetime is the hyperboloid

$$(X^0)^2 - \sum_{r=1}^{n} (X^r)^2 = -a^2$$

in $(n+1)$-dim Minkowski, with the induced metric.



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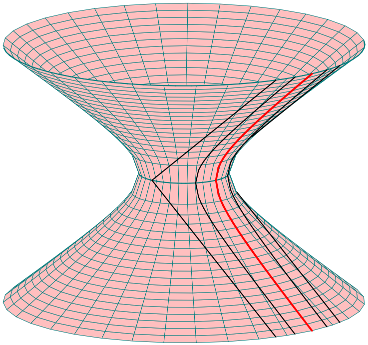

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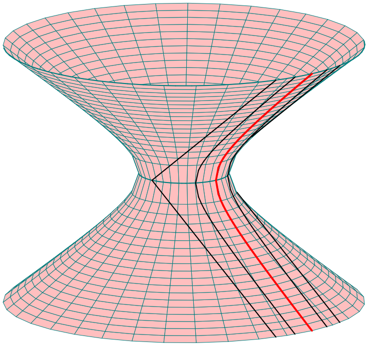

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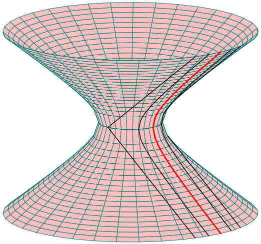
The boundary of the static patch is the observer's cosmological horizon.

Spacetime isometry subgroup $\mathbb{R} \times \mathrm{SO}(3)$ preserves the patch.

The CLPW model Chandrasekaran, Longo, Penington, Witten

Consider QFT representing 'weak quantum gravity' around a de Sitter background, in a de Sitter invariant vacuum state. Consider a timelike geodesic and its static patch.



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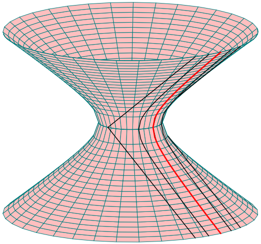

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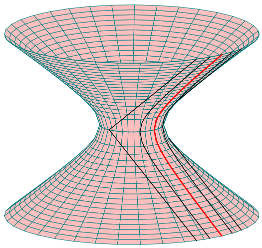

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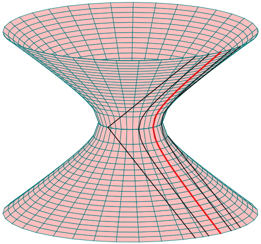

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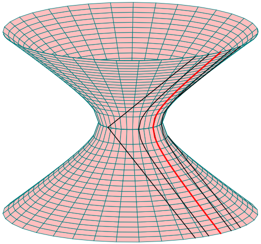

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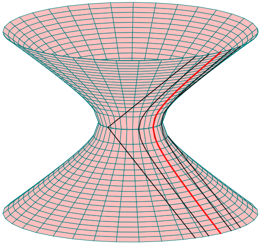

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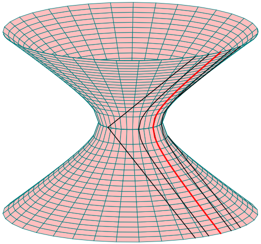
- The resulting algebra has type II_1.
  Uses Takesaki’s theory of vN crossed products.



The CLPW model (continued)

The II_1 factor has a finite trace (unique up to scale, fixed by $\tau(\mathbf{1}) = 1$).
The dS invariant Euclidean vacuum state corresponds to density matrix $\rho_{\mathrm{dS}} = \mathbf{1}$, so

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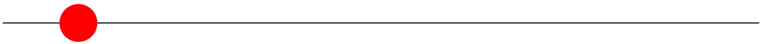
Q How fine-tuned is all this? Other spacetimes? Other symmetries? Why that clock?

(Quantum) reference frames

We cannot measure absolute positions or times – hardwired into microphysical laws.

No absolute position



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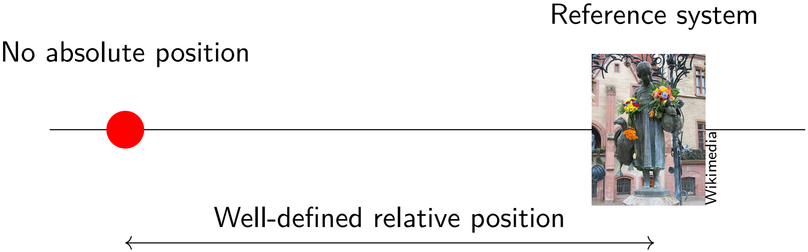

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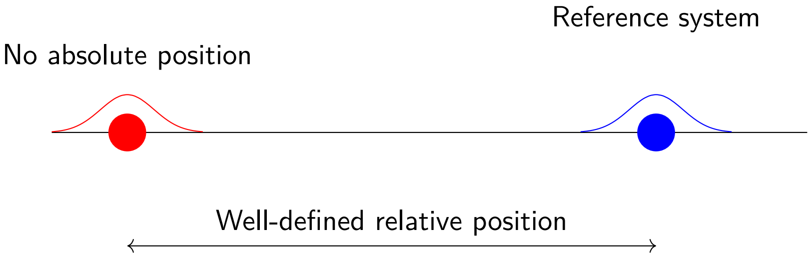

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There are several recent formalisations

- operational [our choice] Busch, Loveridge, Miyadera...
- perspective neutral Höhn, Giacomini, Bruckner...
- information-based Bartlett, Spekkens...

Operational QRFs

Given a quantum system $\mathcal{H}_S$ with symmetry group G represented unitarily by U_S, a QRF is another quantum system $\mathcal{H}_R$, with G represented unitarily by U_R, and a G-covariant observable P, technically, a POVM on value space Σ acted on by G.

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  E.g., relative position of two quantum particles on the line.

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Observation The CLPW clock defines a QRF for time translations $G = \mathbb{R}$, $\Sigma = \mathbb{R}$ $\mathcal{H}_R = L^2(\mathbb{R}^+, dE)$, $H_R = E$, $U_R(t) = e^{iH_R t}$,

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P is a POVM, but not a PVM.

Consistent with Pauli's theorem on nonexistence of time observables if H is semibounded.

Understanding type change CJF + Janssen, Loveridge, Rejzner, Waldron (2025)

The CLPW observer isn't coupled to the system – what (if anything) is it observing?

Our approach: start from a description of measurement theory in QFT CJF & Verch, CMP 2020

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- Invariant elements of the joint QFT-QRF algebra are physical observables.
- Investigate in generality what properties are required for change of algebra type.

Techniques include Mackey theory, and dilation of nonsharp QRFs to sharp ones.

Sample result

Consider situations with a spacetime isometry group $G = \mathbb{R} \times H$ with compact H so that the $\mathbb{R}$ action on the (type III) QFT vN algebra is the modular flow for a β-KMS state (includes CLPW).

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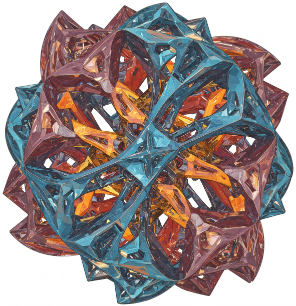
Summary: CLPW is generalised within a broader, operationally motivated, setting, elucidating the roles of good thermodynamic behaviour for the QRF and gravity.

For more detail, see Thursday's talk.

Summary

- Explanation of the role of algebras in QFT
- Explanation of the definition and significance of vN algebra type
- Universal type of local algebras in QFT
- The CLPW model and change of type
- CLPW generalised and more operationally understood.



Hyperfinite type III_1 factor according to Google Gemini