Test of the Noncommutative Standard Model at the LHC

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Analogy to quantum mechanics: postulation of noncommutative space-time

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = i\frac{C_{\mu\nu}}{{\Lambda_{NC}}^2} \quad \Rightarrow \quad \Delta\hat{x}_\mu \cdot \Delta\hat{x}_\nu \geqslant \frac{\theta_{\mu\nu}}{2}$$

Current collider bounds on the noncommutative scale Λ_{NC}:

- $\Lambda_{NC} > 141$ GeV (OPAL: $e^+e^- \to \gamma\gamma$ in NCQED [Abbiendi et al.])

Theoretically, hypothesis/appearance of length/area scale:

- historically regularisation of infinites in QFT [Heisenberg/Pauli/Snyder]
- NCQFT as low energy limit of string theories [Seiberg/Witten]:

String theory and noncommutative geometry.
Nathan Seiberg, Edward Witten (Princeton, Inst. Advanced Study) . IASSNS-HEP-99-74, Aug 1999. 99pp.
Published in **JHEP 9909:032,1999**
e-Print Archive: **hep-th/9908142**

TOPCITE = 1000+
References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 1937 times | More Info
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- possible cut-off for removing divergent contributions to quantum gravity

- special case and useful approximation: $\theta^{\mu\nu}$ constant 4×4-matrix:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} = i\frac{1}{{\Lambda_{NC}}^2} C^{\mu\nu} = i\frac{1}{{\Lambda_{NC}}^2} \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

- effective lagrangians

$$\mathcal{L}_{\text{eff.}} = \cdots + g\bar{\psi}(\hat{x})\gamma_\mu(1-\gamma_5)\psi(\hat{x})W^\mu(\hat{x}) + \cdots$$

with product of functions of noncommuting variables

$$(fg)(\hat{x}) = f(\hat{x})g(\hat{x})$$

realised by Moyal-Weyl $\star$-products of functions of commuting variables:

$$(f\star g)(x) = f(x)e^{\frac{i}{2}\overleftarrow{\partial^\mu}\theta_{\mu\nu}\overrightarrow{\partial^\nu}}g(x) = f(x)g(x) + \frac{i}{2}\theta_{\mu\nu}\frac{\partial f(x)}{\partial x_\mu}\frac{\partial g(x)}{\partial x_\nu} + \mathcal{O}(\theta^2)$$

Note: $[x_\mu \stackrel{\star}{,} x_\nu](x) = (x_\mu \star x_\nu)(x) - (x_\nu \star x_\mu)(x) = i\theta_{\mu\nu} = [\hat{x}_\mu, \hat{x}_\nu]$

1. Replace products in $\mathcal{L}$ with Moyal-Weyl $\star$-products:

$$\mathcal{L}_{SM} = i\bar{\psi}\not{A}\psi + \cdots \longrightarrow \quad \mathcal{L}_{NC} = i\bar{\psi}\star\not{A}\star\psi + \cdots$$

problem: only $U(N)$-gauge theory on NC space-time possible

$$\left([A^a_\mu T^a \overset{\star}{,} A^b_\nu T^b] = \frac{1}{2}[A^a_\mu \overset{\star}{,} A^b_\nu]_- [T^a, T^b]_+ + \frac{1}{2}[A^a_\mu \overset{\star}{,} A^b_\nu]_+ [T^a, T^b]_- \right)$$

2. Express noncommutative fields by commutative fields

$$A_\mu,\ \Psi,\ \lambda \rightarrow \widehat{A}_\mu[A, \theta],\ \widehat{\Psi}[\Psi, A, \theta],\ \widehat{\lambda}[\lambda, A, \theta]$$

and NC gauge transformations through ordinary gauge transformations:

$$\text{e.g. } \hat{A}_\rho(A, \theta) + \delta\hat{A}_\rho(A, \theta) \quad \overset{!}{=} \quad \hat{A}_\rho(A + \delta A, \theta)$$

solution: Seiberg-Witten-maps, as power series in θ

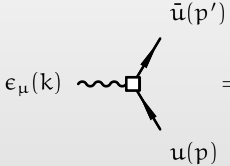
$$\text{e.g. } \widehat{A}_\mu(x) = A_\mu(x) + \frac{1}{4}\theta^{\rho\sigma}\left[A_\sigma(x), \partial_\rho A_\mu(x) + F_{\rho\mu}(x)\right]_+ + \mathcal{O}((\theta^{\mu\nu})^2)$$

allows NC generalization of $SU(N)$

3. Construction of a noncommutative $SU(3)_C \times SU(2)_L \times U(1)_Y$ effective theory [Wess et al.] as expansion in powers of θ

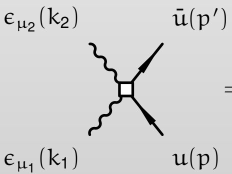
Feynman rules at first and second order in $\theta \propto \frac{1}{\Lambda_{NC}^2}$

- NC-corrections to SM-interactions:



$$= ig \cdot \left\{ \begin{array}{r} \frac{i}{2}\left[(k\theta)_\mu \not{p} + (\theta p)_\mu \not{k} - (k\theta p)\gamma_\mu\right] \\ + \frac{1}{8}(k\theta p)\left[(k\theta)_\mu \not{p} + (\theta p)_\mu \not{k} - (k\theta p)\gamma_\mu\right] \end{array} \right.$$

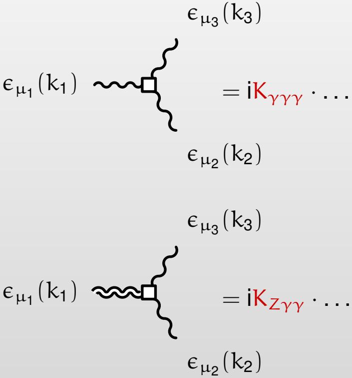
- New interactions:

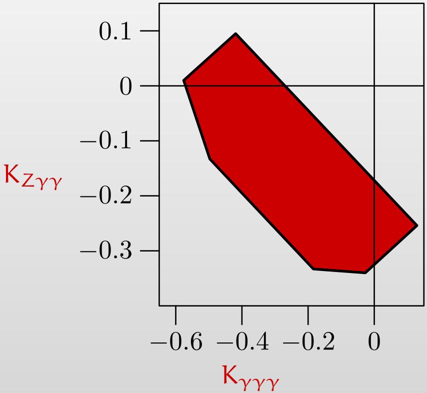


$$= ig^2 \cdot \left\{ \begin{array}{r} \frac{i}{2}\left[(\theta(k_1 - k_2))_{\mu_1}\gamma_{\mu_2} - (\theta(k_1 - k_2))_{\mu_2}\gamma_{\mu_1} \right. \\ \left. -\theta_{\mu_1\mu_2}(\not{k}_1 - \not{k}_2)\right] \\ + \frac{1}{8}\left[k_1\theta k_2\,(3p\theta^{\mu_1} - k_1\theta^{\mu_1} - k_2\theta^{\mu_1})\gamma^{\mu_2} \right. \\ \left. + 2[(k_1 + k_2)\theta^{\mu_2}\, p\theta^{\mu_1}]\not{k}_1 + \ldots\right] \end{array} \right.$$

(all momenta outgoing)

- New interactions: three neutral gauge bosons $\gamma\gamma\gamma$, $Z\gamma\gamma$, $ZZ\gamma$, ... at first order in θ

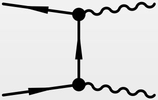


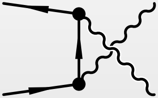


  - coupling constants not unique, yet constrained from matching the SM at $\theta \to 0$
  - no contribution from second order in θ

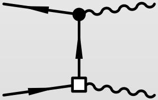
$$pp \to \gamma\gamma,\ Z\gamma,\ ZZ$$

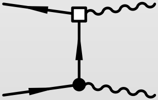
Standard Model:

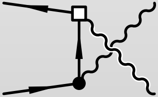
$A_t^{SM} =$ 

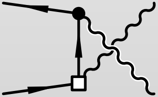
$A_u^{SM} =$ 

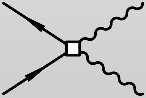
NCSM:

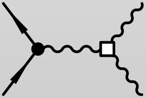
$A_{t,1}^{NC} =$ 

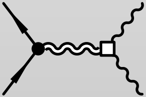
$A_{t,2}^{NC} =$ 

$A_{u,1}^{NC} =$ 

$A_{u,2}^{NC} =$ 

$A_c^{NC} =$ 

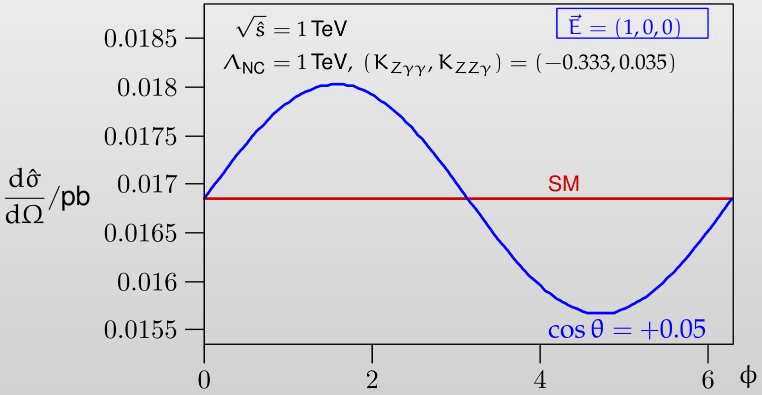
$A_{s,\gamma}^{NC} =$ 

$A_{s,Z}^{NC} =$ 

$q\bar{q} \to Z\gamma$

Discriminative effect: azimuthal dependence of cross sections

- antisymmetric in $\cos\theta \Rightarrow$ integration over one hemisphere
- dependency on $\vec{E}$ stronger than on $\vec{B}$



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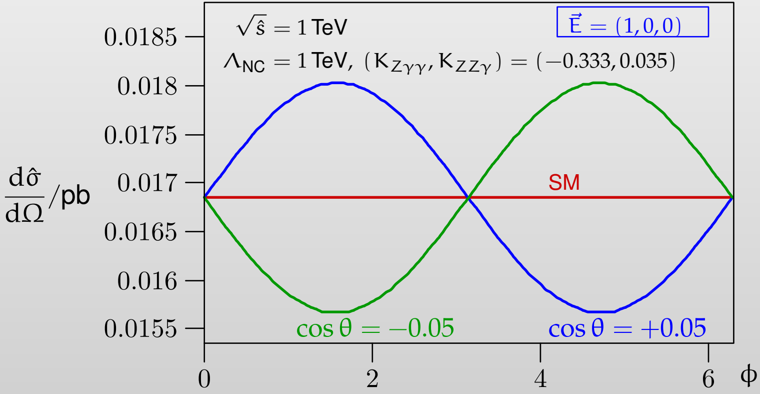

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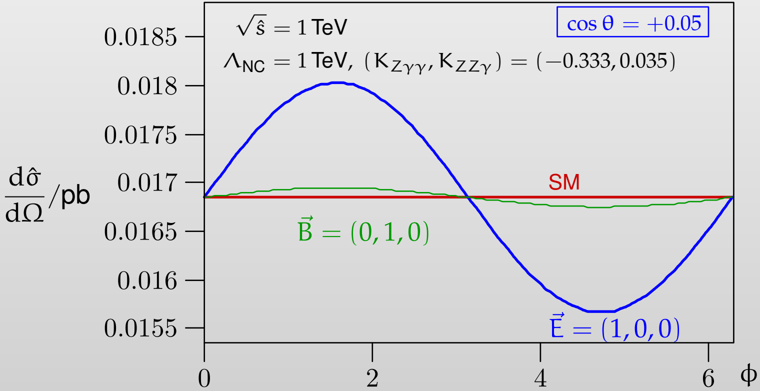
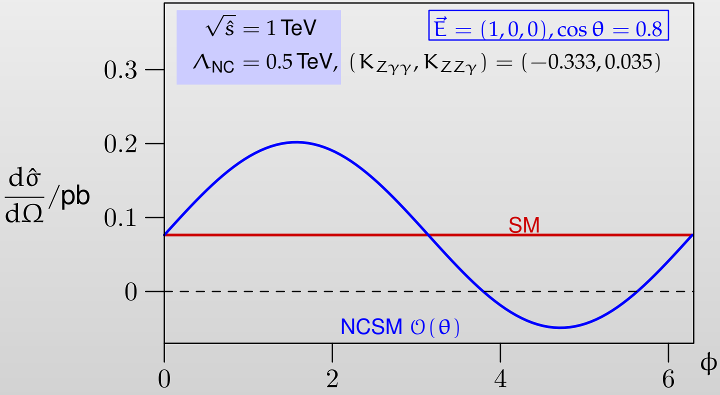


$$q\bar{q} \to Z\gamma$$

Discriminative effect: azimuthal dependence of cross sections

- if $\sqrt{\hat{s}}/\Lambda_{NC}$ large $\Rightarrow$ negative cross section $\Rightarrow$ upper cut on $\sqrt{\hat{s}}$

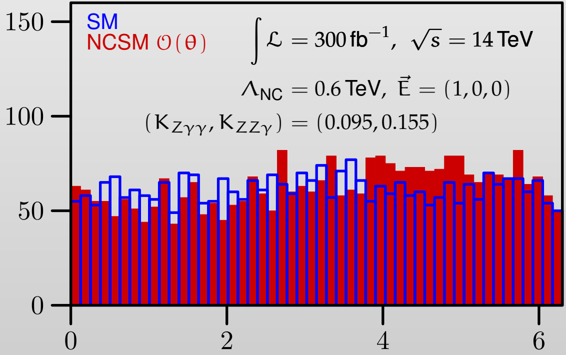


$$pp \to Z\gamma \to e^+e^-\gamma \text{ @LHC}$$

Azimuthal dependence of cross section - cuts:

- Effect $\propto \sqrt{\hat{s}}/\Lambda_{NC} \Rightarrow m_{e^+e^-\gamma} > 220$ GeV
- validity of $\mathcal{O}(\theta)$ approximation $\Rightarrow m_{e^+e^-\gamma} < 1$ TeV
- Effect antisymmetric in $\cos\theta^*_\gamma \Rightarrow 0 < \cos\theta^*_\gamma < 0.9$
- Separation of $q\bar{q}$ from $\bar{q}q$ initial state: $\langle x_{\text{valence}}\rangle \gg \langle x_{\text{sea}}\rangle$
  $\Rightarrow Z$ and γ in the same hemisphere: $\cos\theta_{Z,\gamma} > 0$

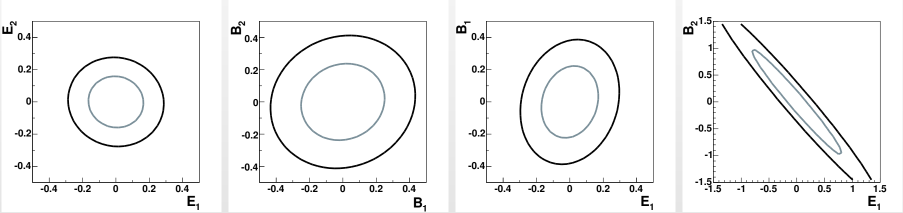
event generation using WHIZARD



azimuthal angle

Likelihood-Fits [A.A., Ohl, Rückl]:

$$e^+e^- \text{ channel}, \quad \sqrt{s} = 14\,\text{TeV}, \quad 100\,\text{fb}^{-1}, \quad \Lambda_{NC} = 500\,\text{GeV}$$



- just kinematical correlations of (E_1, B_2) and (E_2, B_1) because of Lorentz-boost along beam axis x_3

$$\theta^{\mu\nu} = \frac{C^{\mu\nu}}{\Lambda_{NC}^2}: \qquad C^{\mu\nu} \to \Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}C^{\rho\sigma}$$

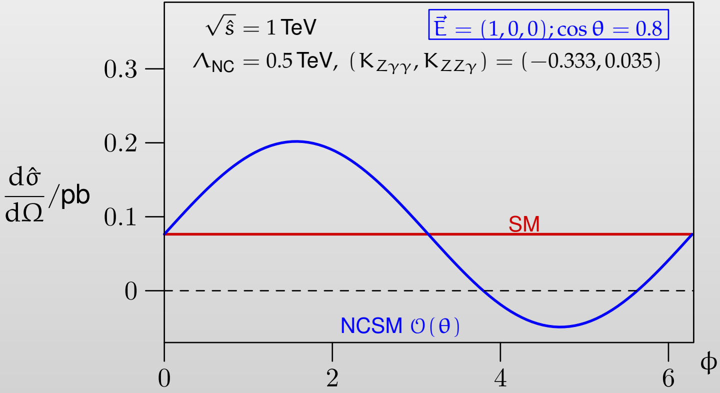
$$\text{i.p. } E_1 \to \gamma(E_1 - \beta B_2) \quad \text{with} \quad \beta = v/c,\ \gamma = 1/\sqrt{1-\beta^2}$$

- Derived bounds:

$$\left.\begin{array}{l} E_i = 1 \text{ all other } 0, \quad i = 1,2 \\ B_i = 1 \text{ all other } 0, \quad i = 1,2 \end{array}\right\} \Rightarrow \Lambda_{NC} \simeq 1.0\,\text{TeV}$$

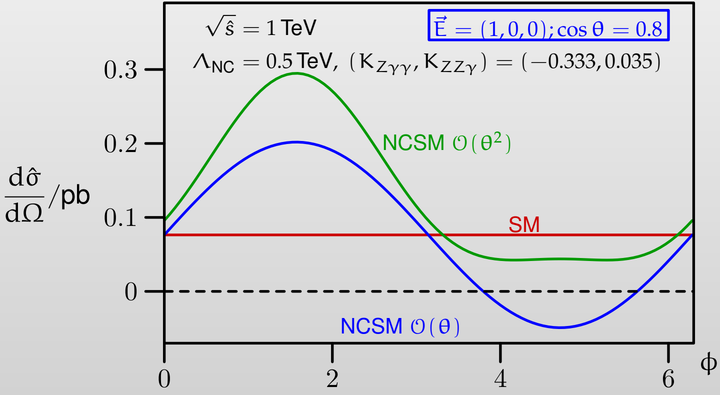
- regions in phase space with large $\sqrt{\hat{s}}/\Lambda_{NC} \Rightarrow$ negative cross section
- interference $\mathcal{A}_{NC}(\theta) \cdot \mathcal{A}^*_{SM}$ dominates $|\mathcal{A}_{SM}|^2$

$$q\bar{q} \rightarrow Z\gamma$$



- regions in phase space with large $\sqrt{\hat{s}}/\Lambda_{NC} \Rightarrow$ negative cross section
- interference $\mathcal{A}_{NC}(\theta) \cdot \mathcal{A}^*_{SM}$ dominates $|\mathcal{A}_{SM}|^2 \Rightarrow$second order in θ needed

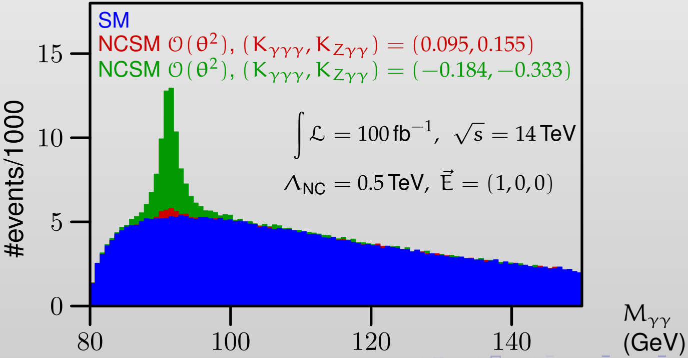
$$q\bar{q} \to Z\gamma$$



$pp \to \gamma\gamma$ @LHC

- at $\mathcal{O}(\theta)$ just minute observable NC effects in unpolarised cross section from interference of Z-exchange diagram with SM
- at $\mathcal{O}(\theta^2)$: peak in $m_{\gamma\gamma}$ distribution at m_Z, from squared Z-exchange diagram
- Cuts (same as for the light Higgs search):
  $p_T > 25\,\text{GeV}, |\eta| < 2.5, 80\,\text{GeV} < M_{\gamma\gamma} < 150\,\text{GeV}, DR_{\gamma\gamma} > 0.4$

event generation using WHIZARD



- QFT on NC space-time realised by means of Moyal-Weyl $\star$ product and Seiberg-Witten maps
- NCSM as expansion in powers of θ:
  - at $\mathcal{O}(\theta)$: unpolarised $pp \to Z\gamma \to e^+e^-\gamma$ phenomenologically richer than $pp \to \gamma\gamma$
  - exclusion limits @LHC: $\Lambda_{NC} \gtrsim 1.0\,\text{TeV}$ [A.A., Ohl, Rückl]
  - no polarisation required, unlike $\gamma\gamma \to e^+e^-$ [Ohl, Reuter, PRD70]
  - at $\mathcal{O}(\theta^2)$: extension of region of validity
  - inclusion $pp \to \gamma\gamma$
    (at $\mathcal{O}(\theta^{\mu\nu})$ only minute observable effect from $K_{Z\gamma\gamma} \neq 0$)

In progress:

- further phenomenological studies including also W's require corresponding Feynman rules up to $\mathcal{O}(\theta^2)$
- ZZZ, WWZ, ... acquire mass dependent NC-corrections coming from the Higgs sector
  $\Rightarrow$ hybrid SWM for Higgs at $\mathcal{O}(\theta^2)$ needed

$$\widehat{\phi} \longrightarrow \widehat{\phi}' = e^{i\widehat{\lambda}_L} \star \widehat{\phi} \star e^{-i\widehat{\lambda}_R}$$

Generalize

$$\widehat{\psi}[\psi, A] = \psi + \frac{1}{2}\theta^{\mu\nu}A_\nu\partial_\mu\psi + \frac{i}{8}\theta^{\mu\nu}\left[A_\mu, A_\nu\right]_- \psi + \mathcal{O}(\theta^2)$$

to a field transforming from the left and on the right under two arbitrary gauge groups:

$$\delta\widehat{\phi} = i\lambda_L \star \widehat{\phi} - i\widehat{\phi} \star \lambda_R$$

Hybrid Seiberg-Witten map:

$$\begin{aligned}\widehat{\phi}[\phi, A_L, A_R] = \phi &+ \frac{1}{2}\theta^{\mu\nu}A_{L\nu}\left(\partial_\mu\phi - \frac{i}{2}\left(A_{L\mu}\phi - \phi A_{R\mu}\right)\right)\\ &+ \frac{1}{2}\theta^{\mu\nu}\left(\partial_\mu\phi - \frac{i}{2}\left(A_{L\mu}\phi - \phi A_{R\mu}\right)\right)A_{R\nu} + \mathcal{O}(\theta^2)\end{aligned}$$