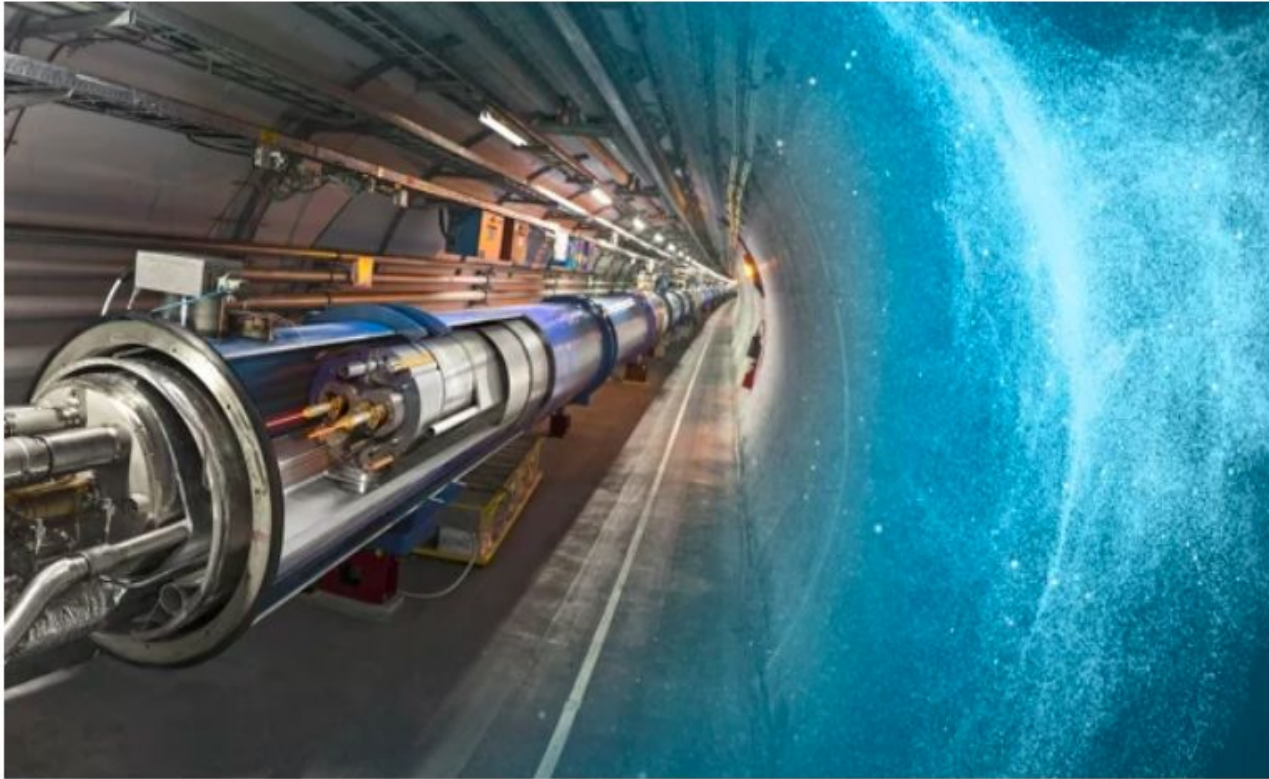


# Introduction to the Standard Model

## Summer Student Lecture 2025 – Part I



Clara Leitgeb

Deutsches  
Elektronen  
Synchrotron

11<sup>th</sup>-13<sup>th</sup> August 2025

# Preface

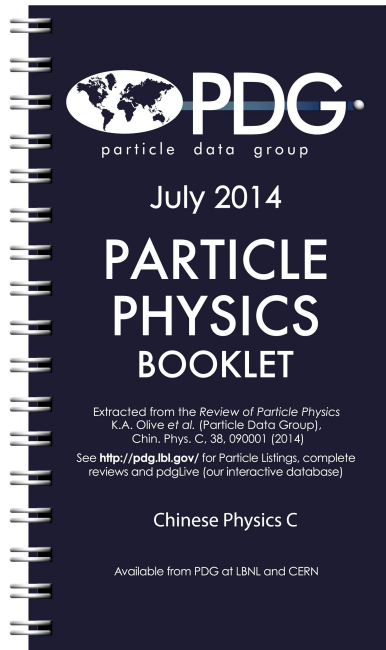
- > Concentrating on *general concepts and a broad overview*.
- > This lecture cannot replace a university course on particle physics
- > Very different level of knowledge: Hard to devise a course that fits all....
  - Some parts more interesting to beginners, some more to the advanced
    - You will see many formulas ! Don't panic and don't focus on them. They are there to support a physics explanation or just to give you a glimpse of what is the theory behind all these concepts (i.e. show that something exists even if we won't talk in depth about it).
- > **Please do ask questions!**
- > **If you have any further question, my email is : [leitgebc@hu-berlin.de](mailto:leitgebc@hu-berlin.de)**



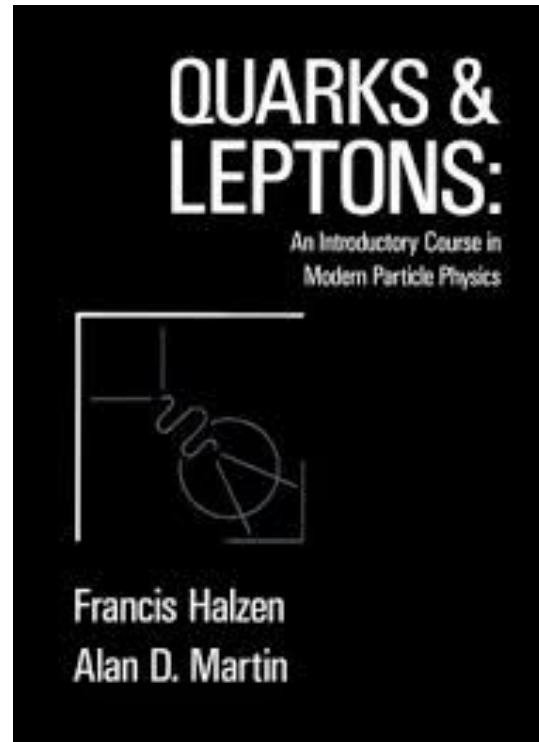
# Literature

> <http://pdg.lbl.gov/>

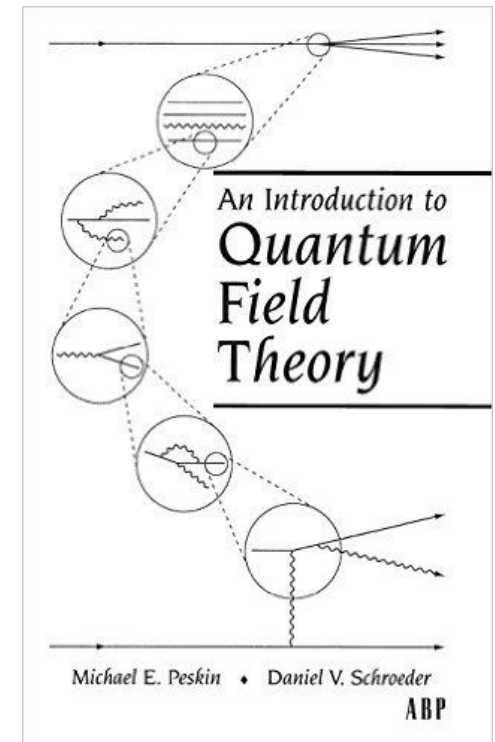
## The summary



## The book



## The BIBLE



## >0) Introduction

- What is the Standard Model?
- Coupling constants, masses and charges
- Units and scales

## >1) Interactions

- Relativistic kinematics
- Symmetries and conserved quantities
- Feynman diagrams
- Running couplings and masses

## >2) Quantum electrodynamics

- Test of QED: Magnetic momentum of the muon
- Test of QED: High energy colliders





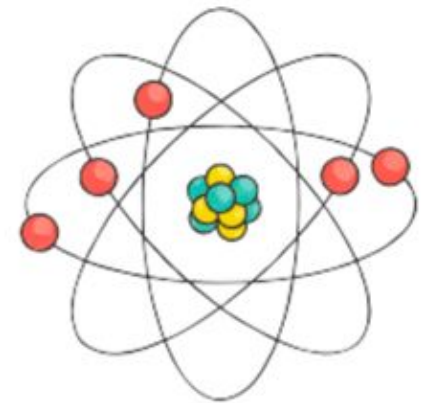
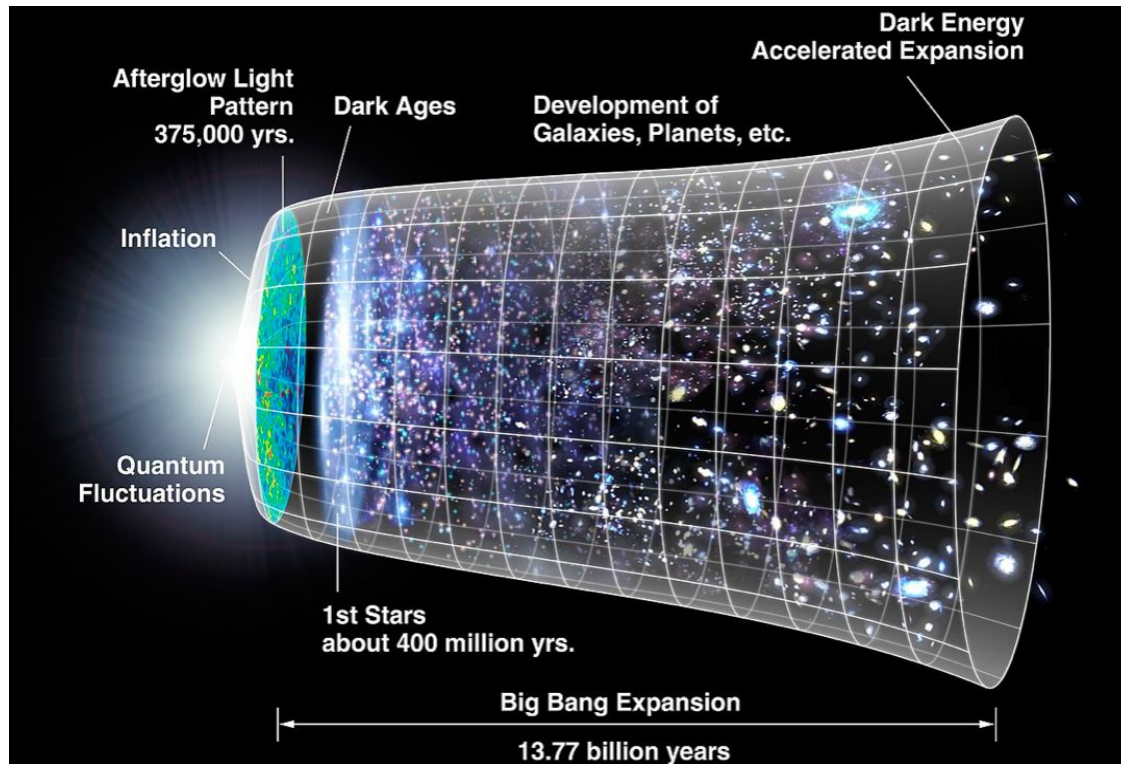
- >3) Strong Interaction: Quantum-Chromodynamics
  - A short history of hadrons and quarks
  - DIS and gluons
  - QCD and its properties
  
- >4) Electroweak interactions
  - Discovery of electroweak bosons
  - Tests of angular distributions
  - Feynman rules
  - Handed-ness of electroweak interactions
  - More tests of the electroweak SM
  
- >5) The Higgs
  - Why was it predicted?
  - How was it found?



# Introduction

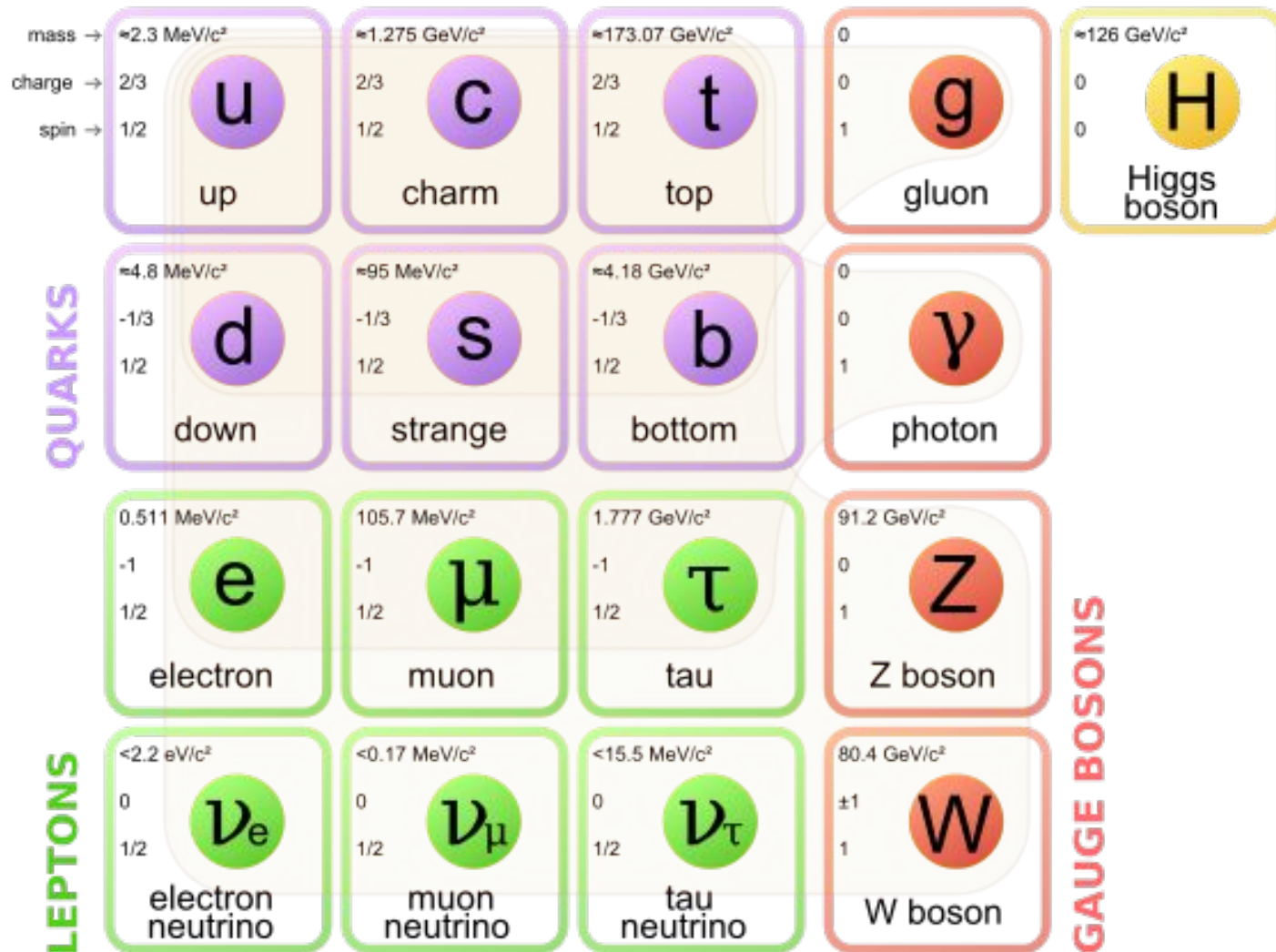
# Introduction to the Standard Model

From atoms to the galaxies and clusters, the history of our Universe and its evolution is determined by the presence of particles and the interactions amongst them



During these days, we will try to summarize our current knowledge of the most basic components of Nature and their interactions

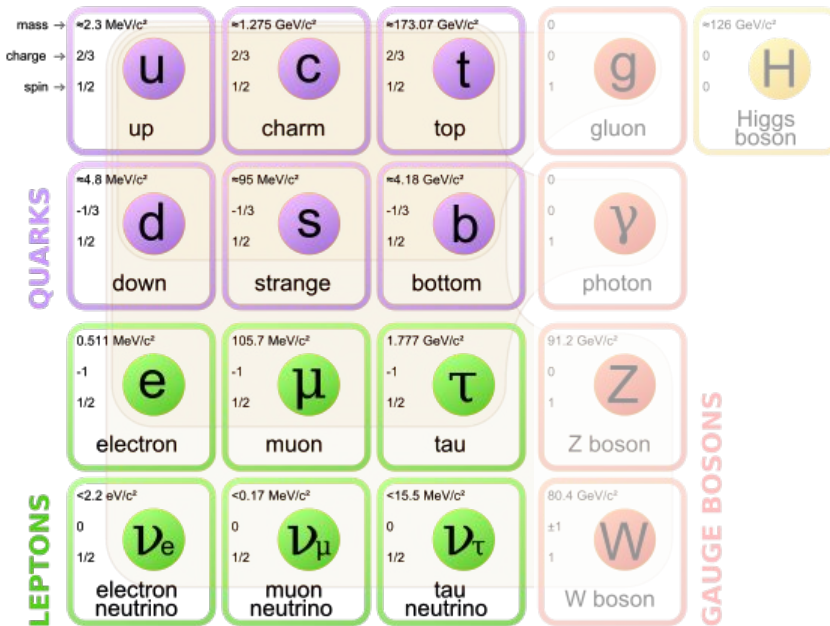
# What is the Standard Model?



>picture from wikipedia

# What is the Standard Model?

➤ Fields described by the Standard Model can be classified as



## Matter fields

These are the fields of which most of the traditional particles (protons, neutrons, electrons) that we know are composed

- **Leptons:** don't interact with strong force
- **Quarks:** interact with strong force
- Spin  $1/2$
- 3 Generations or flavours



# What is the Standard Model?

➤ Fields described by the Standard Model can be classified as

QUARKS	mass → charge → spin →	$\approx 2.3 \text{ MeV}/c^2$ 2/3 1/2 <b>u</b> up	$\approx 1.275 \text{ GeV}/c^2$ 2/3 1/2 <b>c</b> charm	$\approx 173.07 \text{ GeV}/c^2$ 2/3 1/2 <b>t</b> top
LEPTONS				

## Matter fields

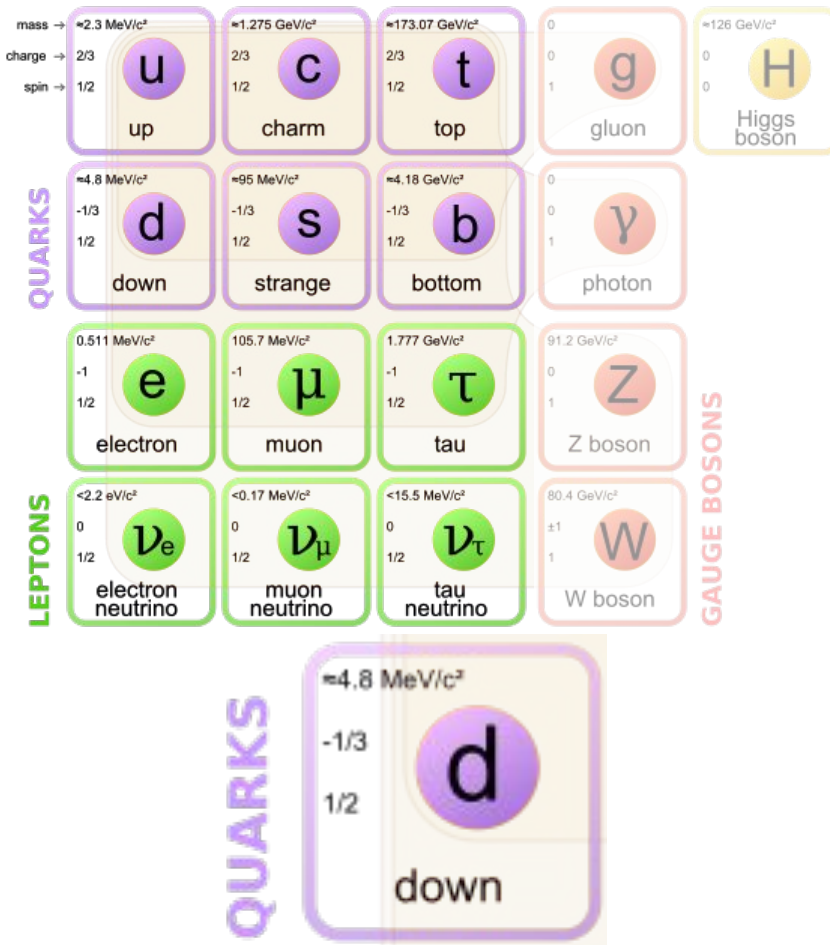
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## Particle properties

- **Baryon number**
- **Lepton number**
- Electric charge, weak isospin and color
- Mass

+Isospin, strangeness, charm, bottomness, topness





# What is the Standard Model?

➤ Fields described by the Standard Model can be classified as

mass →	≈2.3 MeV/c <sup>2</sup>	≈1.275 GeV/c <sup>2</sup>	≈173.07 GeV/c <sup>2</sup>	0	≈126 GeV/c <sup>2</sup>
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS	≈4.8 MeV/c <sup>2</sup>	≈95 MeV/c <sup>2</sup>	≈4.18 GeV/c <sup>2</sup>	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	γ photon	
LEPTONS	0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	0	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
	<2.2 eV/c <sup>2</sup>	<0.17 MeV/c <sup>2</sup>	<15.5 MeV/c <sup>2</sup>	±1	
	0	0	0	1	
	1/2	1/2	1/2	1	
	ν <sub>e</sub> electron neutrino	ν <sub>μ</sub> muon neutrino	ν <sub>τ</sub> tau neutrino	W W boson	
					GAUGE BOSONS

## Force fields

These are the fields exchanged in the interactions between matter fields

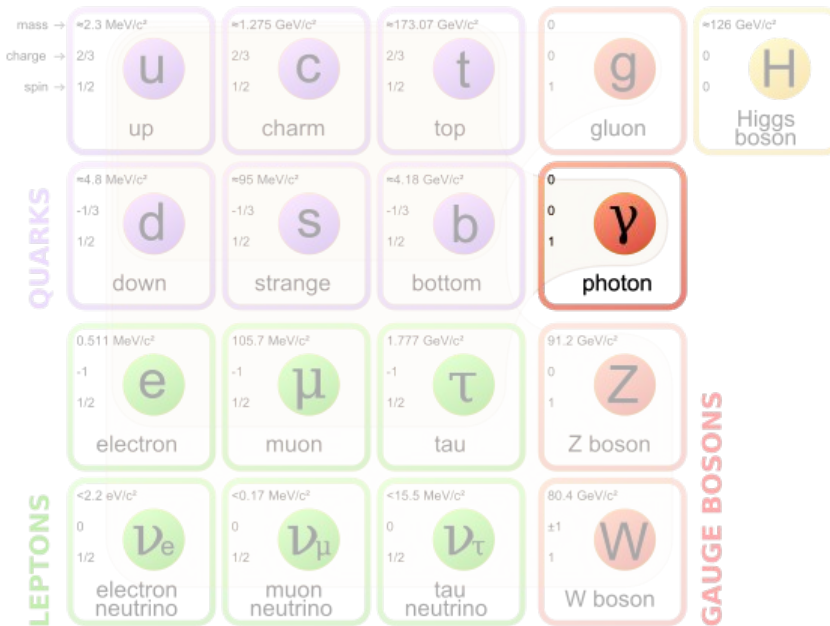
- Gauge bosons
- Spin 1
- Mediator of interactions between particles that are charged under the treated interaction

Three forces currently described by the SM



# What is the Standard Model?

- Fields described by the Standard Model can be classified as

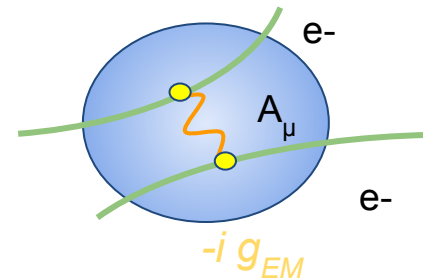
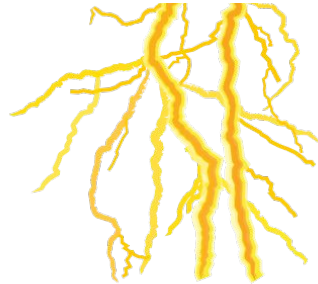


# Force fields

These are the fields exchanged in the interactions between matter fields

- Gauge bosons
- Spin 1
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## Three forces currently described by the SM



## Electromagnetic interaction

Gauge boson: photon ( $A_\mu$ )

Possible between fields that possess electromagnetic charge ( $g_{EM}$ )

## Quantum electrodynamics (QED)

# What is the Standard Model?

➤ Fields described by the Standard Model can be classified as

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	d down	s strange	b bottom	γ photon	
	0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	91.2 GeV/c <sup>2</sup>	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS	<2.2 eV/c <sup>2</sup>	<0.17 MeV/c <sup>2</sup>	<15.5 MeV/c <sup>2</sup>	80.4 GeV/c <sup>2</sup>	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν <sub>e</sub> electron neutrino	ν <sub>μ</sub> muon neutrino	ν <sub>τ</sub> tau neutrino	W W boson	
					GAUGE BOSONS

## Force fields

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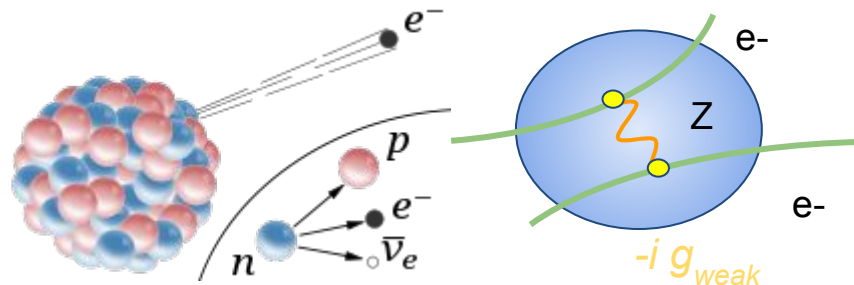
Three forces currently described by the SM

## Weak interaction

Gauge bosons:  $W^+$ ,  $W^-$ ,  $Z^0$  ( $W_\mu$ )

Possible between fields that possess Isospin and/or hypercharge ( $g_{\text{weak}}$ )

GSW mechanism



# What is the Standard Model?

➤ Fields described by the Standard Model can be classified as

mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.07 \text{ GeV}/c^2$	0	$\approx 126 \text{ GeV}/c^2$
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	$\gamma$ photon	
LEPTONS	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	0	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	$\mu$ muon	$\tau$ tau	Z Z boson	
	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$\pm 1$	
	0	0	0	1	
	1/2	1/2	1/2	1	
	$\nu_e$ electron neutrino	$\nu_\mu$ muon neutrino	$\nu_\tau$ tau neutrino	W W boson	
					GAUGE BOSONS

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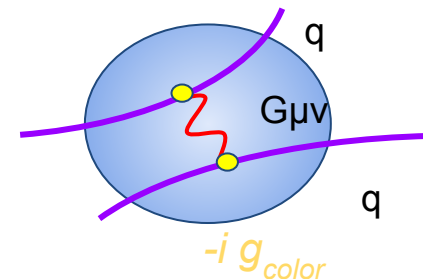
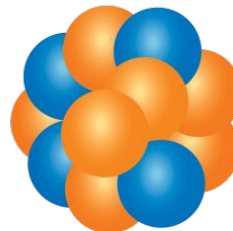
Three forces currently described by the SM

## Strong interaction

Gauge bosons: gluons

Possible between fields that possess color charge ( $g_{\text{color}}$ )

Quantum chromodynamics (QCD)



# What is the Standard Model?

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	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
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LEPTONS	0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	91.2 GeV/c <sup>2</sup>	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
GAUGE BOSONS	<2.2 eV/c <sup>2</sup>	<0.17 MeV/c <sup>2</sup>	<15.5 MeV/c <sup>2</sup>	80.4 GeV/c <sup>2</sup>	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν <sub>e</sub> electron neutrino	ν <sub>μ</sub> muon neutrino	ν <sub>τ</sub> tau neutrino	W W boson	

## Higgs field

Particles in the Standard Model obtain masses via interaction with the Higgs field.

- Excited quanta : Higgs boson
- Spin 0
- The strength of the coupling between the Higgs field is the reason of the mass hierarchy in SM
  - Larger coupling → higher mass of the particle.



# What is the Standard Model?

- Fields described by the Standard Model can be classified as

**Standard Model of Elementary Particles**

	three generations of matter (elementary fermions)			three generations of antimatter (elementary antifermions)			interactions / force carriers (elementary bosons)	
	I	II	III	I	II	III		
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b><math>\bar{u}</math></b> antiup	<b><math>\bar{c}</math></b> anticharm	<b><math>\bar{t}</math></b> antitop	<b>g</b> gluon	<b>H</b> higgs
<b>QUARKS</b>	$\approx 4.7 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ <b>d</b> down	$\approx 96 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ <b>s</b> strange	$\approx 4.18 \text{ GeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$ <b>b</b> bottom	$\approx 4.7 \text{ MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ <b><math>\bar{d}</math></b> antidown	$\approx 96 \text{ MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ <b><math>\bar{s}</math></b> antistrange	$\approx 4.18 \text{ GeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$ <b><math>\bar{b}</math></b> antibottom	0 0 1 <b><math>\gamma</math></b> photon	
	$\approx 0.511 \text{ MeV}/c^2$ -1 $\frac{1}{2}$ <b>e</b> electron	$\approx 105.66 \text{ MeV}/c^2$ -1 $\frac{1}{2}$ <b><math>\mu</math></b> muon	$\approx 1.7768 \text{ GeV}/c^2$ -1 $\frac{1}{2}$ <b><math>\tau</math></b> tau	$\approx 0.511 \text{ MeV}/c^2$ 1 $\frac{1}{2}$ <b><math>e^+</math></b> positron	$\approx 105.66 \text{ MeV}/c^2$ 1 $\frac{1}{2}$ <b><math>\bar{\mu}</math></b> antimuon	$\approx 1.7768 \text{ GeV}/c^2$ 1 $\frac{1}{2}$ <b><math>\bar{\tau}</math></b> antitau	$\approx 91.19 \text{ GeV}/c^2$ 0 1 <b>Z</b> Z <sup>0</sup> boson	<b>GAUGE BOSONS</b> VECTOR BOSONS
<b>LEPTONS</b>	$< 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ <b><math>\nu_e</math></b> electron neutrino	$< 0.17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ <b><math>\nu_\mu</math></b> muon neutrino	$< 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ <b><math>\nu_\tau</math></b> tau neutrino	$< 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$ <b><math>\bar{\nu}_e</math></b> electron antineutrino	$< 0.17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ <b><math>\bar{\nu}_\mu</math></b> muon antineutrino	$< 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$ <b><math>\bar{\nu}_\tau</math></b> tau antineutrino	$\approx 80.39 \text{ GeV}/c^2$ 1 1 <b><math>W^+</math></b> W <sup>+</sup> boson	$\approx 80.39 \text{ GeV}/c^2$ -1 1 <b><math>W^-</math></b> W <sup>-</sup> boson
								<b>SCALAR BOSONS</b>

## Particles and anti-particles

Each of the particles in the SM has its anti-particle

- Same mass
- Same flavour but opposite lepton or baryon number
- Same spin
- Color → anti-color ; negative electric charge → positive electric charge



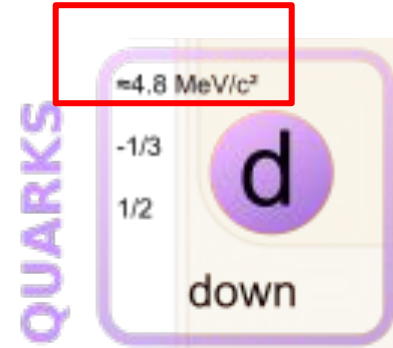
# Units and scales

> “natural units”  $\rightarrow c = 1 \quad \hbar = 1$

(masses, energies and momenta measured in GeV)

Conventional Mass, Length, Time Units, and Positron Charge in Terms of  $\hbar = c = 1$  Energy Units

Conversion Factor	$\hbar = c = 1$ Units	Actual Dimension
$1 \text{ kg} = 5.61 \times 10^{26} \text{ GeV}$	GeV	$\frac{\text{GeV}}{c^2}$
$1 \text{ m} = 5.07 \times 10^{15} \text{ GeV}^{-1}$	$\text{GeV}^{-1}$	$\frac{\hbar c}{\text{GeV}}$
$1 \text{ sec} = 1.52 \times 10^{24} \text{ GeV}^{-1}$	$\text{GeV}^{-1}$	$\frac{\hbar}{\text{GeV}}$
$e = \sqrt{4\pi\alpha}$	—	$(\hbar c)^{1/2}$



## Some Useful Conversion Factors

$$1 \text{ TeV} = 10^3 \text{ GeV} = 10^6 \text{ MeV} = 10^9 \text{ KeV} = 10^{12} \text{ eV}$$

$$1 \text{ fermi} \equiv 1 \text{ F} = 10^{-13} \text{ cm} = 5.07 \text{ GeV}^{-1}$$

$$(1 \text{ F})^2 = 10 \text{ mb} = 10^4 \mu\text{b} = 10^7 \text{ nb} = 10^{10} \text{ pb}$$

$$(1 \text{ GeV})^{-2} = 0.389 \text{ mb}$$

[Taken from:

Quarks and Leptons:

An Introductory Course in Modern Particle Physics

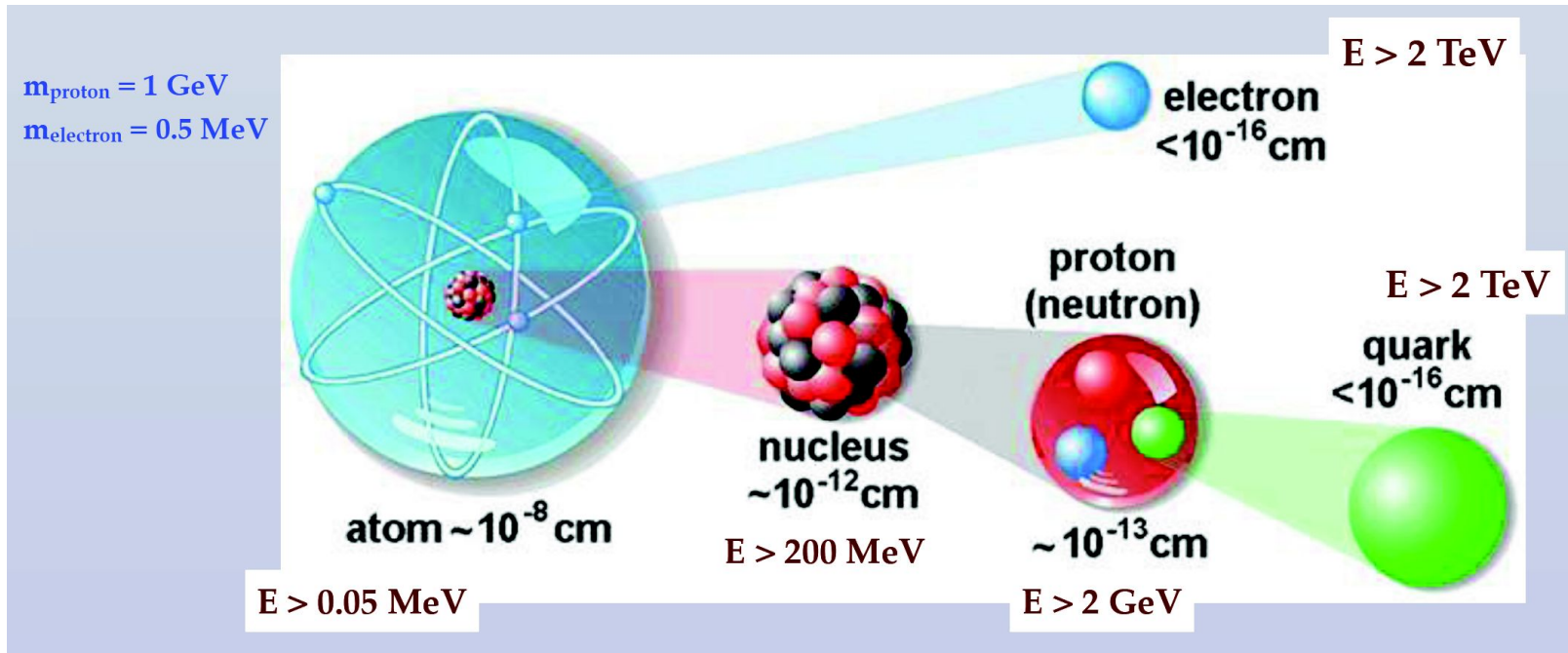
[Francis Halzen/Alan D. Martin](#) ]

$$\hbar c \sim 197 \text{ MeV fm}$$





# Units and scales



$$1 \text{ eV} = 1.602 \cdot 10^{-19} \text{ J}$$

$$\text{keV} = 10^3 \text{ eV}$$

$$\text{MeV} = 10^6 \text{ eV}$$

$$\text{GeV} = 10^9 \text{ eV}$$

$$\text{TeV} = 10^{12} \text{ eV}.$$

$$m_e = 511 \text{ keV}$$

$$m_p = 938 \text{ MeV}$$

$$m_n = 939 \text{ MeV}$$

$$E_e(\text{LEP}) = 104.5 \text{ GeV}$$

$$E_p(\text{Tevatron}) = 980 \text{ GeV}$$

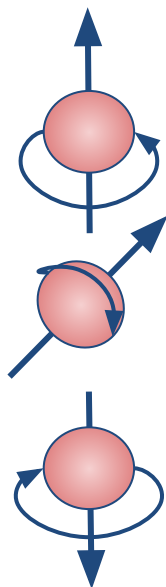
$$E_p(\text{LHC}) = 7 \text{ TeV}.$$

**Basic blocks of the Standard  
Model: conserved quantities,  
interactions, Feynman !**

# Let's start easy: what is a particle in QM ?

In your quantum mechanics courses, a particle is described by a wave-function.

$$|\psi\rangle = \Psi(\underbrace{x, p, t}_{\text{Momentum (or position) and time}}; \underbrace{S, S_Z}_{\text{Spin}}; \underbrace{L, L_z}_{\text{Angular momentum}}; ..)$$



Spin

$$|\uparrow\rangle$$

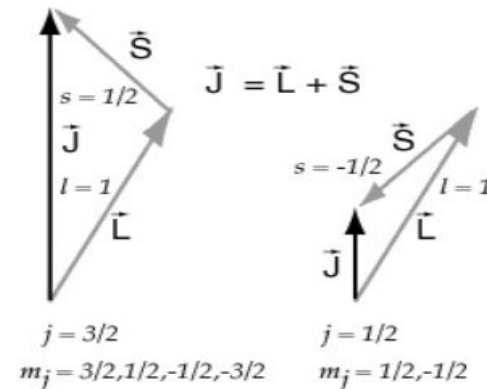
$$\frac{1}{\sqrt{2}} |\uparrow\rangle \pm \frac{1}{\sqrt{2}} |\downarrow\rangle$$

$$|\downarrow\rangle$$

Momentum (or position) and time

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x) e^{-ipx/\hbar} dx$$

Angular momentum



+ other properties

# Let's start easy: what is a particle in QM ?

The evolution of the wave-function is governed by Schroedinger equation

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H} |\Psi\rangle$$

Harmonic oscillator of 1 particle system

Hamiltonian

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2 \hat{X}^2$$

$$\hat{H} = \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \hbar\omega$$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \hat{X} + \frac{i}{\sqrt{2m\omega\hbar}} \hat{P}$$

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \hat{X} - \frac{i}{\sqrt{2m\omega\hbar}} \hat{P}$$

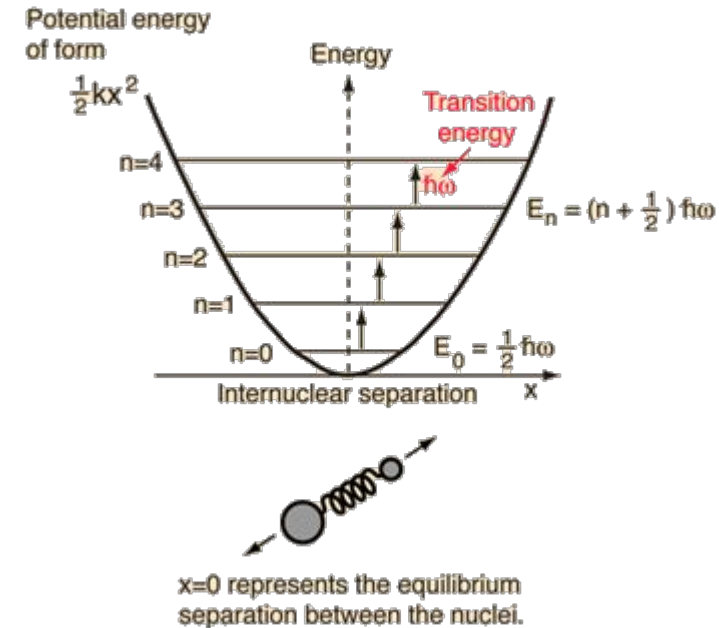
Creation and annihilation

$$\hat{a} |n\rangle = c_n |n-1\rangle$$

$$\hat{a}^\dagger |n\rangle = d_n |n+1\rangle$$

$$\hat{H} |n-1\rangle = E_{n-1} |n-1\rangle = (E_n - \hbar\omega) |n-1\rangle$$

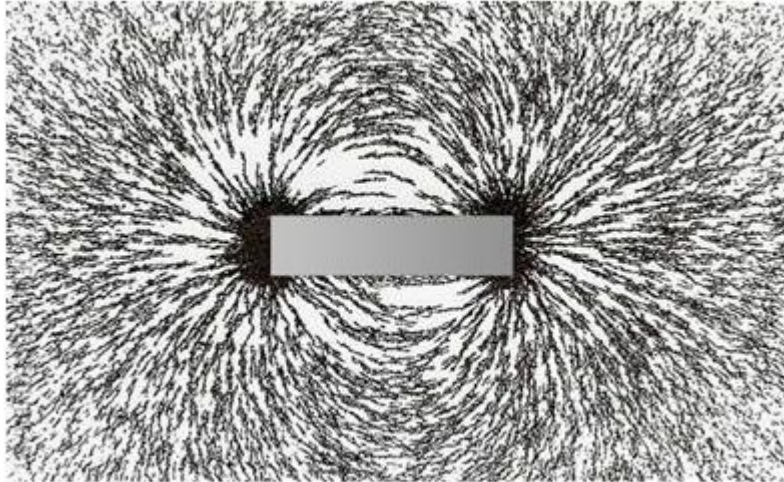
$$\hat{H} |n+1\rangle = E_{n+1} |n+1\rangle = (E_n + \hbar\omega) |n+1\rangle$$



Energy is quantized. Ground state of minimal energy and excited states.

# Let's start easy: what is an interaction in QM ?

Interactions between particles, via classical continuous fields.



## Classical field

Continuous

Mainly describing forces

Following field equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

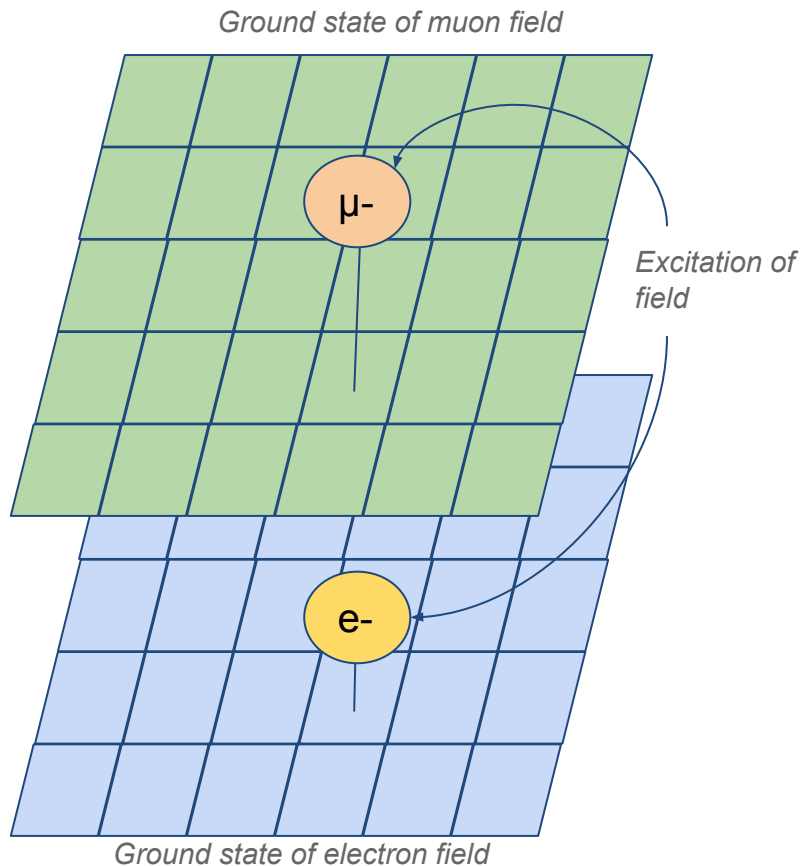
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$$



# Let's start easy: what is a particle ?

In modern particle physics, particles are just excited states of an underlying quantized field !

## Second quantization



$$|\psi(\mathbf{x})\rangle = \phi(\mathbf{x}) |0\rangle$$

$$\phi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left( a_p + a_{-p}^\dagger \right) e^{i\mathbf{p}\cdot\mathbf{x}}$$

Free scalar field for particle creation at position  $x$

- > One field  $\rightarrow$  one particle type
- > Matter fields and force fields
- > Discrete (quantized), not continuous

## Bosonic fields (integer spin)

$$\left[ a_p, a_{p'}^\dagger \right] = i (2\pi)^3 \delta^3 (p - p')$$

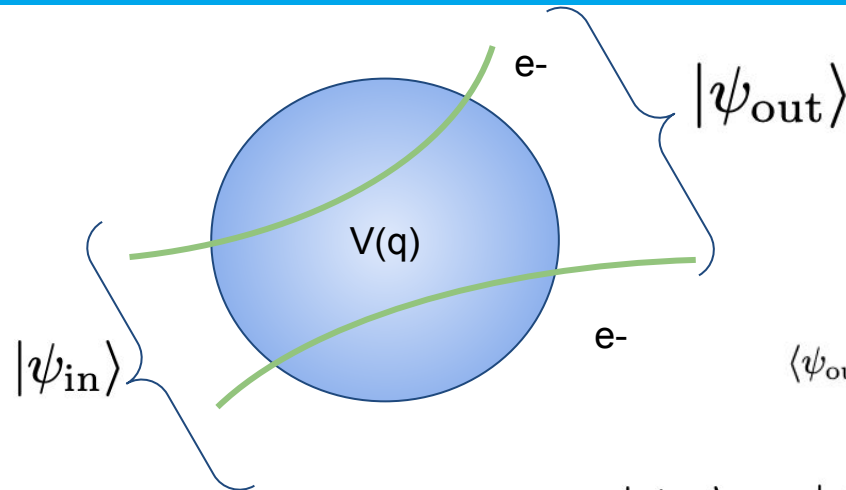
## Fermionic fields (semi-integer spin)

$$\left\{ a_p, b_{p'}^\dagger \right\} = i (2\pi)^3 \delta^3 (p - p')$$

# Let's start easy: what is an interaction between particles ?

## Interactions in QM

In QM, encoded in potential terms in Hamiltonian



$$\langle \psi_{out} | \hat{H} | \psi_{in} \rangle = \langle \psi_{out} | H_0 + H_I | \psi_{in} \rangle = \langle \psi_{out} | -\frac{\hbar^2}{2m} \nabla^2 + V(q) | \psi_{in} \rangle$$

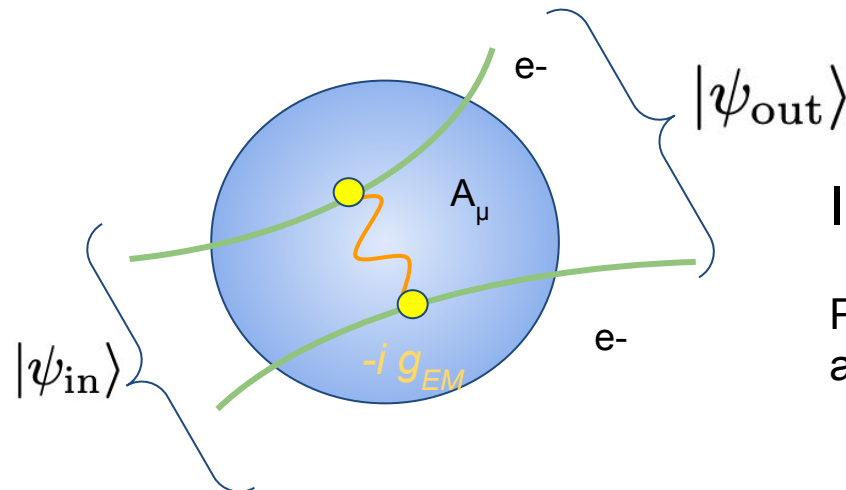
Probability of interaction so state  $|\psi_{in}\rangle \neq |\psi_{out}\rangle$  ?

$$\mathcal{T} \propto |\langle \psi_{out} | V(q) | \psi_{in} \rangle|^2$$

## Forces/interactions in QFT

Interactions are exchanges of “force” fields

Possible between fields that have a coupling/charge associated to this force field in Lagrangian



Probability of interaction so state  $|\psi_{in}\rangle \neq |\psi_{out}\rangle$  ?

$$i\mathcal{M} \propto |g_{EM}^2 \langle \psi_{out} | A_\mu | \psi_{in} \rangle|^2$$



# Let's start easy: what is an interaction between particles ?

## Interactions in QM

In QM. encoded in potential terms in

Quantum field theory is the theoretical framework we use to describe dynamics and interactions of different fields

$$+ V(q) |\psi_{\text{in}}\rangle$$

$|\psi_{\text{in}}\rangle$

$|\psi_{\text{out}}\rangle$

$e^-$

Probability of interaction so state  $|\psi_{\text{in}}\rangle \neq |\psi_{\text{out}}\rangle$  ?

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$|\psi_{\text{in}}\rangle$

$|\psi_{\text{out}}\rangle$

$e^-$

$e^-$

$A_\mu$

$-i g_{EM}$

Probability of interaction so state  $|\psi_{\text{in}}\rangle \neq |\psi_{\text{out}}\rangle$  ?

$$i\mathcal{M} \propto |g_{EM}^2 \langle \psi_{\text{out}} | A_\mu | \psi_{\text{in}} \rangle|^2$$

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$$\mathcal{T} \propto |\langle \psi_{\text{out}} | V(q) | \psi_{\text{in}} \rangle|^2$$

Probability of interaction so state  $|\psi_{\text{in}}\rangle \neq |\psi_{\text{out}}\rangle$  ?

## Forces/interactions in QFT

The Standard Model is the theory that describes all elementary known particles and their interactions.

fields

/charge

$$i\mathcal{M} \propto |g_{\text{EM}}^2 \langle \psi_{\text{out}} | A_\mu | \psi_{\text{in}} \rangle|^2$$

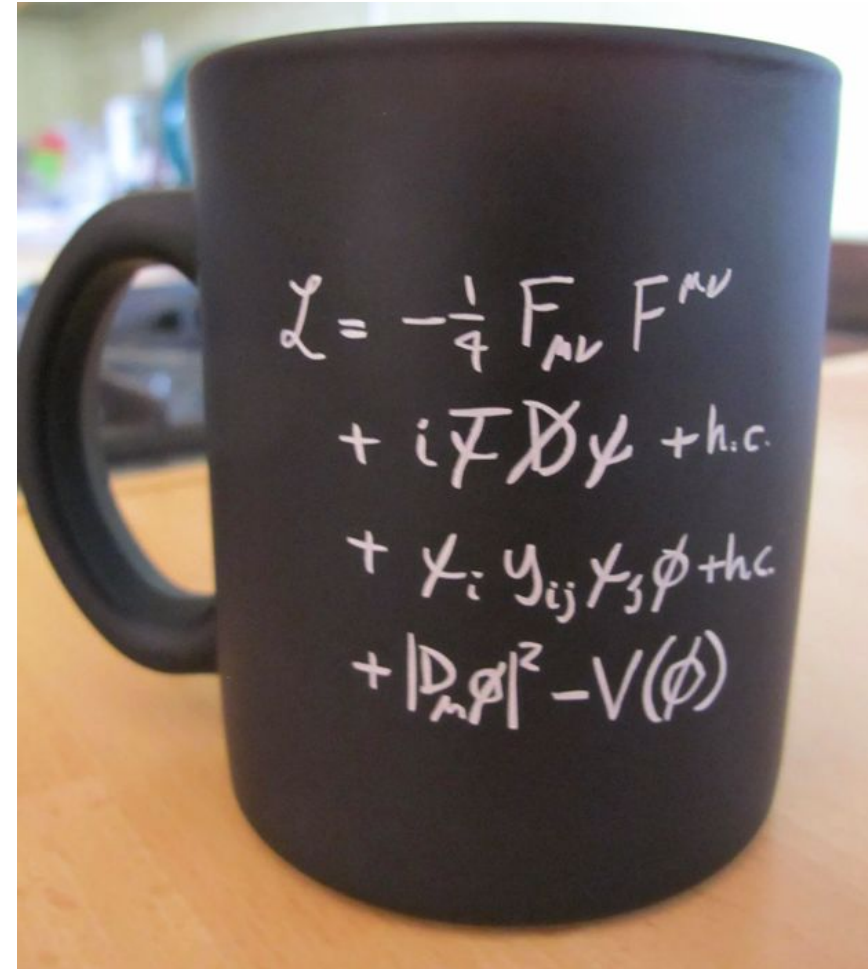
Probability of interaction so state  $|\psi_{\text{in}}\rangle \neq |\psi_{\text{out}}\rangle$  ?

# How do we calculate probabilities ? The big SM monster

## Lagrangian formalism

SM describing dynamics of fields and interactions -> Using a lagrangian

- > Kinematics of spin-1 bosons
- > Kinematics of fermions.
- > Interactions between fields
- > Kinematics of spin-0 fields (no interactions)

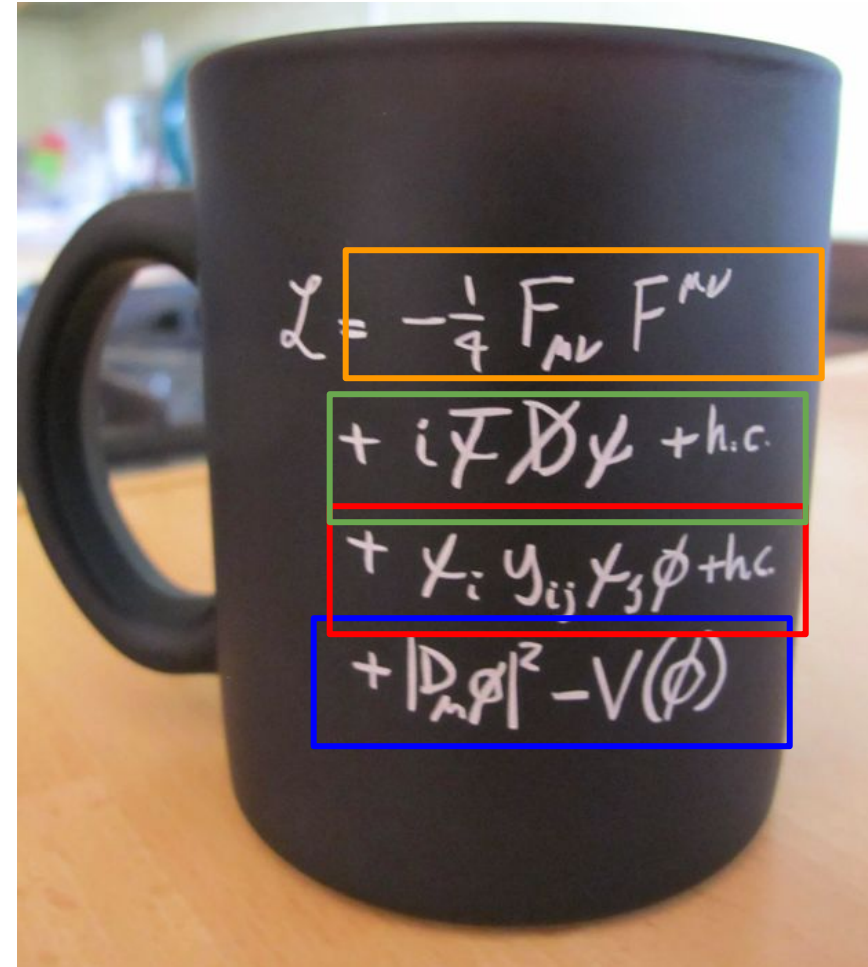


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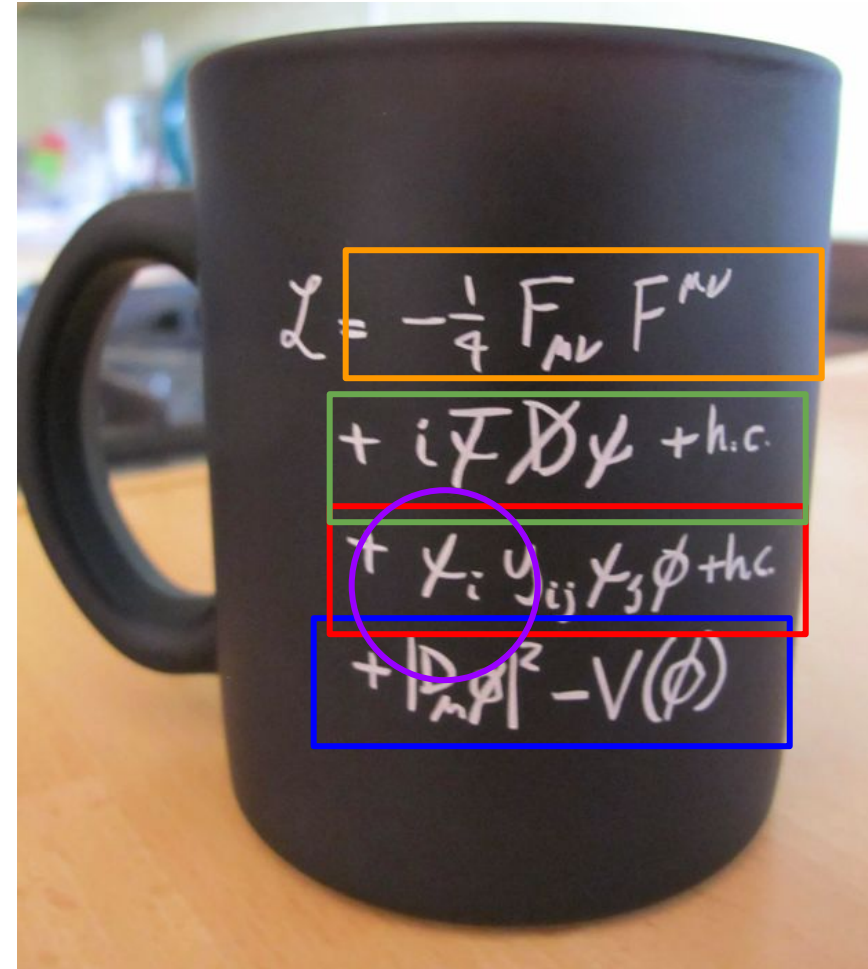
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## Lagrangian formalism

SM describing dynamics of fields and interactions -> Using a lagrangian

- > Kinematics of spin-1 bosons
- > Kinematics of fermions.
- > Interactions between fields
- > Kinematics of spin-0 fields (no interactions)

Coupling between different fields.



# How do we calculate probabilities ? The big SM monster

## Lagrangian formalism

SM describing dynamics of fields and interactions -> Using a lagrangian

- > Kinematics of spin-1 bosons
- > Kinematics of fermions.
- > Interactions between fields
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## Euler-Lagrange

Equations of motion

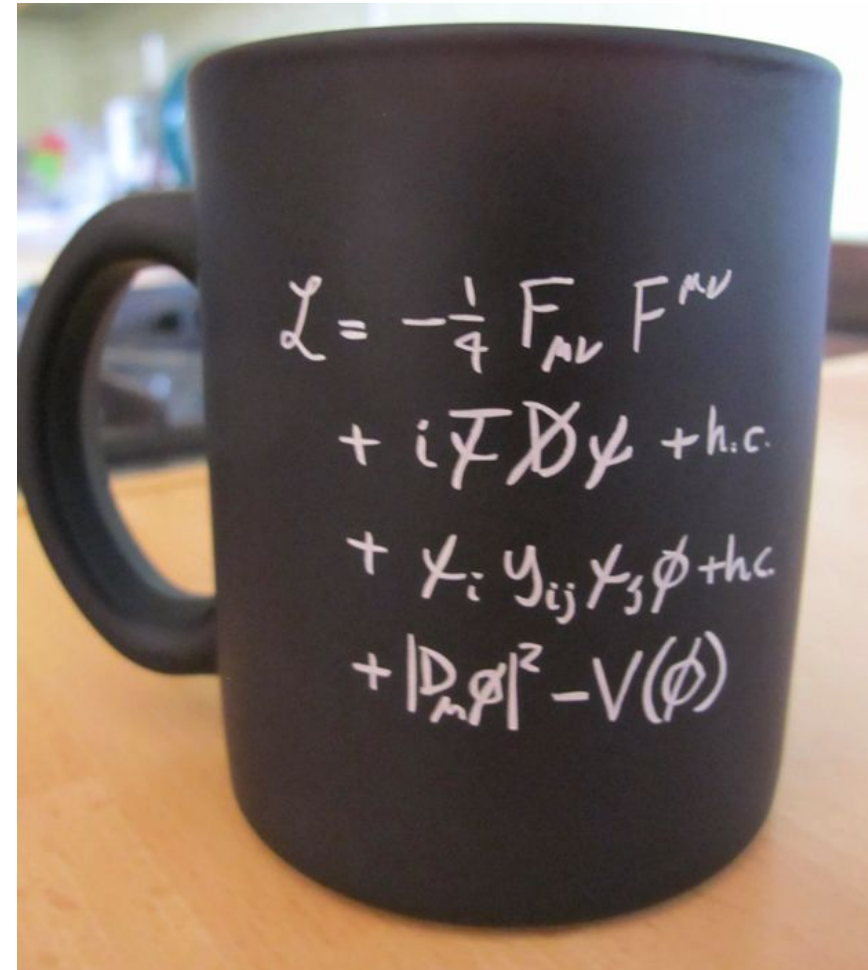
$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi(x))} \right) - \frac{\partial \mathcal{L}}{\partial \phi(x)} = 0$$

Klein-Gordon eq.: kinematics spin-0 fields

$$(\square + m^2) \psi = 0$$

Dirac equation: kinematics spin  $\frac{1}{2}$  field

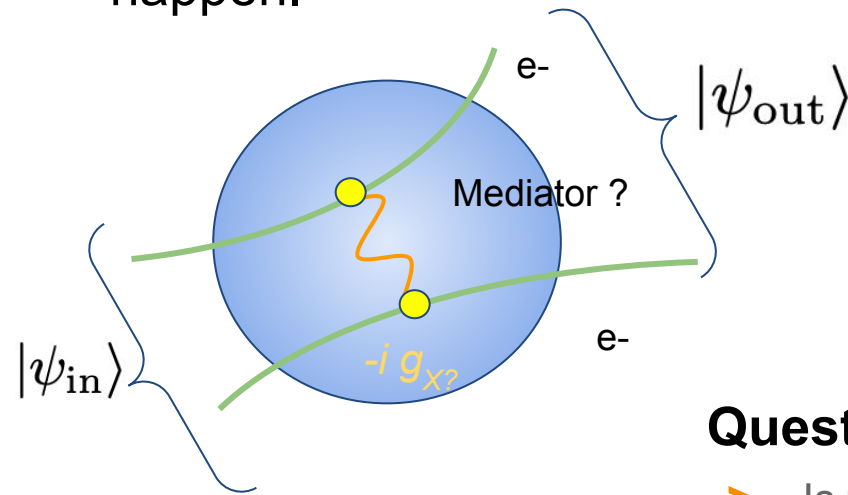
$$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$





# Probability that two particles interact (e.g electrons through QED)?

- > The strength of an interaction depends on the probability of a process to happen.



$$i\mathcal{M} \propto |g_{EM}^2 \langle \psi_{out} | A_\mu | \psi_{in} \rangle|^2$$

## Questions for a possible interaction:

- > Is the process kinematically possible? Do we have enough energy in the interaction to produce a mediator?
- > Are there symmetries of my interaction that wouldn't allow me to have this interactions?
  - E.g. everything seems fine, but spin of the system is not preserved in the interaction.
- > Do my fields have a coupling in the lagrangian?
  - How large is the numerical value of this coupling?





# Interactions and conserved quantities: symmetries

- > From Quantum mechanics: Symmetry connected to conserved quantity
- > Different interactions conserve different quantities

quantity	interaction			invariance
	strong	elm.	weak	
energy	yes	yes	yes	translation in time
momentum	yes	yes	yes	translation in space
angular momentum	yes	yes	yes	rotation in space
P (parity)	yes	yes	no	coordinate inversion
C (charge parity)	yes	yes	no	charge conjugation (particle $\leftrightarrow$ anti-particle)
T (time parity)	yes	yes	no	time inversion
CPT	yes	yes	yes	
lepton number	yes	yes	yes	
baryon number	yes	yes	yes	
isospin	yes	no	no	

+ Flavour : conserved by strong and electromagnetic. Not by Weak interaction.



# Interactions and conserved quantities: Relativistic kinematics

- > particle quantities are described as 4-vectors:

energy/momentum:  $p = (E, \vec{p})$

time/space:  $x = (t, \vec{x})$

- > product of 4-vectors is invariant:  $p_1 \cdot p_2 = (E_1 \cdot E_2 - \vec{p}_1 \cdot \vec{p}_2) = \text{constant}$

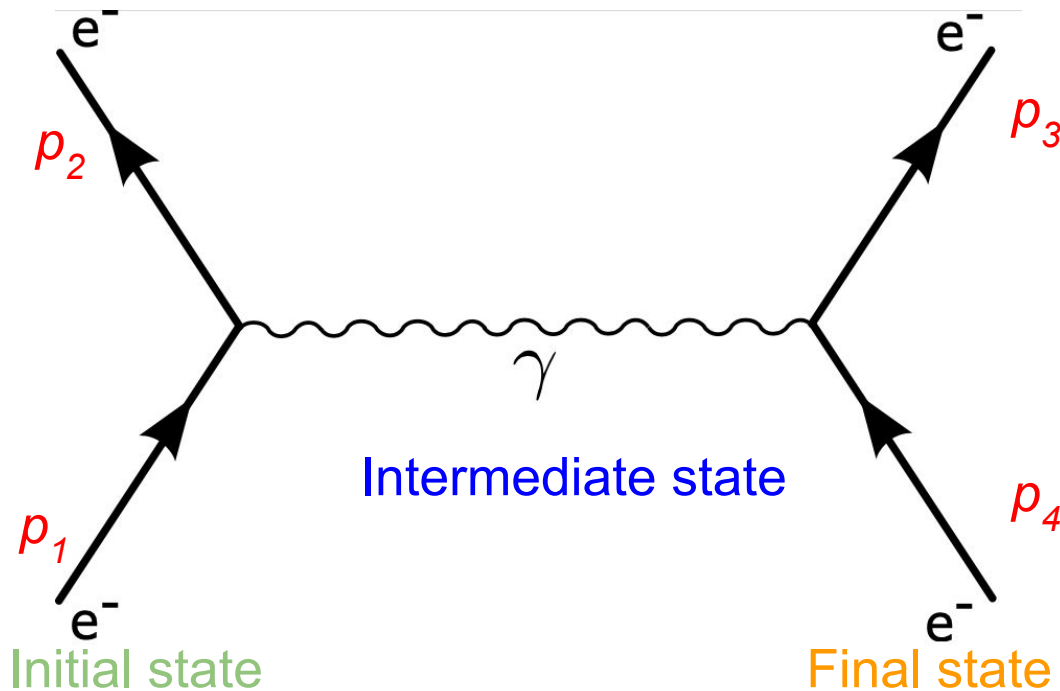
→ can use the “easiest” reference frame for calculations

- > special case:  $p \cdot p = (E^2 - \vec{p} \cdot \vec{p}) = (E_0^2 - 0) = m^2$

with  $\beta = v/c$  and  $\gamma = 1/\sqrt{1-\beta^2}$  :  $E = \gamma m$  and  $|\vec{p}| = \beta \gamma m$



# Interactions and conserved quantities: Relativistic kinematics



## Pair-annihilation/creation

Particle-antiparticle annihilate and produce alternative field

## Scattering

Interacting particles exchange a force field

## Decay

Particle decays into several subparticles

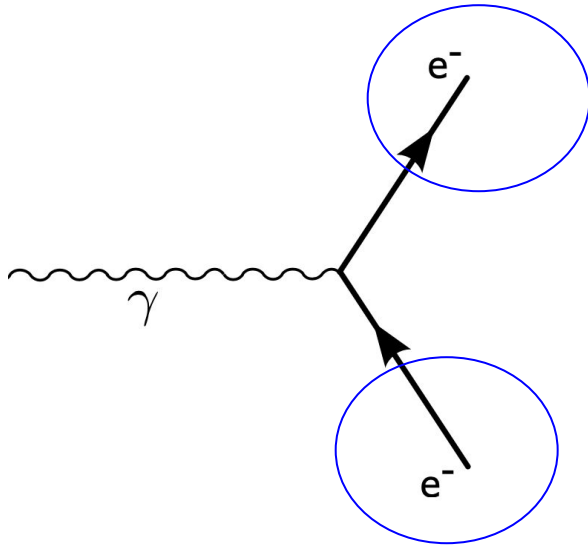
## Radiation

Emission of a final state particle

Kinematically allowed ? → Conservation of total 4-momentum between **initial** and **final** states

$$p_1 + p_2 = p_3 + p_4 \quad \begin{pmatrix} E_1 + E_2 \\ \vec{p}_1 + \vec{p}_2 \end{pmatrix} = \begin{pmatrix} E_3 + E_4 \\ \vec{p}_3 + \vec{p}_4 \end{pmatrix}$$

# Kinematics: on-shell (real) and off-shell (virtual) particles



## On-shell particle or real particle

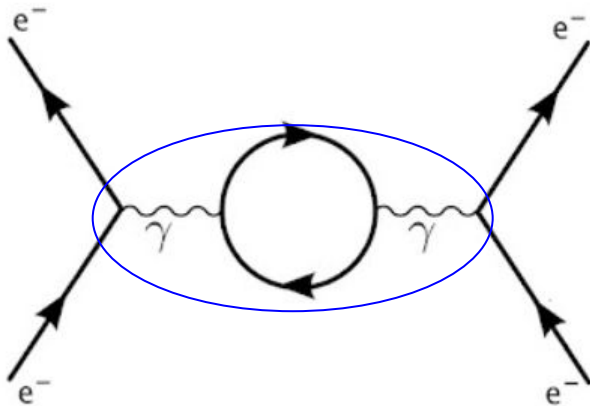
Particles produced in the collision and satisfying momentum-mass relation

$$p \cdot p = m^2$$

## Off-shell particle or virtual particle

Intermediate particles in the interaction. Conserve momentum and energy in the vertices. Quantum fluctuation, possible for very short time thanks to Heisenberg principle.

$$p \cdot p \neq m^2$$

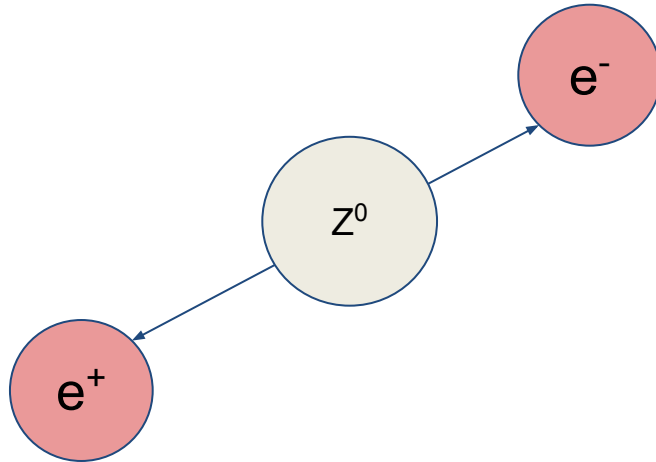


$$\Delta E \Delta t \geq \hbar \approx 6.6 \cdot 10^{-22} \text{ MeV s}$$

Probability of interaction, higher if on-shell particles (resonance)

# Kinematically allowed: decay example

## Decay of on-shell Z-boson



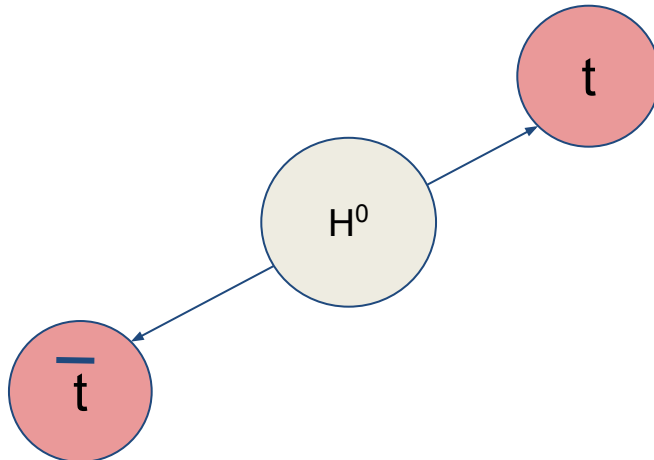
Mass of Z-boson  $\sim 91$  GeV

Mass of electron/positron = 0.511 GeV

$m_Z \gg 2 \times m(\text{electron-positron})$

Allowed !

## Decay of Higgs boson to top quarks



Mass of Higgs boson  $\sim 125$  GeV

Mass of top/anti-top = 175 GeV

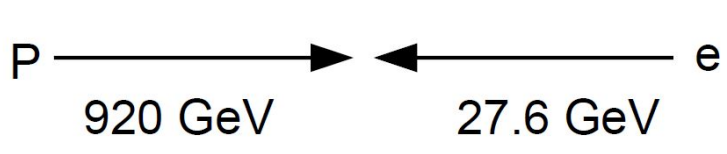
Top-quark has mass (highest mass in SM)  $\rightarrow$  Interaction H-t exists

$m(H) \ll 2 \times m(\text{top})$

Not allowed !

# Relativistic kinematics: additional concepts

- > centre-of-mass energy of a collider, e.g. HERA



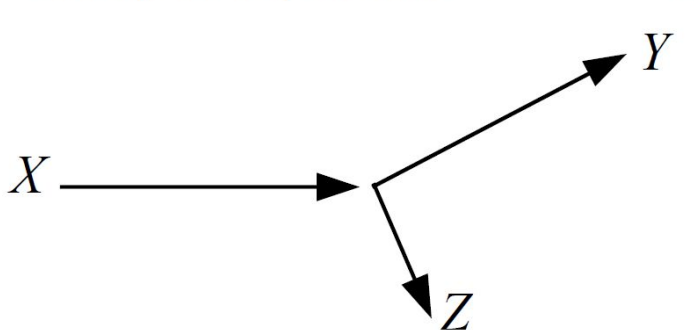
$$p_p = (E_p, \vec{p}_p) = (E_p, 0, 0, E_p)$$

$$p_e = (E_e, \vec{p}_e) = (E_e, 0, 0, -E_e)$$

$$s = (p_p + p_e)^2 = (E_p + E_e)^2 - (E_p - E_e)^2 = 4 E_p E_e \approx 10^5 \text{ GeV}^2$$

$$\Rightarrow \sqrt{s} = 318 \text{ GeV}$$

- > decay of a particle  $X \rightarrow Y Z$  :



$$M_X^2 = (p_X)^2 = (p_Y + p_Z)^2$$

$$= m_Y^2 + m_Z^2 + 2 p_Y p_Z$$

$$= m_Y^2 + m_Z^2 + 2(E_Y E_Z - \vec{p}_Y \cdot \vec{p}_Z)$$

$$\text{and } E_Y^2 = m_Y^2 + |\vec{p}_Y|^2, \quad E_Z^2 = m_Z^2 + |\vec{p}_Z|^2$$

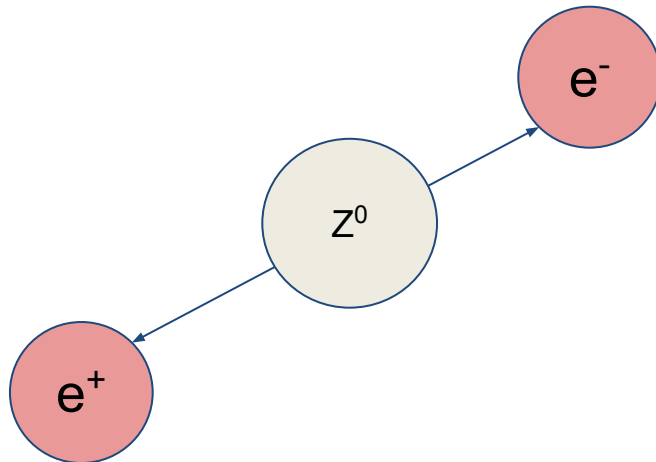
$\Rightarrow$  if daughter particle types are known (or their masses are negligible), mass of decaying mother particle can be reconstructed from the momenta of the daughters (“invariant mass”)



# Symmetries of the interaction: conserved quantities

- Interactions, apart from 4-momentum, conserve properties of the initial and final state.

## Decay of on-shell Z-boson



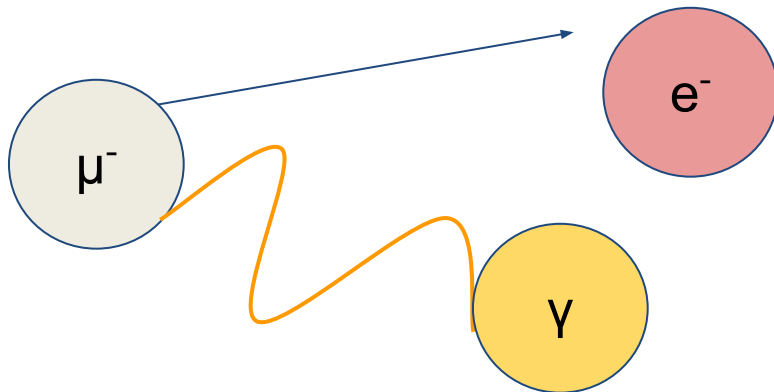
Electric charge of the  $Z^0 = 0$

Electric charge of electron = -1

Electric charge of positron = +1

$$\sum_{\text{initial}} q_i = \sum_{\text{final}} q_j = 0$$

## Muon into electron and photon



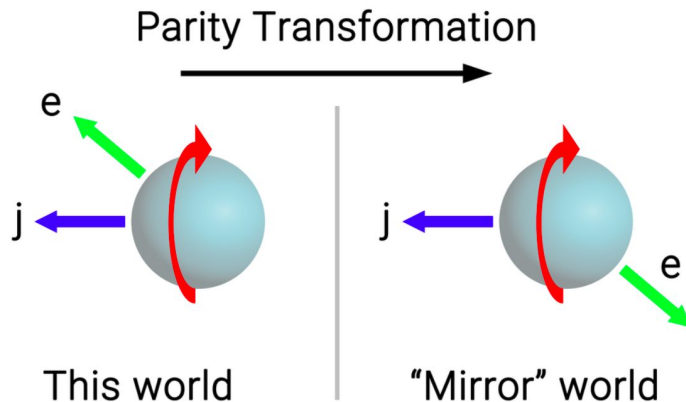
I can clearly write a lagrangian term for this (theory would allow me)

Lepton flavour is violated and SM don't violate flavour

# Other conserved properties of an interaction?

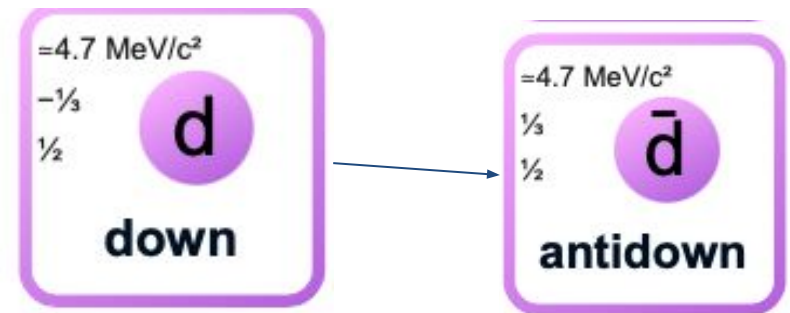
## Parity

Mirror the coordinates of the particle.  
Changes sign of momentum, coordinates  
Spin doesn't change sign.



## Charge conjugation

Change a particle by its anti-particle



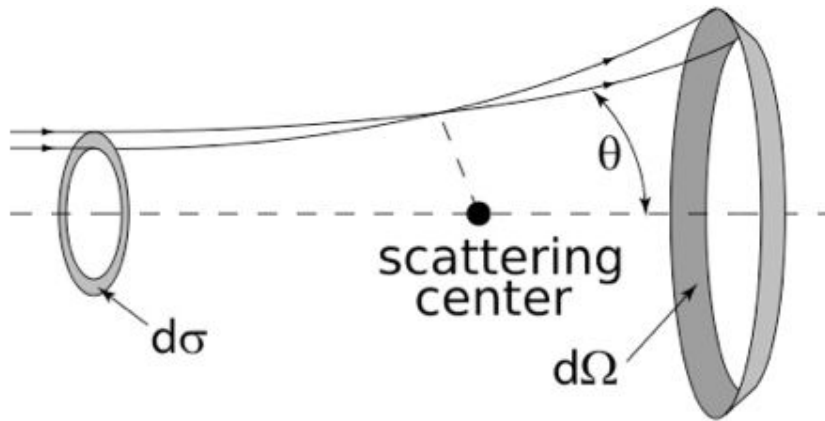
## Time reversal

If I revert the time, would the interaction take place in the same way ?

And combinations: CP, CPT ?

# Cross-section of interaction

- > If my interaction is not violating any symmetry or kinematics, time to make some calculations !



## Cross-section

Encodes the probability of the interaction to happen

Depends on:

- Type and geometry of the interaction
- Incident particles 4-momentum.
- Quantum amplitude of the interaction.

General cross-section formula for processes such that:  $A + B \rightarrow C + D + E + \dots$  (Peskin, Schroeder)

$$d\sigma = \underbrace{\frac{1}{2E_A 2E_B |v_A - v_B|}}_{\text{Momenta and energy of input particles (A,B)}} \underbrace{\left( \prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right)}_{\text{Momenta of outgoing particles}} \underbrace{\times |\mathcal{M}(p_A, p_B \rightarrow p_f)|^2}_{\text{Amplitude of interaction (here's where the SM magic happens)}} \underbrace{(2\pi)^4 \delta^4(p_A + p_B - \sum_f p_f)}_{\text{4-momentum conservation}}$$

# How to calculate the amplitude of the interaction?

$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\mu^a g_\mu^b g_\mu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\mu^c g_\mu^d g_\mu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)) - \\
 & ig s_w (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - \\
 & W_\nu^- \partial_\nu W_\mu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - \\
 & Z_\nu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
 & \beta_h \left( \frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \\
 & \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 & g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \\
 & \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
 & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
 & M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\
 & W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 & \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8}g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 & \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2}ig s_w \lambda_{ij}^a (\bar{q}_i^\alpha \gamma^\mu d_j^\alpha) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + \\
 & m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + ig s_w A_\mu (-\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda) + \\
 & \frac{ig}{4c_w} Z_\mu^0 \{ (\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
 & (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 + \gamma^5) u_j^\lambda) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}_{\lambda\kappa} e^\kappa) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)) + \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep\dagger}_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\kappa\lambda}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\lambda (\bar{\nu}^\lambda U^{lep}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_\kappa^\lambda (\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa} (1 + \gamma^5) \nu^\kappa) - m_\nu^\kappa (\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa} (1 - \gamma^5) \nu^\kappa) - \frac{g}{2} \frac{m_\kappa^\lambda}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
 & \frac{g}{2} \frac{m_\kappa^\lambda}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\kappa^\lambda}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\kappa^\lambda}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa} (1 - \gamma_5) \hat{\nu}_\kappa - \\
 & \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa} (1 - \gamma_5) \hat{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_\kappa^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_\kappa^\lambda}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
 & \frac{g}{2} \frac{m_\kappa^\lambda}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\kappa^\lambda}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\kappa^\lambda}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c + \\
 & \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
 & \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
 & \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H) + \frac{1-2c_w^2}{2c_w} igM (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
 & \frac{1}{2c_w} igM (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + igM s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
 & \frac{1}{2}igM (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
 \end{aligned}$$

$$e^+ e^- \rightarrow \mu^+ \mu^-$$

One of most simple interactions (in QED)

- Annihilation of electron-positron, virtual photon, and pair-creation

Fermion fields

$$\begin{aligned}
 \psi(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_p^s u^s(\mathbf{p}) e^{-iP \cdot x} + b_p^{s\dagger} v^s(\mathbf{p}) e^{+iP \cdot x}) \\
 \psi^\dagger(x) &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_p^{s\dagger} u^{s\dagger}(\mathbf{p}) e^{iP \cdot x} + b_p^s v^{s\dagger}(\mathbf{p}) e^{+iP \cdot x})
 \end{aligned}$$

Boson fields

$$\phi(\mathbf{x}, t_0) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\mathbf{p}}(t_0) e^{i\mathbf{p}\mathbf{x}} + a_{\mathbf{p}}^\dagger(t_0) e^{-i\mathbf{p}\mathbf{x}})$$

Photon field

$$A_\mu(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s (a_{\mathbf{p}}^s \epsilon_\mu^s(p) e^{-ipx} + a_{\mathbf{p}}^{s\dagger} \epsilon_\mu^{s*}(p) e^{ipx})$$

Each step : creation, annihilation for final prob

$$\mathcal{M} \propto \langle 0 | T \phi_I(x) \phi_I(y) \phi_I(z_1) \cdots \phi_I(z_{4n}) | 0 \rangle$$



# How to calculate the amplitude of the interaction?

$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^+ \partial_\nu W_\mu^-) + Z_\mu^0 (W_\nu^+ \partial_\mu W_\nu^- - W_\nu^+ \partial_\mu W_\nu^-) - \\
 & ig s_w (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^+ \partial_\nu W_\mu^-) + A_\mu (W_\nu^+ \partial_\mu W_\nu^- - \\
 & W_\nu^+ \partial_\mu W_\nu^-) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^- W_\nu^+ + g^2 c_w^2 (Z_\mu^0 W_\nu^+ Z_\nu^0 W_\mu^- - \\
 & Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\nu^+ A_\nu W_\mu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
 & \beta_h \left( \frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \\
 & g \alpha_h M (H^3 + H \phi^0 \phi^0 + 2H \phi^+ \phi^-) - \\
 & \frac{1}{8} g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 & g M W_\mu^+ W_\mu^- H - \frac{1}{2} g \frac{M}{c_w^2} Z_\mu^0 Z_\mu^0 H - \\
 & \frac{1}{2} ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
 & \frac{1}{2} g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2} g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
 & M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\
 & W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 & \frac{1}{4} g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8} g^2 \frac{1}{c_w^2} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 & \frac{1}{2} g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2} ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2} g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2} ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{2c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2} ig_s \lambda_{ij}^a (\bar{q}_i^\mu \gamma^\mu d_j^\mu) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + \\
 & m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + m_d^\lambda) d_j^\lambda + ig s_w A_\mu (-\bar{e}^\lambda \gamma^\mu e^\lambda) + \frac{2}{3} (\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3} (\bar{d}_j^\lambda \gamma^\mu d_j^\lambda) + \\
 & \frac{ig}{4c_w} Z_\mu^0 \{ (\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
 & (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 + \gamma^5) u_j^\lambda) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}_{\lambda\kappa} e^\kappa) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)) + \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep\dagger}_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C^\dagger_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\lambda (\bar{\nu}^\lambda U^{lep}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_\lambda^\lambda (\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa} (1 + \gamma^5) \nu^\kappa) - m_\nu^\kappa (\bar{e}^\lambda U^{lep\dagger}_{\lambda\kappa} (1 - \gamma^5) \nu^\kappa) - \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa} (1 - \gamma_5) \hat{\nu}_\kappa - \\
 & \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa} (1 - \gamma_5) \hat{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_d^\lambda (\bar{d}_j^\lambda C^\dagger_{\lambda\kappa} (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C^\dagger_{\lambda\kappa} (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c + \\
 & \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
 & \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
 & \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) - \frac{1}{2} g M (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H) + \frac{1-2c_w^2}{2c_w} ig M (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
 & \frac{1}{2c_w} ig M (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + ig M s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
 & \frac{1}{2} ig M (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
 \end{aligned}$$



$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_p^s u^s(\mathbf{p}) e^{-iP \cdot x} + b_p^{s\dagger} v^s(\mathbf{p}) e^{+iP \cdot x})$$

$$\psi^\dagger(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_p^{s\dagger} u^{s\dagger}(\mathbf{p}) e^{iP \cdot x} + b_p^s v^{s\dagger}(\mathbf{p}) e^{+iP \cdot x})$$

Boson fields

$$\phi(\mathbf{x}, t_0) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} (a_{\mathbf{p}}(t_0) e^{i\mathbf{p}\mathbf{x}} + a_{\mathbf{p}}^\dagger(t_0) e^{-i\mathbf{p}\mathbf{x}})$$

Photon field

$$A_\mu(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E_p}} \sum_s (a_{\mathbf{p}}^s \epsilon_\mu^s(p) e^{-ipx} + a_{\mathbf{p}}^{s\dagger} \epsilon_\mu^{s*}(p) e^{ipx})$$

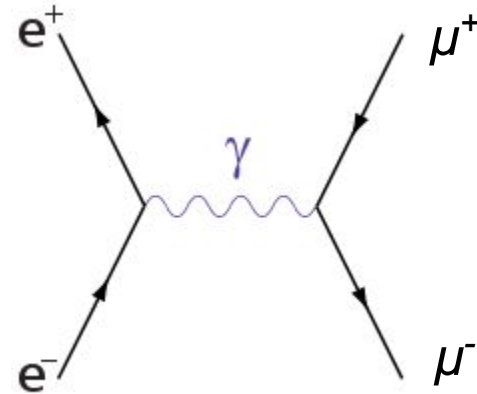
Each step : creation, annihilation for final prob

$$\mathcal{M} \propto \langle 0 | T \phi_I(x) \phi_I(y) \phi_I(z_1) \cdots \phi_I(z_{4n}) | 0 \rangle$$

# Reason why particle physicists LOVE Feynman



## Feynman diagrams



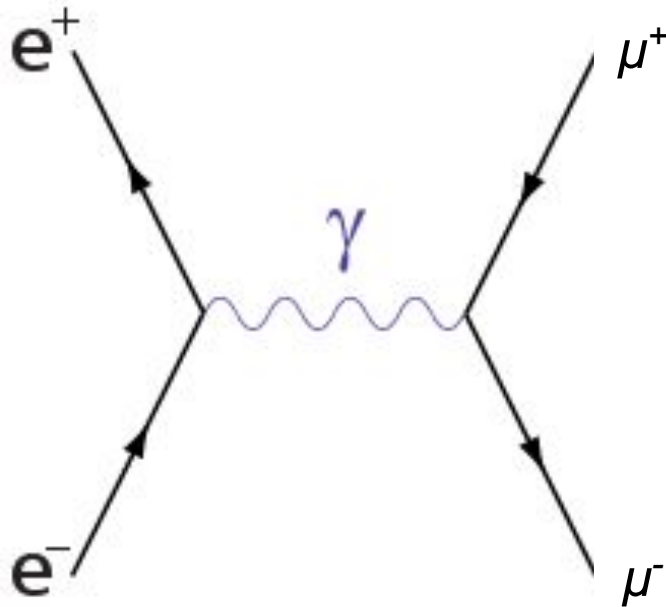
Because these are not just visual representations of the process

They are recipes on how to build the equation of each possible interaction leading to your process.

## Feynman rules



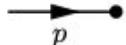
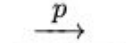

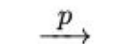
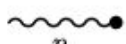
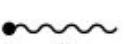
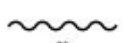
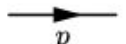

# Feynman rules: example QED



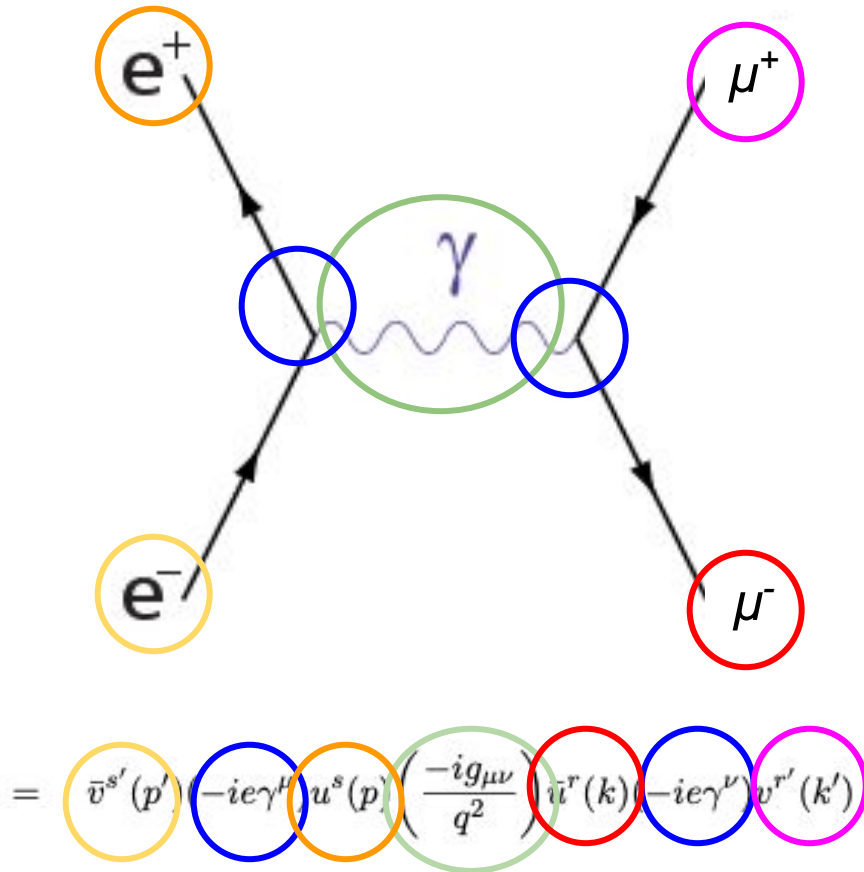
$$= \bar{v}^{s'}(p')(-ie\gamma^\mu)u^s(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^r(k)(-ie\gamma^\nu)v^{r'}(k')$$

Which is what we would've arrived  
with doing it the hard way

$$\langle 0|T\phi_I(x)\phi_I(y)\phi_I(z_1)\cdots\phi_I(z_{4n})|0\rangle$$

1. Incoming fermion  =  $u^s(p)$
2. Incoming antifermion  =  $\bar{v}^s(p)$
3. Outgoing fermion  =  $\bar{u}^s(p)$
4. Outgoing antifermion  =  $v^s(p)$
5. Incoming photon  =  $\epsilon^\mu$
6. Outgoing photon  =  $\epsilon^{\mu*}$
7. Photon propagator  =  $\frac{-ig^{\mu\nu}}{p^2 + i\epsilon}$
8. Fermion propagator  =  $\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$
9. Vertex  =  $-ie\gamma^\mu$
10. Impose 4-momentum conservation at each vertex.
11. Integrate over momenta not determined by 10.:  $\int \frac{d^4p}{(2\pi)^4}$
12. Figure out the overall sign of the diagram.

# Feynman rules: example QED



Which is what we would've arrived  
with doing it the hard way

$$\langle 0 | T \phi_I(x) \phi_I(y) \phi_I(z_1) \cdots \phi_I(z_{4n}) | 0 \rangle$$

1. Incoming fermion  $\rightarrow = u^s(p)$

2. Incoming antifermion  $\leftarrow = \bar{v}^s(p)$

3. Outgoing fermion  $\rightarrow = \bar{u}^s(p)$

4. Outgoing antifermion  $\leftarrow = v^s(p)$

5. Incoming photon  $\rightarrow = \epsilon^\mu$

6. Outgoing photon  $\rightarrow = \epsilon^{\mu*}$

7. Photon propagator  $\rightarrow = \frac{-ig^{\mu\nu}}{p^2 + i\epsilon}$

8. Fermion propagator  $\rightarrow = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$

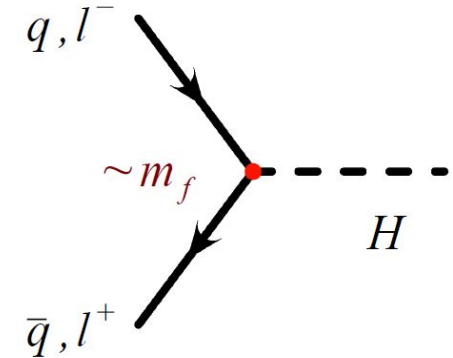
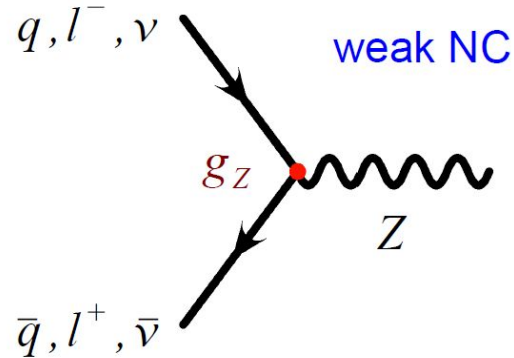
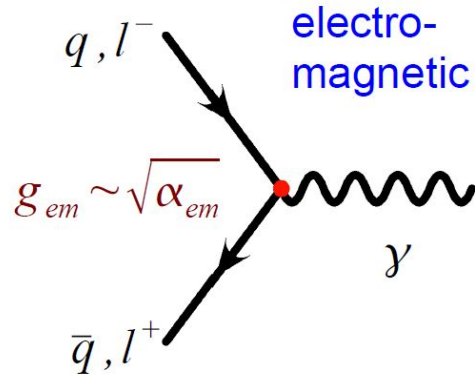
9. Vertex  $\rightarrow = -ie\gamma^\mu$

10. Impose 4-momentum conservation at each vertex.

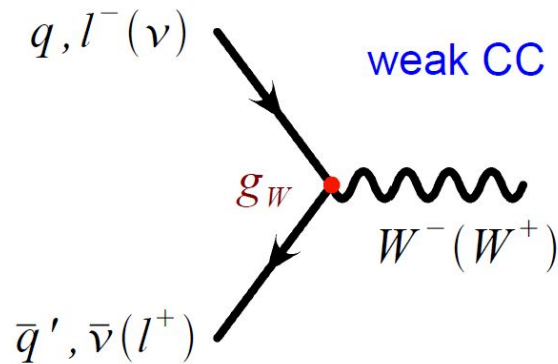
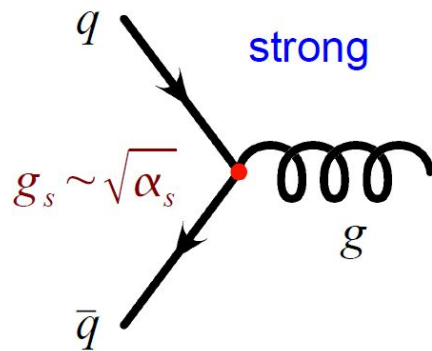
11. Integrate over momenta not determined by 10.:  $\int \frac{d^4p}{(2\pi)^4}$

12. Figure out the overall sign of the diagram.

# Feynman rules: different for each force



neutrinos???



always 2 fermions  
and 1 boson

- mixes generations for quarks
- specific helicity structure

# Leading order and high order diagrams

When collision of particles happen, several kind of Feynman diagrams might appear

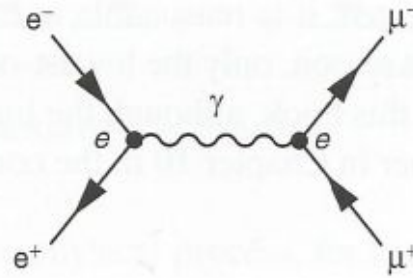


Fig. 6.1

The lowest-order Feynman diagram for the QED annihilation process  $e^+e^- \rightarrow \mu^+\mu^-$ .

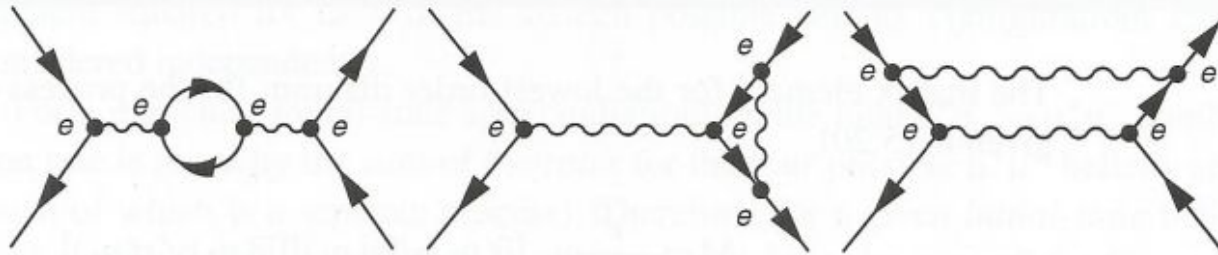


Fig. 6.2

Three of the  $O(\alpha^4)$  Feynman diagrams contributing the QED annihilation process  $e^+e^- \rightarrow \mu^+\mu^-$ .

All of these diagrams contribute to the total amplitude of the process to happen !

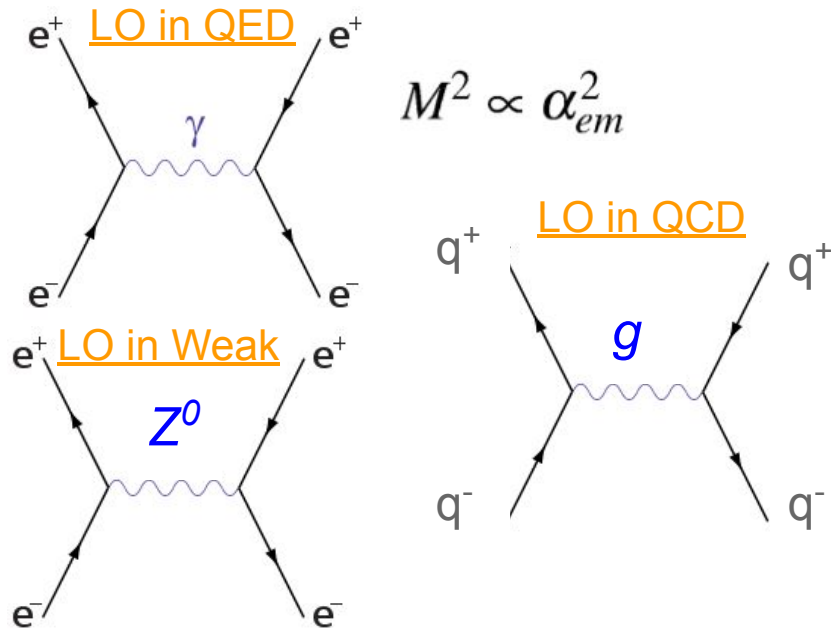
*But the more vertices, the lower is the contribution to the total amplitude (given couplings are  $< 1$ )*

# Leading order and high order diagrams

When collision of particles happen, several kind of Feynman diagrams might appear

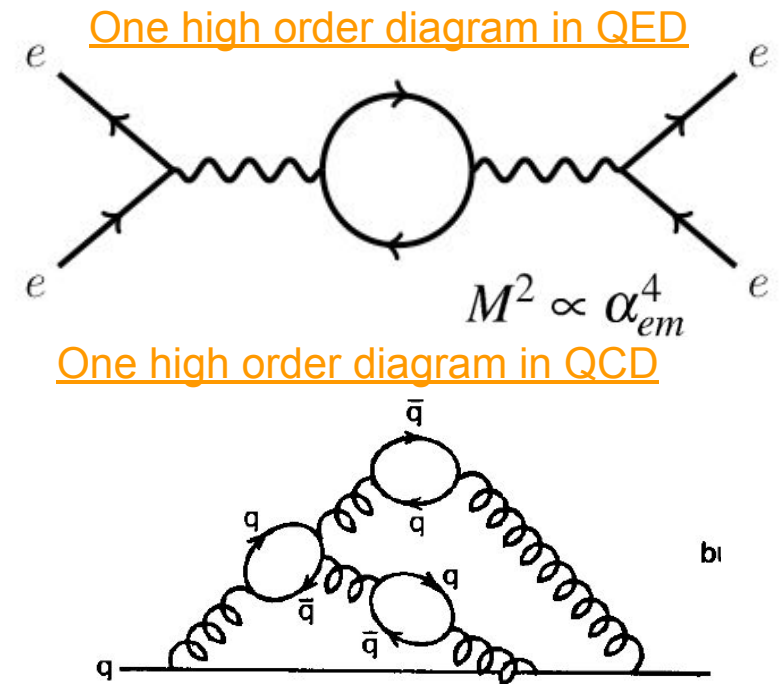
## Leading order (LO) diagrams

Diagrams with less number of vertices possible to allow the interaction by a certain force



## Higher order (NLO, NNLO,...)

Diagrams with larger number of vertices



All of these diagrams contribute to the total amplitude of the process to happen !

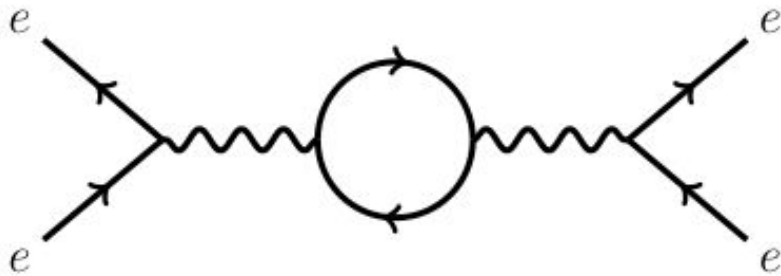
*But the more vertices, the lower is the contribution to the total amplitude (given couplings are  $< 1$ )*

# Couplings, masses and Feynman diagrams

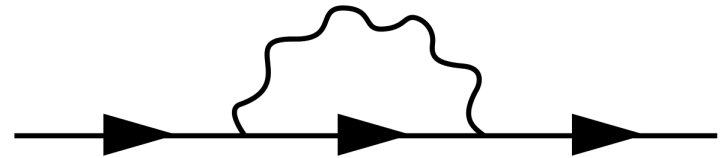
- When calculating the contribution of certain Feynman diagrams to the cross-section of an interaction, divergences (infinities) appear.

According to the laws of statistics, we have to integrate over all possibilities.

→ We have to integrate over all possible momenta the virtual particles in loops → Infinity !



Loop diagrams etc: correction to the interaction coupling (i.e.  $g_{EM}$ )



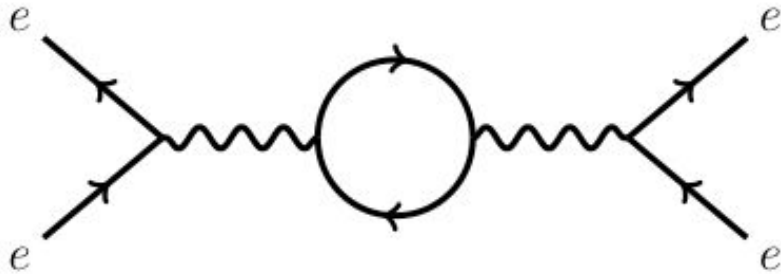
Self-interaction: correction to the mass of the particle



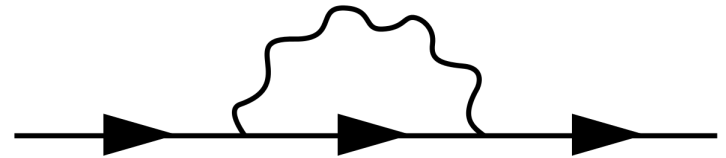
# Couplings, masses and Feynman diagrams

## Regularization and renormalization

Absorb divergences into definition of the physical quantities → Valid up to a certain scale.



Loop diagrams etc: correction to the interaction coupling (i.e.  $g_{EM}$ )  
→ Effective coupling

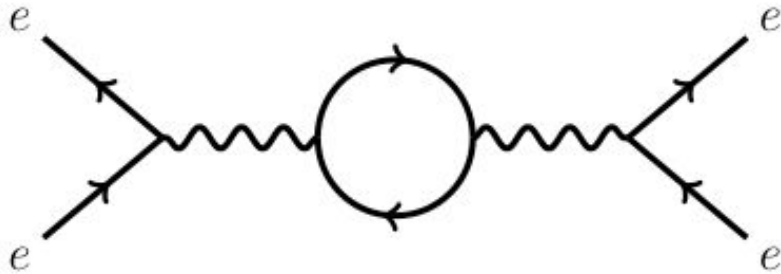


Self-interaction: correction to the mass of the particle → Effective mass

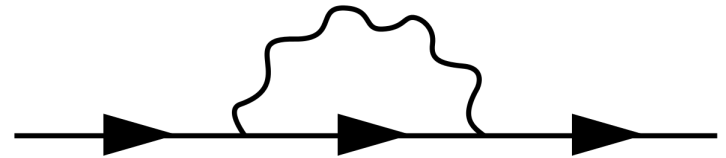
# Couplings, masses and Feynman diagrams

## Regularization and renormalization

Absorb divergences into definition of the physical quantities → Valid up to a certain scale.



Loop diagrams etc: correction to the interaction coupling (i.e.  $g_{EM}$ )  
→ Effective coupling



Self-interaction: correction to the mass of the particle → Effective mass

Message to take home: the observed mass and coupling strength in an interaction depends on the energy exchanged in the interaction

# Backup

# 0) Introduction: SM – A theory of everything?

- > Predictions based on theory found to be experimentally valid, some up to level of  $10^{-6}$
- > Many experimental findings could be well incorporated into theory

## SM does not explain everything:

- > **Gravity.** The standard model does not explain gravity. “Graviton” neither discovered nor does it fit cosmological observations / general relativity.
- > **Dark matter and dark energy.** Standard model describes only 5% of the matter of the universe. Dark matter (25%) and dark energy (70%) [from cosmological observations and general relativity] unexplained – no candidates for dark matter in the standard model
- > **Neutrino masses and oscillations** not explained by SM
- > **Matter-antimatter asymmetry.** SM unable to explain, how and/or why matter dominates over anti-matter in our universe, there is a mechanism included but the effect is much too small

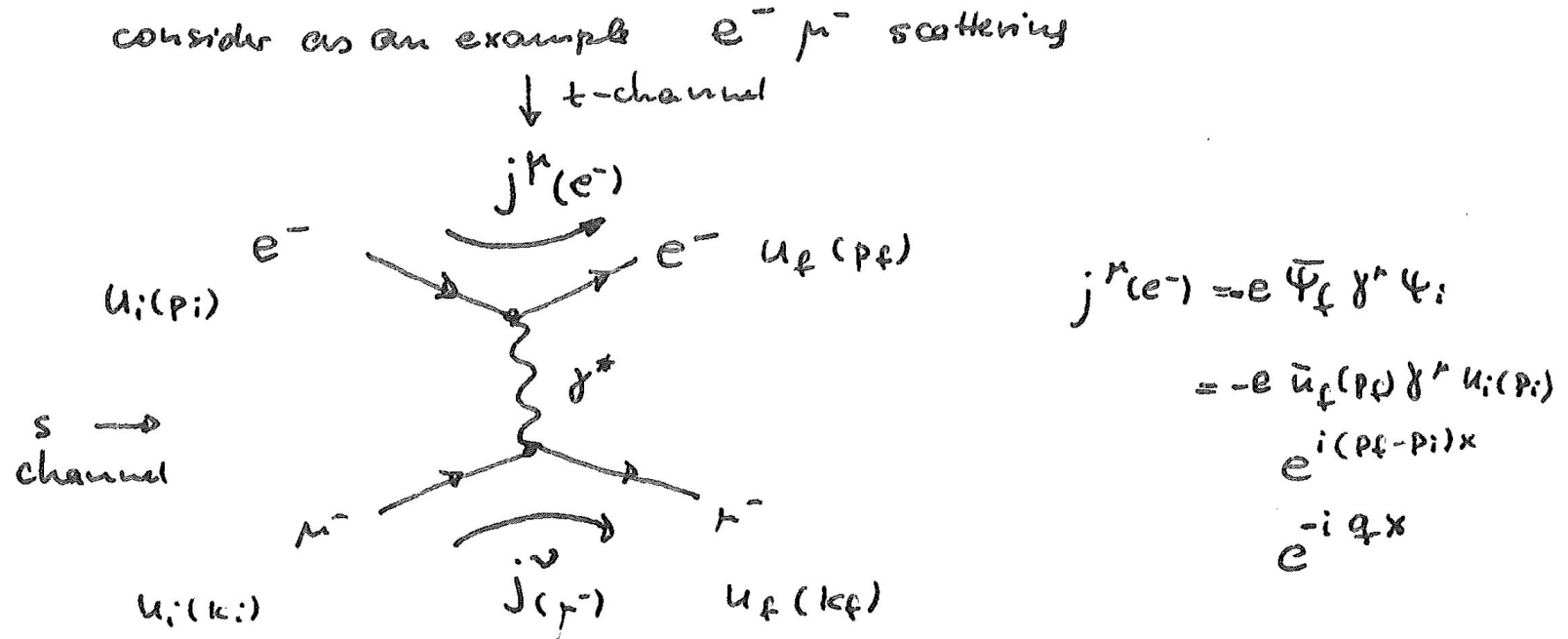


# 0) Introduction: SM – the ugly

- Some parts of the SM are added “ad-hoc” – or “by hand”  
*this means a certain parameter or mechanism needs to be postulated – not in contradiction to any observations and theoretically valid, but still not “satisfying” aesthetically*
- **Number of SM parameters.** Standard model depends on 19 numerical parameters. Their values are known from experiment, but the origin of the values is unknown.
- **Hierarchy problem.** Fine tuning of Higgs mass versus quantum corrections over several orders of magnitude.  
(e.g. cancellation of the size  $\sim 10^{16}$ ) **[see later!]**
- **Strong CP problem.** Theoretically, the SM should contain a term that breaks CP symmetry - relating matter to antimatter – in QCD. Experimentally, however, no such violation has been found, implying that the coefficient of this term is very close to zero → unnatural.
- **Aim of the lecture: Understand better how SM was established and what its problems are**



# 0) Feynman diagrams: example of $e\mu$ scattering



$$M^0 = -j^\mu(e^-) \frac{1}{q^2} j_\mu(\mu^-) = -e^2 \bar{u}(p_f) \gamma^\mu u(p_i) \frac{1}{q^2} \bar{u}(k_f) \gamma_\mu u(k_i)$$

↑  
Photon propagator

- > Average over spins in initial state, sum spins in final state
- > Replace outgoing  $e^-$  by backwards traveling  $e^+ \rightarrow$  amplitude for  $ee \rightarrow \mu\mu$



# 0) Feynman diagrams: example of $e\mu$ scattering I

$$p_i = p, p_f = p'$$



$$|\overline{\mathcal{M}}|^2 = \frac{e^4}{q^4} L_e^{\mu\nu} L_{\mu\nu}^{\text{muon}}$$

> Matrix element:

> Need to sum over spins

$$L_e^{\mu\nu} \equiv \frac{1}{2} \sum_{(e \text{ spins})} [\bar{u}(k') \gamma^\mu u(k)] [\bar{u}(k') \gamma^\nu u(k)]^*$$

> Use “Completeness relation”

$$L_e^{\mu\nu} = \frac{1}{2} \underbrace{\sum_{s'} \bar{u}_{\alpha}^{(s')}(k') \gamma_{\alpha\beta}^{\mu}}_{(\not{k}' + m)_{\delta\alpha}} \underbrace{\sum_s u_{\beta}^{(s)}(k) \bar{u}_{\gamma}^{(s)}(k) \gamma_{\gamma\delta}^{\nu}}_{(\not{k} + m)_{\beta\gamma}} u_{\delta}^{(s')}(k')$$

> Further math

$$|\overline{\mathcal{M}}|^2 = \frac{8e^4}{q^4} [(k' \cdot p')(k \cdot p) + (k' \cdot p)(k \cdot p') - m^2 p' \cdot p - M^2 k' \cdot k + 2m^2 M^2]$$

# 0) Feynman diagrams: example of $e\mu$ scattering I

➤ Neglecting masses, using “Mandelstam variable”

$$s \equiv (k + p)^2 \simeq 2k \cdot p \simeq 2k' \cdot p',$$

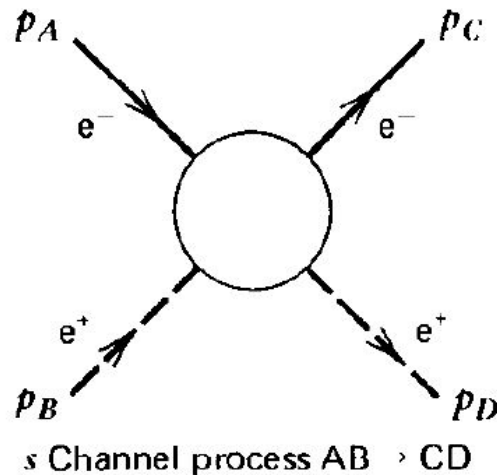
$$t \equiv (k - k')^2 \simeq -2k \cdot k' \simeq -2p \cdot p',$$

$$u \equiv (k - p')^2 \simeq -2k \cdot p' \simeq 2k' \cdot p.$$

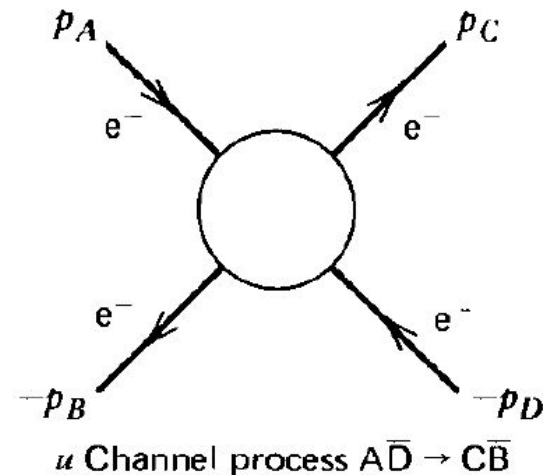
$$|\overline{\mathcal{M}}|^2 = 2e^4 \frac{s^2 + u^2}{t^2}$$



$$|\overline{\mathcal{M}}|^2 = 2e^4 \frac{t^2 + u^2}{s^2}$$



Crossing  $\rightarrow$

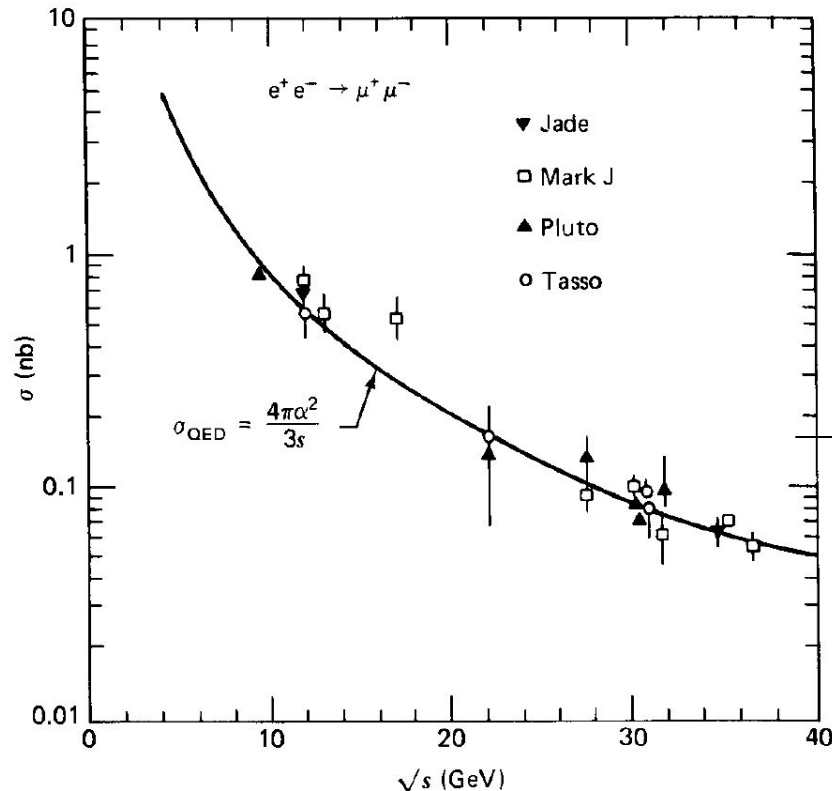


# 0) Feynman diagrams: example of $e\mu$ scattering I

>Measurements:

$$\text{Cross section} = \frac{W_{fi}}{(\text{initial flux})} (\text{number of final states})$$

>Need to integrate over full solid angle here and momenta



How measured?

**Fig. 6.6** The total cross section for  $e^-e^+ \rightarrow \mu^-\mu^+$  measured at PETRA versus the center-of-mass energy.

# How to measure a cross section

> very generally:

$$N_{\text{evt}} = \sigma \int L dt \Rightarrow \sigma = \frac{N_{\text{evt}}}{\int L dt} \quad \text{and} \quad L = \frac{n f N_1 N_2}{\sigma_X \sigma_Y} \quad \text{for a collider}$$

> in practice:

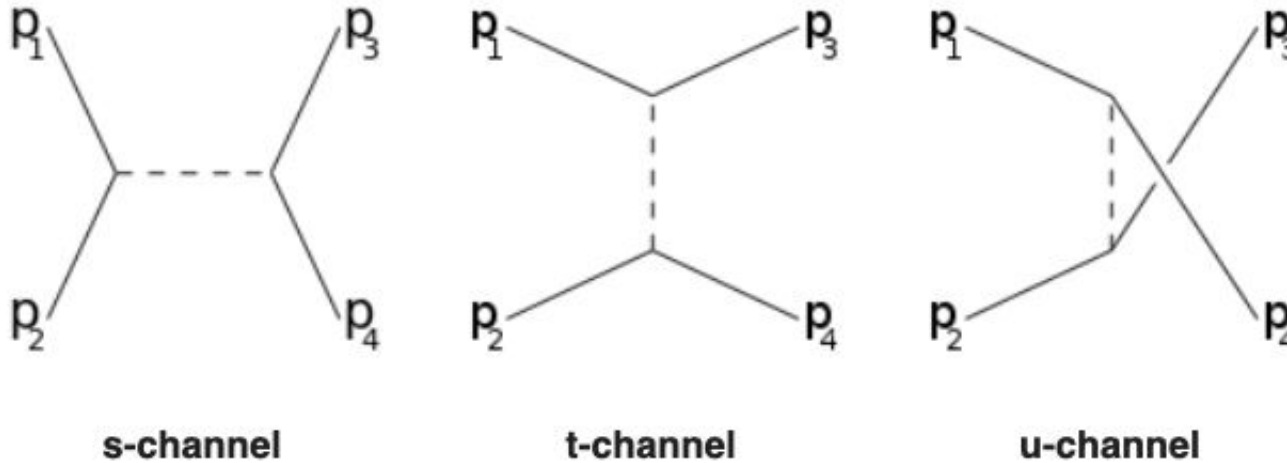
- selected events contain background  $N_{\text{Bkg}}$
- detectors are not perfect, but have an efficiency  $\epsilon$
- events are only measured in a specific decay channel with a branching ratio BR

>

$$\sigma = \frac{(N_{\text{evt}} - N_{\text{Bkg}})}{\epsilon \text{ BR} \int L dt}$$



# Channels and Mandelstam variables



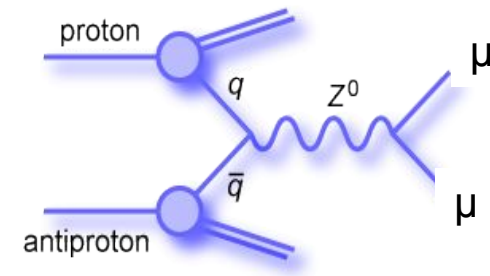
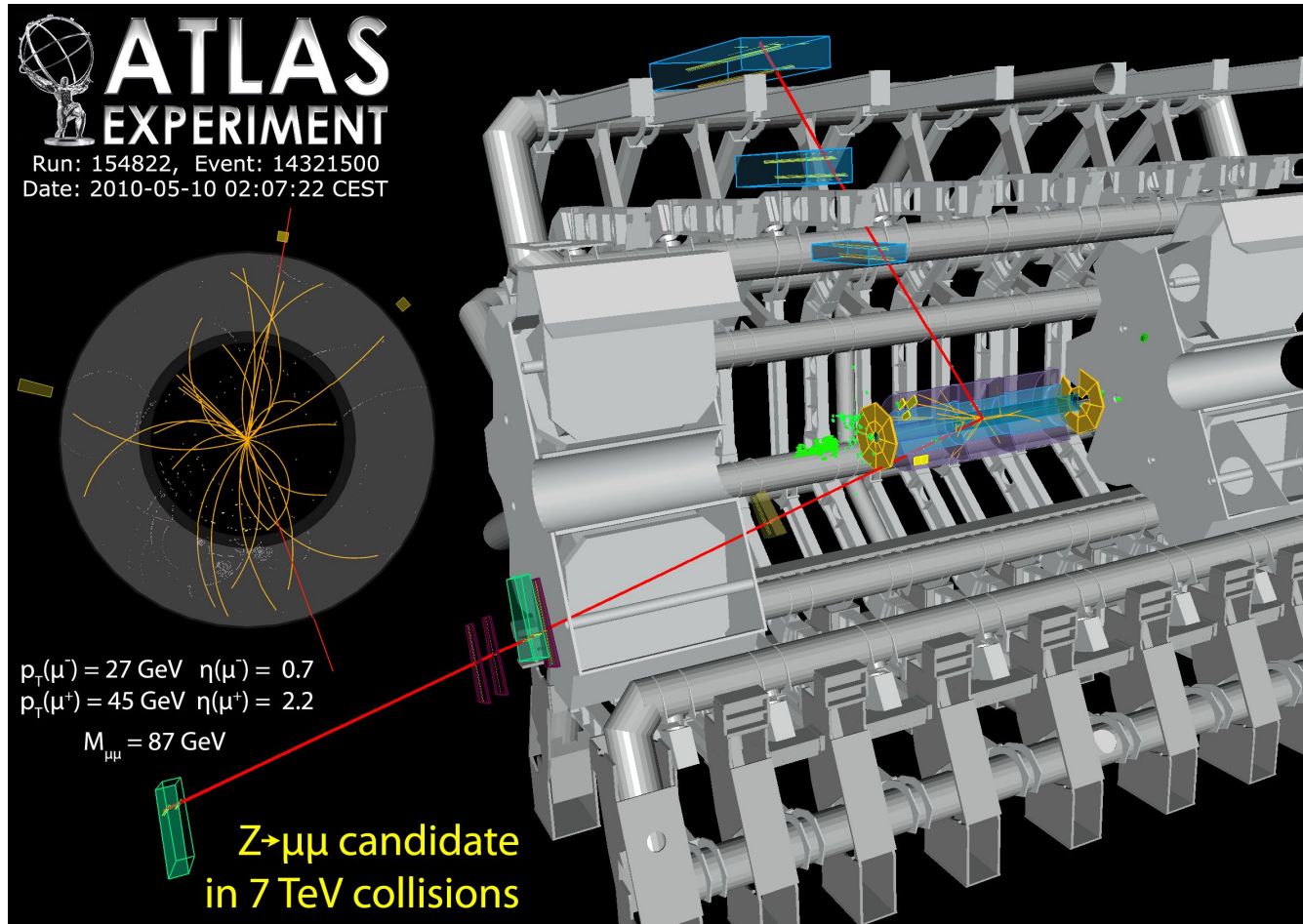
## Mandelstam variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$t = (p_1 - p_3)^2 = (p_4 - p_2)^2$$

$$u = (p_1 - p_4)^2 = (p_3 - p_2)^2$$

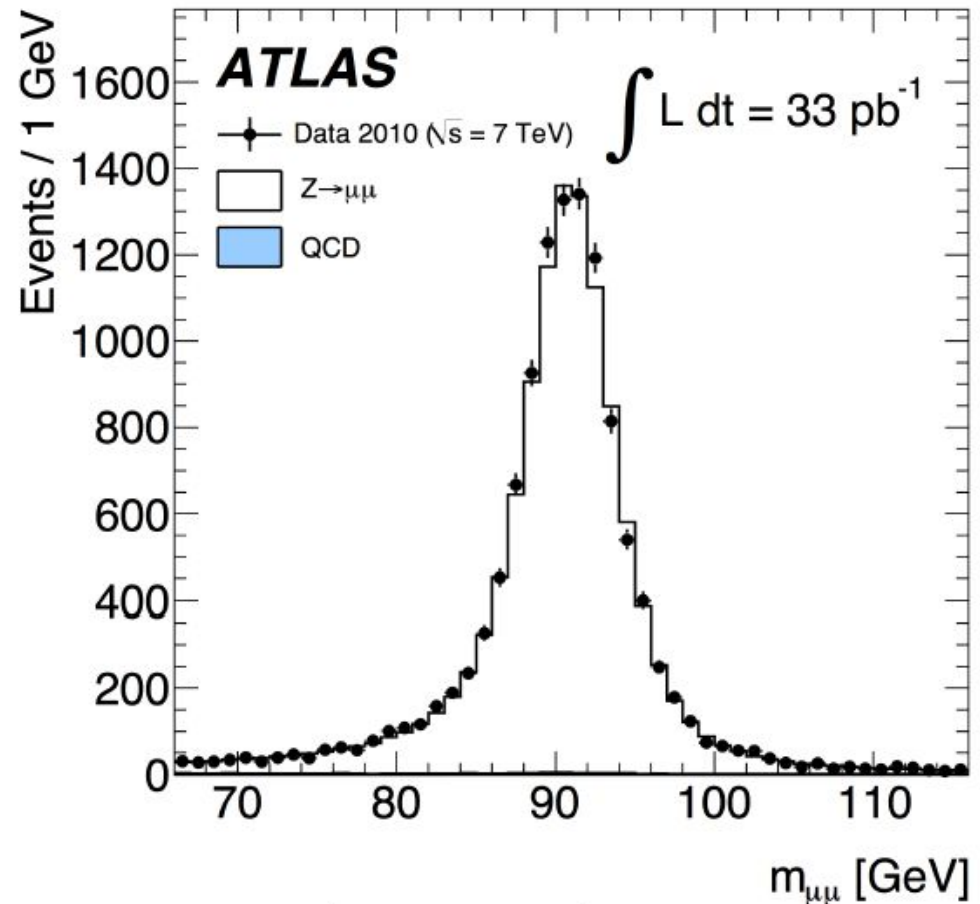
# How to measure a cross section





# How to measure a cross section

- > (simple) example:  
 $Z^0 \rightarrow \mu^+ \mu^-$  from ATLAS
- > identify muons in the detector, select events with at least 2 muons of opposite charge, calculate the invariant mass, fill the mass into a histogram
- > determine how much background
  - use the prediction from a MC simulation (works if background is well known)
  - fit a function of the form  $f(m) = \text{signal}(m) + \text{bkg}(m)$  to the data (works if expected shapes are known)

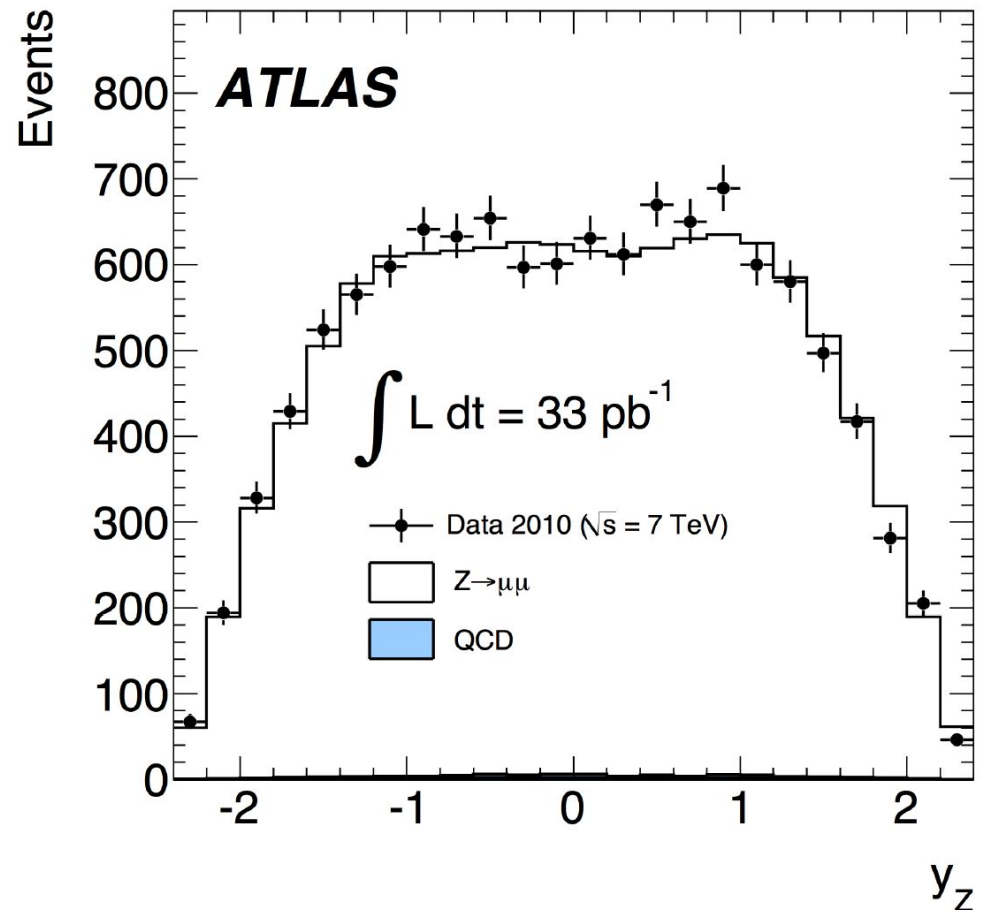


$$\sigma = \frac{(N_{\text{evt}} - N_{\text{Bkg}})}{\epsilon \text{ BR } \int L dt}$$

# How to measure a differential cross section

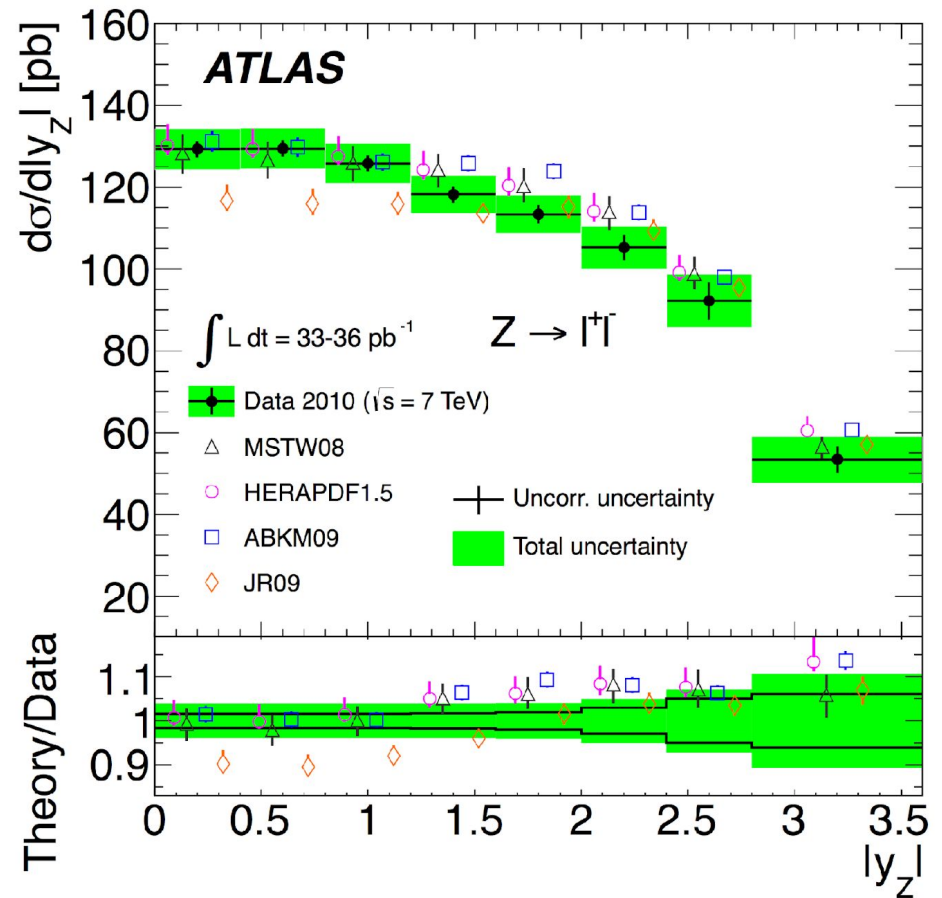
- > quite often more can be learned by the dependence of the cross section on some quantities
- > determine the number of signal (and background) events in bins of that quantity and fill it into a histogram (here: rapidity  $y$ )
- > but now the number of events depends on the bin size!
- > take bin size into account by measuring differential cross section

$$\frac{d\sigma}{dy} \approx \frac{\Delta\sigma}{\Delta y} = \frac{(N_{\text{evt}} - N_{\text{Bkg}})_{\text{bin}}}{\Delta y \epsilon BR \int L dt}$$



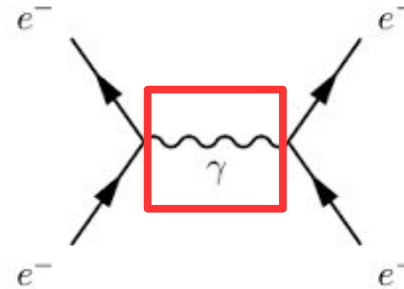
# Theory comparisons

- > • differential cross section can be compared to theory predictions
  - can exclude predictions that describe the total cross section but differ in shape
- > • meaningful comparison only possible if uncertainties are known!
  - statistical uncertainties (from signal and background events!)
  - systematic uncertainties (efficiency, branching ratio, luminosity)
  - are there correlations between the bins? (e.g. uncertainty on luminosity shifts all data points the same way → correlated)



# 0) The Heisenberg principle

> Interaction carried by “force carrier”

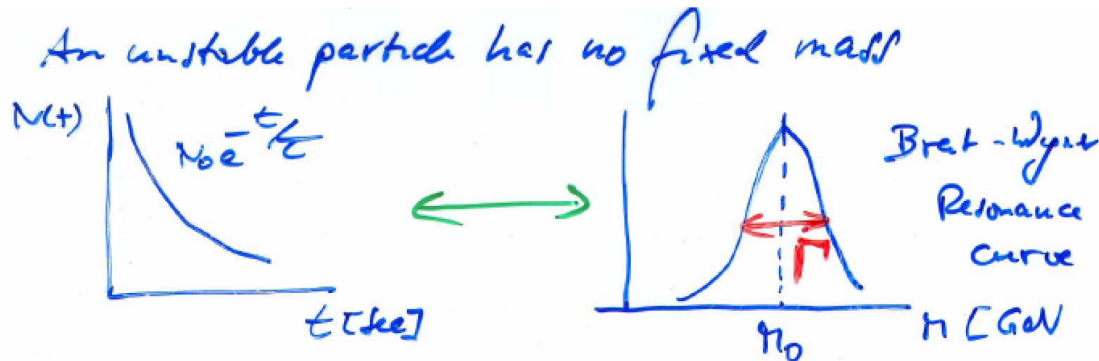


> Heisenberg principle: limits the precision with which (certain) pairs of physical quantities can be determined

> relation energy  $\leftrightarrow$  times:  $\Delta E \Delta t \geq \hbar \approx 6.6 \cdot 10^{-22} \text{ MeV s}$

> since in a particle's rest frame the energy is given by the mass, this implies that only stable particles have an exact mass!

Life time  $\tau$

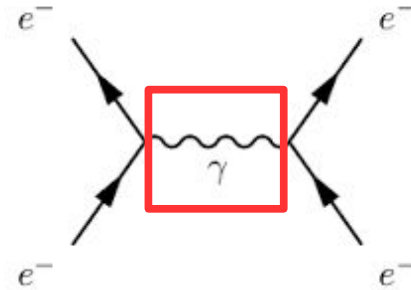


Decay width  $\Gamma$

$$\Gamma \tau = \hbar \approx 6.6 \cdot 10^{-22} \text{ MeV s}$$

# 0) The Heisenberg principle II

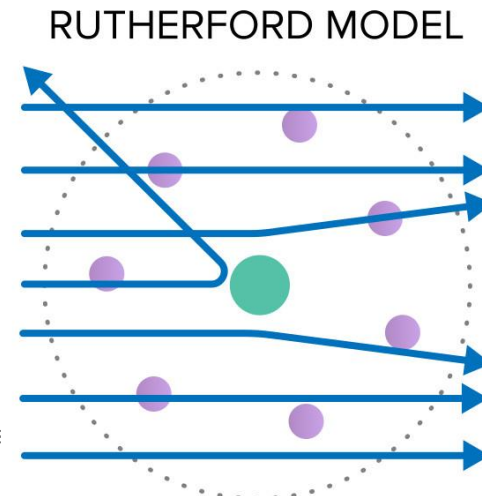
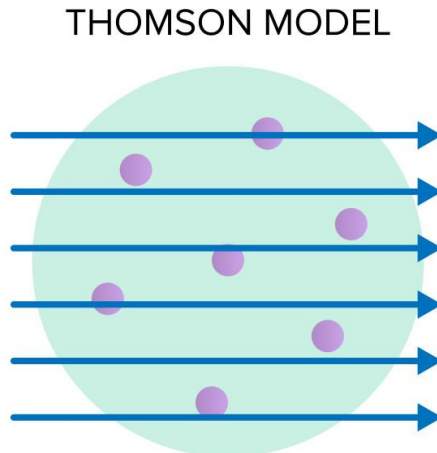
> Interaction carried by “force carrier”



> relation momentum  $\leftrightarrow$  position:  $\Delta p \Delta x \geq \hbar c \approx 200 \text{ MeV fm}$

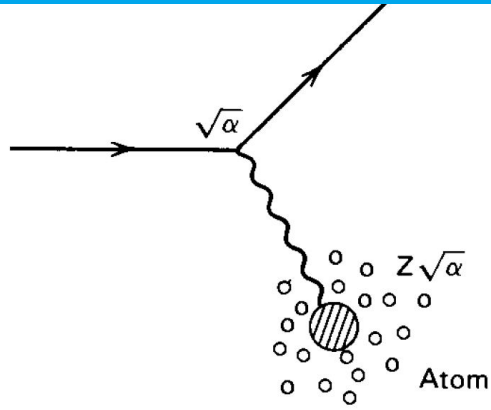
> Another possible application: the maximum possible momentum transfer in a reaction limits the size of structures you can resolve (what counts is the momentum transfer in the center-of-mass frame)

> Examples: you need a momentum transfer of 200 MeV to resolve structures of the size of a proton (1 fm)



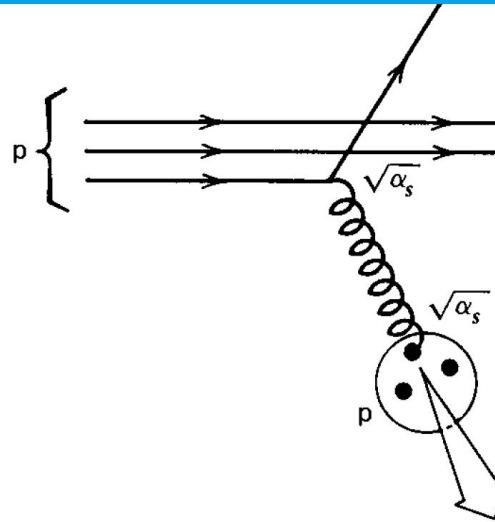


# 0) The Heisenberg principle III



Probability  $Z^2 \alpha^2$

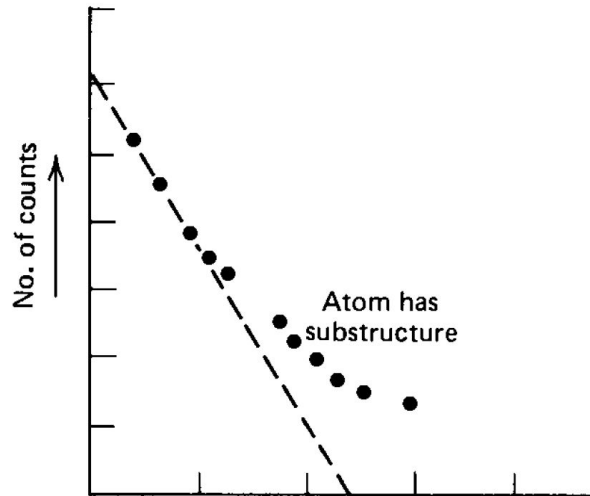
(a)



Probability  $\alpha_s^2$

(b)

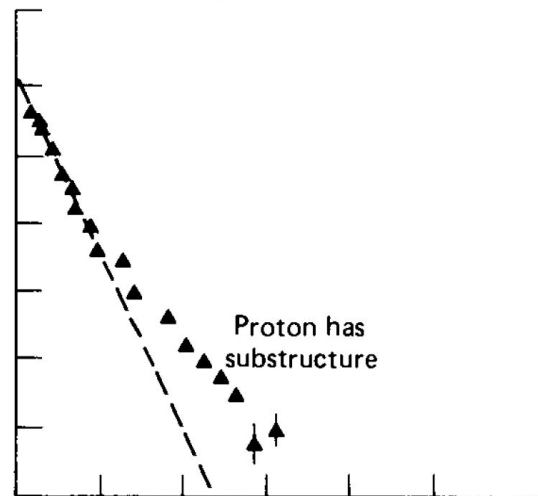
Au target *Phil. Mag.* xxi, 669 (1911)



Transverse momentum  
or scattering angle

(c)

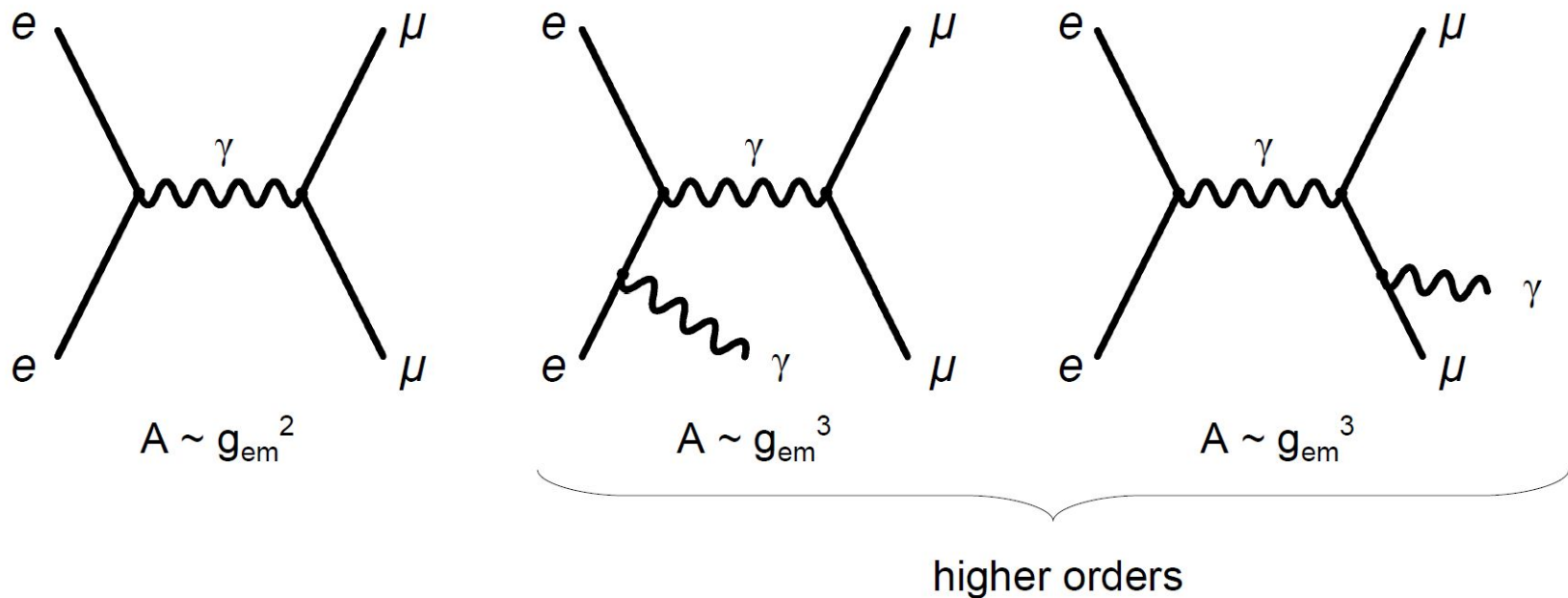
Proton target *Phys. Lett.* 46B, 471 (1973)





# 1) Higher orders

- > • in principle, all possible Feynman diagrams contribute to a reaction
- > • practically, those with the smallest number of vertices are most relevant
- > • those with more vertices are referred to as “higher orders” since they correspond to terms with higher order in the coupling if you write the cross section as a perturbation series



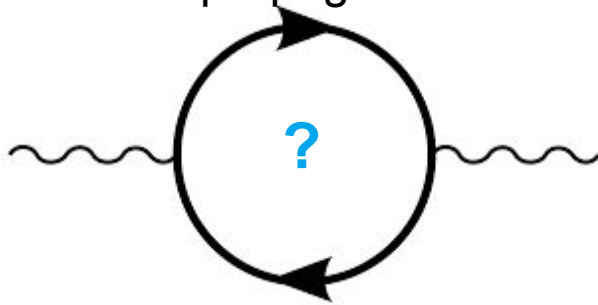
Initial state radiation

Final state radiation

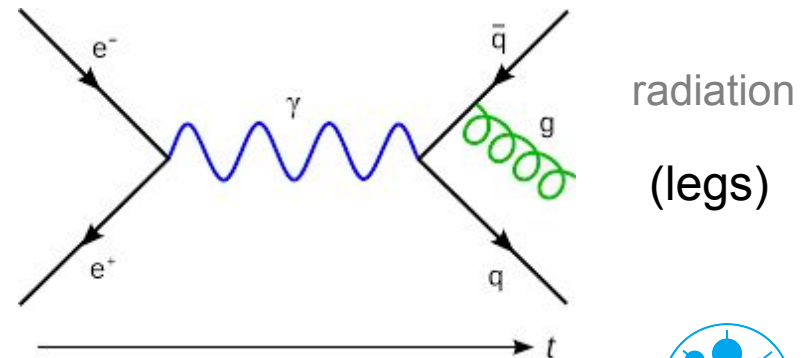
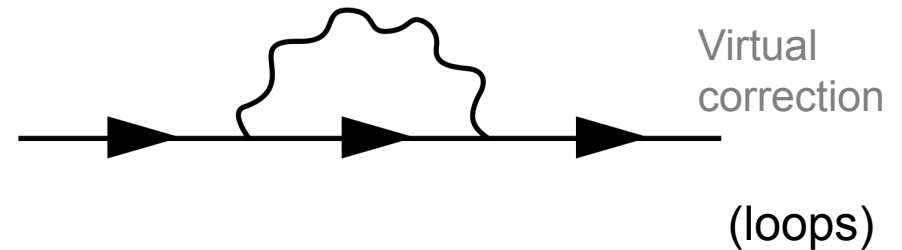
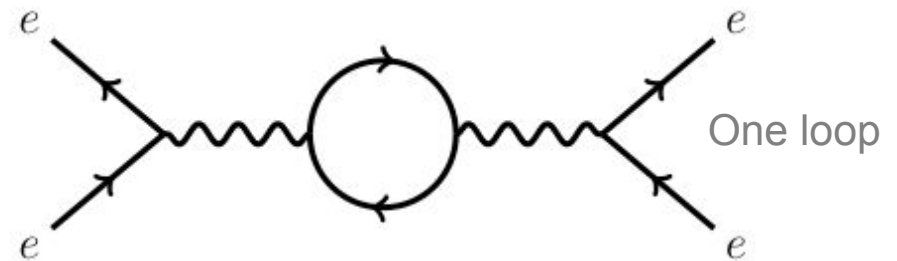
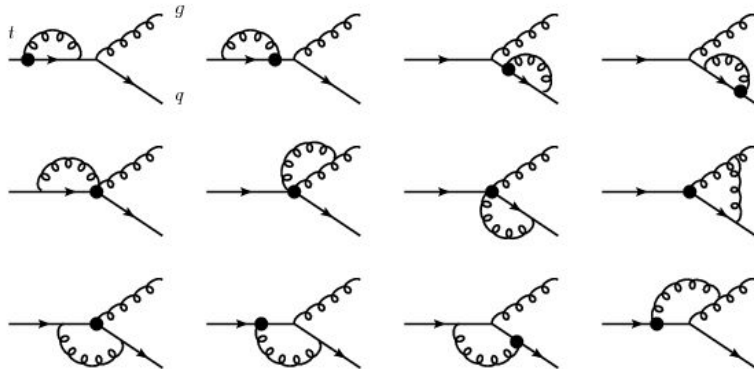
# Loop diagrams

- Possible is also the occurrence of “virtual” particles in loops, corrections to particles mass and couplings

Need to be able  
to couple to  
propagator

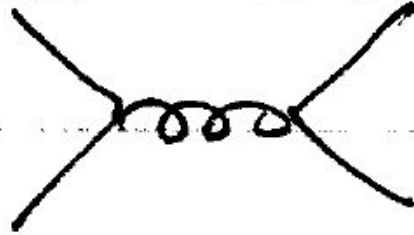


One loop correction to decay

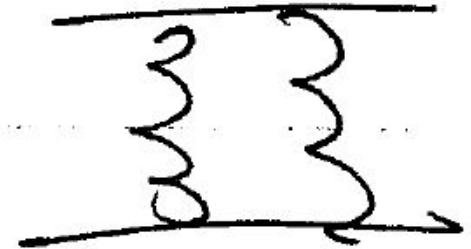
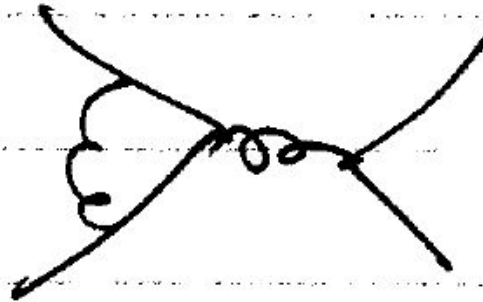


# Examples for Loop diagrams

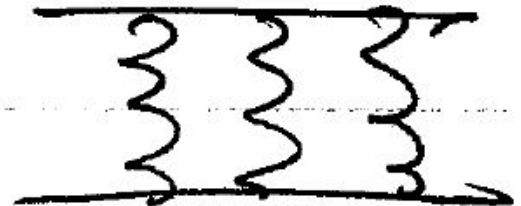
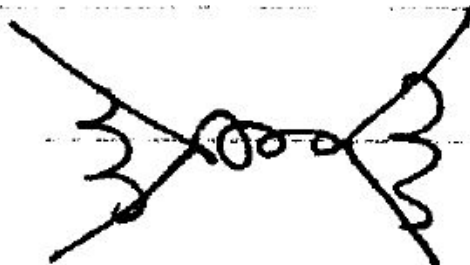
tree



loop



2-loop



Note: not all diagrams in a row are equivalent to each other

# Let's go back to the properties of an interaction

## Chirality

Identical to helicity in the massless case but something more complicated

It tells how two separate components of a fermionic field change under Lorentz boost (space-time change) → Weyl spinors. Each fermion has a left-handed component and a right-handed one

Parity transformations change chirality

General Lorentz transformation

$$S = \exp \begin{bmatrix} \frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi} & 0 \\ 0 & \frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi} \end{bmatrix}$$

$\theta$ : angle in space rotations

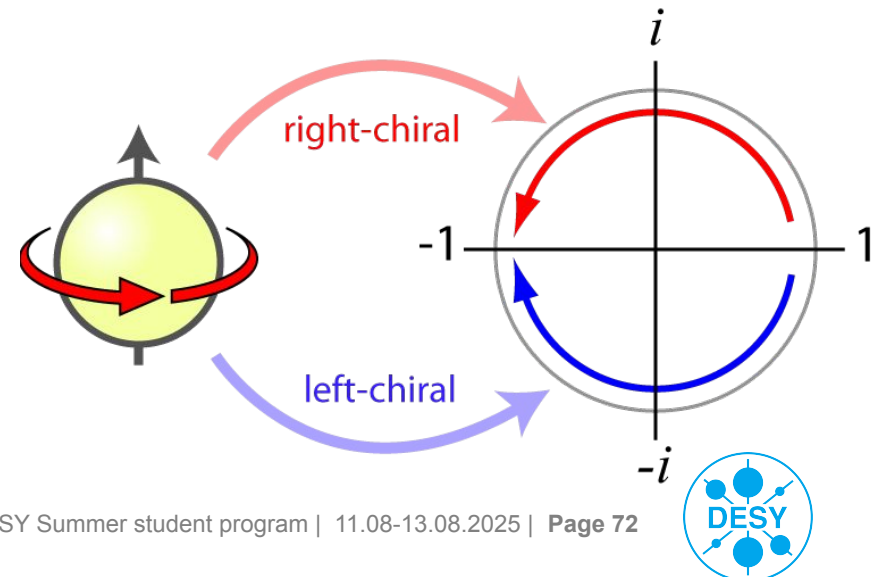
$\Phi$ : boost (time and space rotation)

Spinor of fermion: 2 terms (Weyl spinor) with 2 components

$$\Psi = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix} \quad \begin{aligned} \psi'_R &= \exp\left(\frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi}\right) \psi_R \\ \psi'_L &= \exp\left(\frac{1}{2}i\boldsymbol{\sigma} \cdot \boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\sigma} \cdot \boldsymbol{\phi}\right) \psi_L \end{aligned}$$

Mass terms in Lagrangian

$$-m\bar{\Psi}\Psi = -m(\bar{e}_L e_R + \bar{e}_R e_L)$$

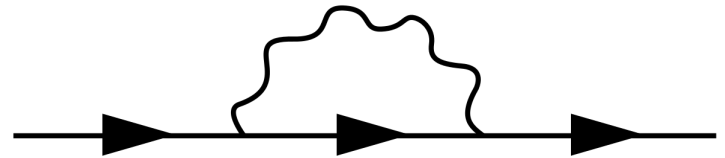
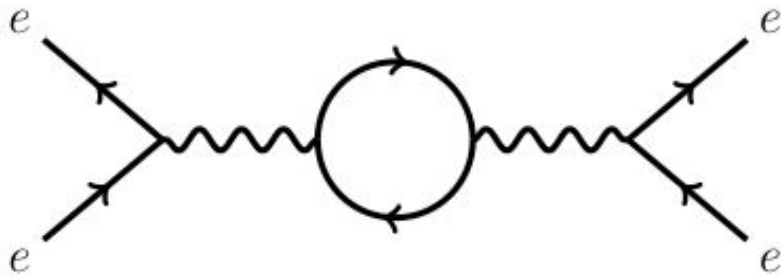


# Couplings, masses and Feynman diagrams

- When calculating the contribution of certain Feynman diagrams to the cross-section of an interaction, divergences (infinities) appear.

According to the laws of statistics, we have to integrate over all possibilities.

→ We have to integrate over all possible momenta the virtual particles in loops → Infinity !

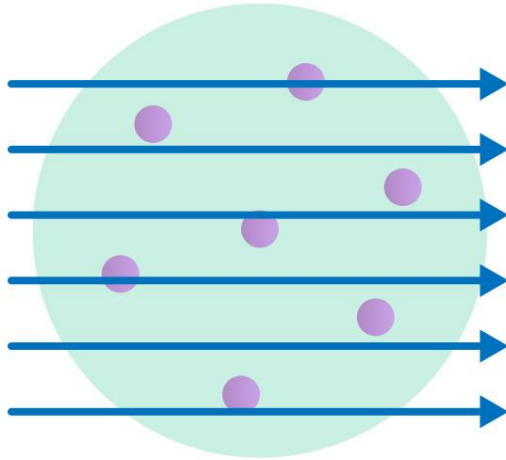


- What does it mean when you have a virtual particle creation close to infinity? If you are just making a collision at LHC, is it true that the possibility of creating virtual particles with infinity momenta will affect you?

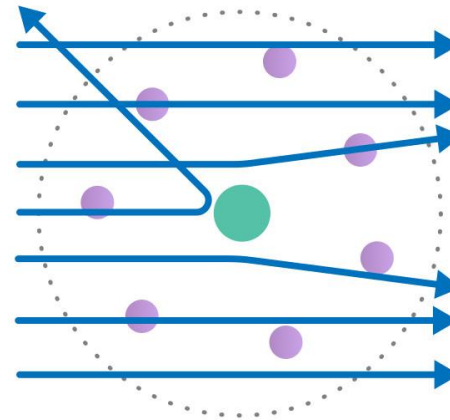
# Let's go back to the Heisenberg principle

$$\Delta p \Delta x \geq \hbar c \approx 200 \text{ MeV fm}$$

THOMSON MODEL



RUTHERFORD MODEL



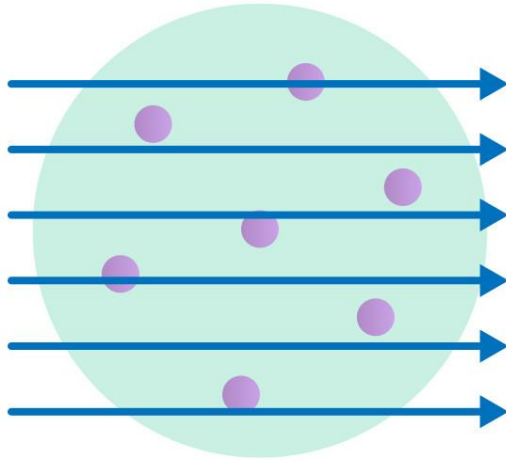
- > If I want to have a good resolution of the atom structure (1fm), at least the momentum transfer between the incoming particle and the atom has to be beyond 200 MeV.
- > If I want a better resolution, momentum exchange has to be higher !



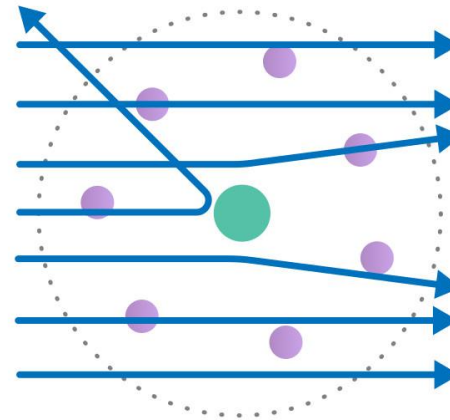
# Let's go back to the Heisenberg principle

$$\Delta p \Delta x \geq \hbar c \approx 200 \text{ MeV fm}$$

THOMSON MODEL



RUTHERFORD MODEL



- > Conversely, diagrams with virtual particles with very high momentum (energy) are diagrams that are happening in very short scales (times)
- > If my interaction is happening at momentum 200 GeV and therefore can structures and processes at  $x = 1 \text{ am}$

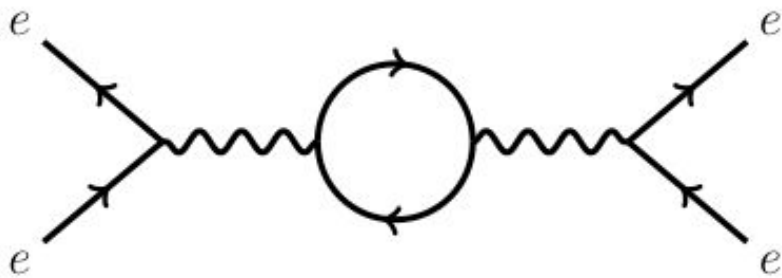
Can my measurements be affected by things happening at  $\Delta x \sim 0$  ?

-> the assumption is that not !  $\rightarrow$  Energy cut-off  $\Lambda$

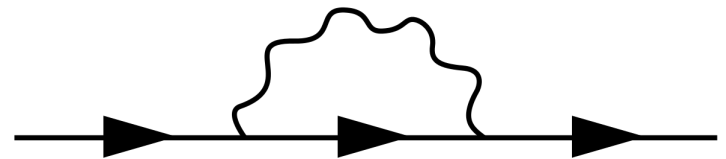
# Effective couplings and masses

- Regularization and renormalization: the procedure on which I take these divergences and cut-offs and absorb them into definition of the physical quantities

Some of these infinities might cancel out due to symmetries or because similar contributions but opposite sign.



Loop diagrams etc: correction to the interaction coupling (i.e.  $g_{EM}$ )  
→ Effective coupling

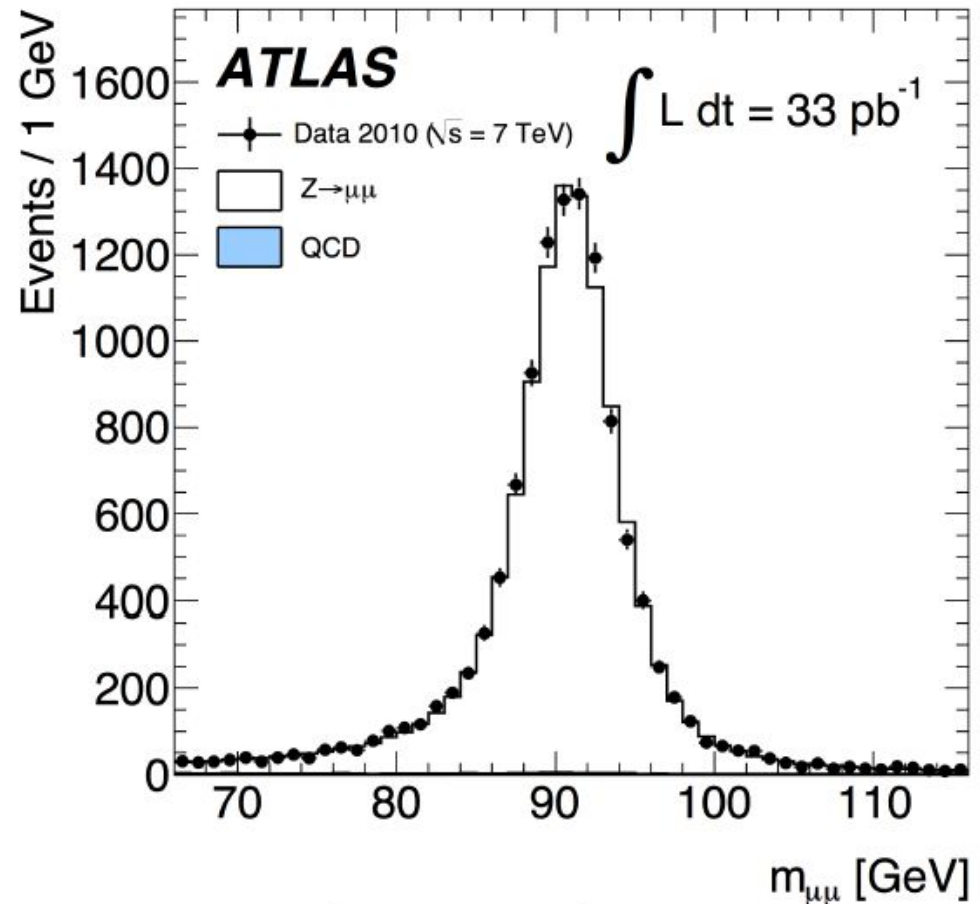


Self-interaction: correction to the mass of the particle → Effective mass

Message to take home: the observed mass and coupling strength in an interaction depends on the energy exchanged in the interaction

# How to measure a cross section

- > (simple) example:  
 $Z^0 \rightarrow \mu^+ \mu^-$  from ATLAS
- > identify muons in the detector, select events with at least 2 muons of opposite charge, calculate the invariant mass, fill the mass into a histogram
- > determine how much background
  - use the prediction from a MC simulation (works if background is well known)
  - fit a function of the form  $f(m) = \text{signal}(m) + \text{bkg}(m)$  to the data (works if expected shapes are known)



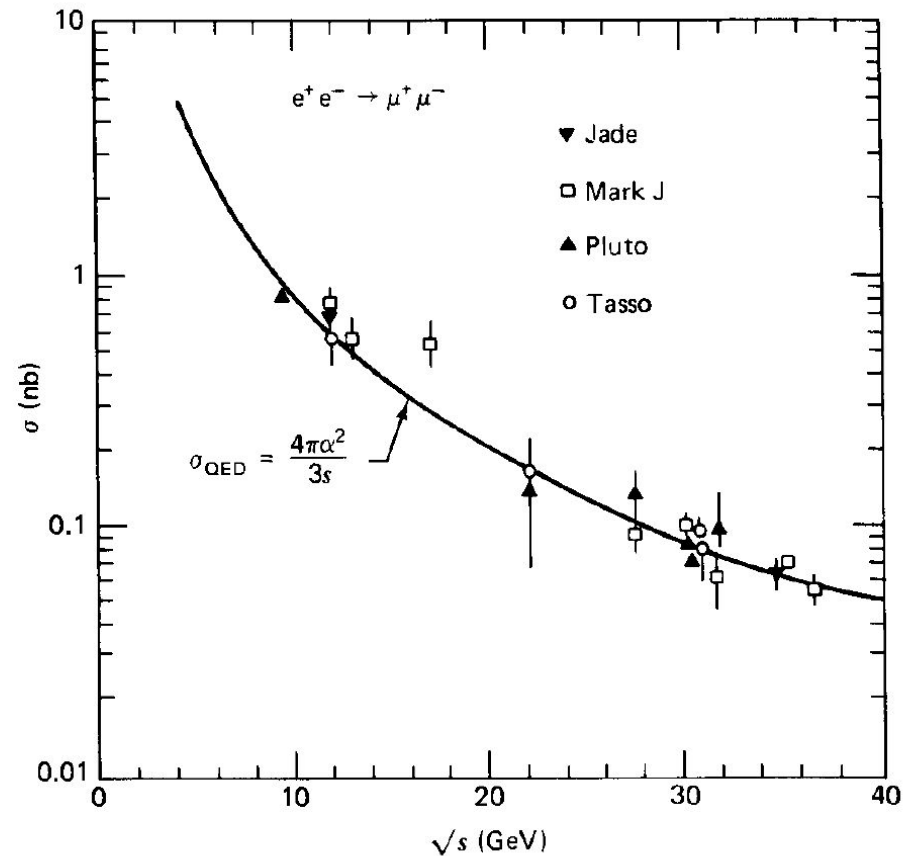
$$\sigma = \frac{(N_{\text{evt}} - N_{\text{Bkg}})}{\epsilon \text{ BR } \int L dt}$$

# Feynman diagrams: example of $e\mu$ scattering

>Measurements:

$$\text{Cross section} = \frac{W_{fi}}{(\text{initial flux})} (\text{number of final states})$$

>Need to integrate over full solid angle here and momenta



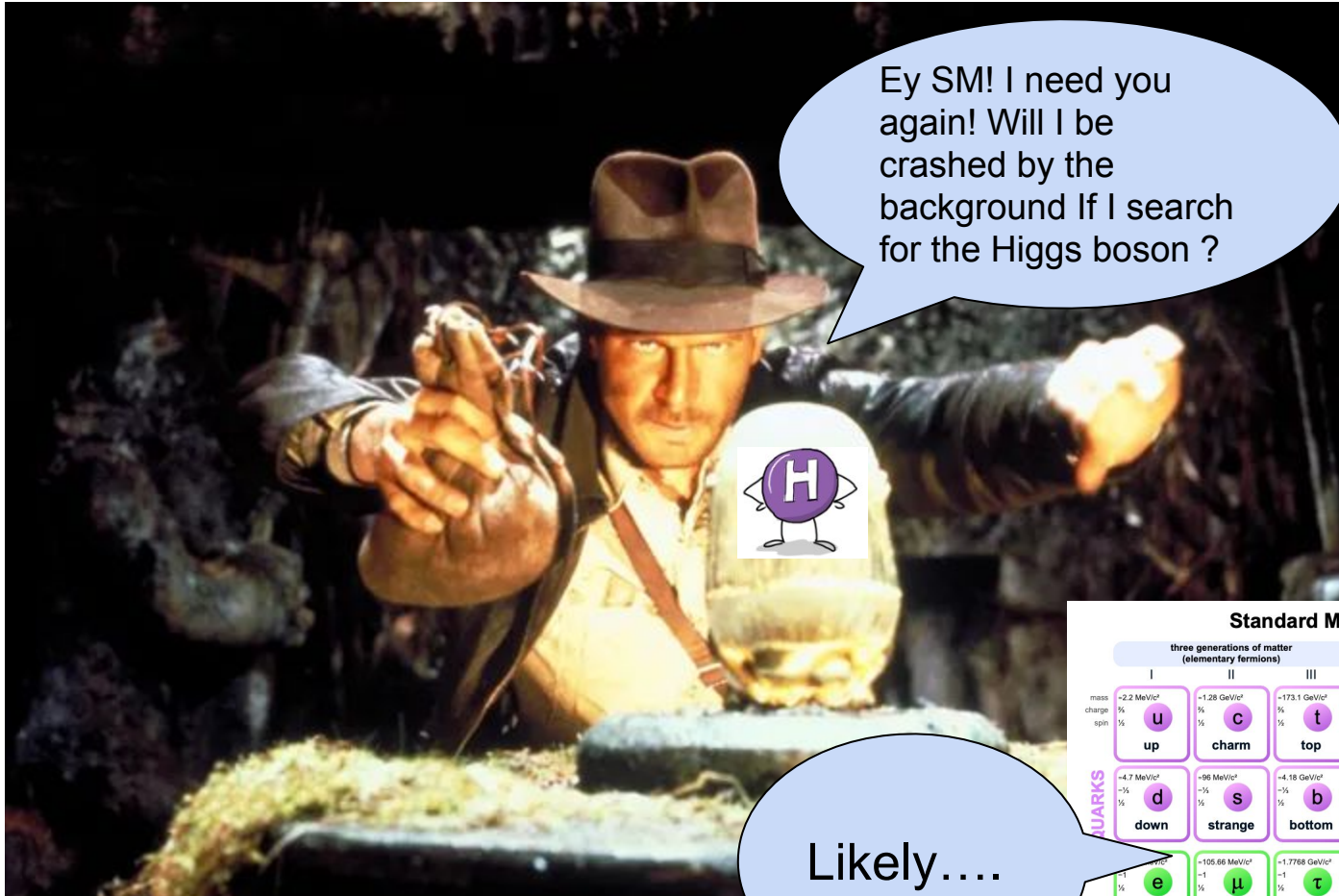
**Fig. 6.6** The total cross section for  $e^-e^+ \rightarrow \mu^-\mu^+$  measured at PETRA versus the center-of-mass energy.





# We have the Standard Model. Why did we do that ?

>We have formulated how fields interact in a consistent way. Why ?



Likely....

## Standard Model of Elementary Particles

three generations of matter (elementary fermions)									three generations of antimatter (elementary antifermions)									interactions / force carriers (elementary bosons)	
I			II			III			I			II			III				
mass	~2.2 MeV/c <sup>2</sup>	1/2	~1.28 GeV/c <sup>2</sup>	1/2	~173.1 GeV/c <sup>2</sup>	1/2	~2.2 MeV/c <sup>2</sup>	1/2	~1.28 GeV/c <sup>2</sup>	1/2	~173.1 GeV/c <sup>2</sup>	1/2	~2.2 MeV/c <sup>2</sup>	1/2	~173.1 GeV/c <sup>2</sup>	1/2	0	0	1
charge	2/3		2/3		2/3		-2/3		-2/3		-2/3		-2/3		-2/3		0	0	0
spin	1/2		1/2		1/2		1/2		1/2		1/2		1/2		1/2		1	1	1
	<b>u</b>		<b>c</b>		<b>t</b>		<b>ū</b>		<b>c̄</b>		<b>t̄</b>		<b>d</b>		<b>s̄</b>		<b>g</b>		<b>H</b>
	up		charm		top		antiup		anticharm		antitop		down		strange		gluon		higgs
	~4.7 MeV/c <sup>2</sup>	1/2	~96 MeV/c <sup>2</sup>	1/2	~4.18 GeV/c <sup>2</sup>	1/2	~4.7 MeV/c <sup>2</sup>	1/2	~96 MeV/c <sup>2</sup>	1/2	~4.18 GeV/c <sup>2</sup>	1/2	~4.7 MeV/c <sup>2</sup>	1/2	~96 MeV/c <sup>2</sup>	1/2	0	0	1
	2/3		-1/3		-1/3		1/3		1/3		1/3		2/3		2/3		0	0	0
	1/2		1/2		1/2		1/2		1/2		1/2		1/2		1/2		1	1	1
	<b>d</b>		<b>s</b>		<b>b</b>		<b>d̄</b>		<b>s̄</b>		<b>b̄</b>		<b>u</b>		<b>c̄</b>		<b>γ</b>		
	down		strange		bottom		antidown		antistrange		antibottom		up		charm		photon		
	~0.511 MeV/c <sup>2</sup>	1/2	~105.66 MeV/c <sup>2</sup>	1/2	~1.7768 GeV/c <sup>2</sup>	1/2	~0.511 MeV/c <sup>2</sup>	1/2	~105.66 MeV/c <sup>2</sup>	1/2	~1.7768 GeV/c <sup>2</sup>	1/2	~0.511 MeV/c <sup>2</sup>	1/2	~105.66 MeV/c <sup>2</sup>	1/2	~91.19 GeV/c <sup>2</sup>	0	1
	-1		-1		-1		1		1		1		-1		-1		0	0	0
	1/2		1/2		1/2		1/2		1/2		1/2		1/2		1/2		1	1	1
	<b>e</b>		<b>μ</b>		<b>τ</b>		<b>e<sup>+</sup></b>		<b>μ̄</b>		<b>τ̄</b>		<b>ν<sub>e</sub></b>		<b>ν̄<sub>e</sub></b>		<b>Z</b>		
	electron		muon		tau		positron		antimuon		antitau		electron neutrino		electron antineutrino		Z <sup>0</sup> boson		
	~0.511 MeV/c <sup>2</sup>	1/2	~105.66 MeV/c <sup>2</sup>	1/2	~1.7768 GeV/c <sup>2</sup>	1/2	~0.511 MeV/c <sup>2</sup>	1/2	~105.66 MeV/c <sup>2</sup>	1/2	~1.7768 GeV/c <sup>2</sup>	1/2	~0.511 MeV/c <sup>2</sup>	1/2	~105.66 MeV/c <sup>2</sup>	1/2	~91.19 GeV/c <sup>2</sup>	0	1
	0		0		0		0		0		0		0		0		0	0	0
	1/2		1/2		1/2		1/2		1/2		1/2		1/2		1/2		1	1	1
	<b>ν<sub>e</sub></b>		<b>ν̄<sub>e</sub></b>		<b>ν̄<sub>μ</sub></b>		<b>ν̄<sub>τ</sub></b>		<b>ν̄<sub>e</sub></b>		<b>ν̄<sub>μ</sub></b>		<b>ν̄<sub>τ</sub></b>		<b>ν̄<sub>e</sub></b>		<b>W<sup>+</sup></b>		<b>W<sup>-</sup></b>
	electron neutrino		electron antineutrino		muon antineutrino		tau antineutrino		electron antineutrino		muon antineutrino		tau antineutrino		electron antineutrino		W <sup>+</sup> boson		W <sup>-</sup> boson

**QUARKS**

**LEPTONS**

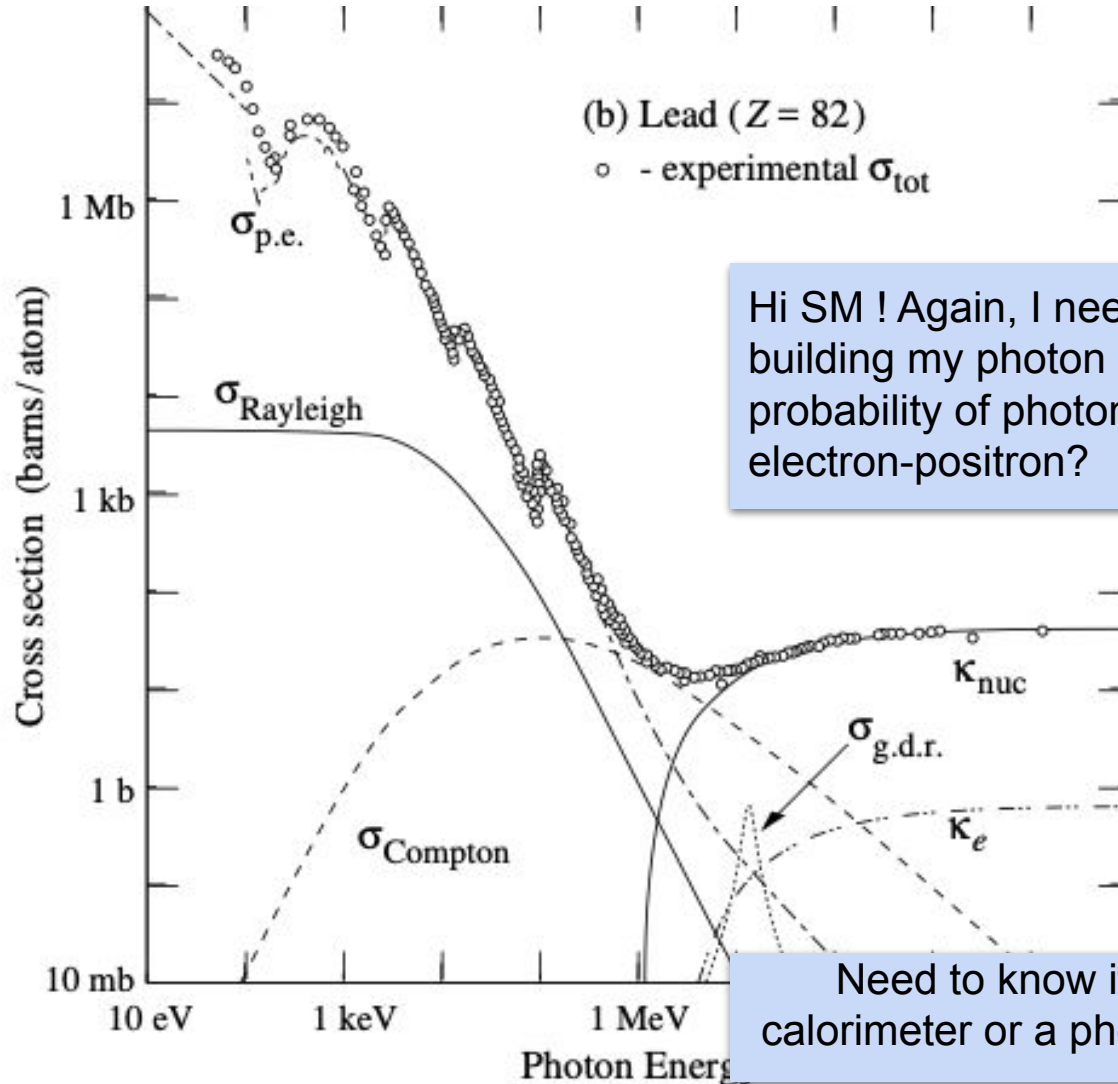
**GAUGE BOSONS VECTOR BOSONS**

**SCALAR BOSONS**



# We have the Standard Model. Why did we do that ?

> We have formulated how fields interact in a consistent way. Why ?



Hi SM ! Again, I need your magic. I am building my photon detector. What is the probability of photons to decay into electron-positron?

Need to know if I need a calorimeter or a photo-multiplier.

# We have the Standard Model. Why did we do that ?

> We have formulated how fields interact in a consistent way. Now, what ?

mass →	≈2.3 MeV/c <sup>2</sup>	≈1.275 GeV/c <sup>2</sup>	≈173.07 GeV/c <sup>2</sup>	0	≈126 GeV/c <sup>2</sup>
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> Higgs boson
<b>QUARKS</b>					
	≈4.8 MeV/c <sup>2</sup>	≈95 MeV/c <sup>2</sup>	≈4.18 GeV/c <sup>2</sup>	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b>γ</b> photon	
	0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	91.2 GeV/c <sup>2</sup>	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	<b>e</b> electron	<b>μ</b> muon	<b>τ</b> tau	<b>Z</b> Z boson	
<b>LEPTONS</b>					
	<2.2 eV/c <sup>2</sup>	<0.17 MeV/c <sup>2</sup>	<15.5 MeV/c <sup>2</sup>	80.4 GeV/c <sup>2</sup>	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	<b>ν<sub>e</sub></b> electron neutrino	<b>ν<sub>μ</sub></b> muon neutrino	<b>ν<sub>τ</sub></b> tau neutrino	<b>W</b> W boson	
					<b>GAUGE BOSONS</b>

Knowledge of what interactions we can expect from every particle.

Predictions + measurements of masses and couplings.

Predictions of how likely an interaction is to happen.

.....

Deviations from the Standard Model predictions → New Physics !

New particles (DM..) or forces (super-QED ? ) → New Physics !