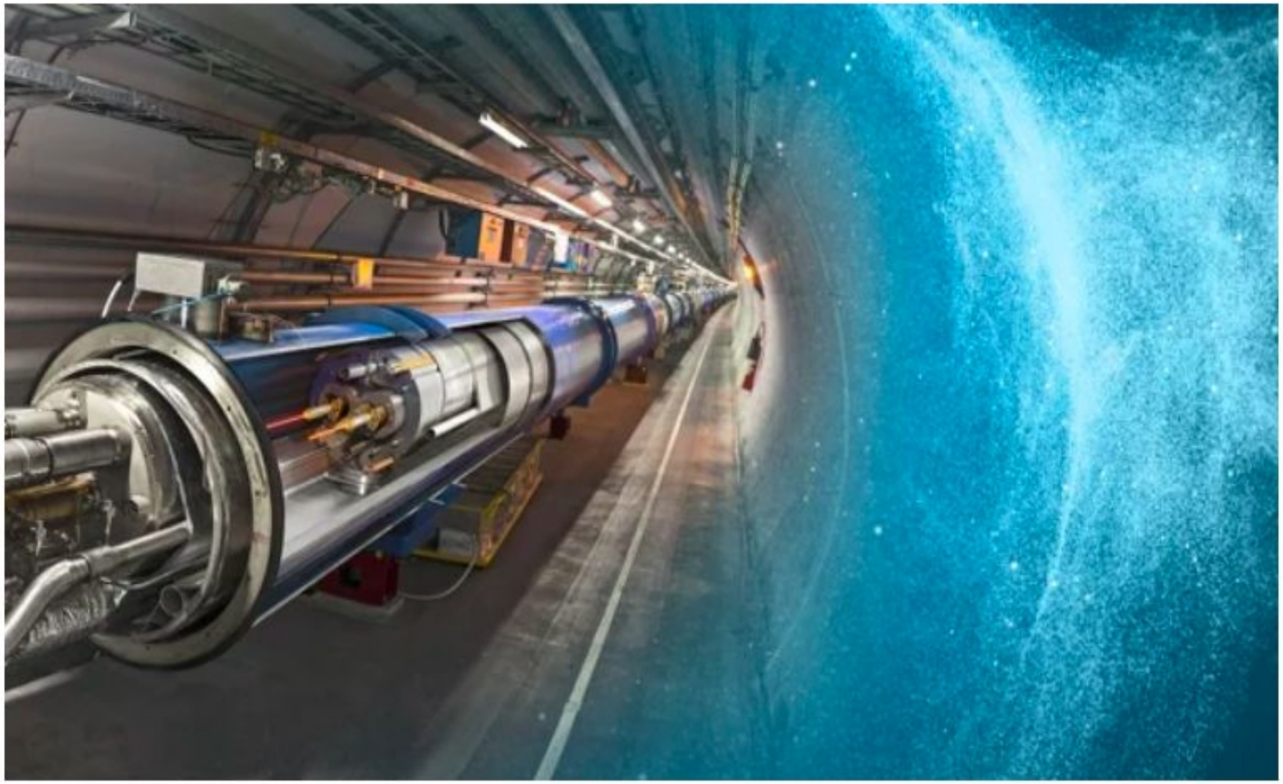
Introduction to the Standard Model

Summer Student Lecture 2025 – Part III



Clara Leitgeb

Deutsches Elektronen Synchrotron

11th-13th August 2025

HELMHOLTZ ASSOCIATION

Many thanks to Thorsten Kuhl and Alvaro Lopez Solis for their lectures and help

DESY

Content

- 0) Introduction
  - What is the Standard Model?
  - Coupling constants, masses and charges
  - Units and scales

- 1) Interactions
  - Relativistic kinematics
  - Symmetries and conserved quantities
  - Feynman diagrams
  - Running couplings and masses

- 2)Quantum electrodynamics
  - Test of QED: Magnetic momentum of the muon
  - Test of QED: High energy colliders

Content

- 3) Strong Interaction: Quantum-Chromodynamics
  - A short history of hadrons and quarks
  - DIS and gluons
  - QCD and its properties

- **4) Electroweak interactions**
  - **History of the weak interaction : β-decays**
  - **Parity violation**
  - **CP-violation**
  - **GSW mechanism and CKM**
  - **Experimental verification**

- **5) The Higgs**
  - **Why is it necessary ?**
  - **How was it found ?**

- **Beyond the Standard Model (brief)**

Weak interaction

A little bit of history: the β⁻ decays and radiation

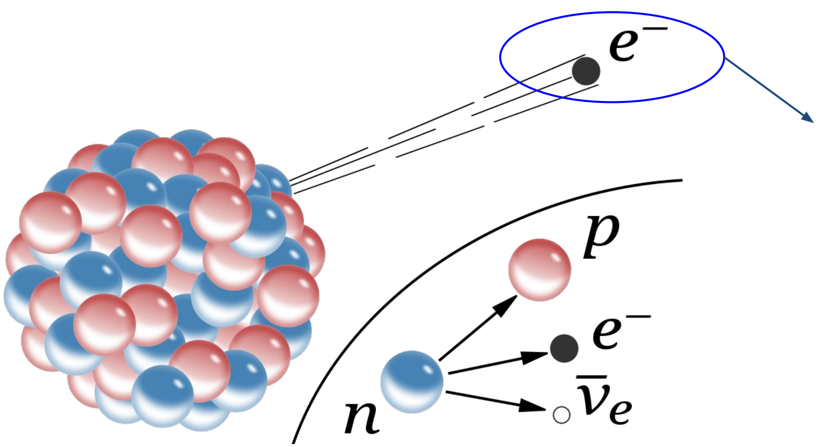
> Discovered with radioactivity. Initially, only observed that nuclei emitted one electron and the atomic number was unchanged.

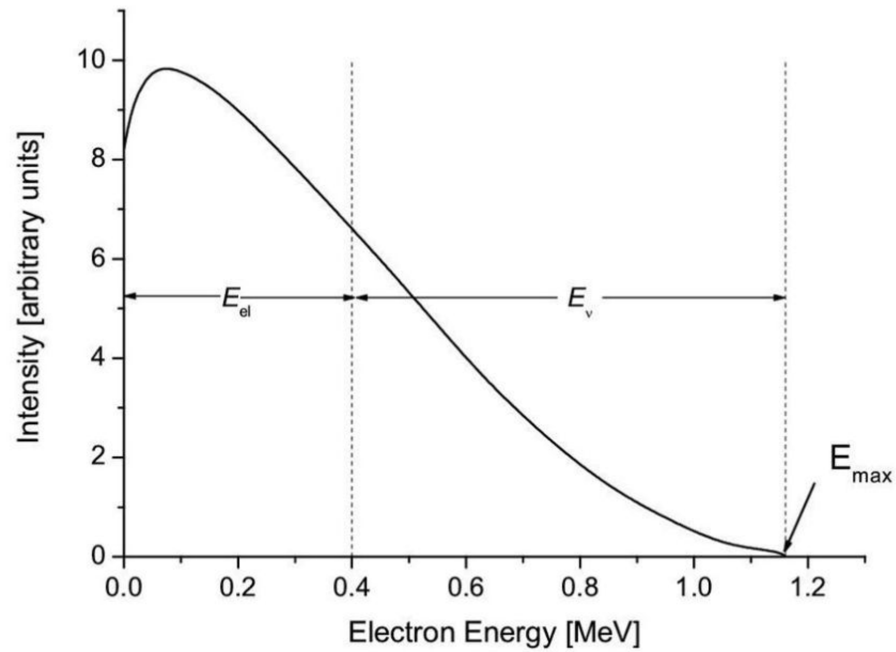
> Puzzling at the time:

- Spin of the nuclei unchanged or integer change → electron with spin ½. How ?
- Energy conservation: if only electron is emitted, energy should have a defined value → But continuous

> Pauli's proposal: neutrino

Detected in 1956 (Cowan,Reines)

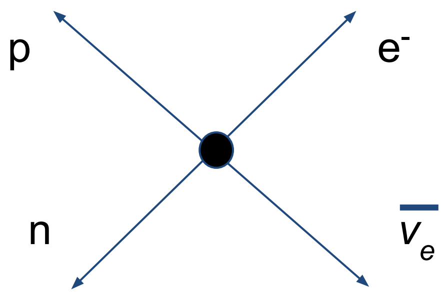




A little bit of history: Weak interactions

- β⁻ decays usually have a long lifetime (e.g. isolated neutron having a half life of 10 mins)
  - Lifetime depends on the probability of the decay to happen
  - Probability of decay depends on interaction strength → Weak interaction !
- Before full QFT developed, Fermi theory proposed as explanation of beta-decay

  → four point interaction



$$\mathcal{M} = \frac{G_F}{\sqrt{2}}(\bar{\psi}_p \gamma^\mu \psi_n)(\bar{\psi}_e \gamma_\mu \psi_{\bar{\nu}}),$$

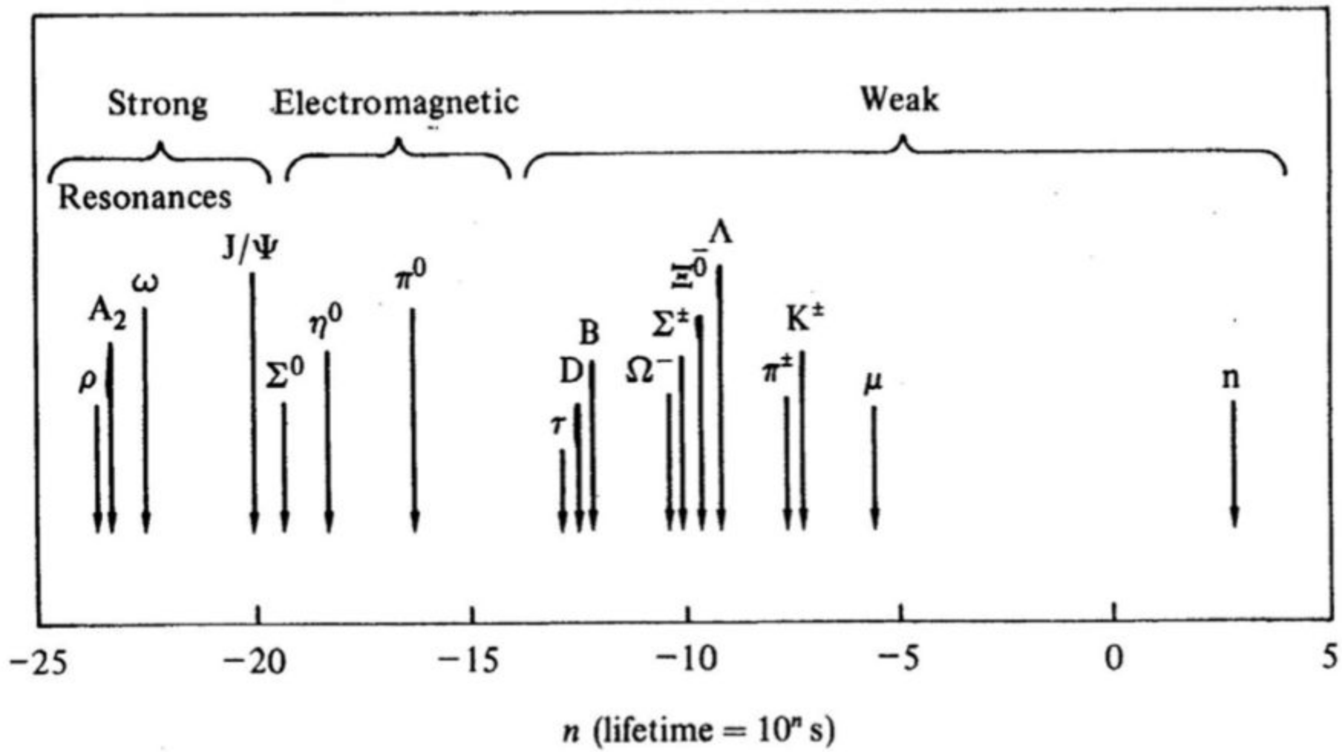
$G_F = 1.6637 \times 10^{-5}\ \mathrm{GeV}^{-2}$
measured from lifetime of muon

- Fermi's theory successfully described decays but incomplete.

Weak interaction decays started to show strange behaviours w.r.t electromagnetic and strong interactions

Hadrons and weak interactions

- Yesterday we saw the appearances of hadrons in the 50's and 60's. At the same time, these hadrons are not stable and decay
- Lifetimes very different.



Hadrons and weak interactions

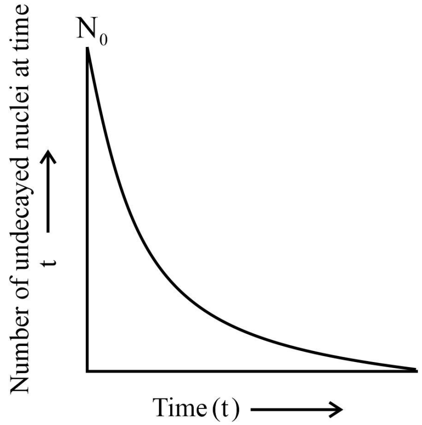
> The probability of finding a particle after a certain time is given by:

$$\psi(t) \propto e^{-iMt} e^{-\Gamma t/2} = e^{-i(M - i\Gamma/2)t},$$

$$P(t) = |\psi(t)|^2 \propto e^{-\Gamma t}$$

$$\Gamma = 2\Delta E = \frac{\hbar}{\tau}$$

Decay width



Decay width proportional to the amplitude of the process to happen

$$d\Gamma = \frac{(2\pi)^4 \delta^{(4)}(P_f - p_R)}{2E_R} \overline{|\mathcal{M}_{fi}|^2} \prod_{j=f} \frac{d^3 p_j}{2E_j (2\pi)^3}.$$

- More likely process (e.g strong)→ Larger decay width → shorter lifetime
- Less likely process (e.g weak interaction) → Smaller decay width → larger lifetime

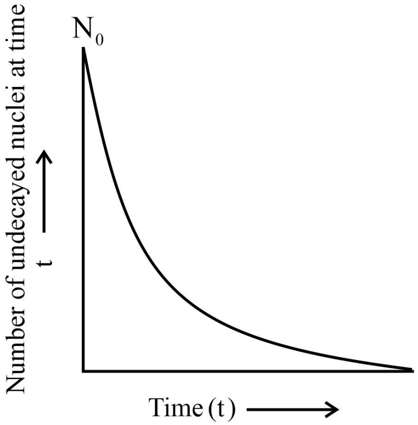
> The probability of finding a particle after a certain time is given by:

$$\psi(t) \propto e^{-iMt} e^{-\Gamma t/2} = e^{-i(M - i\Gamma/2)t},$$

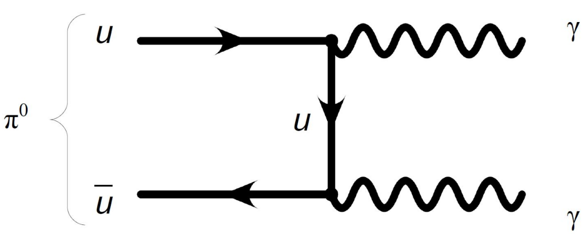
$$P(t) = |\psi(t)|^2 \propto e^{-\Gamma t}$$

$$\Gamma = 2\Delta E = \frac{\hbar}{\tau}$$

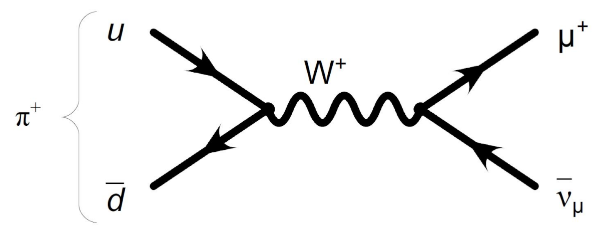
Decay width



Decay width proportional to the amplitude of the process to happen



electromagnetic interaction



weak interaction

Mean life $\tau = (8.52 \pm 0.18) \times 10^{-17}$ s

Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s

Reminder 1st lecture

- From Quantum mechanics: Symmetry connected to conserved quantity
- Different interactions conserve different quantities

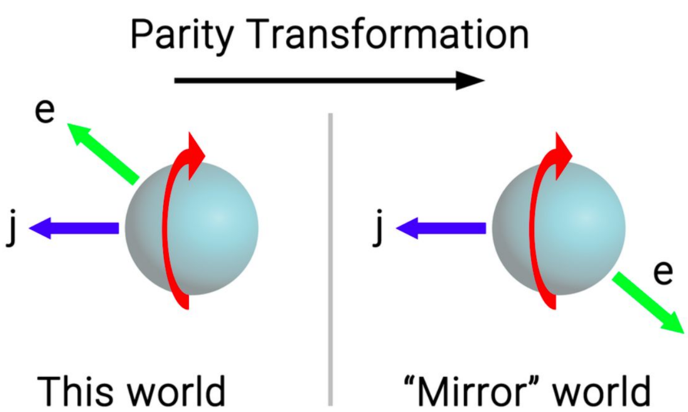
quantity	interaction			invariance
	strong	elm.	weak	
energy	yes	yes	yes	translation in time
momentum	yes	yes	yes	translation in space
angular momentum	yes	yes	yes	rotation in space
P (parity)	yes	yes	no	coordinate inversion
C (charge parity)	yes	yes	no	charge conjugation (particle ↔ anti-particle)
T (time parity)	yes	yes	no	time inversion
CPT	yes	yes	yes	
lepton number	yes	yes	yes	
baryon number	yes	yes	yes	
isospin	yes	no	no	

+ Flavour : conserved by strong and electromagnetic. Not by Weak interaction.

Reminder 1st lecture

Parity

Mirror the coordinates of the particle.
Changes sign of momentum, coordinates
Spin doesn't change sign.



Time reversal

If I revert the time, would the interaction take place in the same way ?

Charge conjugation

Change a particle by its anti-particle

And combinations: CP, CPT ?

The τ-θ puzzle (1956)

- In the 50's, two particles were observed: τ^+ and θ^+.τ-θ puzzle

$$\tau^+ \rightarrow \pi^+\pi^+\pi^- \qquad P(\tau^+) = P(\pi^+\pi^+\pi^-) = -1$$

$$\theta^+ \rightarrow \pi^+\pi^0 \qquad P(\theta^+) = P(\pi^+\pi^0) = +1$$

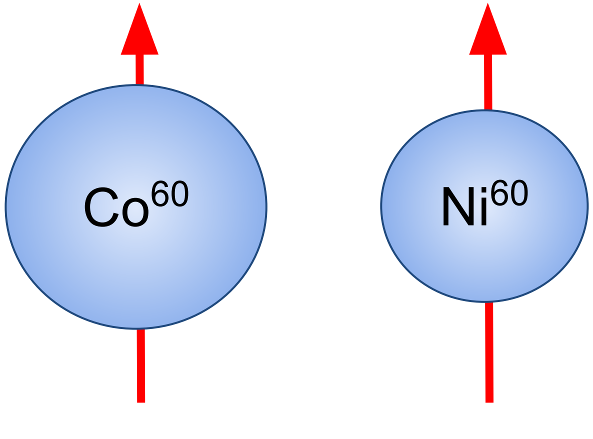
- Decaying into different states and different parity …. Same mass, lifetime, charge, spin ….

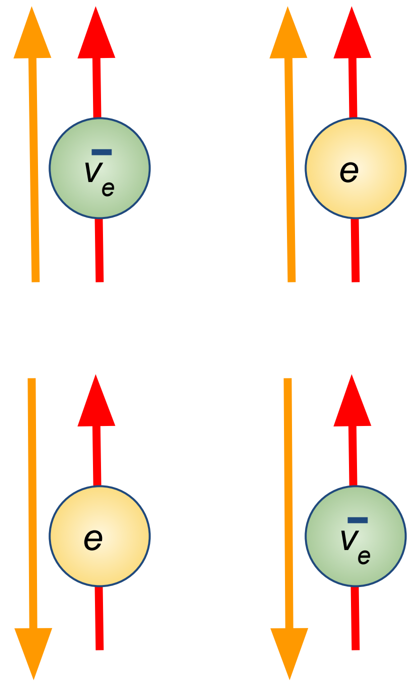
- Proposal that both particles are actually the same particle (K^+) but parity violation in the interaction.

β-decays of Co^{60}: Parity violation of the weak interaction

Co^{60} atoms aligned with magnetic field

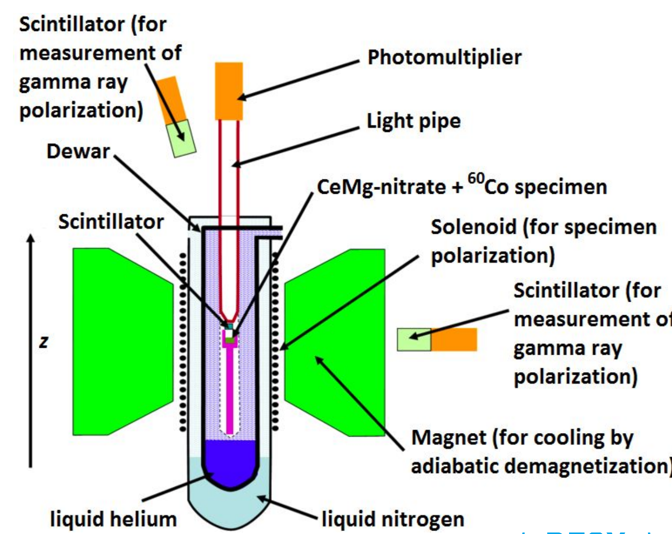
$$^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$





Experiment of Madame Wu

Check decays of Co^{60} into Ni^{60}. Align Co^{60} using a uniform magnetic field and reduce thermal motion with low-temperature experiment. 2γ from Ni* de-excitation, isotropic. Anisotropy would show how Co^{60} was aligned. If parity symmetry, no preferred direction of the electron from this decay.

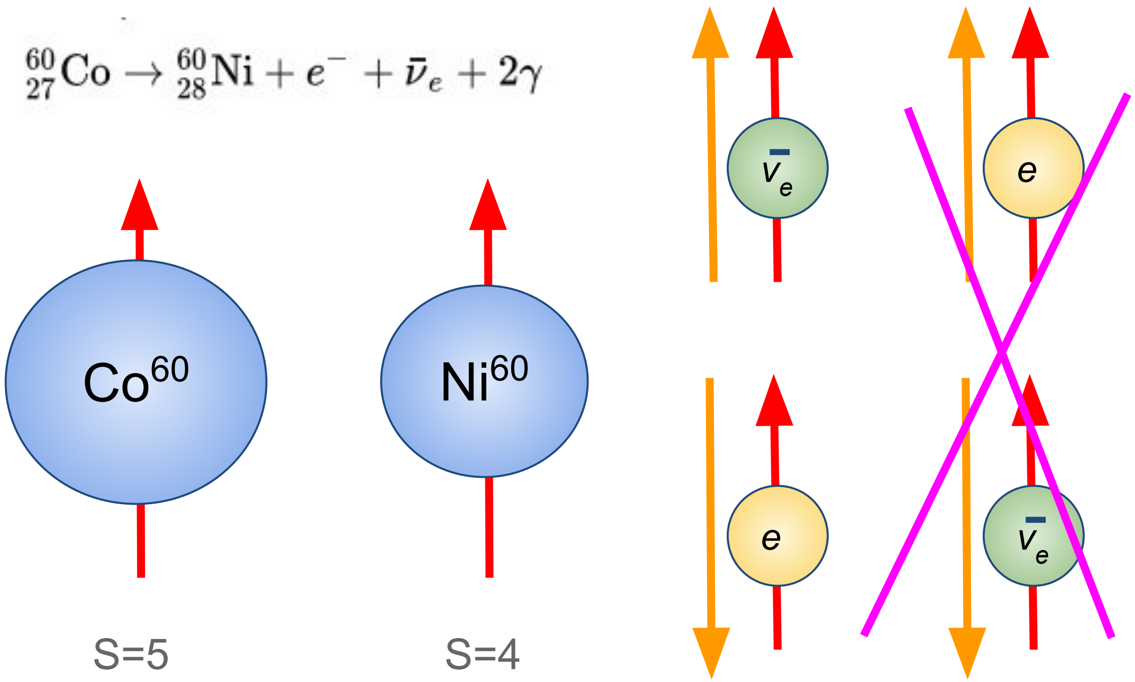


β-decays of Co^{60}: Parity violation of the weak interaction

Co^{60} atoms aligned with magnetic field

If parity symmetry, no preferred direction of flight for the electron

$$^{60}_{27}\mathrm{Co} \rightarrow ^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$



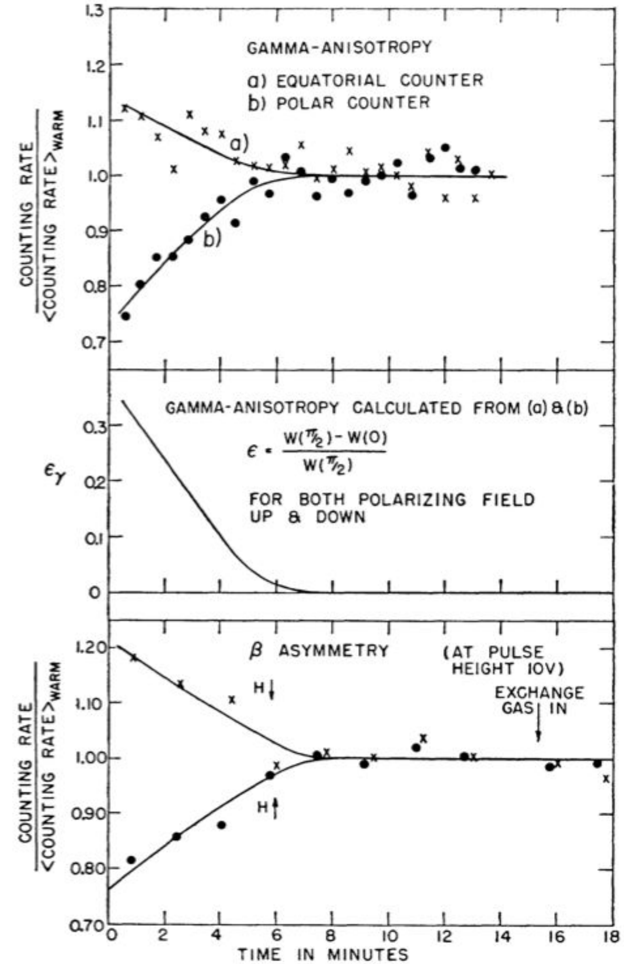


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

Observed that electrons are preferentially emitted in opposite direction to nucleus spin. At the same time, C-symmetry violated in
https://journals.aps.org/pr/abstract/10.1103/PhysRev.105.1415

Weak interaction violates parity and C-symmetry.. What about CP ?

Further problem: K^0_S and K^0_L and CP-violation

Experiment of Christenson-Fitch-Cronin-Turlay: link

Two neutral kaons (meson with one strange quark) were known with same mass and properties but two different lifetimes and decay types: K^0_S and K^0_L

K^0_S, $\tau = 9.0 \cdot 10^{-11}$ s ($c\tau = 2.7$ cm)

K^0_L, $\tau = 5.1 \cdot 10^{-8}$ s ($c\tau = 15$ m)

$$K^0_S \to \pi^0\pi^0/\pi^+\pi^- \quad \mathrm{CP} = +1$$

$$K^0_L \to \pi^0\pi^0\pi^0/\pi^+\pi^-\pi^0 \quad \mathrm{CP} = -1$$

- Lifetimes typical of weak interaction
- Experiment with a beam of neutral kaons. If beam long enough, enriched with K^0_L.
- If only 3π decays, no CP violation.

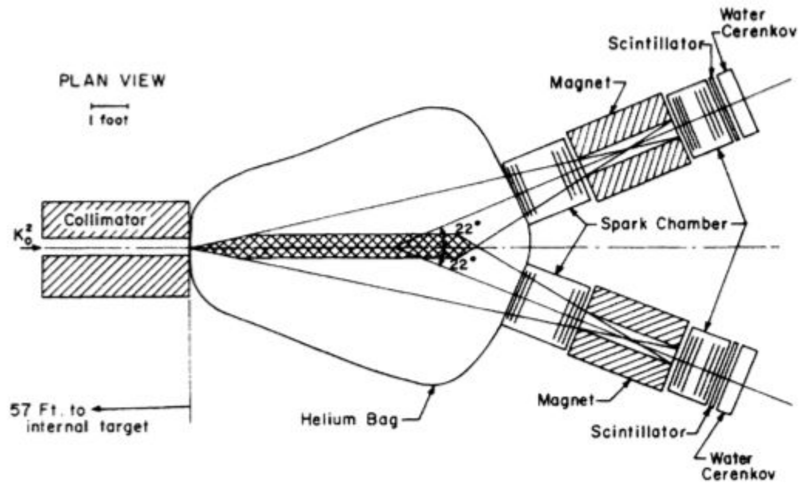


FIG. 1. Plan view of the detector arrangement.

Further problem: K^0_S and K^0_L and CP-violation

Experiment of Christenson-Fitch-Cronin-Turlay: link

Two neutral kaons (meson with one strange quark) were known with same mass and properties but two different lifetimes and decay types: K^0_S and K^0_L

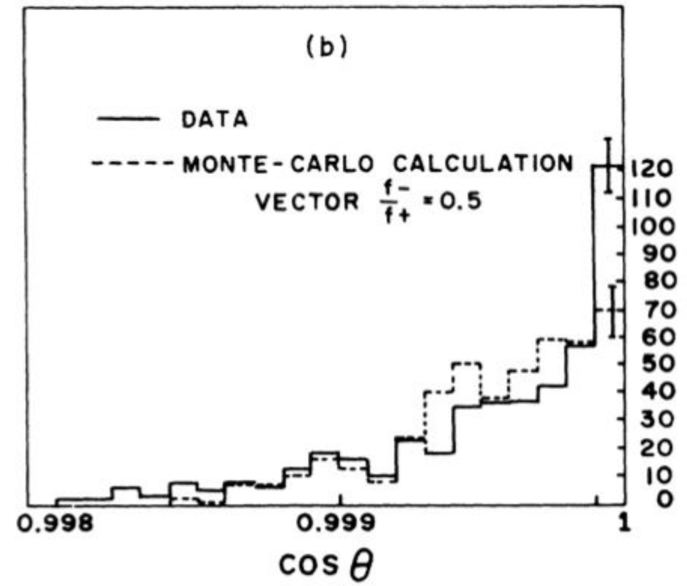
K^0_S, $\tau = 9.0 \cdot 10^{-11}$ s ($c\tau = 2.7$ cm)

K^0_L, $\tau = 5.1 \cdot 10^{-8}$ s ($c\tau = 15$ m)

$$K^0_S \to \pi^0\pi^0/\pi^+\pi^- \quad CP = +1$$

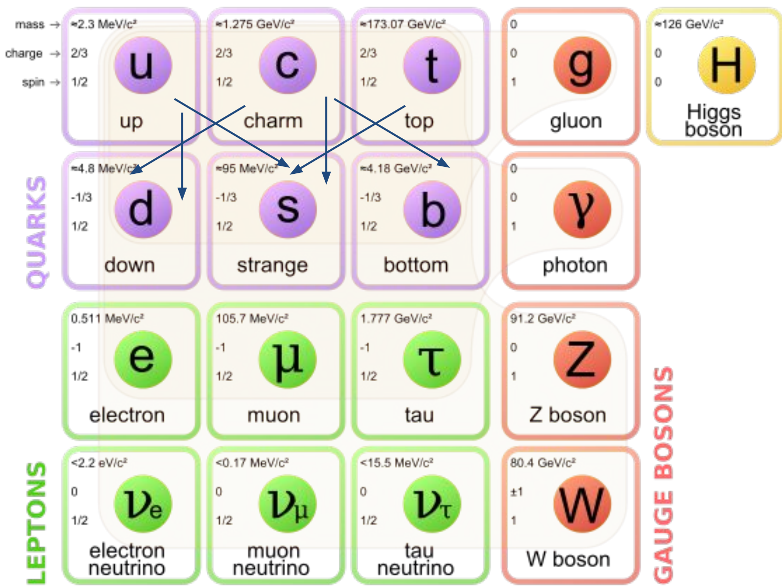
$$K^0_L \to \pi^0\pi^0\pi^0/\pi^+\pi^-\pi^0 \quad CP = -1$$

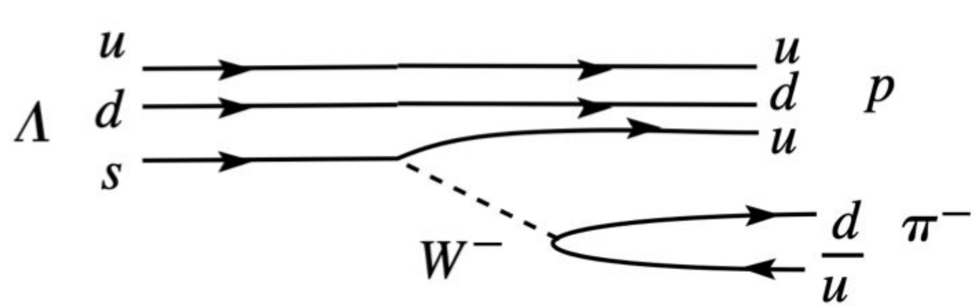
- Lifetimes typical of weak interaction
- Experiment with a beam of neutral kaons. If beam long enough, enriched with K^0_L.
- Observed more events than expected and associated to production of 2π! → CP-violation !



Flavour, quarks and weak interaction: CKM matrix

- In the 60’s, together with parity and CP-violation, observed:
  - $u - d$, $e^- - \nu_e$, $\mu^- - \nu_\mu$ transitions with weak interaction had same probability to happen
  - Charged currents do not seem to conserve flavour
  - $\Delta S = 1$ transitions had ¼ of probability of occuring than $\Delta S = 0$





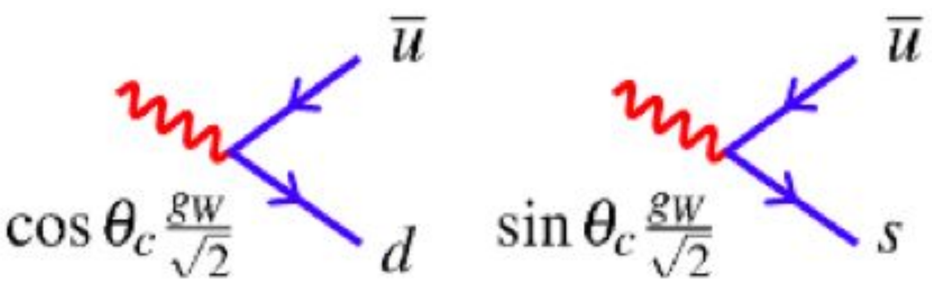
Flavour, quarks and weak interaction: Cabibbo angle

- Reason for the change in flavour → weak interaction eigenstates are not mass eigenstates → mixture of quarks
- Nicola Cabibbo introduced mixing angle

Weak eigenstate

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Mass eigenstate

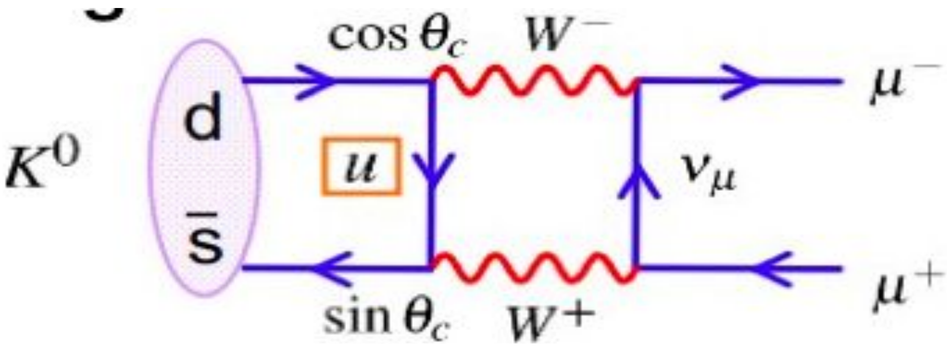


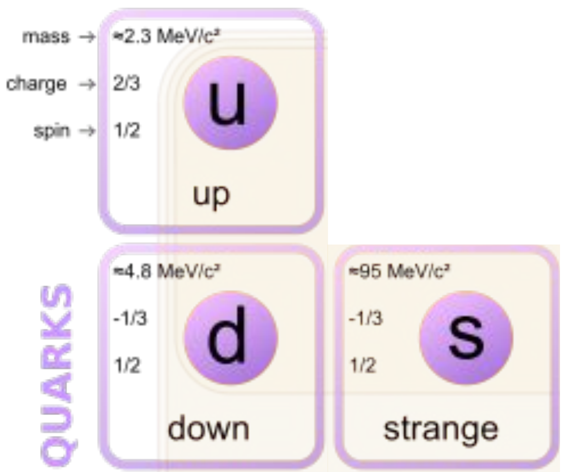
- Interaction of the u-quarks with the d-quark and s-quark dependent on the θ_C.
- Allows change in flavour and generations.
- Small angle allows to explain the different probability in ΔS transitions

Flavour, quarks and weak interaction: GIM mechanism

- At the time, calculated probability of some processes had large discrepancies with observations → Flavour changing neutral currents

$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c$$





FCNC are experimentally suppressed

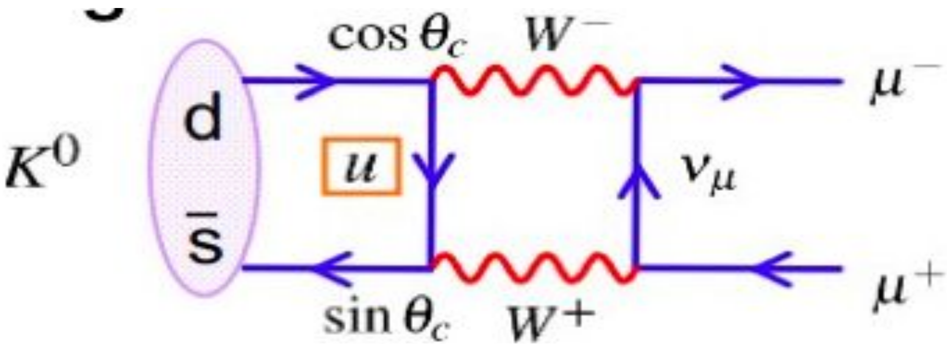
- We don't observe change in the flavour (e.g. $\Delta S = 1$) unless W^- or W^+.

$Br(K^0 \rightarrow \mu^+\mu^-)$ (exp.) $< 10^{-9}$
$Br(K^0 \rightarrow \mu^+\mu^-)$ (theo) $< 10^{-4}$

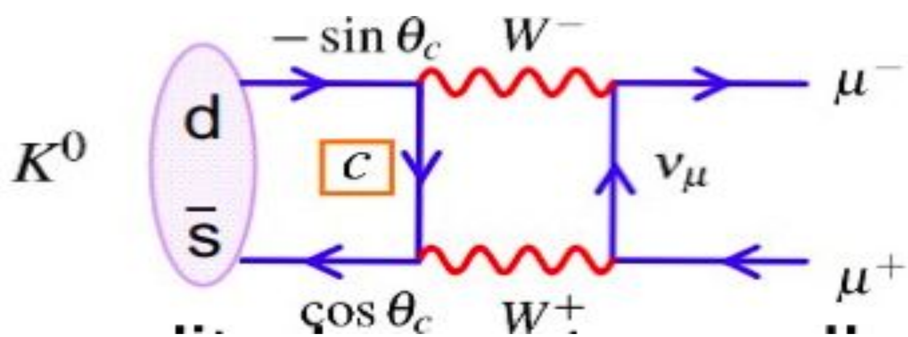
Flavour, quarks and weak interaction: GIM mechanism

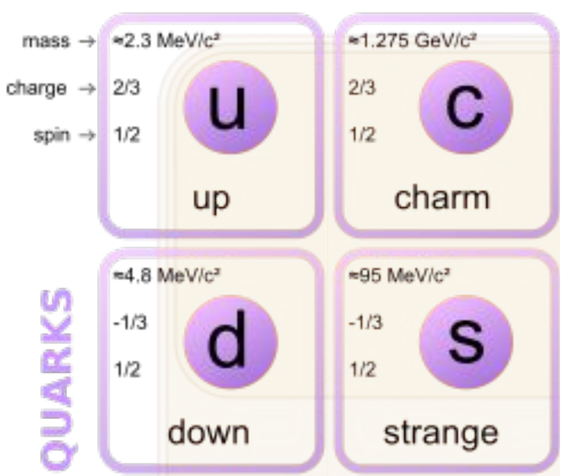
- GIM mechanism: introduction of the charm-quark, which suppresses flavour changing neutral currents.

$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c$$



$$M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c$$





c-quark discovered in 1974

Br ($K^0 \rightarrow \mu^+\mu^-$) (exp.) < 10^{-9}
Br ($K^0 \rightarrow \mu^+\mu^-$) (theo) < 10^{-9}

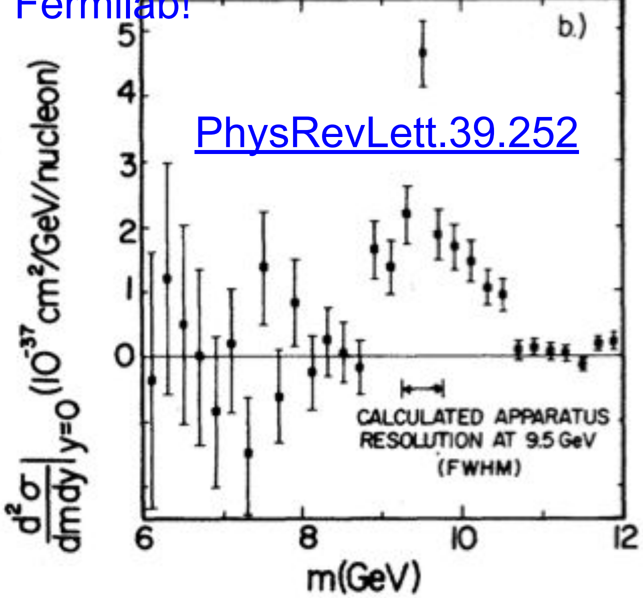
Flavour and weak interaction: CKM matrix

- Expansion of GIM mechanism to include CP-violating effects in weak interactions led to CKM matrix: Cabibbo-Kobayashi-Maskawa ability

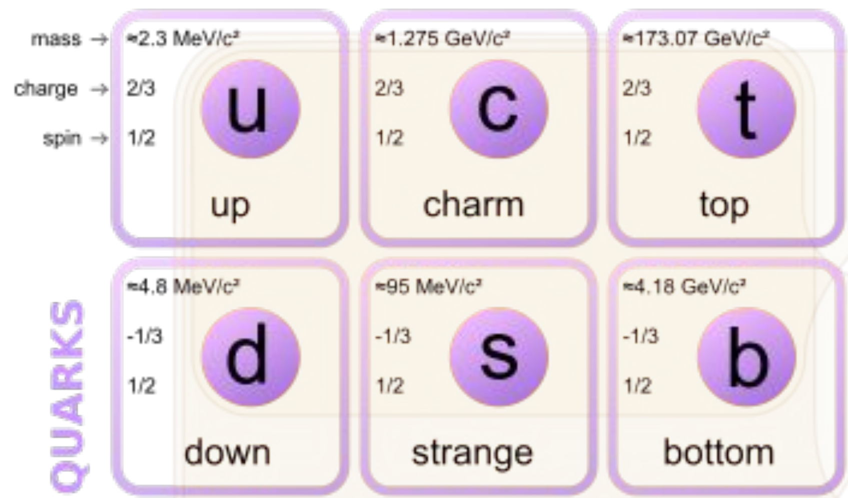
$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{\text{ud}} & V_{\text{us}} & V_{\text{ub}} \\ V_{\text{cd}} & V_{\text{cs}} & V_{\text{cb}} \\ V_{\text{td}} & V_{\text{ts}} & V_{\text{tb}} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Discovery of bottom-quark in 1977 at Fermilab!



PhysRevLett.39.252



Flavour and weak interaction: CKM matrix

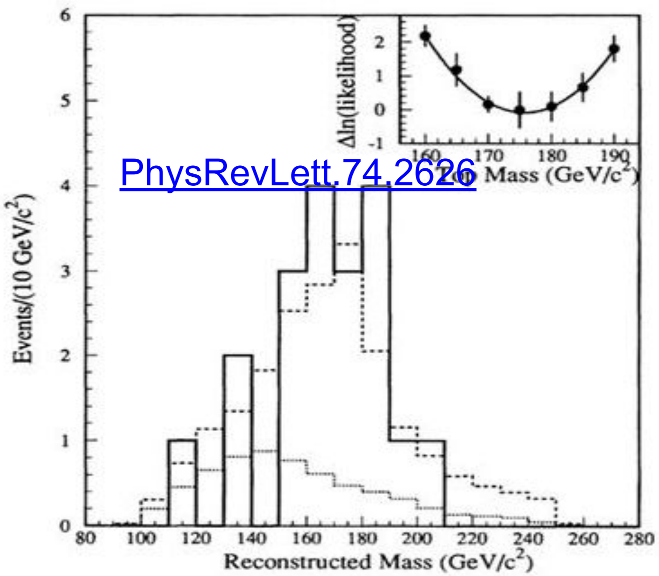
- Expansion of GIM mechanism to include CP-violating effects in weak interactions led to CKM matrix: Cabibbo-Kobayashi-Maskawa ability

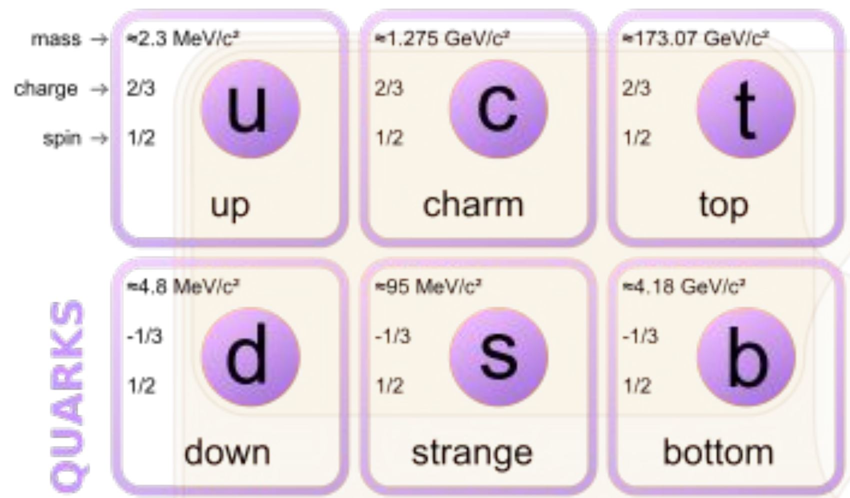
$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Discovery of the top-quark at Tevatron in 1995

PhysRevLett.74.2626

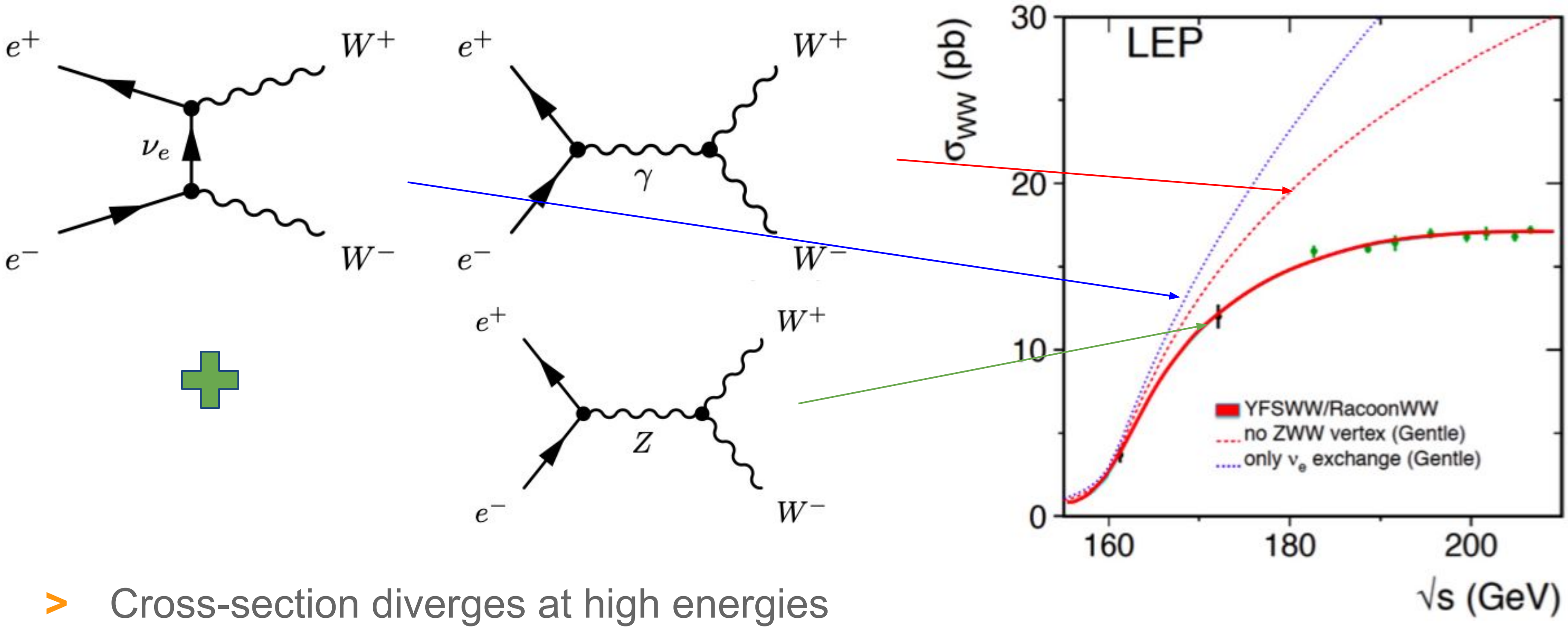




Electroweak unification

Towards a QFT of weak interactions → Electroweak

- Weak interactions present so far via a charge current (exchange of a charged mediator)
  - Mediators might be massive → W+,W- bosons
- W-boson charged → interaction with the photon. Consider ee → WW



- Cross-section diverges at high energies
- Cured if introducing a coupling to a neutral current
- Only possible if γ, W and Z boson couplings are related → Electroweak unification

Electroweak unification !!!

- Glashow, Salam and Weinberg proposed a gauge theory with two separate symmetry groups SU(2) x U(1)
- SU(2) : interactions of particles that have a weak isospin $I_{weak.}$ Coupling g
  - Three bosons mediating this force : W_1, W_2, W_3
- U(1): interaction of particles that have an hypercharge Y. Coupling g'
  - One single boson mediating this force : B-boson

$$D_\mu \equiv \partial_\mu - i\,\frac{g'}{2}\,Y\,B_\mu - i\,\frac{g}{2}\,T_j\,W^j_\mu$$

$$\mathcal{L}_g = -\tfrac{1}{4} W_a^{\mu\nu} W^a_{\mu\nu} - \tfrac{1}{4} B^{\mu\nu} B_{\mu\nu},$$

Electroweak unification !!!

> Rearranging the fields, we find two charged gauge bosons and two neutral gauge bosons

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$$

→ > Charged gauge bosons mediating weak interaction

$$Z_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu)$$

→ > Neutral gauge boson mediating weak interaction → Never observed before !

$$A_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu)$$

→ >Identified with the photon

Weinberg angle

$$\begin{pmatrix} \gamma \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^0 \\ W^0 \end{pmatrix}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^{\,2}}}$$ and

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^{\,2}}}\,.$$

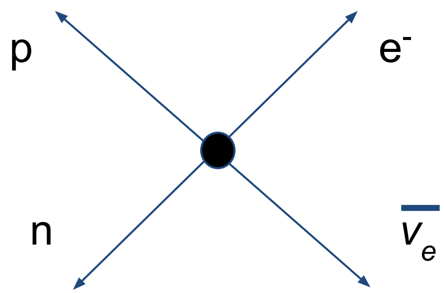
Electroweak unification !!!

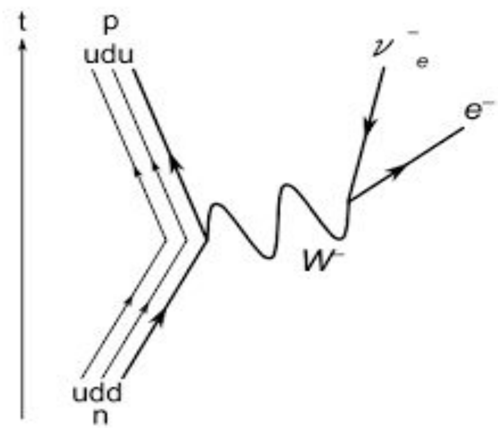
> Found relations between the couplings of the different gauge bosons

$$Q = T_3 + \frac{1}{2} Y_{W},$$

electric charge of a particle is related to particle's hypercharge and isospin.

Fermi constant and EWK theory





$$\mathcal{M} = \frac{G_F}{\sqrt{2}} (\bar{\psi}_p \gamma^\mu \psi_n)(\bar{\psi}_e \gamma_\mu \psi_{\bar{\nu}}),$$

$$G_F^0 = \frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2}}{8} \frac{g^2}{M_W^2 c^4} = 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Electroweak interactions

Fermion family		Left-chiral fermions			Right-chiral fermions			
		Electric charge Q	Weak isospin T_3	Weak hyper-charge Y_W		Electric charge Q	Weak isospin T_3	Weak hyper-charge Y_W
Leptons	ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	−1	No interaction, if they even exist			0
	e^-, μ^-, τ^-	−1	$-\frac{1}{2}$	−1	e^-_R, μ^-_R, τ^-_R	−1	0	−2
Quarks	u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$	u_R, c_R, t_R	$+\frac{2}{3}$	0	$+\frac{4}{3}$
	d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$	d_R, s_R, b_R	$-\frac{1}{3}$	0	$-\frac{2}{3}$

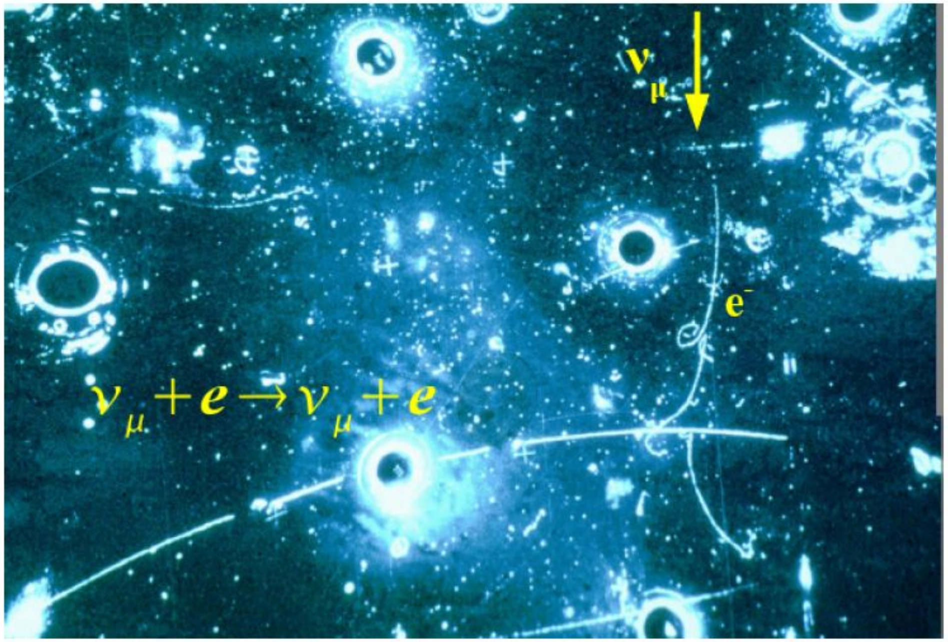
Interaction mediated	Boson	Electric charge Q	Weak isospin T_3	Weak hypercharge Y_W
Weak	$W^\pm$	±1	±1	0
	Z^0	0	0	0
Electromagnetic	γ^0	0	0	0

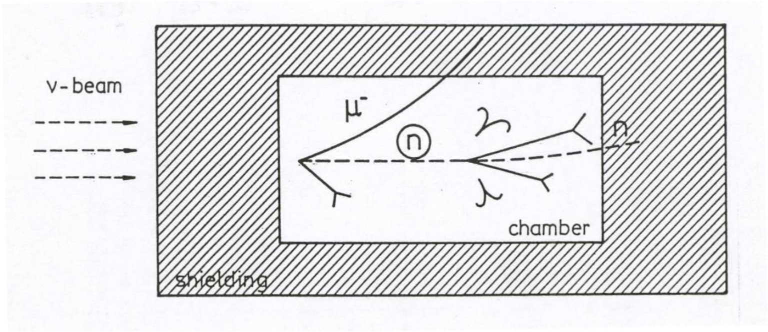
from wikipedia

Evidence of GSW validity: neutral weak interaction

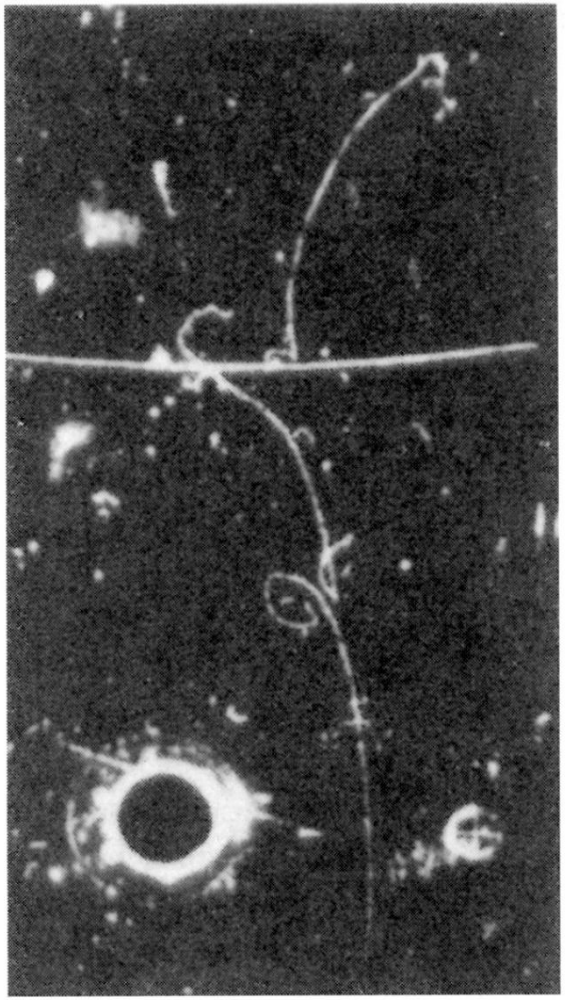
- Neutral current discovered in 1973 with *Gargamelle* at CERN by observing eν → eν.
- First evidence of neutral current in leptons

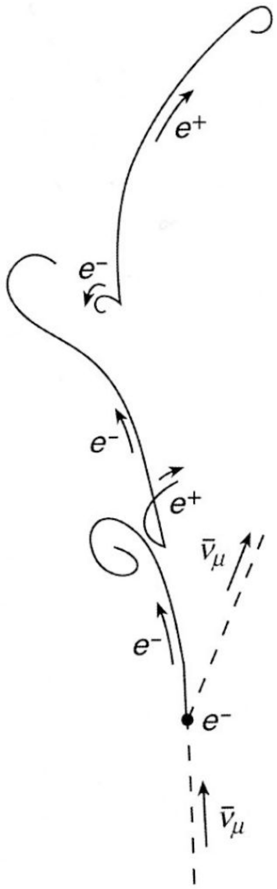
Confirmation of the existence of neutral weak currents !





Evidence of GSW validity: neutral weak interaction



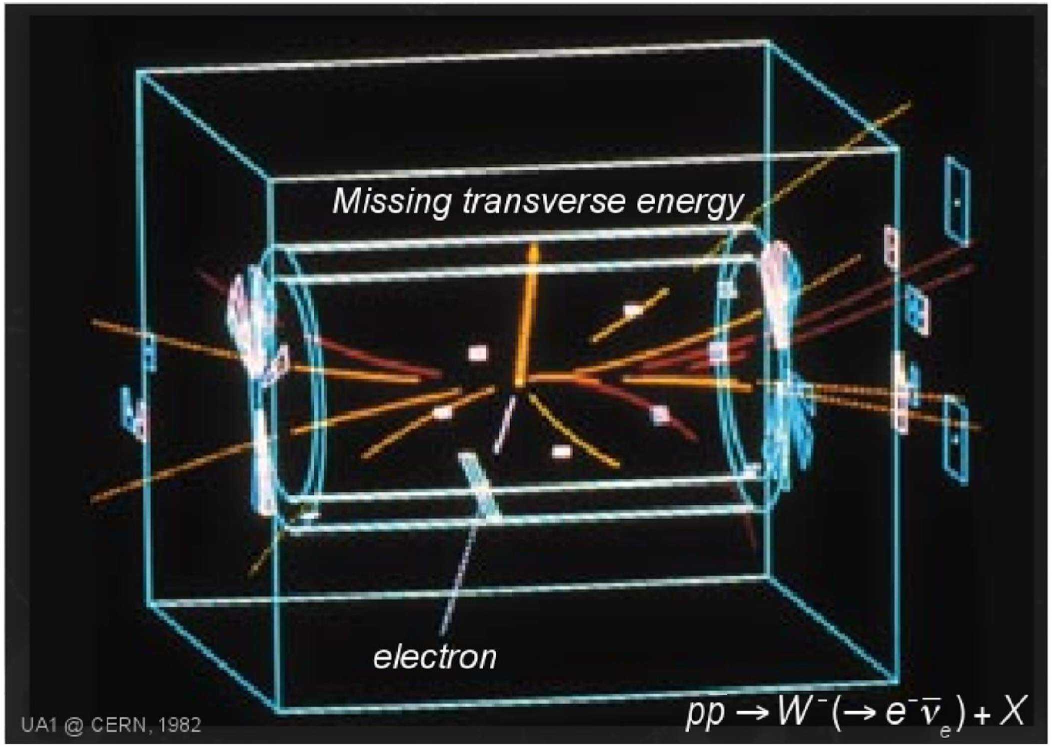


The first picture of a neutral weak process

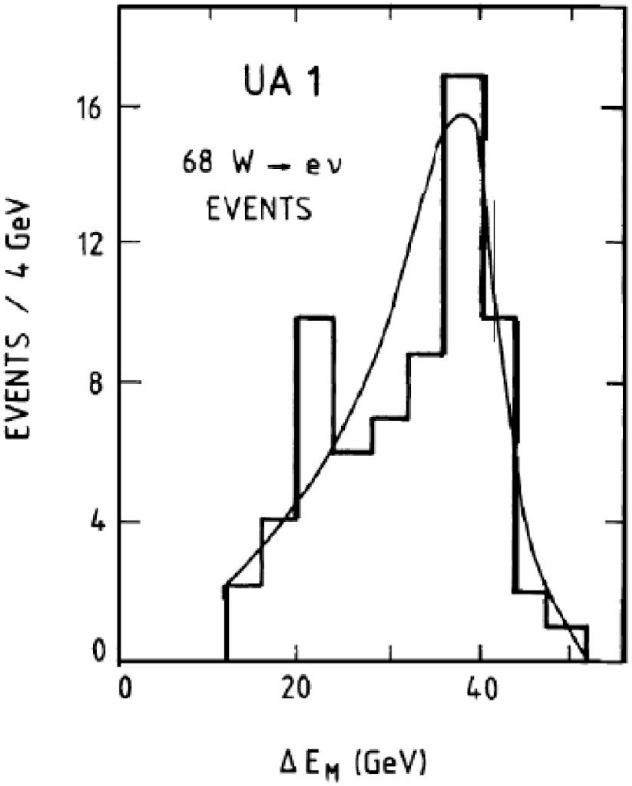
$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$.

The neutrino enters from below (leaving no track), and strikes an electron. which moves upwards, emitting two photons (visible via the e+e- pairs from subsequent conversions)

Evidence of GSW validity: Discovery of W boson



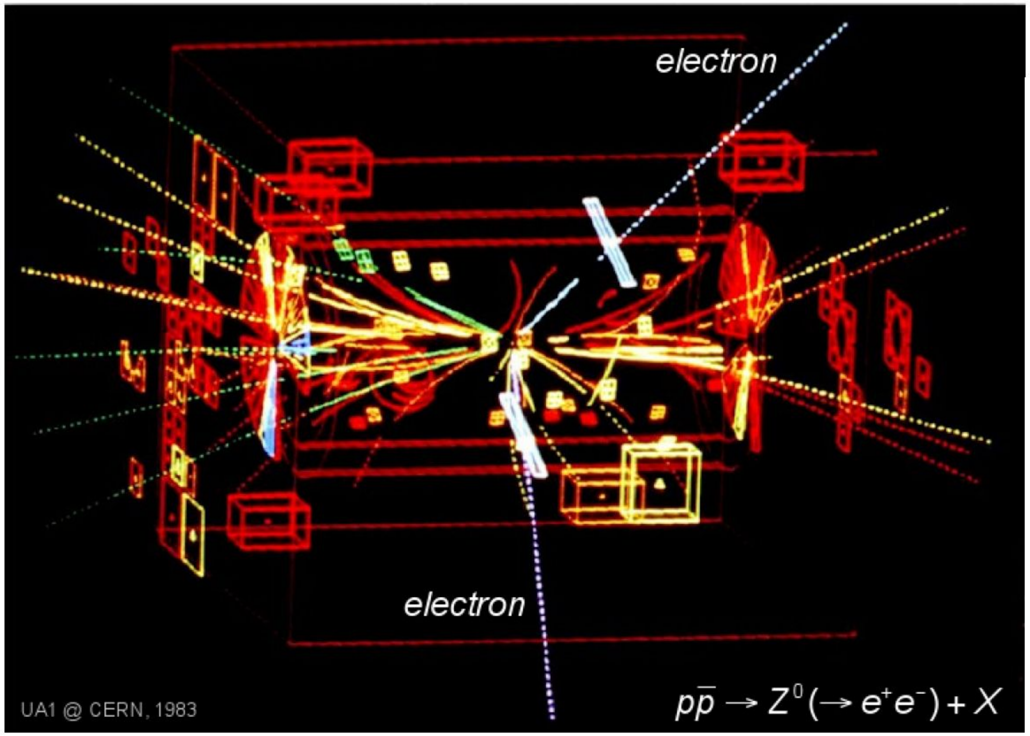
Missing transverse energy in events with E_e > 15 GeV

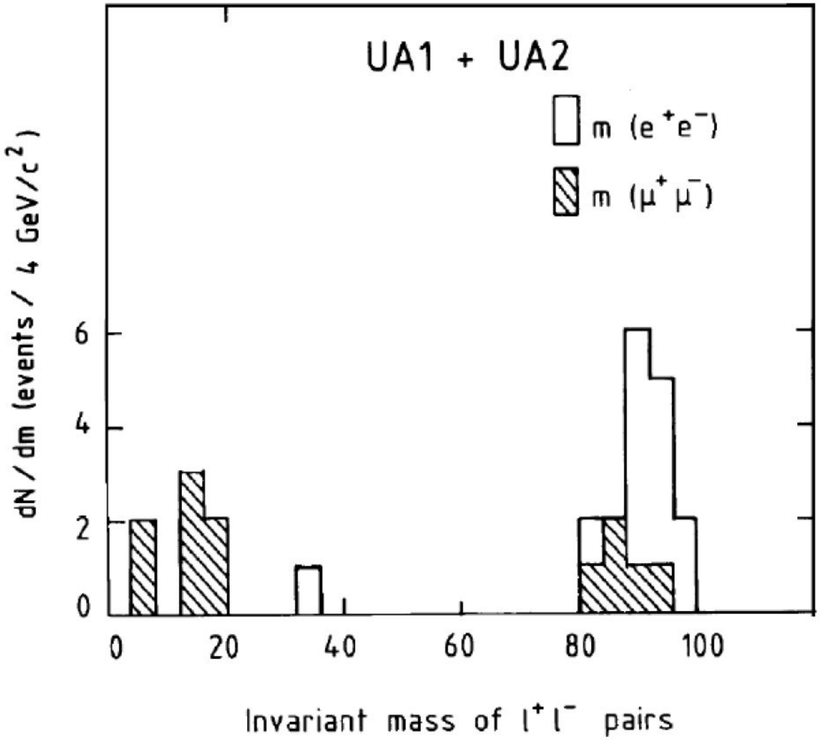


$$m_W = (80.9 \pm 1.5 \pm 2.4)\ \text{GeV}$$

C. Rubbia, Nobel Lecture, 1984

Evidence of GSW validity: Discovery of Z boson

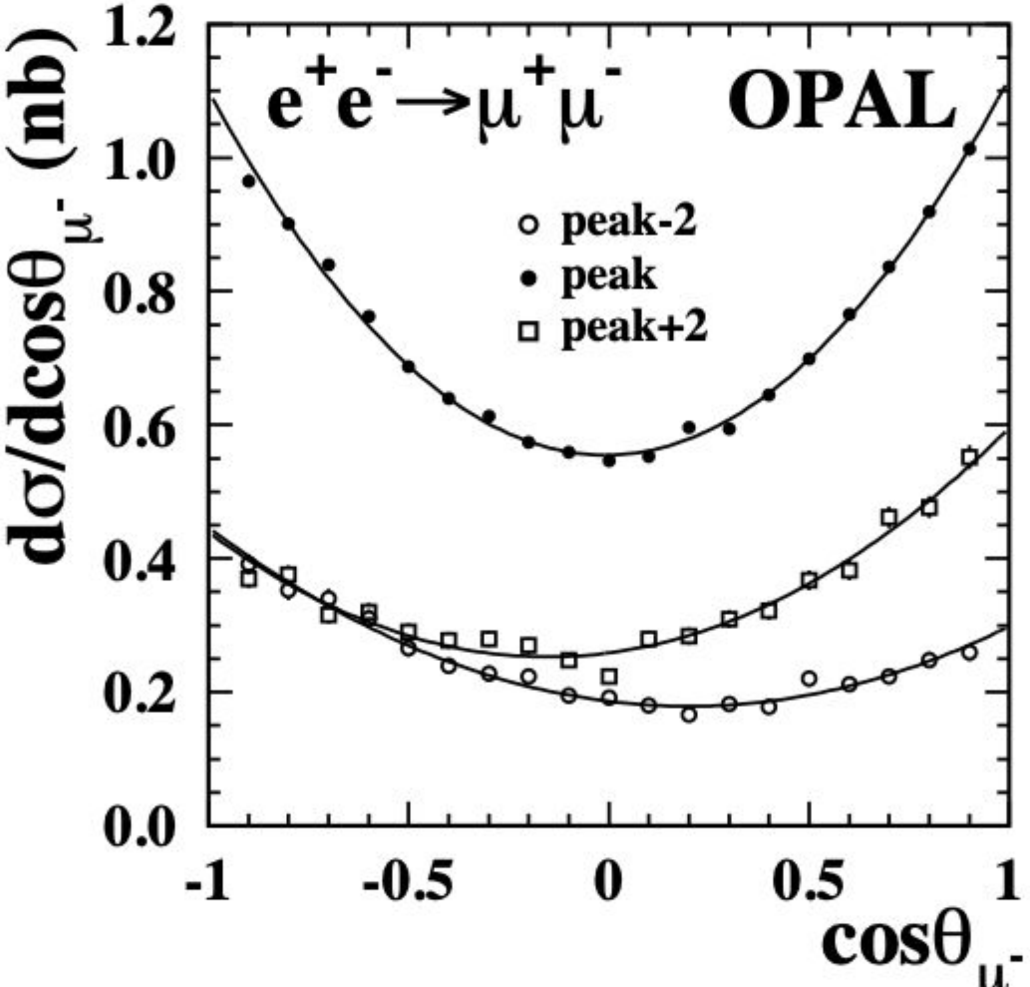


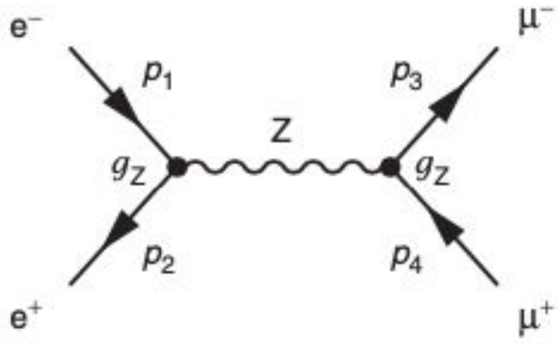


$$m_Z = (95.1 \pm 2.5)\ \text{GeV}$$

- 1983: first signals with 6 W → eν and 4 Z → ee events
- 1984: Nobel prize for C. Rubbia (UA1) and S. van der Meer

Evidence of GSW validity: Angular relations



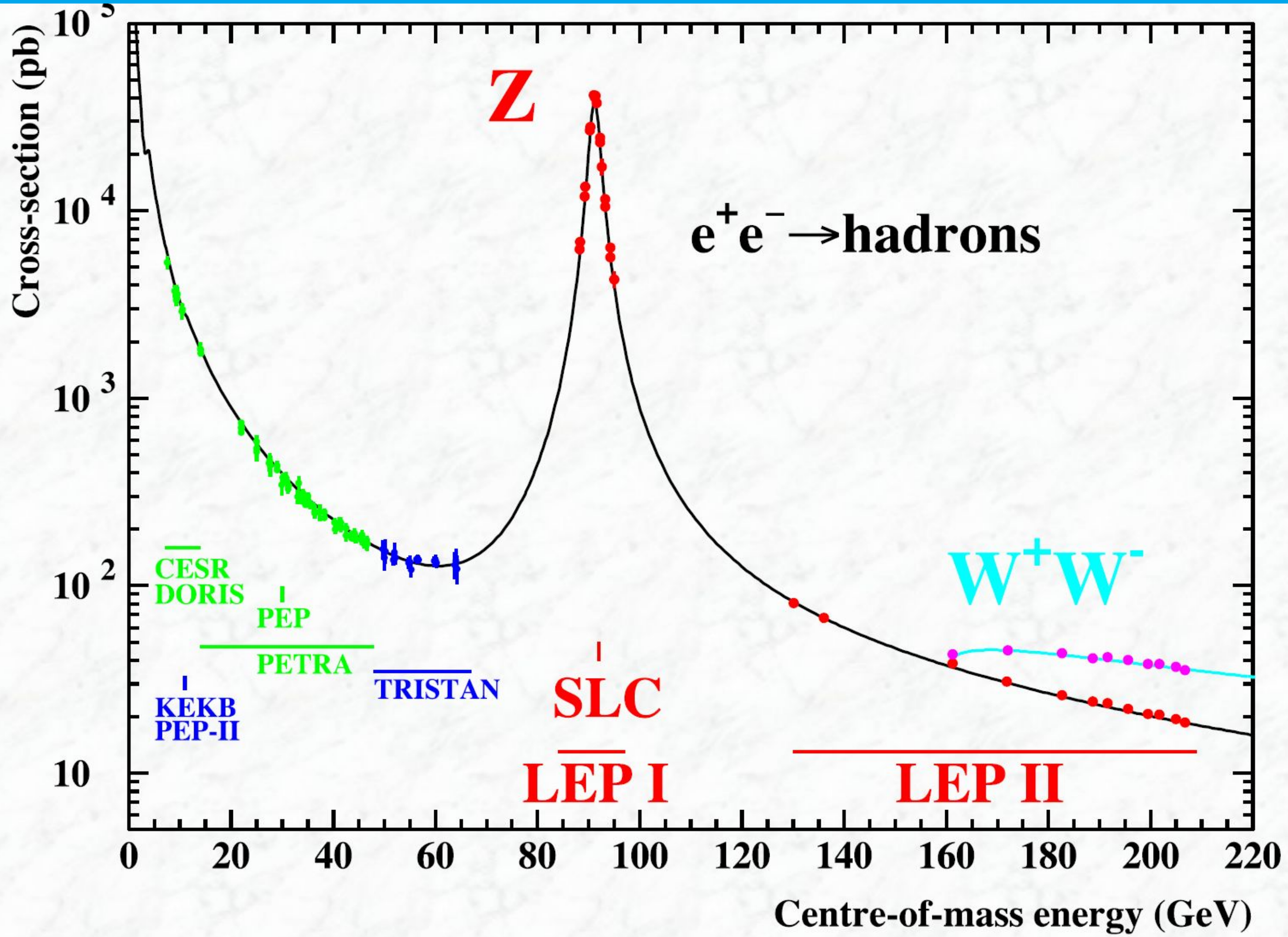


- Angular distributions changed by electroweak interactions
- Reason: V-A structure of neutral current (NC)

→ Forward-backward asymmetry

$$\frac{d\sigma}{d\Omega} \propto a(1 + \cos^2\theta) + 2b\cos\theta$$

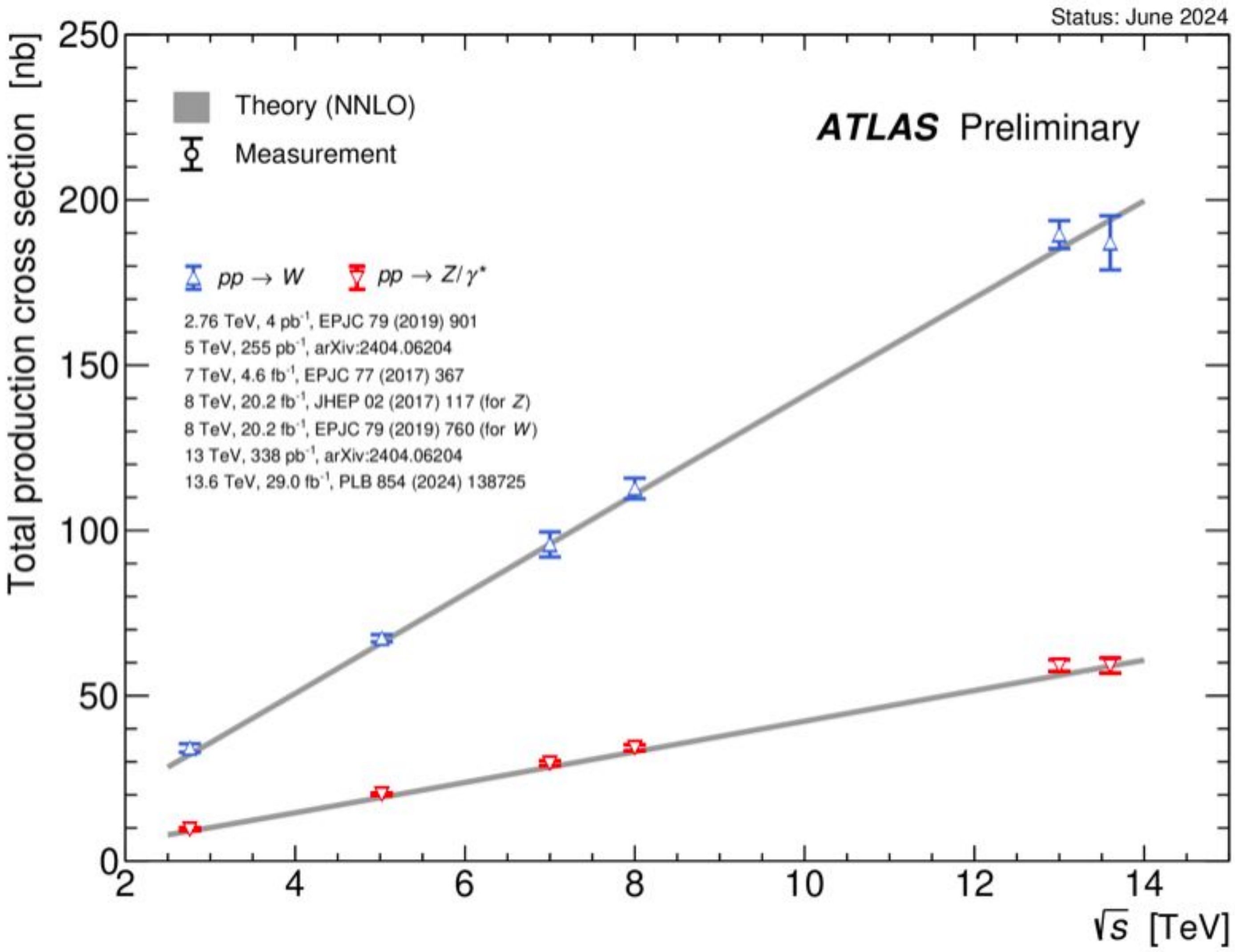
EWK tests: $\sigma(e^+e^- \rightarrow W/Z)$ production at LEP



Precision tests of the Z sector

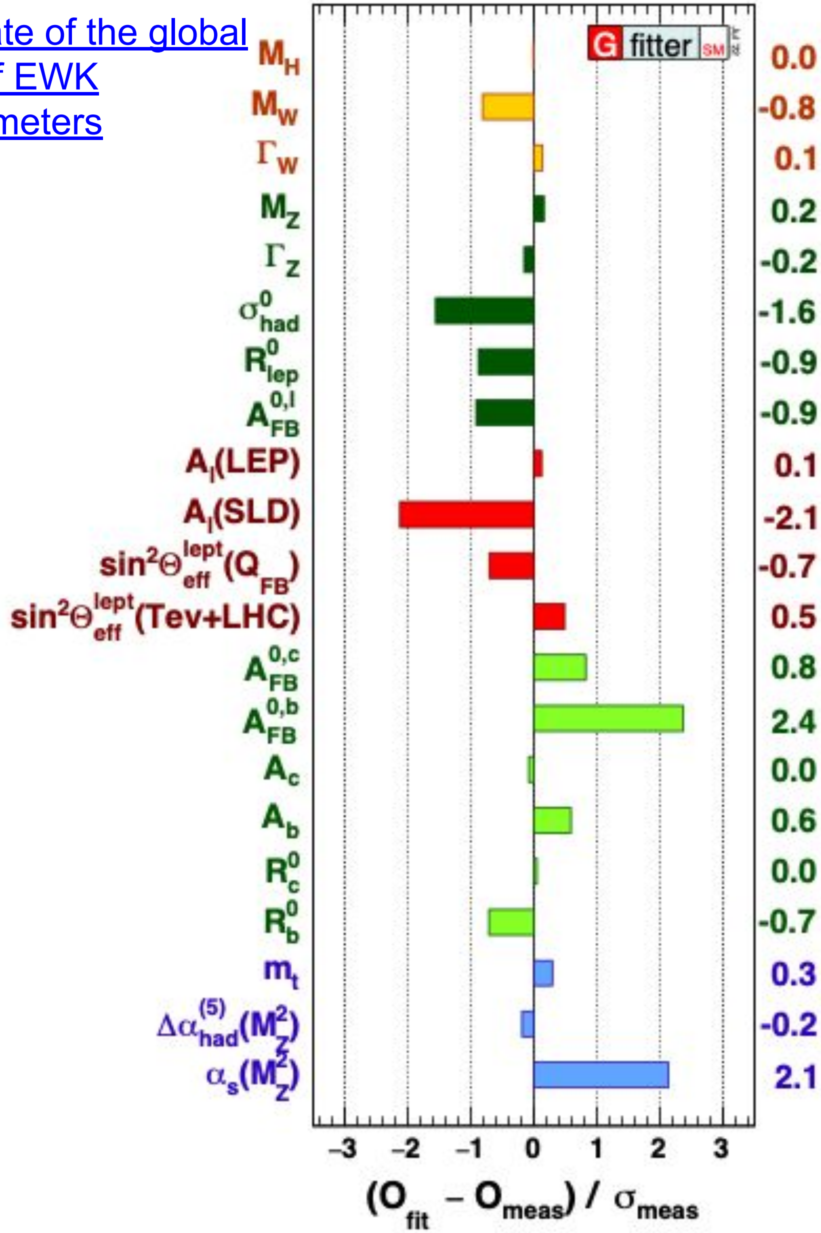
Tests of the W sector

EWK tests: $\sigma(pp \rightarrow W/Z)$ production at LHC

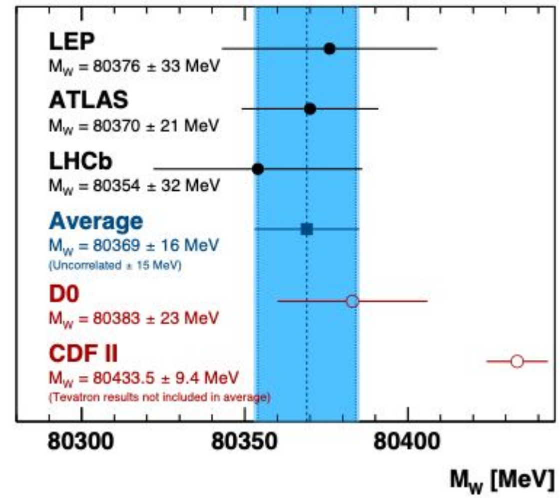


Consistent picture of electroweak parameters

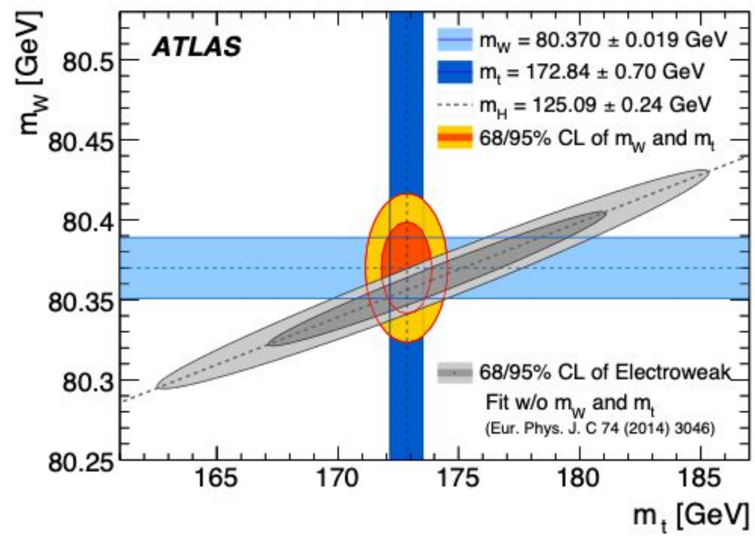
Update of the global fits of EWK parameters



ATLAS W-mass measurement



m_W-m_t-m_H dependence



Higgs boson mechanism

But why was the Higgs predicted?

- **Problem with electroweak unification:** Gauge invariance implied massless gauge bosons and fermions.
  - Mass terms are not allowed for gauge bosons as they violate gauge invariance (and fermions in SU(2) weak interaction)

Lagrangian of the QED (U(1))

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu D_\mu - mc^2)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

+ mass term for gauge bosons

$$+\frac{1}{2}m^2 A_\mu A^\mu$$

But why was the Higgs predicted?

- **Problem with electroweak unification:** Gauge invariance implied massless gauge bosons and fermions.
  - Mass terms are not allowed for gauge bosons as they violate gauge invariance (and fermions in SU(2) weak interaction)

Lagrangian of the QED (U(1))

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu D_\mu - mc^2)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

+ mass term for gauge bosons

$$+\frac{1}{2}m^2 A_\mu A^\mu$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda(x) \longrightarrow +\frac{1}{2}m^2 A_\mu A^\mu \rightarrow +\frac{1}{2}m^2 A_\mu A^\mu + m^2 A_\mu \partial^\mu \Lambda + \frac{1}{2}m^2 \partial_\mu \Lambda \partial^\mu \Lambda$$

Mass term

Additional terms in the Lagrangian

As the SM was being formulated, terms in the lagrangian that would lead to a mass for the particles were forbidden.

How can we get massive particles → BEH mechanism

We know weak interaction must have massive gauge bosons but gauge theories don't allow them→ How can we get mass terms leaving the theory invariant ?

How can we get massive particles → BEH mechanism

We know weak interaction must have massive gauge bosons but gauge theories don't allow them→ How can we get mass terms leaving the theory invariant ?

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \mathcal{L}_{Higgs} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

Proposed solution: add scalar field with a potential that is invariant under gauge transformations.

How can we get massive particles → BEH mechanism

This scalar would interact with the gauge bosons (it is charged under the weak interaction) and will interact with the fermions in a gauge invariant way.

$$\mathcal{L}_{Higgs} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W^i_\mu + i\frac{1}{2} g' B_\mu \right) \phi$$

Interaction with weak gauge bosons

$$\mathcal{L}_{Yuk} = f_e\, \bar{l}_L\, \phi\, e_R + f_u\, \bar{q}_L\, \tilde{\phi}\, u_R + f_d\, \bar{q}_L\, \phi\, d_R + h.c.$$

Interaction via Yukawa couplings

How can we get massive particles → BEH mechanism

The key feature of this new field is the potential term V(Φ).
If $\mu^2 > 0$, potential follows the “Mexican” hat form

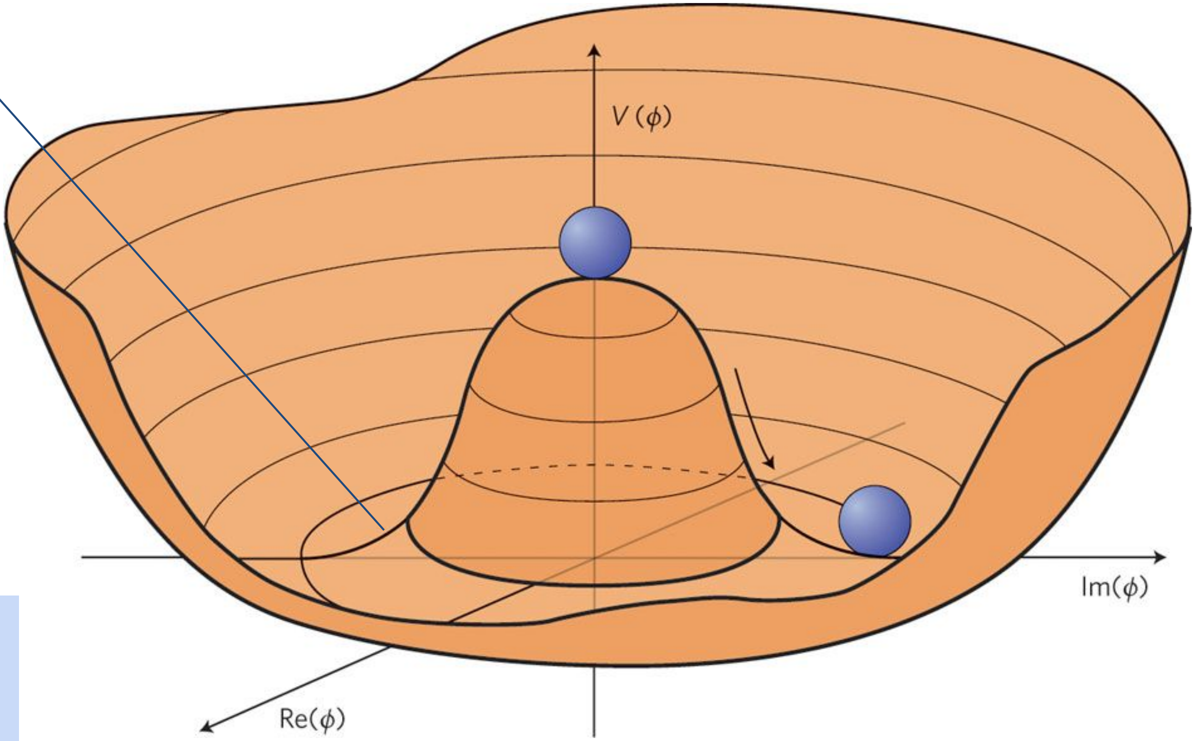
$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Minimum of energy of the theory, this field present a value v

$$v = \mu^2/\lambda$$

Vacuum expectation value (VEV)

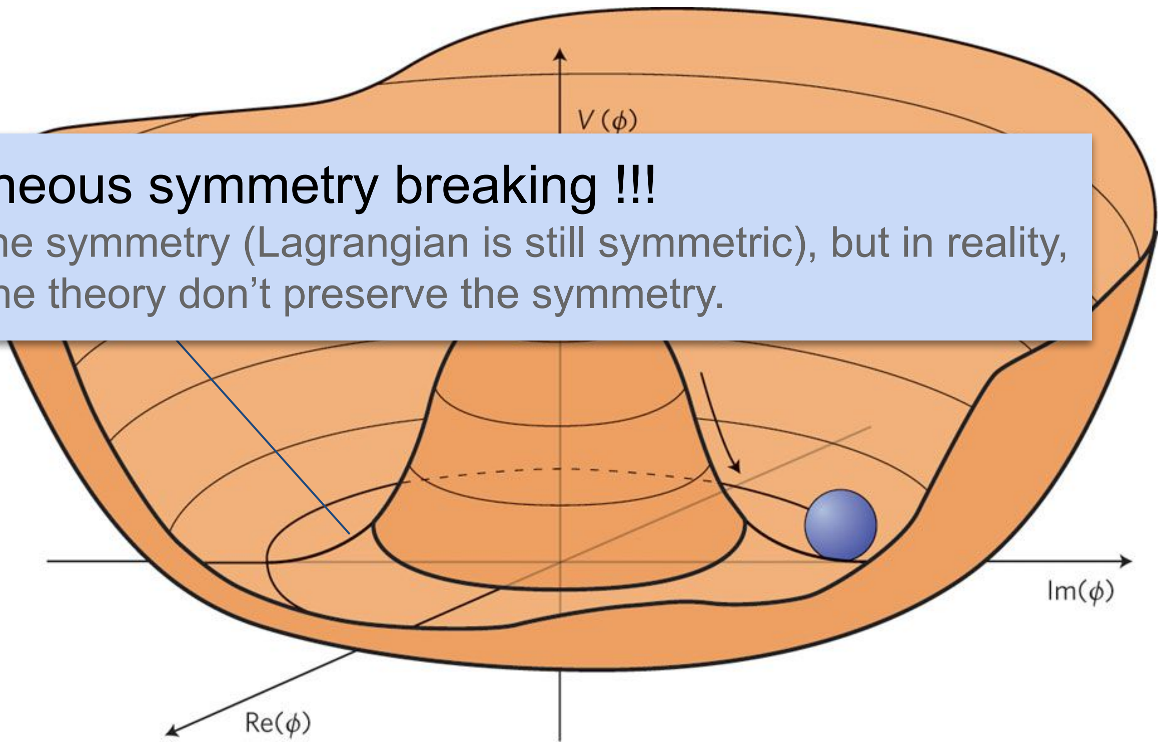


How can we get massive particles → BEH mechanism

The key feature of this new field is the potential term V(Φ).
If $\mu^2 > 0$, potential follows the “Mexican” hat form

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$



Spontaneous symmetry breaking !!!

Underlying physics law keeps the symmetry (Lagrangian is still symmetric), but in reality, the lowest energy solutions of the theory don’t preserve the symmetry.

present a value v

$$v = \mu^2/\lambda$$

Vacuum expectation value (VEV)

How can we get massive particles → BEH mechanism

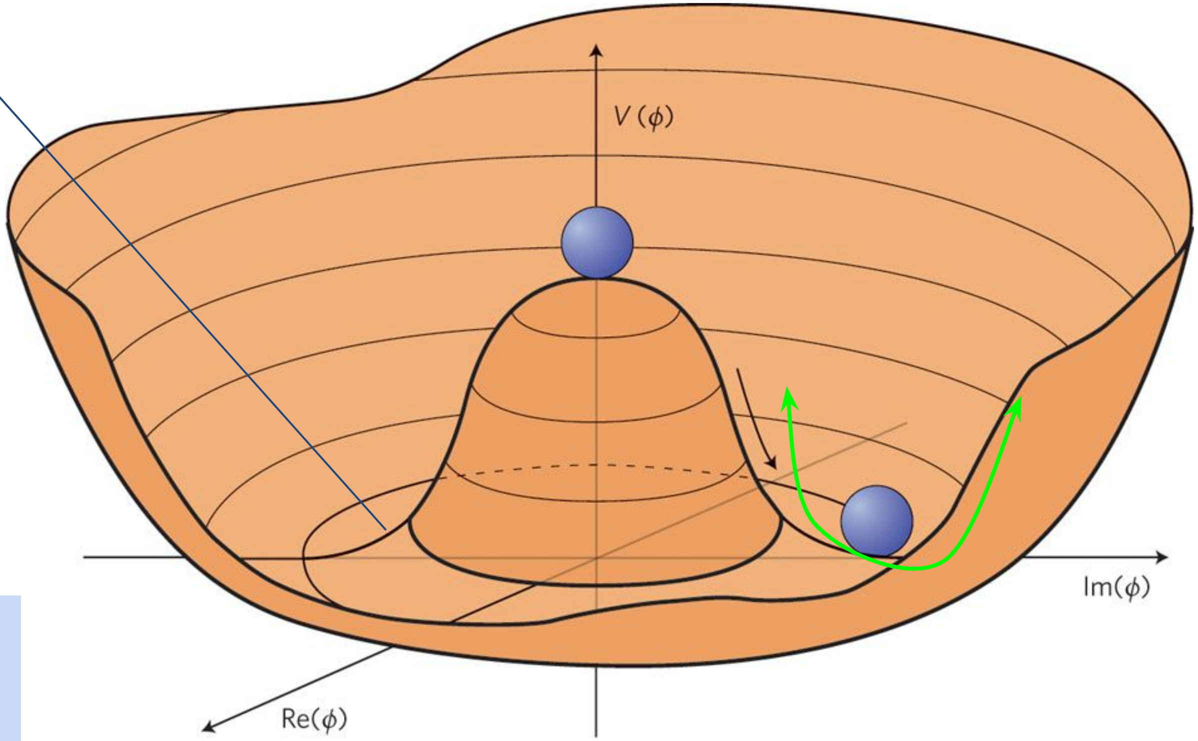
The key feature of this new field is the potential term V(Φ).
If $\mu^2 > 0$, potential follows the “Mexican” hat form

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

Minimum of energy of the theory, this field present a value v

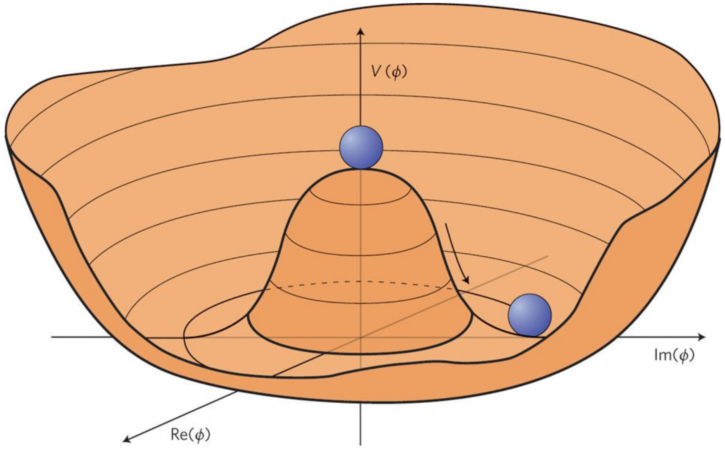
$$v = \mu^2/\lambda$$



Excited states around the minimum → Higgs boson!

Consequences of the shape of the potential

Interaction terms of Higgs with gauge bosons



$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W_\mu^i + i\frac{1}{2} g' B_\mu \right) \phi$$

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$= \frac{v^2}{8} \left[g^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + (g W_\mu^3 - g' B_\mu)^2 \right]$$

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2)$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}} (g W_\mu^3 - g' B_\mu)$$

$$A_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}} (g' W_\mu^3 + g B_\mu)$$

Masses of gauge bosons

$$m_Z = \frac{v}{2} \sqrt{g^2 + g'^2}$$

$$m_A = 0$$

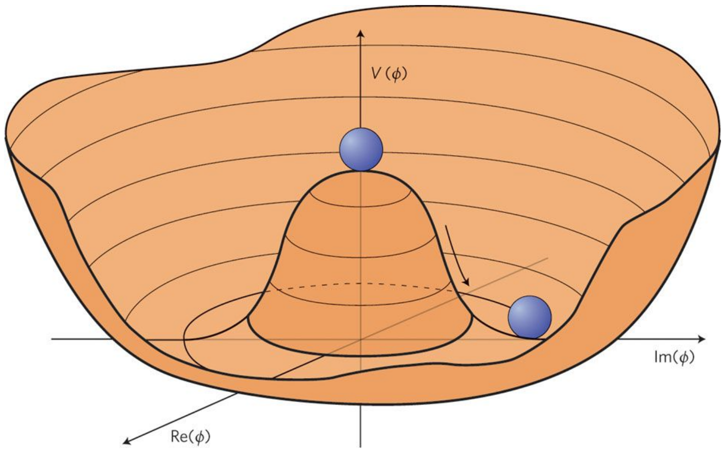
$$m_W = \frac{g\ v}{2}$$

Consequences of the shape of the potential

Yukawa interactions of Higgs field and fermions

$$\mathcal{L}_{Yuk} = f_e\, \bar{l}_L\, \phi\, e_R + f_u\, \bar{q}_L\, \tilde{\phi}\, u_R + f_d\, \bar{q}_L\, \phi\, d_R + h.c.$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$



$$\mathcal{L}_{Yuk} = \frac{f_e\; v}{\sqrt{2}}\underbrace{(\bar{e}_L e_R + \bar{e}_R e_L)} + \frac{f_u\; v}{\sqrt{2}}(\bar{u}_L u_R + \bar{u}_R u_L) + \frac{f_d\; v}{\sqrt{2}}(\bar{d}_L d_R + \bar{d}_R d_L)$$

Masses of fermions

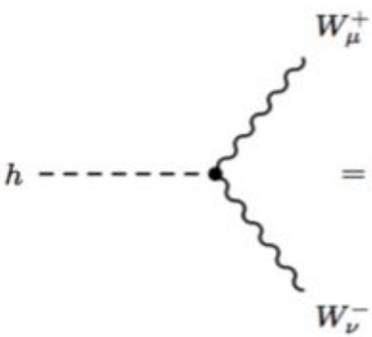
$$m_i = -\frac{f_i\; v}{\sqrt{2}}$$

Brout-Englert-Higgs mechanism → Higgs boson !

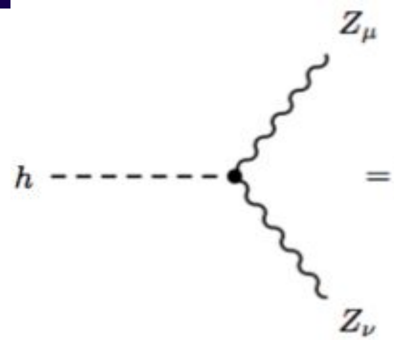
- Oscillations around VeV → Higgs bosons !
- Due to the interaction with gauge fields/fermions → Higgs couplings

$$\mathcal{L}_{\mathcal{H}-\mathcal{W}/\mathcal{Z}} = \frac{1}{2}(\mathrm{v}+\mathrm{H})^2\left[\frac{\mathrm{g}_2^2}{2}\mathrm{W}_\mu^+\mathrm{W}^{-\mu} + \frac{\mathrm{g}_2^2+\mathrm{g}_1^2}{4}\mathrm{Z}_\mu\mathrm{Z}^\mu\right]$$

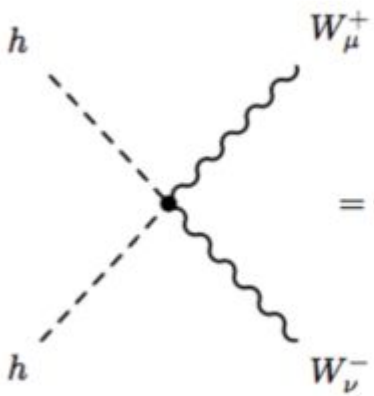
$\mathrm{g}^2 = \mathrm{g}$; $\mathrm{g}^1 = \mathrm{g}'$



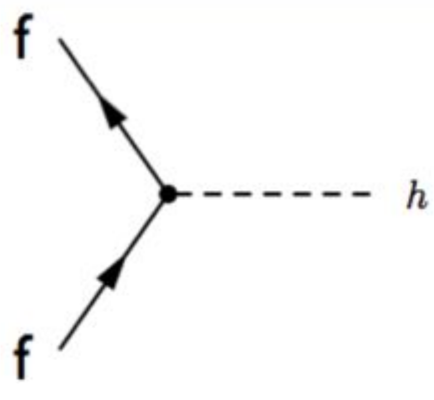
$$= i\frac{g^2 v}{2} g_{\mu\nu} = 2i\frac{M_W^2}{v} g_{\mu\nu}$$



$$= i\frac{(g^2+g'^2)v}{4}\cdot 2g_{\mu\nu} = 2i\frac{M_Z^2}{v} g_{\mu\nu}$$



$$= i\frac{g^2}{4}\cdot 2g_{\mu\nu} = 2i\frac{M_W^2}{v^2} g_{\mu\nu}$$



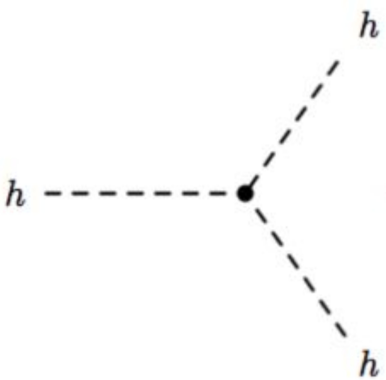
$$= -i\frac{h_f}{\sqrt{2}} = -i\,\frac{m_f}{v}$$

Brout-Englert-Higgs mechanism → Higgs boson!

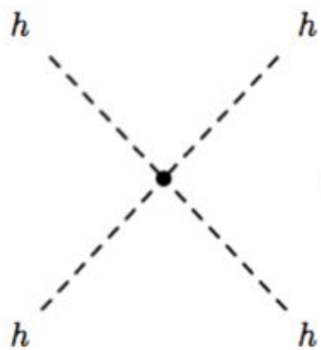
- Oscillations around VeV → Higgs bosons !
- Due to the V(Φ)potential shape, Higgs interacting with itself → Higgs self-couplings

$$V = \lambda\, v^2 h^2 + \lambda\, v\, h^3 + \frac{\lambda}{4}\, h^4$$

$$m_{\mathrm{H}} = \sqrt{2\mu_{\mathrm{H}}^2} \equiv \sqrt{2\lambda v^2}$$



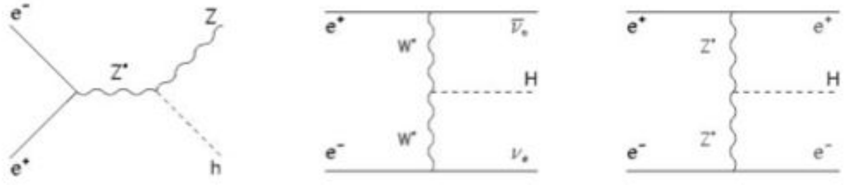
$$= -i\lambda v \cdot 3! = -6i\lambda v = -3i\frac{m_h^2}{v}$$

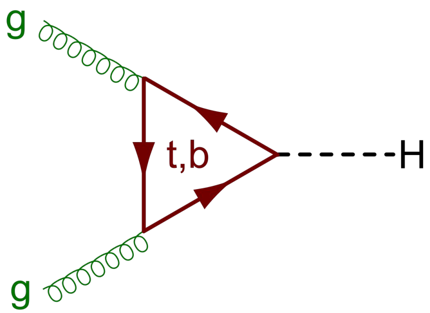


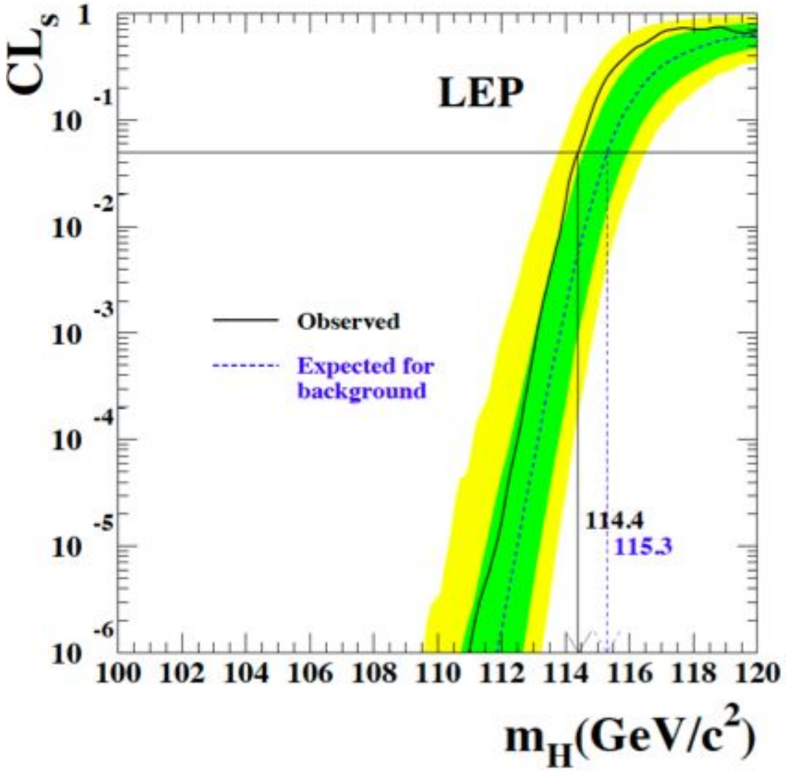
$$= -i\frac{\lambda}{4} \cdot 4! = -6i\lambda = -3i\frac{m_h^2}{v^2}$$

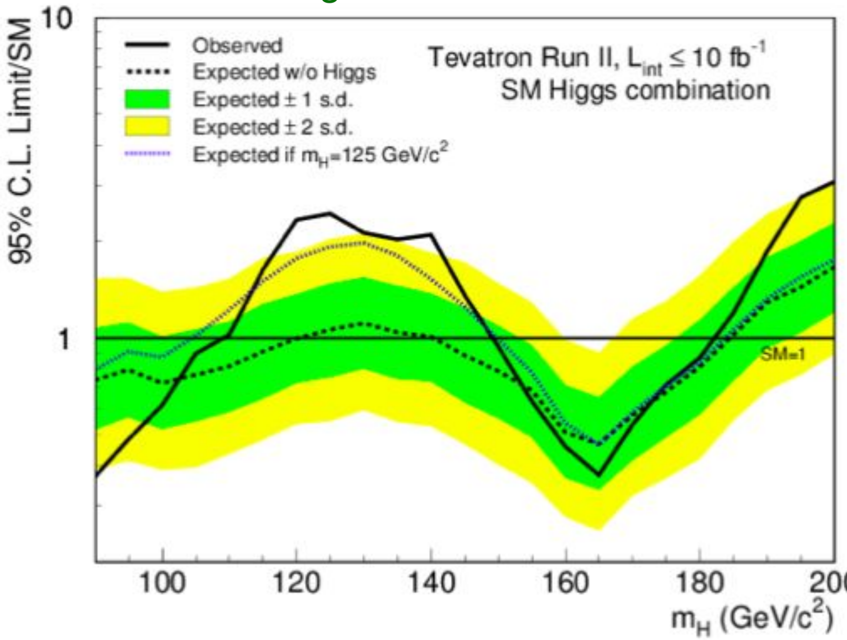
The Higgs boson before LHC

LEP (e^+e^-) and Tevatron ($p\bar{p}$) indicated Higgs around 120-130 GeV









Where is it ? The Standard Model's biggest triumph

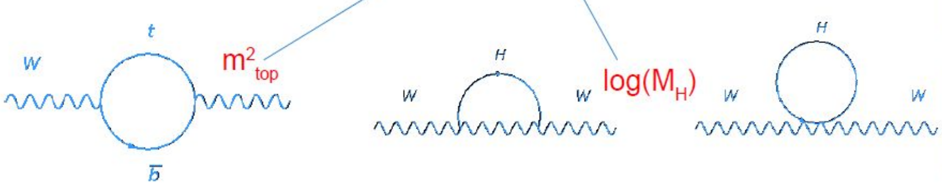
As you may have observed, the mass of the Higgs boson depends on λ
→ Mass of the Higgs boson is a free parameter of the SM

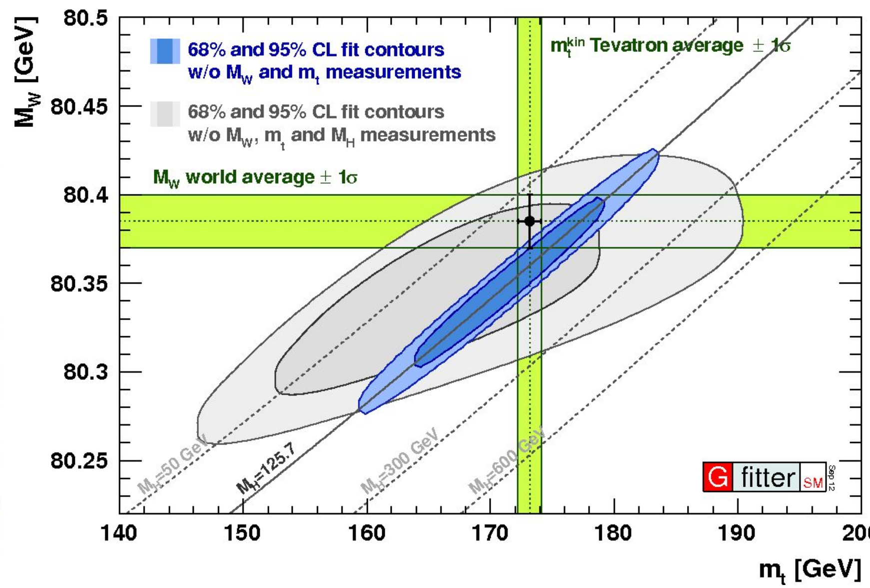
However, indirect constraints on the Higgs mass could be searched for

Radiative corrections to W-boson mass

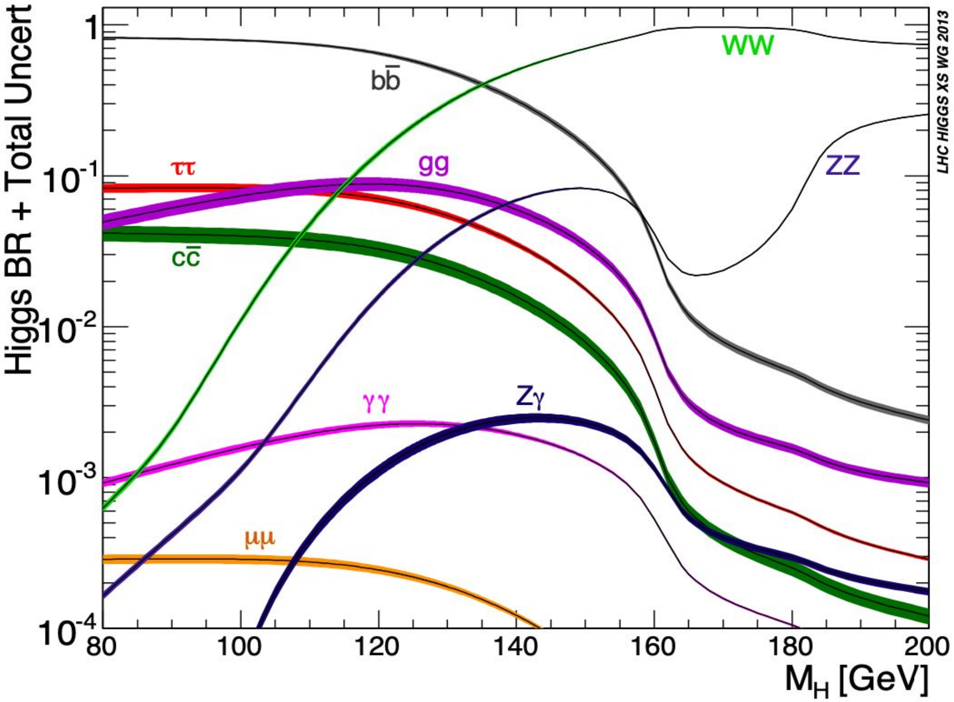
$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2}\, G_F}\right)^{1/2} \frac{1}{\sin\theta_W \sqrt{1-\Delta r}}$$

radiative corrections





Measurements of the Higgs boson at LHC



Low mass ($\lesssim \mathbf{140\,GeV}$)

$H \to \gamma\gamma$

Rare decay, but distinct signal

$H \to \tau\tau$

Enhanced in MSSM, also contributes to SM search

$H \to b\bar{b}$

Main search channel at LEP and Tevatron, important to study Higgs properties

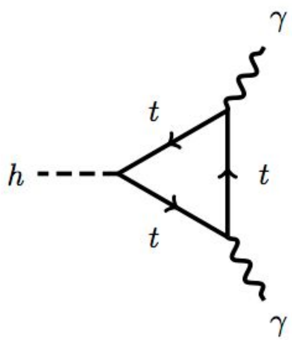
Intermediate and large m_H ($\gtrsim \mathbf{130\,GeV}$)

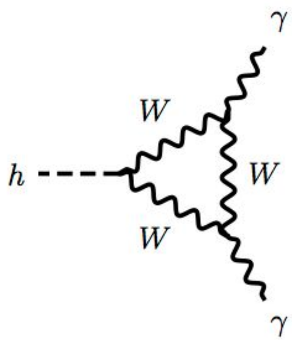
$H \to WW$

Large signal yield

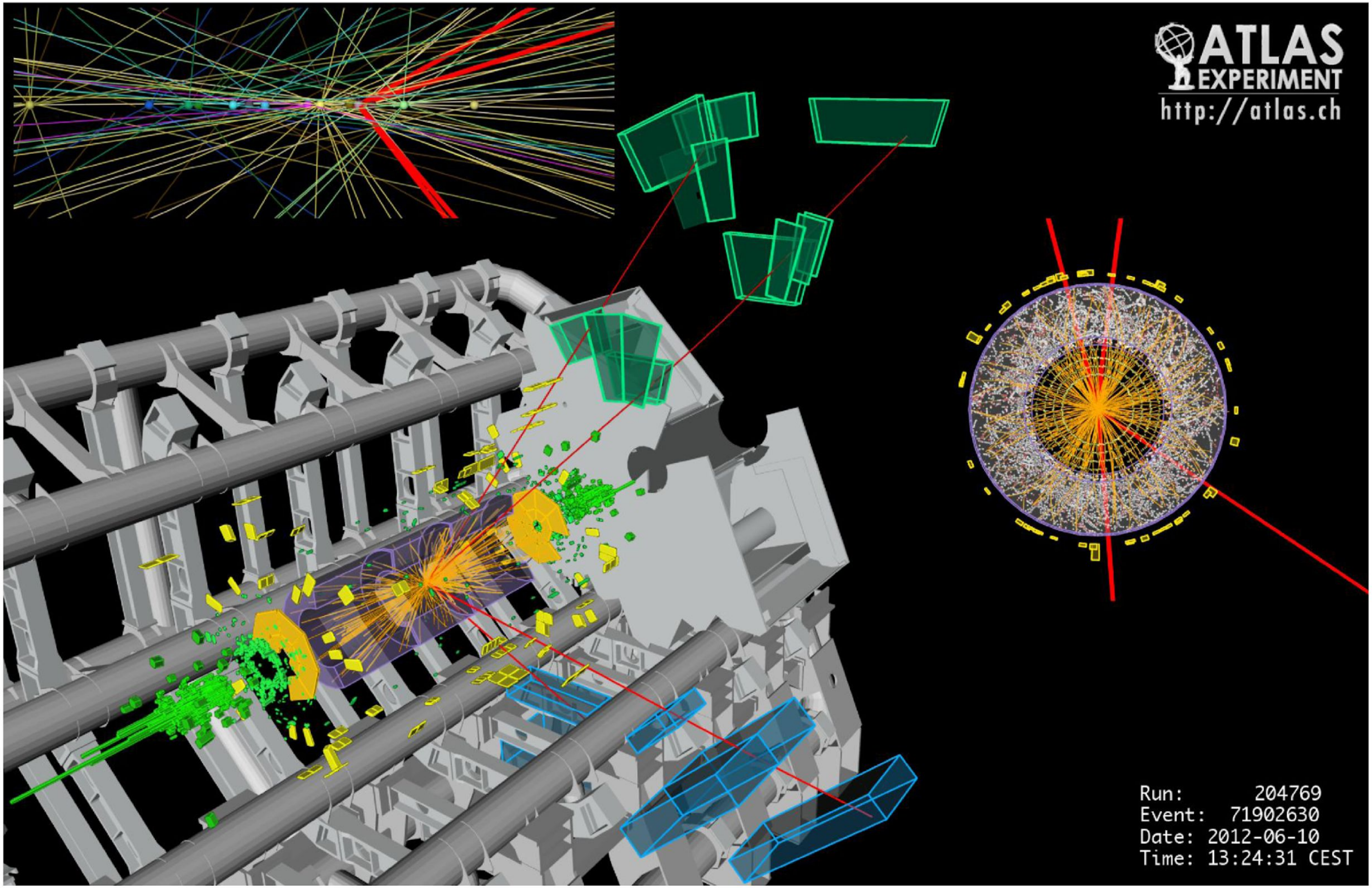
$H \to ZZ$

Very clean signal if both $Z \to \ell\ell$



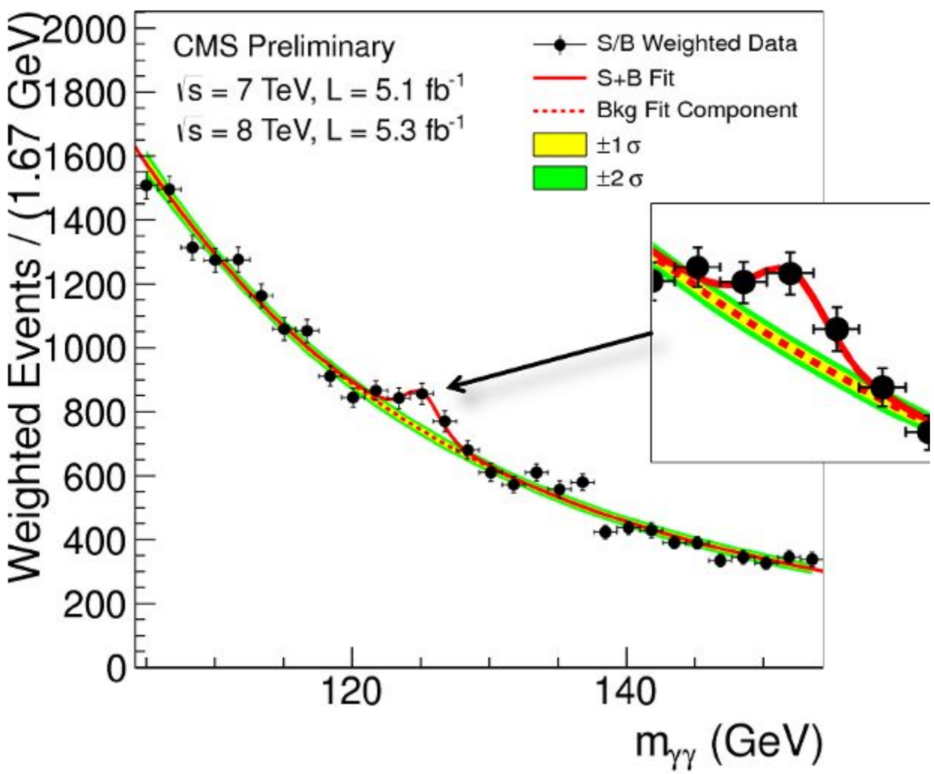


The Higgs is there !

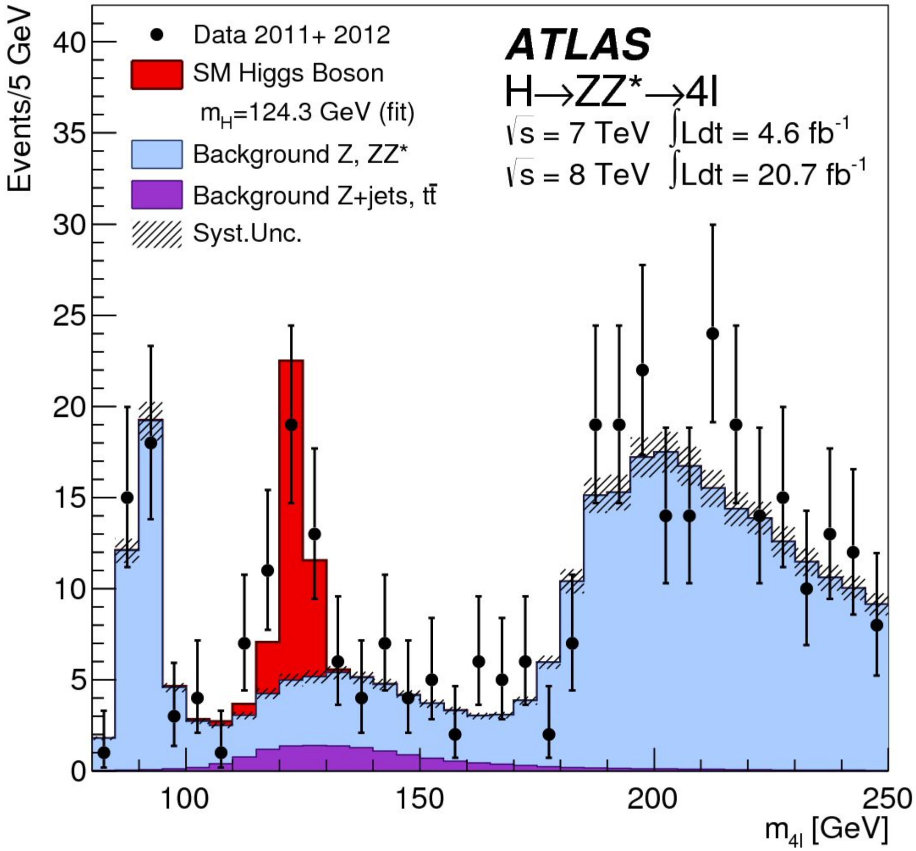


The Higgs is there !

$H \rightarrow \gamma\gamma$

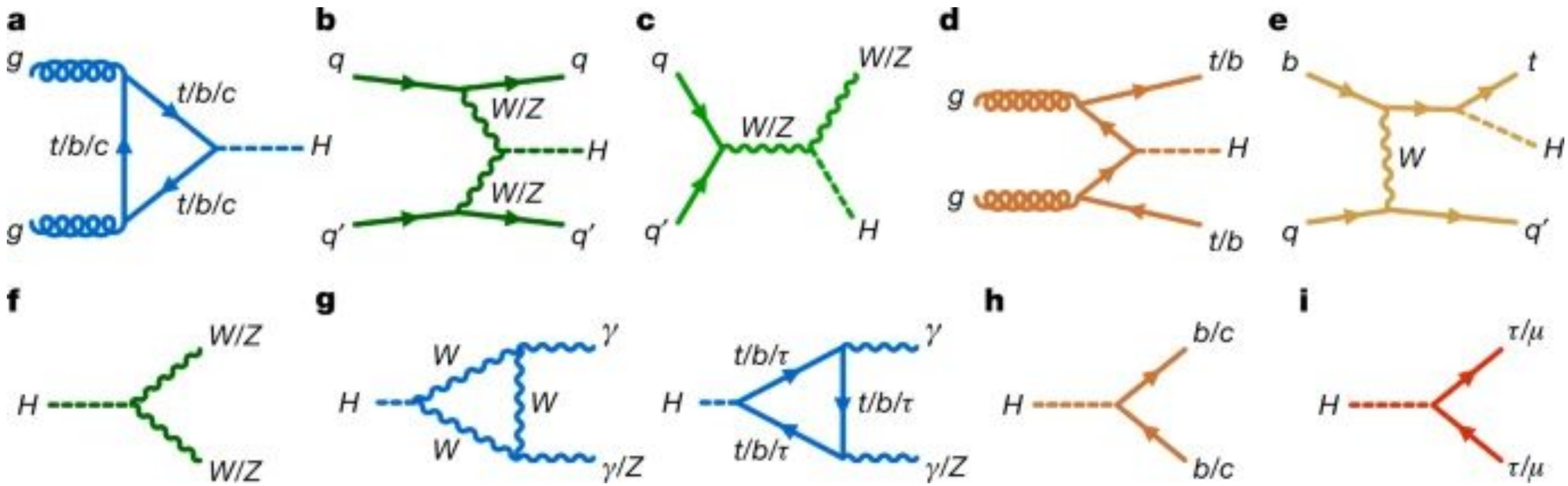


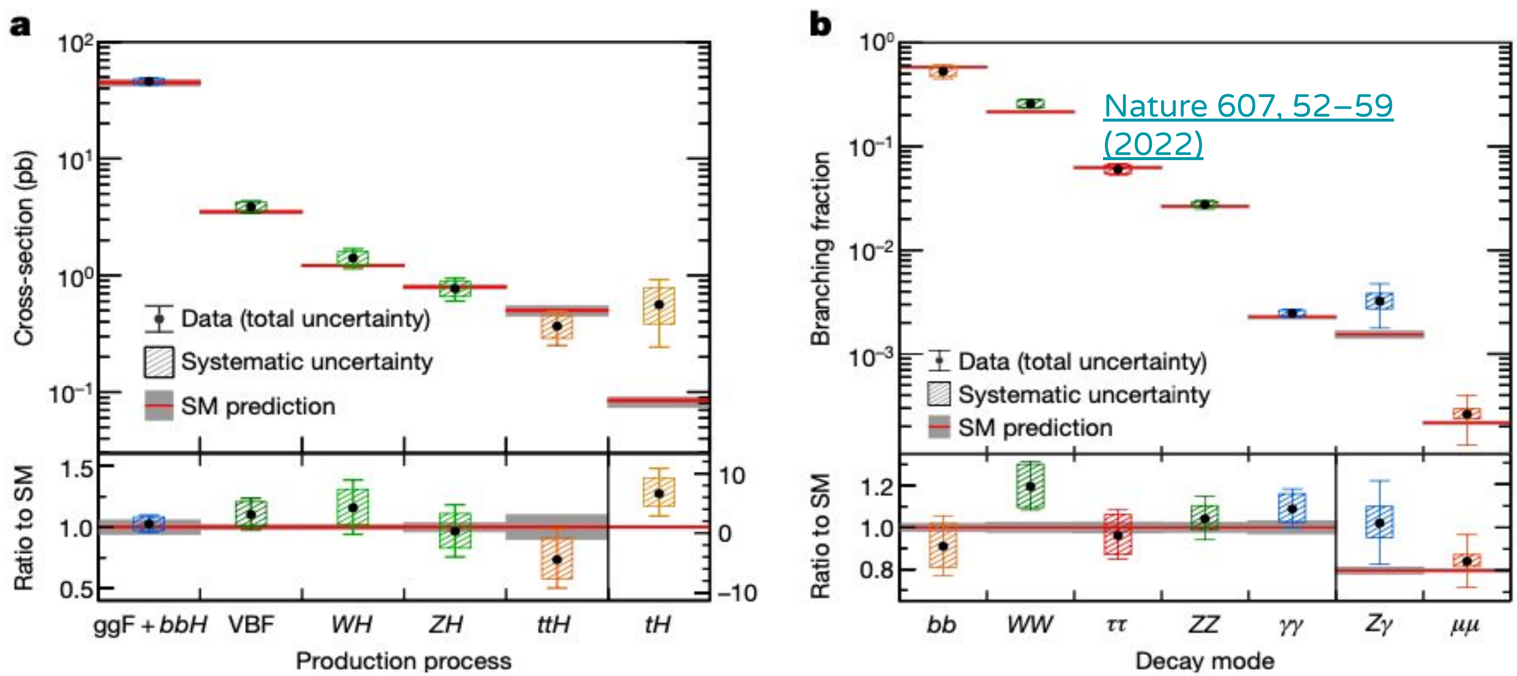
$H \rightarrow ZZ$



The last missing piece in the Standard Model

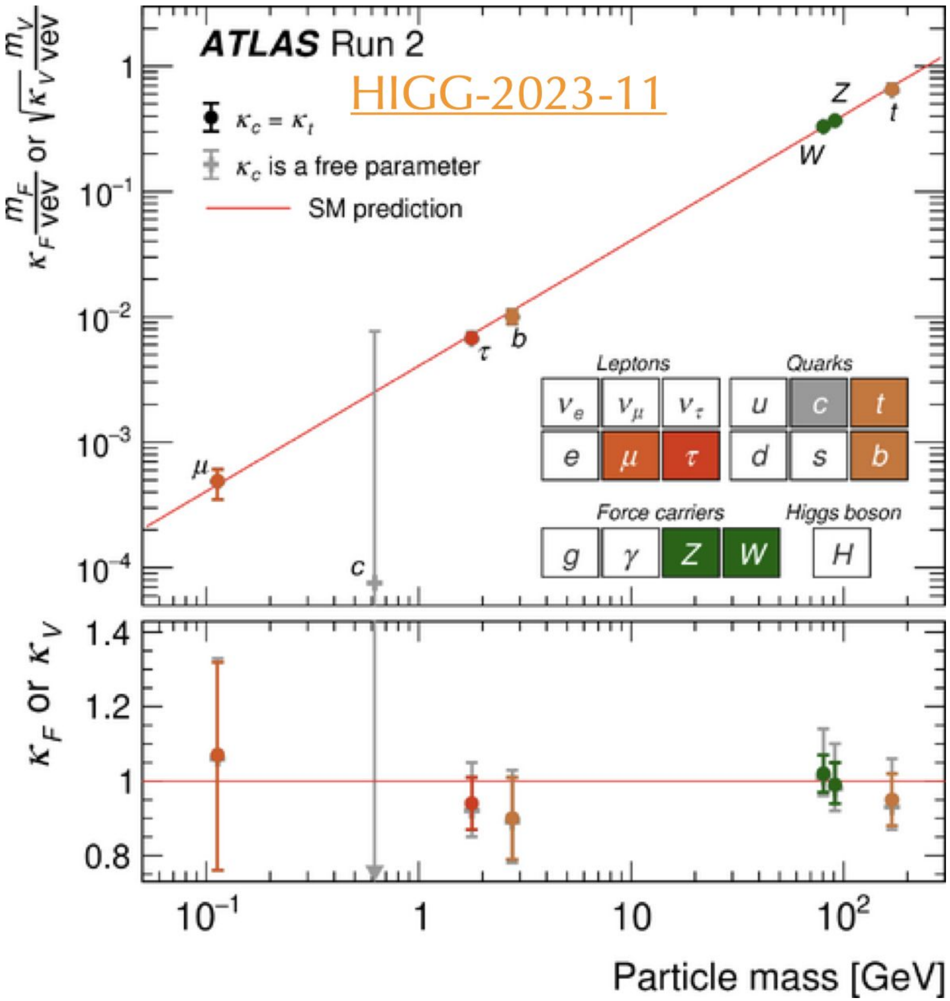
Measured it in many of its production and decay modes

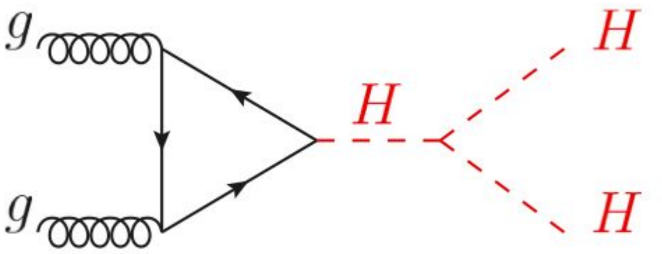


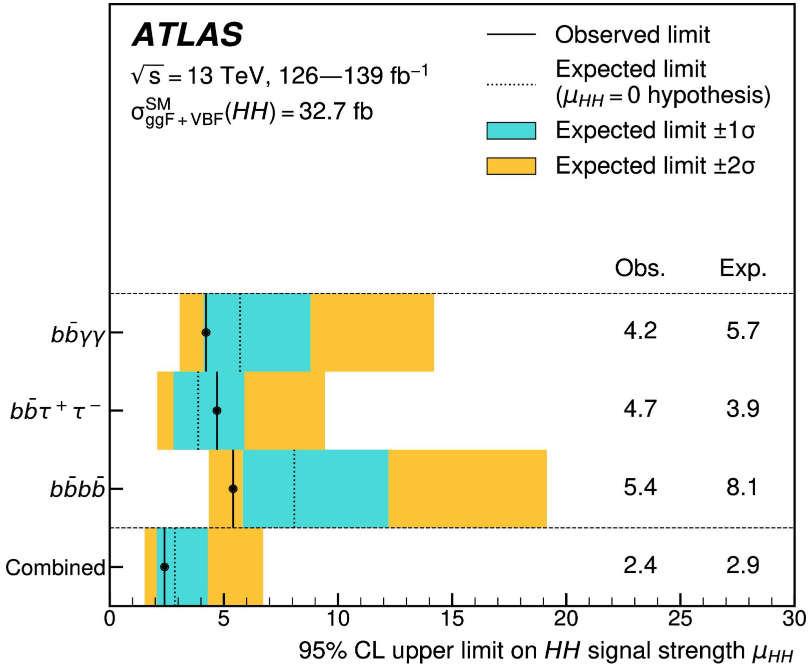


Nature 607, 52–59 (2022)

But still many predictions to be tested !







Beyond Standard Model

The Standard Model: Free parameters

The standard model establish relations between different parameters

However, some of its parameters cannot be known a priori → Experiments

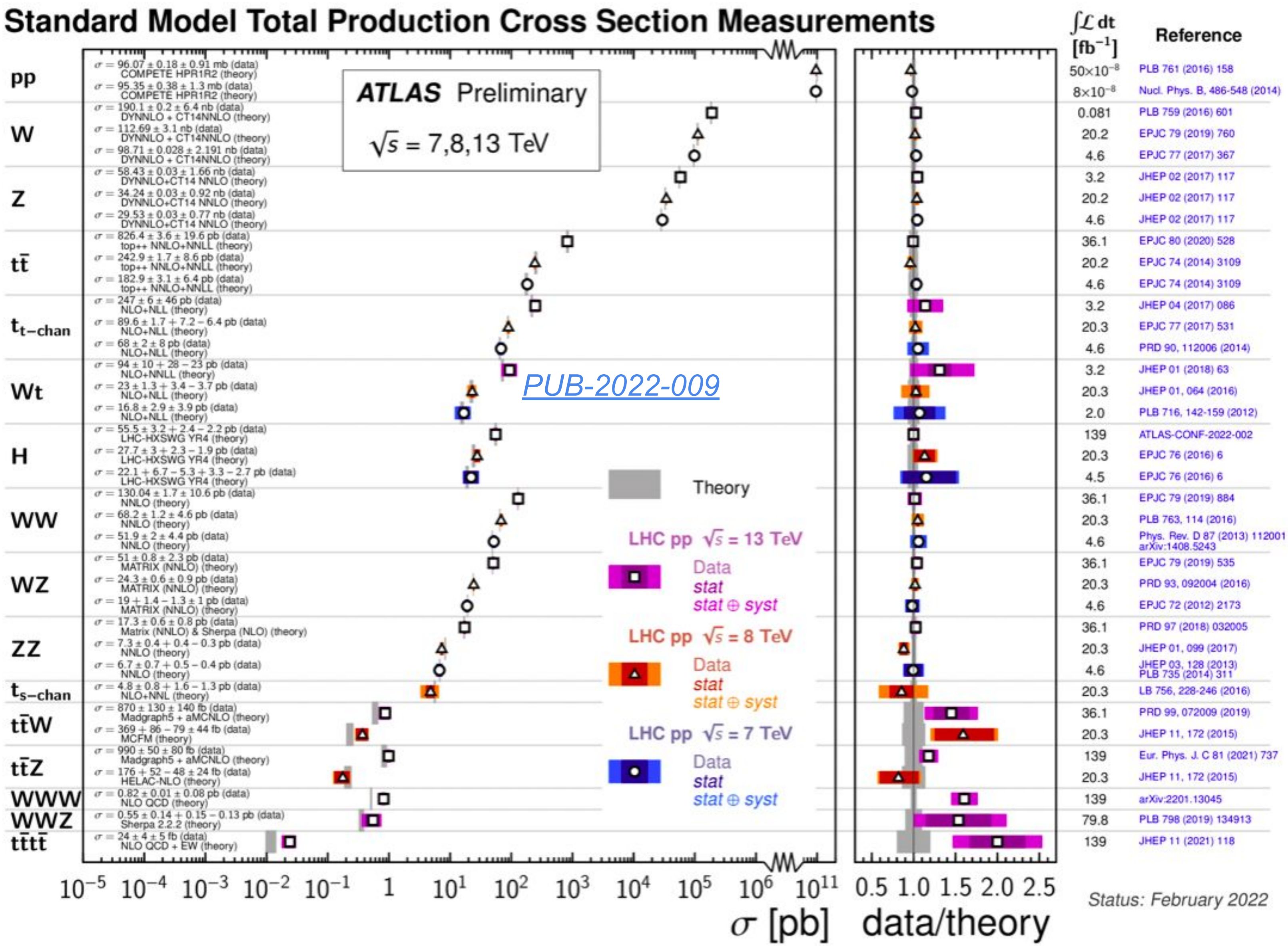
- Particle masses
- CKM parameters
- Gauge couplings at a given energy: strength of forces
- CP properties of QCD
- Parameters of electroweak symmetry breaking: v and m_H

Parameters of the Standard Model [hide]

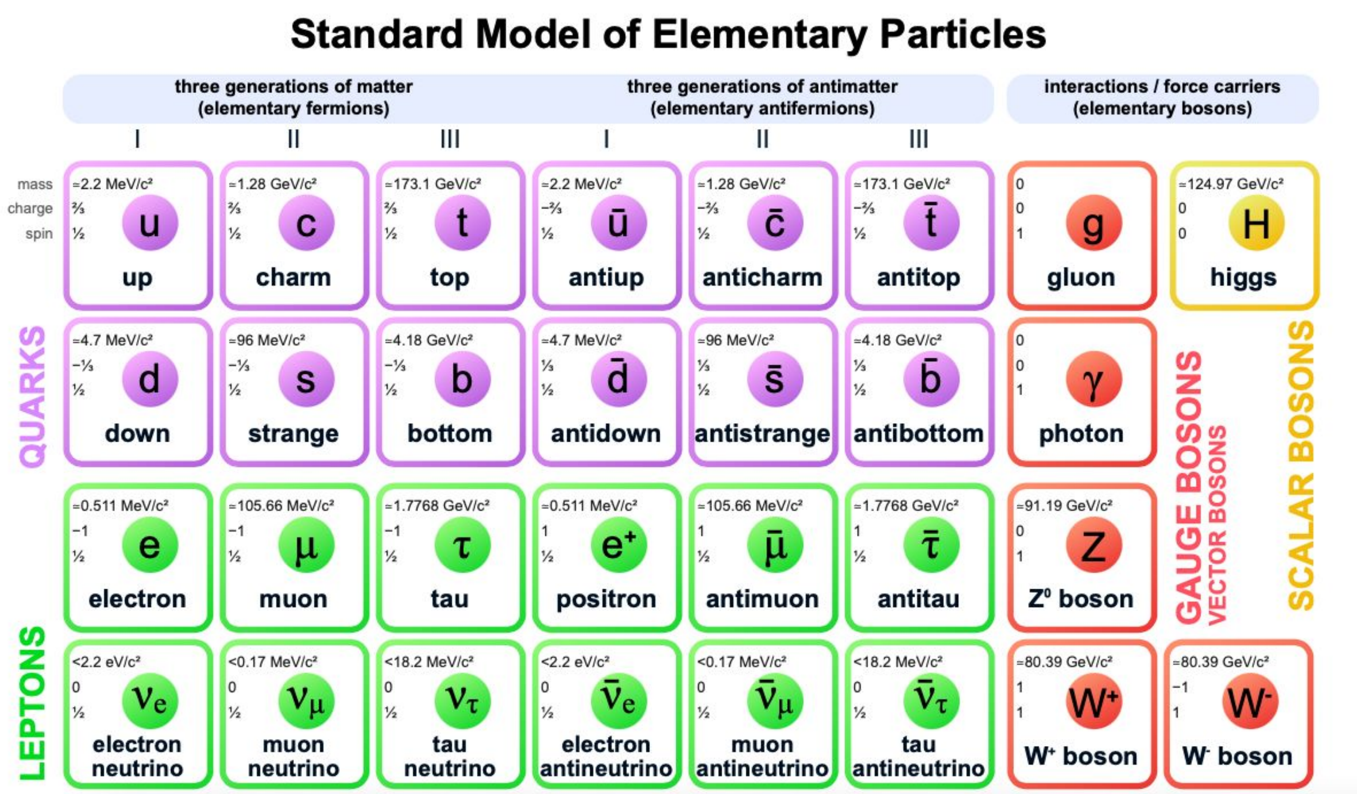
#	Symbol	Description	Renormalization scheme (point)	Value
1	m_e	Electron mass		0.511 MeV
2	m_μ	Muon mass		105.7 MeV
3	m_τ	Tau mass		1.78 GeV
4	m_u	Up quark mass	$\mu_{\overline{MS}} = 2$ GeV	1.9 MeV
5	m_d	Down quark mass	$\mu_{\overline{MS}} = 2$ GeV	4.4 MeV
6	m_s	Strange quark mass	$\mu_{\overline{MS}} = 2$ GeV	87 MeV
7	m_c	Charm quark mass	$\mu_{\overline{MS}} = m_c$	1.32 GeV
8	m_b	Bottom quark mass	$\mu_{\overline{MS}} = m_b$	4.24 GeV
9	m_t	Top quark mass	On shell scheme	173.5 GeV
10	θ_{12}	CKM 12-mixing angle		13.1°
11	θ_{23}	CKM 23-mixing angle		2.4°
12	θ_{13}	CKM 13-mixing angle		0.2°
13	δ	CKM CP violation Phase		0.995
14	g_1 or g'	U(1) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.357
15	g_2 or g	SU(2) gauge coupling	$\mu_{\overline{MS}} = m_Z$	0.652
16	g_3 or g_s	SU(3) gauge coupling	$\mu_{\overline{MS}} = m_Z$	1.221
17	θ_{QCD}	QCD vacuum angle		~0
18	v	Higgs vacuum expectation value		246 GeV
19	m_H	Higgs mass		125.09 ±0.24 GeV

The Standard Model: Extremely predictive

Once parameters are known, everything else is "fixed"



What is missing ? Beyond Standard Model Physics



Is the SM complete ?

Presented the SM during the last days. Very successful and predictive theory but …..

- We know gravity to be one interaction of nature. Why is it not included ?
- Naturalness problem
- Hierarchy problem
- Matter-antimatter asymmetry

H. Murayama

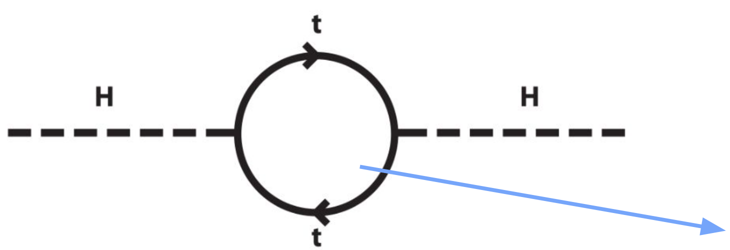
Naturalness problem (some might call it hierarchy)

Mass corrections to the Higgs

As we have seen, some Feynman diagrams might diverge and renormalization of couplings and masses helps to remove these divergences.

- Correction to the Higgs mass include loops with creation of fermions.
- Cannot absorb this correction. Dependent on cut-off Λ.

Radiative correction to Higgs mass very large, if no other new physics of mass Λ



$$\Delta m_h^2 \sim \frac{3}{4\pi^2}\Big(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda\Big)\Lambda^2$$

$$M < \left(\frac{10\%}{\text{tuning}}\right)\ 1\ \text{TeV}$$

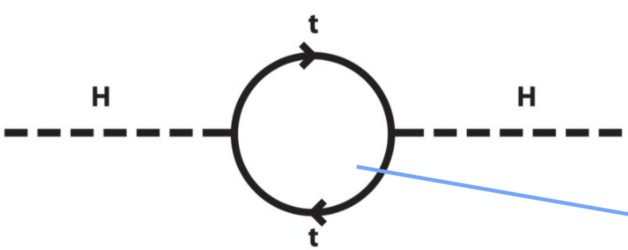
Naturalness problem (some might call it hierarchy)

Mass corrections to the Higgs

As we have seen, some Feynman diagrams might diverge and renormalization of couplings and masses helps to remove these divergences.

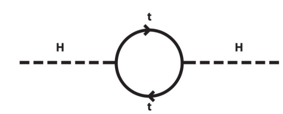
- Correction to the Higgs mass include loops with creation of fermions.
- Cannot absorb this correction. Dependent on cut-off Λ.
- Very typical new theory to solve Naturalness problem : Supersymmetry !

Radiative correction to Higgs mass very large, if no other new physics of mass Λ



$$\Delta m_h^2 \sim \frac{3}{4\pi^2}\left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2\theta_W} + \lambda\right)\Lambda^2$$

(-1)*



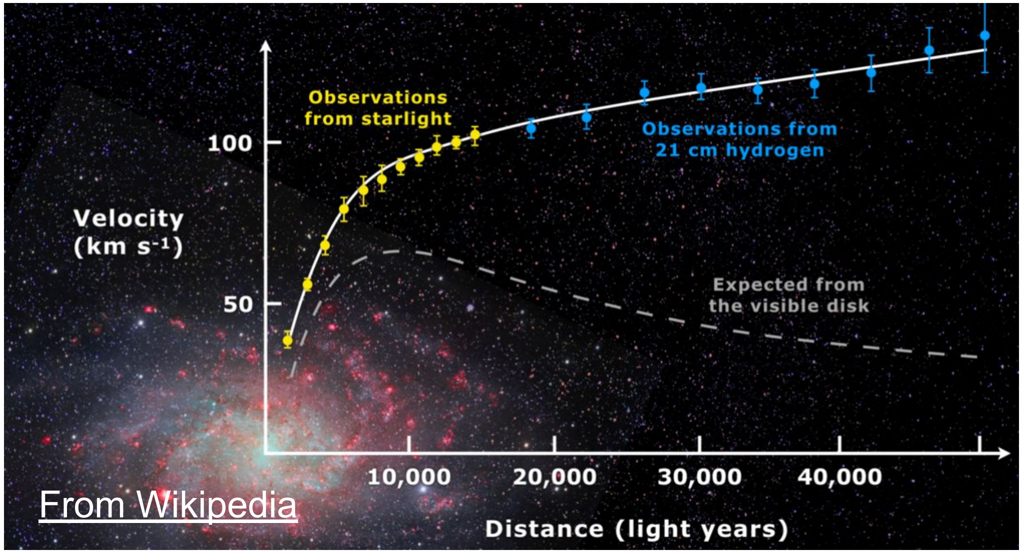
A super-partner of the top (boson) would generate same correction but with negative value → Cancellation

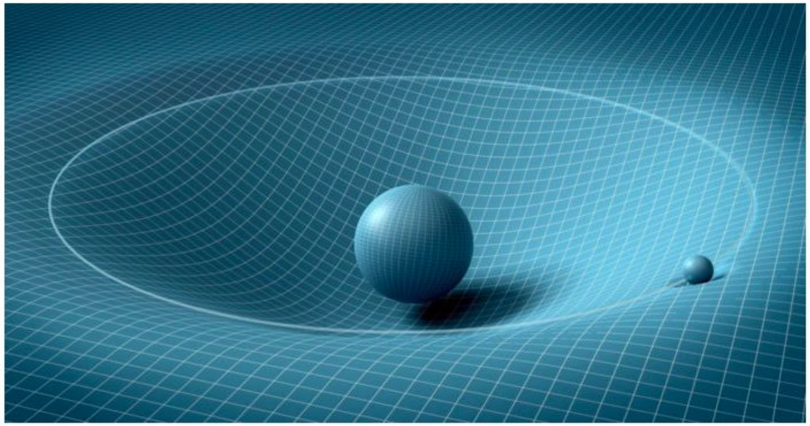
Gravity, dark matter, matter-antimatter asymmetry, …..

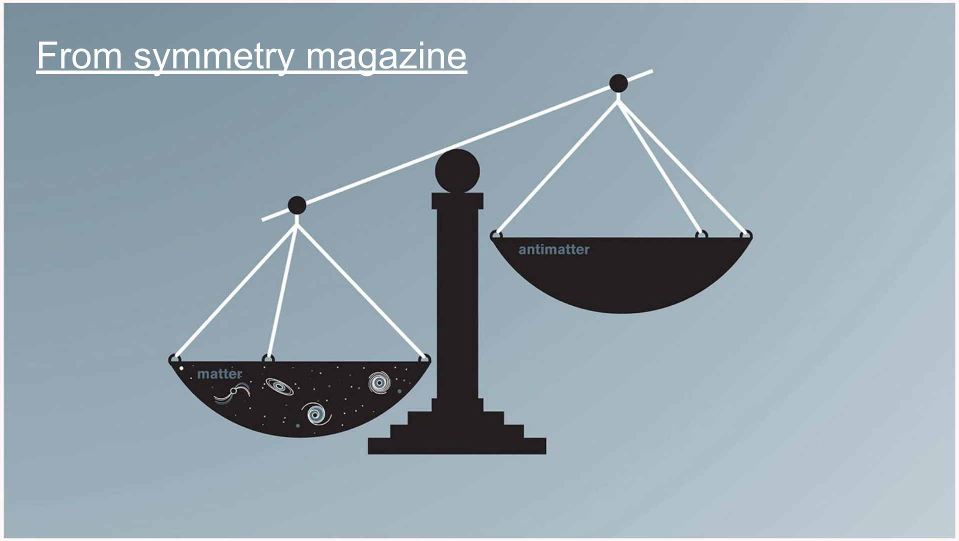
More missing pieces

- Gravity: non-renormalizable theory
- Dark Matter: no candidate particle in SM
- We live in a matter dominated Universe. CP violation in EWK and CKM/PMNS cannot explain it. Why ?
- Strong CP problem

And many more missing pieces !







So what else is out there?

More and better in the BSM talks by Thorsten Kuhl!

So what else is out there?

More and better in the BSM talks by Thorsten Kuhl!

Many exciting questions to answer and discoveries awaiting us !!

Backup

Further problem: K^0_S and K^0_L and CP-violation

Experiment of Christenson-Fitch-Cronin-Turlay: link

Two neutral kaons (meson with one strange quark) were known with same mass and properties but two different lifetimes and decay types: K^0_S and K^0_L

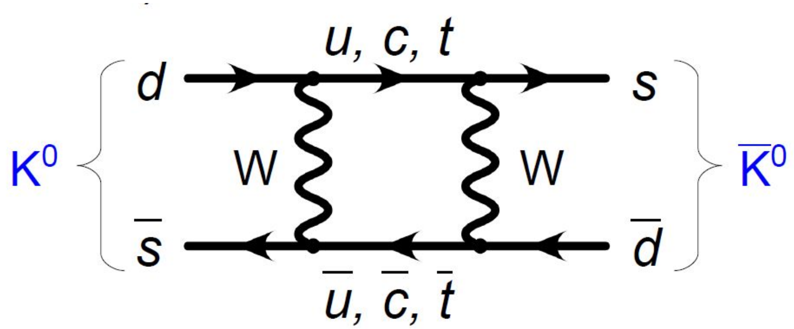
K^0_S, $\tau = 9.0 \cdot 10^{-11}$ s ($c\tau = 2.7$ cm) $\quad K^0_S \rightarrow \pi^0\pi^0/\pi^+\pi^-$; $CP = +1$

K^0_L, $\tau = 5.1 \cdot 10^{-8}$ s ($c\tau = 15$ m) $\quad K^0_L \rightarrow \pi^0\pi^0\pi^0/\pi^+\pi^-\pi^0$; $CP = -1$

- As we know today, the reason is that K^0_S and K^0_L are actually a mixing of the strong interaction eigenstates K^0 and $\overline{K}^0$
- Mass and charge of the interaction determined by QCD and QED (this is the particle we see).
- But QCD eigenstate is not weak eigenstate. QCD eigenstate = composition of weak eigenstates → Turn CP=1 state into CP = -1 → **CP-violation!**

K^0 and $\overline{K}^0$ can turn from one into the other:

"oscillation"



First proposal of a QFT for weak interactions

Feynman and Gell-Mann proposed a QFT where the force field is V-A interaction

Explanation of Wu's experiment : weak interaction only with left-handed states of particles (and right-handed anti-particles)

Vector-axial symmetry → Interaction only happens between left-handed particles (or right-handed antiparticles).

$$e_L = \frac{1}{2}(1-\gamma_5)e$$

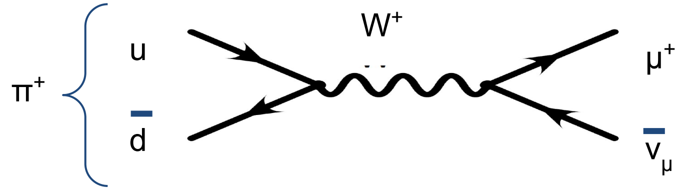
$$e_R = \frac{1}{2}(1+\gamma_5)e$$

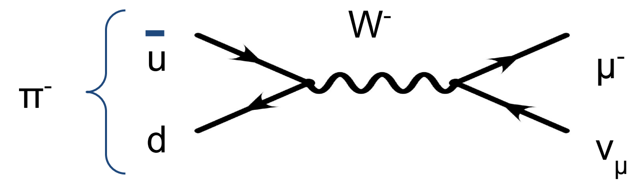
$$\mathcal{L}_\mu = \frac{G_\mu}{\sqrt{2}}\left[\bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\mu\right]\left[\bar{e}\gamma_\lambda(1-\gamma_5)\nu_e(x)\right] + \text{c.c.}\ .$$

QFT theory still allows the CP-symmetry

Change parity of the interaction and change particles by anti-particles → Same probability.

Experiment of Wu didn't show any problem CP-symmetry





Handed-ness and hadronic decays: The Pion

π^0

$\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ $\qquad I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9766 \pm 0.0006$ MeV (S = 1.1)

$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV

Mean life $\tau = (8.52 \pm 0.18) \times 10^{-17}$ s (S = 1.2)

$c\tau = 25.5$ nm

π^0 DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
2γ	(98.823 ± 0.034) %	S=1.5	67
$e^+ e^- \gamma$	(1.174 ± 0.035) %	S=1.5	67

$\pi^\pm$

$\pi^+ = u\bar{d},\ \pi^- = \bar{u}d$ $\qquad I^G(J^P) = 1^-(0^-)$

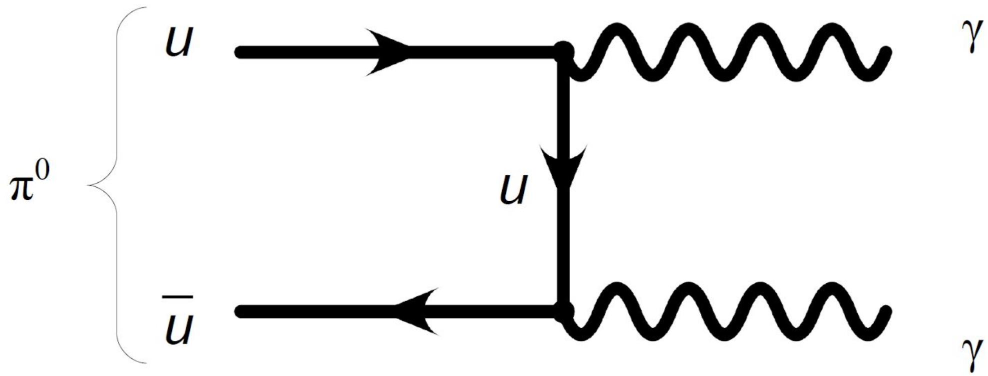
Mass $m = 139.57018 \pm 0.00035$ MeV (S = 1.2)

Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)

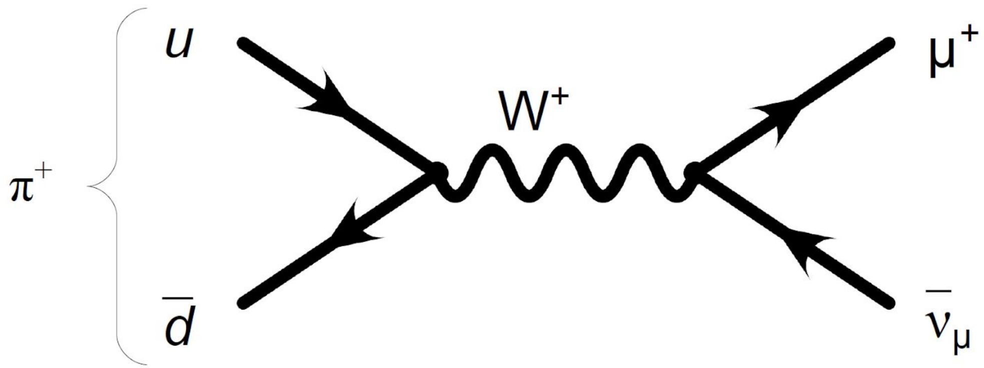
$c\tau = 7.8045$ m

π^+ DECAY MODES		Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\mu^+ \nu_\mu$	[b]	(99.98770 ± 0.00004) %		30
$\mu^+ \nu_\mu \gamma$	[c]	$(2.00 \pm 0.25) \times 10^{-4}$		30

Handed-ness and hadronic decays: The Pion



electromagnetic interaction



weak interaction

Handed-ness and hadronic decays: The Pion

$\pi^{\pm}$ $\qquad I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57018 \pm 0.00035$ MeV $\quad (S = 1.2)$
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s $\quad (S = 1.2)$
$c\tau = 7.8045$ m

⋮

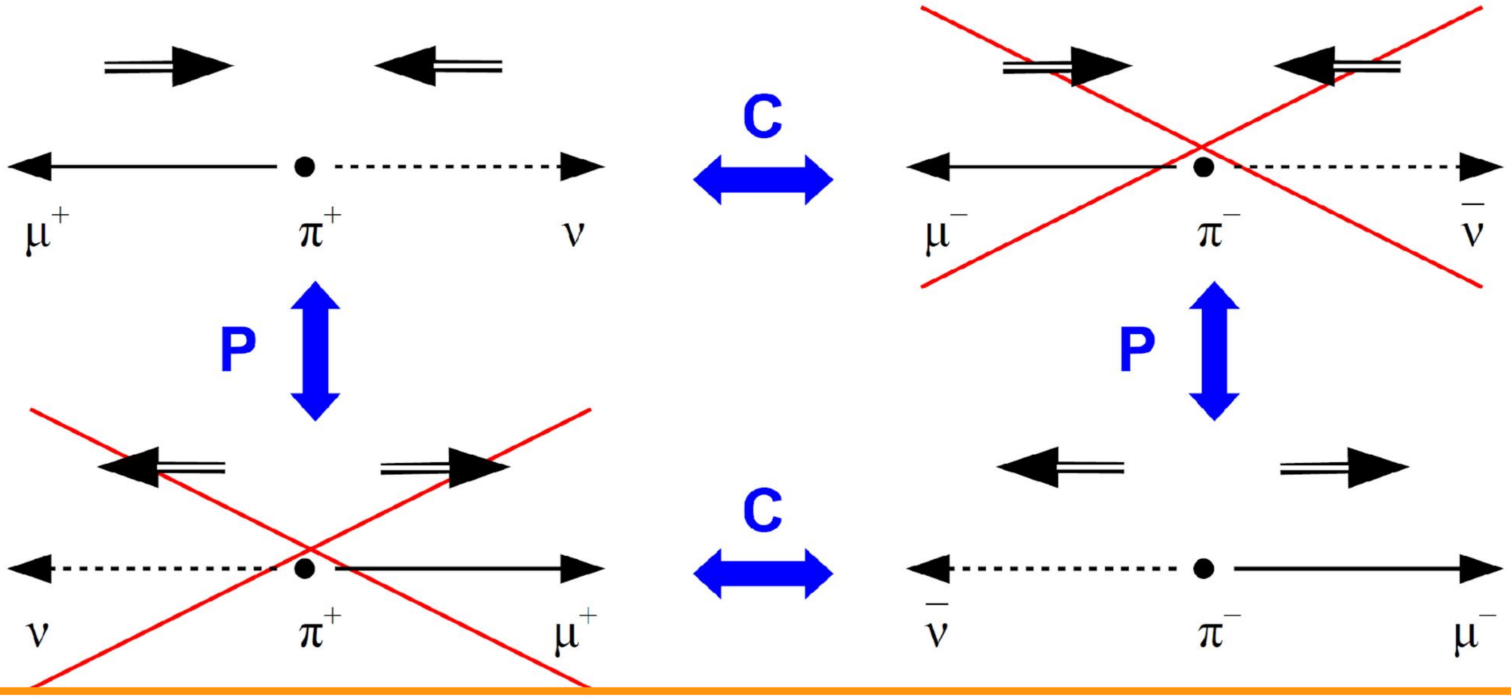
π^+ DECAY MODES		Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
$\mu^+ \nu_\mu$	[b]	(99.98770 ± 0.00004) %		30
$\mu^+ \nu_\mu \gamma$	[c]	$(2.00 \pm 0.25) \times 10^{-4}$		30
$e^+ \nu_e$	[b]	$(1.230 \pm 0.004) \times 10^{-4}$		70
$e^+ \nu_e \gamma$	[c]	$(7.39 \pm 0.05) \times 10^{-7}$		70
$e^+ \nu_e \pi^0$		$(1.036 \pm 0.006) \times 10^{-8}$		4

⋮

why is the decay to muon and neutrino so much more likely than the decay to electron and neutrino, although the muon is much heavier than the electron?

Handed-ness and hadronic decays: The Pion

- neutrino is left-handed, π has spin 0
  $\Rightarrow$ charged lepton also has to be left-handed, which is the “wrong” spin
- the heavier the charged lepton, the less suppressed is the wrong helicity, proportional to (1-v/c)



- left-handedness of neutrinos also means that weak interaction violates C, but CP can be conserved (and indeed CP violation is much smaller)

LEP: Cross section of $e^+e^- \rightarrow \mu^+\mu^-$

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right]$$

γ — γ/Z interference — Z

γ/Z interference: vanishes at $\sqrt{s} \approx M_Z$

$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1+\cos^2\theta) = (1+\cos^2\theta)$$

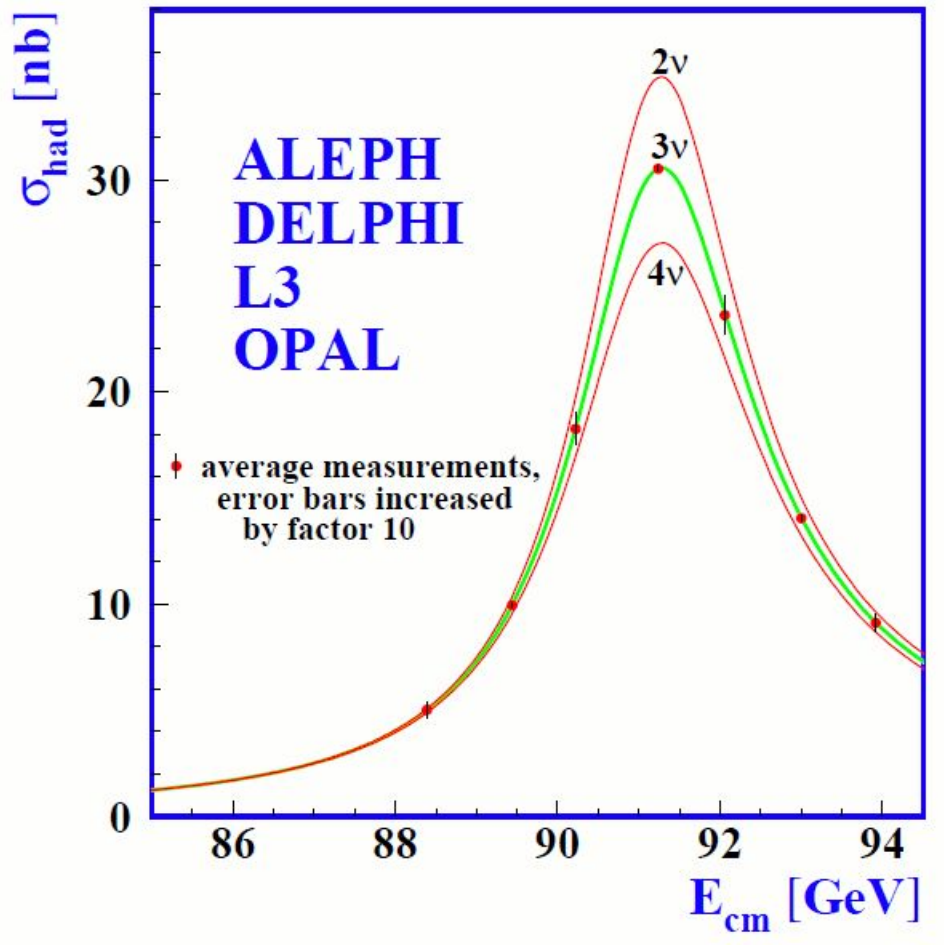
$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W\cos^2\theta_W}\left[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^\mu\cos\theta\right]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W\cos^4\theta_W}\left[({g_V^e}^2 + {g_A^e}^2)({g_V^\mu}^2 + {g_A^\mu}^2)(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu\cos\theta\right]$$

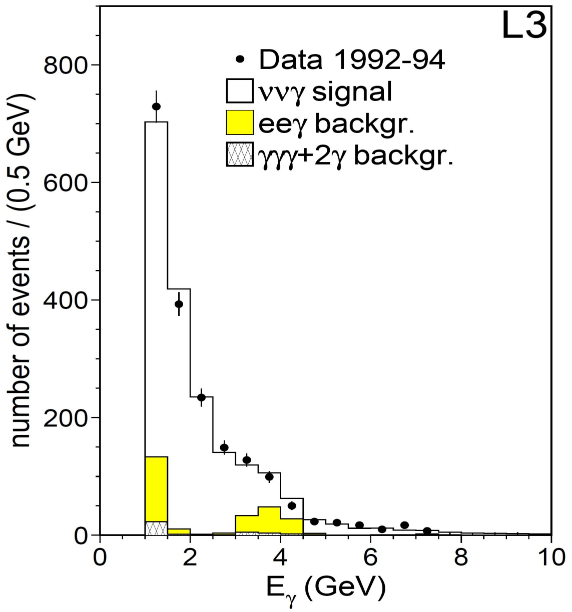
$\alpha = \alpha(m_Z)$: running electromagnetic coupling [$\alpha(m_Z) = \alpha/(1-\Delta\alpha)$ with $\Delta\alpha \approx 0.06$]

g_V, g_A = c_V, c_A: effective coupling constants (vector and axial vector)

LEP: Number of light neutrinos



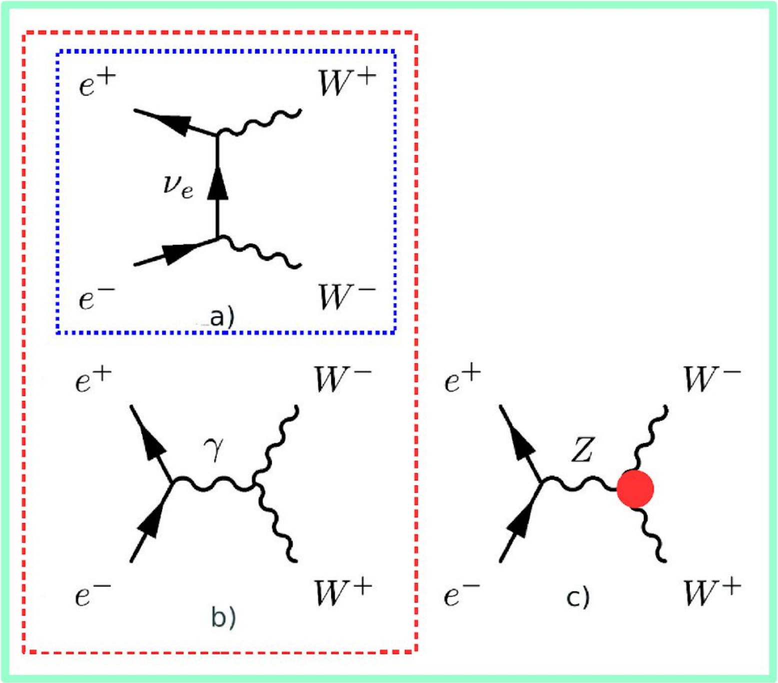
> Data selected using invisible Z decays with photon radiation

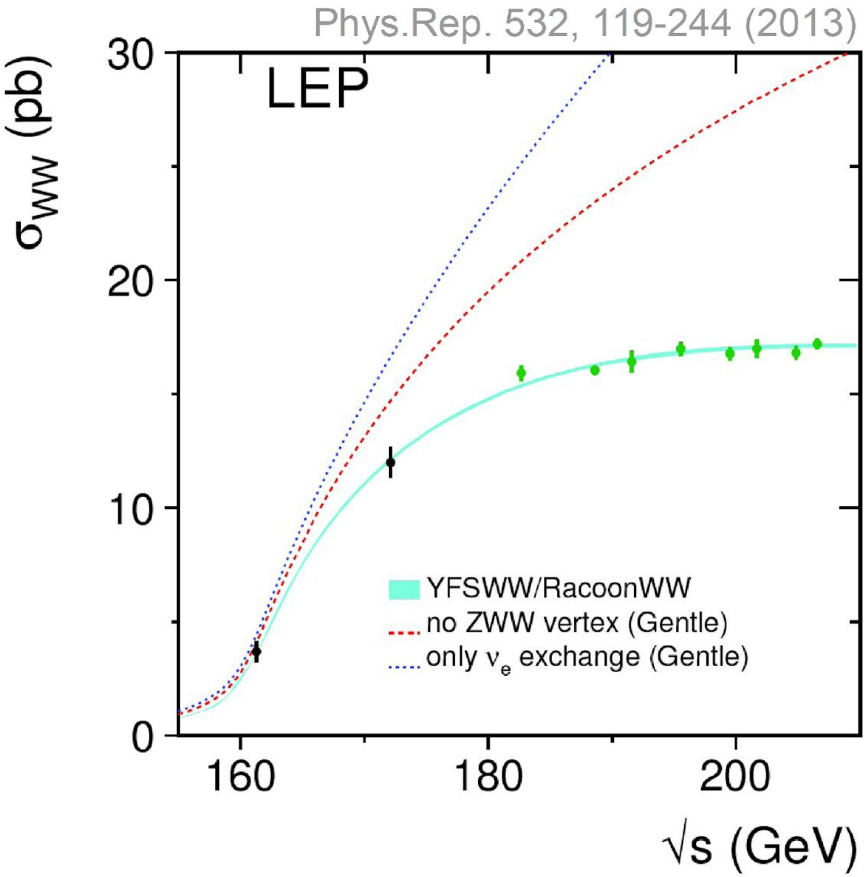


> Width of Z boson depends of the number of decay channels

LEP: WW production and the TGC vertex

- LEP also proved self-interaction of weak bosons through indirect measurement of triple gauge coupling vertex
- Interference between all three diagrams leads to "safe" energy behavior





Gauge invariance and Noether's theorem

Noether's theorem (informal version):
If a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved in time.

quantity	interaction			invariance
	strong	elm.	weak	
energy	yes	yes	yes	translation in time
momentum	yes	yes	yes	translation in space
angular momentum	yes	yes	yes	rotation in space

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0,$$

As in classical mechanics, equations of motions (e.g. Dirac equation) can be obtained from Lagrangian (or rather: can construct Lagrangian to further analyse equations of motions)

Gauge invariance and Noether's theorem

- Invariance of field equations describing electromagnetic and weak interaction (→ Lagrangian)

- Gauge (phase) transition possible (without changing effect of interaction) for electron (→ U(1) symmetry: U(A)U(B) = U(B)U(A):
-

$$\psi(x) \rightarrow e^{i\alpha}\psi(x),$$

$$\partial_\mu \psi \rightarrow e^{i\alpha}\partial_\mu \psi,$$

$$\bar{\psi} \rightarrow e^{-i\alpha}\bar{\psi}.$$

- Apply to field questions

- These extra phase terms ca  ns and we are left with:

$$\partial_\mu j^\mu = 0,$$

- Global gauge invariance → α fixed to one value everywhere

>Conservation of electrical charge!

Gauge invariance and Noether's theorem

- More general: local gauge invariance, α depends on space points

- With some tweaking to make this gauge invariance work, we arrive at the Lagrangian for the QED (Quantum electrodynamics):
- 

- Note: This was done posteriori (i.e. after QED was already know)

- One take away message however for theorists: It only works, if the photon is massless (and this also explains problems with the weak boson masses)
- Works well also for QCD (and naturally requires gluon self-interaction)

- **Find a solution that allows for gauge invariance in EWK interactions including massive W and Z bosons**

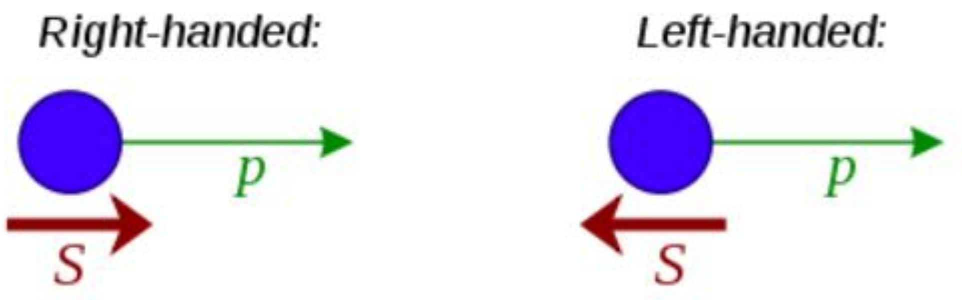
Let's go back to the properties of an interaction

Helicity

Particles whose momentum direction aligns with spin → right-handed particles

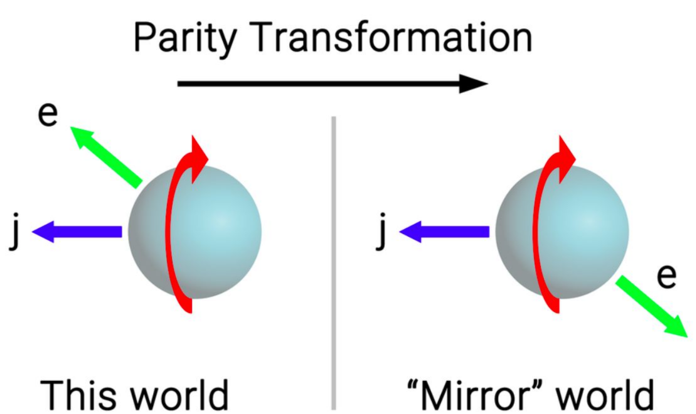
Particles whose momentum direction aligns with spin → left-handed particles

Dependent on the reference frame.



Parity

Mirror the coordinates of the particle.
Changes sign of momentum, coordinates
Spin doesn't change sign.



Charge conjugation

Change a particle by its anti-particle

Let's go back to the properties of an interaction

Chirality

Identical to helicity in the massless case but something more complicated

It tells how two separate components of a fermionic field change under Lorentz boost (space-time change) → Weyl spinors. Each fermion has a left-handed component and a right-handed one

Parity transformations change chirality

General Lorentz transformation

$$S = \exp\begin{bmatrix} \frac{1}{2}i\boldsymbol{\sigma}\cdot\boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\phi} & 0 \\ 0 & \frac{1}{2}i\boldsymbol{\sigma}\cdot\boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\phi} \end{bmatrix}$$

θ: angle in space rotations

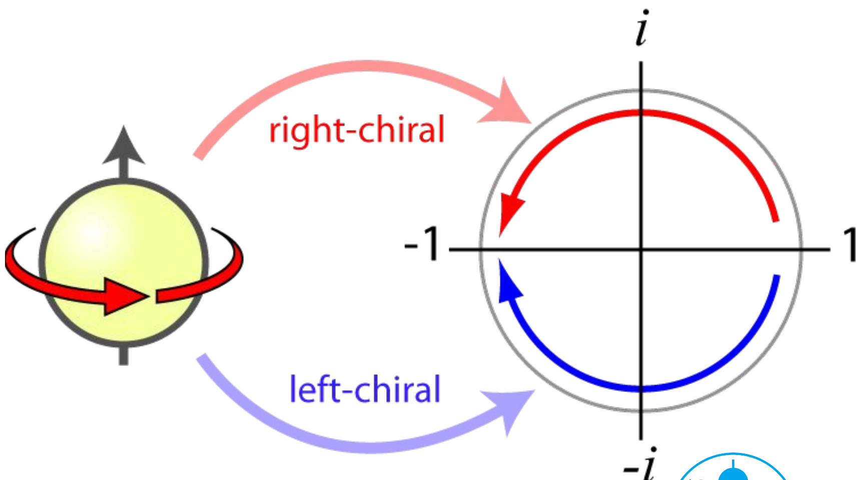
Φ: boost (time and space rotation)

Spinor of fermion: 2 terms (Weyl spinor) with 2 components

$$\Psi = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix}$$

$$\psi'_R = \exp\left(\frac{1}{2}i\boldsymbol{\sigma}\cdot\boldsymbol{\theta} - \frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\phi}\right)\psi_R$$

$$\psi'_L = \exp\left(\frac{1}{2}i\boldsymbol{\sigma}\cdot\boldsymbol{\theta} + \frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\phi}\right)\psi_L$$



Mass terms in Lagrangian

$$-m\bar{\Psi}\Psi = -m(\bar{e}_L e_R + \bar{e}_R e_L)$$

Towards a QFT of weak interactions → Electroweak

> Problem: Divergences! Theory only valid at low energies

$$\mathcal{L}_\mu = \frac{G_\mu}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\mu\right] \left[\bar{e}\gamma_\lambda (1-\gamma_5)\nu_e(x)\right] + \text{c.c.}\ .$$

→

$$\sigma^{e^- + \nu_e \to e^- \nu_e} = \frac{4G_F^2}{\pi} E_{\text{CM}}^2$$

Additional problems:

- Radiative corrections divergent (but needed)
- Unitarity violated at high energy (cross-section goes to very large values)
- Loop processes such as K^0 mixing meaningless/incorrect
- Need theory which includes Feynman theory but does not diverge at high energies

1960-1968: Formulation of electroweak unification
(Glashow, Salam, Weinstein) → **massive W/Z bosons + massless γ**

Gauge theory for the weak interaction → Electroweak

A gauge theory is a QFT theory that is invariant under local transformations
Local transformation = transformation that is not the same in all space → x+Δx with Δx = f(x)

This is the way that interactions are introduced in the SM → Find a variable/symmetry of the interaction and formulate a theory including a new gauge boson that would make the lagrangian invariant.

Lagrangian of the free Dirac field

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu\partial_\mu - mc^2)\psi$$

Invariance under phase transformations?
Physics should be similar

$$\psi(x) \rightarrow e^{iq\Lambda(x)/\hbar}\psi(x)$$
$$\bar{\psi}(x) \rightarrow \bar{\psi}(x)e^{-iq\Lambda(x)/\hbar}$$

$$\mathcal{L} \rightarrow \bar{\psi}e^{-iq\Lambda(x)/\hbar}(i\hbar c\gamma^\mu\partial_\mu - mc^2)e^{iq\Lambda(x)/\hbar}\psi =$$
$$= \bar{\psi}(i\hbar c\gamma^\mu(\partial_\mu + iq\partial_\mu\Lambda(x)/\hbar) - mc^2)\psi$$

Introduce A_μ to absorb δΛ(x)

$$A_\mu \rightarrow A_\mu + \partial_\mu\Lambda(x) \qquad D_\mu = \partial_\mu - \frac{i}{\hbar}qA_\mu$$

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu(\partial_\mu - iqA_\mu/\hbar) - mc^2)\psi$$

Lagrangian of the QED (U(1))

$$\mathcal{L} = \bar{\psi}(i\hbar c\gamma^\mu D_\mu - mc^2)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Gauge theory for the weak interaction → Electroweak

Transformations are described by a Lie group of transformations → Includes the whole ensemble of NxN matrices that make a transformation possible in a space defined by N-sized vectors.

In QFT, we us unitary matrices U(N) and SU(N) (with det(U))=1

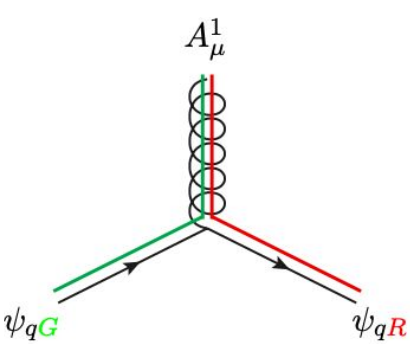
- Each transformation can be described by a set of N^2-1 matrices → generators
- In the context of QFT, one gauge boson per generator

A complex example: SU(3) or also called QCD !!

- Strong interaction behaves equally for 3 different colors → Gauge invariance from color transformations → 3 different directions : red, blue and green
- Transformations between 3 colors leave invariant the lagrangian
- 8 different generators → One gauge boson per generator ! 8 glu

gluon

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - ig_s t^a_{ij} A^a_\mu$$



$$\propto \; -\tfrac{i}{2}g_s \quad \bar{\psi}_{qR} \quad \lambda^1 \quad \psi_{qG}$$

$$= -\tfrac{i}{2}g_s \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

generators of rotation in SU(3)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

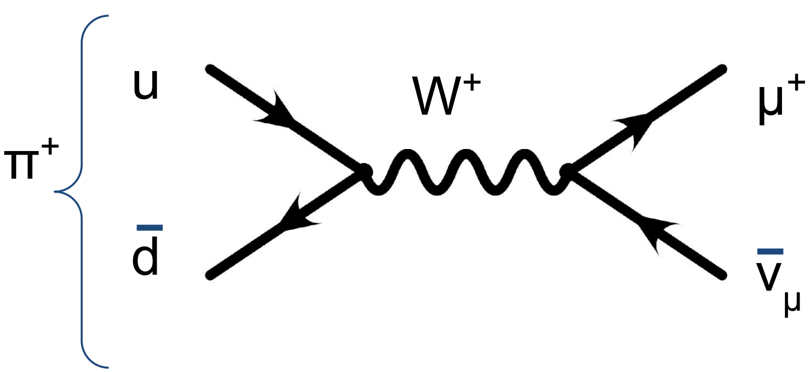
Electroweak interactions

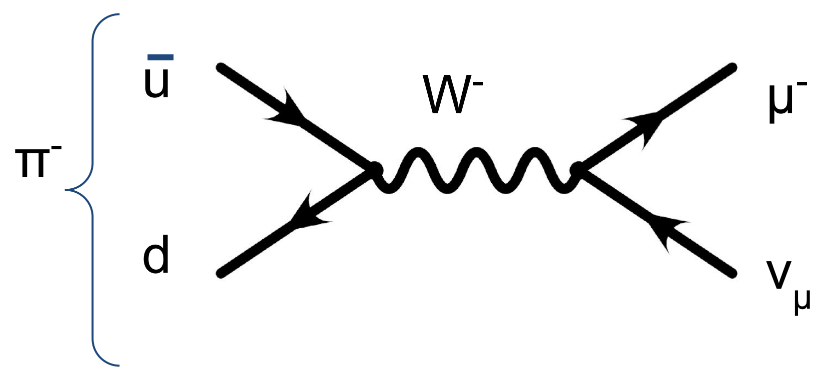
Usually observed charged (positive and negative) interactions with long-lifetime → Propose mediated by two charged heavy bosons!

- Consider W+/W- as doublets of the charge current
- Introduce in QFT → Postulate $SU(2)_L$ symmetry acting only on the left-handed fermions.
  - Observed that weak interaction between two distinct particles → Introduction of weak isospin
  - A 3rd gauge field → Introduction of *neutral current !*

$$\psi_L = \begin{pmatrix} e_L \\ \nu_e \end{pmatrix}$$

- Try preserving $SU(2)_L$ and $U(1)_Q$ symmetry → Introduce Hypercharge Y_L to preserve $U(1)_Y$
- Arrive at a unified interaction with massive W/Z boson +massless γ





Flavour and weak interaction: CKM matrix

> Expansion of GIM mechanism to include CP-violating effects in weak interactions led to CKM matrix: Cabibbo-Kobayashi-Maskawa ability

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} = \begin{bmatrix} 0.97370 \pm 0.00014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 0.221 \pm 0.004 & 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\ 0.0080 \pm 0.0003 & 0.0388 \pm 0.0011 & 1.013 \pm 0.030 \end{bmatrix}$$

> observed that the current Cabibbo
explain CP violation (can rotate 2
- Added a 3rd generation in
- CKM matrix explaining mix

> Measurement show diagonal te
prob

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{bmatrix}$$

> Reduce the amount of information

> Change style of subsequent slides

$$\mathcal{L}_C = -\frac{g}{\sqrt{2}} \left[\overline{u}_i \gamma^\mu \frac{1-\gamma^5}{2} M_{ij}^{CKM} d_j + \overline{\nu}_i \gamma^\mu \frac{1-\gamma^5}{2} e_i \right] W_\mu^+ + \text{h.c.} ,$$

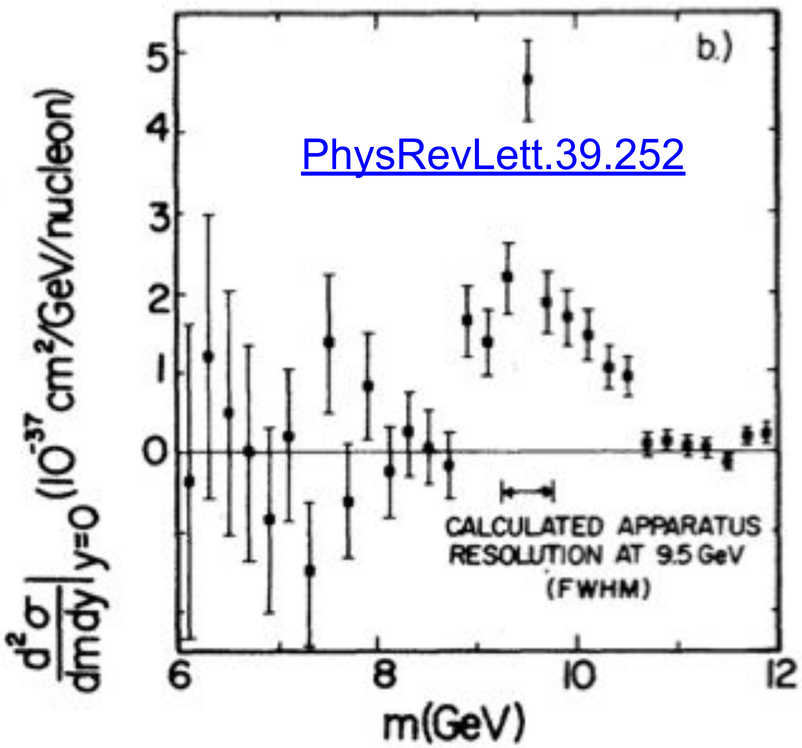
Flavour and weak interaction: CKM matrix

- **CKM matrix:** Cabibbo-Kobayashi-Maskawa observed that the current Cabibbo matrix (2d and u,d,s,c) couldn't explain CP violation (can rotate 2d matrix to absorb any phase)
  - Added a 3rd generation in order to include a CP-violation phase
  - CKM matrix explaining mixing in charged currents with quarks
- Mea… diagonal, lower probability

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \quad \ldots \begin{matrix} 0014 & 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 04 & 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\ 003 & 0.0388 \pm 0.0011 & 1.013 \pm 0.030 \end{matrix} \Bigg] .$$

$$\begin{bmatrix} 1 & & \\ 0 & c & \\ 0 & - & \end{bmatrix} \ldots \begin{matrix} s_{12} & 0 \\ c_{12} & 0 \\ 0 & 1 \end{matrix} \Bigg]$$

$$= \Bigg[\begin{matrix} \\ - \\ s \end{matrix} \ldots \begin{matrix} s_{13} e^{-i\delta_{13}} \\ s_{23} c_{13} \\ c_{23} c_{13} \end{matrix} \Bigg] .$$



Discovery of bottom-quark in 1977 at Fermilab !

$$\mathcal{L}_C = -\frac{g}{\sqrt{2}} \left[\overline{u}_i \, \gamma^\mu \, \frac{1-\gamma^5}{2} \, M^{\mathrm{CKM}}_{ij} \, d_j + \overline{\nu}_i \, \gamma^\mu \, \frac{1-\gamma^5}{2} \, e_i \right] W^+_\mu + \mathrm{h.c.} \, ,$$

Flavour and weak interaction: CKM matrix

> **CKM matrix:** Cabibbo-Kobayashi-Maskawa observed that the current Cabibbo matrix (2d and u,d,s,c) couldn't explain CP violation (can rotate 2d matrix to absorb any phase)

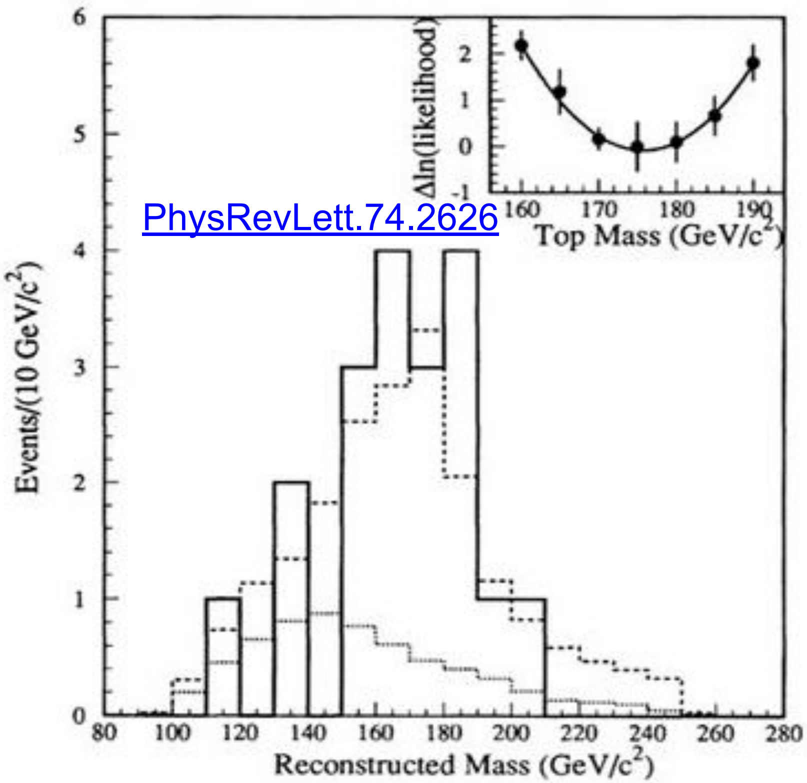
- Added a 3rd generation in order to include a CP-violation phase
- ... s with quarks

> Meas ... onal, lower probability

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \Bigg[\quad \cdots \quad \begin{matrix} 0.2245 \pm 0.0008 & 0.00382 \pm 0.00024 \\ 0.987 \pm 0.011 & 0.0410 \pm 0.0014 \\ 0.0388 \pm 0.0011 & 1.013 \pm 0.030 \end{matrix} \Bigg] .$$

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & c_2 & \cdots & 0 \\ 0 & -s & \cdots & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cdots & \cdots & {}_{13}e^{-i\delta_{13}} \\ -s_1 & \cdots & s_{23}c_{13} \\ s_{12} & \cdots & c_{23}c_{13} \end{bmatrix} .$$



PhysRevLett.74.2626

Discovery of the top-quark at Tevatron in 1995 !

$$\mathcal{L}_C = -\frac{g}{\sqrt{2}} \left[\overline{u}_i \, \gamma^\mu \, \frac{1-\gamma^5}{2} \, M_{ij}^{\mathrm{CKM}} \, d_j + \overline{\nu}_i \, \gamma^\mu \, \frac{1-\gamma^5}{2} \, e_i \right] W_\mu^+ + \mathrm{h.c.} \, ,$$

How can we get massive gauge bosons → BEH mechanism

We know weak interaction must have massive gauge bosons but gauge theories don't allow them→ Spontaneous symmetry breaking

Underlying physics law keeps the symmetry (Lagrangian is still symmetric), but in reality, the ground state of the theory doesn't preserve the symmetry.

- Add scalar field with a particular potential
- If $\mu^2 > 0$, potential follows the "Mexican" hat form
    - Minimum of potential is not for <Φ> = 0 but for:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

v = Vacuum expectation value (VEV)

- Ground state of field, a certain value with $v = \mu^2/\lambda$
- Excitations around VEV. Quanta of the field → Higgs bosons

$$\mathcal{L}_{Higgs} = (D^\mu\phi)^\dagger (D_\mu\phi) - V(\phi)$$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$