QCD axion dark matter from parametric resonance

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Collaborators: Marco Gorghetto, Mariachiara Ingicco, Geraldine Servant

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

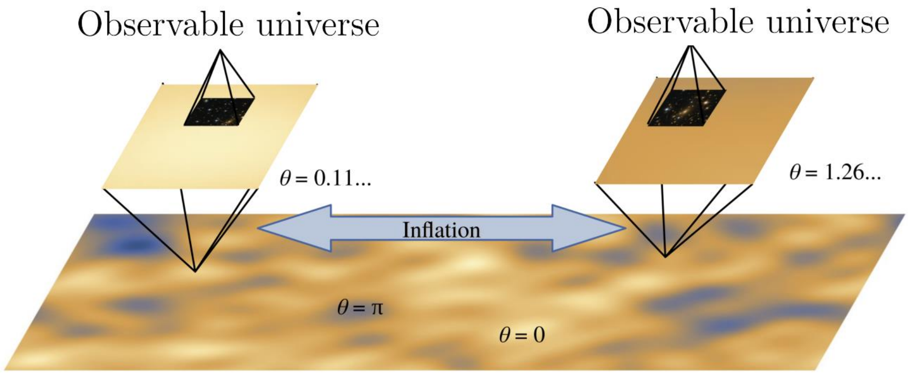
Outline

Goal: In this work, we investigate the evolution of the Peccei–Quinn field following a **large displacement** from its minimum, and the subsequent **growth of inhomogeneities** induced by **parametric resonance**.

1. **Axion Cosmology**
2. **Initial Field Displacement**
3. **Parametric Resonance**
4. **Axion Dark Matter production**
5. **String Loops Formation**

Axion Cosmology

Pre-inflationary



$$\theta \equiv \frac{a}{f_a} \in [-\pi, \pi]$$

$$\Omega_a \simeq \theta_0^2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{1.2} \Omega_{\mathrm{DM}}$$

- The axion field is homogeneous in our universe
- No presence of topological defects
- Isocurvature problem
- Different values of f_a can lead to the correct abundance

Pictures made by Marco Gorghetto

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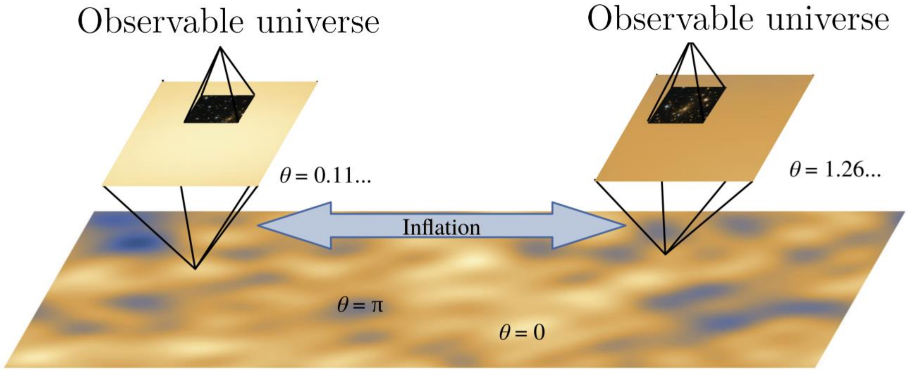

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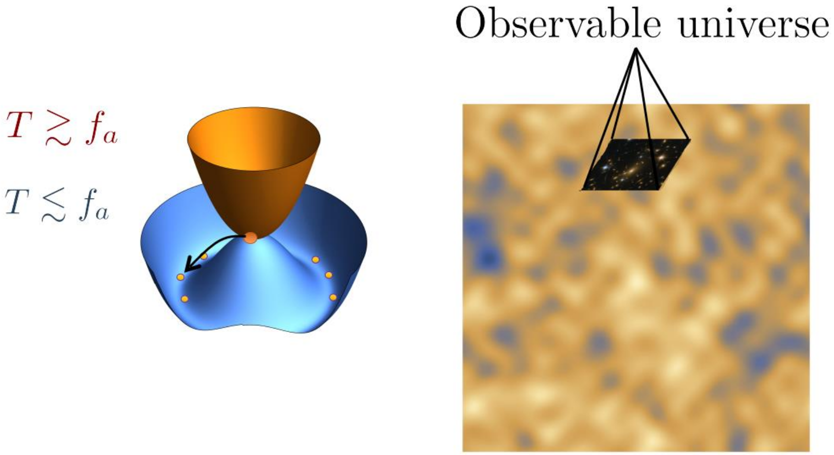
Post-inflationary



- Axion field is completely randomized
- Axion strings and domain walls are formed
- Only one value of f_a leads to the correct abundance

Pictures made by Marco Gorghetto

Initial Field Displacement

$$\Phi = \frac{\overset{\text{Saxion}}{\phi}}{\sqrt{2}} e^{i\underset{\text{Axion}}{\theta}}$$

Saxion

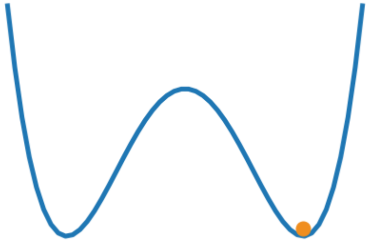
Axion

$$S = \int d^4x\sqrt{-g}\left(\partial_\mu\Phi\partial^\mu\Phi^* + cH^2|\Phi|^2 - V_{PQ}(|\Phi|)\right)$$

Peccei-Quinn Field

Hubble-induced mass

Peccei-Quinn Symmetry Breaking Potential



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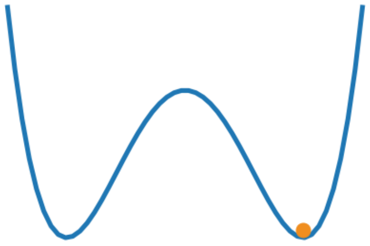

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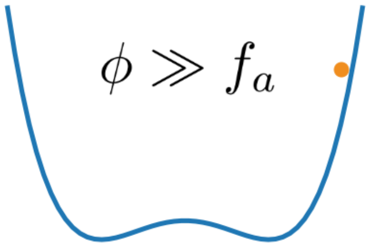

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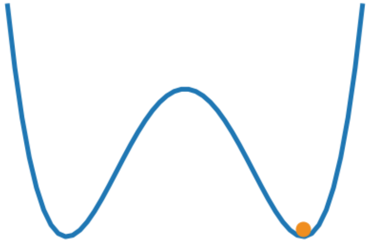

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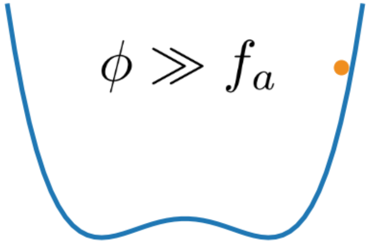

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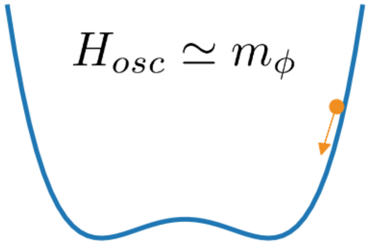
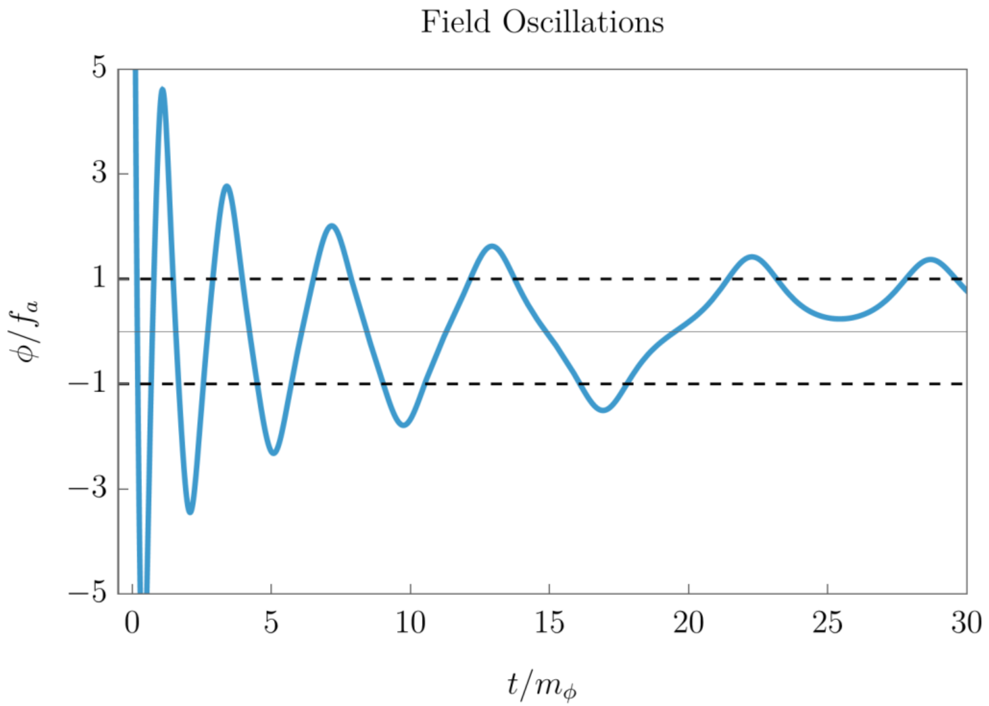


When the Hubble friction is weak enough, around $H_{osc} \simeq m_\phi$, the field starts oscillating

Parametric Resonance



We can decompose the Peccei-Quinn field in a ***homogenous component*** **+** ***spatial perturbations***

$$\phi(t,x) = \bar{\phi}(t) + \delta\phi(t,x)$$

Letting the homogeneous field oscillating and treating it as a background field, we can now study the dynamics of **perturbations**

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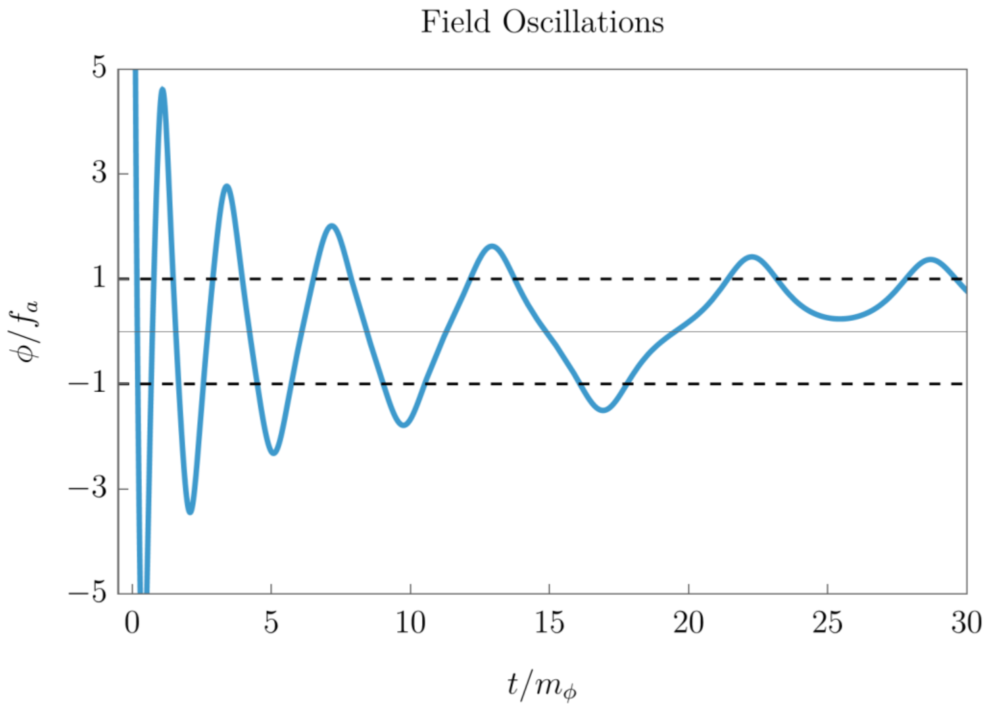

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Decomposing perturbations into Fourier modes we notice the arising of a **Hill differential equation**

$$\delta\ddot{\phi}_k + [k^2 + F(t)]\delta\phi_k = 0$$

$F(t+T) = F(t)$ periodic function

This equation admits **exponentially growing solutions** for those modes whose wave numbers resonate with the oscillation frequency of the background field

Parametric Resonance in a Quartic Potential

Let’s take the case of a **quartic potential** and decompose the field into its ***real*** and ***imaginary*** components

$$\frac{\lambda}{4}\left(|\Phi|^2 - f_a^2\right)^2$$

Cartesian basis: $\Phi = X + iY$

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$$\boxed{\begin{aligned} &x_k'' + [k^2 + 3\lambda\phi_i^2\cos(\sqrt{\lambda}\phi_i\tau)]x_k = 0 \\ &y_k'' + [k^2 + \lambda\phi_i^2\cos(\sqrt{\lambda}\phi_i\tau)]y_k = 0 \end{aligned}}$$

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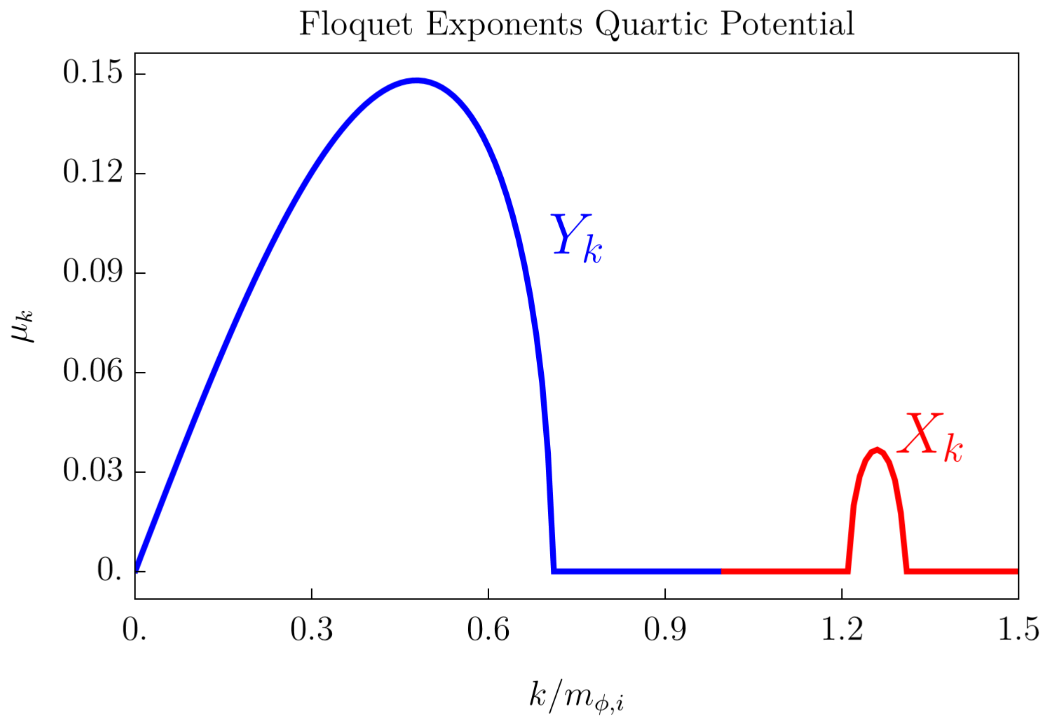
These equations are also known as Mathieu equations and are characterized by **instability bands** where the solutions exhibit **exponential growth**.

$$y_k \sim \exp\left(\mu_k\tau\right)$$

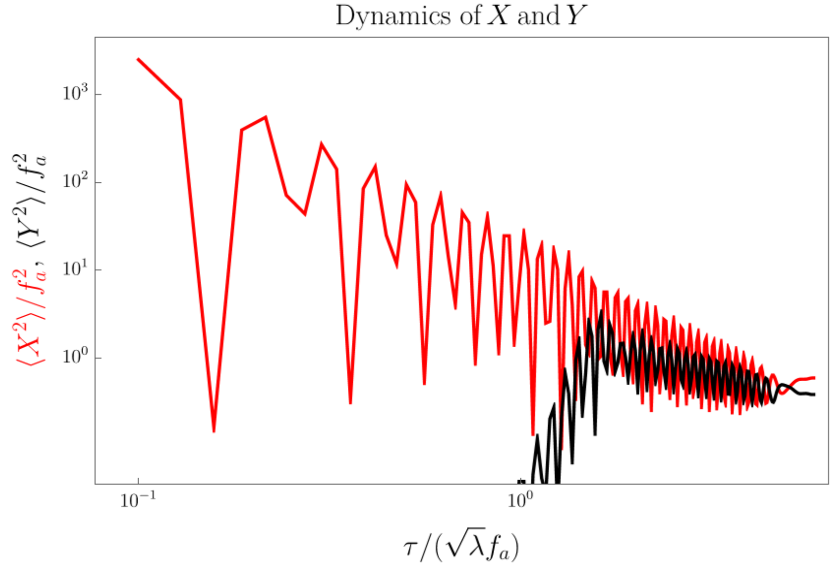
$$x_k \sim \exp\left(\mu_k\tau\right)$$



Axion Production from Parametric Resonance

The exponential growth provides a good approximation until **perturbations backreact** on the background field. At this point, we can no longer rely on linear analysis and need to perform **lattice simulations**.

$$X \sim \sqrt{\langle \delta Y^2 \rangle}$$



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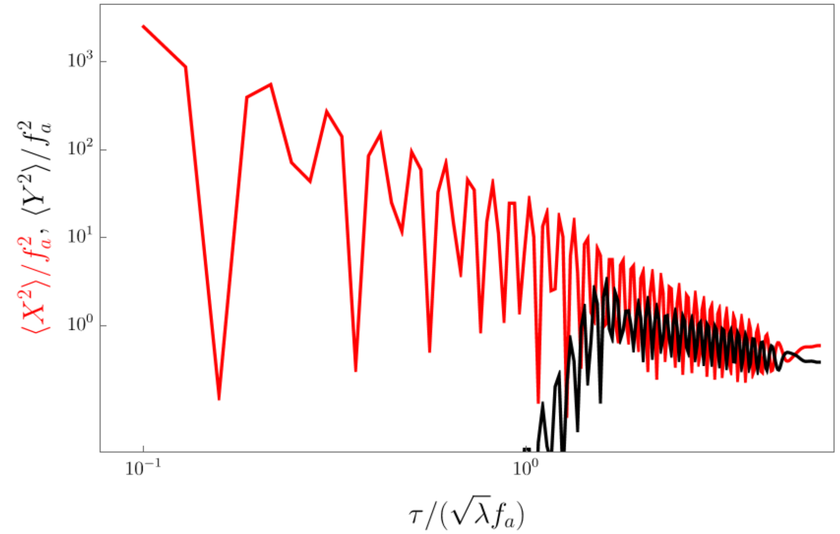
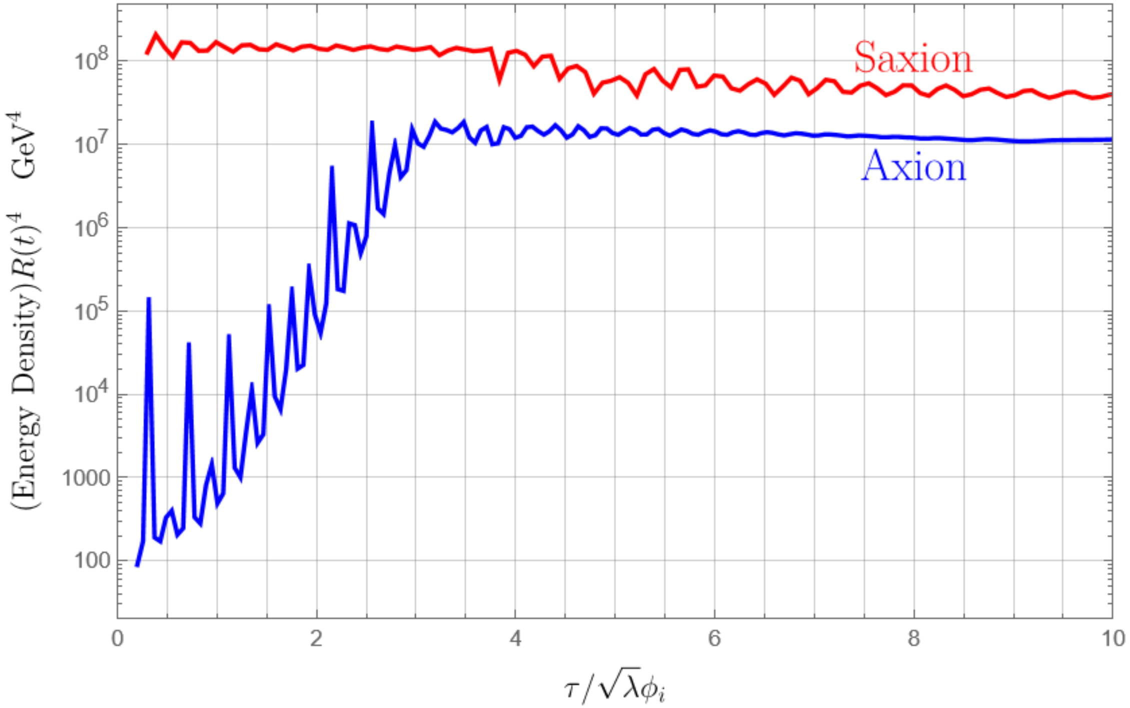
The growth of perturbations results into a **production of axions**

$$\frac{a}{f_a} = \arctan\left(\frac{Y}{X}\right)$$

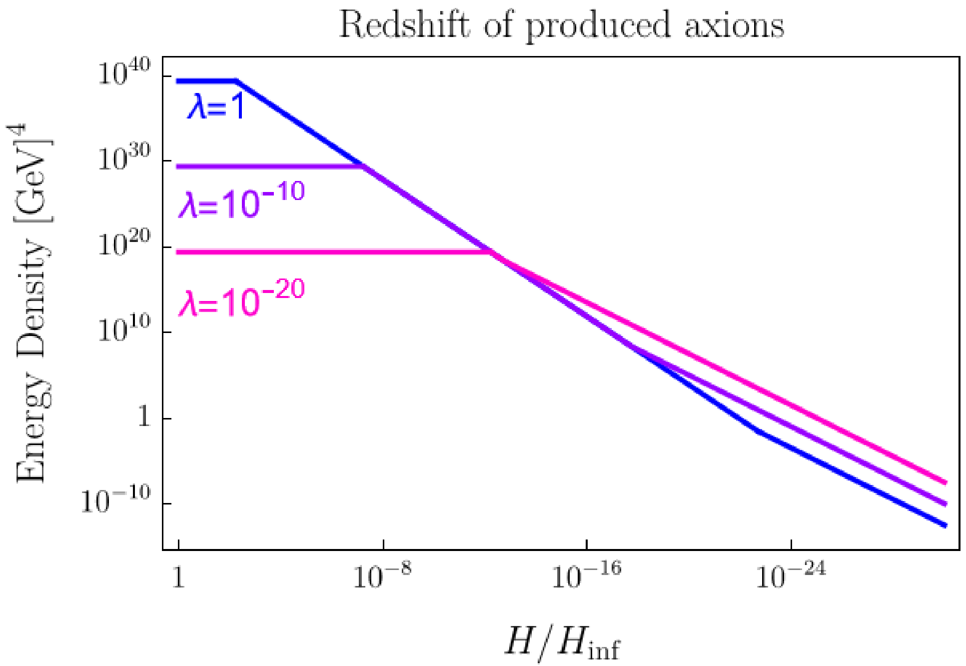
At the end of the process the axion energy density becomes **comparable** to the saxion energy density

$$\rho_a \sim \rho_\phi$$


Dynamics of X and Y



Axion Dark Matter



Axions are produced with a **momentum equal to the saxion mass** and redshift until they become **cold dark matter**.

$$k \simeq m_\phi \gg m_a$$

Avoiding bounds on warm dark matter results into requiring **small saxion masses**, this may be motivated by ***SUSY models***

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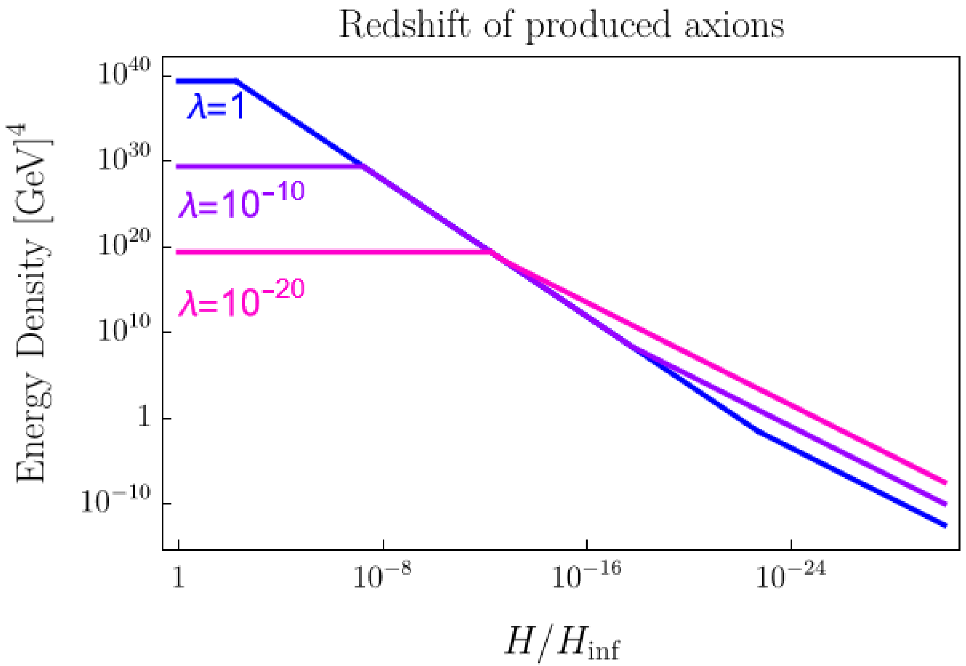

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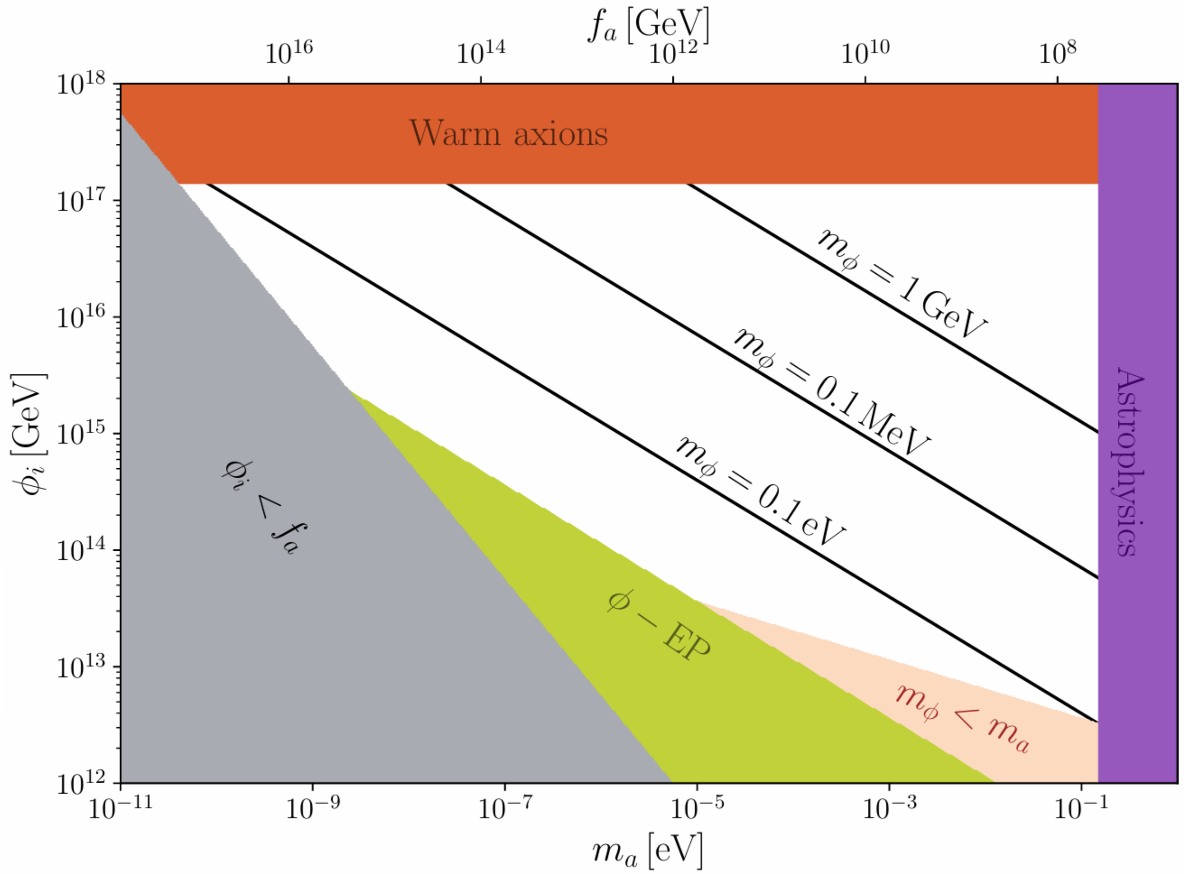


In conclusion, the **relic axion abundance**, produced by **parametric resonance** can account to cold dark matter via the relation:

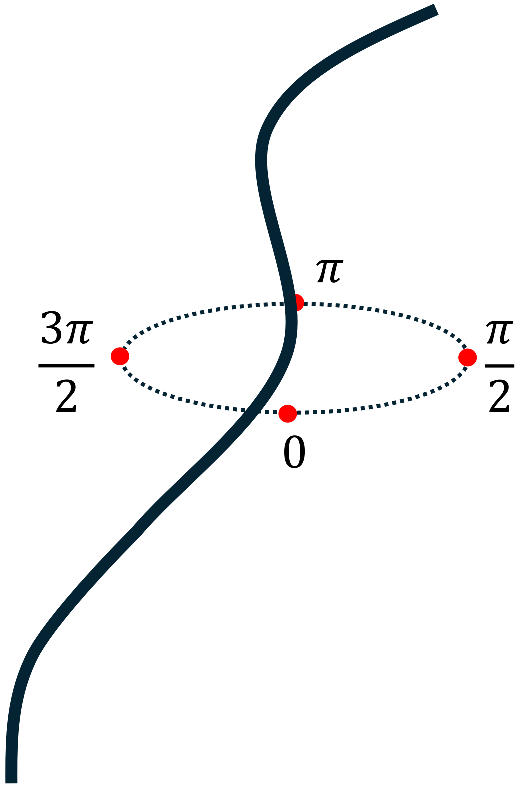
$$h^2\Omega_{axion} \simeq 0.12\left(\frac{\phi_i}{10^{-2}M_{Pl}}\right)^2\left(\frac{m_a}{10^{-3}\,eV}\right)\left(\frac{10^2\,GeV}{m_{\phi_i}}\right)^{1/2}$$

Axion Strings Formation

If the growth of perturbations is sufficiently efficient, the inhomogeneities of the field lead to the formation of **axion strings**

The condition to be checked, however, is whether, when the field settles down to the minimum, perturbations in the axion field are large enough to **wrap the U(1) vacuum manifold**.

$$\langle \delta a^2 \rangle \sim f_a^2$$



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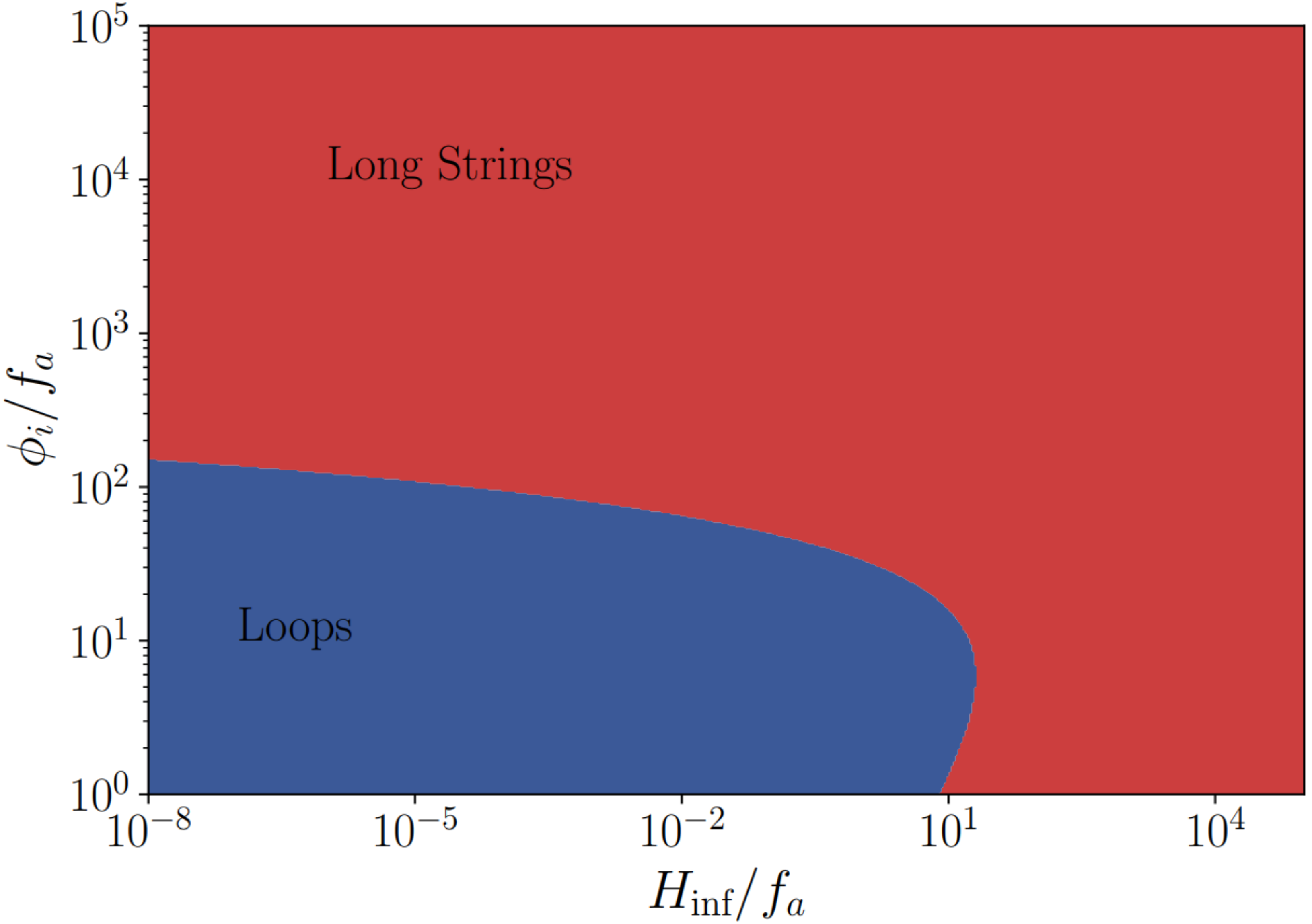

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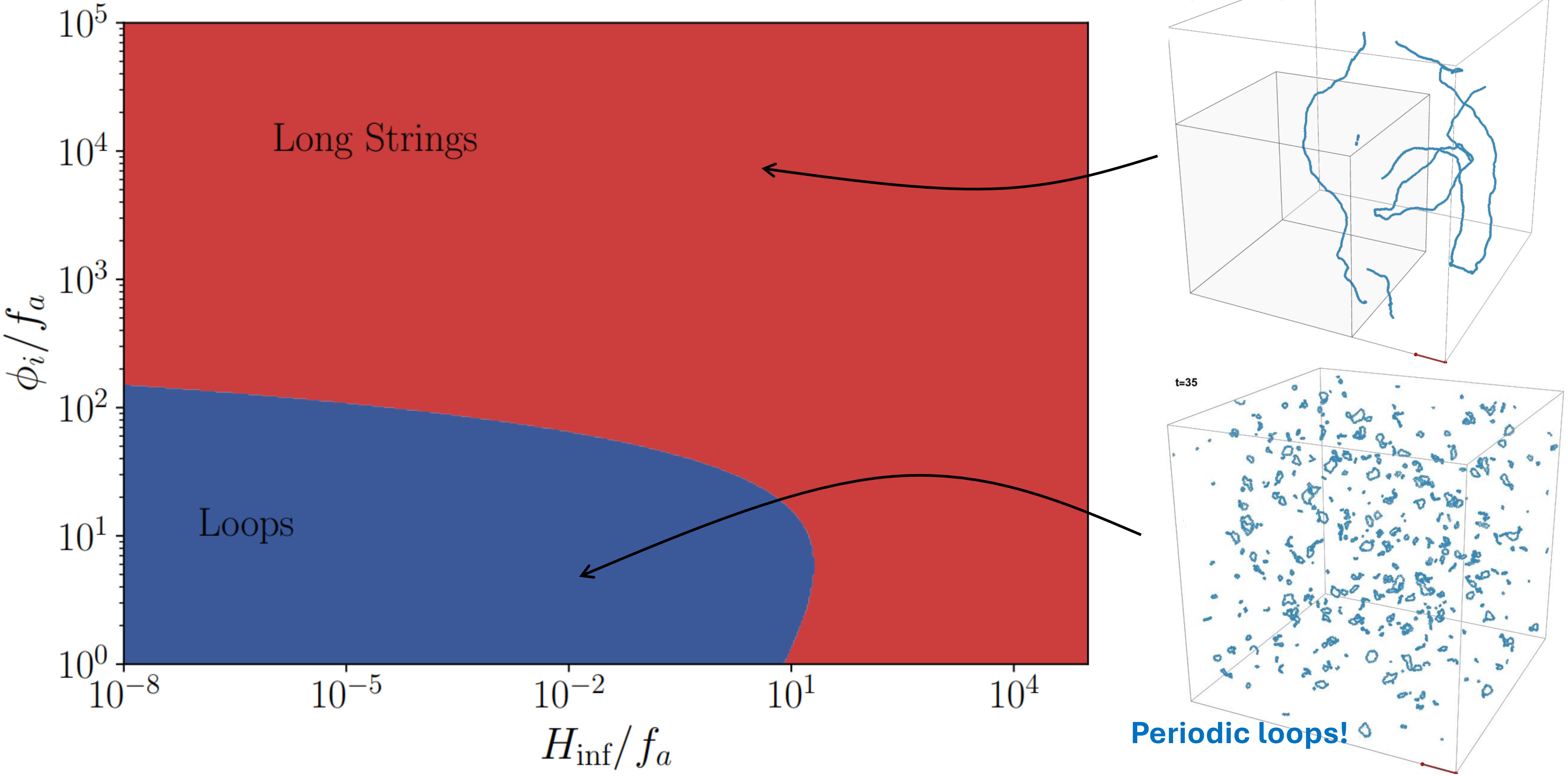
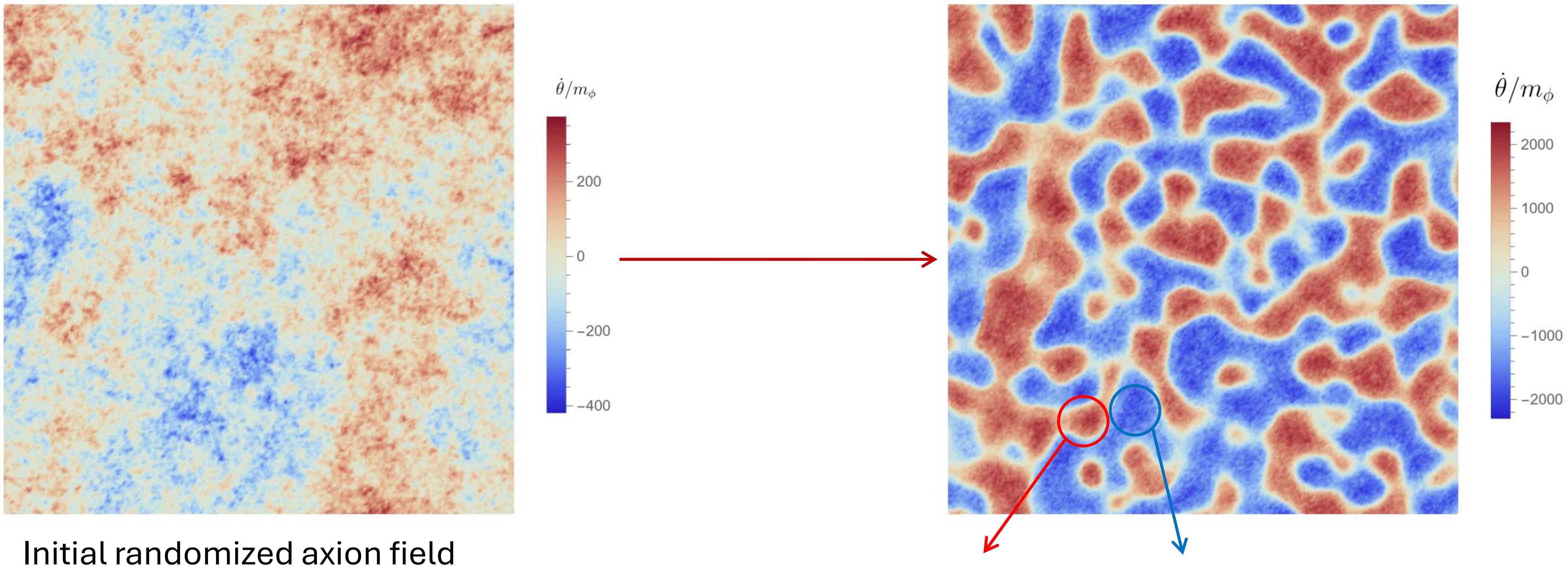


Long strings from pre-inflationary!

Periodic loops!

Periodic Loops

What we observe is the formation of many well-defined **domains**, in which the axion field is either rotating **clockwise** or **counterclockwise**.



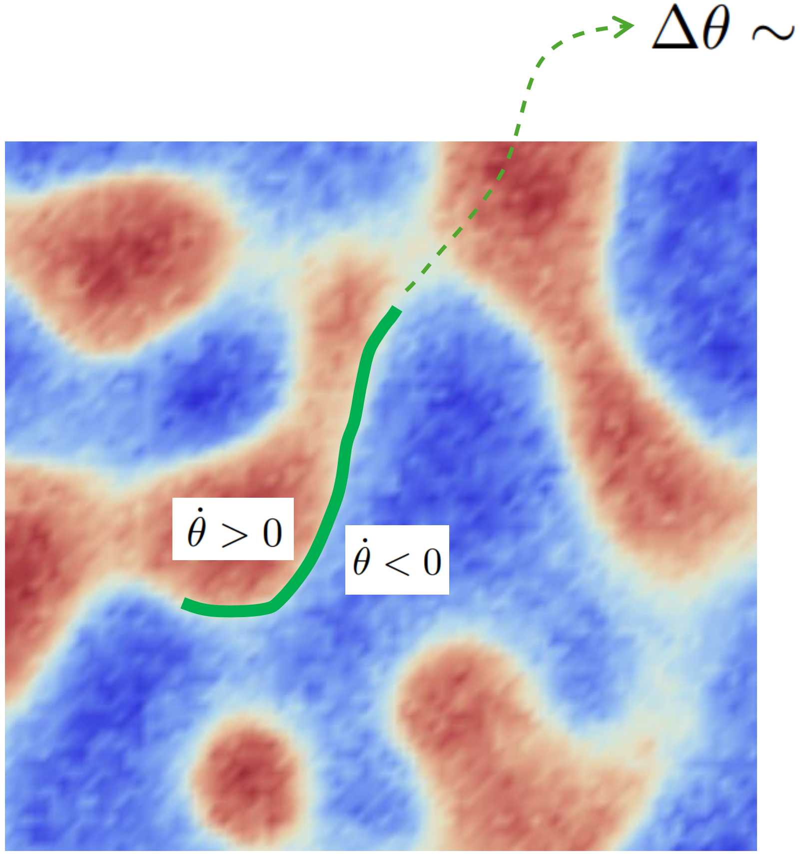
Initial randomized axion field

Counterclowise rotation

Clockwise rotation

Periodic Loops

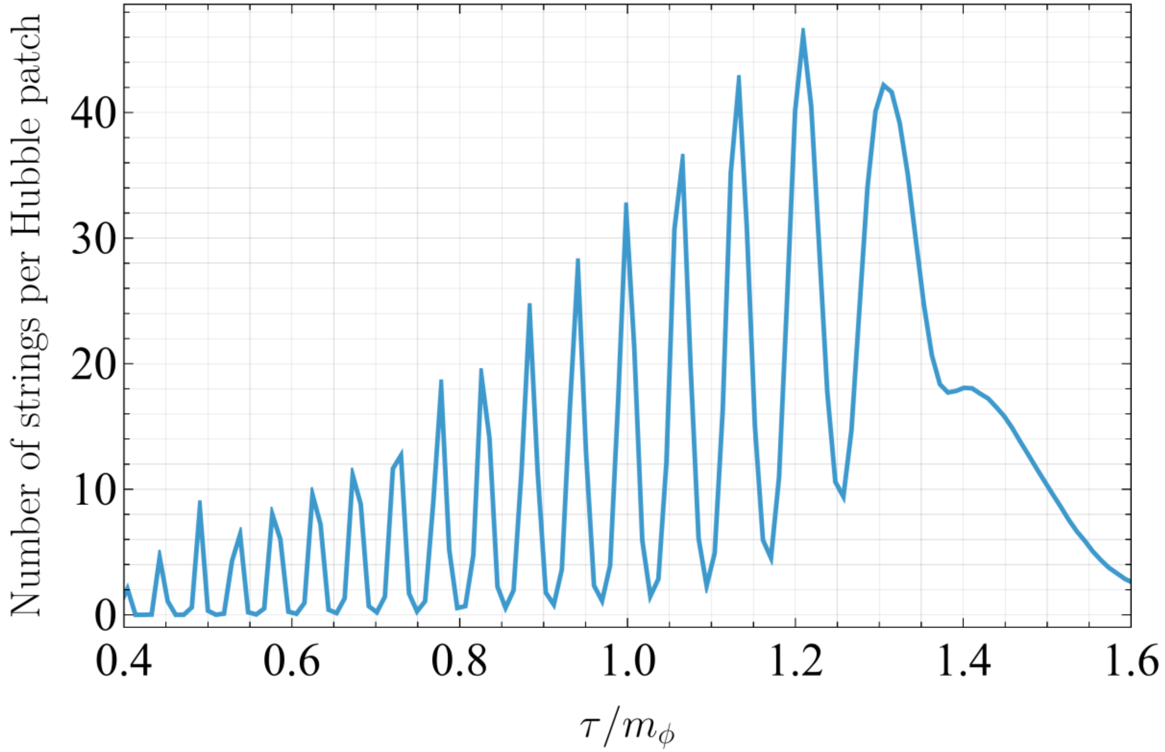
$$\Delta\theta \sim 2\dot{\theta}\Delta\tau > 2\pi$$

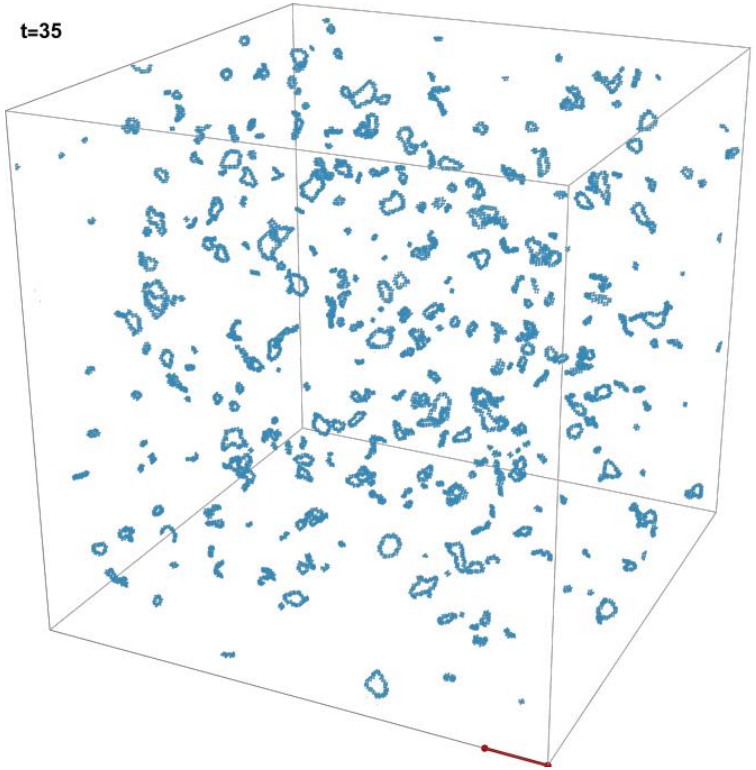


The presence of domains with **opposite axion velocities** forces the axion field to undergo **large excursions** at the **boundaries** between these domains. Each time it reaches the value 2π, axion loops are produced.

Periodic Cosmic Strings

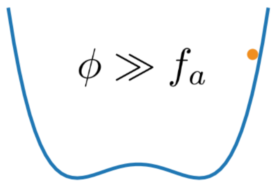
The axion field periodically wraps around its interval across the boundary, leading to the **periodic formation of axion string loops**, with a period determined by the oscillation of the radial mode.





Summary and Conclusion

- We start from a **pre-inflationary scenario** with a Hubble-induced term that provides a minimum at **large field values**



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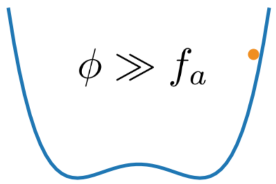

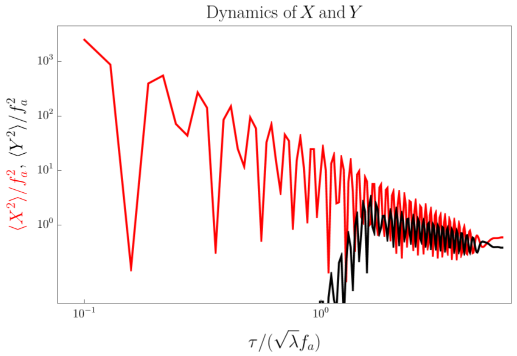

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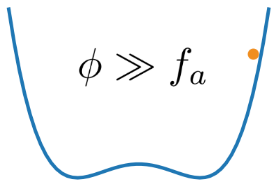

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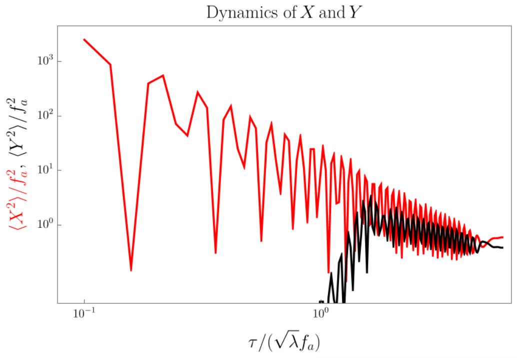
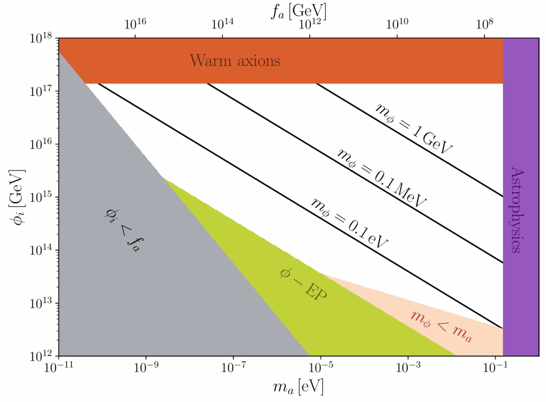


- **Axions are produced** and eventually contribute to CDM if they have enough time to redshift



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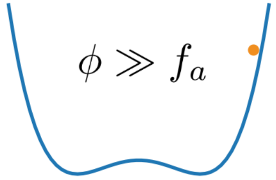

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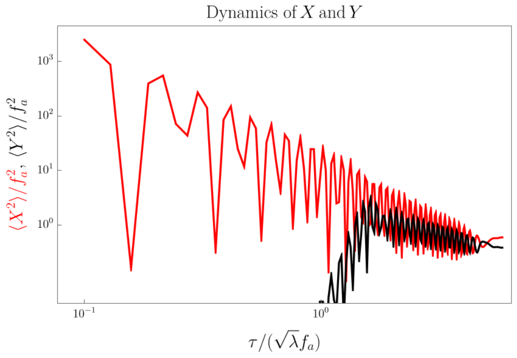

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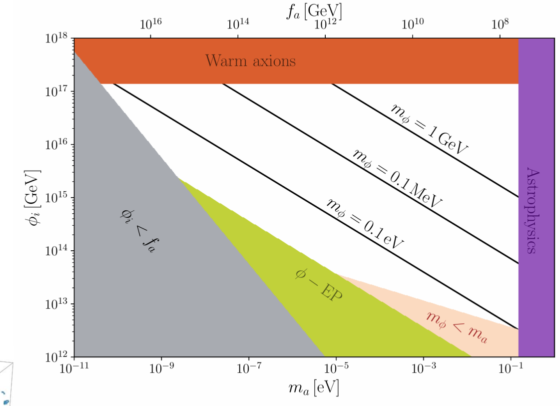
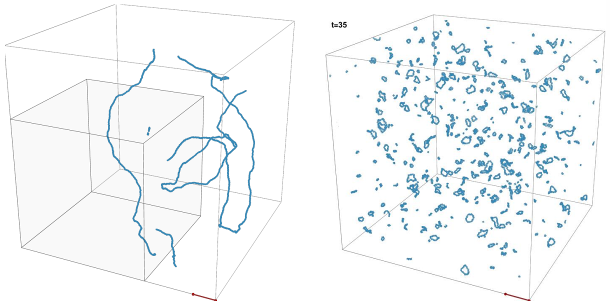


- **Topological defects** are formed due to the highly inhomogeneous field



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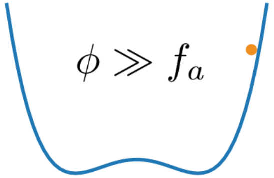

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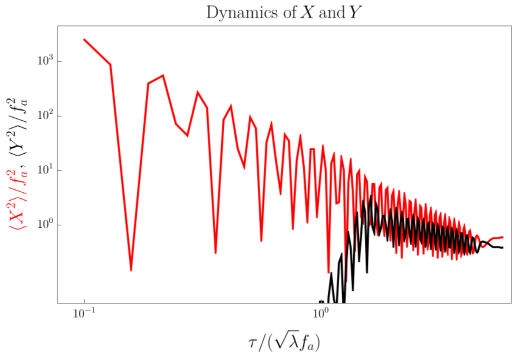

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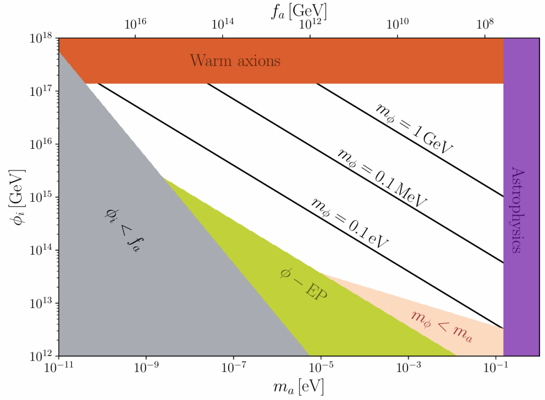

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