Physics of θ-Vacuum and Inevitability of Axions

in Standard Model and Gravity

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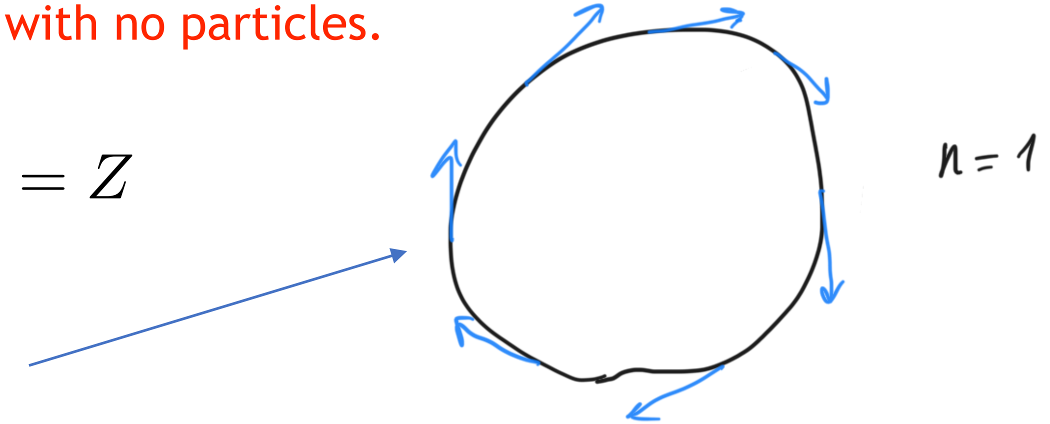
We start by discussing the beautiful physics of the QCD vacuum (our vacuum).

QCD is a SU(N) gauge theory with N=3 colors. The mediators are 8 gluons, which carry both color and anti-color. They mediate interactions among themselves as well as among quarks.

At distances larger than the QCD length-scale, $1/\Lambda$, QCD confines and the right degrees of freedom are the ``composites": glueballs, mesons, baryons.

The vacuum of QCD, as usual, corresponds to a state with no particles.
However, this vacuum is topologically non-trivial:

$$\pi_3(SU(N)) = Z$$



Classically, it is described by configurations with integer winding number n:

$$|n\rangle$$

Locally, these configurations look pure-gauge (no field strengths) but cannot be removed by vacuum field deformations.

However, in quantum theory there are transitions generated by instantons, that change n.

Instantons: Euclidean configurations describing a process that change winding number n

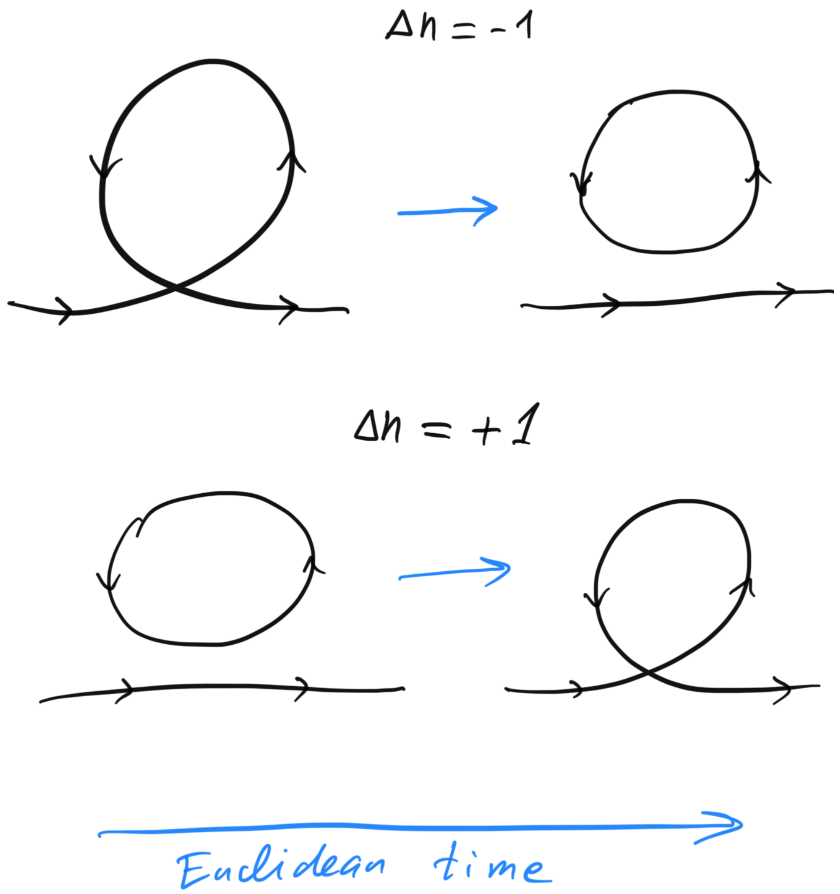
(Belavin, Polyakov, Schwarz and Tyupkin)

Their classical Euclidean action is scale invariant, i.e., independent of size. E.g., for $\Delta n = 1$

$$S_E = \frac{8\pi^2}{g^2}$$

Correspondingly, the transition rate is:

$$\Gamma \propto \mathrm{e}^{-S_E} = \mathrm{e}^{-\frac{8\pi^2}{g^2}}$$



Thus, the topology of the QCD vacuum

$$\pi_3(SU(N)) = Z$$

Instanton transitions with the rate

$$\Gamma \sim e^{-\frac{8\pi^2}{g^2}}$$

Thus, the states $|n\rangle$ are not vacua.

Instead, the right vacua are the θ -vacua: (Callan, Dashen, Gross '76; Jackiw, Rebbi '76)

$$|\theta\rangle = \sum_n e^{i\theta n}|n\rangle$$

The choice of the vacuum is described by the θ -term:

$$\mathcal{L}_\theta = \theta \frac{g^2}{16\pi^2} G\tilde{G}$$

Notice: it is a total derivative and thus a boundary term $G\tilde{G} = \epsilon^{\alpha\beta\mu\nu}\partial_\alpha C_{\beta\mu\nu}$

This leads to the strong-CP problem. In the standard discussion (which ignores gravity) the strong-CP puzzle is formulated as the following naturalness problem.

QCD has a continuum of vacua conventionally labelled by the CP-violating vacuum angle (Callan, Dashen, Gross '76; Jackiw, Rebbi '76).

$$|\theta\rangle = \sum_n e^{i\theta n}|n\rangle$$

These vacua belong to different superselection sectors. In fact, in theory with massive quarks, the physically measurable parameter is the quantity

$$\bar{\theta} \equiv \theta + \mathrm{arg.detM_q}$$

which induces the electric dipole moment of neutron (EDMN) (Baluni, '79, Crewther et al '79, erratum '80). The comparison of the resulting theoretical value with the current experimental limit (Baker et al '06),

$$|d_n| < 2.9 \times 10^{-26} e\,\mathrm{cm}$$

gives the bound:

$$|\bar{\theta}| \lesssim 10^{-9}$$

Notice that there exists an additional contribution to EDMN, coming from the breaking of CP-symmetry by the weak interaction (Ellis, Gaillard and Nanopoulos '76; Shabalin '79; Ellis and Gaillard '79).
However, this correction is too small for affecting the current bound.

$$|\bar{\theta}| \lesssim 10^{-9}$$

Thus, the observations indicate that we live in a sector with a minuscule or zero $\bar{\theta}$

This is puzzling.

Peccei-Quinn solution: Removing $\bar{\theta}$ by anomalous chiral symmetry $U(1)_{PQ}$

$$\psi \to e^{-i\frac{1}{2}\alpha\gamma_5}\psi$$

Anomalous non-conservation of the current

$$\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = G\tilde{G}$$

and Lagragian shifts

$$\delta L = \alpha\, G\tilde{G}$$

Thus, $\bar{\theta}$ -term can be shifted away

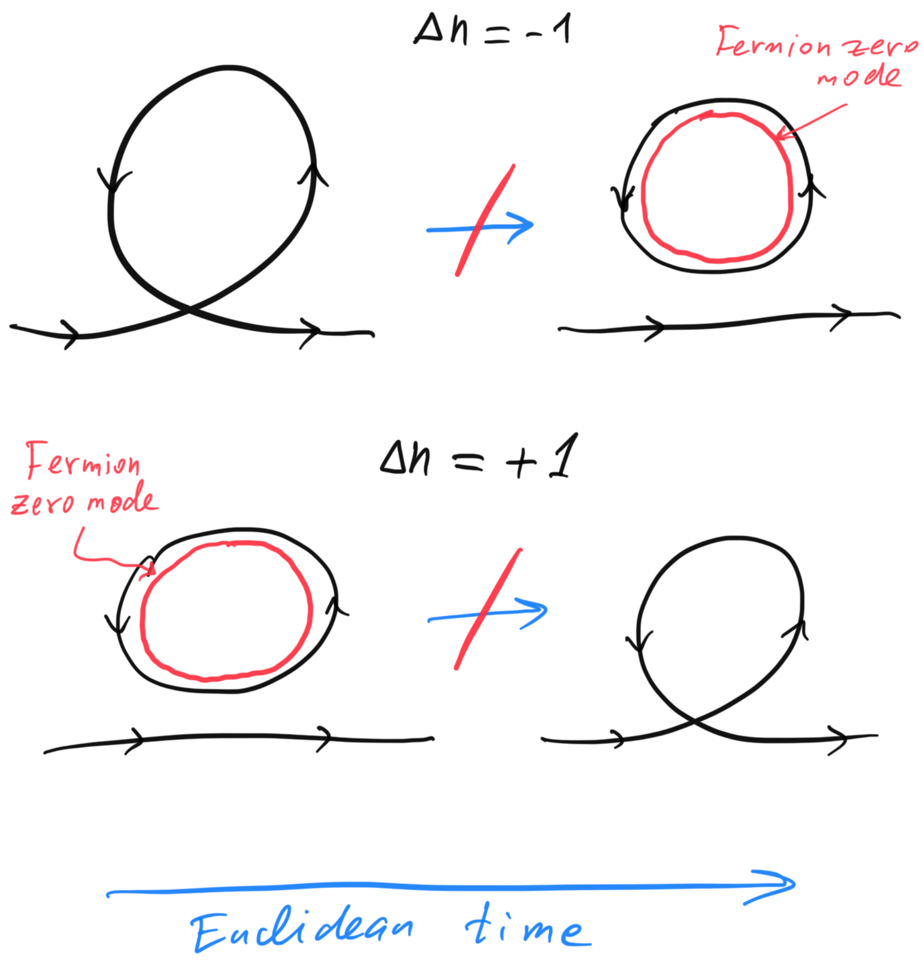
$$\bar{\theta} \to \bar{\theta} + \alpha$$

Thus, $\bar{\theta}$ is unphysical!

This matches the knowledge from index theorem: According to index theorem each spin-1/2 quark deposits a chiral zero mode in the instanton background. That is, a zero action mode that solves the Dirac equation in instanton background:

$$\gamma^\mu D_\mu^{(inst)} \psi = 0$$

this suppresses the transitions, since the vacuum cannot support the chiral charge.



However, in nature there are no massless quarks. So the $U(1)_{PQ}$ - symmetry must be spontaneously broken by the VEV of a complex scalar field

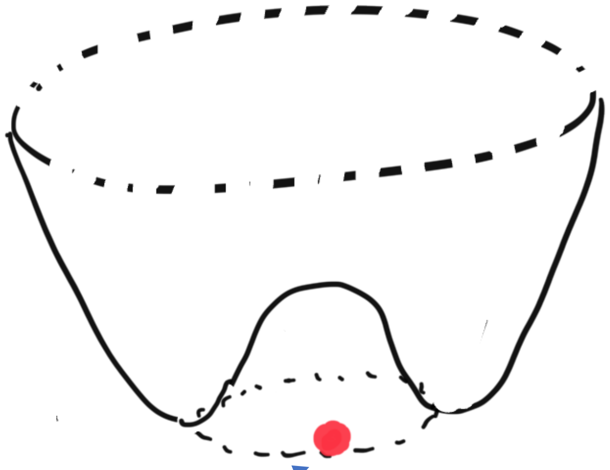
$$\Phi = \rho(x) \mathrm{e}^{i \frac{a(x)}{f_a}}$$

which under $U(1)_{PQ}$ transforms as

$$\Phi \to \mathrm{e}^{i\alpha} \Phi$$

With the Mexican-hat (Goldstone) potential

$$V(\Phi) = \lambda^2 (\Phi^\dagger \Phi - f_a^2)^2$$



Nambu-Goldstone phase is the axion (Weinberg '78; Wilczek '78).

The fermions (for simplicity take one) get mass from

$$L_\psi \;=\; i\bar{\psi}\gamma^\mu D_\mu\psi - \lambda\Phi\,\bar{\psi}\psi$$

Taking into account anomaly, the effective theory is:

$$L_a = \frac{1}{2}(\partial_\mu a)^2 \;-\; \left(\frac{a}{f_a} - \bar{\theta}\right) G\tilde{G}$$

Thus, $\bar{\theta}_{eff} = \dfrac{a}{f_a} - \bar{\theta}$ became a dynamical variable.

The global minimum is at (Vafa, Witten '84)

$$\bar{\theta}_{eff} = 0$$

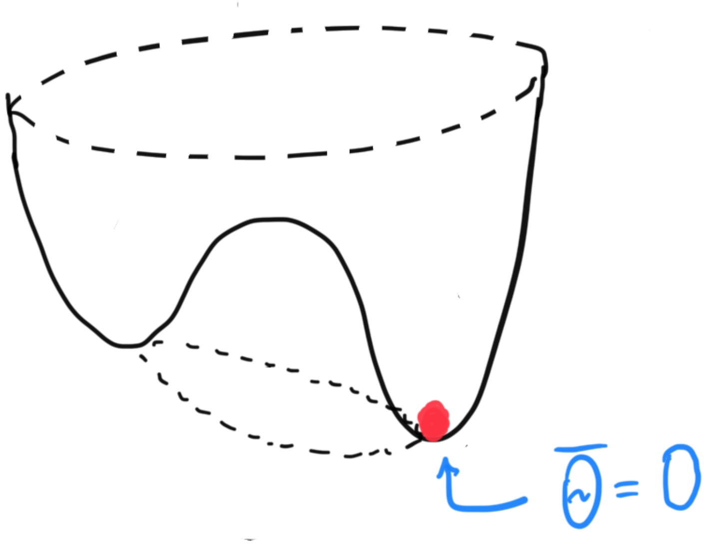
And the axion relaxes there dynamically.

Indeed, the non-perturbative effects (instantons) generate the potential for the axion. In dilute instanton gas approximation,

$$V(a) = -\Lambda^4 \cos\left(\frac{a}{f_a} - \bar{\theta}\right)$$

which indeed forces

$$\bar{\theta}_{eff} = 0$$



Anomalous symmetry

Fermionic zero mode on instanton

$$\bar{\theta} = 0$$

Dynamical relaxation by axion

Axion skeptics:

Is strong-CP even a problem? Let me simply tune $\theta = 0$.

OK, let us accept the problem. But in axion solution you rely on an (approximate) global symmetry which must be explicitly broken, exclusively, by the QCD anomaly.

What kind of a deal is this?

Why should this symmetry be respected by other forces of nature (e.g., by gravity) or simply even be there to start with?

The PQ solution is unstable with respect to an arbitrary continuous deformation of the theory that breaks the PQ symmetry explicitly. e.g.,

$$(\Phi^\dagger)^n(\Phi)^m \to \Delta V(a) \propto \cos\left((m-n)\frac{a}{f_a} - (?)\right)$$

These competing terms induce $\bar{\theta}_{eff} \neq 0$

This is the axion quality problem.

Other questions: What about beyond the dilute instanton gas approximation?

What if there exist local minima with $\bar{\theta}_{eff} \neq 0$?

In order to address these questions and understand how deep and profound the story is, let us start by showing that QCD already contains an axion, albeit of poor quality.

This axion is the η'-meson.

Consider QCD with massless (or light) quarks. It exhibits axial symmetry $U(1)_A$

$$\psi_i \to \mathrm{e}^{-i\frac{1}{2}\alpha\gamma_5}\psi_i \quad i = 1, 2, ...N_f \ .$$

In real world $N_f = 3$ with 3 light quarks $\psi_i = u, d, s$

However, let us keep the discussion generic with all quarks massless.

Now, the axial symmetry is spontaneously broken by the quark condensate

$$\langle \bar{\psi}_i \psi_i \rangle = \Lambda^3$$

But, the corresponding massless (light) Nambu-Goldstone boson was nowhere to be found.

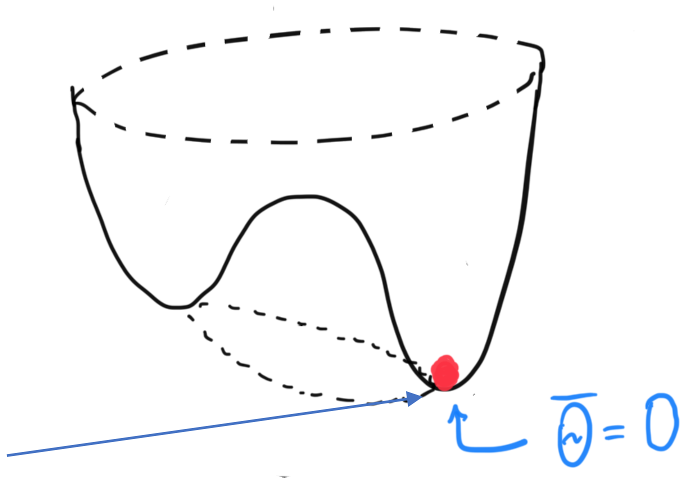
This is the famous axial $U(1)_A$ -problem.

This problem was solved by 't Hooft, who understood that the would-be Goldstone boson was getting mass from the $U(1)_A$-anomaly through the instantons.

The resulting potential for η' (again in dilute instanton gas approximation) is:

$$V(\eta') = -\Lambda^4 \cos\left(\frac{\eta'}{f_\eta} - \bar{\theta}\right)$$

which gives mass to η'



Interestingly, 't Hooft did not point out that simultaneously with generating this mass, the η' solves the strong-CP problem:

the vacuum relaxes to $\bar{\theta}_{eff} = 0$

In other words, in QCD with massless quarks η' is a full-fledged axion (G.D., '05, G.D., Jackiw, Pi '06).

Unfortunately (or fortunately?), in the real world $U(1)_A$ is also explicitly broken by quark masses. Therefore, η' cannot enforce $\bar{\theta}_{eff} = 0$

Thus, η' is a poor-quality axion!

The lessons we learn from η' are extremely important:

The mass of η' represents an experimental proof of the existence of the θ-vacuum.

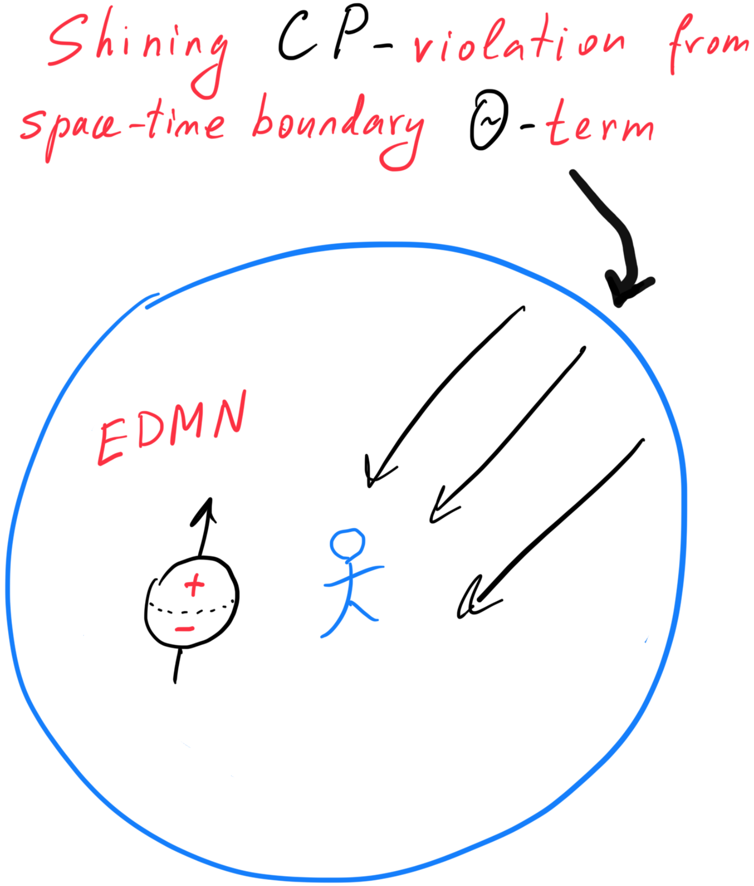
η' illustrates the reality of the axion dynamics:
η' would be an exact quality axion, if at least one quark would be massless.

Since quarks are massive, there must exist a good-quality axion.

In order to gain a wider perspective, we shall switch to the description based on most fundamental aspects of QFT.

There exist a powerful QFT language for describing the story.

Let us ask the following question. QCD is a theory with a mass gap. How, in such a theory, can the boundary term be locally-observable?



The answer is that there exist a long-range correlator that brings the message from the boundary. This is the so-called topological susceptibility of the vacuum (TSV):

$$\langle G\tilde{G}, G\tilde{G}\rangle_{p\to 0} \equiv \lim_{p\to 0} \int d^4x\, e^{ipx}\, \langle T[G\tilde{G}(x), G\tilde{G}(0)]\rangle \;=\; \mathrm{const} \neq 0,$$

The job of the instantons boils down to creating the above correlator. Any other way of describing it, e.g., via the glueball tower (Witten '80, Veneziano '80) does the same job.

This fact liberates us from the need of knowing the details of sub-structure and endows us with the full power of EFT.

In particular, this language allows to understand the solution of strong-CP by axion (or by η' in case of massless quark) as well as the generation of their masses, without relying on the accuracy of the instanton calculus.

θ-vacuum = non-zero topological susceptibility of the vacuum

$$\langle G\tilde{G}, G\tilde{G}\rangle_{p\to 0} \;=\; \mathrm{const} \neq 0,$$

We recall that

$$G\tilde{G} = \epsilon^{\alpha\beta\mu\nu}\partial_\alpha C_{\beta\mu\nu}$$

where the Chern-Simons 3-form:

$$C_{\mu\nu\alpha} \equiv \mathrm{tr}(A_{[\mu}\partial_\nu A_{\alpha]} + \frac{2}{3}A_{[\mu}A_\nu A_{\alpha]})$$

Thus, the non-zero topological susceptibility = massless pole in Kallen-Lehmann spectral representation:

$$\langle C, C\rangle = \frac{1}{p^2} + \sum_{m\neq 0}\frac{\rho(m^2)}{p^2 - m^2}$$

Thus, in order to solve strong-CP, the massless pole in the Chern-Simons correlator

$$\langle C, C \rangle = \frac{1}{p^2} + \sum_{m \neq 0} \frac{\rho(m^2)}{p^2 - m^2}$$

must be eliminated.

By gauge invariance, this requires the existence of a pseudo-scalar that gets its mass from topological susceptibility of the vacuum.

This pseudo-scalar is the axion

$$\mathcal{L} = \frac{g^2}{16\pi^2} \frac{a}{f_a} G\tilde{G}$$

With axion (of exact quality) the massless pole in the topological susceptibility is removed and the correlator vanishes

$$\mathrm{FT}\langle G\tilde{G}(x)\ G\tilde{G}(0)\rangle_{p\to 0} \propto \left.\frac{p^2}{p^2 - m^2}\right|_{p\to 0} = 0,$$

mass of the axion

(This can be viewed as 3-form Higgs effect: 0+1=1 G.D.'05)

In Peccei-Quinn scenario, axion is an elementary pseudo-scalar.

However, in case of a massless quark:

$$axion = \eta' - meson$$

The axion quality problem justifies an alternative (exact quality) formulation of axion:

The Gauge Axion (G.D., '05).

In this formulation axion is introduced as intrinsic part of QCD gauge redundancy, without need of any global symmetry.

Under the QCD gauge redundancy gluons transform as:

$$A_\mu \rightarrow U(x) A_\mu U^\dagger(x) + U \partial_\mu U^\dagger \quad \text{where} \quad U(x) \equiv e^{-i\omega(x)^b T^b}$$

The Chern-Simons 3-form shift as: $C_{\mu\nu\beta} \rightarrow C_{\mu\nu\beta} + \partial_{[\mu}\Omega_{\nu\beta]}$

The axion emerges as a 2-form (Stuckelberg) field of this redundancy:

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \frac{1}{f_a}\Omega_{\mu\nu},$$

Thus, the axion appears as organic part of QCD, which enters the Lagrangian through the following (unique) gauge invariant:

$$C_{\mu\nu\beta} - f_a \partial_{[\mu} B_{\nu\beta]}$$

This guarantees the exact solution of strong-CP!

The lowest order term:

$$L = \frac{1}{f_a^2}(C - f_a dB)^2$$

This removes the massless pole in the 3-form propagator and makes the topological susceptibility of the vacuum zero, thereby eliminating theta-vacua.

This is a 3-form version of the Higgs effect: 0 + 1 = 1

By the power of gauge redundancy, the gauge axion scenario predicts

$$\bar{\theta} = 0$$

to all orders in operator expansion (G.D., '05, '22; Sakhelashvili '21)

Any experimental indication of EDMN will be a signal of new physics beyond the Standard Model.

Gravity

Gravity sheds the whole new light at θ - vacua

(G.D., Gomez, Zell ' 18, G.D., '22, G.D., Kobakhidze, Sakhelashvili ' 24)

The reason: the valid vacuum of gravity is Minkowski.

(Or a weaker version: If there exist other vacua, they must not be obtainable from Minkowski by continuous deformations of parameters.)

This is justified by number of considerations:

1) S-matrix formulation of gravity (G.D., Comez'13, ..., G.D. '20)
2) Inconsistencies with de Sitter (G.D., Gomez '13,'14, + Zell '17) and Minkowski to AdS transitions (G.D., '11)
3) BRTS quantization of gravity (Berezhiani, G.D., Sakhelashvili '24)

We shall refer to this as Minkowski criterion.

Minkowski criterion is incompatible with the existence of θ- vacua in any sector of the theory.

As we already explained in QCD, the vacuum energy is a periodic function of theta, with global minimum at (Vafa, Witten '84)

$$\theta = 0$$

E.g., in dilute instanton gass approximation:

$$E_{vac}(\theta) \propto -\cos\theta$$

Since vacua are not degenerate, only one of them can be Minkowski (even at the expense of fine tuning). Correspondingly only one of them can satisfy the Minkowski criterion.

We thus arrive to the conclusion that gravity demands theta to be unphysical.

In summary: gravity necessitates the existence of exact quality axion per each gauge sector with topologically non-trivial vacuum.

θ -vacuum of gravity and evidence for supersymmetry

(G.D., Kobakhidze and Sakhelashvili '24)

We now apply the same reasoning to the θ -vacuum of GR, which originates from Eguchi-Hanson instantons with the following metric:

$$ds^2 = \left(1 - \frac{a^4}{r^4}\right)^{-1} dr^2 + r^2 \left(\sigma_x^2 + \sigma_y^2\right) + r^2 \left(1 - \frac{a^4}{r^4}\right) \sigma_z^2,$$

There are two topological invariants. The Euler characteristics:

$$\chi = \frac{1}{8\pi^2} \int d^4x \sqrt{g} \left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\alpha\beta}^2\right) + \text{bound.terms} = 2.$$

and gravitational Pontryagin index:

$$\tau = -\frac{1}{24\pi^2} \int d^4x \, R\tilde{R} = 1. \qquad \tilde{R}R = \epsilon^{\mu\nu\alpha\beta} R^{\kappa}_{\gamma\mu\nu} R^{\gamma}_{\kappa\alpha\beta}$$

Notice that in pure GR, the EH instanton has zero action

$$S_{EH} = 0\,.$$

The GB-term has to be added to the Euclidean action

$$\Delta S = c\,\frac{\chi}{2}$$

The transition rate then becomes

$$\Gamma \sim e^{-c}$$

The validity of EFT demands: $c \gg 1$

The parameter c encodes information about the cutoff scale:

$$c \sim \left(\frac{M_{pl}}{\Lambda_{gr}}\right)^2$$

EH instanton is a fully trustable configuration mediating vacuum transitions and generating the topological susceptibility of the gravitational vacuum

$$\mathrm{FT}\langle \tilde{R}R(x)\ \tilde{R}R(0)\rangle_{p\to 0} \neq 0$$

This creates a gravitational analog of theta-vacua

$$S = \frac{\theta}{24\pi^2}\int d^4x R\tilde{R}$$

The energy of the ground state depends on theta. Thus, starting from a ``naive" semi-classical Minkowski vacuum, we obtained a landscape of theta-vacua.

This is incompatible with the Minkowski criterion. Thus, gravity requires a mechanism that eliminates theta-vacua.

The physicality of the gravitational theta-term and the necessity to make it unphysical constitutes the gravity CP-problem.

Elimination of the gravitational theta-vacuum requires a fermion with chiral gravitational anomaly (regardless of elementary axion that couples via such a fermion) Delbourgo, Salam '72

$$\partial_\mu j_5^\mu \propto R\tilde{R}$$

However, this role cannot be assumed by a spin-1/2 fermion. The general index theorem (Atiyah '75) states the absence of zero modes on the Eguchi-Hanson background for a spin-1/2 fermion. This is confirmed by explicit computation of index in Eguchi-Hanson background which shows (Eguchi, Hanson, '78)

$$Q_5(t=\infty) - Q_5(t=-\infty) = 0.$$

The fermion of the lowest spin that is capable of eliminating the gravitational theta-vacua is a chiral fermion with spin=3/2. Indeed, the index theorem shows a nontrivial index of such a fermion in the Eguchi-Hanson background (Eguchi,Hanson '78). The chirality of the massless Rarita-Schwinger field in the above background is broken by two units

$$I_{3/2} = -2$$

As it is well known, a theory that includes a fermion spin=3/2 coupled to gravity incorporates local supersymmetry, supergravity (Freedman, Ferrara, Nieuwenhuizen '76)

The anomalous U(1)-symmetry that renders the gravitational theta-vacua unphysical is the R-symmetry under which gravitino transforms as,

$$\psi_\mu \to e^{i\alpha\gamma_5}\psi_\mu$$

The corresponding shift is

$$\theta \to \theta + 2\alpha$$

which shows that theta is unphysical. However, as in case of QCD, by consistency, there must exist a pseudoscalar degree of freedom that eliminates the massive pole in the topological susceptibility of gravitational vacuum.

$$\langle C, C\rangle = \frac{1}{p^2} + \sum_{m\neq 0}\frac{\rho(m^2)}{p^2 - m^2}$$

where

$$C_{\mu\nu\alpha} \equiv \mathrm{tr}(\Gamma_{[\mu}\partial_\nu\Gamma_{\alpha]} + \frac{2}{3}\Gamma_{[\mu}\Gamma_\nu\Gamma_{\alpha]})\,,$$

is the gravitational Chern-Simons 3-form.

And indeed, we can identify such a degree of freedom.

It has been shown (Hawking '78, Konishi ' 88mb, Konishi '89) that EH instantons form a bilinear condensate of gravitino

$$\langle \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu} \rangle \neq 0$$

Then, the phase of this condensate is the right candidate for a composite R-axion. We shall denote it by η_R

Indeed, the condensate must be accompanied by the appearance of a composite multiplet, which consists of a pseudoscalar, a dilaton and a dilatino. The above is in agreement with the index. The condensate includes two fermions and violates the R-charge by two units.

The existence of zero modes in the Eguchi-Hanson background generates a corresponding 't Hooft vertex for gravitino,

$$\frac{W_{3/2}^{*}}{M_{pl}^{2}} \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu}$$

which gives mass to η_R -meson. Situation is fully analogous to QCD η'

The operator

$$\frac{W_{3/2}^*}{M_{pl}^2} \bar{\psi}^\mu \sigma_{\mu\nu} \psi^\nu$$

per se does not break supersymmetry. However, the dynamically generated superpotential would lower the vacuum to AdS via the negative cosmological term:

$$-3|W_{3/2}|^2/M_{pl}^2$$

This contribution, if not balanced, would violate the criterion of Minkowski.

In order to avoid this, supersymmetry must be broken (super-Higgsed).

Thus, we arrive to conclusion that topological structure of GR vacuum demands existence of spontaneously broken supersymmetry:

supergravity in superHiggs phase.

Most elegant scenario would be in which the composite gravitino multiplet breaks SUSY with no external help (G.D., Kobakhidze, Sakhelashvili, '24).

Alternatively, we must employ one of the conventional mechanisms of SUSY breaking.

In order to break supersymmetry and to uplift the vacuum of the theory to Minkowski, it is sufficient to add a single chiral superfield X, that enters linearly in the superpotential,

$$W = \hat{X}\Lambda_X^2 + W_{3/2}$$

We thus effectively end up with the Polonyi model, with the difference that the constant term in the superpotential is dynamically generated by EH instantons an in addition there is a composite gravitino multiplet. For

$$W_{3/2} \ll M_P^3$$

the Minkowski vacuum is achieved similarly to Polonyi case. SUSY is broken by the F-term of X, and the R-axion is coming mostly from the phase of X with small admixture from η_R.

This is very similar to Peccei-Quiinn in QCD, where hidden axion has a small admixture from η'.

Electroweak θ -vacuum and evidence for weak axion η_w

(G.D., Kobakhidze and Sakhelashvili, '24)

We now move to the electroweak vacuum. It is well known that this vacuum has similar topological structure as QCD. However, since SU(2) gauge theory is in the Higgs phase, the instantons are constrained. This however does not preclude the topological structure. It is well established that the gauge sector of electroweak theory exhibits the topological susceptibility of the vacuum, Anselm, Johansen '93,'94

$$\mathrm{FT}\langle W\tilde{W}(x)\; W\tilde{W}(0)\rangle_{p\to 0} \;\sim\; e^{-\frac{2\pi}{\alpha_w}}$$

Correspondingly the weak-theta term

$$\theta_w \frac{g_w^2}{16\pi^2} W\tilde{W}$$

is physical. This creates theta-vacuum with all usual consequences. In particular, the vacuum energy depends on theta.

Gravity demands the existence of a boson that makes theta unphysical.

This is achieved either by a speudo-Goldstone transforming under anomalous B+L symmetry

$$\frac{\eta_w}{f_w} \to \frac{\eta_w}{f_w} - \alpha$$

and shifting theta

$$\theta_w \to \theta_w + \alpha$$

Or by a gauge axion which transforms under SU(2) gauge symmetry

$$B_{\mu\nu} \to B_{\mu\nu} + \frac{1}{f_w}\Omega_{\mu\nu}$$

in accordance with SU(2) Chern-Simons

$$C^{(w)}_{\mu\nu\beta} \to C^{(w)}_{\mu\nu\beta} + \partial_{[\mu}\Omega_{\nu\beta]}$$ where $$\Omega_{\mu\nu} = \mathrm{tr} W_{[\mu}\partial_{\nu]}\omega$$

However, the story splits in two according to the quality of the B+L symmetry

We shall say that B+L symmetry is of poor quality if it is explicitly broken by sources beyond the electroweak anomaly. In the opposite case, B+L is of good quality.

Poor quality B+L

Let us assume that B+L symmetry of the standard model which acts on lepton and quark doublets as

$$l \rightarrow e^{i\alpha} l, \quad q \rightarrow e^{i\frac{\alpha}{3}} q$$

is of poor quality. For example, is explicitly broken by gauge-invariant operators of the type

$$qqql$$

In such a case, the electroweak sector has theta-vacuum and gravity requires η_w in form of a gauge axion for eliminating it. An ordinary Peccei-Quinn type axion (probably) will not work due to the poor quality of B+L.

Good quality B+L

In such a case, theta can be eliminated by B+L transformation,

$$l \to e^{i\alpha} l, \quad q \to e^{i\frac{\alpha}{3}} q$$

under which it shifts:

$$\theta_w \to \theta_w + \alpha$$

Correspondingly, the topological susceptibility of the vacuum vanishes

$$\mathrm{FT}\langle W\tilde{W}(x)\, W\tilde{W}(0)\rangle_{p\to 0} \,=\, 0$$

and there are no theta-vacua. Gravity should be satisfied.

But, where is a particle?

The presence of a pseudoscalar is necessary for eliminating the massless pole in the spectral representation of Chern-Simons correlator:

$$\langle C^{(w)}, C^{(w)} \rangle = \frac{1}{p^2} + \sum_{m \neq 0} \frac{\rho(m^2)}{p^2 - m^2}$$

What kills this?

Important point is that at finite Planck mass, poles cannot decouple. Thus, the only way compatible with gauge invariance for eliminating the massless pole is by making it massive. This requires a propagating pseudo-scalar degree of freedom, η_w-meson.

The possible origins of this particle are:

1) External, likely in form of a gauge axion;

2) Internal, from the fermion condensate triggered by instantons;

3) Both.

Calculation in the toy version of the Standard Model with one lepton and single color quark doublet, shows the presence of the B+L-violating condensate of 't Hooft determinant,

$$\langle ql \rangle \neq 0$$

the phase of this condensate has all the right properties for contributing into η_w, however, some open questions remain.

Whatever is its origin, one thing is certain: Standard Model must be accompanied by η_w -particle by consistency of Minkowski criterion.

Some remarks on phenomenology/cosmology:

The gauge axion UV-completes in gravity. In this setup, the axion experiments (Madmax, IAXO, ADMX, HAYSTAC, CULTASK, ALPS-II, ...) represent probes of the scale of QEFT cutoff of gravity.

Prediction of the gauge axion: $\bar{\theta} = 0$

Any observation of EDMN will be a signature of BSM physics.

The QCD axion scale (and therefore the mass $m_a = \Lambda^2/f_a$) is unknown.
For gauge axion version of QCD axion, we know it is the scale of quantum gravity, but the value is unknown.

For gravitational axion, we know the scale very well:

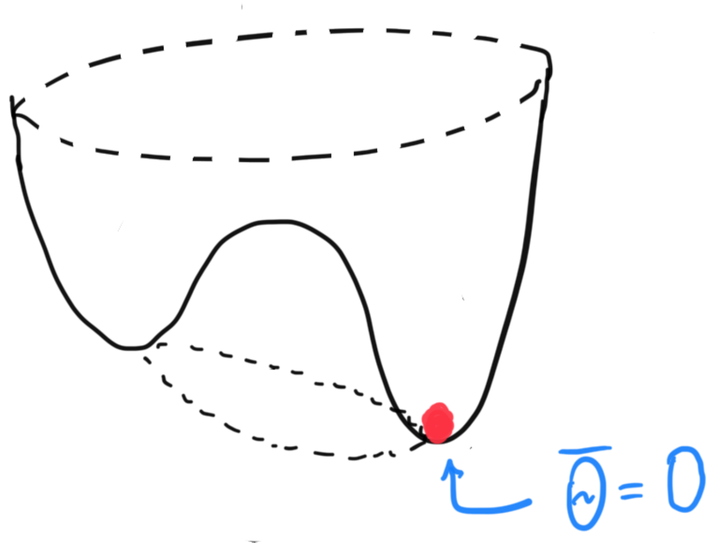
$$f_a = M_P$$

but not the mass. However, it is directly linked with the scale of SUSY-breaking.

Axion (both QCD and gravitational R-axion) is a great candidate for dark matter, since the energy density of its coherent oscillations redshift as matter.

Notice that the ``standard'' cosmological constraints on the scale of QCD axion $f_a < 10^{12} GeV$

(Preskill, Wise and F. Wilczek '83; Abbott and P. Sikivie, '83; Dine and W. Fischler '83) are not robust:



The axion likely has learned about its minimum way before the QCD phase transition. Within the inflationary cosmology (but not only) this is generic. This liberates the QCD axion scale from cosmological constraints.

(G.D., '95; for recent review of this mechanism, see Koutsangelas '22, and references therein)

Domain walls and strings for gauge axion (G.D., Komisel, Stuhlfauth '25)

Summary and Outlook:

1) η'-meson of QCD is axion, albeit of poor-quality. Its mass represent the experimental proof of the θ-vacua.

2) There exists a pure gauge formulation of axion (without need of global symmetry) which is of exact quality.

Gravity brings a whole new dimension to axion physics. Applying the Minkowski criterion to the topological structure of vacuum, we learned the following:

1) Any gauge sector with topological structure of vacuum must incorporate axion of exact quality

2) For QCD this necessitates the presence of axion by consistency and motivated the gauge axion which does not rely on gauge symmetry and has exact quality to all orders in operator expansion.

3) Applied to θ -vacuum of GR generated by Eguchi-Hanson instantons, the presence of a spin-3/2 particle is inevitable. This justifies supersymmetry. Moreover, SUSY must be in the super-Higgs phase.

4) Analogously, the topological structure of the electroweak vacuum, demands the existence of a new particle, η_w-meson, which gets its mass from the toplogical susceptibility.

5) Its origin depends of the quality of the B+L symmetry.
For a good quality B+L , the particle exist regardless of the Minkowski criterion.