Towards a quantum treatment of leptogenesis

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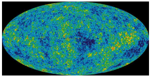
based on work done in collaboration with A. De Simone, A. Ibarra, C. Weniger; M. M. Müller; A. Hohenegger, A. Kartavtsev, M. Lindner

Physics beyond the Standard Model

Collider exp.

Baryon asymmetry



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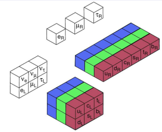

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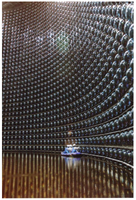
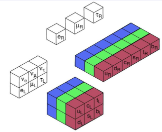


Neutrino exp.

Dark matter

Physics of the Early Universe



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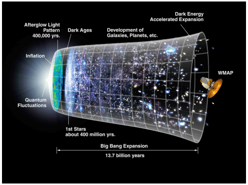
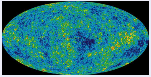
$\leftrightarrow$



$\leftrightarrow$



- Inflation, Reheating
- Baryogenesis
- Thermal relics (gravitino)
- Dark matter freeze-out
- ...

Nonequilibrium dynamics at high energy

Outline

Towards a quantum treatment of leptogenesis

- Leptogenesis
- Quantum fields out of equilibrium
- Application to leptogenesis

Leptogenesis

Standard Model (SM) extended by three heavy singlet neutrino fields $N_i = N_i^c$, $i = 1, 2, 3$ with Majorana masses $\hat{M} = \text{diag}(M_i)$ in the mass eigenbasis

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} + \tfrac{1}{2}\bar{N} i\not{\partial} N - \tfrac{1}{2}\bar{N}\hat{M}N - \bar{\ell}\widetilde{\phi} h P_R N - \bar{N} P_L h^\dagger \widetilde{\phi}^\dagger \ell$$

Light neutrino masses via seesaw mechanism

$$m_\nu = -v_{EW}^2 h \hat{M}^{-1} h^T$$

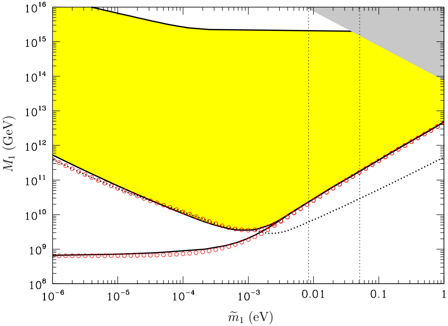
Baryogenesis via leptogenesis *Fukugita, Yanagida 86*

- B-violation via L-violating Majorana masses M_i
- CP-violation via Yukawa couplings $\text{Im}[(h^\dagger h)_{ij}] \neq 0$
- Out-of-equilibrium (inverse) decay $N_i \leftrightarrow \ell\phi^\dagger$ and $N_i \leftrightarrow \ell^c \phi$

$$\begin{aligned} (\Gamma_i/H)|_{T=M_i} &\simeq \tilde{m}_i/\text{meV} \quad \sim \mathcal{O}(1) \quad \text{where } \tilde{m}_i = v_{EW}^2 (h^\dagger h)_{ii}/M_i \\ (\Gamma_{SM}/H)|_{T=M_i} &\sim g^4 M_{pl}/M_i \gg 1 \qquad \text{for } M_i \ll 10^{16}\text{GeV} \end{aligned}$$

Leptogenesis

Vanilla leptogenesis for hierarchical spectrum $M_1 \ll M_{2,3}$ requires large values of the reheating temperature $T_R \gtrsim \mathcal{O}(M_1) \gtrsim 10^9$GeV, leading to potential conflicts with gravitino production in supersymmetric scenarios *Davidson, Ibarra*



Buchmüller, Di Bari, Plümacher

$$\Omega^{th}_{3/2} h^2 \simeq 0.27 \left(\frac{T_R}{10^9\ \mathrm{GeV}}\right)\left(\frac{10\ \mathrm{GeV}}{m_{3/2}}\right)\left(\frac{m_{\widetilde{g}}}{1\ \mathrm{TeV}}\right)^2$$

Moroi, Murayama, Yamaguchi, 93; Bolz, Brandenburg, Buchmüller, 01; Pradler, Steffen, 06; Rychkov, Strumia, 07

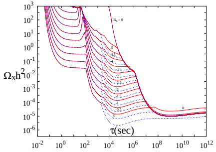
Leptogenesis

- If the gravitino is the LSP, it is a natural candidate for the Dark Matter

$$\Omega_{3/2} h^2 = 0.11$$

- NLSP (e.g. stau/neutralino) is long-lived

$$\tau_{\tilde{\tau}_1} \simeq 3\,\mathrm{days}\left(\frac{m_{3/2}}{10\,\mathrm{GeV}}\right)^2 \left(\frac{250\,\mathrm{GeV}}{m_{\tilde{\tau}_1}}\right)^5$$



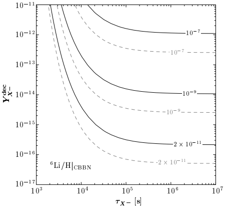
Jedamzik 2006

- Hadronic energy release during BBN (p-n conversion, D by hadrodissociation)

$$\tau_{\chi_1^0} \lesssim 1 - 100\mathrm{sec}$$

- Bound state $^4\mathrm{He}\,X^- + \mathrm{D} \to {}^6\mathrm{Li} + X^-$ catalyzes ^{6}Li production; spallation

$$\tau_{\tilde{\tau}_1} \lesssim 2000\mathrm{sec}$$



Pradler, Steffen 2008

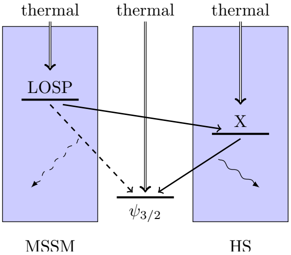
Leptogenesis

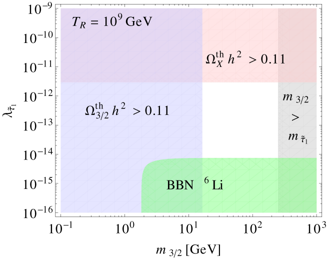
Some proposed solutions (incomplete list...)

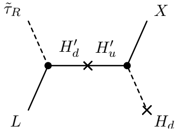
- Violation of R-parity
  Buchmüller, Covi, Hamaguchi, Ibarra, Yanagida 07; Bobrovskyi, Buchmüller, Hajer, Schmidt 10,11
- Entropy production
  Pradler, Steffen 09; Hasenkamp, Kersten 10
- Sneutrino or stop LOSP
  Kanazaki, Kawasaki, Kohri, Moroi 07; Diaz-Cruz, Ellis, Olive, Santoso 07
- LR mixing
  Ratz, Schmidt-Hoberg, Winkler
- Degenerate LOSP and gravitino
  Boubekeur, Choi, de Austri, Vives 10; Pradler, Steffen 09
- Degenerate Gluino and bino
  Covi, Olechowski, Pokorski, Turzynski, Wells 10
- ...
- Light hidden sector
  De Simone, MG, Ibarra, Weniger 10

Supersymmetric Leptogenesis with a light hidden sector

GUT-suppressed NLSP decay into light hidden fermion can reconcile leptogenesis, gravitino DM and BBN (R conserved) *De Simone, MG, Ibarra, Weniger 10*







$$\Gamma_{\tilde{\tau}_1 \to \psi_{3/2}\tau} \simeq \frac{1}{48\pi} \frac{m_{\tilde{\tau}_1}^5}{m_{3/2}^2 m_P^2} \left(1 - \frac{m_{3/2}^2}{m_{\tilde{\tau}_1}^2}\right)^4$$

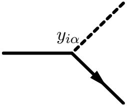
$$\Gamma_{\tilde{\tau}_1 \to X\tau} \simeq \frac{|\lambda_{\tilde{\tau}_1}|^2 m_{\tilde{\tau}_1}}{8\pi} \left(1 - \frac{m_X^2}{m_{\tilde{\tau}_1}^2}\right)^2$$

$$\lambda_{\tilde{\tau}_1} \sim \frac{\langle H_d \rangle}{M} \sim 10^{-14} \left(\frac{M \cos\beta}{10^{16}\,\text{GeV}}\right)^{-1}$$

$$\Rightarrow \tilde{\tau}_1 \to (\tau, \mu, e)X; \tau h X$$

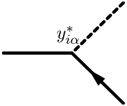
L-violating decay of heavy right-handed neutrino N_i

$$\mathcal{M}_{N_i \to \ell_\alpha h^\dagger} =$$



$$+ \ldots$$

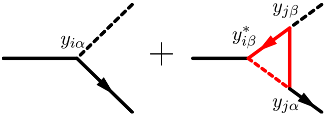
$$\mathcal{M}_{N_i \to \ell^c_\alpha h} =$$



$$+ \ldots$$

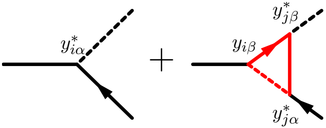
L-violating decay of heavy right-handed neutrino N_i

$$\mathcal{M}_{N_i \to \ell_\alpha h^\dagger} \quad = \quad$$



$$+ \ldots$$

$$\mathcal{M}_{N_i \to \ell^c_\alpha h} \quad = \quad$$

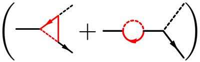


$$+ \ldots$$

Matter-antimatter (CP) asymmetry

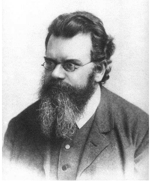
⇔ interference of tree and loop processes

$$\Gamma(N_i \to \ell_\alpha h^\dagger) - \Gamma(N_i \to \ell^c_\alpha h) \;\sim\; \mathrm{Im}(y_{i\alpha} y_{i\beta} y^*_{j\alpha} y^*_{j\beta}) \cdot \mathrm{Im}$$



Standard Boltzmann approach

$$N_L(t) = \int d^3x \sqrt{|g|} \int \frac{d^3p}{(2\pi)^3} \sum_\ell \left[f_\ell(t, \mathbf{x}, \mathbf{p}) - f_{\bar{\ell}}(t, \mathbf{x}, \mathbf{p}) \right]$$



$$\begin{aligned} p^\mu \mathcal{D}_\mu f_\ell(t, \mathbf{x}, \mathbf{p}) \; &= \; \sum_i \int d\Pi_{N_i} d\Pi_h \\ &\times (2\pi)^4 \delta(p_\ell + p_h - p_{N_i}) \\ &\times \Big[|\mathcal{M}|^2_{N_i \to \ell h^\dagger} \; f_{N_i}(1 - f_\ell)(1 + f_h) + \dots \\ &\quad - \; |\mathcal{M}|^2_{\ell h^\dagger \to N_i} \; f_\ell f_h (1 - f_{N_i}) + \dots \Big] \end{aligned}$$

$f_\psi(t, \mathbf{x}, \mathbf{p})$: distribution function of on-shell particles

$|\mathcal{M}|^2$: matrix elements computed in *vacuum*, off-shell effects

Corrections within Boltzmann framework

- Bose-enhancement, Pauli-Blocking; kinetic (non-)equilibrium
  - quantum statistical factors $1 \pm f_k$
  - non-integrated Boltzmann equations

  Hannsestad, Basbøll 06; Garayoa, Pastor, Pinto, Rius, Vives 09; Hahn-Woernle, Plümacher, Wong 09

- Thermal corrections via thermal QFT
  - medium correction to CP-violating parameter $\epsilon = \epsilon^{vac} + \delta\epsilon^{th}$
  - thermal masses, decay width, ...

  Covi, Rius, Roulet, Vissani 98; Giudice, Notari, Raidal, Riotto, Strumia 04; Besak, Bödeker 10

  Kiessig, Thoma, Plümacher 10; Salvio Lodone Strumia 11...

- Flavour effects

  Nardi, Nir, Roulet, Racker 06; Abada, Davidson, Ibarra, Josse-Micheaux, Losada, Riotto 06; Blanchet, diBari 06; ...

- Spectator processes, scatterings, N_2, ...

Double Counting Problem

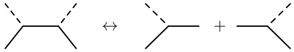
Naive contribution from decay/inverse decay

$$|\mathcal{M}|^2_{N_i\to\ell h^\dagger} = |\mathcal{M}_0|^2(1+\epsilon_i) \qquad |\mathcal{M}|^2_{\ell h^\dagger\to N_i} = |\mathcal{M}_0|^2(1-\epsilon_i)$$

$$|\mathcal{M}|^2_{N_i\to\ell^c h} = |\mathcal{M}_0|^2(1-\epsilon_i) \qquad |\mathcal{M}|^2_{\ell^c h\to N_i} = |\mathcal{M}_0|^2(1+\epsilon_i)$$

$$\begin{aligned}\frac{dN_{B-L}}{dt} &\propto (|\mathcal{M}|^2_{N_i\to\ell h^\dagger} - |\mathcal{M}|^2_{N_i\to\ell^c h})N_{N_i} \\ &\quad - (|\mathcal{M}|^2_{\ell h^\dagger\to N_i} - |\mathcal{M}|^2_{\ell^c h\to N_i})N^{eq}_{N_i} \\ &\propto \epsilon_i(N_{N_i} + N^{eq}_{N_i})\end{aligned}$$

$\Rightarrow$ spurious generation of asymmetry even in equilibrium

Origin: Double Counting Problem 

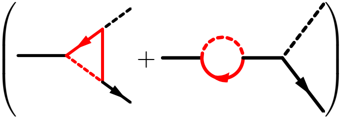
$\to$ need real intermediate state subtraction

... justification from first principles / generalization ?

Leptogenesis - resonant enhancement

Efficiency of leptogenesis depends on CP-violating parameter, which is one-loop suppressed

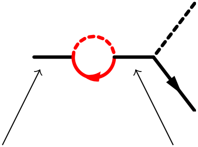
$$\epsilon_{N_i} = \frac{\Gamma(N_i \to \ell\phi^\dagger) - \Gamma(N_i \to \ell^c\phi)}{\Gamma(N_i \to \ell\phi^\dagger) + \Gamma(N_i \to \ell^c\phi)} \propto \text{Im}$$



Self-energy (or 'wave') contribution to CP-violating parameter features a resonant enhancement for a quasi-degenerate spectrum $M_1 \simeq M_2 \ll M_3$

$$\epsilon_{N_i}^{wave} = \frac{\text{Im}[(h^\dagger h)_{12}^2]}{8\pi(h^\dagger h)_{ii}} \times \frac{M_1 M_2}{M_2^2 - M_1^2}$$

Flanz Paschos Sarkar 94/96; Covi Roulet Vissani 96;



On-shell initial N_1: $p^2 = M_1^2$ Internal N_2: $\frac{i}{p^2 - M_2^2}$

Resonant leptogenesis

- *Flanz Paschos Sarkar Weiss 96;* effective Hamiltonian approach

$$\epsilon_{N_i} = -\frac{\mathrm{Im}[(h^\dagger h)^2_{12}]}{16\pi(h^\dagger h)_{ii}} \frac{M_1(M_2 - M_1)}{(M_2 - M_1)^2 + M_1^2(\mathrm{Re}(h^\dagger h)_{12}/(16\pi))^2}$$

- *Covi Roulet 96;* CP violating decay of mixing scalar fields described by effective mass matrix; formalism as in Liu Segre 93
- *Pilaftsis 97; Pilaftsis Underwood 03;* Pole mass expansion of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)^2_{ij}]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + M_i^2\Gamma_j^2}$$

- *Buchmüller Plümacher 97;* Diagonalization of resummed propagators

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)^2_{12}]}{8\pi(h^\dagger h)_{ii}} \frac{M_1 M_2(M_2^2 - M_1^2)}{(M_2^2 - M_1^2 - \frac{1}{\pi}\Gamma_i M_i \ln\frac{M_2^2}{M_1^2})^2 + (M_1\Gamma_1 - M_2\Gamma_2)^2}$$

- *Rangarajan Mishra 99;* comparison of different approaches
- *Anisimov Broncano Plümacher 05;* Reconciliation of diagonalization approach with the pole mass expansion approach
- Invariant quantity $M_1 M_2(M_2^2 - M_1^2)\mathrm{Im}(h^\dagger h)^2_{12}$ related to CP violation appears in the enumerator

Resonant leptogenesis

The results can be summarized (neglecting log-corrections) as

$$\epsilon_{N_i} = \frac{\mathrm{Im}[(h^\dagger h)^2_{12}]}{8\pi (h^\dagger h)_{ii}} \times R, \qquad R \equiv \frac{M_1 M_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + A^2}$$

Different calculations correspond to different expressions for the 'regulator' A^2

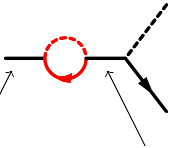
$$A^2 = \begin{cases} \frac{1}{4}(M_1 + M_2)^4 \left(\frac{\mathrm{Re}(h^\dagger h)_{12}}{16\pi}\right)^2 & \textit{Flanz Paschos Sarkar Weiss 96} \\ M_i^2 \Gamma_j^2 & \textit{Pilaftsis 97; Pilaftsis Underwood 03} \\ (M_1\Gamma_1 - M_2\Gamma_2)^2 & \textit{Buchmüller Plümacher 97; Anisimov Broncano Plümacher 05; \ldots} \\ \ldots & \end{cases}$$

The regulator is relevant for determining the maximal possible resonant enhancement, which occurs for $M_2^2 - M_1^2 = \pm A$, and is given by

$$R_{max} = \frac{M_1 M_2}{2|A|}$$

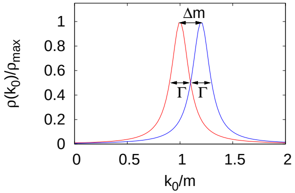
Resonant leptogenesis

The origin of the regulator is the finite width of N_1 and N_2



Off-shell initial N_1: $p^2 = M_1^2 + iM_1\Gamma_1$ Internal N_2: $\frac{i}{p^2 - M_2^2 - iM_2\Gamma_2}$

In the maximal resonant case $M_2 - M_1 = \mathcal{O}(\Gamma_i)$, the spectral functions overlap



$\Rightarrow$ Need to go beyond the quasi-particle approximation which underlies the conventional semi-classical Boltzmann approach.

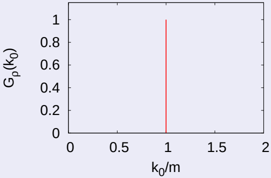
Going beyond the Boltzmann picture

Statistical propagator $G_F^{ij}(x,y) = \langle \Phi^i(x)\Phi^j(y) + \Phi^j(y)\Phi^i(x)\rangle/2$

Spectral function $G_\rho^{ij}(x,y) = i\langle \Phi^i(x)\Phi^j(y) - \Phi^j(y)\Phi^i(x)\rangle$

Boltzmann limit

- on-shell quasi-stable particles



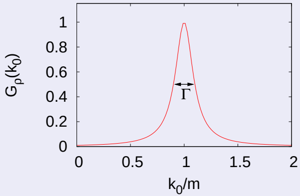
$$G_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like
  fluctuation-dissipation relation

$$G_F^{ij}(t,k) = \left(f_k^i(t) + \frac{1}{2}\right) G_\rho^{ij}(k)$$

Propagation beyond Boltzmann

- spectrum with (thermal) width



$$G_\rho^{ij}(t,k) \sim \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2\Gamma_i(t)^2} + \ldots$$

- on-/off-shell, cross-correlations

$$G_F^{ij}(t,k) = \begin{pmatrix} G_F^{11} & G_F^{12} \\ G_F^{21} & G_F^{22} \end{pmatrix}$$

Kadanoff-Baym equations

$$\left(\partial_{x^0}^2 - \nabla_x^2 + m_i^2(x)\right) G_F^{ij}(x,y) = \int_0^{y^0} d^4z\, \Pi_F^{ik}(x,z) G_\rho^{kj}(z,y) - \int_0^{x^0} d^4z\, \Pi_\rho^{ik}(x,z) G_F^{kj}(z,y)$$

$$\left(\partial_{x^0}^2 - \nabla_x^2 + m_i^2(x)\right) G_\rho^{ij}(x,y) = \int_{x_0}^{y^0} d^4z\, \Pi_\rho^{ik}(x,z) G_\rho^{kj}(z,y)$$

- Statistical propagator encodes time-evolution of the state
- Spectral function includes off-shell effects self-consistently
- Memory integrals

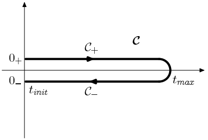
Kadanoff-Baym equations

- Obtained from stationarity condition of the 2PI effective action...

  Cornwall, Jackiw, Tomboulis (1974)

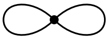
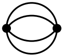
$$\frac{\delta\Gamma[\phi, G]}{\delta G} = 0 \quad \Leftrightarrow \quad G^{-1} = G_0^{-1} - \Pi[G]$$

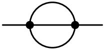
...evaluated on the closed Schwinger-Keldysh time contour



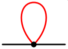
$$\begin{aligned} G(x,y) &= \langle T_{\mathcal{C}}\Phi(x)\Phi(y)\rangle \\ &= G_F(x,y) - \frac{i}{2}\,\mathrm{sign}_{\mathcal{C}}(x^0 - y^0)\,G_\rho(x,y) \end{aligned}$$

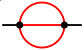
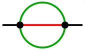
- Conserving approximation by truncation of the 2PI functional $\Gamma_2[\phi, G]$
- Example: Three-loop truncation in $\lambda\Phi^4$-theory (for $\langle\Phi\rangle = 0$)

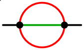
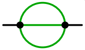
$\Gamma_2[G] =$  $+$ 

$\Pi[G] = \frac{2i\delta\Gamma_2}{\delta G} =$  $+$ 

Important: Π contains resummed propagator (self-consistent)

$$\left(\partial_{x^0}^2 + \mathbf{k}^2 + m^2 + \right.$$  $$\left. \right) G_F(x^0, y^0, \mathbf{k}) \quad =$$

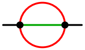
$$\int_0^{y^0} dz^0 \left(\right.$$  $$+$$  $$\left. \right) G_\rho(z^0, y^0, \mathbf{k})$$

$$- \int_0^{x^0} dz^0 \left(\right.$$  $$+$$  $$\left. \right) G_F(z^0, y^0, \mathbf{k})$$

Mixed two-time/momentum representation (spatially homogeneous)

$$G(x, y) = G(x^0, y^0, \mathbf{x} - \mathbf{y}) \quad \to \quad G(x^0, y^0, \mathbf{k}), \quad \mathbf{k} = (k_x, k_y, k_z)$$

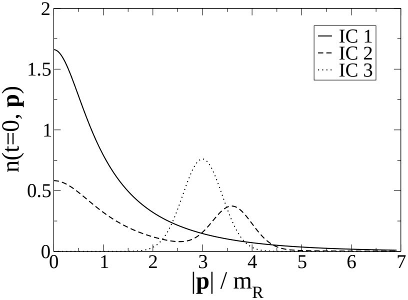
 $$= \frac{\lambda}{2} \int \frac{d^3p}{(2\pi)^3} G_F(x^0, x^0, \mathbf{p})$$

 $$= -\frac{\lambda^2}{6} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} G_F(x^0, z^0, \mathbf{p}) G_\rho(x^0, z^0, \mathbf{q}) G_F(x^0, z^0, \mathbf{k} - \mathbf{p} - \mathbf{q})$$

Numerical solution of Kadanoff-Baym equations

Initial condition (example): $\phi = \dot{\phi} = 0$,

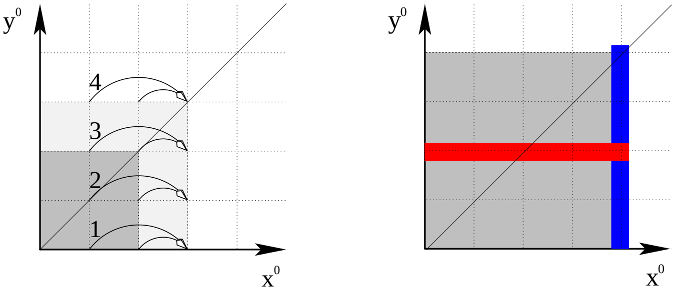
$$
\begin{aligned}
G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= \frac{n_{\mathbf{p}}(t_{init}) + 1/2}{\omega_{\mathbf{p}}(t_{init})} \\
(\partial_{x^0} + \partial_{y^0}) G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= 0 \\
\partial_{x^0}\partial_{y^0} G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= \omega_{\mathbf{p}}(t_{init}) \left(n_{\mathbf{p}}(t_{init}) + 1/2\right)
\end{aligned}
$$



$$\omega_{\mathbf{p}}(t_{init}) = \sqrt{m_R^2 + \mathbf{k}^2}$$

Numerical solution of Kadanoff-Baym equations

- Time-stepping in the two-time plane for $G(x^0, y^0, \hat{\mathbf{k}})$



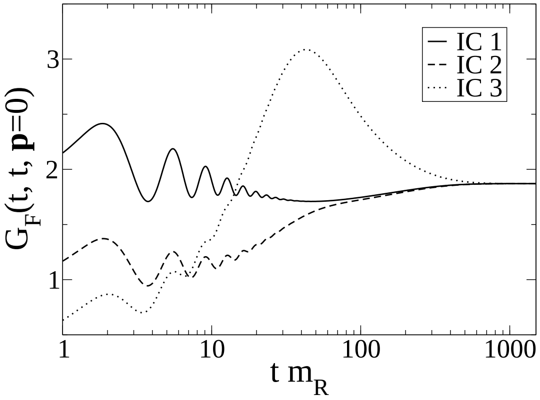
- History matrix for computing memory integrals

$$MEMINT\left(x^0, y^0, \hat{\mathbf{k}}\right) = \sum_{z^0} \Pi(x^0, z^0, \hat{\mathbf{k}})\, G(z^0, y^0, \hat{\mathbf{k}})$$

Danielewicz (1983); Köhler (1994); Berges, Cox (2001); Aarts, Berges (2002); Berges, Borsanyi, Serreau (2003); Juchem, Cassing, Greiner (2004); Arrizabalaga, Smit, Tranberg (2005); Lindner, Müller (2006); . . .

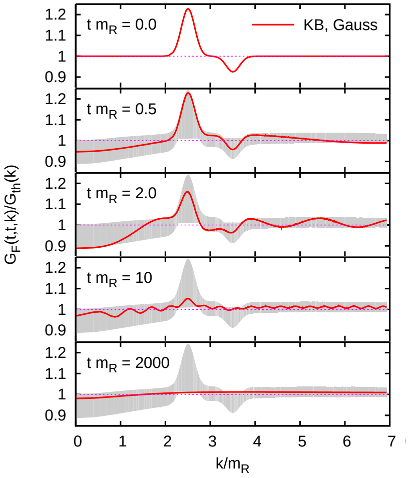
Numerical solution of Kadanoff-Baym equations

Quantum thermalization in Φ^4-theory *Lindner, Müller 2006*



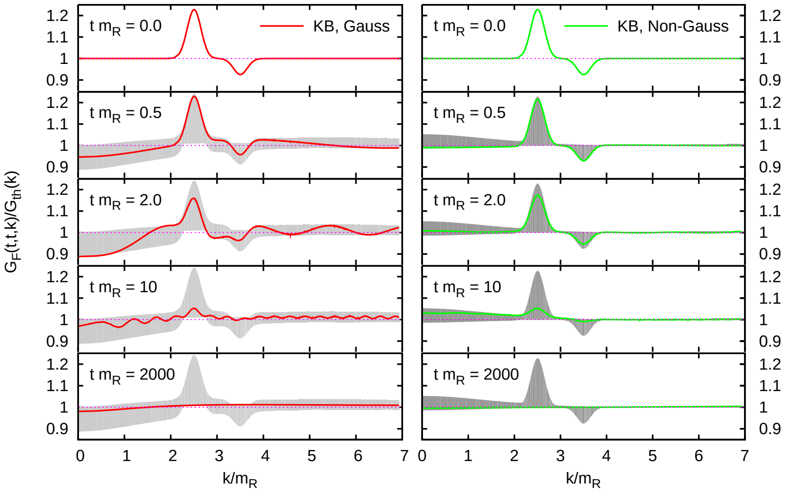
Numerical solution of Kadanoff-Baym equations

- Standard KBEs: interactions 'switched on' at $t = t_{init} \Rightarrow$ transients



Numerical solution of Kadanoff-Baym equations

- Standard KBEs: interactions 'switched on' at $t = t_{init}$ $\Rightarrow$ transients
- $\Rightarrow$ Extended KBEs with initial 4-point correlations *MG, M. Müller 2009*



CTP/Kadanoff-Baym approach to leptogenesis

Goal

- derivation of kinetic equations starting from first principles
- on-/off-shell treated in a unified way (avoid double-counting)
- thermal medium corrections, resonant leptogenesis, coherent flavor transitions

Buchmüller, Fredenhagen 00

De Simone, Riotto 07

Anisimov, Buchmüller, Drewes, Mendizabal 08,10

MG, Kartavtsev, Hohenegger, Lindner 09,10

Beneke, Fidler, Garbrecht, Herranen, Schwaller 10

CTP/Kadanoff-Baym approach to leptogenesis

Lepton current

$$j_L^\mu(x) = \left\langle \sum_\alpha \bar{\ell}_\alpha(x)\gamma^\mu \ell_\alpha(x) \right\rangle = -\mathrm{tr}\left[\gamma^\mu S_\ell{}^{\alpha\beta}(x,x)\right]$$

Lepton asymmetry

$$n_L(t) = \frac{1}{V}\int_V d^3x\, j_L^0(t,\mathbf{x})$$

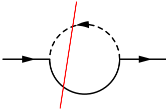
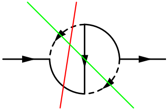
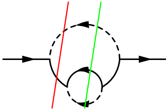
Equation of motion

$$\frac{dn_L}{dt} = \frac{1}{V}\int_V d^3x\, \partial_\mu j_L^\mu(x) = -\frac{1}{V}\int_V d^3x\, \mathrm{tr}\left[\gamma_\mu(\partial_x^\mu + \partial_y^\mu) S_\ell{}^{\alpha\beta}(x,y)\right]_{x=y}$$

Use KB equations for leptons on the right-hand side $\Rightarrow$

$$\begin{aligned}\frac{dn_L}{dt} \;=\; & i\int_0^t dt' \int \frac{d^3p}{(2\pi)^3}\, \mathrm{tr}\Big[\Sigma_{\ell\,\rho\,\mathbf{p}}^{\alpha\gamma}(t,t') S_{\ell\,F\,\mathbf{p}}^{\gamma\beta}(t',t) - \Sigma_{\ell\,F\,\mathbf{p}}^{\alpha\gamma}(t,t') S_{\ell\,\rho\,\mathbf{p}}^{\gamma\beta}(t',t) \\ & - S_{\ell\,\rho\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell\,F\,\mathbf{p}}^{\gamma\beta}(t',t) + S_{\ell\,F\,\mathbf{p}}^{\alpha\gamma}(t,t')\Sigma_{\ell\,\rho\,\mathbf{p}}^{\gamma\beta}(t',t)\Big]\end{aligned}$$

CTP/Kadanoff-Baym approach to leptogenesis

			
$N \leftrightarrow \ell\phi^\dagger$ $N \leftrightarrow \ell^c\phi$	$\lvert tree \rvert^2$	tree × vertex-corr.	tree × wave-corr.
$\ell\phi^\dagger \leftrightarrow \ell^c\phi$		s × t, s × u, t × u	s × s, t × t, u × u

- unified description of CP-violating decay, inverse decay, scattering
- dn_L/dt vanishes in equilibrium due to KMS relations

$$S_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) S_\rho^{eq} \qquad \Sigma_F^{eq} = \frac{1}{2}\tanh\left(\frac{\beta k^0}{2}\right) \Sigma_\rho^{eq}$$

⇒ consistent equations free of double-counting problems

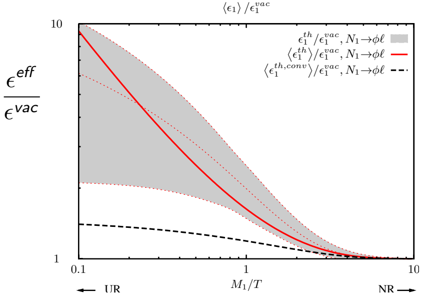
MG, Kartavtsev, Hohenegger, Lindner 09;

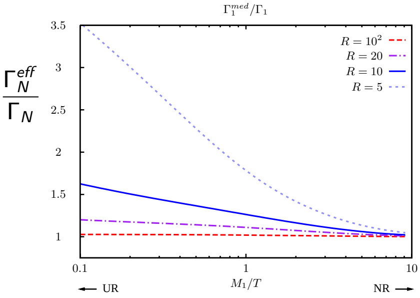
Beneke, Garbrecht, Herranen, Schwaller 10;

CTP/Kadanoff-Baym approach to leptogenesis

Quantum-corrected Boltzmann equations *MG, Kartavtsev, Hohenegger, Lindner 2010*

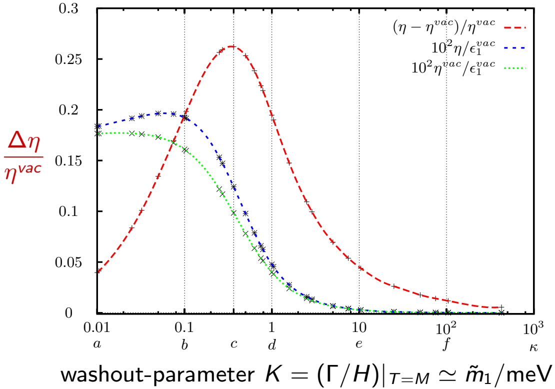
$$\epsilon^{vac} \mapsto \epsilon^{eff}(p, \ldots, T) \quad \Gamma_N \mapsto \Gamma_N^{eff}(p, \ldots, T)$$





- Enhancement of the effective CP-violating parameter
- Applicable in hierarchical and mildly degenerate case $M_2 - M_1 \gg \Gamma_i$

Hierarchical case



thermal initial abundance

MG, Kartavtsev, Hohenegger, Lindner 10

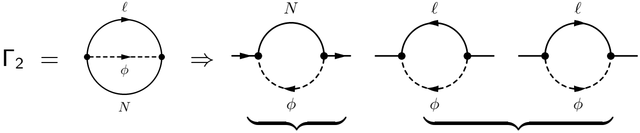
Resonant enhancement

- Statistical propagator S_F and spectral function S_ρ are matrices in N_1, N_2, N_3 flavor space. We consider the sub-space N_1, N_2 of the quasi-degenerate states.

$$S^{ij}(x,y) = \langle T_{\mathcal{C}} N_i(x) \bar{N}_j(y) \rangle = \begin{pmatrix} S^{11} & S^{12} \\ S^{21} & S^{22} \end{pmatrix}$$

$\Rightarrow$ coherent N_1–N_2 transitions out-of-equilibrium

- Self-energies for leptons and for Majorana neutrinos



$$\Sigma_\ell{}^{\alpha\beta}(x,y) = \frac{\partial i\Gamma_2}{\partial S_\ell{}^{\beta\alpha}(y,x)} \qquad \Sigma_N{}^{ij}(x,y) = \frac{\partial i\Gamma_2}{\partial S^{ji}(y,x)}$$

- Solve KBEs in Breit-Wigner approximation treating lepton and Higgs as a thermal bath *[hierarchical case: Anisimov, Buchmüller, Drewes, Mendizabal 08,10]*
- Important: lepton self-energy contains full Majorana propagator-matrix

Resonant enhancement

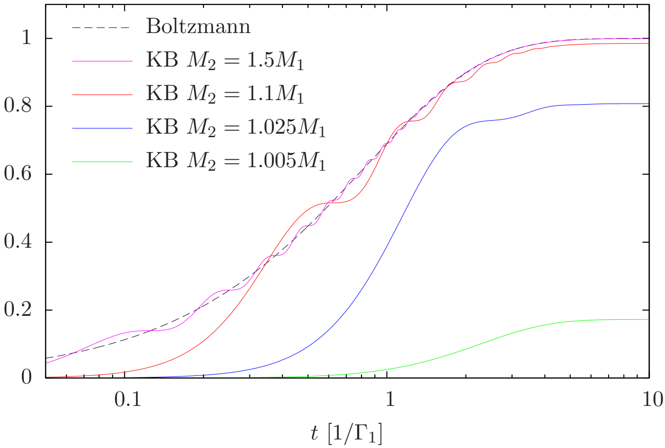
Kadanoff-Baym Equations

$$
\begin{aligned}
(i\not\partial_x - M_i) S_F^{ij}(x,y) &= \int_0^{x^0} dz^0 \int d^3z \, \Sigma_{N\rho}^{ik}(x,z) S_F^{kj}(z,y) \\
&\quad - \int_0^{y^0} dz^0 \int d^3z \, \Sigma_{NF}^{ik}(x,z) S_\rho^{kj}(z,y) \\
(i\not\partial_x - M_i) S_\rho^{ij}(x,y) &= \int_{y^0}^{x^0} dz^0 \int d^3z \, \Sigma_{N\rho}^{ik}(x,z) S_\rho^{kj}(z,y)
\end{aligned}
$$

Resonant enhancement

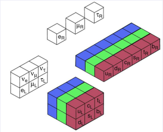
MG, Kartavtsev, Hohenegger 11xx.yyyy

$n_L(t, \mathbf{p})/n_L^{Boltzmann}(t = \infty, \mathbf{p})$



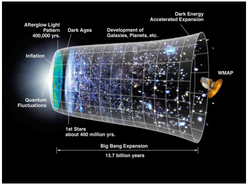
$n_L(t) = \int \frac{d^3p}{(2\pi)^3}\, n_L(t, \mathbf{p}), \Gamma_1 = 0.01 M_1, \Gamma_2 = 0.015 M_1, \Gamma_{\ell\phi} \to 0$

Conclusions

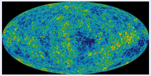


$+ \, ?$

$\leftrightarrow$



$\leftrightarrow$



$\tilde{m}, \theta_{13}, \sigma v, \ldots$

microscopic description of out of equilibrium processes in the Early Universe

$\eta_b, \Omega_{dm}, n_s, \ldots$

- First-principles methods like Kadanoff-Baym equations are important to describe quantum effects and to scrutinize classical approximations
- Leptogenesis
  - resolve double counting issues
  - quantum-corrected Boltzmann equations
  - size of the resonant enhancement

thank you!

Thermal bath limit

Analytical solution in the hierarchical case $M_1 \ll M_2$

Anisimov, Buchmüller, Drewes, Mendizabal 08,10 Phys.Rev.Lett. 104 (2010) 121102

$$
\begin{aligned}
S_{N,\mathcal{A}}(\Delta t) &= \left(i\gamma_0 \cos(\omega \Delta t) + \frac{M - \mathbf{p}\gamma}{\omega} \sin(\omega \Delta t) \right) e^{-\Gamma|\Delta t|/2} \\
S_{N,F}(t, \Delta t) &= -\left(i\gamma_0 \cos(\omega \Delta t) - \frac{M - \mathbf{p}\gamma}{\omega} \sin(\omega \Delta t) \right) \\
&\quad \times \Big[\underbrace{\frac{\tanh(\frac{\beta\omega}{2})}{2} e^{-\Gamma|\Delta t|/2}}_{\substack{\text{Equilibrium} \\ \text{Damped w.r.t } \Delta t}} - \underbrace{\delta f_N(\omega) e^{-\Gamma t}}_{\substack{\text{Non-equilibrium} \\ \text{Undamped w.r.t } \Delta t}} \Big]
\end{aligned}
$$

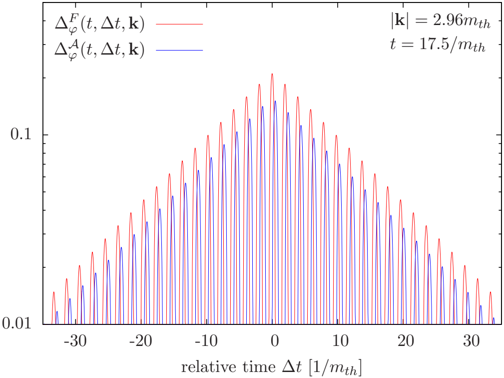
$\Delta t = x^0 - y^0$, $t = (x^0 + y^0)/2$

$S_{\mathcal{A}} = i(S_> - S_<)/2, S_F = (S_> + S_<)/2$

Thermal bath limit

Comparison with a full numerical solution of the two-time KBEs (for ϕ^4-theory)

Garbrecht, MG 2011

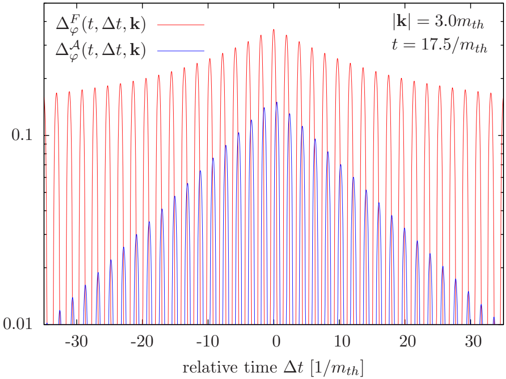


Statistical propagator $\Delta_\varphi^F = (\Delta_\varphi^> + \Delta_\varphi^<)/2$ (red) and spectral function $\Delta_\varphi^{\mathcal{A}} = \frac{\mathrm{i}}{2}(\Delta_\varphi^> - \Delta_\varphi^<)$ (blue) obtained from a numerical solution of the KBEs .

Dependence on the relative time Δt for fixed central time t.

Thermal bath limit

Garbrecht, MG 2011

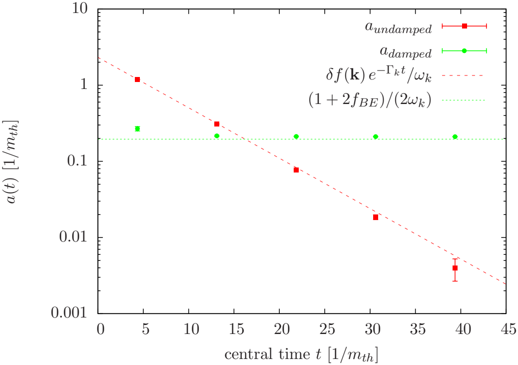


Excited momentum mode $|\mathbf{k}| = 3.0 m_{th}$. The statistical propagator $\Delta_\varphi^F = (\Delta_\varphi^> + \Delta_\varphi^<)/2$ can be described by the sum of an exponentially damped equilibrium contribution and an undamped non-equilibrium contribution.

Thermal bath limit

Evolution of damped and un-damped components with the central time t

Garbrecht, MG 2011



$$\Delta^F_{\varphi,fit} \equiv (a_{damped}(t)e^{-\Gamma_k|\Delta t|/2} + a_{undamped}(t))\cos(\omega_k \Delta t)$$