Waist size issues

Tom Blackburn

The normalised amplitude (as used in Ptarmigan) is defined in terms of the laser's peak electric field E_0 and angular frequency ω_0 as

$$a_0 = \xi = \frac{eE_0}{mc\omega_0}.$$ (1)

The peak electric field depends on the laser intensity, but also the laser polarisation:

$$E_0 = \sqrt{\frac{I_0}{\epsilon_0 c}} \times \begin{cases} \sqrt{2} & \text{LP} \\ 1 & \text{CP} \end{cases},$$ (2)

because in linear polarisation, the electric field magnitude oscillates [$\vec{E}_\perp = E_0(\sin\phi, 0)$], whereas for circular polarisation, it is fixed [$\vec{E}_\perp = E_0(\sin\phi, \cos\phi)$]. In other words, switching between CP and LP at fixed pulse energy increases the electric field (and so a_0) by a factor of $\sqrt{2}$. Using this convention is why the mass shift in nonlinear Compton scattering is proportional to $1 + a_0^2/2$ in linear polarisation, but $1 + a_0^2$ in circular. The alternative would be to define a_0 in terms of the r.m.s. electric field, in which case LP and CP have the same a_0 at the same intensity, but then one loses the direct connection to the maximum quantum parameter as $\chi = a_0\eta$.

The next step is to assume that the laser has a Gaussian temporal envelope,

$$I(\phi) = I_0 \exp\left(-\frac{4\ln 2\, \phi^2}{\omega_0^2 \tau^2}\right),$$ (3)

where τ is the full width at half maximum duration, and that it is focused to a *perfect* Gaussian spot,

$$I(r, z = 0) = I_0 \exp\left(-\frac{2r^2}{w_0^2}\right)$$ (4)

where w_0 is the waist.

The total energy contained in the pulse is

$$\mathcal{E} = \frac{\pi^{3/2}}{4\sqrt{\ln 2}} I_0 w_0^2 \tau = \frac{\pi^{7/2}}{2\sqrt{\ln 2}} \frac{\epsilon_0 m^2 c^5}{e^2} \left(\frac{a_0 w_0}{\lambda}\right)^2 \tau \times \begin{cases} 1 & \text{LP} \\ 2 & \text{CP} \end{cases}.$$ (5)

Rearranging for w_0, and in engineering units:

$$w_0[\mu\text{m}] = 148 \frac{\mathcal{E}^{1/2}[\text{J}]\ \lambda[\mu\text{m}]}{a_0\ \tau^{1/2}[\text{fs}]} \times \begin{cases} \sqrt{2} & \text{LP} \\ 1 & \text{CP} \end{cases}.$$ (6)

The values for pulse energy etc. in the Ptarmigan simulations of the LUXE IP were inherited from the configuration of Tony Hartin's original IPstrong simulations, where the polarisation was fixed to be circular, and have never been changed since.