VECTOR-BOSON FUSION AT MUON COLLIDERS: ELECTROWEAK FACTORISATION AND PDFS

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based on work with B. Dahlén, M. Loeschner, J. Reuter, P. Stylianou & T. Han, Y. Ma, J. Reuter, K. Xie

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MUON COLLIDER PHYSICS PROGRAMME

Direct searches
e.g. DM, HNL

Muon physics
e.g. muon g-2, LFV

High-intensity SM
e.g. rare decays

High-energy SM
e.g. quartic coupling, VBF

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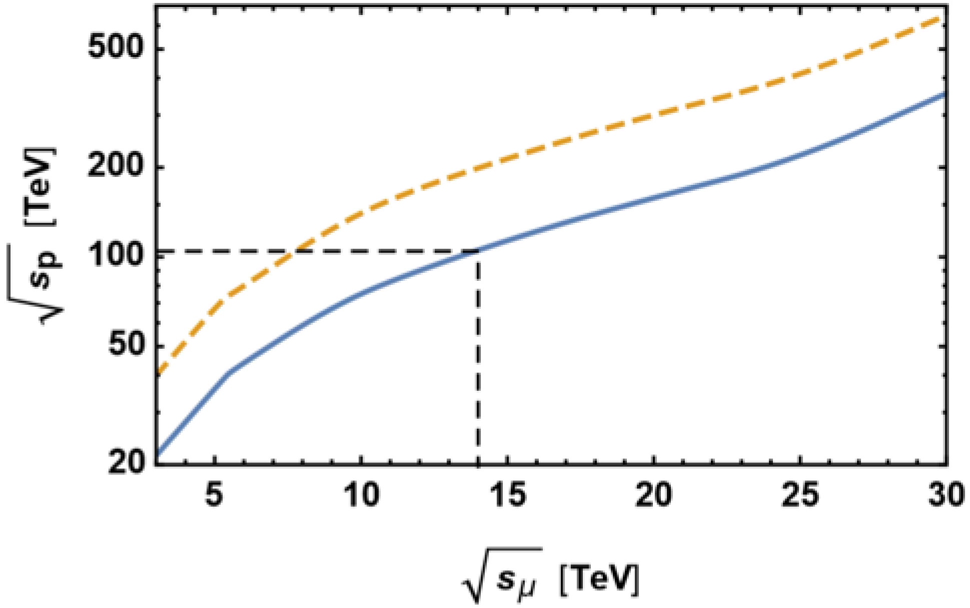
ENERGY-FRONTIER COLLIDERS

Muons can be accelerated to energies comparable to those of protons:

- As they are point-like particles, the *entire* nominal centre-of-mass energy is available for interactions.
- For protons, only a part of the energy is available since it is distributed statistically among partons.



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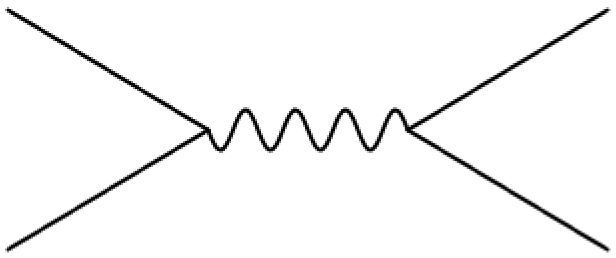
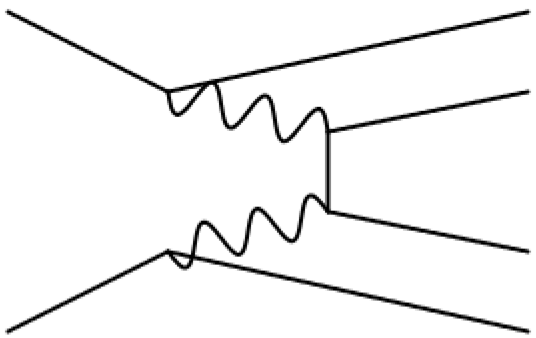
ANNIHILATION & VECTOR-BOSON FUSION

- VBF topologies are contaminated by annihilation topologies, as neutrinos cannot be observed.
- VBF is suppressed by additional powers of couplings but at the same time, enhanced by longitudinal vector bosons and soft/collinear logs.





ANNIHILATION & VECTOR-BOSON FUSION

- For SM processes well above treshold:

$$\frac{\sigma_{\rm VBF}^{\rm SM}}{\sigma_{\rm ann}^{\rm SM}} \propto \alpha_W^2 \frac{s}{m_V^2} \log^3 \frac{s}{m_V^2}$$

- Similarly, for BSM processes:

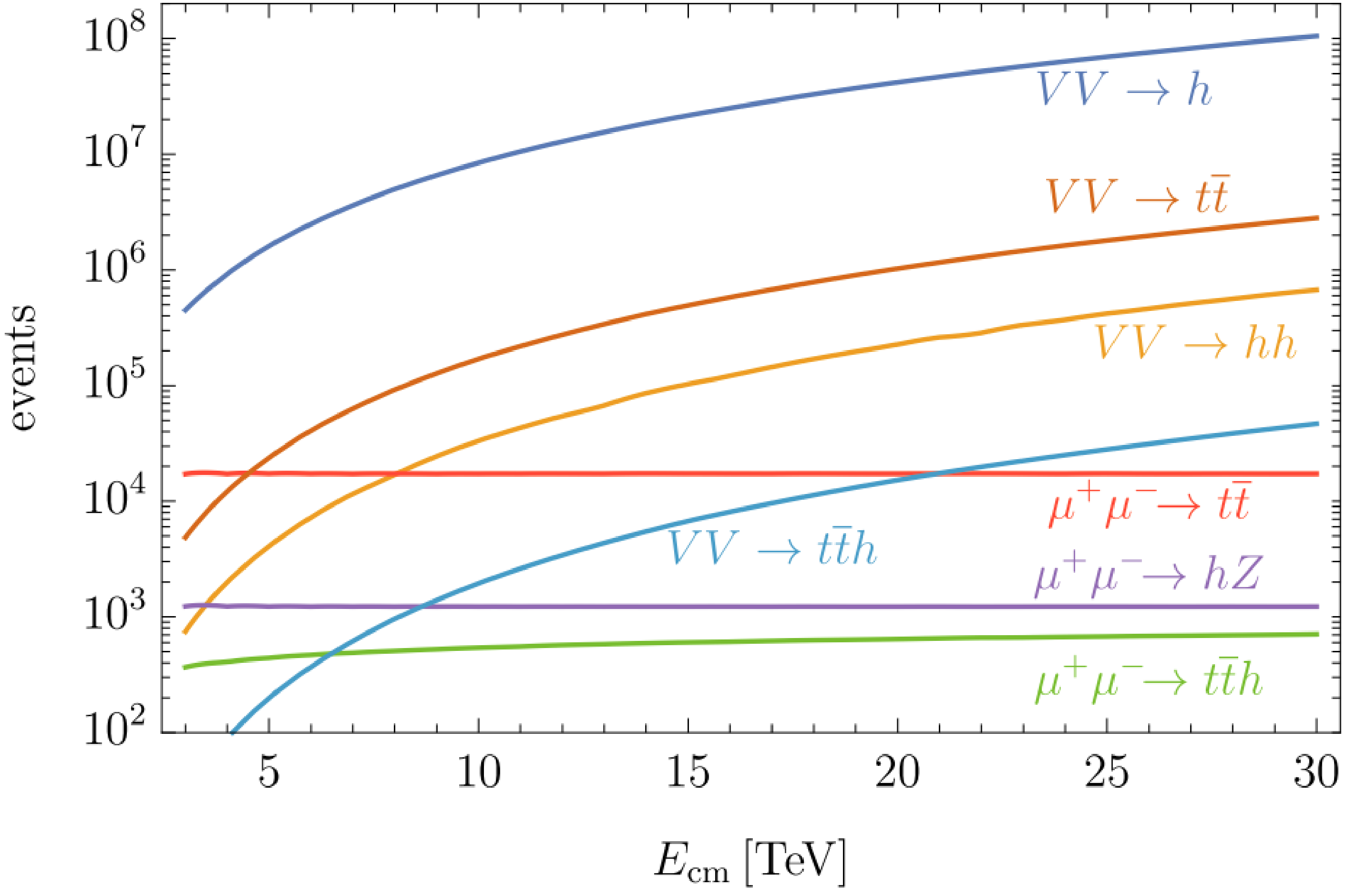
$$\frac{\sigma_{\rm VBF}^{\rm BSM}}{\sigma_{\rm ann}^{\rm BSM}} \propto \alpha_W^2 \frac{s}{m_X^2} \log^2 \frac{s}{m_V^2} \log \frac{s}{m_X^2}$$

VBF subprocesses grow with *s*!

[2005.10289]

ANNIHILATION & VECTOR-BOSON FUSION

$\mathcal{L}_{int} \sim E^2$



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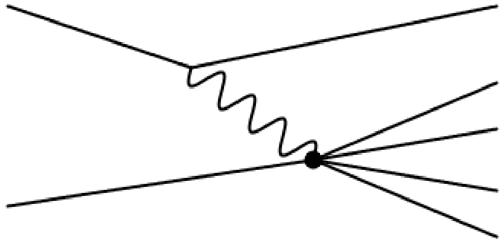
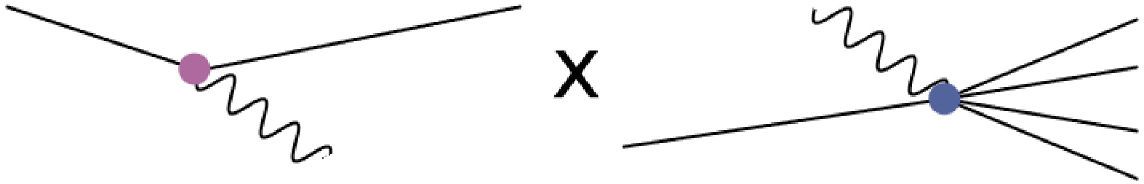
ELECTROWEAK FACTORISATION: EQUIVALENT VECTOR-BOSON APPROXIMATION

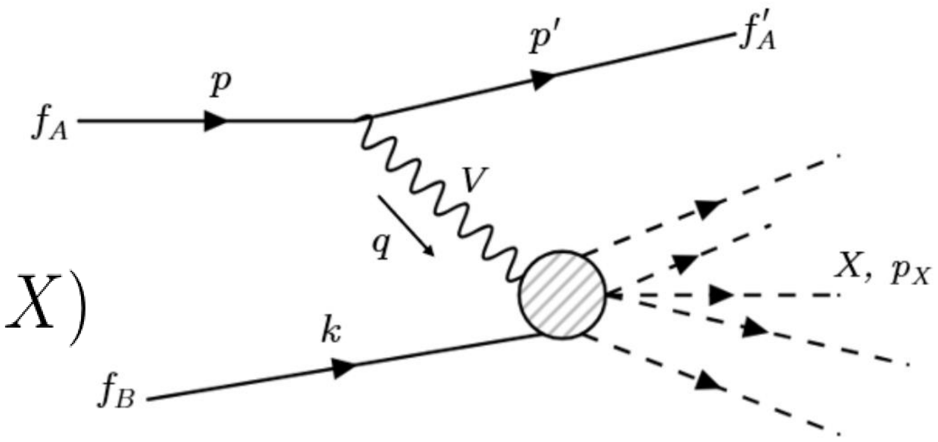
- Let us consider a boson-initiated process, for example:



- Ideally, we would like to factorise the process into some universal structure function describing the emission and a hard-process Matrix Element.



EVA DERIVATION



In general, full ME is given by:

$$\mathcal{M}_{\text{full}} = \mathcal{M}^{\mu}(f_A \to V f'_A)\Delta^V_{\mu\nu}(q)\mathcal{M}^{\nu}(f_B V \to X)$$

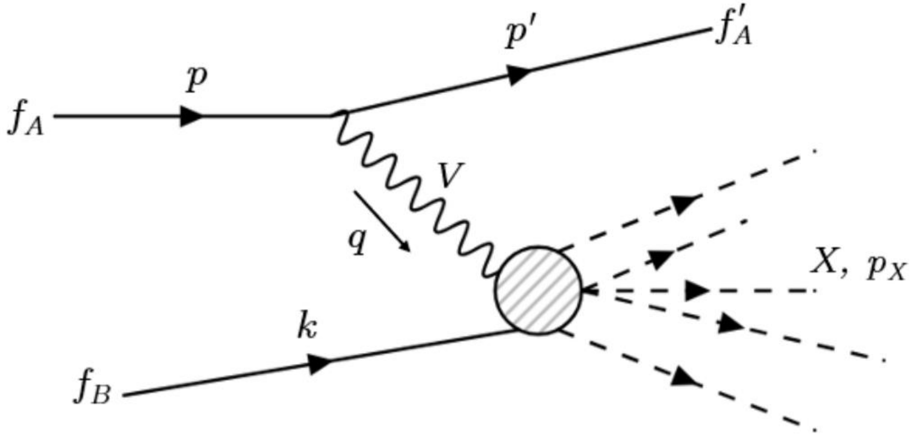
In the weak-boson case, the emission ME is:

$$\mathcal{M}^{\mu}(f_\lambda \to f'_{\lambda'} V_{\lambda_V}) = \bar{u}(p', \lambda')\varepsilon^*_\mu(q, \lambda_V)\gamma^\mu(g_V - g_A\gamma^5)u(p, \lambda)$$

The propagator term reads:

$$\Delta^V_{\mu\nu}(q) = \frac{i\sum_{\lambda_V} \varepsilon^*_\mu(q, \lambda_V)\varepsilon_\nu(q, \lambda_V)}{q^2 - M_V^2}$$

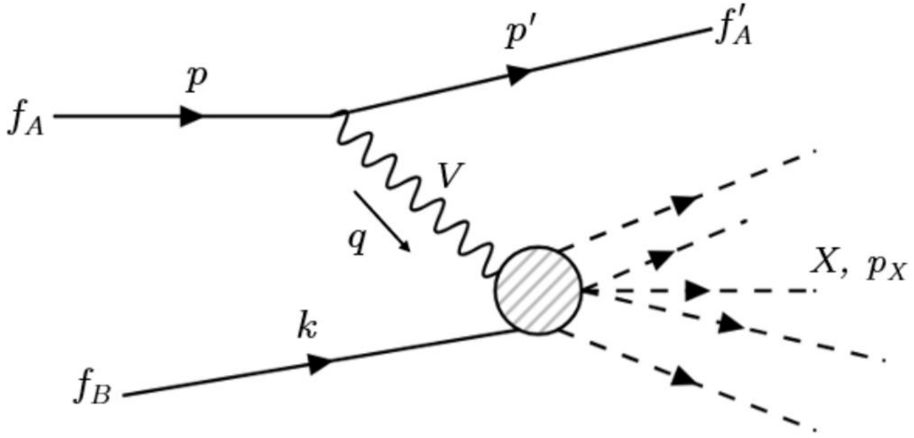
EVA DERIVATION II



The differential cross section is given by:

$$\mathrm{d}\sigma = \frac{x}{2s}\sum_{\lambda_V}|\overline{\mathcal{M}}(f_A \to f'_A V_{\lambda_V})|^2 \frac{1}{(q^2 - M_V^2)^2}|\overline{\mathcal{M}}(f_B V_{\lambda_V} \to X)|^2\,\mathrm{d}PS$$

EVA DERIVATION II



The differential cross section is given by:

$$d\sigma = \frac{x}{2s} \sum_{\lambda_V} |\overline{\mathcal{M}}(f_A \to f'_A V_{\lambda_V})|^2 \frac{1}{(q^2 - M_V^2)^2} |\overline{\mathcal{M}}(f_B V_{\lambda_V} \to X)|^2 \, dPS$$

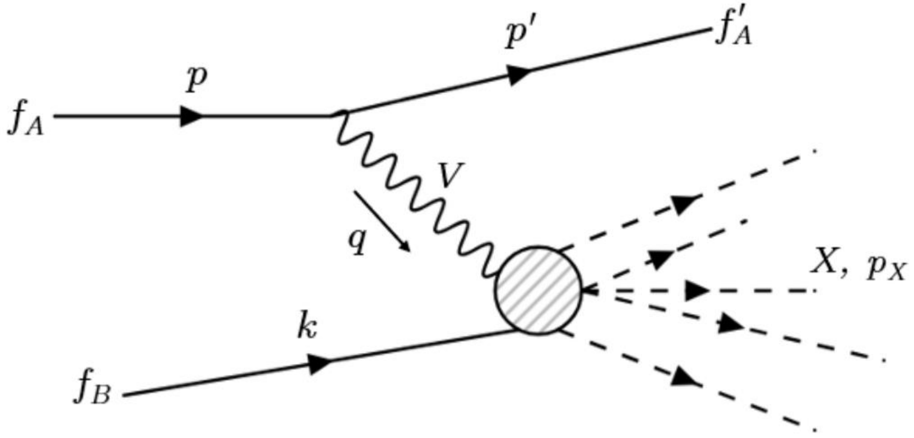
boost of the splitting to the hard-process frame

flux factor

phase-space element

EVA DERIVATION III

Kinematics of the process can be expressed in terms of:



- E – beam energy,
- x – fraction of the energy carried by the boson,
- p_T – transverse momentum of the radiated particle.

Assuming $p_T/E \ll 1$ (*collinear approximation*), one can integrate over p_T and obtain:

$$\sigma(f_A f_B \to f'_A X) = \sum_{\lambda_V} \int_{x_{min}}^{1} \mathrm{d}x F_{\lambda_V}(x, p_T^{max}) \times \hat{\sigma}(f_B V_{\lambda_V} \to X)$$

EVA STRUCTURE FUNCTIONS

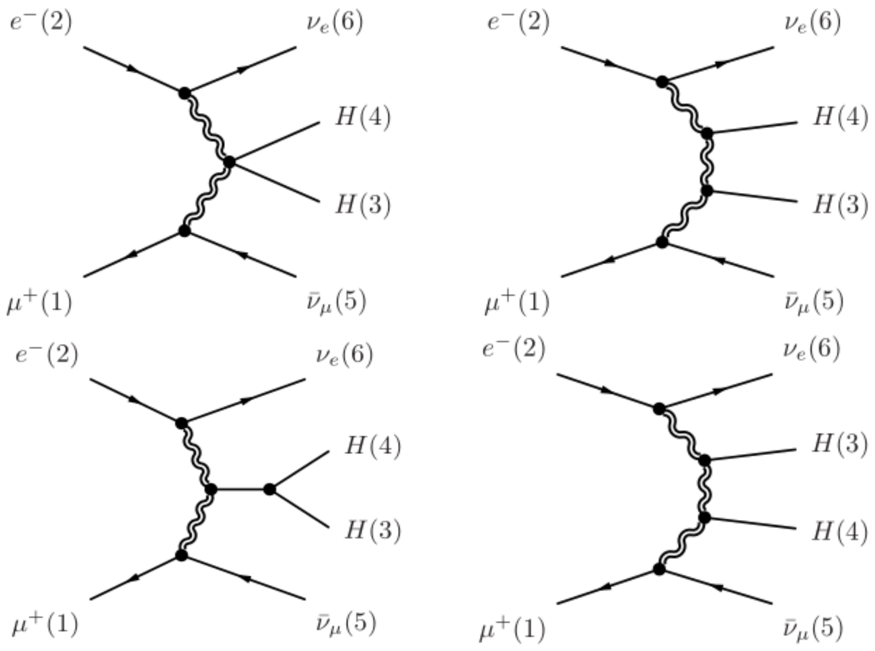
$$F_+(x) = \frac{1}{16\pi^2} \frac{(c_v - c_a)^2 + (c_v + c_a)^2 \bar{x}^2}{x} \left[\ln\left(\frac{p_{T,\max}^2 + \bar{x} M^2}{\bar{x} M^2} \right) - \frac{p_{T,\max}^2}{p_{T,\max}^2 + \bar{x} M^2} \right]$$

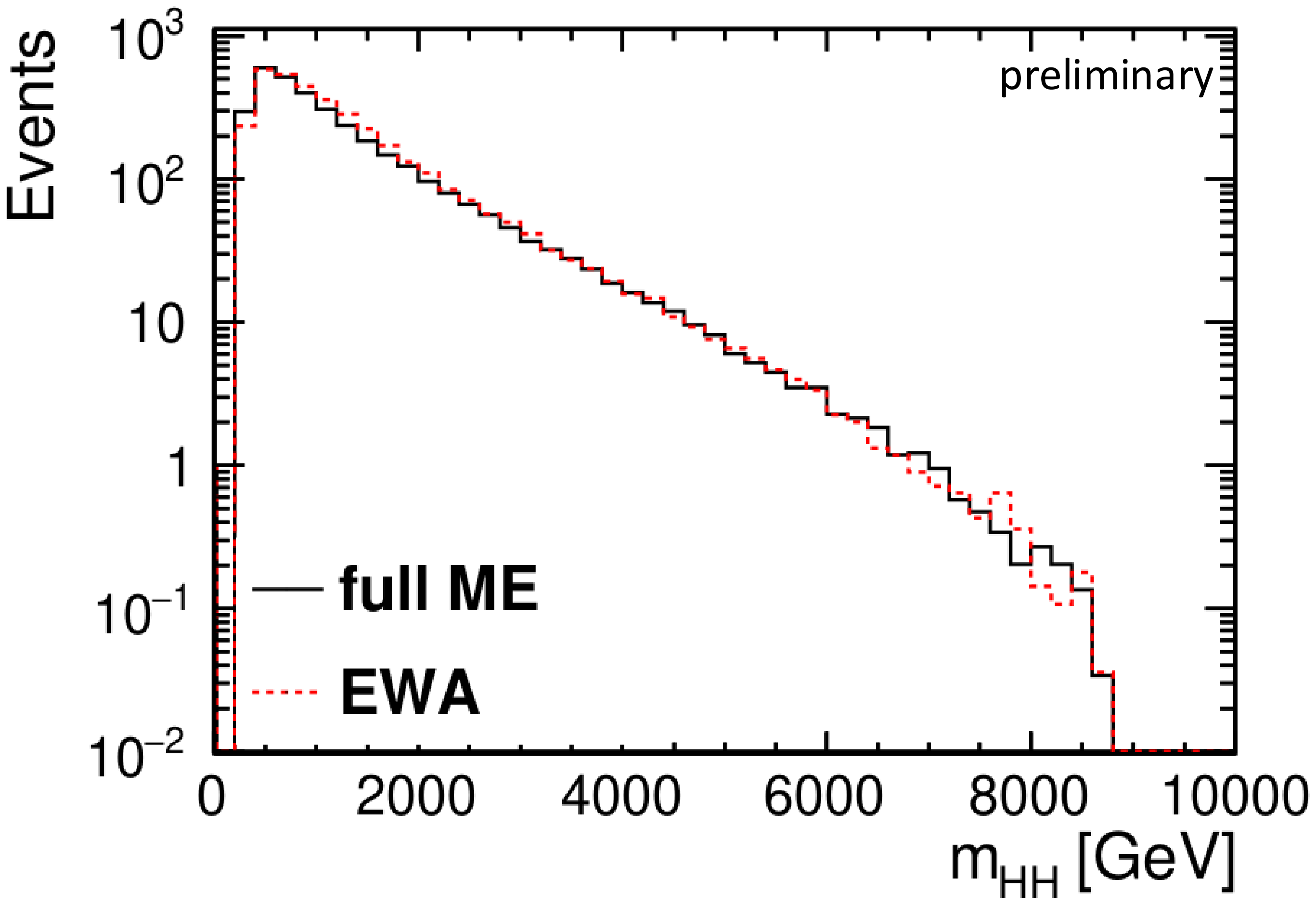
$$F_-(x) = \frac{1}{16\pi^2} \frac{(c_v + c_a)^2 + (c_v - c_a)^2 \bar{x}^2}{x} \left[\ln\left(\frac{p_{T,\max}^2 + \bar{x} M^2}{\bar{x} M^2} \right) - \frac{p_{T,\max}^2}{p_{T,\max}^2 + \bar{x} M^2} \right]$$

$$F_0(x) = \frac{c_v^2 + c_a^2}{8\pi^2} \frac{2\bar{x}}{x} \frac{p_{T,\max}^2}{p_{T,\max}^2 + \bar{x} M^2}$$

EVA VS. FULL PROCESS

Double Higgs production at a 10-TeV $\mu^+ e^-$ collider*





*- for validation purposes

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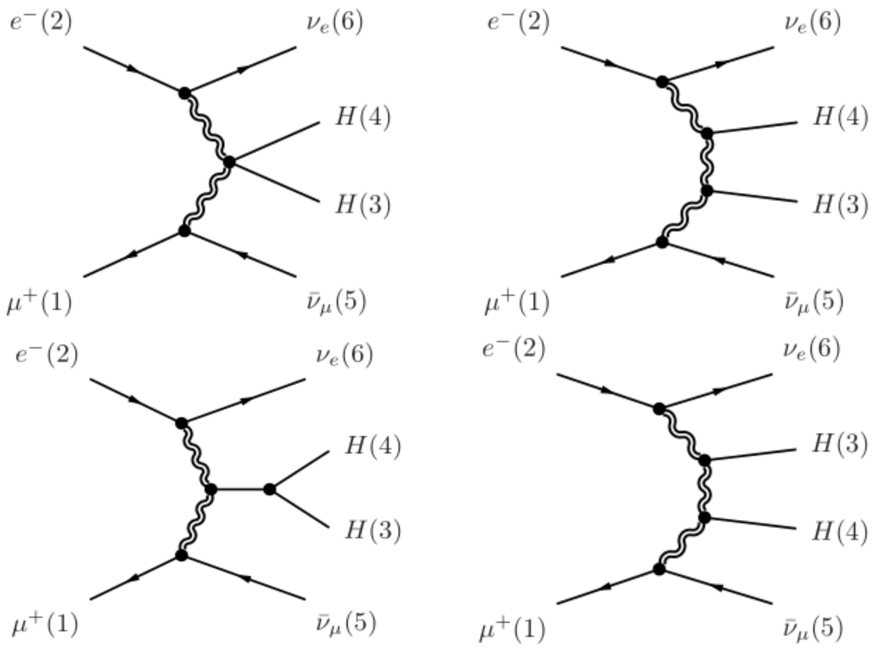
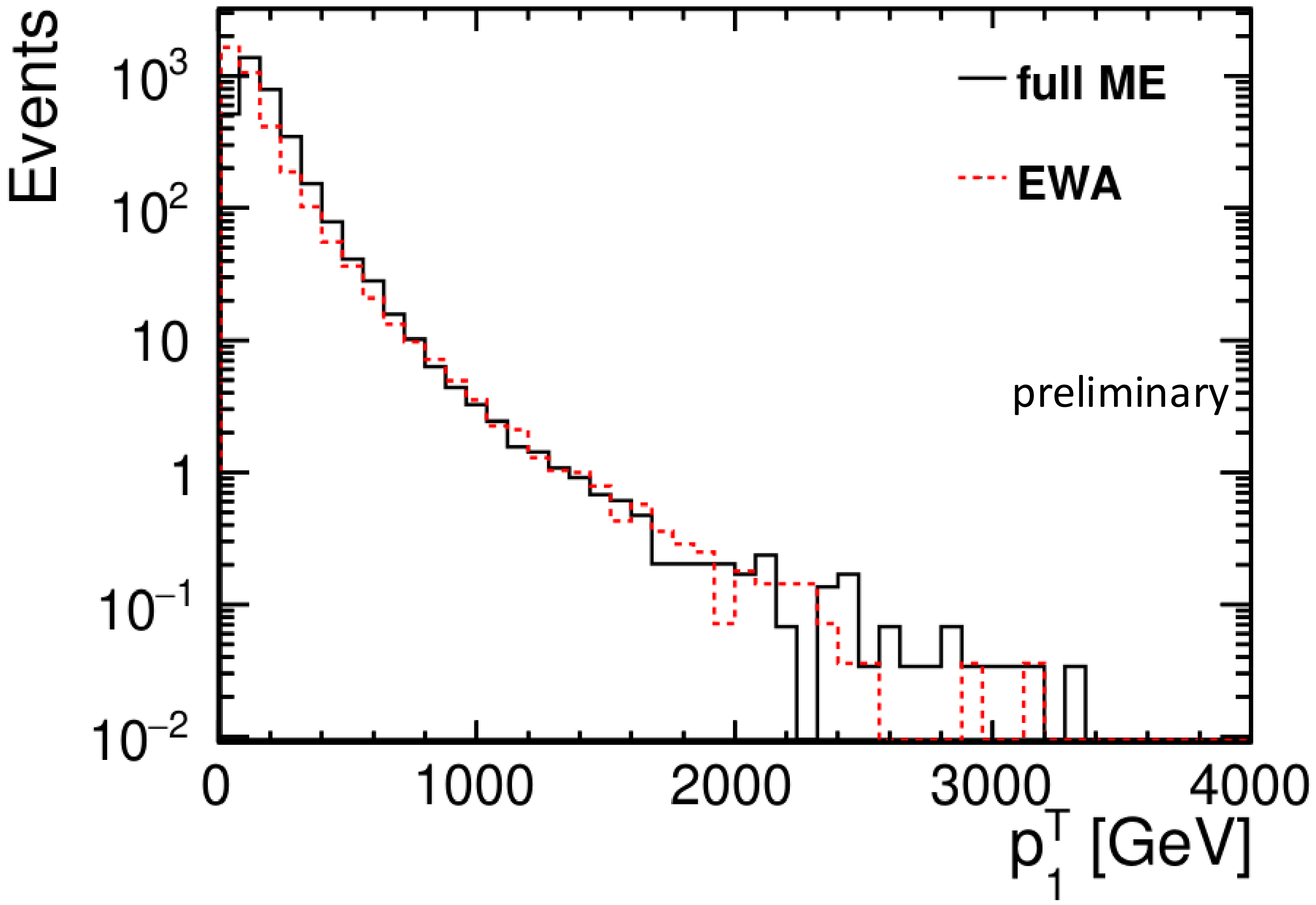
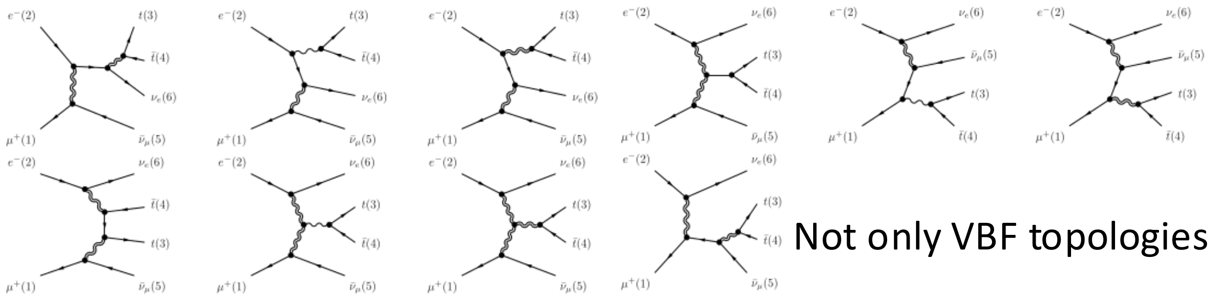




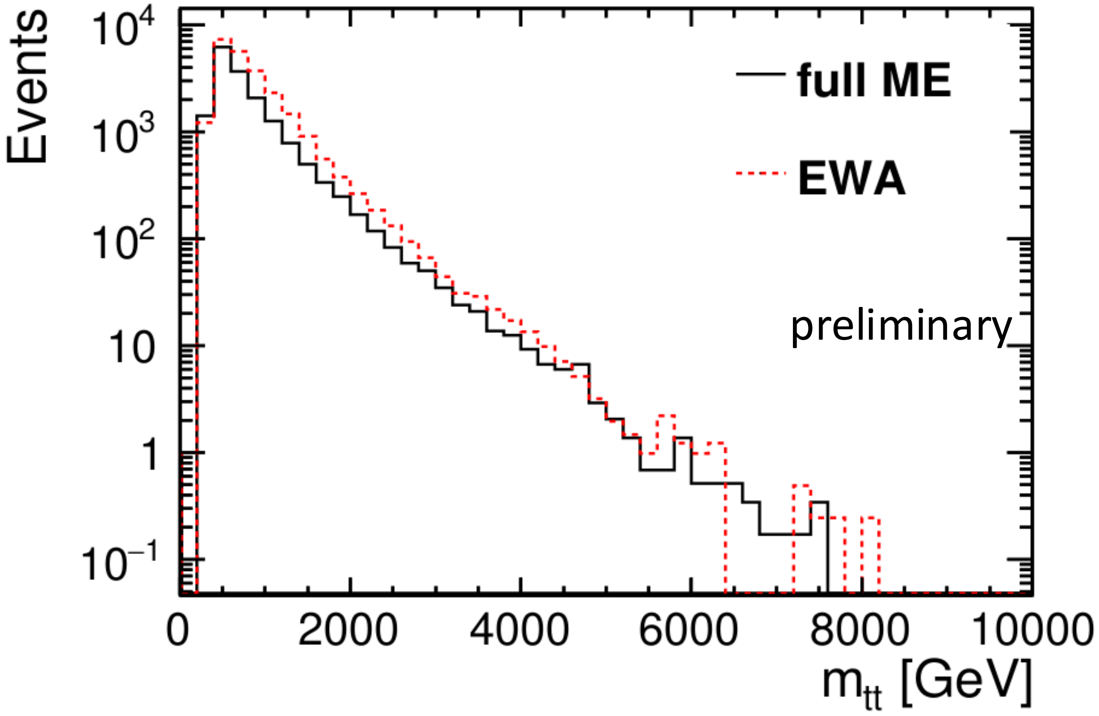
*- for validation purposes

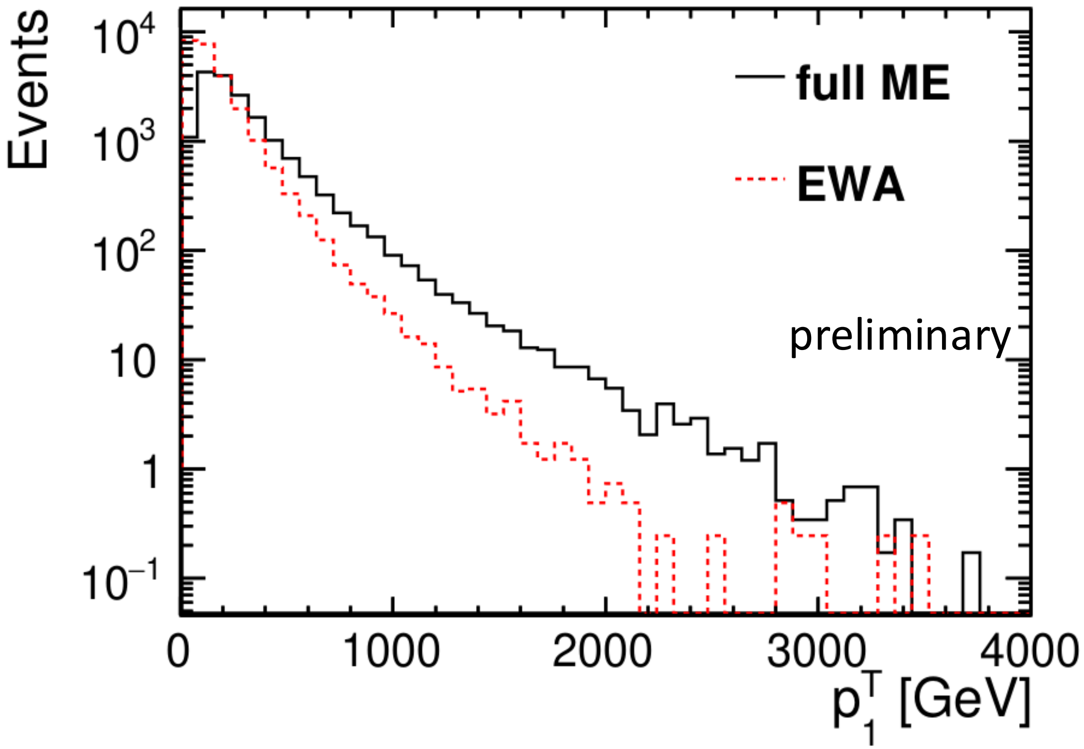
EVA VS. FULL PROCESS

t-tbar production at a 10-TeV μ^+e^- collider



Not only VBF topologies!





EVA MOTIVATION

1. Faster computation (2 → 2 instead of 2 → 4, ...)

   crucial in the past, nowadays of less importance

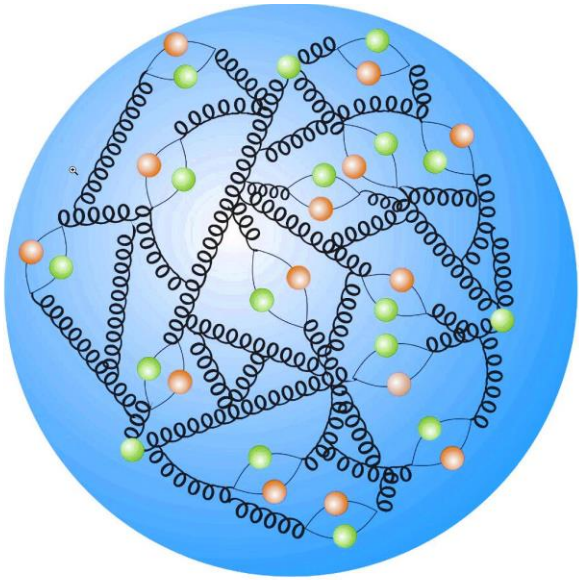
2. Factorisation as an interesting concept on its own

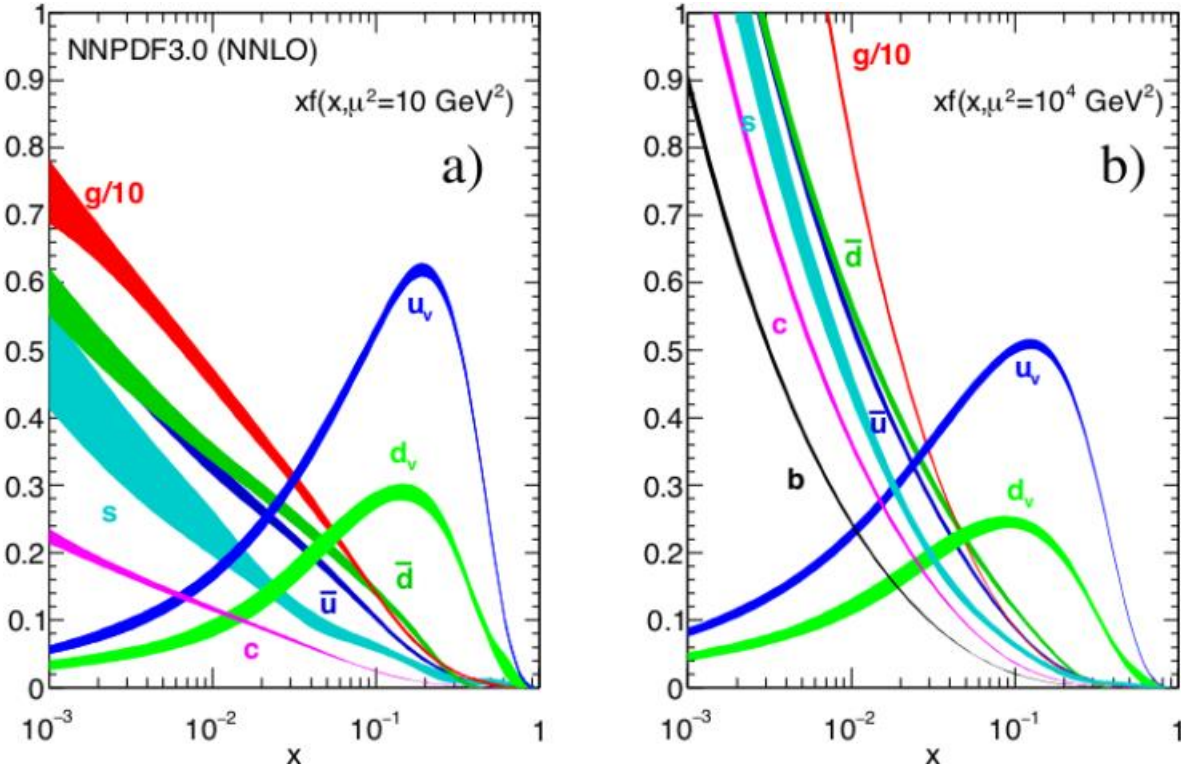
   theoretical studies: validation of the approximation

3. Transition to the EW PDF picture

   path to full SM spectrum as partons

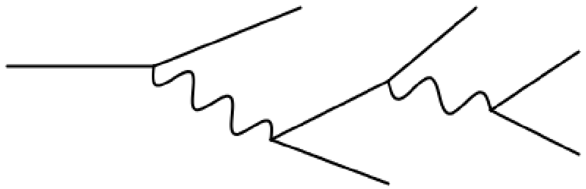
PARTON DISTRIBUTION FUNCTIONS IN QCD





EW RADIATION AT MUON COLLIDERS

- At high energies, muons emit multiple EW bosons: the coupling suppresion is compensated by logs of the ratios of scales between the collision energy and particle masses.
- The radiation is an interesting phenomenon on its own (precision studies, mono-X searches, …), but can also dramatically change the picture of muon collisions.
- The emitted bosons can initiate a hard collision or decay. The products of the decay can initiate a hard collision or decay. The products of this decay can also initiate a hard collision or decay further...



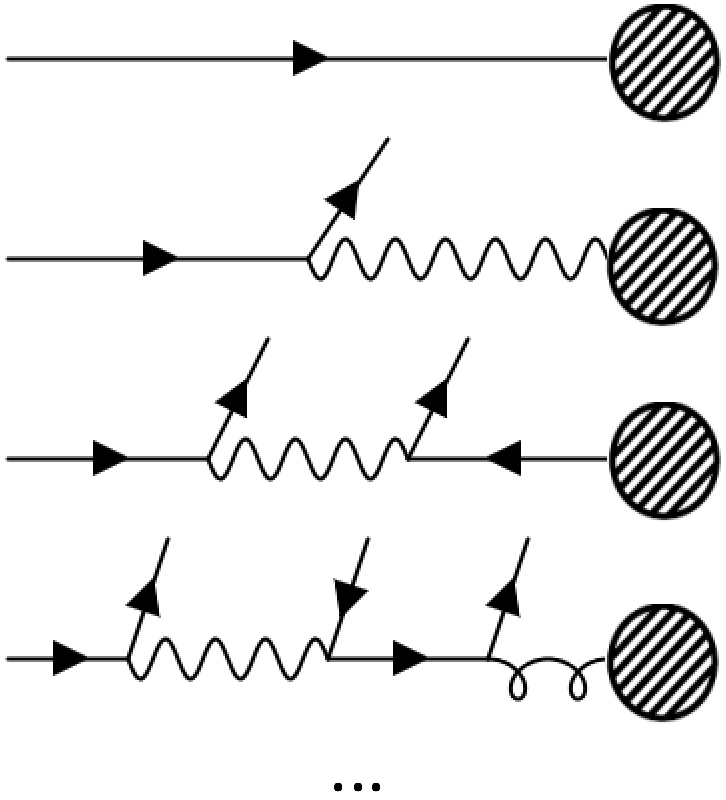
PARTON DISTRIBUTION FUNCTIONS IN SM

- This leads to the PDF-like picture of a muon: large logs of scales have to be resummed for reliable predictions.
- The full EW DGLAP equation has to be employed:

$$\frac{\mathrm{d}f_i}{\mathrm{d}\ln Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P^I_{i,j} \otimes f_j$$

- At the EW scale and above, all electroweak states in the **unbroken** SM are dynamically activated. As the SM is a chiral theory, **parton polarisation** emerges naturally.

FULL PARTICLE SPECTRUM IN EW PDFS



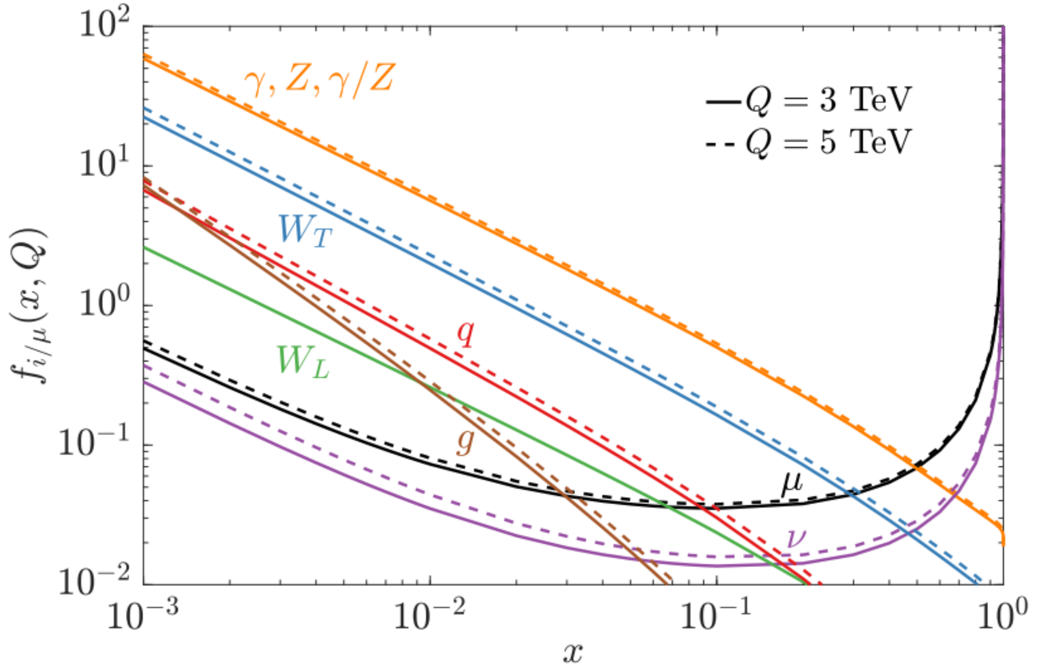
e_L	eL	11	-
e_R	eR	11	+
ν_e	nue	12	-
μ_L	muL	13	-
μ_R	muR	13	+
ν_μ	numu	14	-
τ_L	taL	15	-
τ_R	taR	15	+
ν_τ	nuta	16	-
$\bar{e}_L$	eLb	-11	+
$\bar{e}_R$	eRb	-11	-
$\bar{\nu}_e$	nueb	-12	+
$\bar{\mu}_L$	muLb	-13	+
$\bar{\mu}_R$	muRb	-13	-
$\bar{\nu}_\mu$	numub	-14	+
$\bar{\tau}_L$	taLb	-15	+
$\bar{\tau}_R$	taRb	-15	-
$\bar{\nu}_\tau$	nutab	-16	+

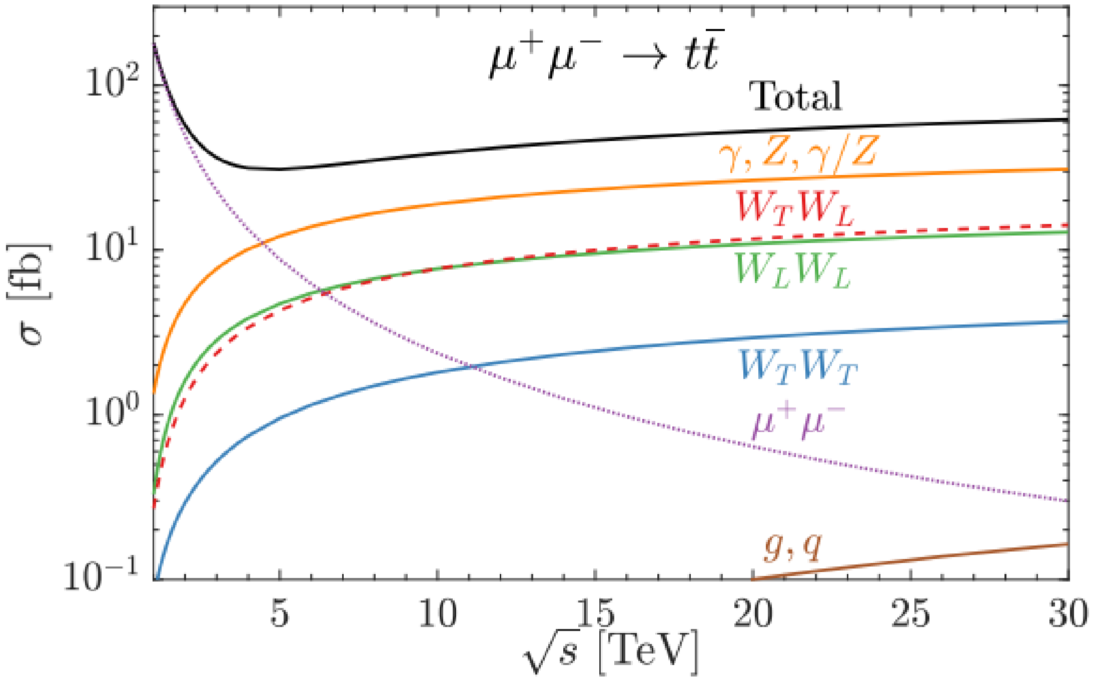
d_L	dL	1	-
d_R	dR	1	+
u_L	uL	2	-
u_R	uR	2	+
s_L	sL	3	-
s_R	sR	3	+
c_L	cL	4	-
c_R	cR	4	+
b_L	bL	5	-
b_R	bR	5	+
t_L	tL	6	-
t_R	tR	6	+
$\bar{d}_L$	dLb	-1	+
$\bar{d}_R$	dRb	-1	-
$\bar{u}_L$	uLb	-2	+
$\bar{u}_R$	uRb	-2	-
$\bar{s}_L$	sLb	-3	+
$\bar{s}_R$	sRb	-3	-
$\bar{c}_L$	cLb	-4	+
$\bar{c}_R$	cRb	-4	-
$\bar{b}_L$	bLb	-5	+
$\bar{b}_R$	bRb	-5	-
$\bar{t}_L$	tLb	-6	+
$\bar{t}_R$	tRb	-6	-

g_+	gp	21	+
g_-	gm	21	-
γ_+	gap	22	+
γ_-	gam	22	-
Z_+	Zp	23	+
Z_-	Zm	23	-
Z_L	ZL	23	0
Z/γ_+	Zgap	2223	+
Z/γ_-	Zgam	2223	-
W_+^+	Wpp	24	+
W_-^+	Wpm	24	-
W_L^+	WpL	24	0
W_+^-	Wmp	-24	+
W_-^-	Wmm	-24	-
W_L^-	WmL	-24	0
h	h	25	0
h/Z_L	hZL	2523	0

[2303.16964]

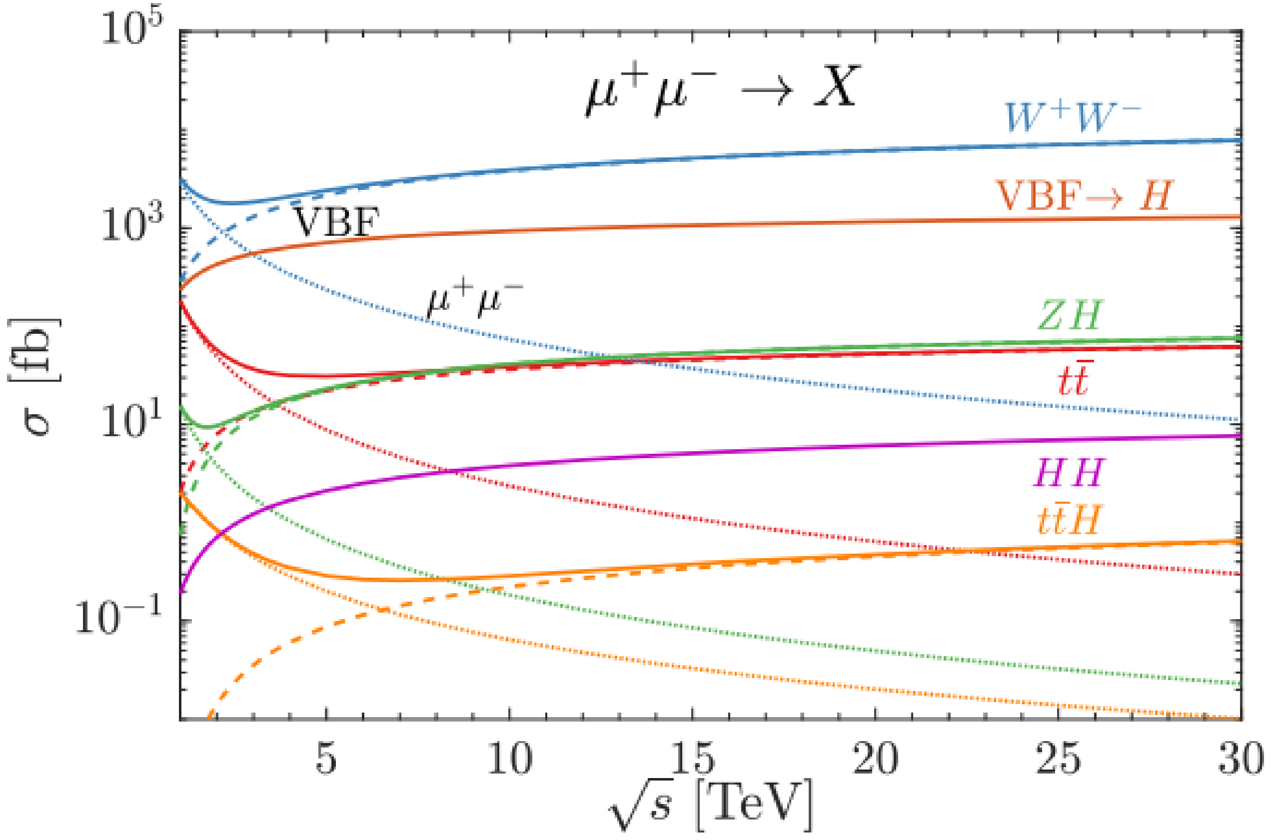
PARTON DISTRIBUTION FUNCTIONS FOR MUONS





[2007.14300]

CROSS SECTION – ANNIHILATION VS. VBF



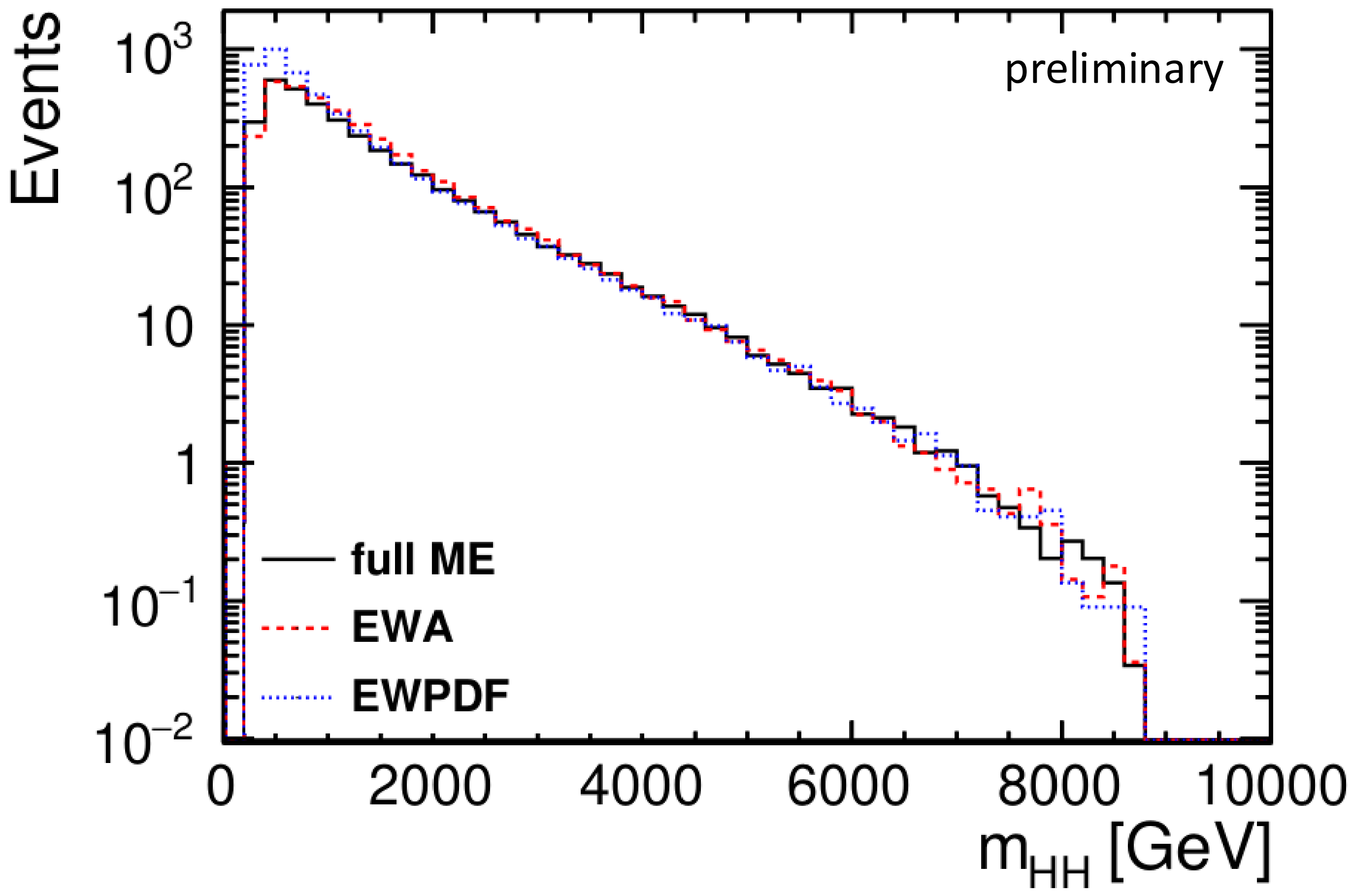
[2007.14300]

FULL ME VS. EVA VS. EWPDF

Double Higgs production at a 10-TeV μ^+e^- collider:

a very good agreement!

Technical implementation in *Whizard*



EW PDF MOTIVATION

1. Faster computation (2 → 2 instead of 2 → 4, ...)
2. Factorisation as an interesting concept on its own
3. Convenient way to look for specific processes, e.g. BSM
4. The so far most promising way to automate precision resummation of threshold logs for 10-TeV+ colliders

CONCLUSIONS & OUTLOOK

- The collinear emission of nearly on-shell massive vector bosons makes any multi-TeV machine a **vector-boson collider**!
- EVA and EW PDFs offer an interesting framework to study physics in the collinear approximation.
- Many new results expected soon...