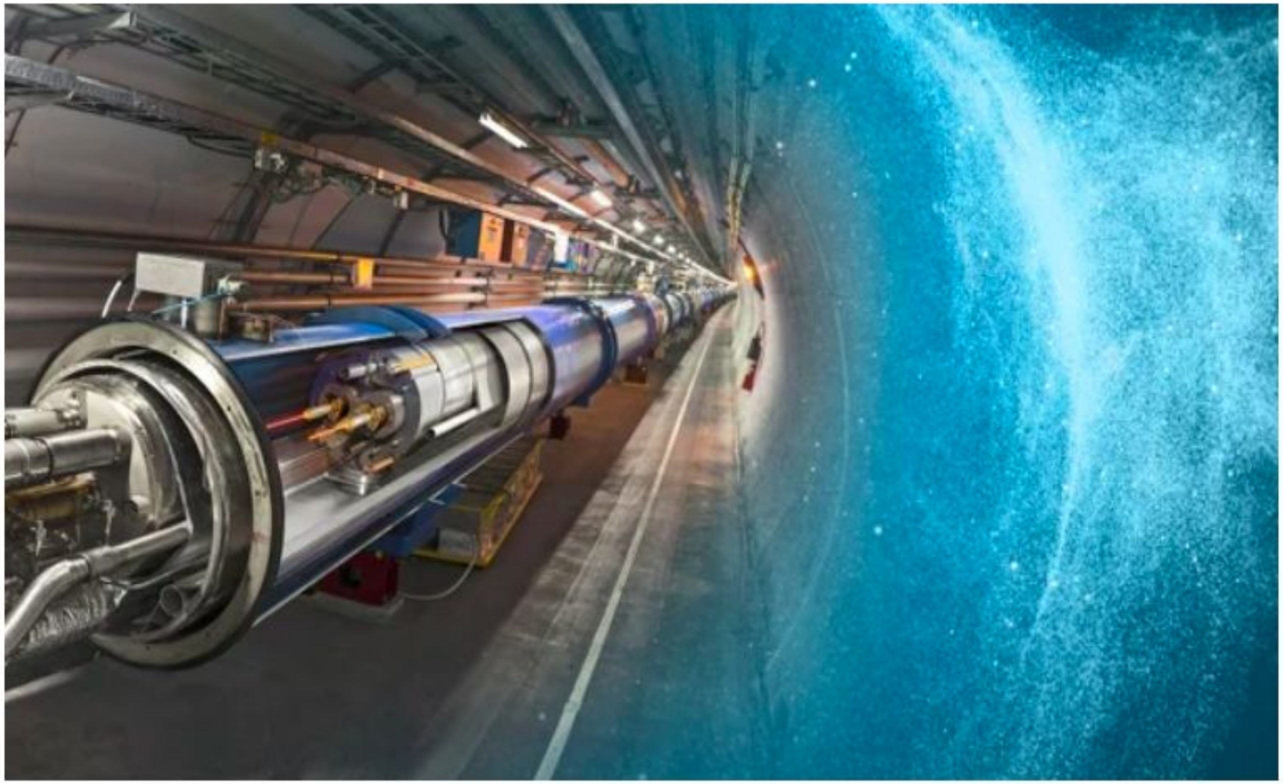
Introduction to the Standard Model

Summer Student Lecture 2025 – Part II



Clara Leitgeb

Deutsches Elektronen Synchrotron

HELMHOLTZ | ASSOCIATION

Many thanks to Thorsten Kuhl and Alvaro Lopez Solis for their lectures and help

DESY

Content

- 0) Introduction
  - What is the Standard Model?
  - Coupling constants, masses and charges
  - Units and scales

- 1) Interactions
  - Relativistic kinematics
  - Symmetries and conserved quantities
  - Feynman diagrams
  - Running couplings and masses

- **2)Quantum electrodynamics**
  - **Tests of QED: Magnetic momentum of the leptons**
  - **Tests of QED: High energy colliders**

Content

- **3) Strong Interaction: Quantum-Chromodynamics**
  - **A short history of hadrons and quarks**
  - **Deep inelastic scattering and gluons**
  - **QCD and its properties**

- 4) Electroweak interactions
  - Discovery of electroweak bosons
  - Tests of angular distributions
  - Feynman rules
  - Handed-ness of electroweak interactions
  - More tests of the electroweak SM

- 5) The Higgs
  - Why was it predicted?
  - How was it found?

Quantum electrodynamics
QED

Let’s start from the beginning

It was a warm summer in ancient Greece. Someone, for no particular reason, approached iron to a stone….

And 3000 years later + some very intelligent people

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)
\end{aligned}
$$

Covariant formalism→ Classical field theory

$$
F_{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & -iE_x \\ -B_z & 0 & B_x & -iE_y \\ B_y & -B_x & 0 & -iE_z \\ iE_x & iE_y & iE_z & 0 \end{pmatrix}
$$

Second quantization

$$
\mathbf{A}(\mathbf{r}) = \sum_{\mathbf{k},\mu} \sqrt{\frac{\hbar}{2\omega V \epsilon_0}} \left(\mathbf{e}^{(\mu)} a^{(\mu)}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} + \bar{\mathbf{e}}^{(\mu)} a^{\dagger(\mu)}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{r}} \right)
$$

$$
\mathbf{E}(\mathbf{r}) = i \sum_{\mathbf{k},\mu} \sqrt{\frac{\hbar\omega}{2V \epsilon_0}} \left(\mathbf{e}^{(\mu)} a^{(\mu)}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} - \bar{\mathbf{e}}^{(\mu)} a^{\dagger(\mu)}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{r}} \right)
$$

$$
\mathbf{B}(\mathbf{r}) = i \sum_{\mathbf{k},\mu} \sqrt{\frac{\hbar}{2\omega V \epsilon_0}} \left((\mathbf{k} \times \mathbf{e}^{(\mu)}) a^{(\mu)}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} - (\mathbf{k} \times \bar{\mathbf{e}}^{(\mu)}) a^{\dagger(\mu)}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{r}} \right),
$$

$$
\mathcal{L}_{\mathcal{EM}} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{c} j_\mu A_\mu
$$

Quantum electrodynamics in a nutshell

> Gauge theory (lagrangian symmetric under local transformations) including the electromagnetic interaction

Treating only electromagnetic field interactions with particles charged under the electromagnetic field → electric charge

Gauge boson mediating interaction: photon (A_μ,γ)

Quarks:

Up/charm/top : q = ⅔

Down/Strang/Bottom: q = -⅓

Anti-quarks:

Exactly opposite sign charge of their anti-particle

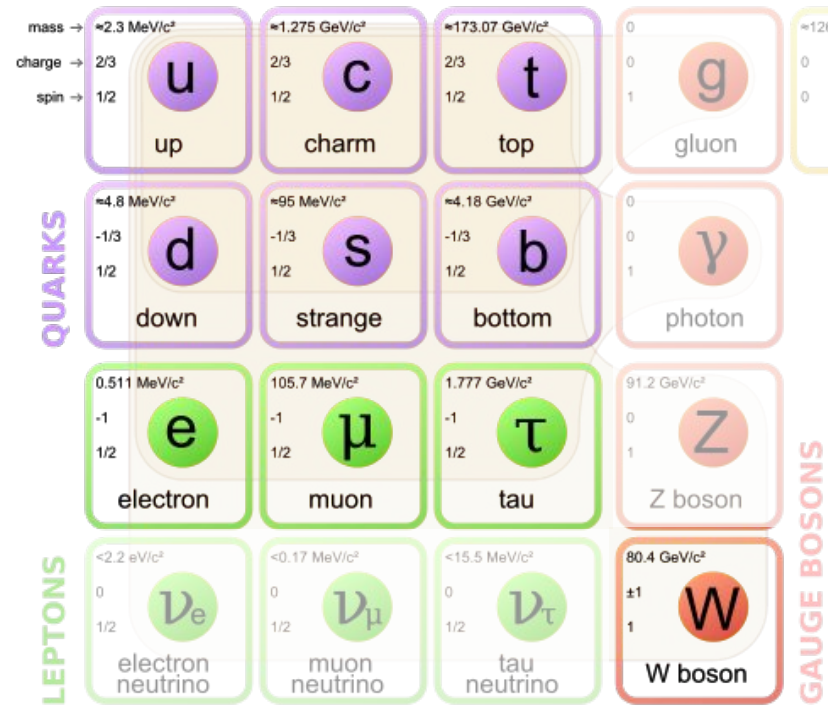
Leptons:

Electron/muon/tau : q = -1

Neutrinos: q = 0

Anti-leptons:

Exactly opposite sign charge of their anti-particle



W-bosons: q = +-1 (particle-anti-particle)

Quantum electrodynamics in a nutshell

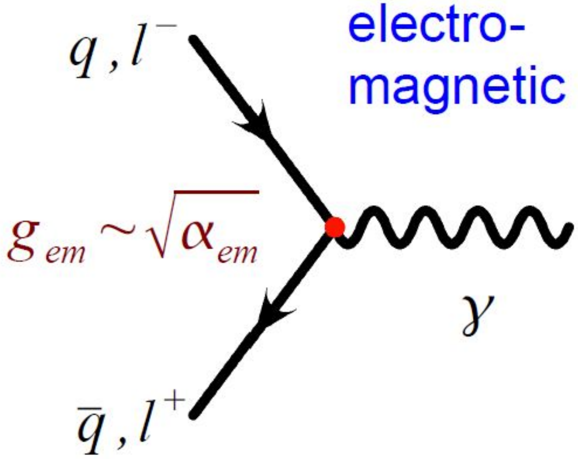
- Gauge theory (lagrangian symmetric under local transformations) including the electromagnetic interaction
  - Treating only electromagnetic field interactions with particles charged under the electromagnetic field → electric charge

Coupling constant

Universal (not different coupling depending on the particle)
At low energies, fine structure constant

$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi\epsilon_0 \hbar c}$$

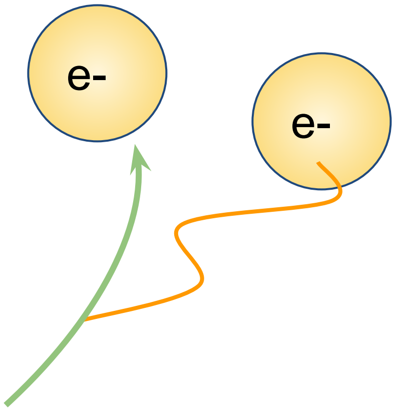
$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi} = \frac{1}{137}$$



Running coupling of QED

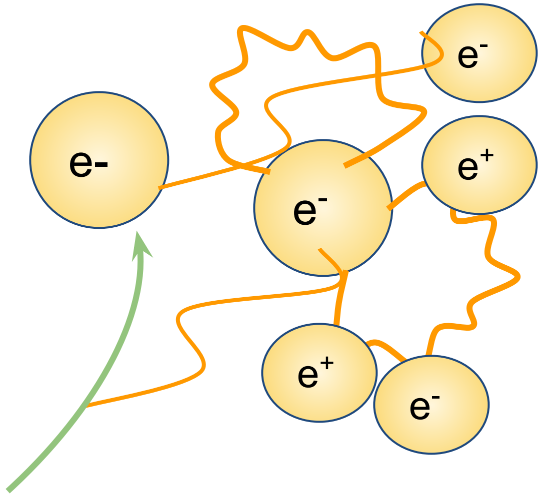
Low energy QED interaction

An incident particle would see or would be more affected by is the tree level diagrams



High energy QED interaction

An incident particle would start to feel the effects of high-energy (short wave-lengths) interactions



At different interaction energies, the QED coupling that a incident electron sees is different !

Quantum electrodynamics in a nutshell

- Gauge theory (lagrangian symmetric under local transformations) including the electromagnetic interaction
  - Treating only electromagnetic field interactions with particles charged under the electromagnetic field → electric charge

Coupling constant

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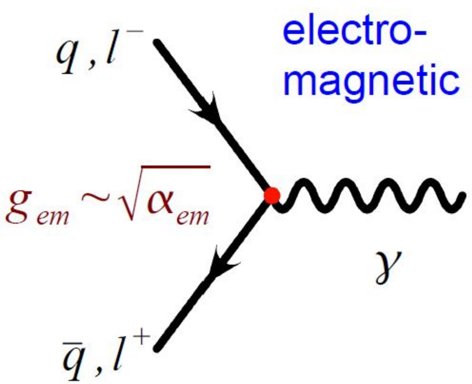
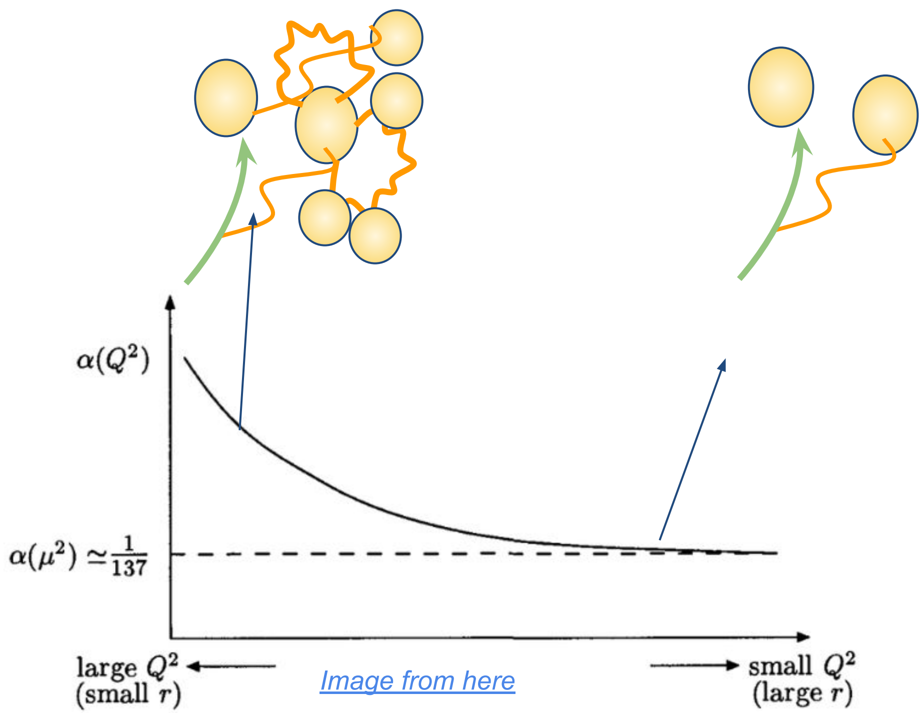


$$\alpha_{\text{EM}} = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$\alpha_{\text{EM}} = \frac{e^2}{4\pi} = \frac{1}{137}$$



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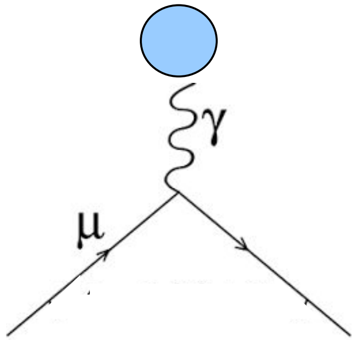
Current tests of quantum electrodynamics

- Electromagnetism is very well-known at low energies and for a high range of energies → Extremely precise calculations up to several orders of loops needed

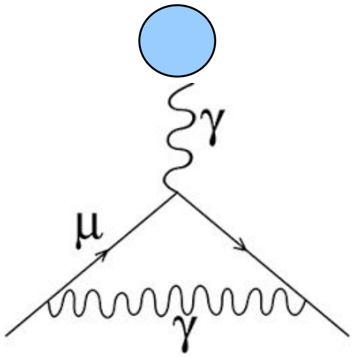
Low energy: magnetic moment of charged particles

Magnetic moment due to charged body with angular momentum and/or spin

$$\mu = -g\frac{e}{2m}s = -\frac{g}{2}\frac{e}{2m}$$



Dirac theory predicts g=2

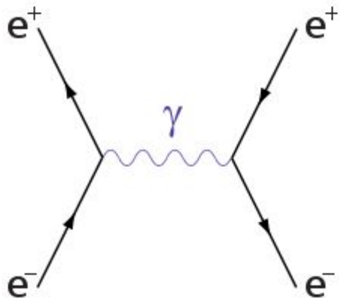


Quantum corrections

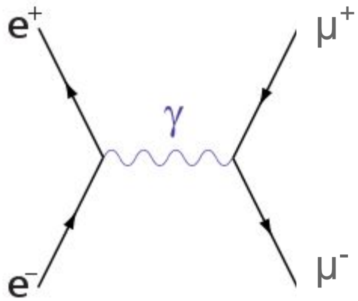
High energy tests:

Verify cross-section predictions at high energies
QED running coupling
QED lepton coupling universality

$\sigma(e^+e^- \rightarrow e^+e^-)$



$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$

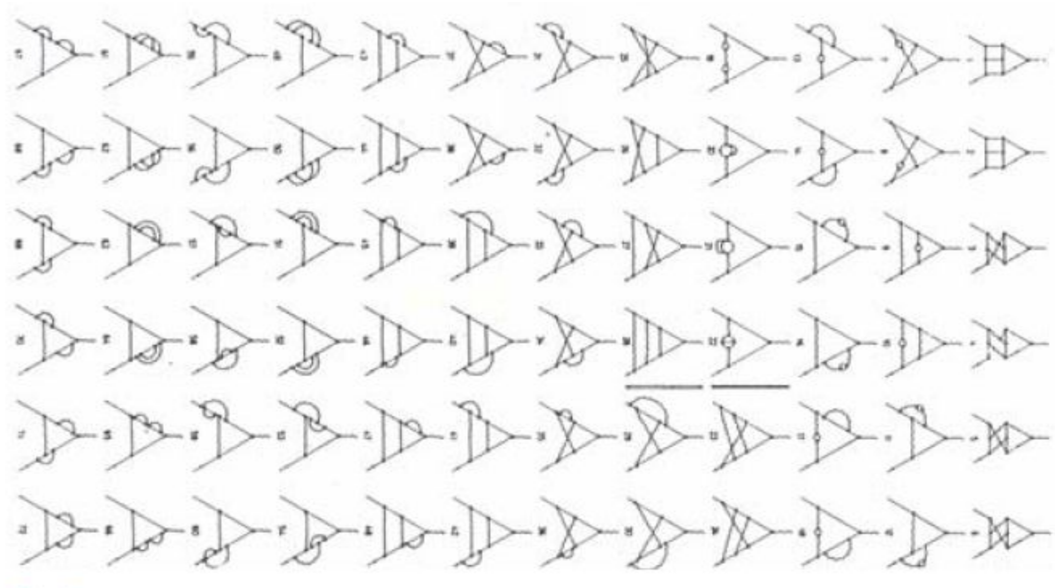


Low energy QED tests: electron magnetic moment

> Summing Feynman diagrams up to 4th order in the EM coupling

Feynman Graphs		
$O(\alpha)$		1
$O(\alpha^2)$		7
$O(\alpha^3)$	analytically	72
$O(\alpha^4)$	numerically	891
til $O(\alpha^4)$		971

Most precise calculations:
T. Kinoshita et al.



> Capable of producing results up to an accuracy of 10^{-12}

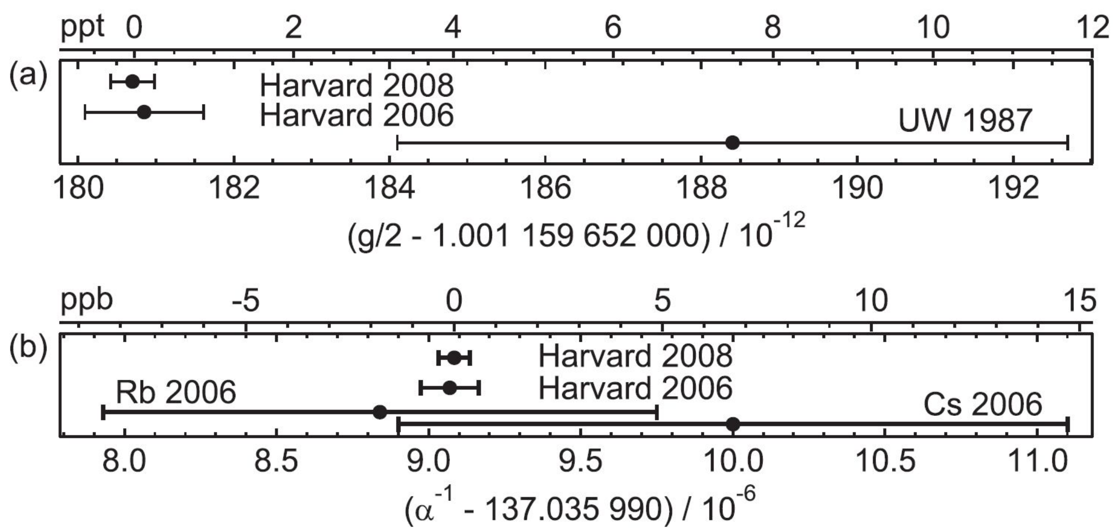
$$(g/2 - 1.001\ 159\ 652\ 000)\ /\ 10^{-12}$$

Low energy QED tests: electron magnetic moment

- Precision measurements of electron magnetic moment show a good agreement with predictions !

JHEP11 (2012) 113

Phys. Rev. Lett. **100**, 120801



Reference 1:

Probability that the SM can explain this is 0.19

Reference 2:

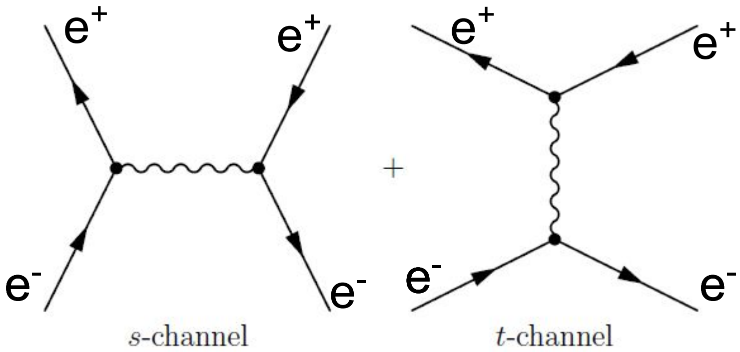
To even consider that there is some evidence of BSM, HEP needs ≥ 2 σ - 3 σ

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -10.5\,(8.1) \times 10^{-13}$$

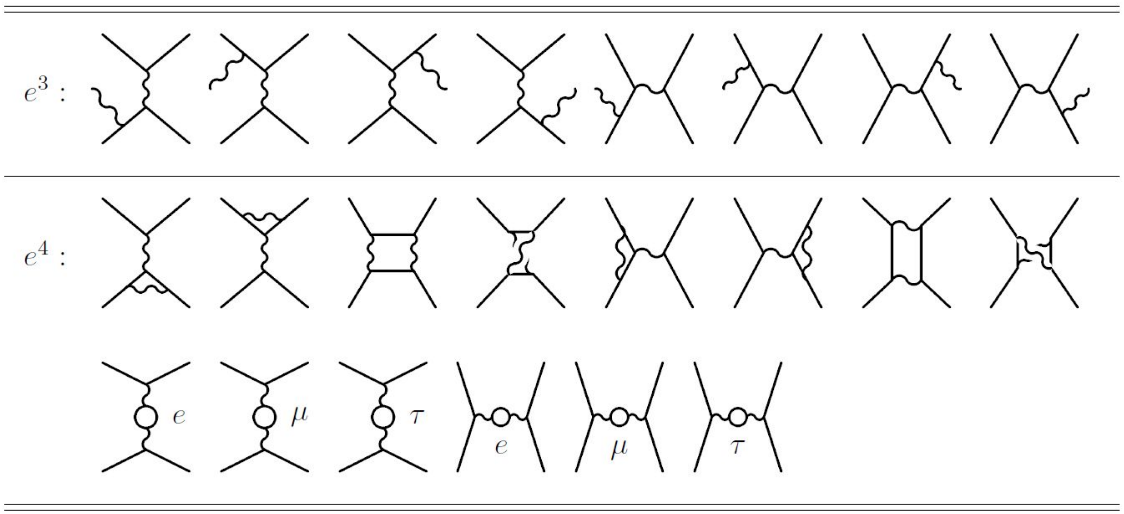
Difference of around 1.3 σ

High-energy tests: Bhabha scattering ($e^+e^- \rightarrow e^+e^-$)

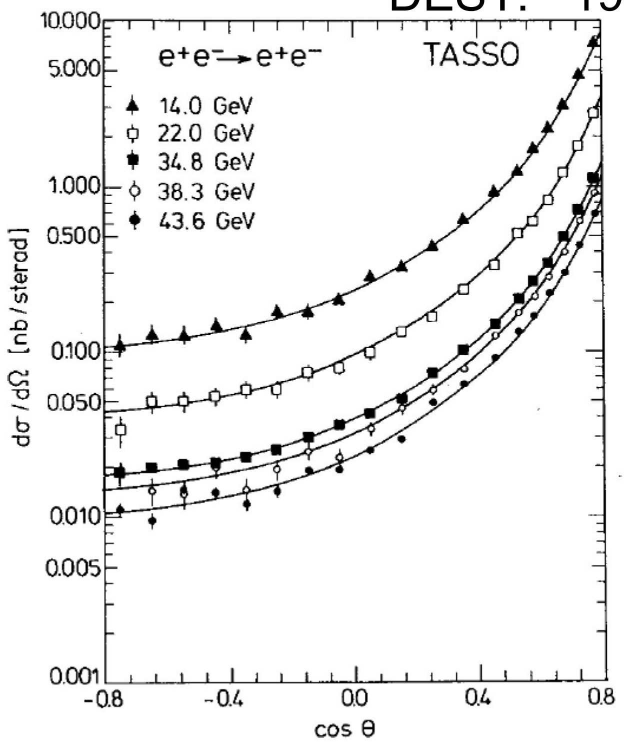
- High-energy colliders probe the following processes:



$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{4s}\left(\underbrace{\frac{t^2+s^2}{u^2}}_{t\text{-channel}} + \underbrace{\frac{2t^2}{us}}_{\text{interference}} + \underbrace{\frac{t^2+u^2}{s^2}}_{s\text{-channel}}\right)$$
$$= \frac{\alpha^2}{4s}\left(\frac{3+\cos^2\vartheta}{1-\cos\vartheta}\right)^2 .$$

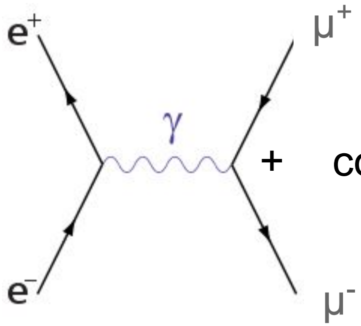


DESY: ~1979



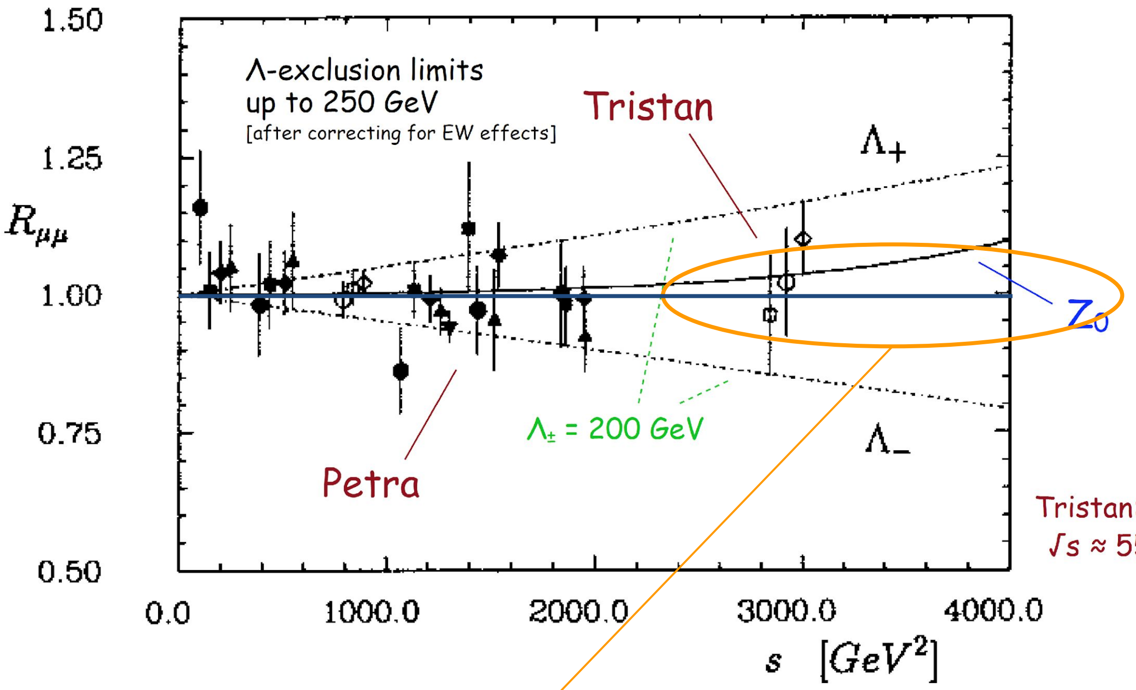
High-energy tests: lepton pair production

QED process



+ corrections

$$R_{\mu\mu} = \frac{\sigma_{\text{meas}}^{e^+e^- \to \mu^+\mu^-}}{\sigma_{\text{QED}}^{e^+e^- \to \mu^+\mu^-}},$$

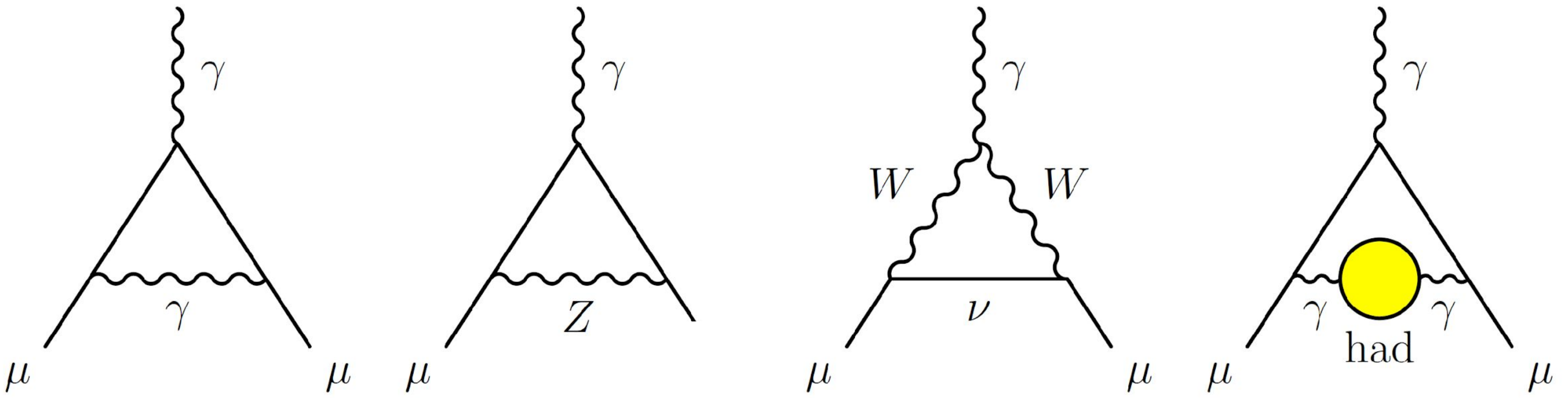


Tristan:
√s ≈ 55 GeV

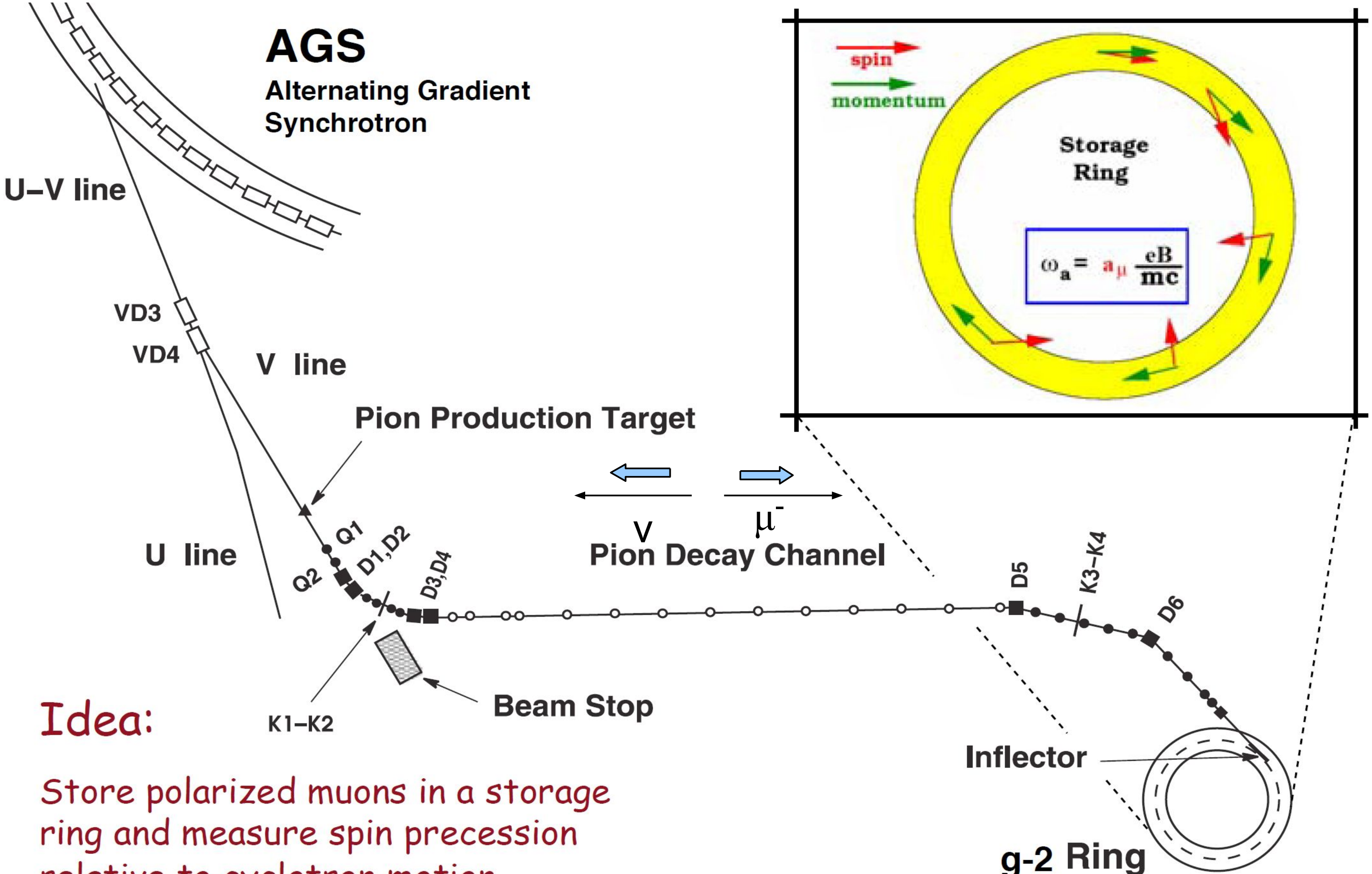
Deviations mean that there is something that we don't know!
First indication of a unified EW force → Tomorrow's lecture

Current challenges to QED: muon-magnetic moment

- Similar to electron's, muon's magnetic moment with larger corrections due to QCD and EWK.



Tests of QED: magnetic momentum for muons



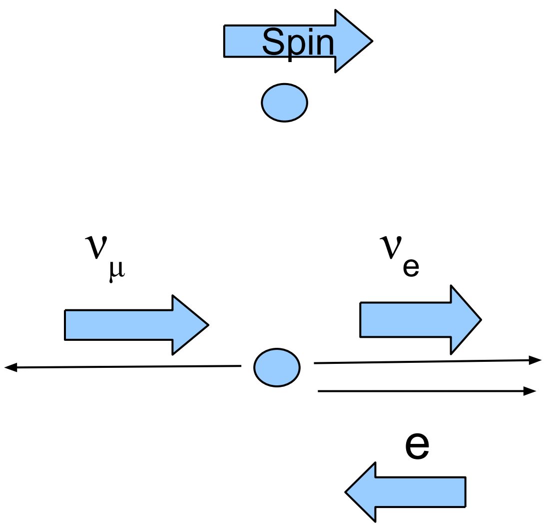
Idea:

Store polarized muons in a storage ring and measure spin precession relative to cyclotron motion

$$\vec{\omega}_{a_\mu} = \vec{\omega}_S - \vec{\omega}_C = -g_\mu \frac{Qe\vec{B}}{2m} - (1-\gamma)\frac{Qe\vec{B}}{\gamma m} + \frac{Qe\vec{B}}{\gamma m} = -\left(\frac{g_\mu - 2}{2}\right)\frac{Qe}{m}\vec{B} = -a_\mu \frac{Qe}{m}\vec{B}$$

Tests of QED: magnetic momentum for muons

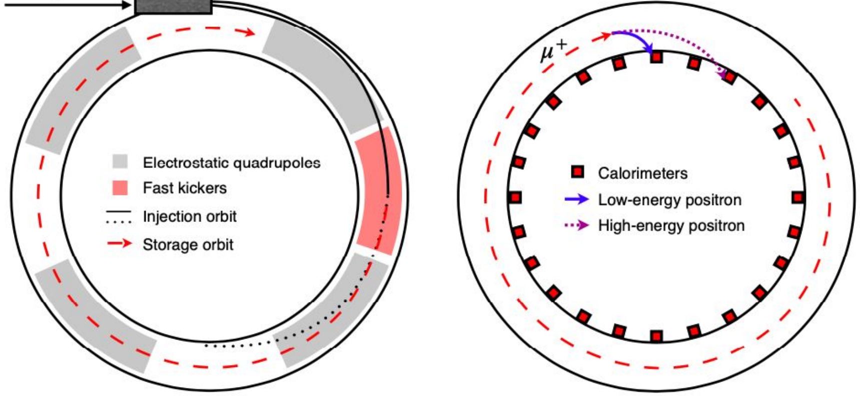
Muon-Decay in Rest

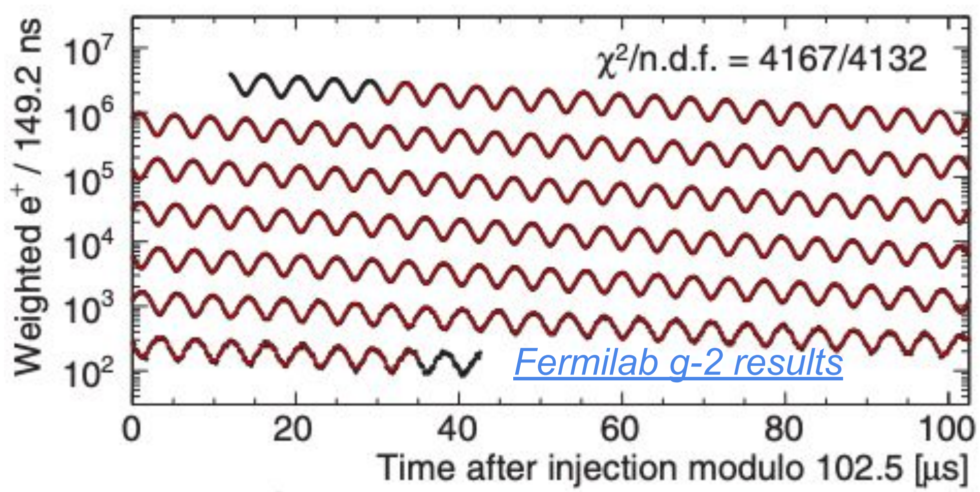


Electrons from Muon decay prefer to flight in Muon spin direction
→ Electron energy gives information of muon spin

Count electrons above an energy threshold vs time at fixed position

[arxiv:2106.06723](https://arxiv.org/abs/2106.06723)





Fermilab g-2 results

$$N(t) = N_0 e^{-t/\tau}[1 + A\cos(\omega_a t + \phi)]$$

Tests of QED: magnetic momentum for muons

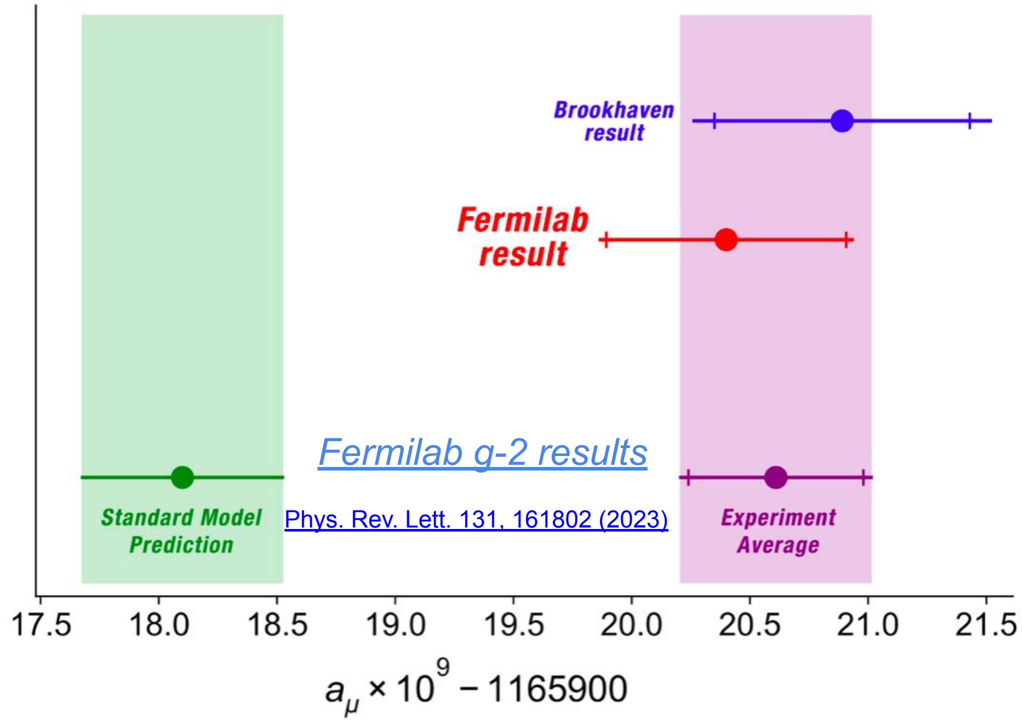
- Results with precision to 10^{-11}
- Combined measurements of BNL and Fermilab disagree **by 4.2 - 5 (debated) standard deviations of Standard Model prediction**

Reference 1:

Probability that the SM can explain this is 2.7×10^{-5}

Reference 2:

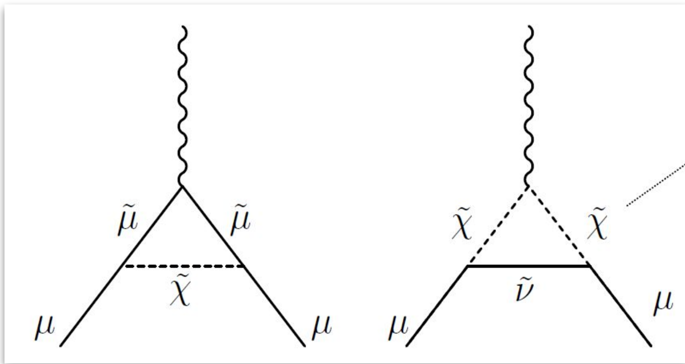
Discovery in HEP is claimed if 5σ



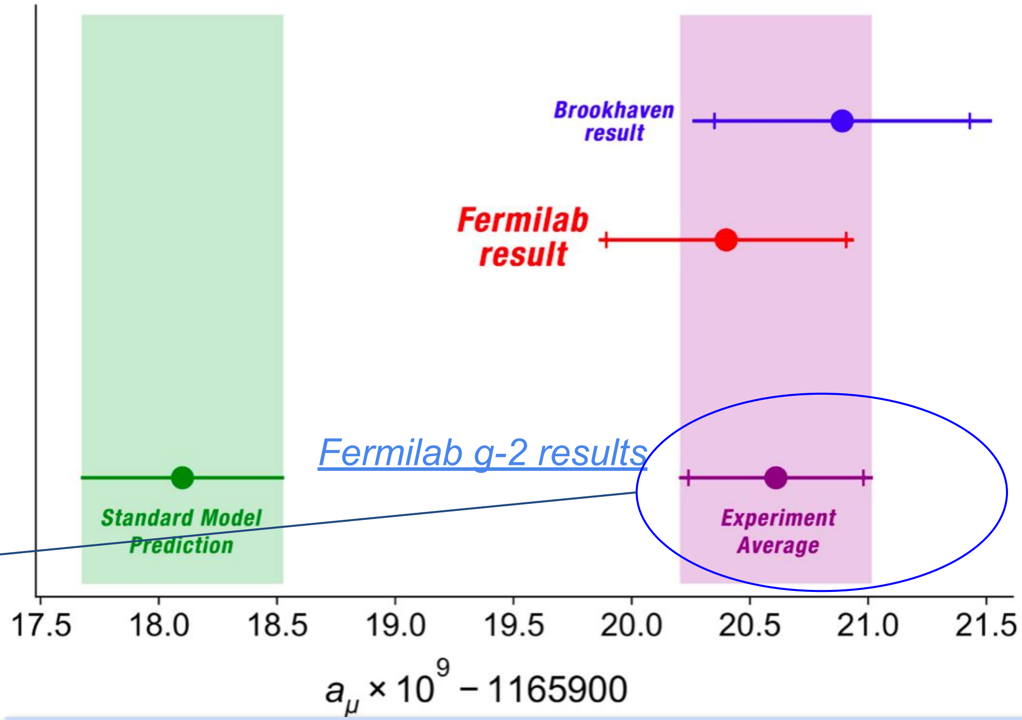
e.g. corrections from Higgs: $a_\mu^{\mathrm{EW}}[\text{2-loop}] = -41.2(1.0) \times 10^{-11}$

Tests of QED: magnetic momentum for muons

- Results with precision to 10^{-11}
- Combined measurements of BNL and Fermilab disagree **by 4.2 - 5 (debated) standard deviations of Standard Model prediction**



$$\lambda_{\rm NP} = \left(\frac{m_\mu^2}{M_{\tilde{\chi}}^2} \right)$$

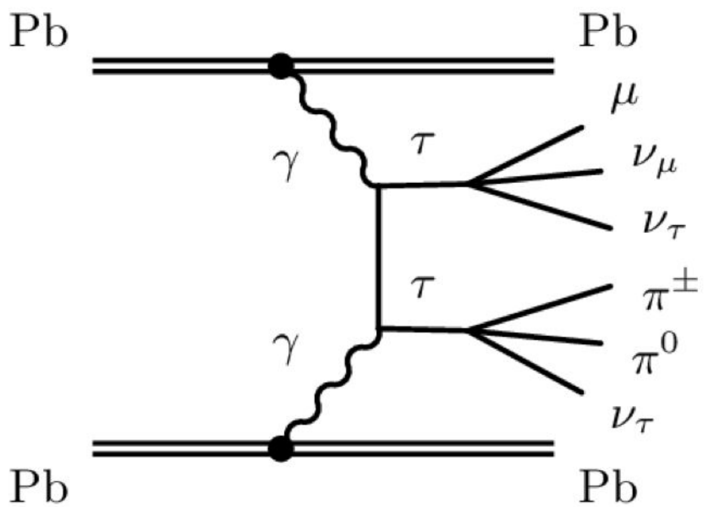


New physics or theory calculations ?

https://arxiv.org/abs/2203.15810

Tests of QED: magnetic momentum for τ-leptons

- Magnetic moment of taus not so well known as the muon's or electron's.
- Expected to be sensitive to BSM effects in many theories

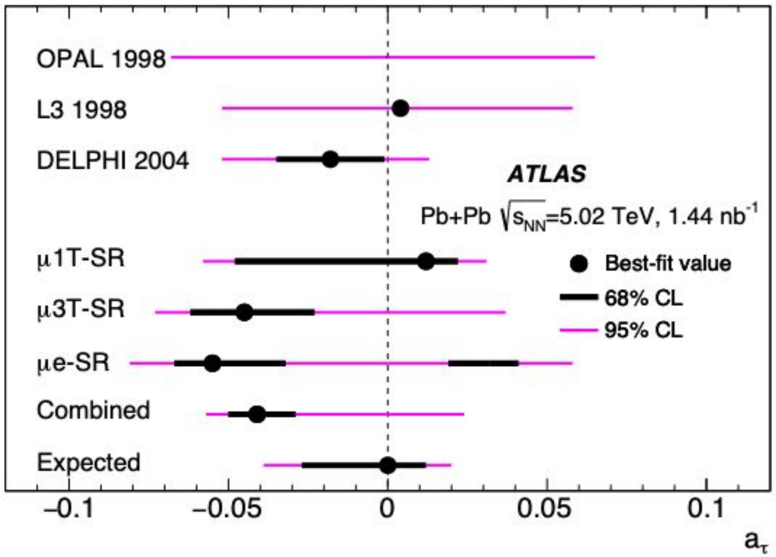


Images shamelessly taken from Lydia's talk

DELPHI (2004): Eur. Phys. J. C **35** (2004) 159

$$a_\tau^{\exp} = -0.018\ (17)$$

$$a_\tau^{\text{theory}} = 0.00117721\ (5)$$

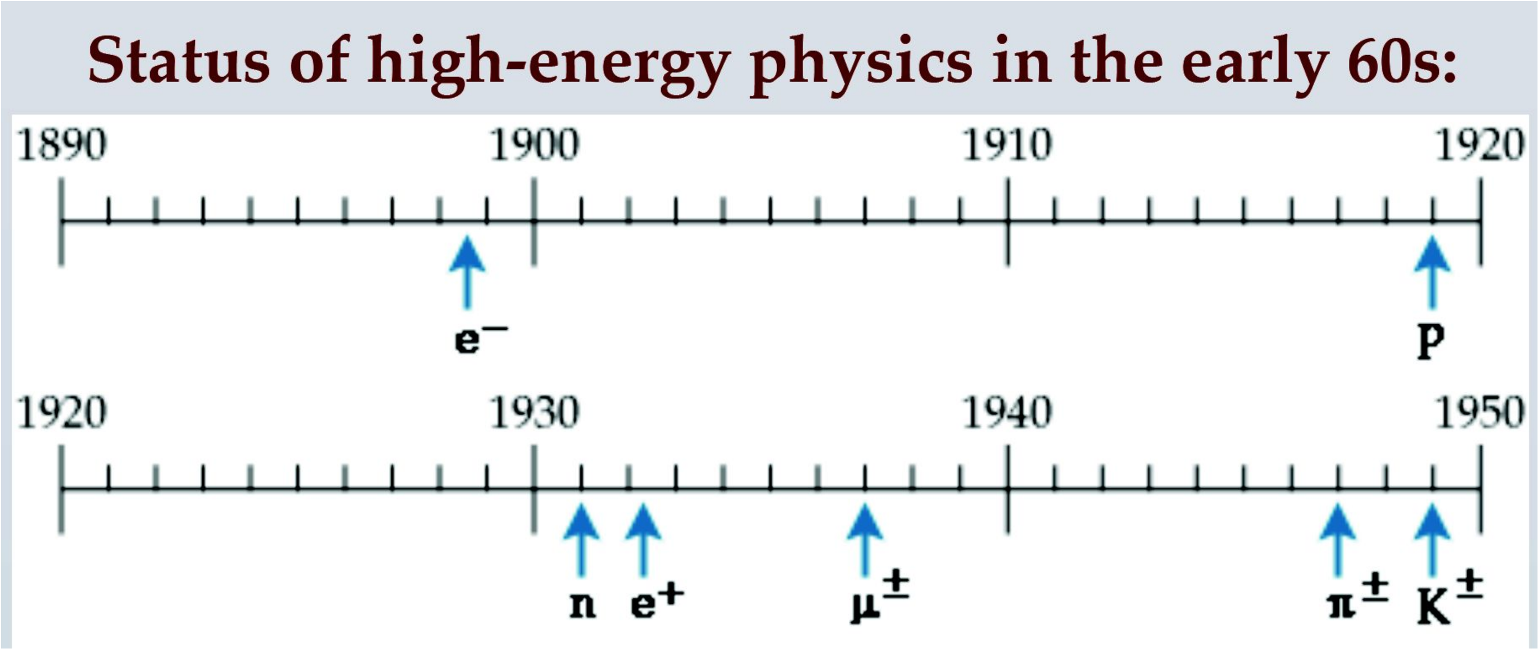


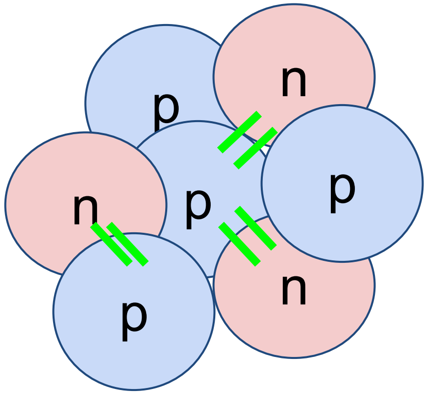
ATLAS(2022): Phys. Rev. Lett. 131 (2023) 151802

Experimentally, very difficult to reach same precision than muons. Pb+Pb collisions at colliders

Quantum chromodynamics
QCD

A historical perspective: the strong force

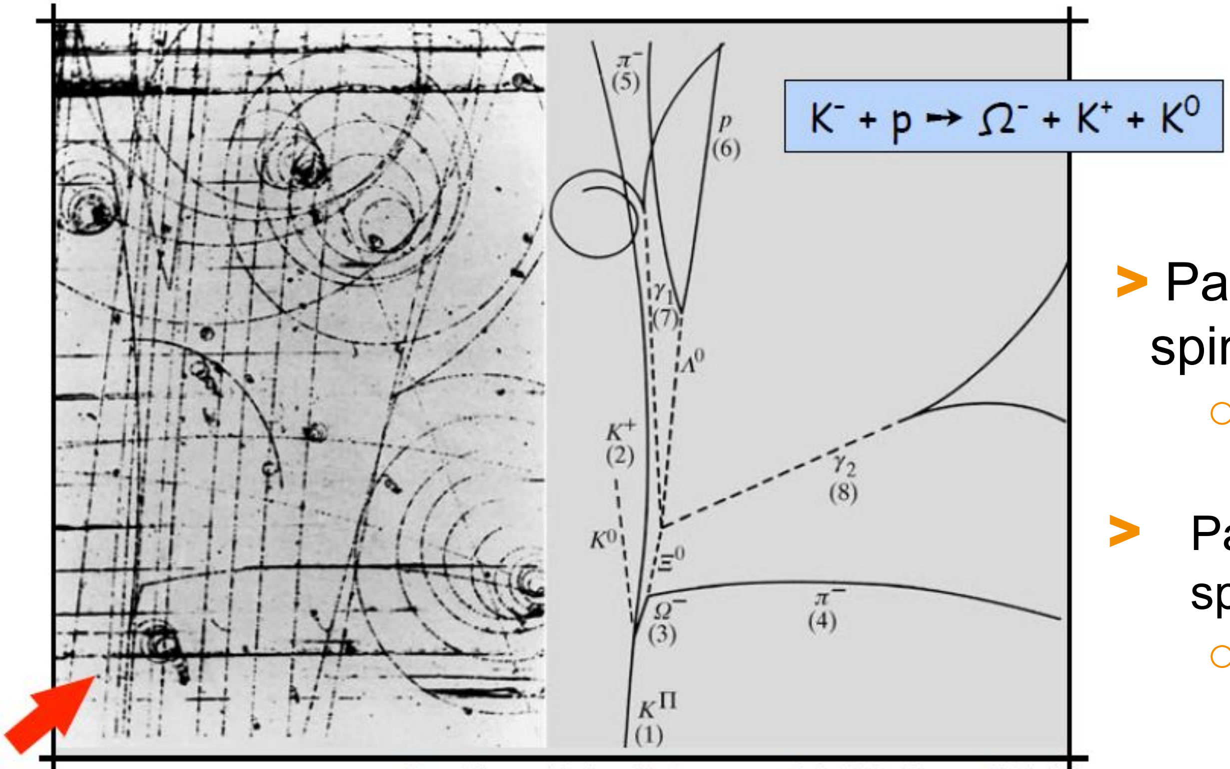




- Early 20th century, limited knowledge of particles.
- Atomic nuclei composed of protons (+1 charge) and neutrons.
- How do they hold together ? → **Strong force** !

A historical perspective: the particle zoo

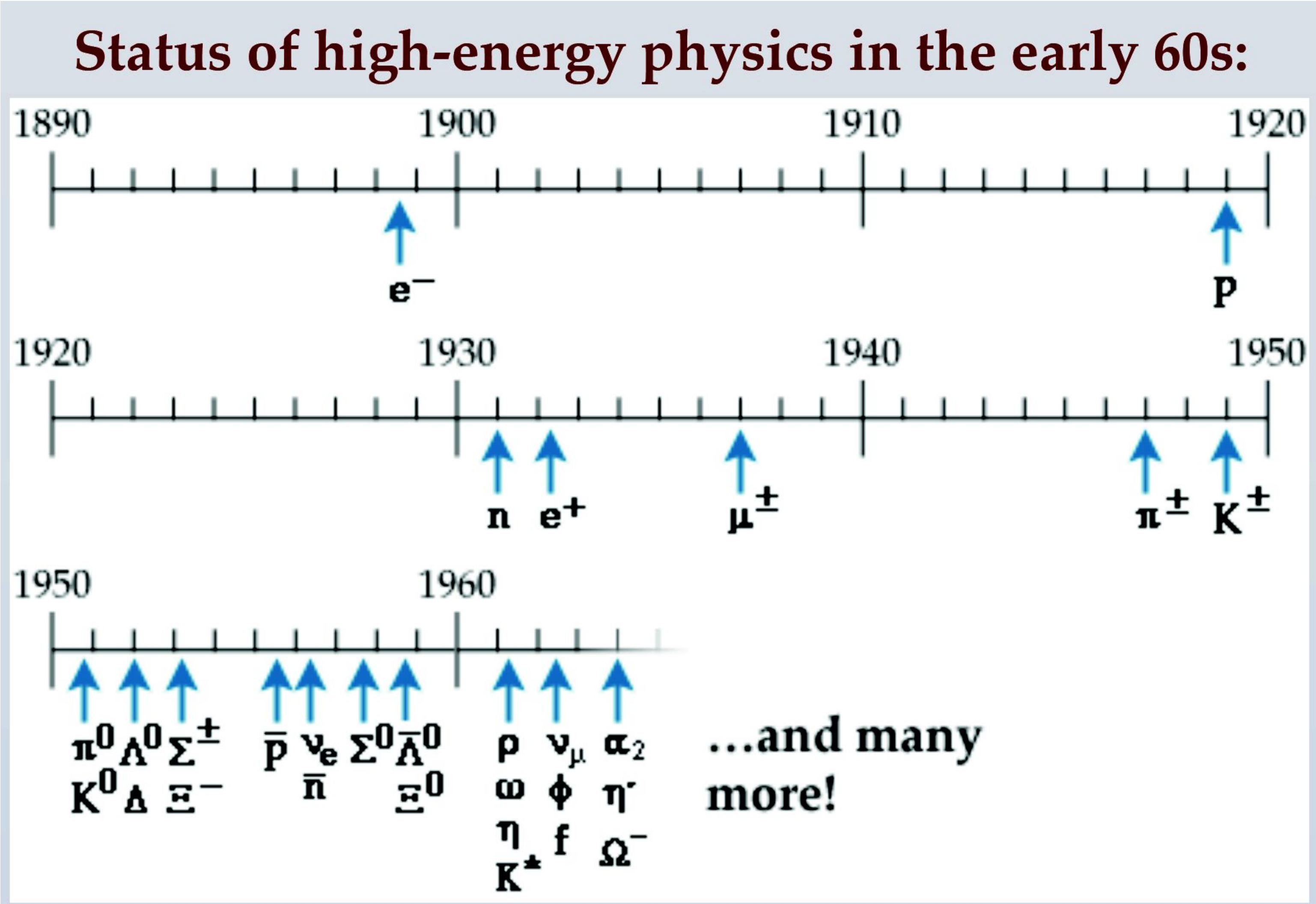
> However, in the 50's and 60's, experiments in bubble and spark chambers were showing the creation of new particles → Particle Zoo



Brookhaven National Laboratory, 5 GeV K⁻-Mesons [1964]

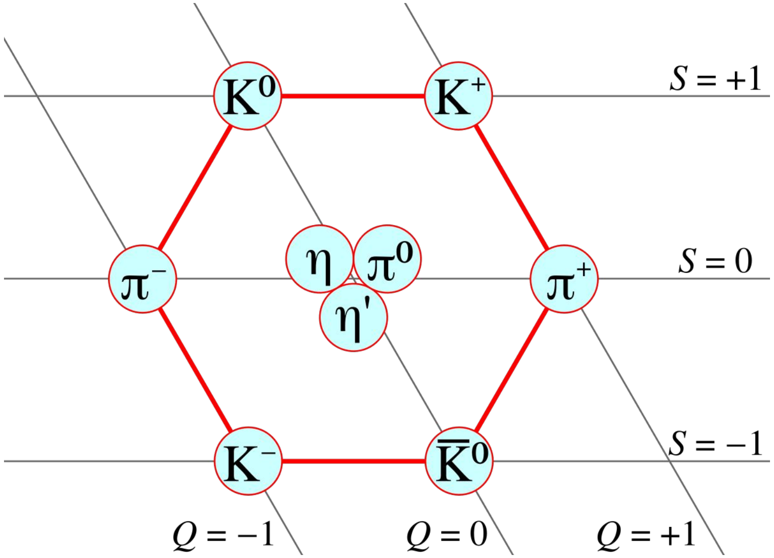
- Particles with integer spin (0,1)
  - Mesons: pions, kaons, …
- Particles with half-integer spin (½, 3/2)
  - Baryons: protons, neutrons …

A historical perspective: the particle zoo



- New particles similar in properties to protons and neutrons → Not alone anymore

Classification of new particles. Any pattern in this zoo ?



- Physicists tried to order this zoo based on properties of the particles
- Strangely, several properties were similar
  - Some particles have very similar masses, spin with different charges.
  - Thought initially to be elementary particles

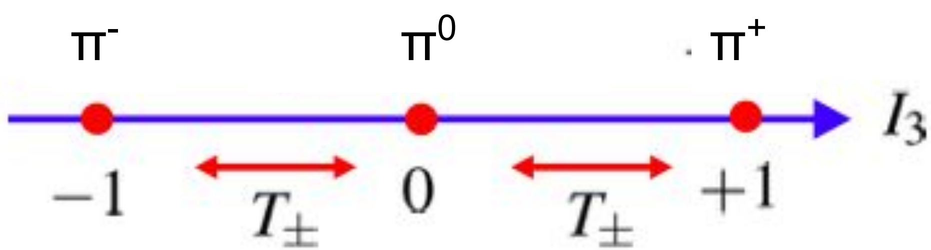
Isospin, strangeness and baryon decuplet

$\pi^\pm$ $I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57039 \pm 0.00018$ MeV (S = 1.8)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

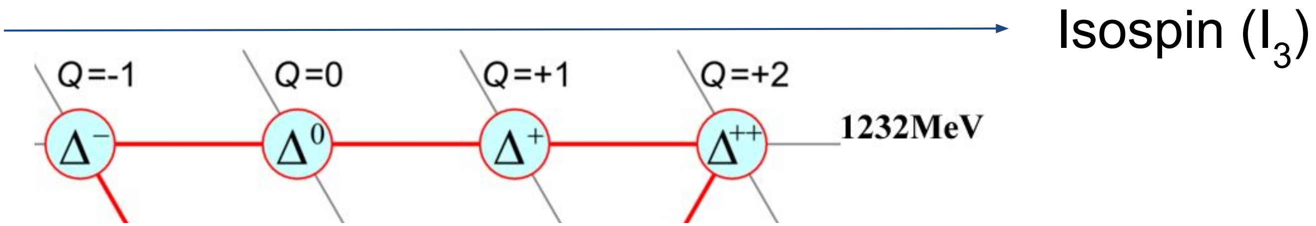
π^0 $I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9768 \pm 0.0005$ MeV (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.43 \pm 0.13) \times 10^{-17}$ s (S = 1.2)
$c\tau = 25.3$ nm



- They can be ordered by mass and electric charge
  - If mass similar but different particles, a symmetry is conserved → Isospin (e.g. pions)

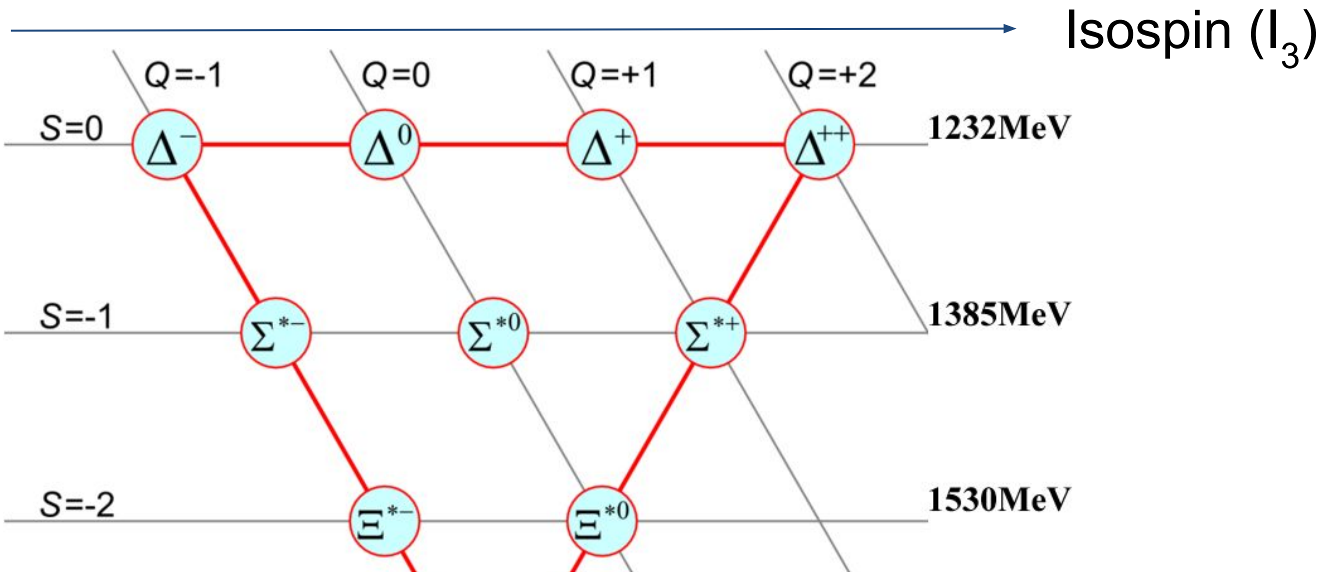
Isospin, strangeness and baryon decuplet



Let's take for instance the baryons with spin 3/2

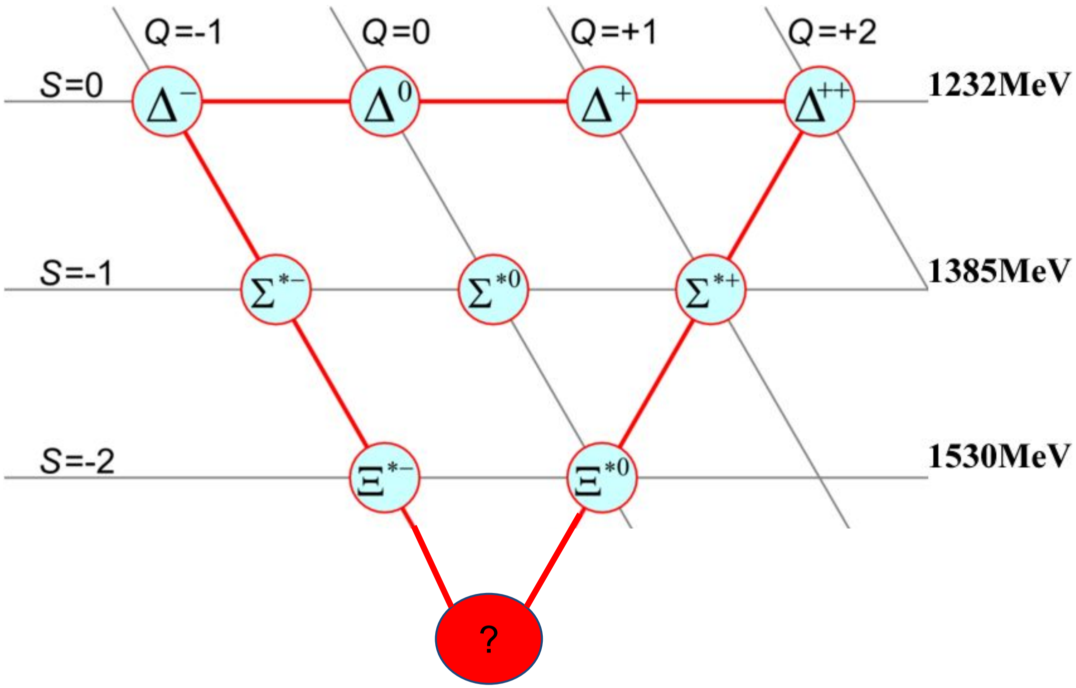
- They can be ordered by mass and electric charge
  - If mass similar but different particles, a symmetry is conserved → Isospin (e.g. pions or spin 3/2)

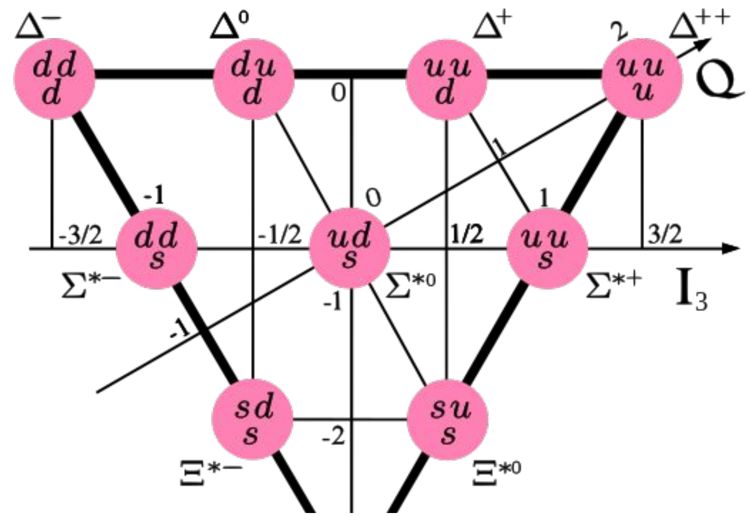
Isospin, strangeness and baryon decuplet



- Another set of spin 3/2 showed different masses and longer lifetime than other counterparts
  - Another quantum number called strangeness
  - Symmetrical patterns appear if have their strangeness plotted against their electric charge.

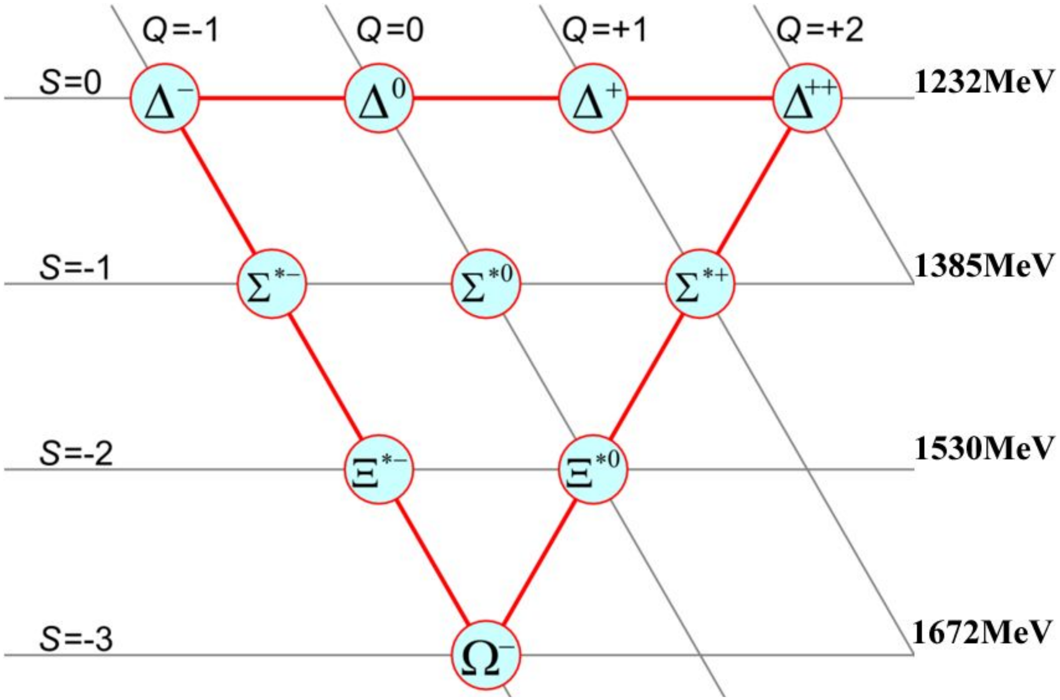
Isospin, strangeness and baryon decuplet

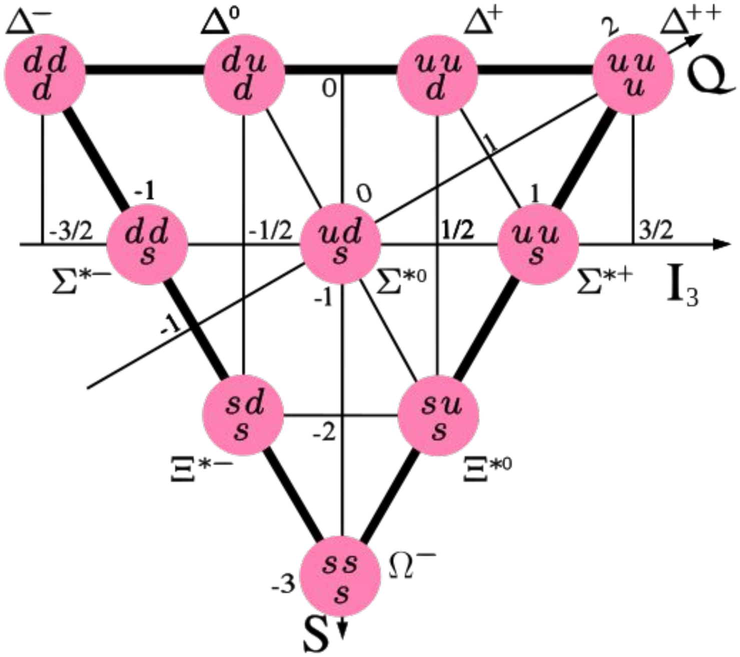




- Lead to definition of these particles as composed particles → Quarks!
  - 3 quarks: u, d and s → spin ½
  - s carries the quantum number strange.
  - u,d are the particles carrying the isospin ½ and -½
- Prediction of the Ω^- with 3 s-quarks, no isospin and negative charge ..

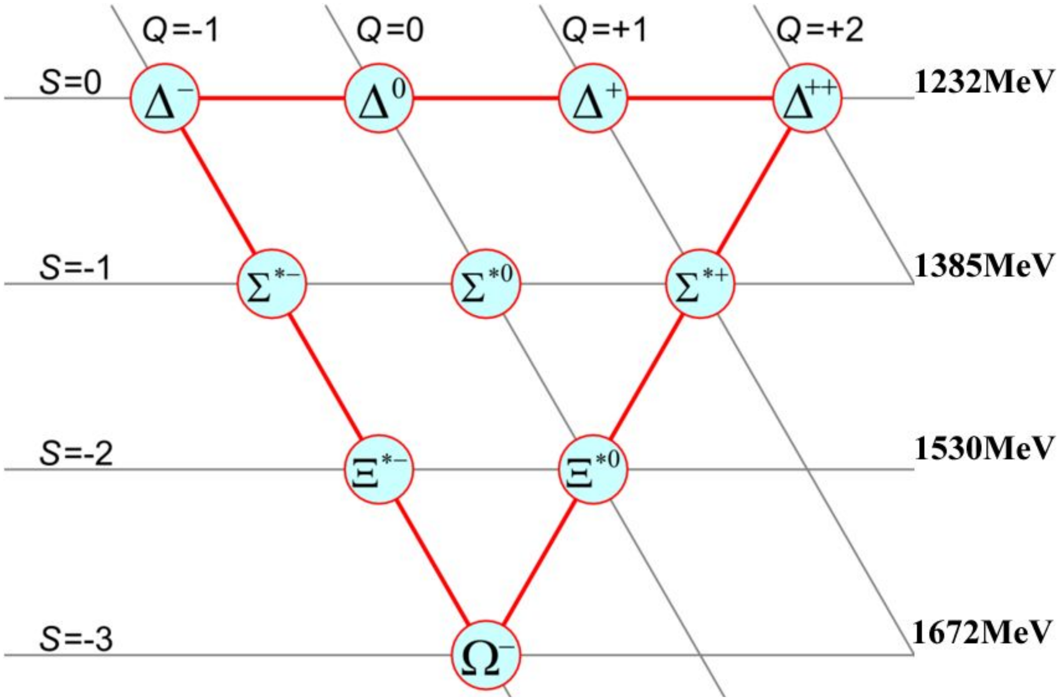
Isospin, strangeness and baryon decuplet

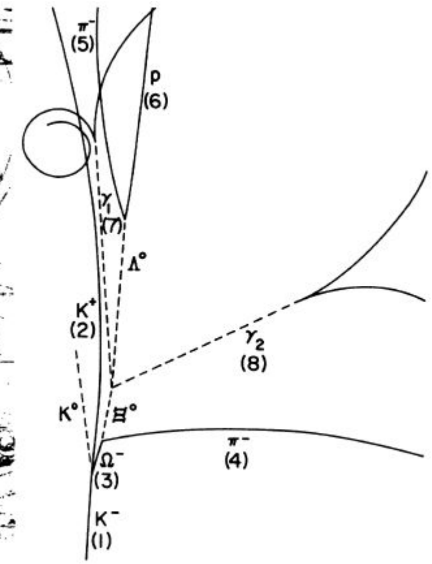




- > Lead to the definition of the quarks as constituents of these particles
  - 3 quarks: u, d and s → spin ½
  - s carries the quantum number strange.
  - u,d are the particles carrying the isospin ½ and -½
- > Prediction of the Ω^- with 3 s-quarks, no isospin and negative charge ..
  - Success !!! (1964 link)

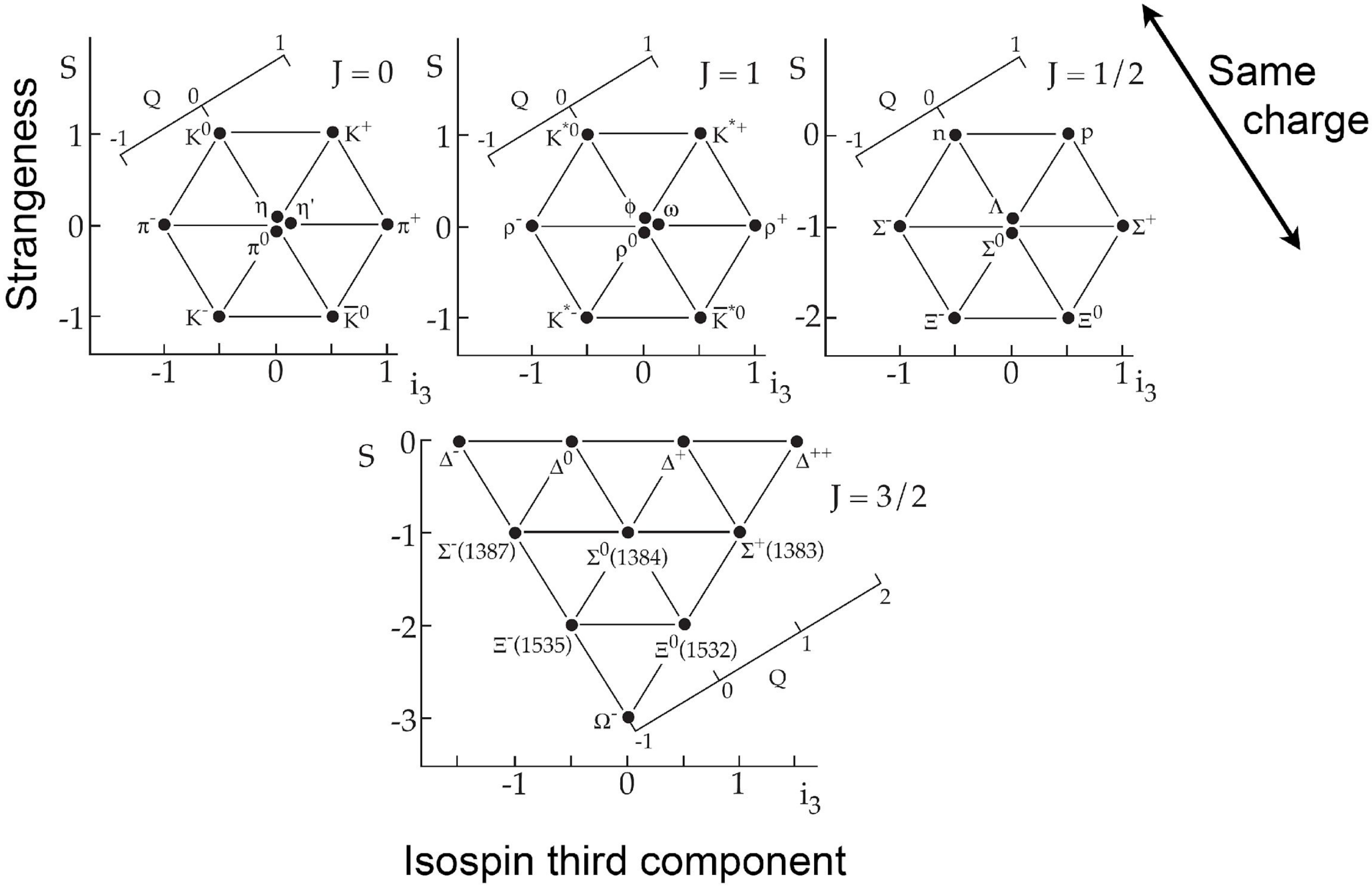
Isospin, strangeness and baryon decuplet



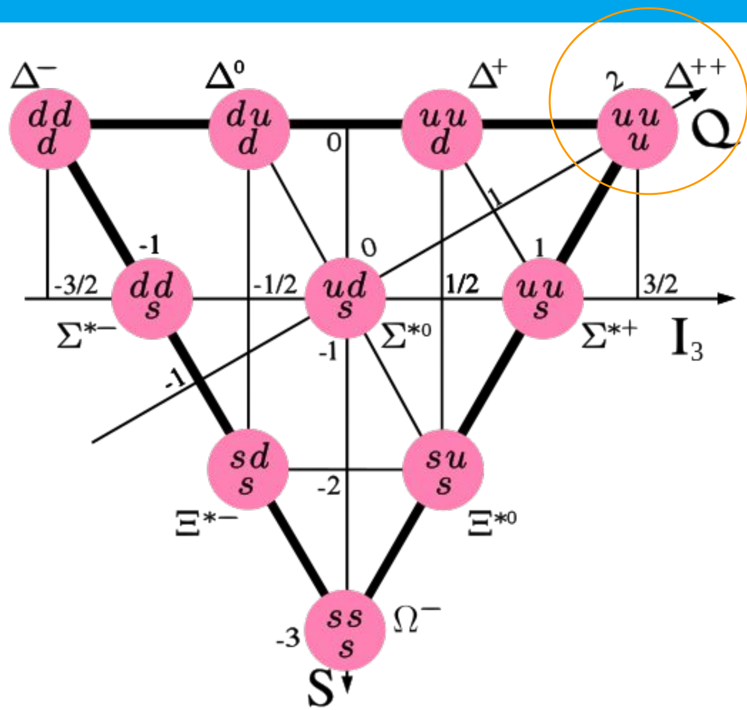


- Lead to the definition of the quarks as constituents of these particles
  - 3 quarks: u, d and s → spin ½
  - s carries the quantum number strange.
  - u,d are the particles carrying the isospin ½ and -½
- Prediction of the Ω^- with 3 s-quarks, no isospin and negative charge ..
  - Success !!! (1964 link)

Quarks


Strangeness
S
J = 0
K^0
K$^+$
π$^-$
η
η'
π^0
π$^+$
K$^-$
$\bar{K}^0$
J = 1
K^{*0}
K^{*+}
ρ$^-$
ϕ
ω
ρ^0
ρ$^+$
K^{*-}
$\bar{K}^{*0}$
J = 1/2
n
p
Σ$^-$
Λ
Σ^0
Σ$^+$
Ξ$^-$
Ξ^0
Q
i$_3$
Same charge
J = 3/2
Δ$^-$
Δ^0
Δ$^+$
Δ^{++}
Σ$^-$(1387)
Σ^0(1384)
Σ$^+$(1383)
Ξ$^-$(1535)
Ξ^0(1532)
Ω$^-$
Isospin third component

A story of spin and color



In the quark model, Δ++ posed a problem.

This is a particle with spin 3/2, with spin I_3 component +- 3/2 composed by same particles

Symmetric wave-function but it is a fermion !
Violation of Pauli principle !

$$|\Delta^{++}\rangle = |u_\uparrow\, u_\uparrow\, u_\uparrow\rangle$$

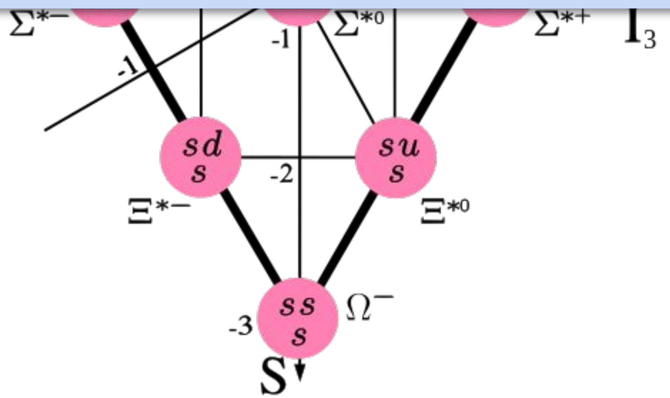
Solution: another quantum number → Color !

$$|\Delta^{++}\rangle = |u,\uparrow,g\rangle + |u,\uparrow,r\rangle + |u,\uparrow,b\rangle$$

Initially postulated 3 values of color: red, blue and green.

A story of spin and color

Organized the particle zoo as being composed of 3 different quarks. Introduced isospin, strangeness and color. Explained charge, masses, spin and lifetimes of some particles.



Symmetric wave-function but it is a "fermion" !
Violation of Pauli principle !

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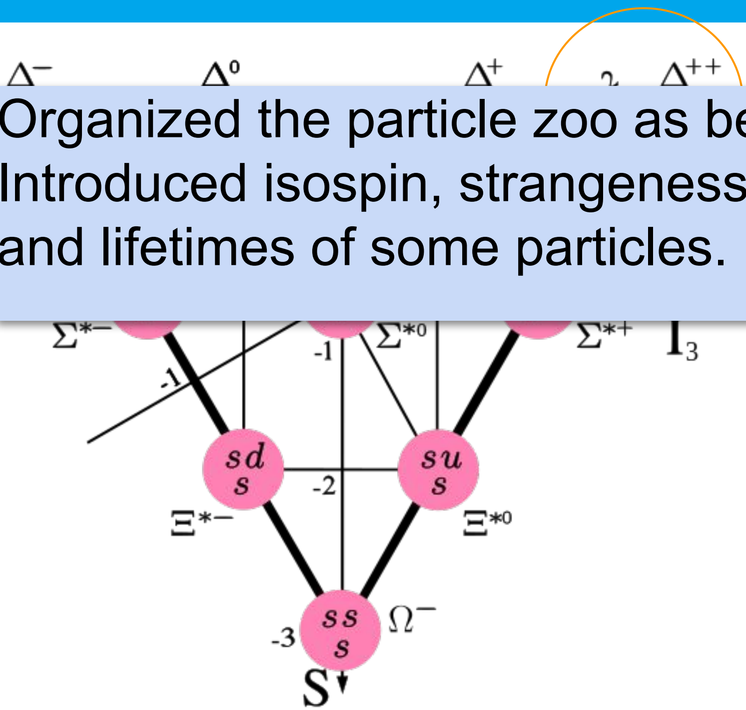


Symmetric wave-function but it is a "fermion" !
Violation of Pauli principle !

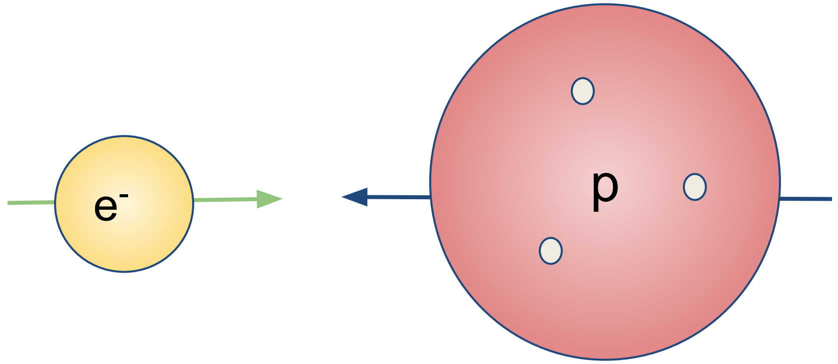
$$|\Delta^{++}\rangle = |u_\uparrow\, u_\uparrow\, u_\uparrow\rangle$$

Solution: another quantum number → Color !

But we have never seen a single spin ½ particle alone. Where are they ?
Answer to this question came from deep inelastic scattering measurements on the proton! (SLAC 1969) e + p → e + X

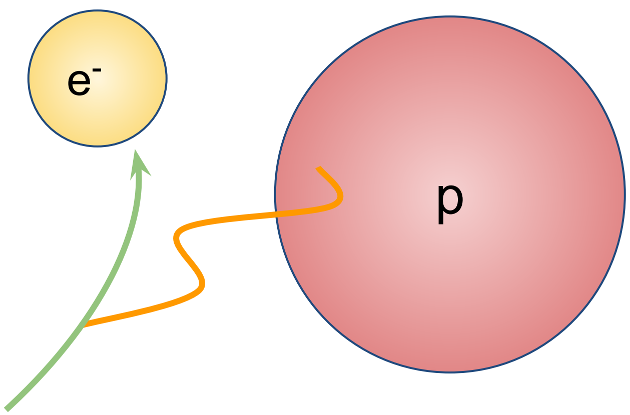
Deep inelastic scattering

Collision of an electron beam with a proton beam.



Deep inelastic scattering

Collision of an electron beam with a proton beam.



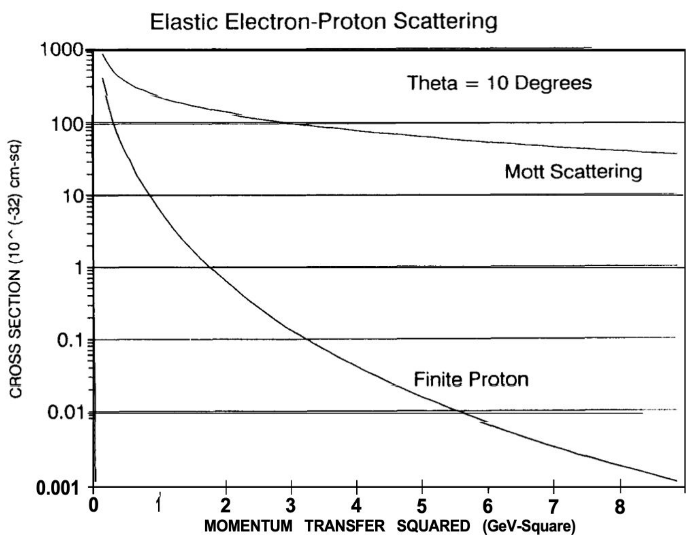
At low e-beam energies (low Q^2)

Resolution is close to the size of the proton

Scattering of the electron will show patterns consistent with the ones of a spin-½ particle scattering against another spin-½ particle

Mott scattering or elastic scattering

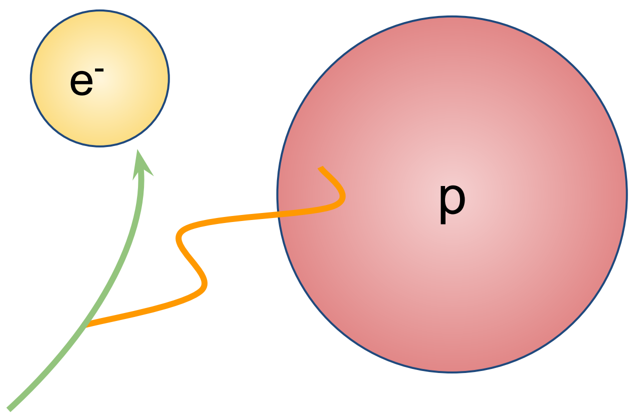
Inelastic electron scattering against point-like proton

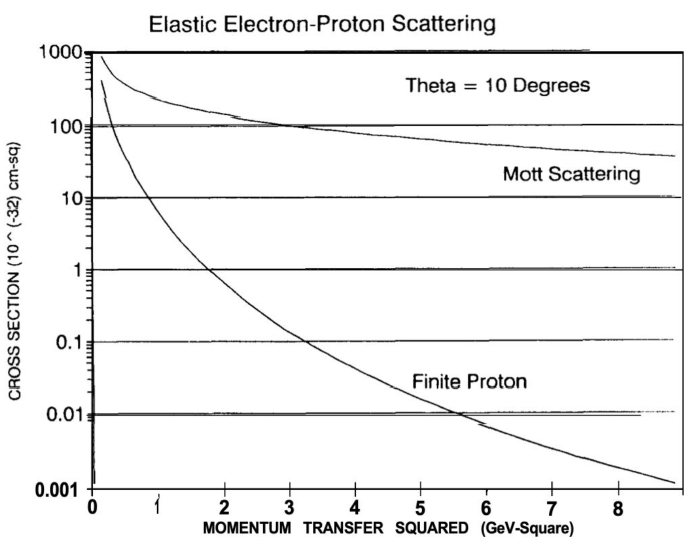


$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}$$

Deep inelastic scattering

Collision of an electron beam with a proton beam.





At mid/high e-beam energies ($Q^2 \geq M_p$)

Can see the structure of the proton!

If the proton is a point-particle, similar results than at low energy.

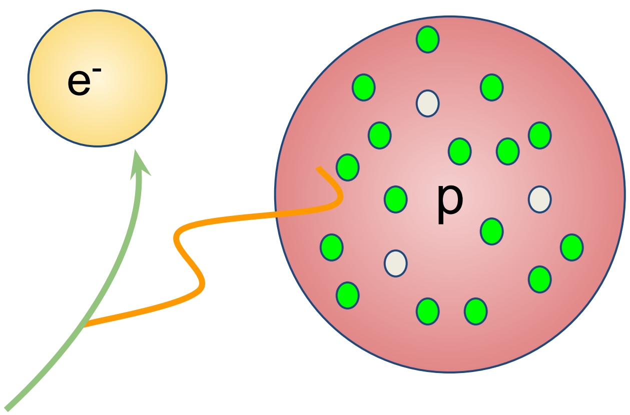
Mott scattering or elastic scattering

Inelastic electron scattering against point-like proton

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2 \frac{\theta}{2}}{4E^2 \sin^4 \frac{\theta}{2}}$$

Deep inelastic scattering

Collision of an electron beam with a proton beam.



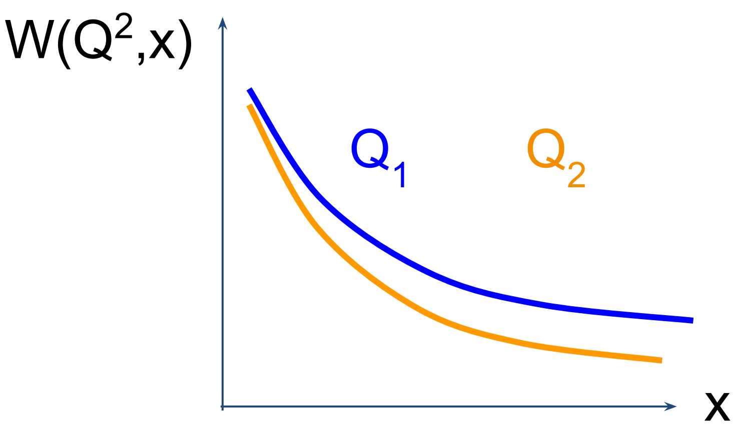
At mid/high e-beam energies ($Q^2 \geq M_p$)

Can see the structure of the proton!

If the proton is composed, divergence from Mott scattering !

Mott scattering + Form factors

Form factors describe the charge distribution inside proton.

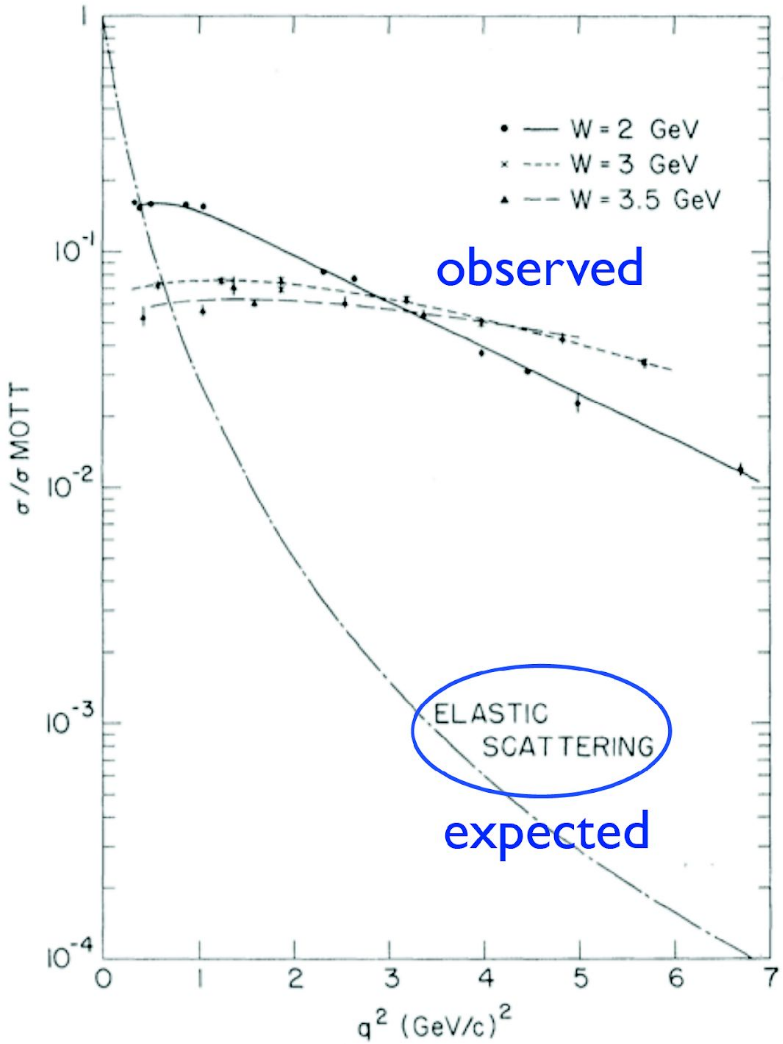


$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}} \left[W_2(\nu, q^2) + W_1(\nu, q^2) \tan^2\frac{\theta}{2} \right]$$

x : momentum fraction of the charge inside proton

Q,q: momentum exchange in scattering

Deep inelastic scattering experiment at SLAC in 1969



Early results from SLAC (1969):

$E = 7$ - 17.7 GeV

$\theta = 10°$

Elastic cross section falls off rapidly due to the proton not being point-like

Inelastic: $W > M$

Ratio to Mott cross section nearly flat in Q^2

Q^2 dependence becomes weaker for increasing W

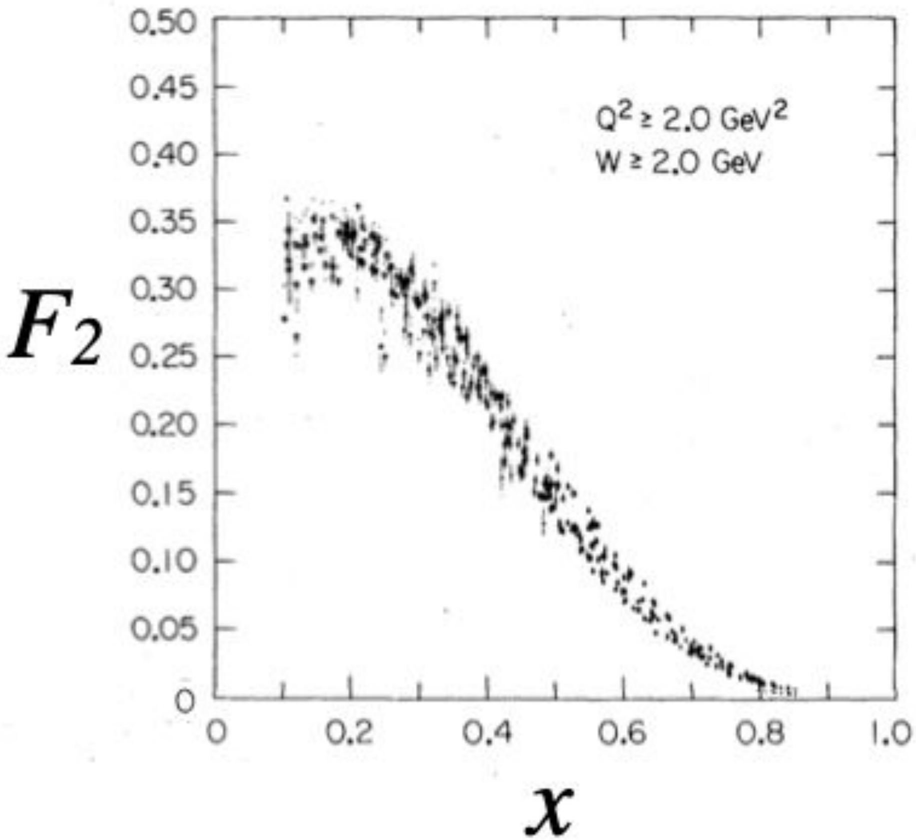
Proton a composite particle!

M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969)

DIS experiment additional results: The gluon !

Experimental studies of the neutron and proton electromagnetic structure functions continued.

Integral of form factors over quarks charge distribution in proton → Total momentum of the proton !

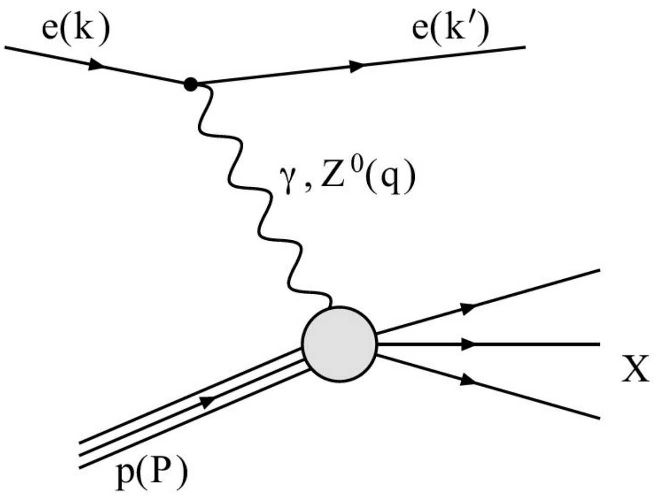


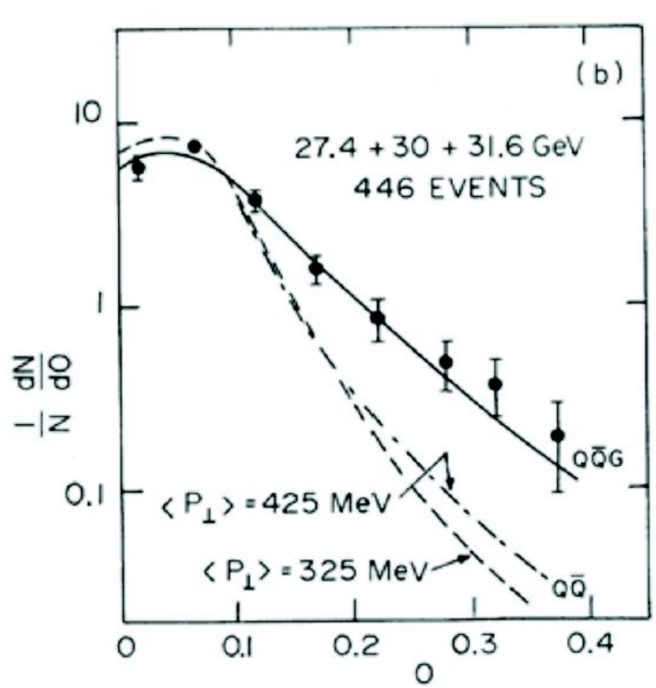
$$\int_0^1 F_2^p(x)\mathrm{d}x \approx 0.18 \quad \text{and} \quad \int_0^1 F_2^n(x)\mathrm{d}x \approx 0.12$$

$$\Rightarrow \boxed{f_u = 0.36 \;\; \text{and} \;\; f_d = 0.18}$$

~ 50%. Where is the rest of the proton momentum ???

DIS experiment additional results: The gluon !





D. P. Barber (Mark-J), Phys.Lett.B89, 139(1979)

DIS rely on electromagnetic interactions of an electron with the proton charged components.

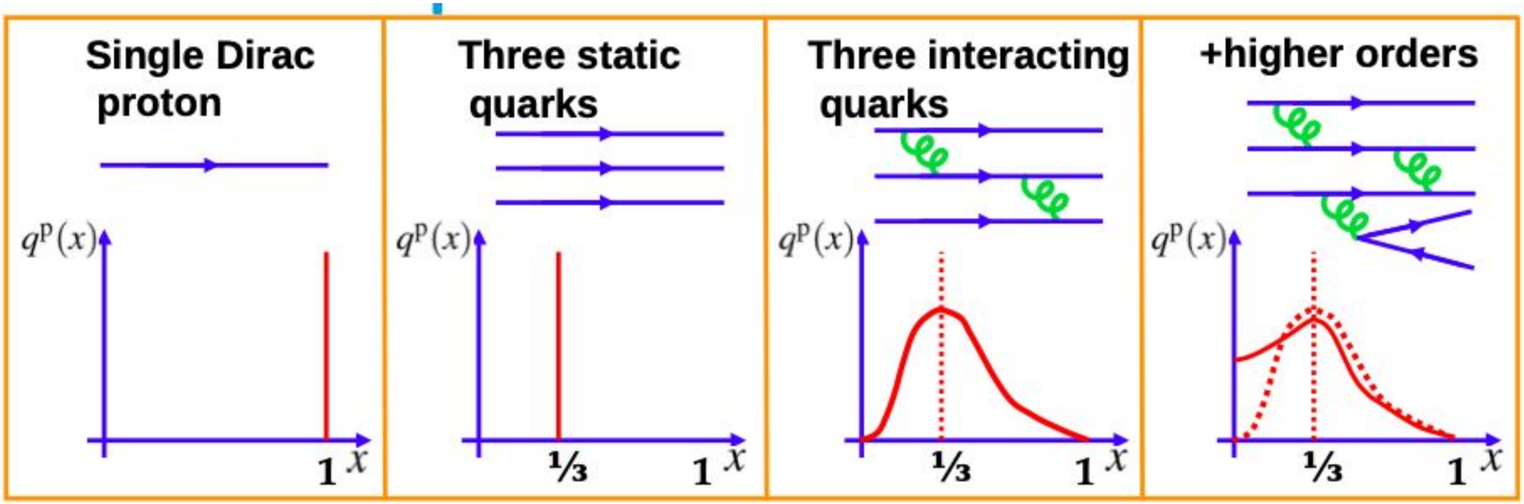
Proton must have another component but neutral → Gluon !

Hadrons composed of quarks and gluons → partons !
Quark Parton Model success

Discovered at PETRA in 1979

DIS experiment additional results: Parton density functions

Form factors describing the charge distribution inside proton → PDFs



Instagram picture of a proton

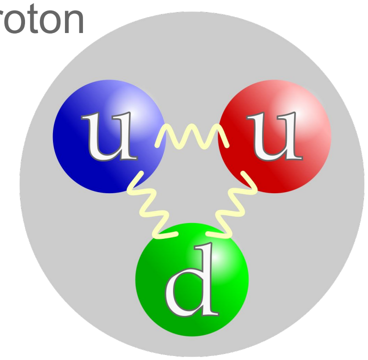


image from official HEP instagram (wikipedia)

Real picture: Parton density functions

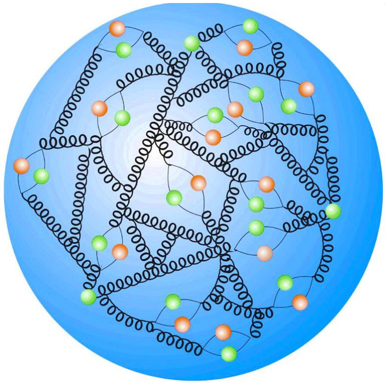
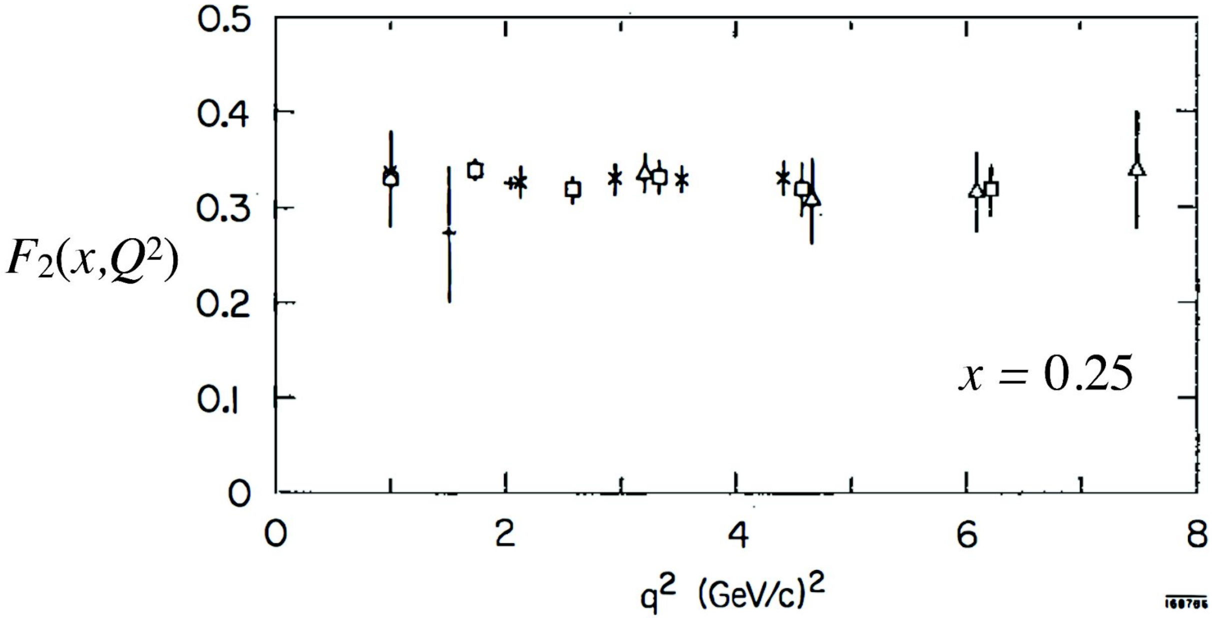


image from here

Scaling violation: Parton density functions

Bjorken scaling: quarks behave as point-like constituents at high energies

J.T. Friedman, H.W. Kendall, Ann. Rev. Nucl. Sci. 22, 203 (1972)



Scaling!

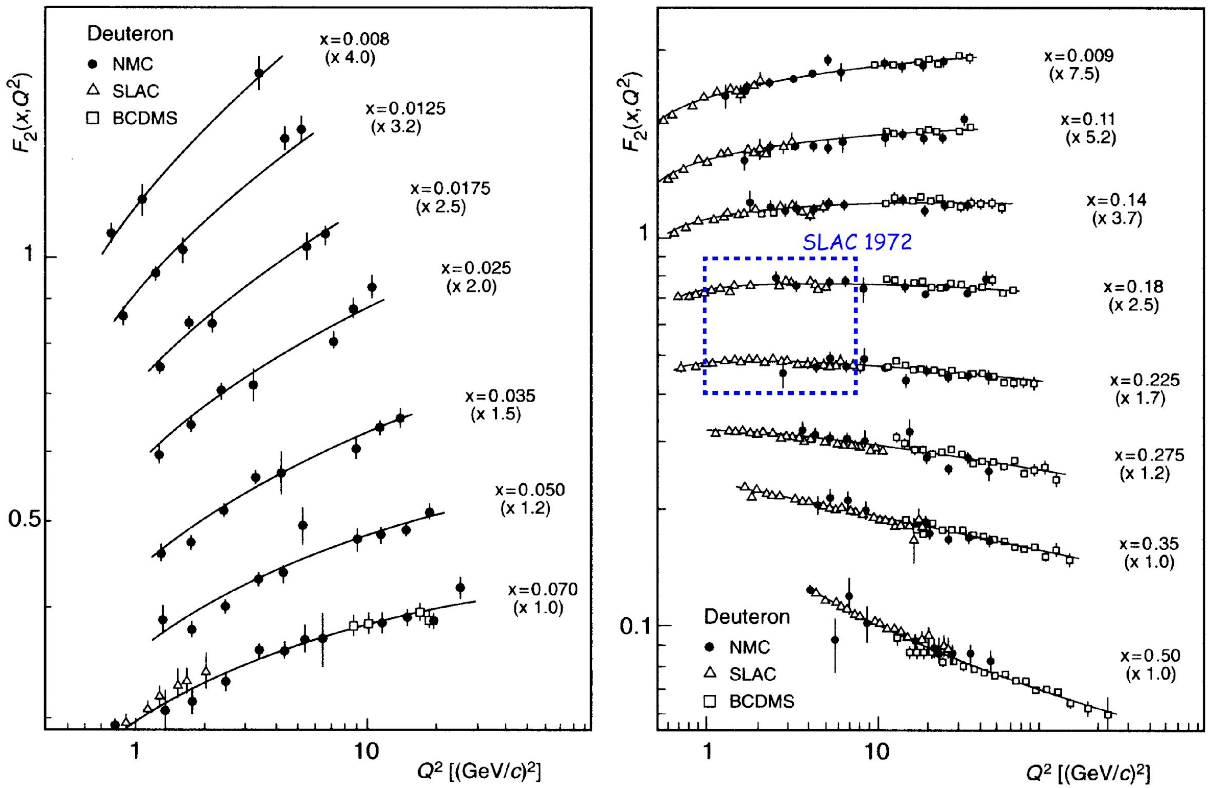
Independence of the structure functions of Q^2: $F_i(x,Q^2) = F_i(x)$

J.D. Bjørken predicted scaling for $Q^2 \to \infty$ as x stays fixed.
Scaling is obtained using Gell-Mann's current algebra in the quark model.

Scaling violation: Parton density functions

Bjorken scaling: quarks behave as point-like constituents at high energies

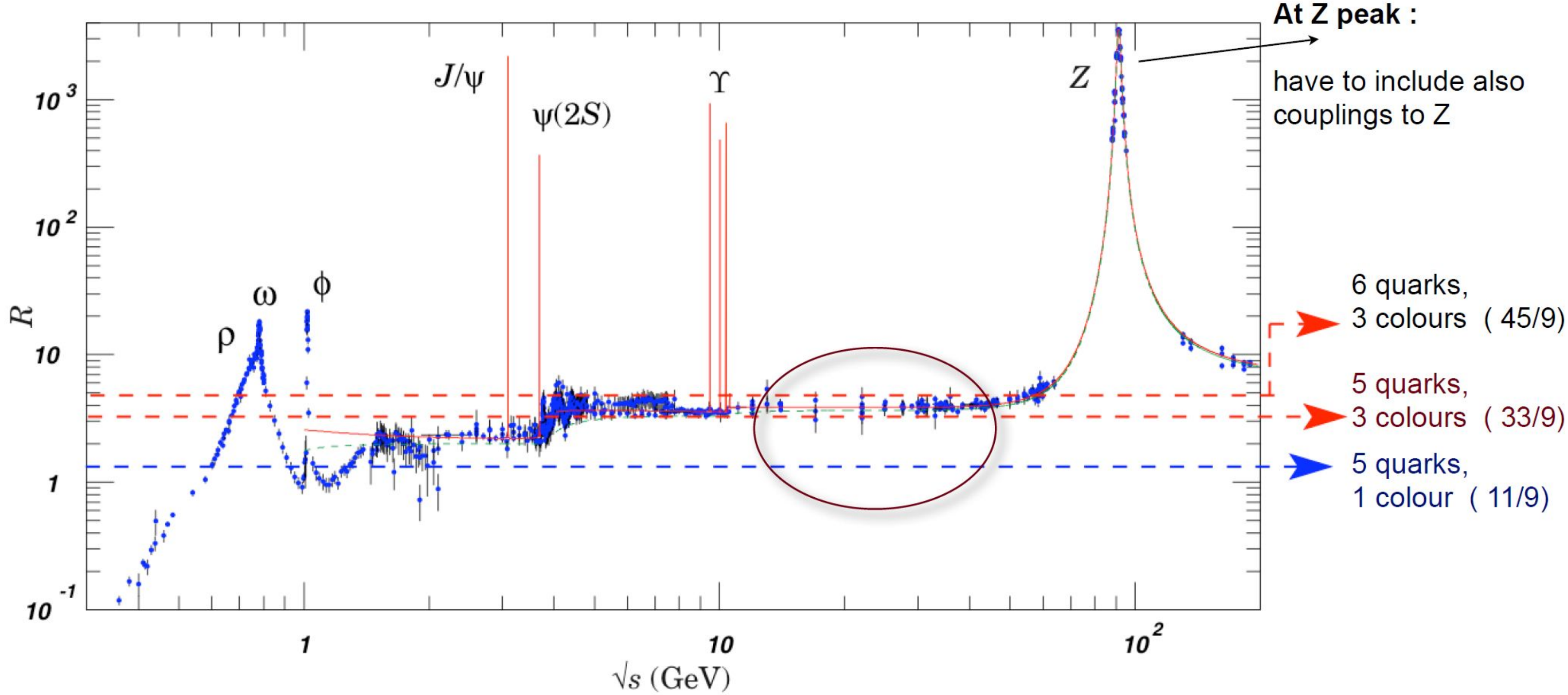
$$F_2(x,Q^2) = \sum e_q^2 x q(x,Q^2)$$



Scaling violation → small, only appears in QFTs with asymptotic freedom → Might the strong force be explained as a perturbation theory (like QED) but with asymptotic freedom ?

Number of colors

> Prediction for Ratio: $$R = \frac{\sigma^{e^+e^-\to\text{hadrons}}}{\sigma^{e^+e^-\to\mu^+\mu^-}} = N_c \sum_q e_q^2$$



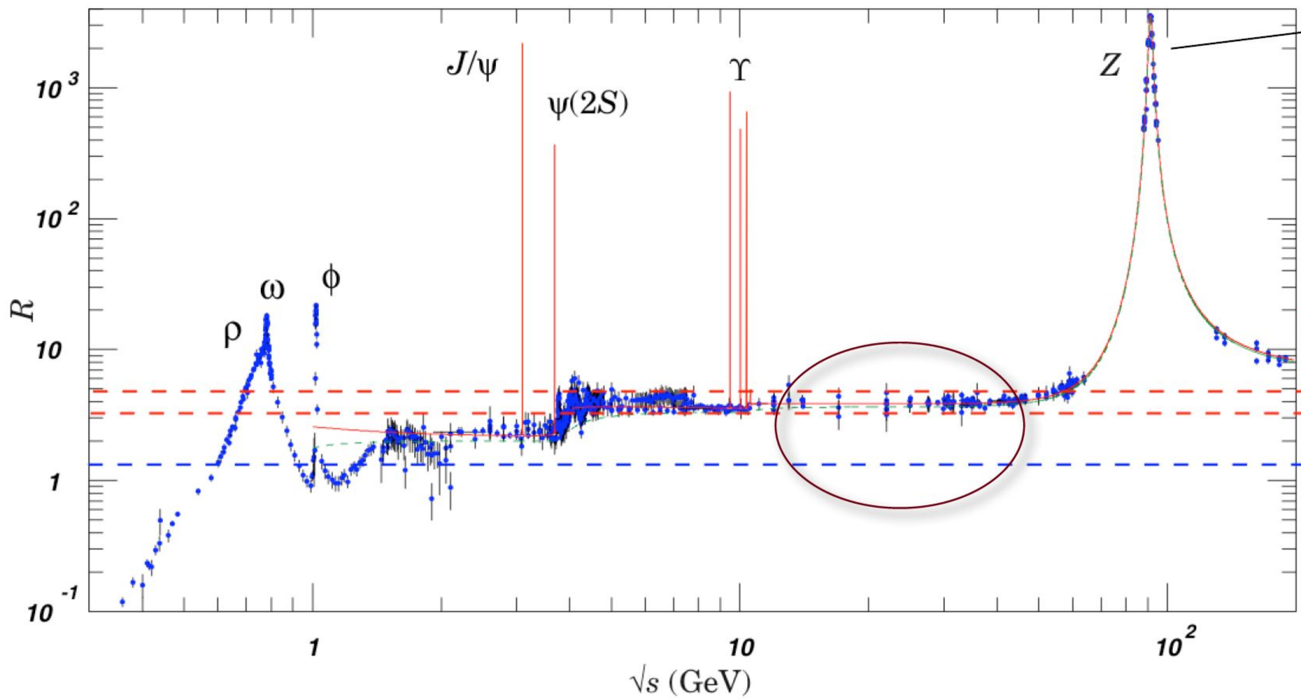
$$R = N_c \sum_q e_q^2 = N_c \left[\underbrace{\left(\frac{2}{3}\right)^2}_{u} + \underbrace{\left(-\frac{1}{3}\right)^2}_{d} + \underbrace{\left(-\frac{1}{3}\right)^2}_{s} + \underbrace{\left(\frac{2}{3}\right)^2}_{c} + \underbrace{\left(-\frac{1}{3}\right)^2}_{b} \right] = N_c \frac{11}{9}.$$

c-quark discovered in 1974

b-quark discovered in 1977

t-quark discovered in 1995

> Prediction for Ratio: $$R = \frac{\sigma^{e^+e^- \to \text{hadrons}}}{\sigma^{e^+e^- \to \mu^+\mu^-}} = N_c \sum_q e_q^2$$



At Z peak :

have to include also couplings to Z

6 quarks, 3 colours (45/9)

5 quarks, 3 colours (33/9)

5 quarks, 1 colour (11/9)

$$R = N_c \sum_q e_q^2 = N_c \left[\underbrace{\left(\frac{2}{3}\right)^2}_{u} + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = N_c \frac{11}{9}.$$

c-quark discovered in 1974

b-quark discovered in 1977

quark discovered in 1995

Quantum chromodynamics

Quantum Chromodynamics

- Gauge theory invariant under transformations in the color space
- Kinematics and interactions of particles charged under the strong force → color charge

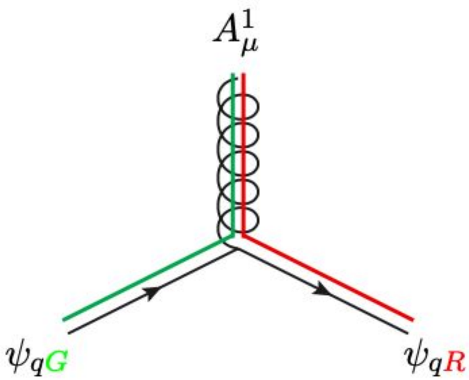
This is the perturbative story of quarks and gluons !

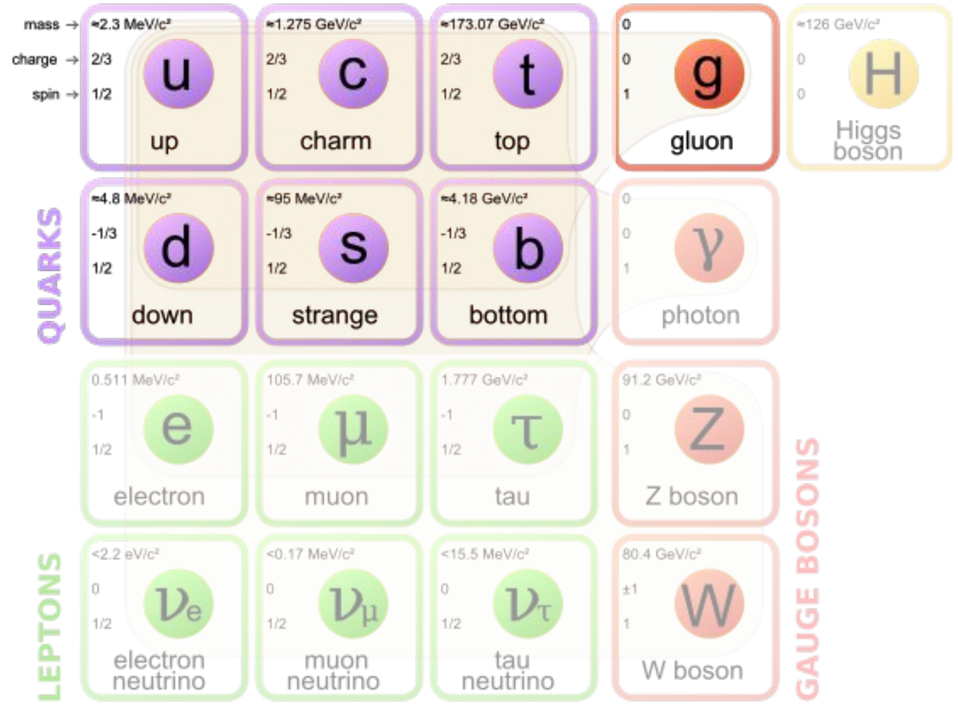
Quarks and anti-quarks

Exactly one single color, can be either of the three available

Gluons:
Two colour numbers

qqg vertex



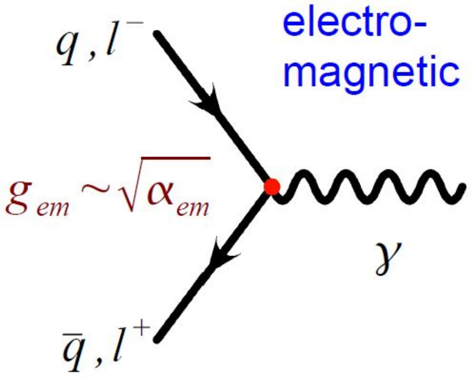


Quantum Chromodynamics

- Gauge theory invariant under transformations in the color space
- Kinematics and interactions of particles charged under the strong force → color charge

Coupling constant (Reminder QED)

Universal (not different magnitude of coupling depending on the particle or color)
Similar to the electromagnetic coupling, expressed in terms of an α constant ….



$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$\alpha_{\mathrm{EM}} = \frac{e^2}{4\pi} = \frac{1}{137}$$

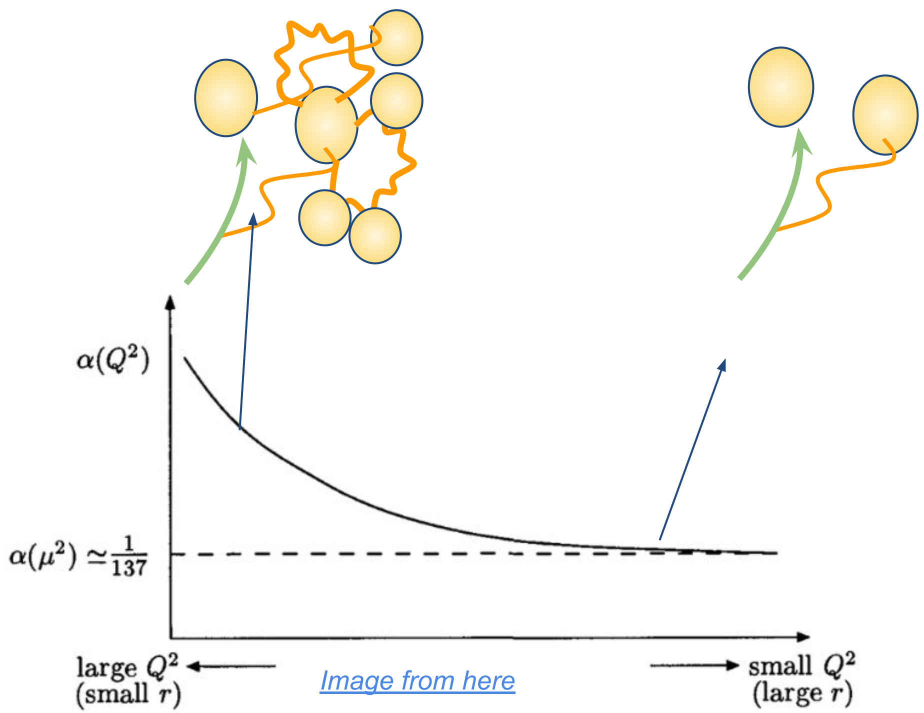
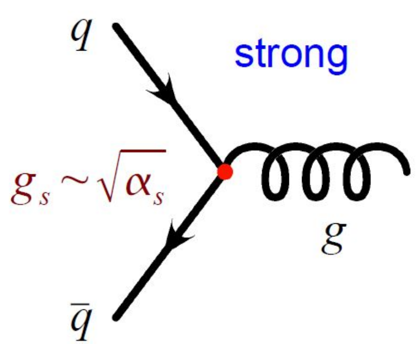


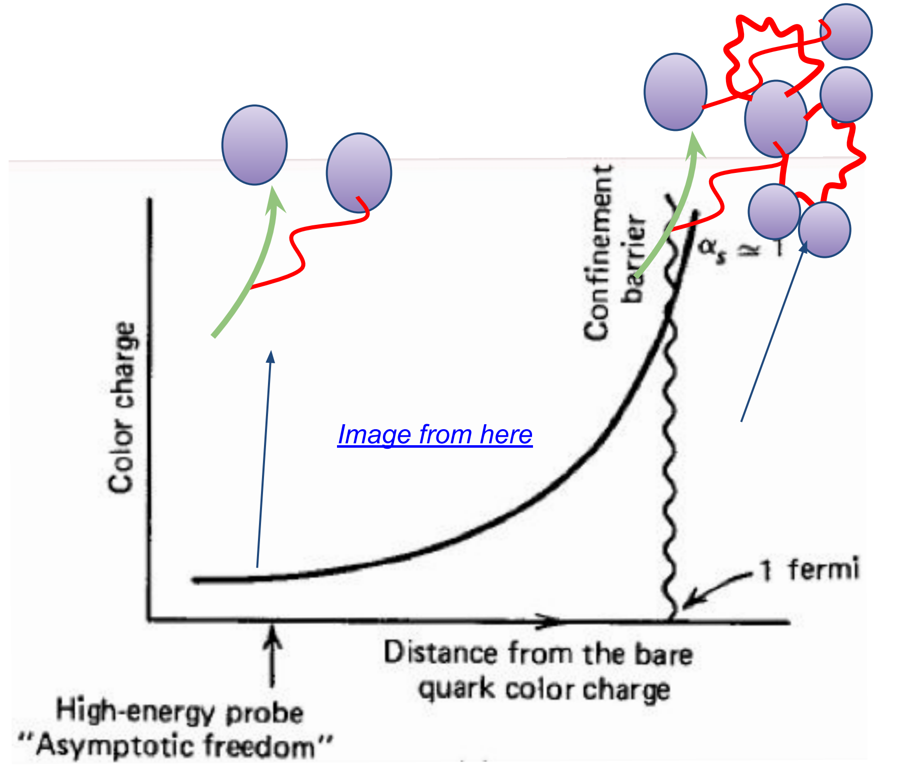
Image from here

- Gauge theory invariant under transformations in the color space
- Kinematics and interactions of particles charged under the strong force → color charge

Coupling constant

Universal (not different magnitude of coupling depending on the particle or color)
Similar to the electromagnetic coupling, expressed in terms of an α constant …..
BUT ! Different evolution with Q^2

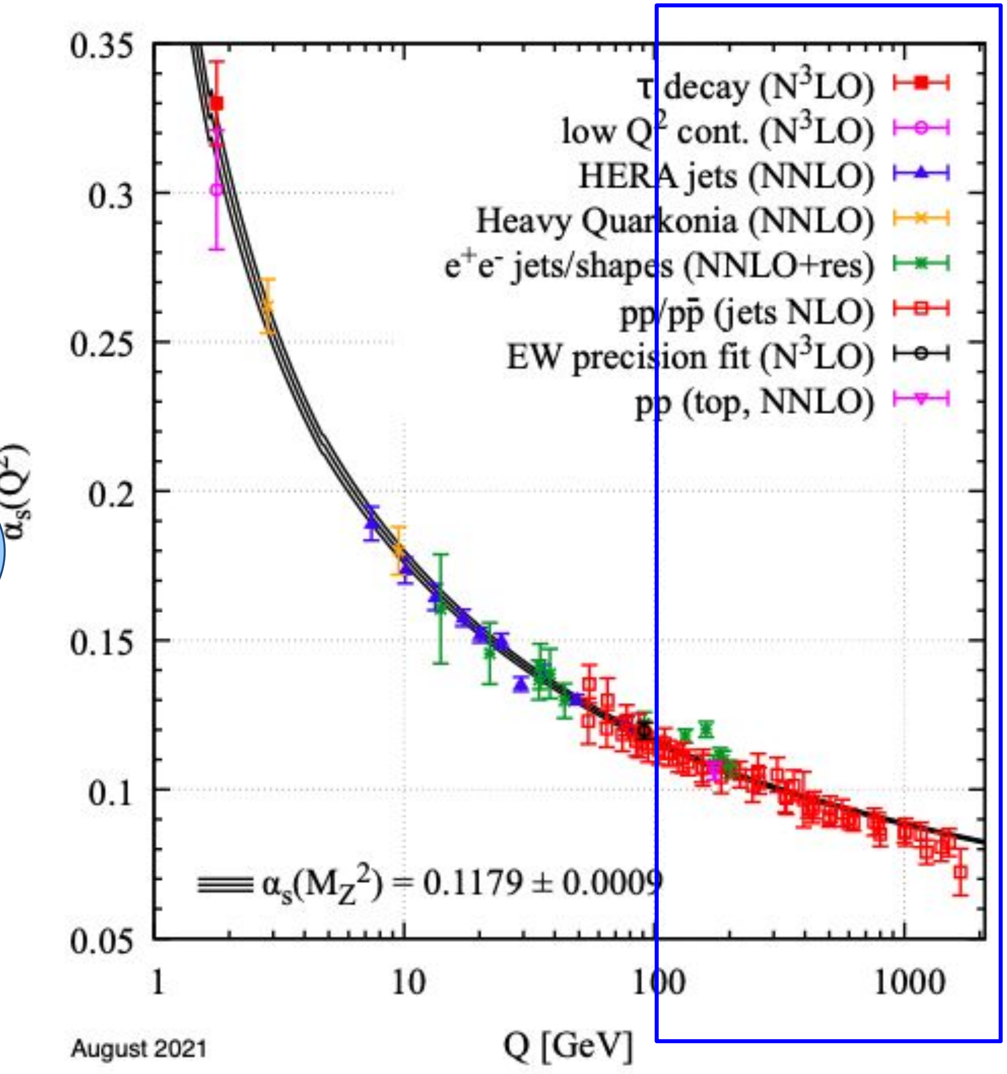




High Q^2 (small r) Small Q^2(large r)

Asymptotic freedom

In QCD interactions with high energy, coupling tends to zero
→ At high energies, quarks and gluons would behave as if the strong force doesn't exist

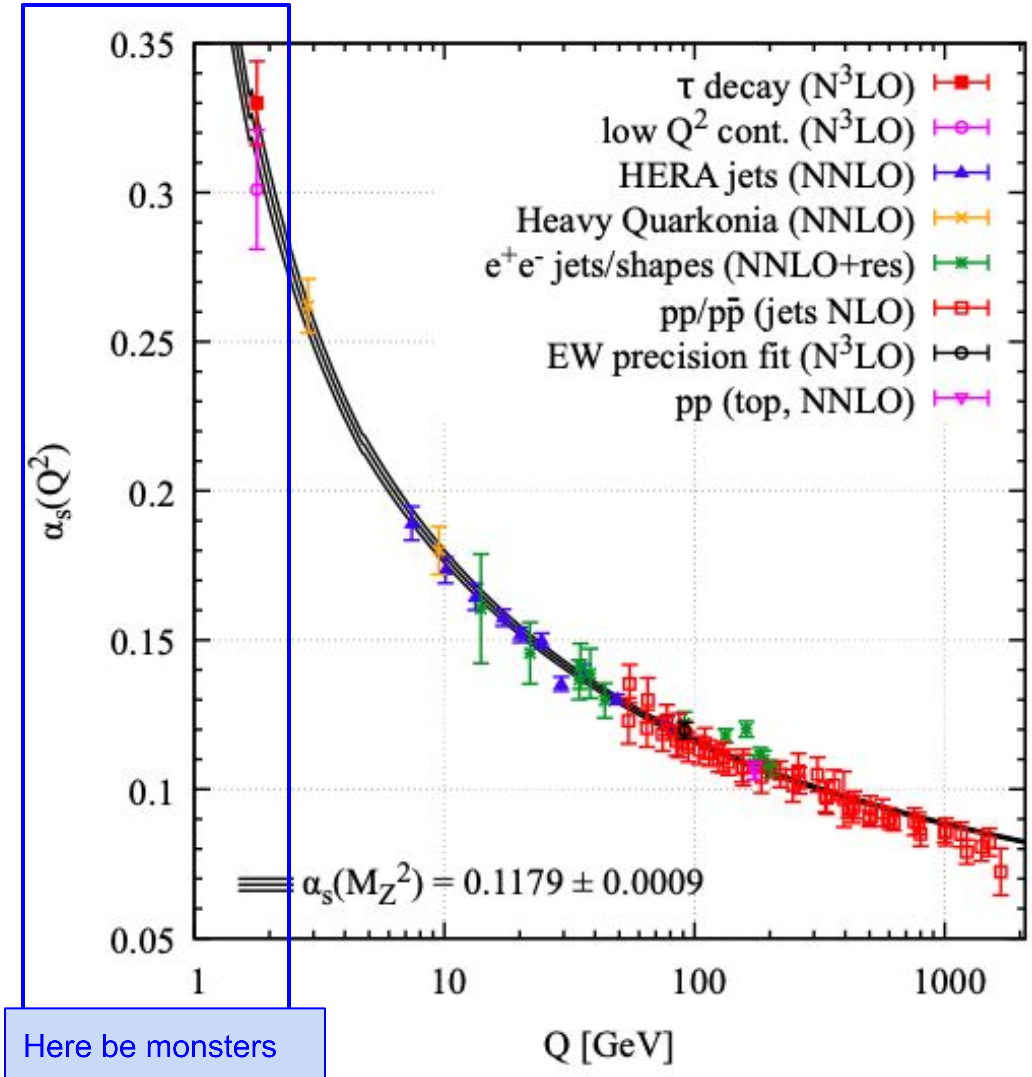


QCD at low energies

At low energies, coupling increases to very high values

- At Λ ~ 200 MeV → non-perturbative QCD

Now you know why we won't see a quark alone!

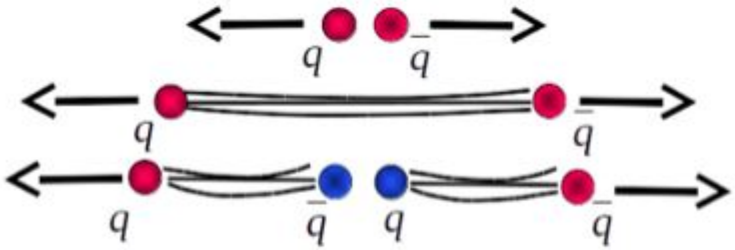


Here be monsters

QCD at low energies: some consequences

Confinement

Quarks and/or gluons move away
- Energy between partons become high
- Spontaneous creation of pair quark-anti-quark
- Hadronization (shower of particles)



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Parton density functions

Charge distribution within hadrons. Constant production and annihilation of partons.
- If high energy interaction with proton, interaction with a distribution of particles → PDFs.
- PDFs: experimental knowledge
- Evolution of PDFs with energy → DGLAP equations.

Instagram picture of a proton

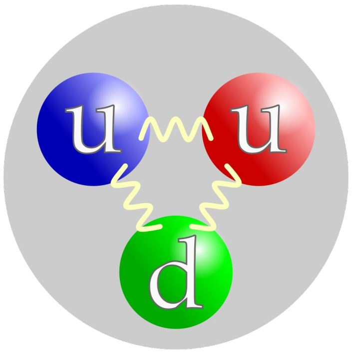
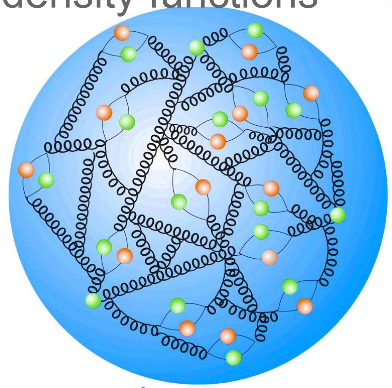


image from official HEP instagram (wikipedia)

Real picture: Parton density functions



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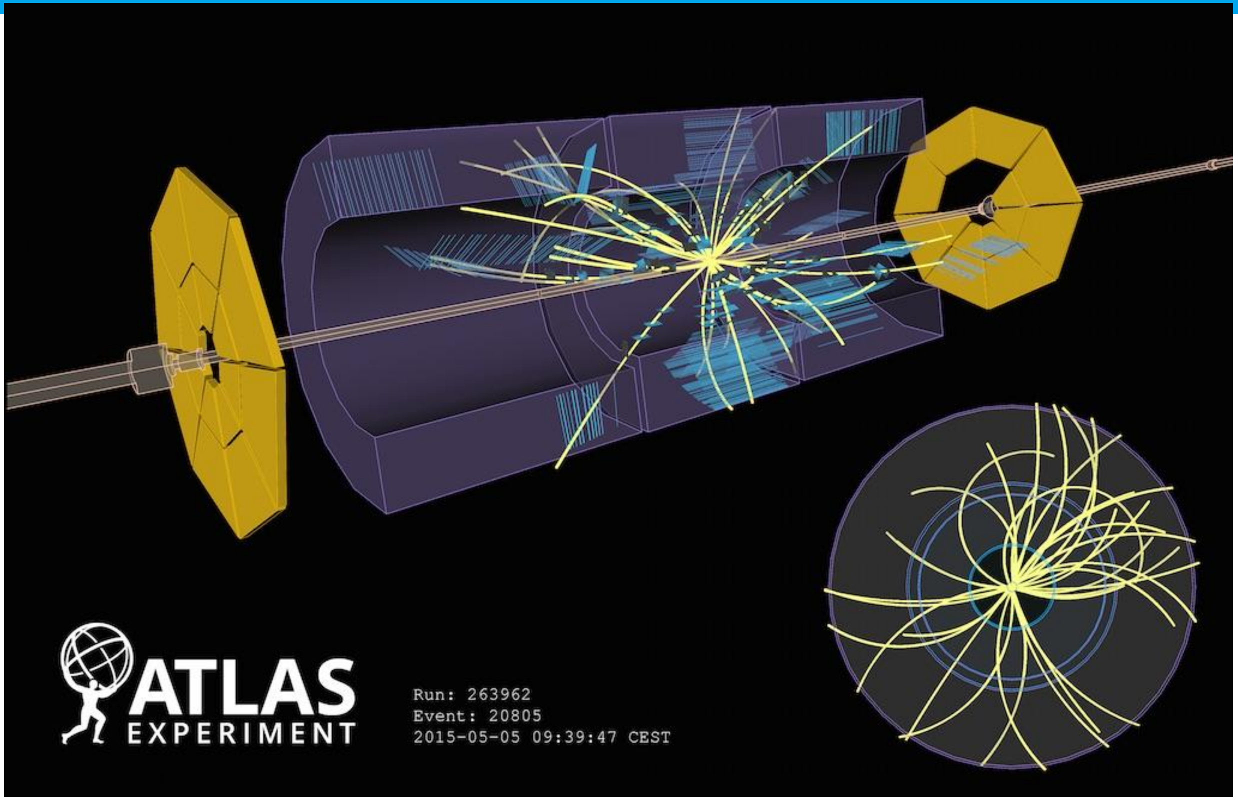
SM predictions fail here → Can only know through experiments by now

Miscellanea: calculations in proton-proton collisions



$$\sigma_{PP\to X} = PDF \otimes \sigma_{\text{hardscatter}} = \sum_q \int dx_1 dx_2 f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \otimes \hat{\sigma}_{q\bar{q}\to X}(\alpha, Q^2)$$

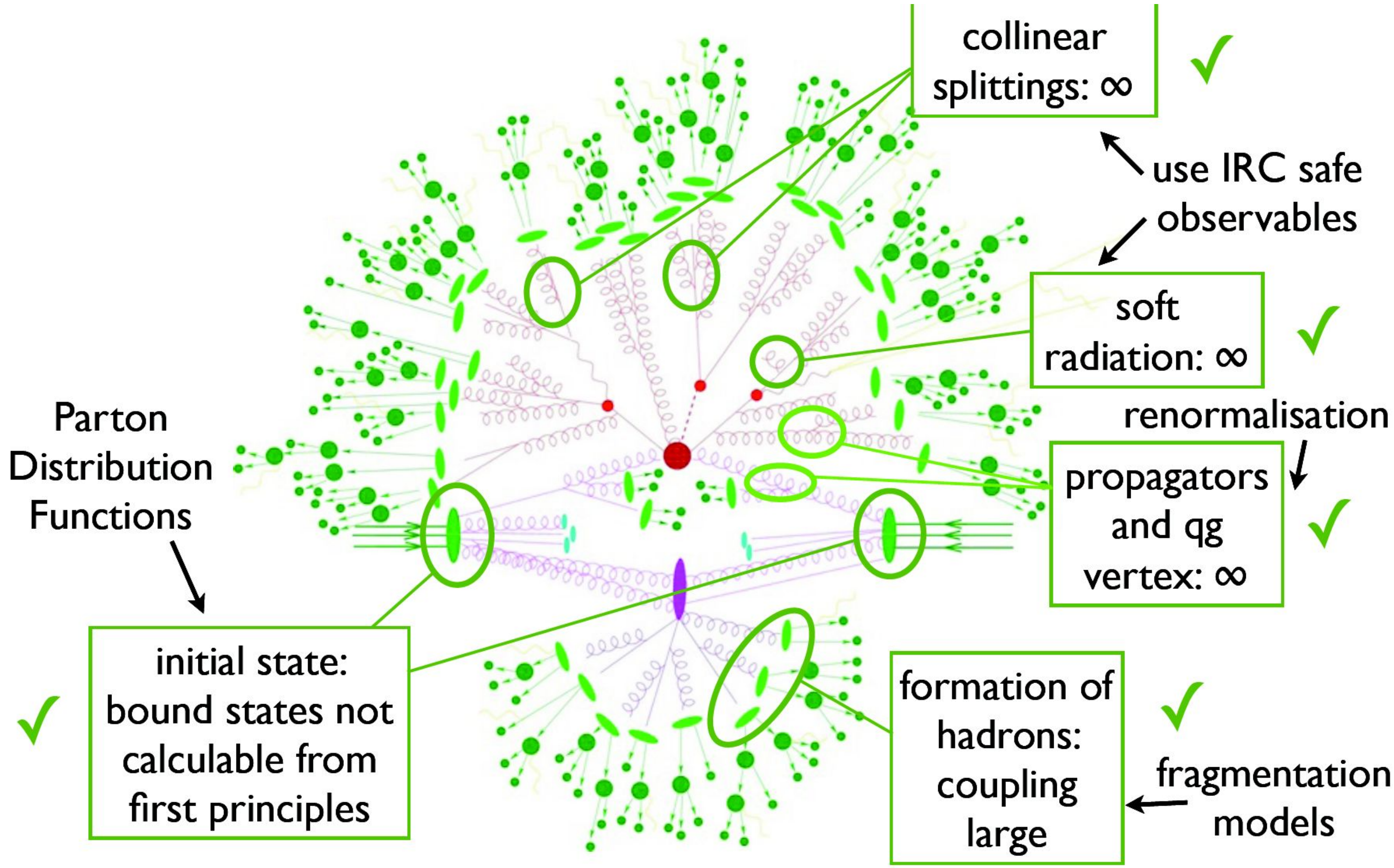
Perturbative QCD

- Cross-section
- Infrared radiation
- Evolution of PDFs at high energy

Non-perturbative QCD

- Proton structure (PDFs)
- Parton shower
- Hadronization
- Jets

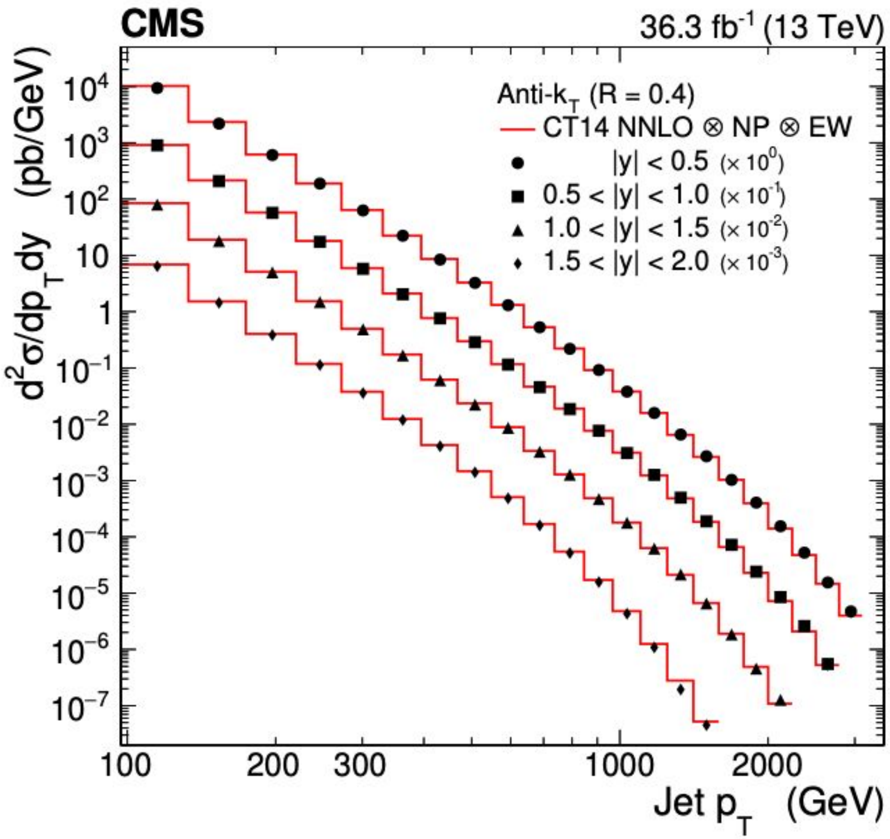
Miscellanea: calculations in proton-proton collisions



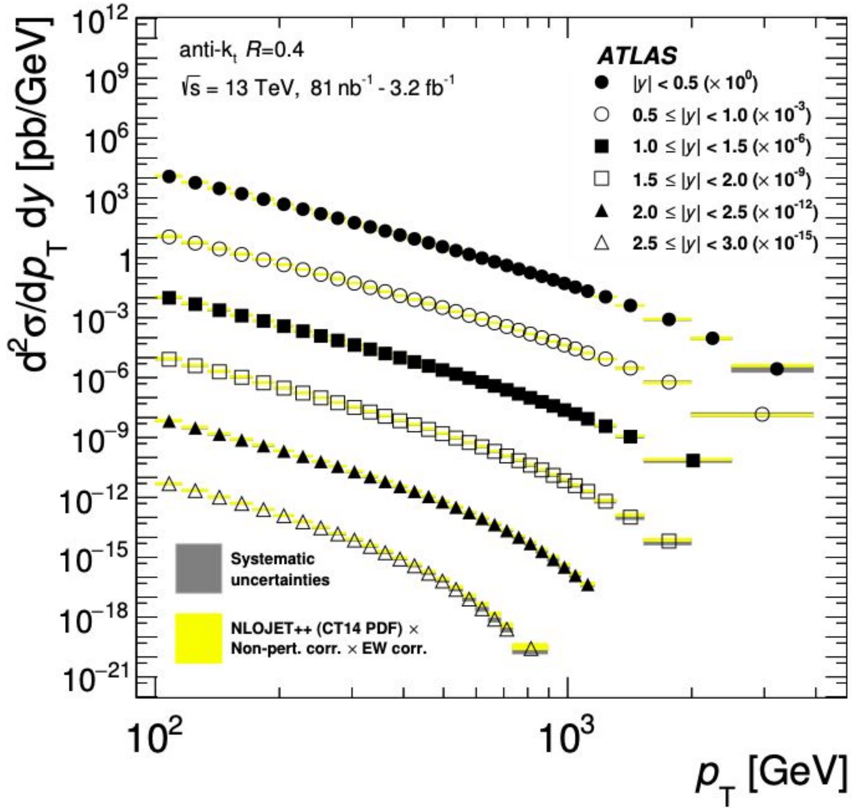
$$\sigma_{PP\to X} = PDF \otimes \sigma_{\text{hardscatter}} = \sum_q \int dx_1 dx_2 f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \otimes \hat{\sigma}_{q\bar{q}\to X}(\alpha, Q^2)$$

Measuring QCD properties: Differential jet cross-section

[JHEP02(2022)142](#)



[JHEP05(2018)195](#)



Measuring QCD properties: measurement of α_s

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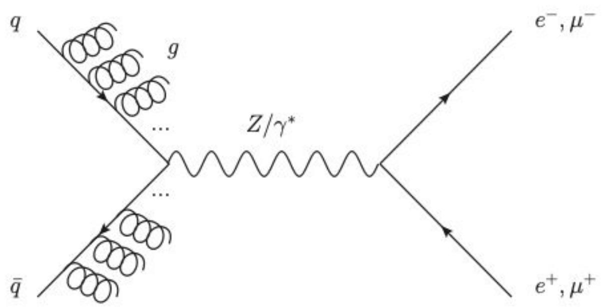
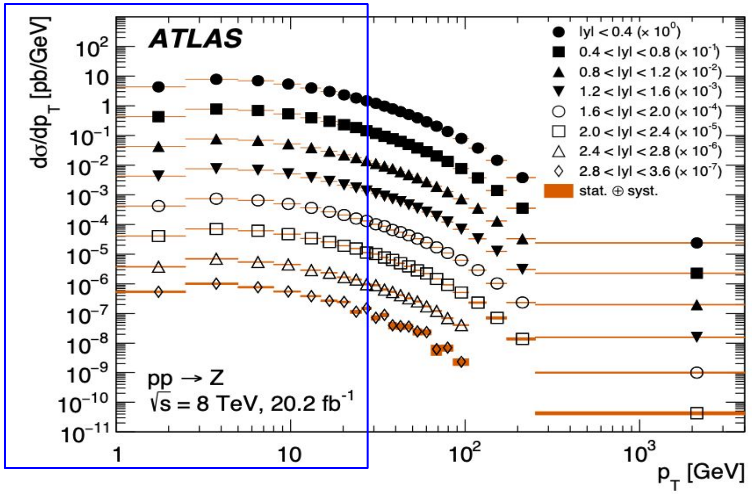
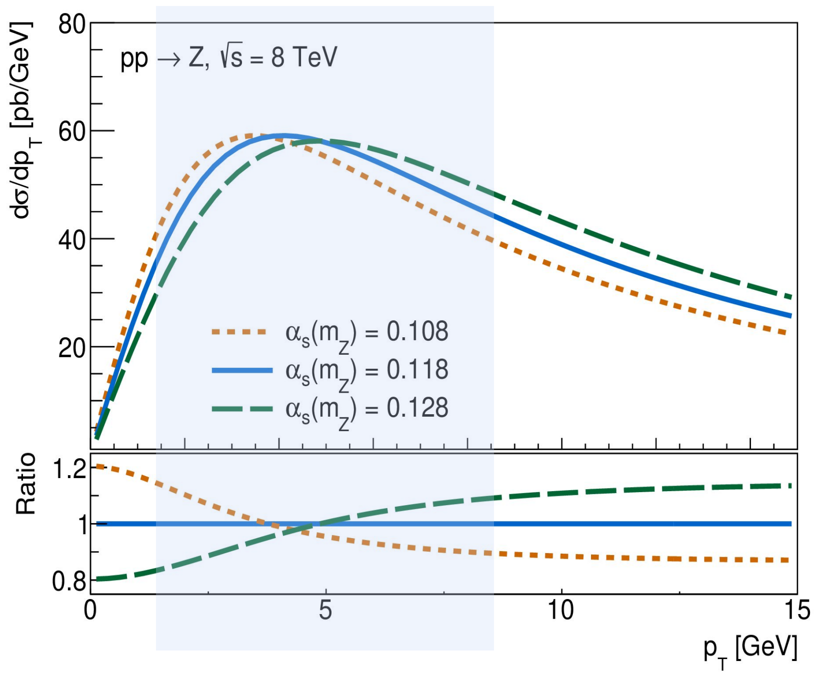
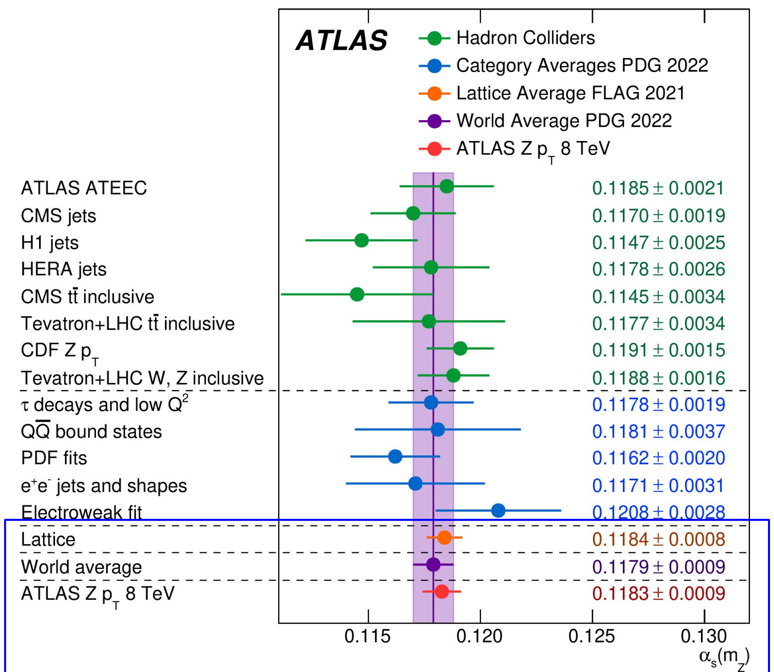








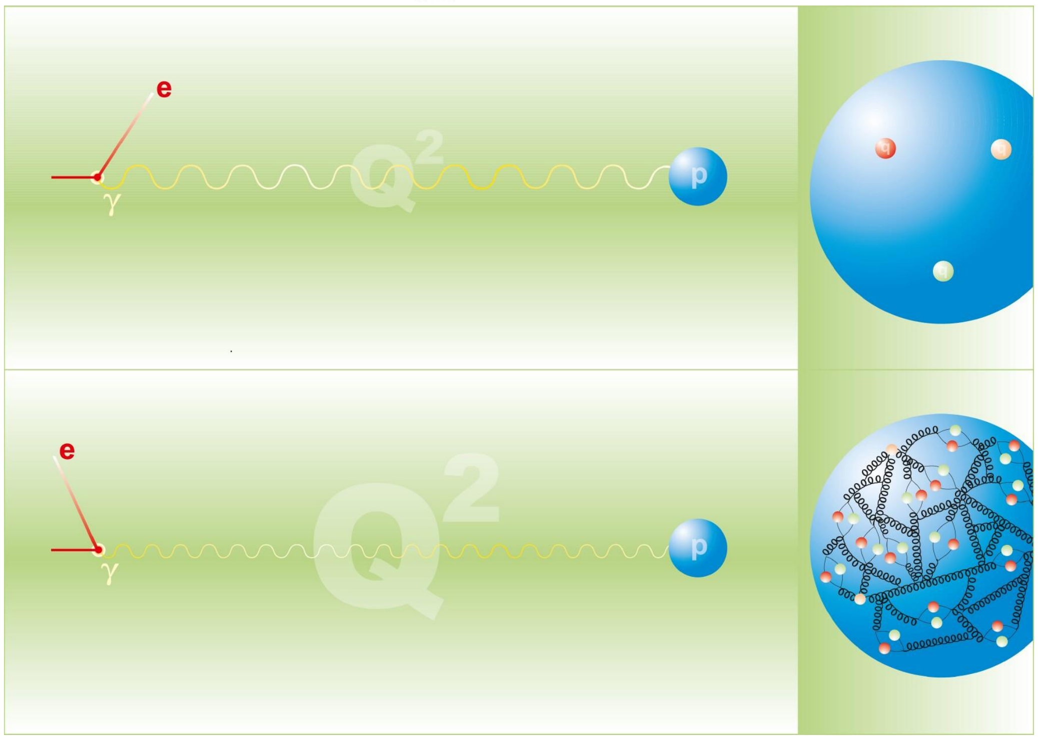
Backup

Two additional basic concepts
Gauge theory and running couplings

PDFs at Hadron colliders

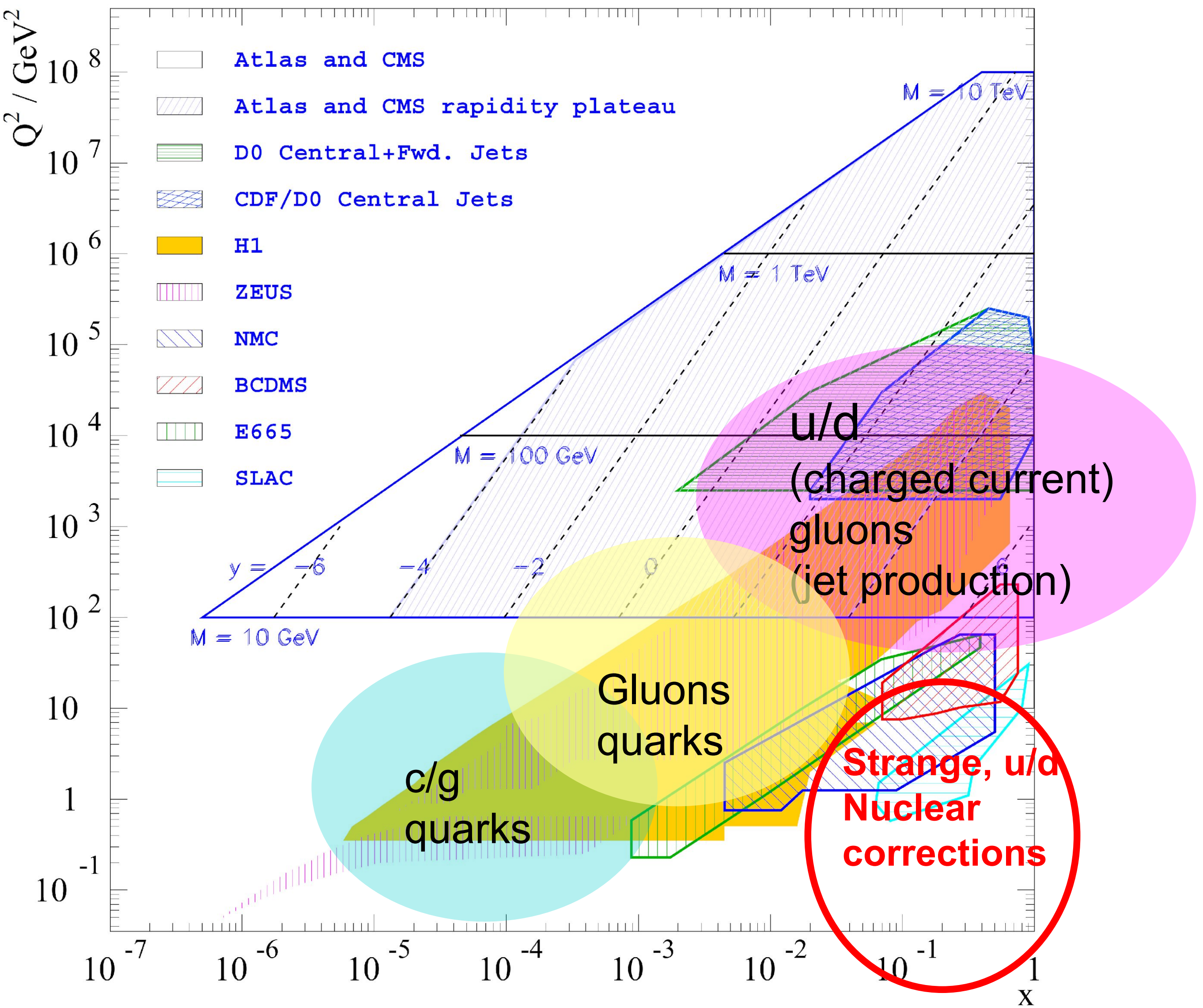
- Probability to find a parton q carrying momentum fraction x of the proton momentum to enter a collision at a momentum transfer squared Q^2

$$f_q(x_1, \boxed{Q^2})$$

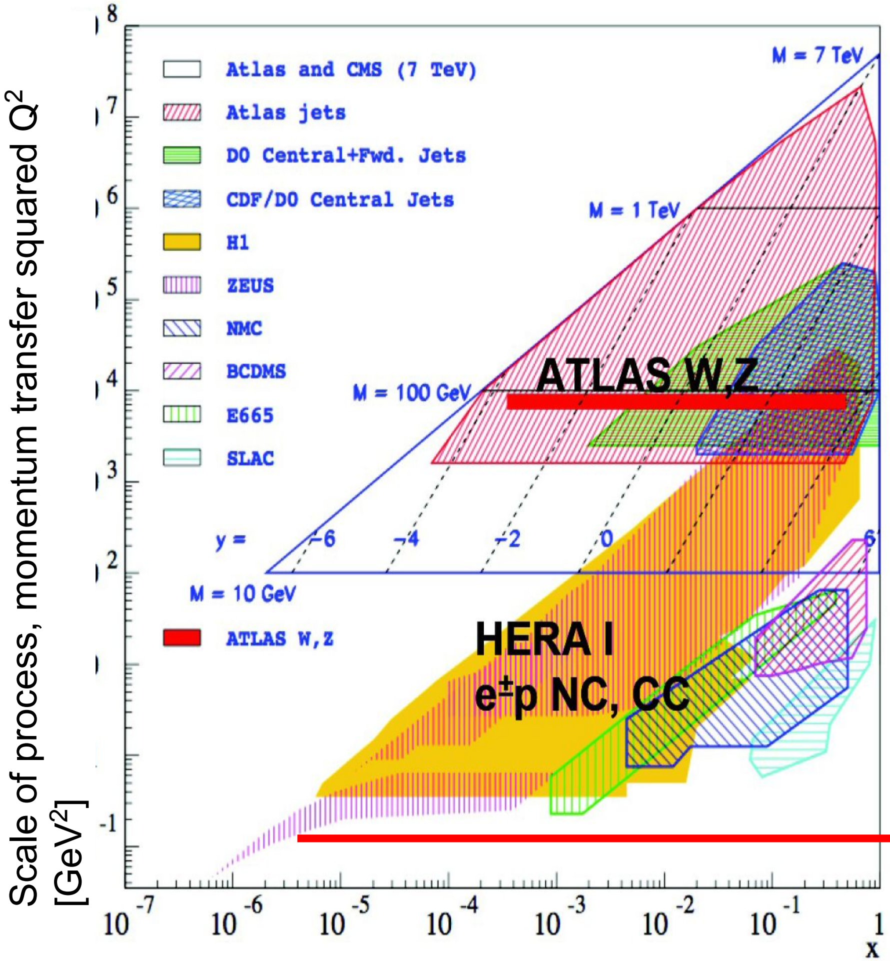


$$\frac{\Delta E}{\Delta t} \leq \hbar/2$$

Input measurements



Procedure of PDF fits I



starting scale Q^2 ~ 1.5 - 2 GeV

- Evolve input PDFs with DGLAP equations
- Calculate observables using (N)(N)LO and compare to experiments
- Minimize global Chi2 between data and theory

- Groups: MSTW/MRST (global fit, up to NNLO)
  CTEQ / CT (global fit, up to NLO, now NNLO)
  NNPDF (global fit, neural network PDFs)
  HERA PDFs (Hera collider data only so far)

What is inside the proton ?? Scattering experiments

- Transferred momentum:

  $$q = k - k'$$

- Virtuality of exchanged boson:

  $$Q^2 = -q^2 > 0$$

- Squared centre-of-mass energy:

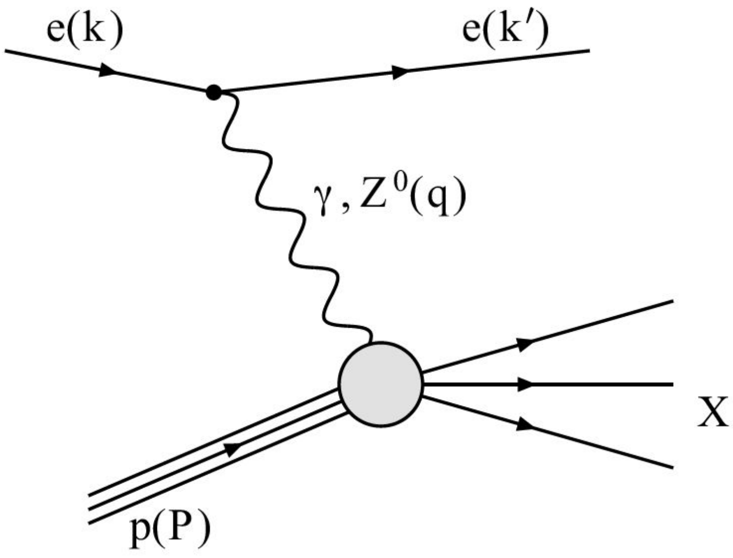
  $$s = (P + k)^2$$

- Squared mass of the hadronic final state:

  $$W^2 = (P + q)^2 = M^2 + 2q \cdot P - Q^2$$

- Inelasticity: $y = \dfrac{q \cdot P}{k \cdot P}$ with $0 \leq y \leq 1$

- Scaling variable: $x = \dfrac{Q^2}{2q \cdot P}$ with $0 \leq x \leq 1$

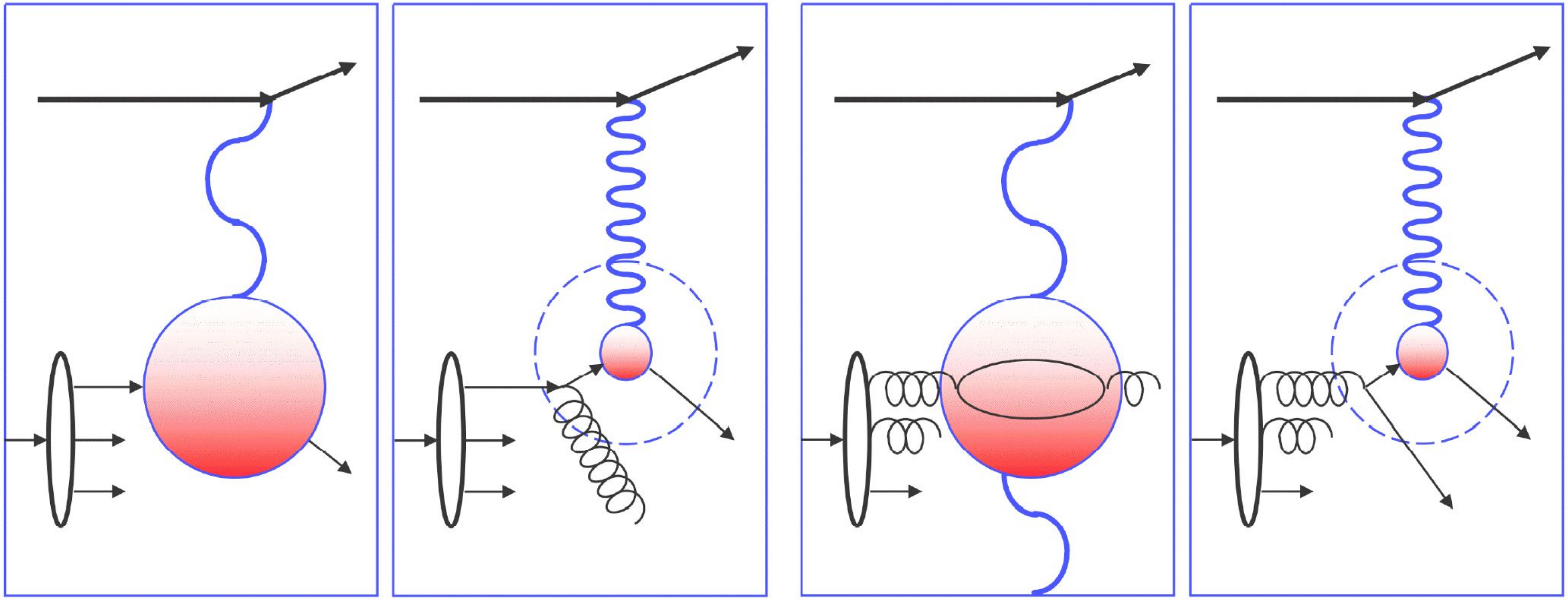


Deep: $Q^2 \gg M^2$

Inelastic: $W > M$

Because increasing energy implies potentially improved spatial resolution, scaling implies independence of the absolute resolution scale, and hence effectively point-like substructure.

- Proton quark dominated: $Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x
- Proton gluon dominated: $Q^2 \uparrow \Rightarrow F_2 \uparrow$ for fixed x



Q^2-evolution described by DGLAP equations

“Reality” of Quarks

- originally, when proposed in 1964 by Gell-Mann and Zweig, “quarks” were considered by many physicists just a principle for ordering the new-found particle zoo
- if the quarks really correspond to constituents of the hadrons was not clear
- in 1968, deep inelastic electron-proton scattering at SLAC showed that the proton consisted of smaller constituents, then called “partons” by Feynman
- only slowly it was accepted that the partons in the proton correspond to the *u* and *d* quarks

→ Quark-Parton-Model (QPM)

Procedure of PDF fits II

- Parameterize x distributions for all parton flavours

  - $x \to 0$ u = d, $q \propto x^{a1}$
  - $x \to 1$ q $(1 - x)^{a2}$ (quark counting rules)
  - $P(x, \ldots)$ medium-x range, just convenient form

Example (NNPDF): 29 input parameters

$$xu_v(x,Q^2) = A_u x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u x)$$
$$xd_v(x,Q^2) = A_d x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d x)$$
$$xS(x,Q^2) = A_S x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\sqrt{x}+\gamma_S x)$$
$$x\Delta(x,Q^2) = A_\Delta x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta+\delta_\Delta x^2)$$

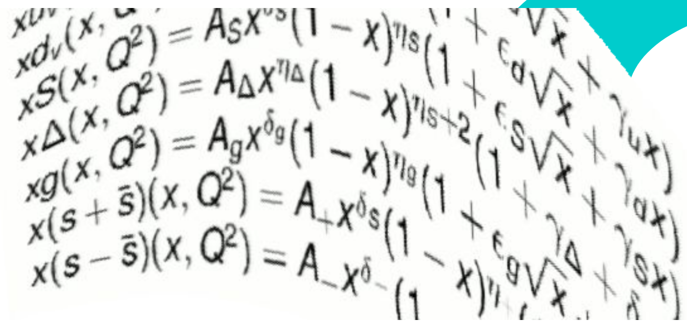
Δ = Sea asymmetry u - d

$$xg(x,Q^2) = A_g x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) + A_{g'}x^{\delta_{g'}}(1-x)^{\eta_{g'}}$$
$$x(s+\bar{s})(x,Q^2) = A_+ x^{\delta_S}(1-x)^{\eta_+}(1+\epsilon_S\sqrt{x}+\gamma_S x)$$
$$x(s-\bar{s})(x,Q^2) = A_- x^{\delta_-}(1-x)^{\eta_-}(1+x/x_0)$$

- Create PDFs with default starting values at given scale Q^2

Error estimation

- Use **Hessian Approach** (most PDF groups):
  Transform original PDF parametrizations into eigenvector basis

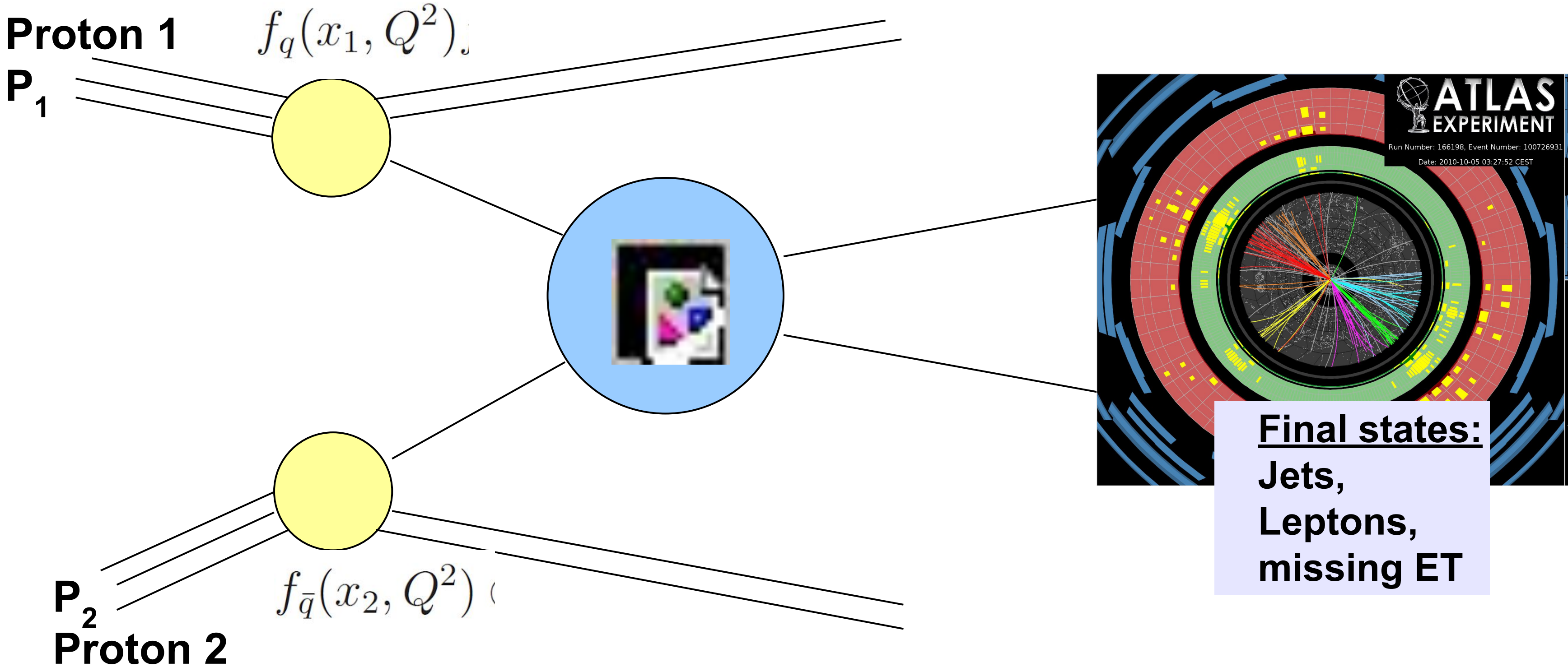


- ~40 eigenvectors (combinations of PDF parameters)
- orthogonal!!
- changing one eigenvector cannot be compensated in terms of Chi2 by changing another one as well
- Reflect correlations between input observables

- Use **MC replica approach** (mostly NNPDF):
  Prepare pseudo data replicas of the input data samples, which are randomly varied within their errors,
  Fit them and extract PDF and errors from mean + RMS of replica PDFs

PDFs at Hadron colliders

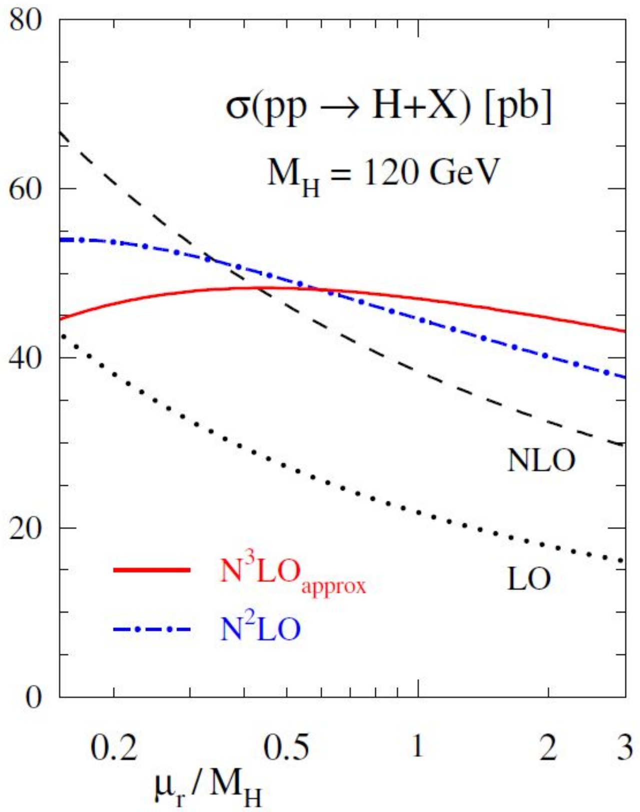
$f_q(x_1, Q^2)$ **Probability to find parton with momentum fraction x in proton**



phenomenological part Analytical part

$$\sigma_{PP \to X} = PDF \otimes \sigma_{\text{hardscatter}} = \sum_q \int dx_1 dx_2 f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \otimes \hat{\sigma}_{q\bar{q} \to X}(\alpha, Q^2)$$

Analytical part

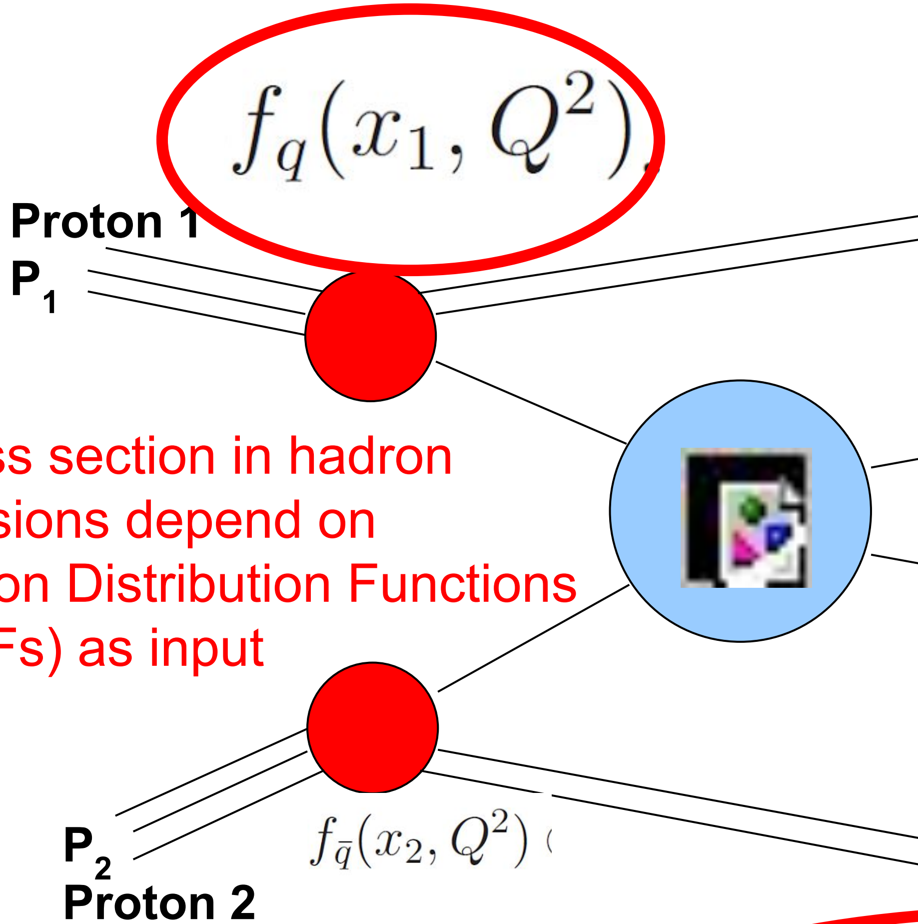


- Renormalization Scale dependence
- Factorization Scale dependence
- Electroweak input-parameter scheme
- ....

phenomenological part   Analytical part

$$\sigma_{PP\to X} = PDF \otimes \sigma_{\text{hardscatter}} = \sum_q \int dx_1 dx_2 f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \otimes \hat{\sigma}_{q\bar{q}\to X}(\alpha, Q^2)$$

PDFs at Hadron colliders



Cross section in hadron collisions depend on Parton Distribution Functions (PDFs) as input

- Not calculable from first principles!!!
- Dependence on input data
- Fitting scheme
- PDF parametrization
- ....

phenomenological part

Analytical part

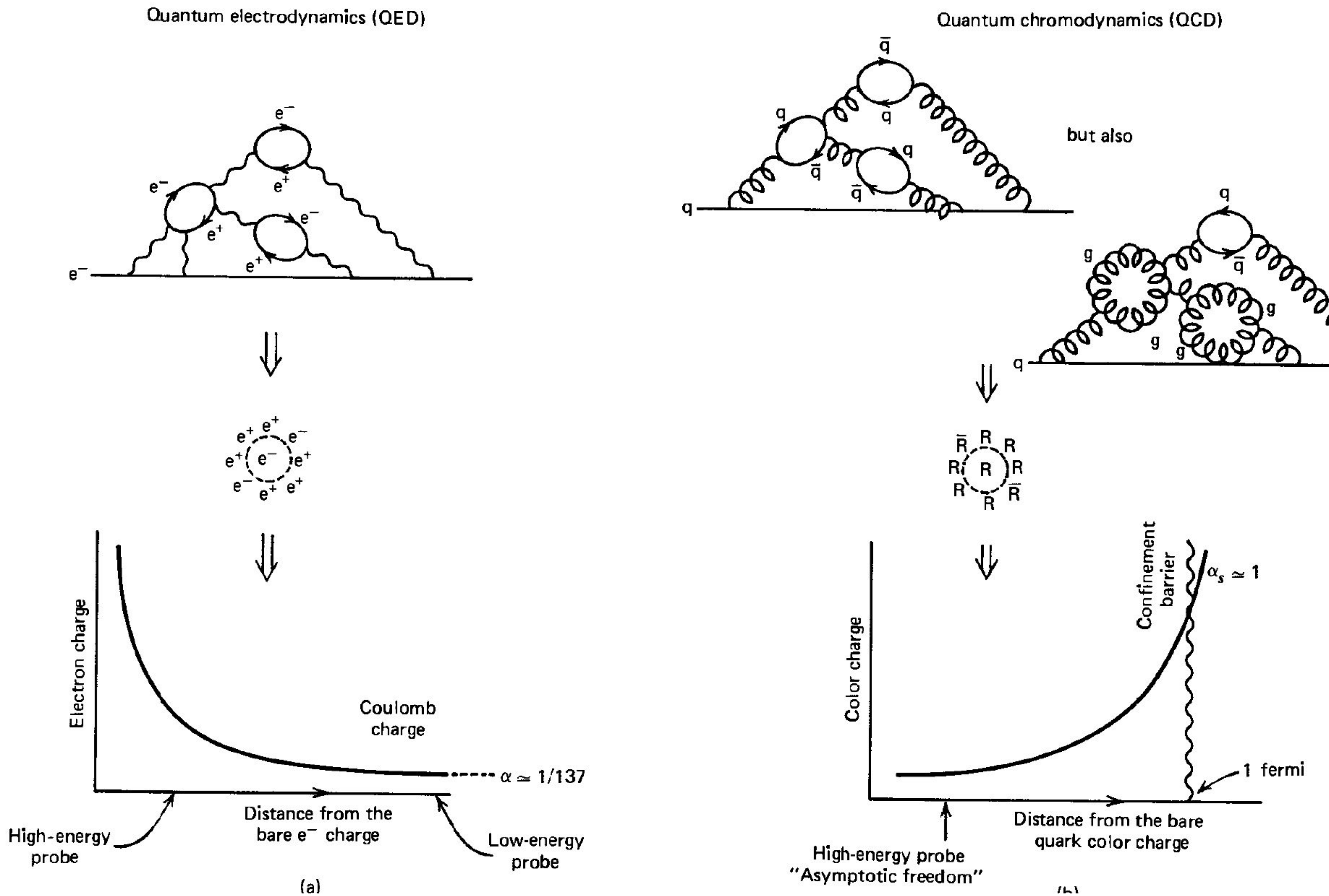
$$\sigma_{PP\to X} = PDF \otimes \sigma_{\text{hardscatter}} = \sum_q \int dx_1 dx_2 f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \otimes \hat{\sigma}_{q\bar{q}\to X}(\alpha, Q^2)$$

Confinement

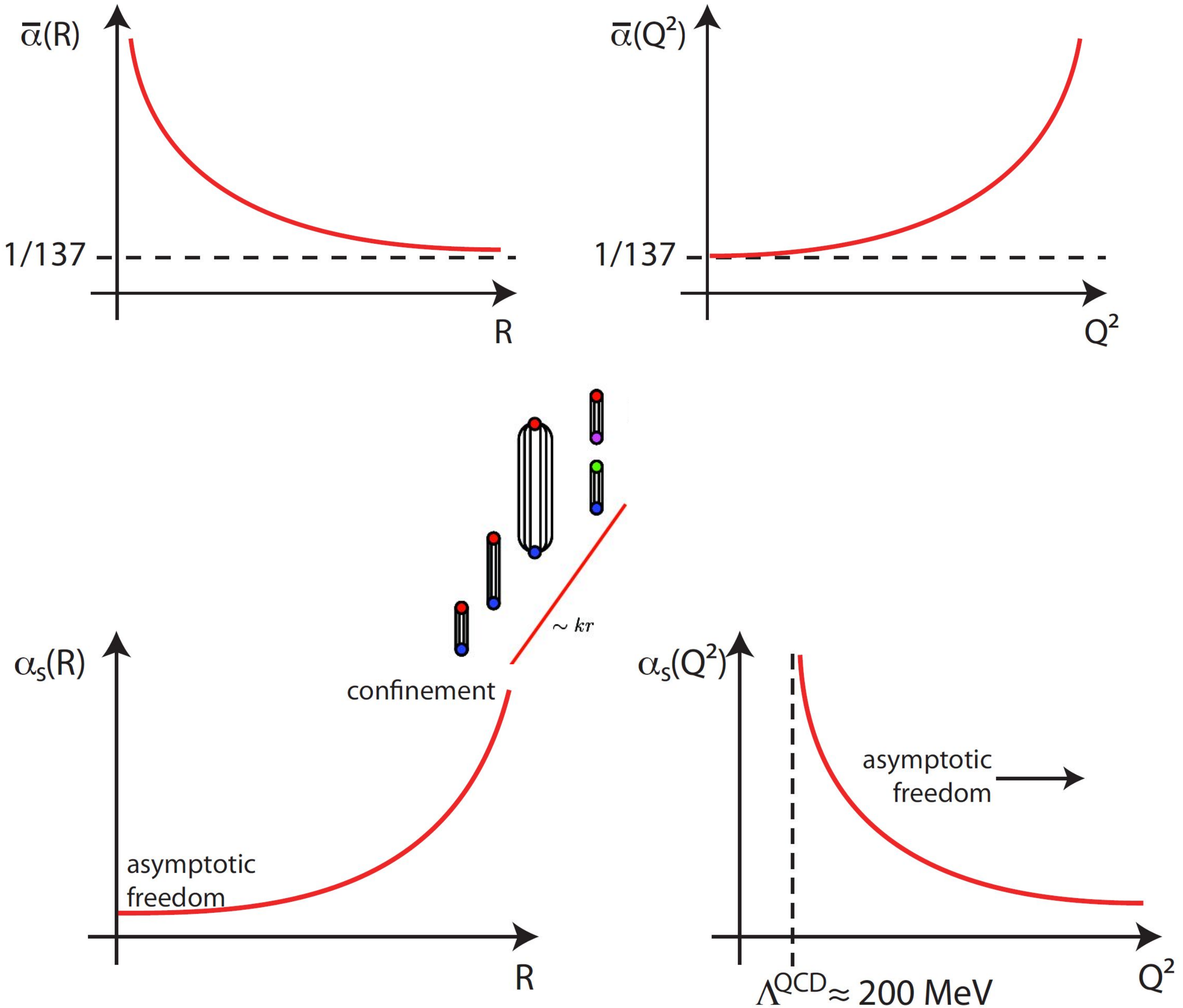
$\overline{\alpha}(R)$

1/137

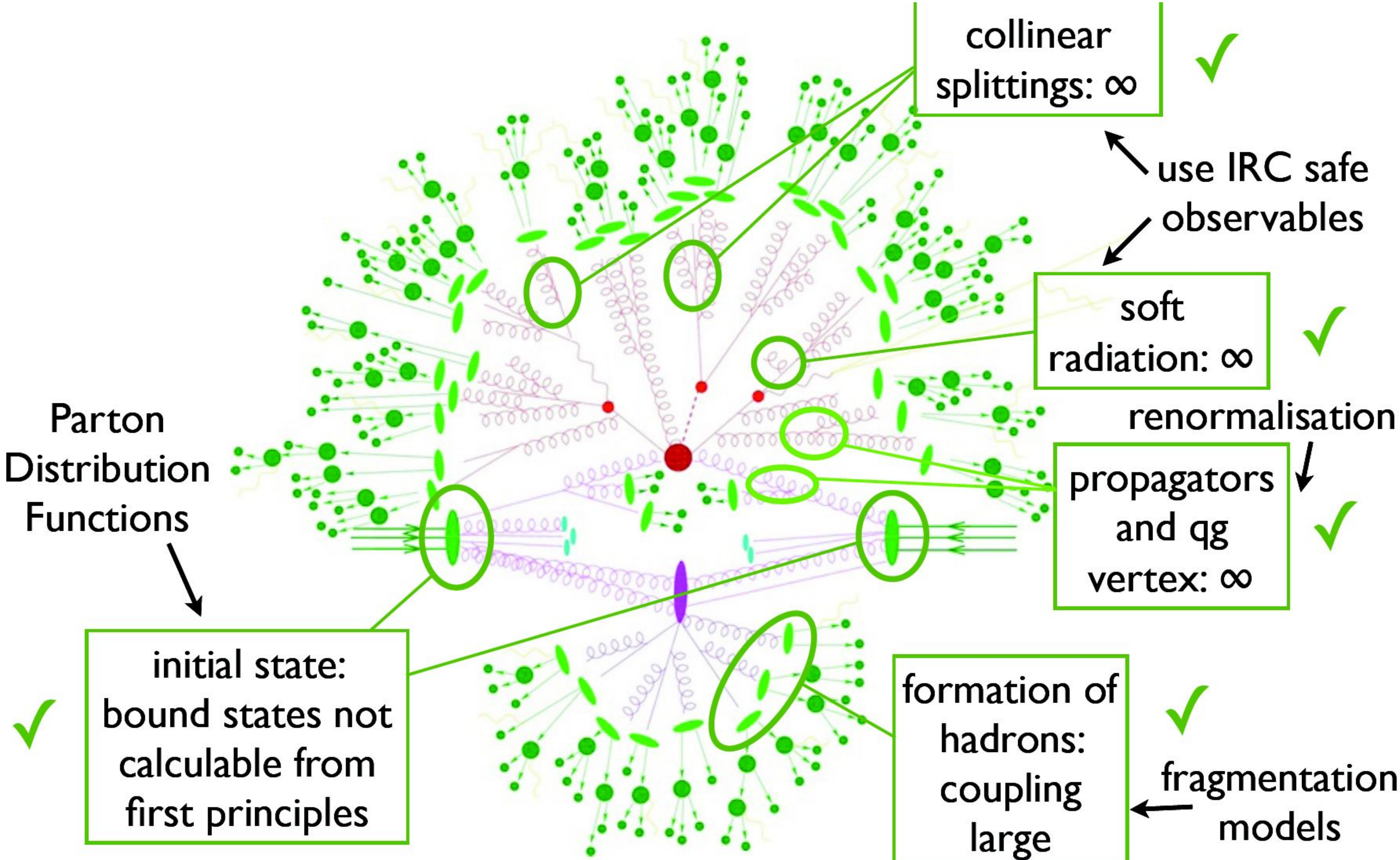
$\alpha_s(R)$



Confinement



QCD is messy


collinear splittings: ∞
use IRC safe observables
soft radiation: ∞
renormalisation
propagators and qg vertex: ∞
Parton Distribution Functions
initial state: bound states not calculable from first principles
formation of hadrons: coupling large
fragmentation models

How to measure QCD? – Jets

Last missing piece before we can calculate real-life cross sections

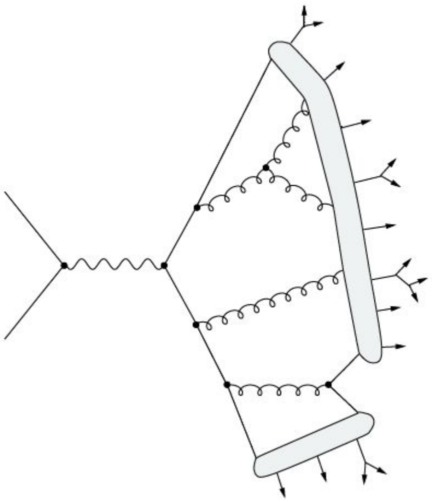
Full-scale event generators generate QCD branching according to emission probabilities - the parton shower approach

Once the scale of the emitted partons becomes small, perturbative QCD is not applicable anymore

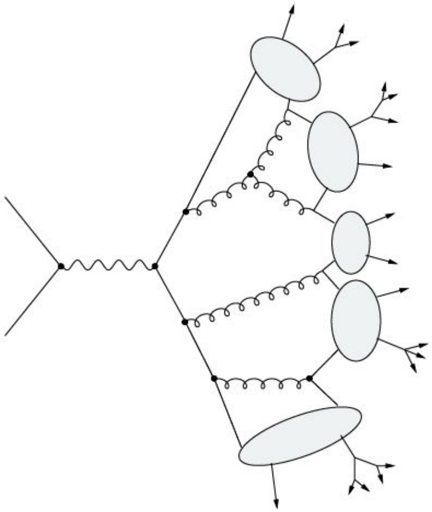
Model the formation of hadrons with phenomenological approaches

Based on the idea of the QCD potential

$$V(r) \propto k \cdot r$$



String Fragmentation
(Pythia and friends)



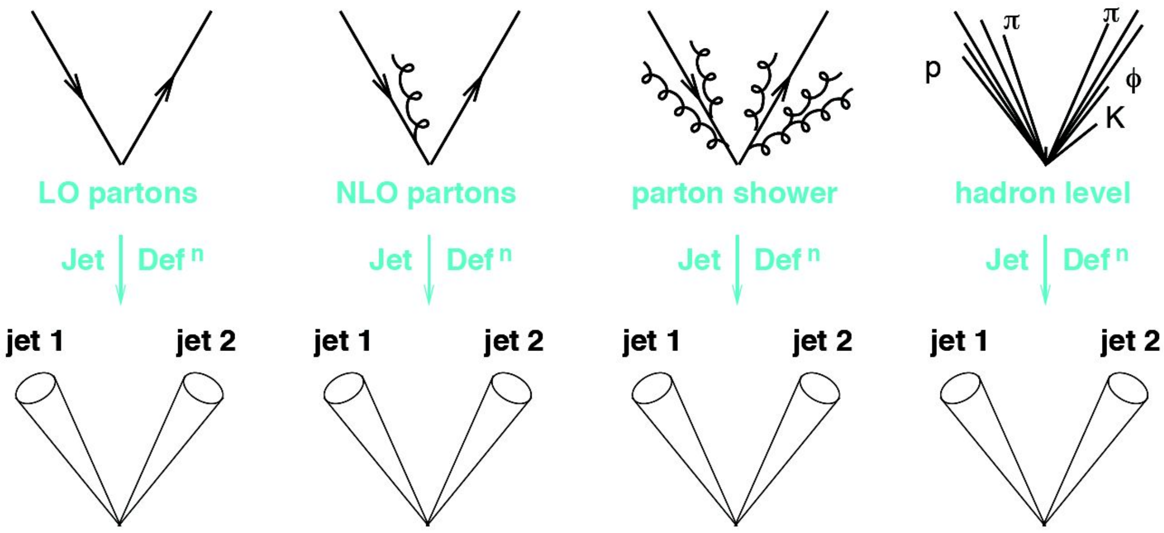
Cluster Fragmentation
(Herwig)

→ don't forget to model particle decays

How to measure QCD? – Jets

A jet algorithm combines objects (partons, hadrons, detector deposits) which are "close" together

Different choices for infrared and collinear (IRC) safe jet algorithms exist, with different distance definitions, but the working principle is:

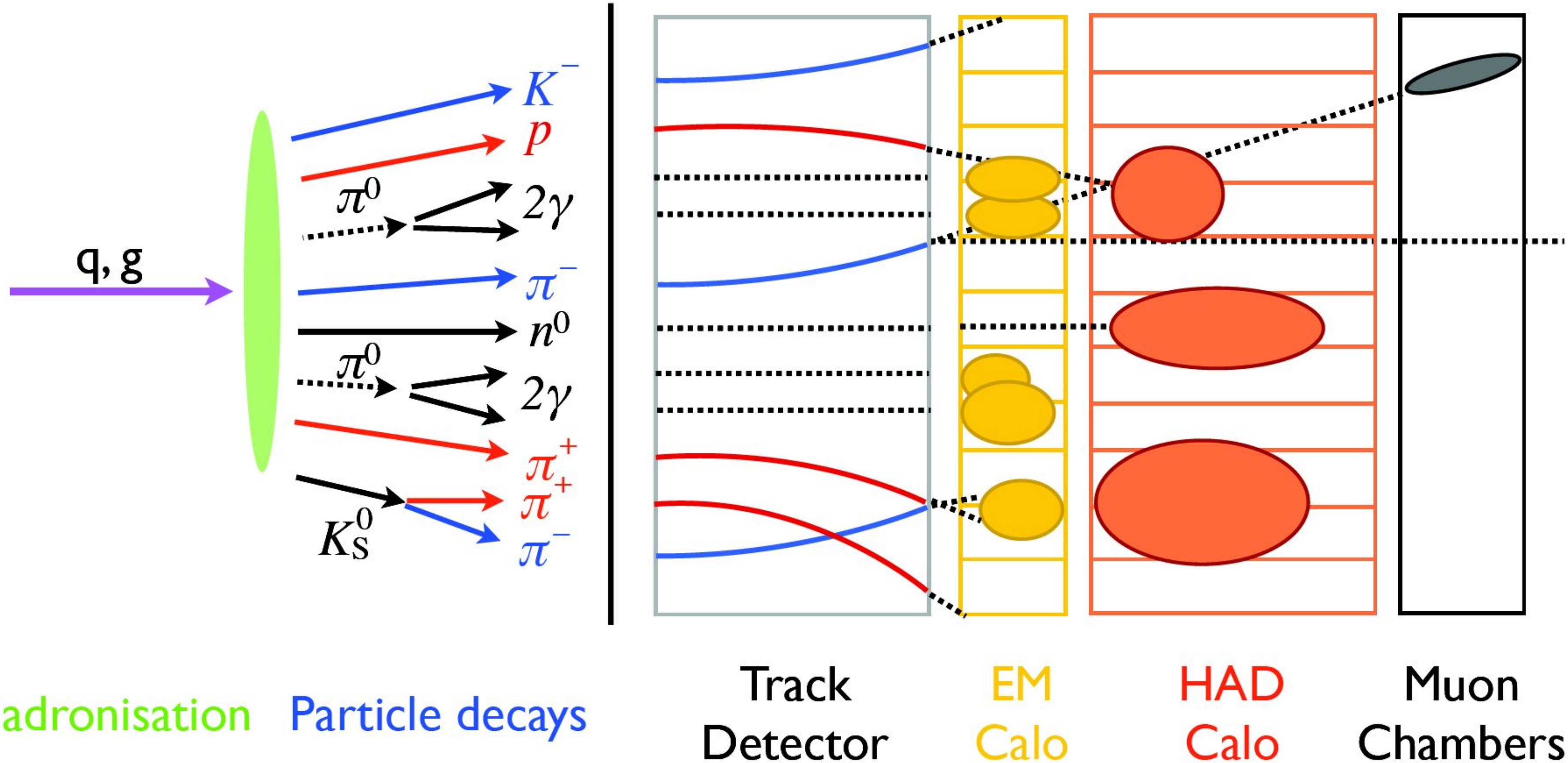


Projection to jets should be resilient to QCD and detector effects

Jets help us to study the underlying parton dynamics

(courtesy of Gavin Salam)

How to measure QCD? – Jets



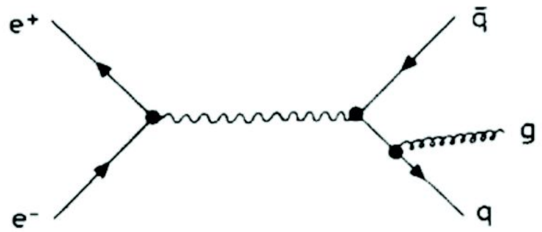
After the hadronisation and the detector effects it is virtually impossible to reconstruct all particles which originated from a single quark or gluon

The total deposited energy can be well measured

The gluon

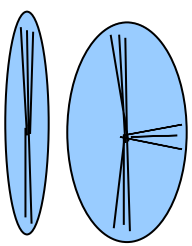
Three-Jet Events in e^+e^-

Radiation of a gluon leads to 3-jet structure

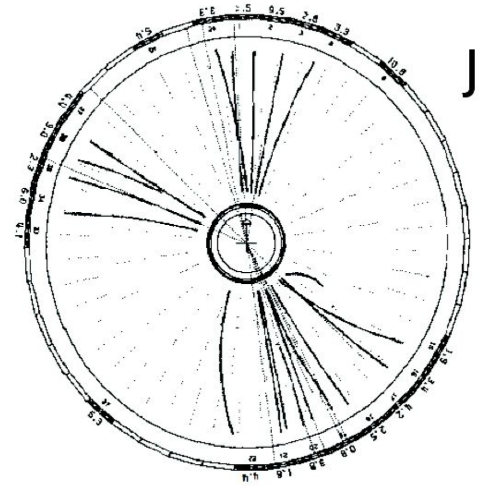


First observed at PETRA (higher CMS energy than at DORIS)

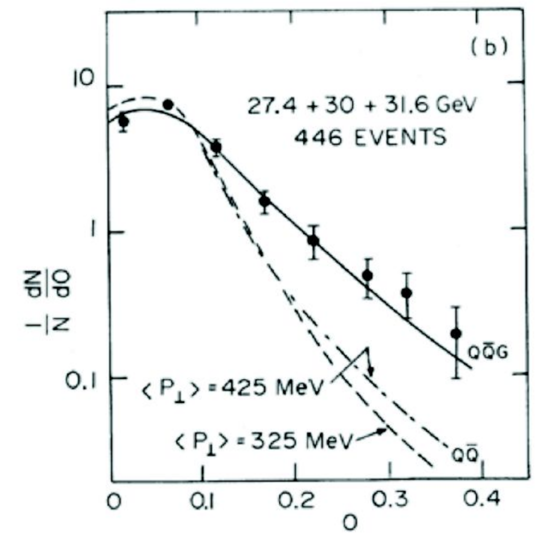
Oblateness: $O = F_{\mathrm{major}} - F_{\mathrm{minor}}$



O is small for 2-jet events and becomes larger for 3-jet events, proportional to the P_T of the radiated gluon



JADE

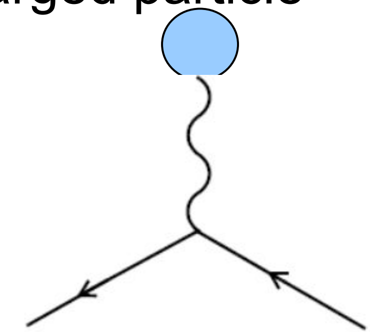


D. P. Barber (Mark-J), Phys.Lett.B89, 139(1979)

Tests of QED: magnetic momentum

- Magnetic moment due to rotating charge body → spin of charged particle

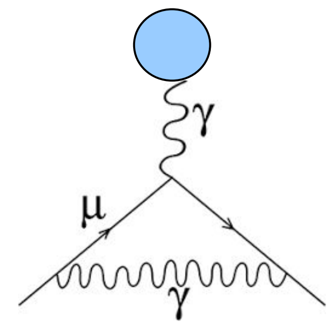
$$\mu = -g\frac{e}{2m}s = -\frac{g}{2}\frac{e}{2m}$$



- Classical result by Dirac: g-factor = 2

$$a \equiv \frac{g-2}{2} = 0$$

- Deviations from classical result caused by quantum corrections



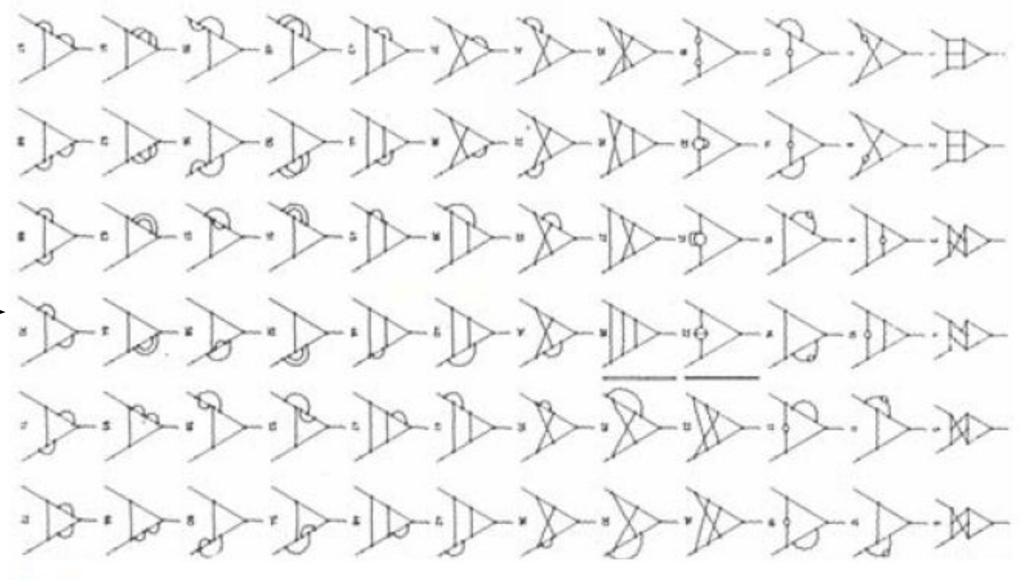
$$g(\alpha)/2 = 1.001\ 159\ 652\ 177\ 60\ (520) \quad [5.2\ \text{ppt}]\ (\text{predicted}).$$

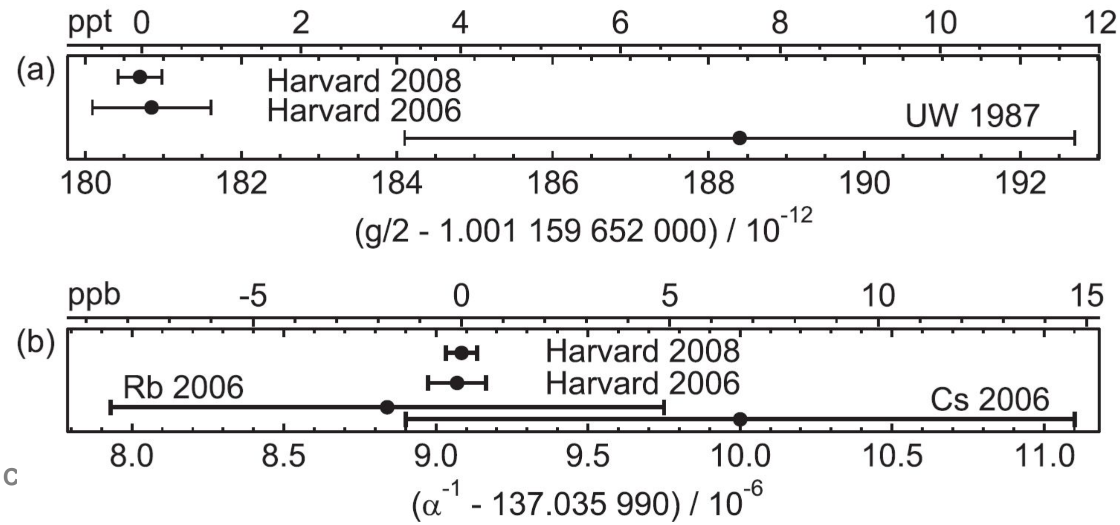
Tests of QED: electron magnetic momentum

> Extremely precise calculations up to several orders of loops needed

Feynman Graphs		
$O(\alpha)$		1
$O(\alpha^2)$		7
$O(\alpha^3)$	analytically	72
$O(\alpha^4)$	numerically	891
til $O(\alpha^4)$		971

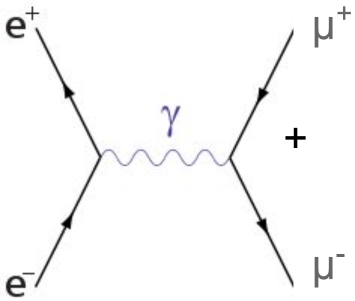
Most precise calculations:
T. Kinoshita et al.





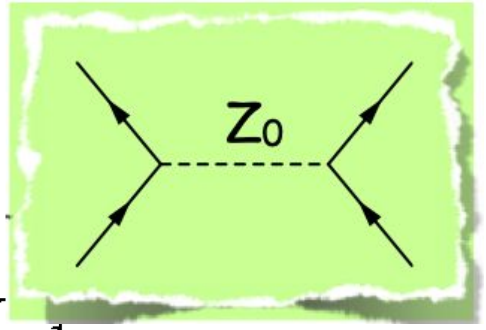
High-energy tests: lepton pair production

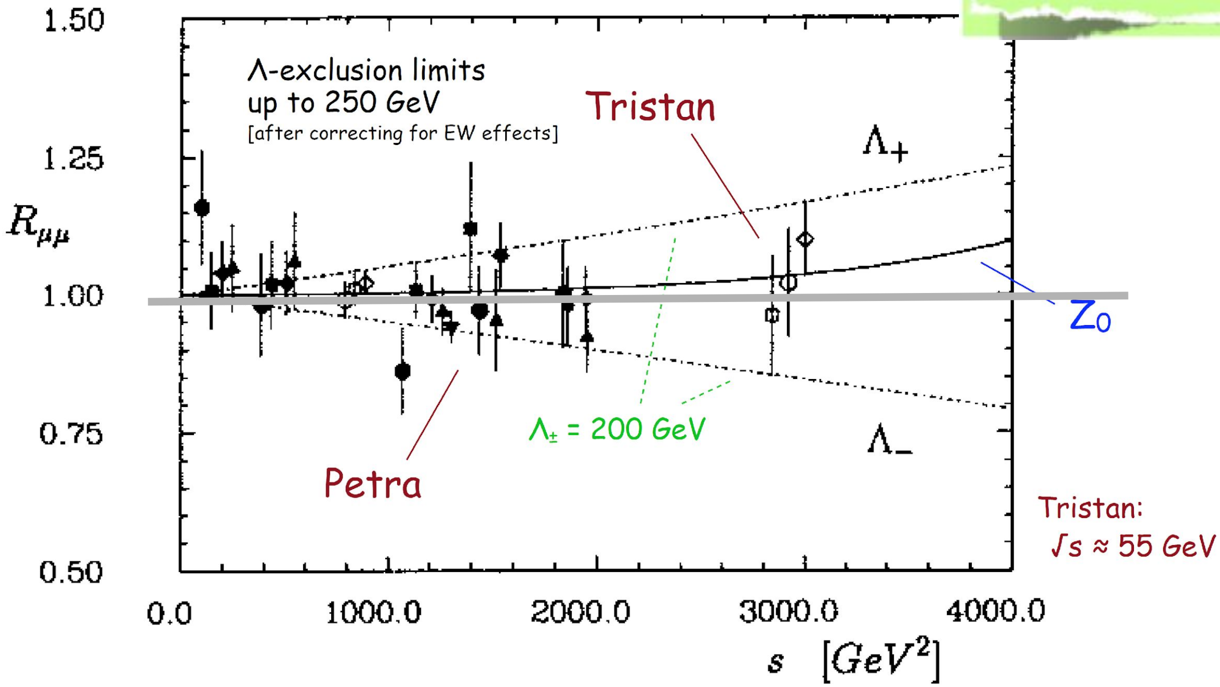
QED process



+ corrections

$$R_{\mu\mu} = \frac{\sigma_{\text{meas}}^{e^+e^-\to\mu^+\mu^-}}{\sigma_{\text{QED}}^{e^+e^-\to\mu^+\mu^-}},$$



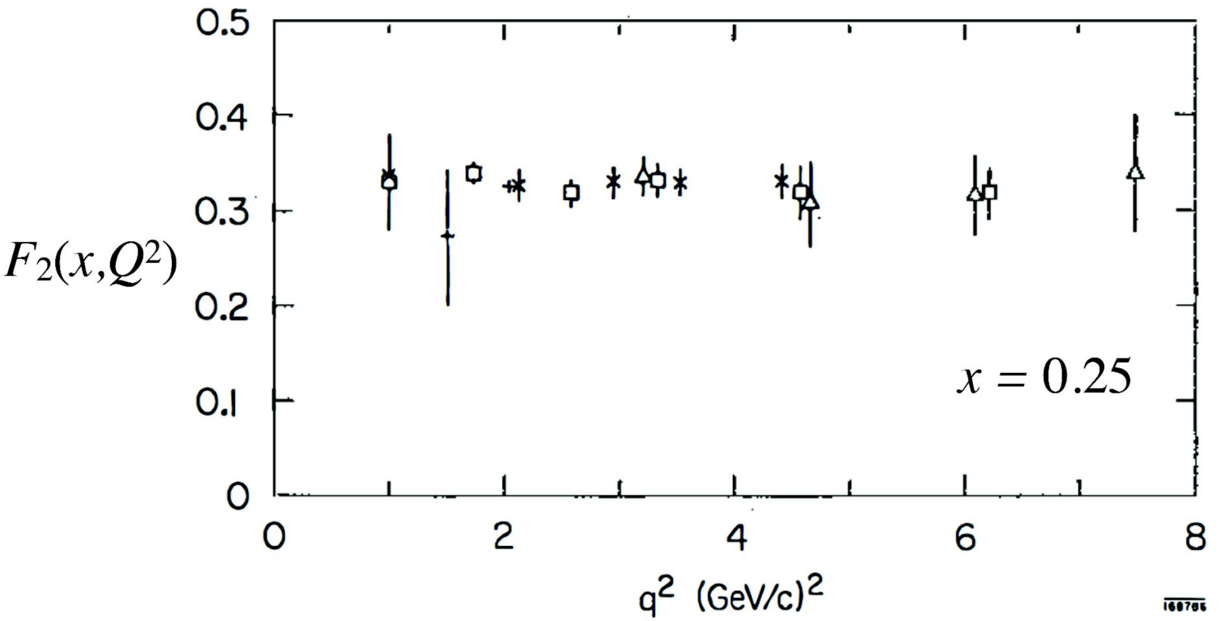


Deviations mean that there is something that we don't know!

First indication of a unified EW force → Tomorrow's lecture

DIS experiment additional results: asymptotic freedom

J.T. Friedman, H.W. Kendall, Ann. Rev. Nucl. Sci. 22, 203 (1972)



Scaling!

Independence of the structure functions of Q^2: $F_i(x,Q^2) = F_i(x)$

J.D. Bjørken predicted scaling for $Q^2 \to \infty$ as x stays fixed.
Scaling is obtained using Gell-Mann's current algebra in the quark model.

Scattering from point-like constituents of the proton!

And at high energies, they behave like free particles