CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

DESY THEORY WORKSHOP

SYNERGIES TOWARDS THE FUTURE STANDARD MODEL

HELMHOLTZ

23 – 26 September 2025 DESY Hamburg, Germany

Inverse phase transitions

aka False Vacuum Cleaners

IFT Madrid

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based on

JCAP 10 (2024) 042, 2503.01951 and **2510.xxxxx**
with *Simone Blasi, Miguel Vanvlasselaer and Eric Madge*

2508.08362
with *Andrea Tesi*

CSIC
Consejo Superior de Investigaciones Científicas

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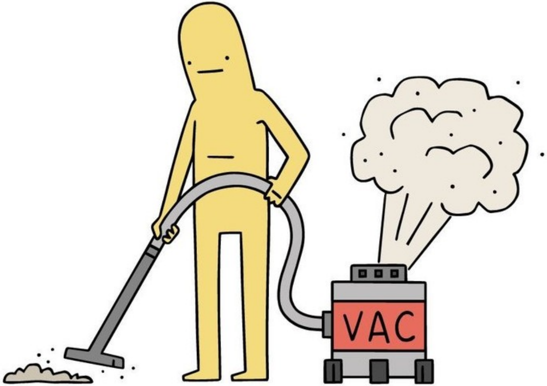
Outline

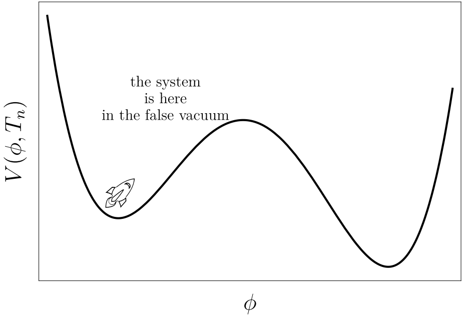
1. FOPTs: *direct* vs *inverse*
2. Hydrodynamic description
3. Supercooling vs Superheating 

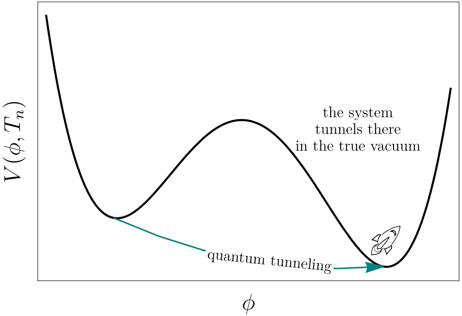


Sketch of an *Inverse* PT

FOPTs: *direct* vs *inverse*

Let us consider a system described by the scalar potential $V(\phi, T_n)$

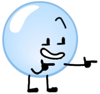




Tunneling decay rate of the false vacuum

$\Gamma \sim A e^{-S_E}$, Euclidean action

S_E computed on the sol. of the EOMs



The solutions with the least action are **spherically symmetric**

Coleman, Callan ('77)

FOPTs: *direct* vs *inverse*

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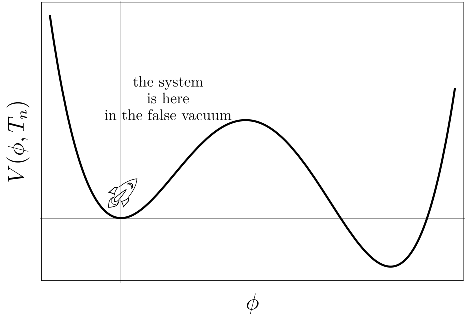

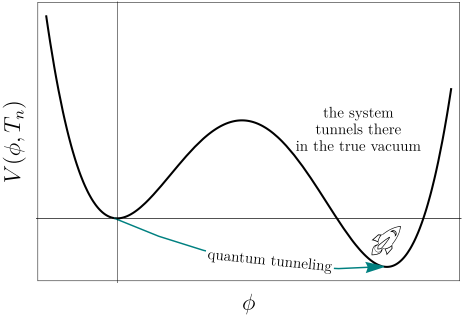

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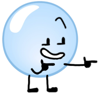

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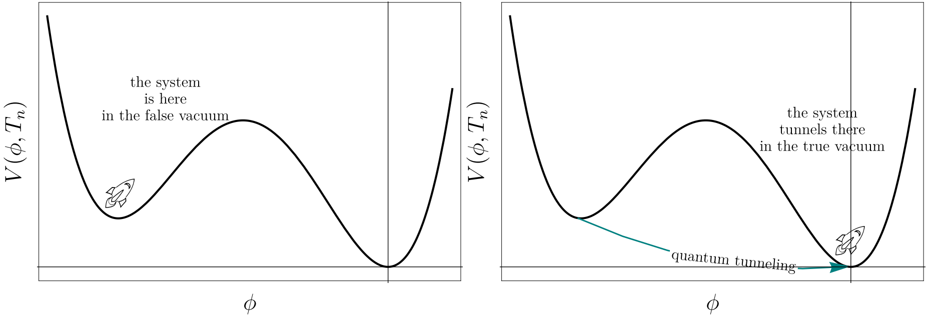

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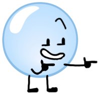

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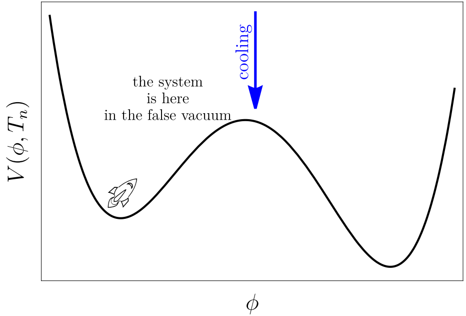

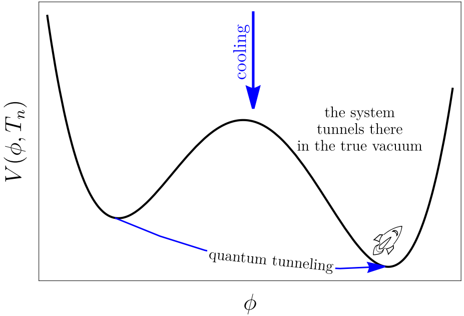

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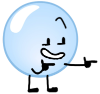

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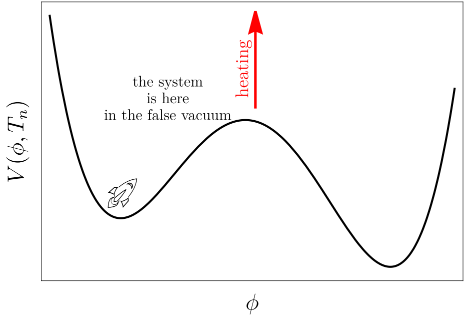

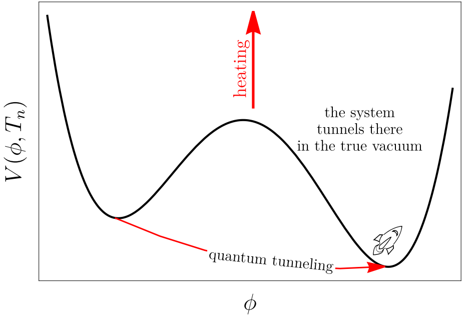

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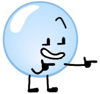

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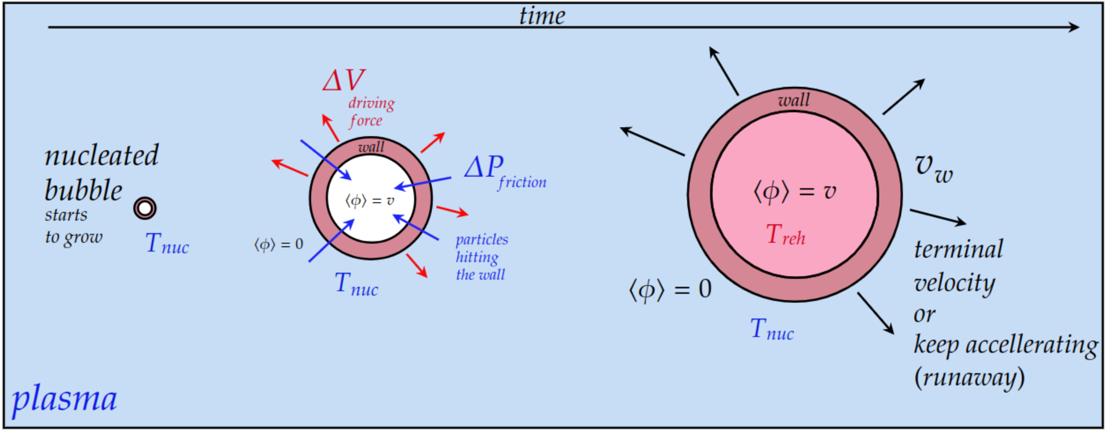
How?

FOPTs: How?


time
nucleated bubble starts to grow
T_{nuc}
ΔV
driving force
wall
$\langle\phi\rangle = v$
$\Delta P_{friction}$
$\langle\phi\rangle = 0$
particles hitting the wall
T_{nuc}
wall
$\langle\phi\rangle = v$
T_{reh}
v_w
$\langle\phi\rangle = 0$
T_{nuc}
terminal velocity or keep accellerating (runaway)
plasma

Hydrodynamic description

Coupled system of the scalar background and the plasma

$$T^{\mu\nu} = T_\phi^{\mu\nu} + T_p^{\mu\nu}, \qquad \begin{cases} T_\phi^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu} \left[\frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \\ T_p^{\mu\nu} = (e+p) u^\mu u^\nu - p\, g^{\mu\nu} \end{cases}$$

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Energy conservation: $\nabla_{\mu}T^{\mu\nu} = 0 \quad \rightarrow \quad \begin{cases} \text{Continuity eq.} \\ \text{Euler eq.} \end{cases}$ (for continuous waves)

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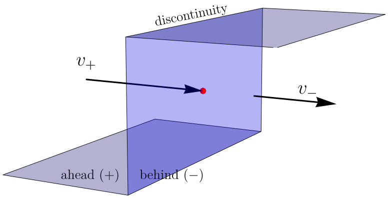
Energy conservation: $\nabla_\mu T^{\mu\nu} = 0 \quad \rightarrow \quad \begin{cases} \text{Continuity eq.} \\ \text{Euler eq.} \end{cases}$ (for continuous waves)

Hydrodynamical flows can develop **discontinuities** such as shocks or reaction fronts $\rightarrow$ matching conditions across discontinuities ($\pm$ bubble wall frame)



$$w_+\gamma_+^2 v_+ = w_-\gamma_-^2 v_-$$
$$w_+\gamma_+^2 v_+^2 + p_+ = w_-\gamma_-^2 v_-^2 + p_-$$

where $w = e + p =$ enthalpy

Thermodynamics

Once the microphysics is specified (i.e., a model is chosen), we can compute the free energy, related to the pressure via:

$$p = -\mathcal{F} = -V_{\rm eff} = -\left(V_0 + V_{\text{1-loop}} + V_T\right)$$

From the pressure, other thermodynamic quantities follow:

$$w = T\frac{\partial p}{\partial T}, \qquad e = w - p, \qquad c_s^2 = \frac{\partial p}{\partial e}$$

Matching conditions: $\quad v_+ v_- = \dfrac{p_+ - p_-}{e_+ - e_-}\ , \qquad \dfrac{v_+}{v_-} = \dfrac{e_- + p_+}{e_+ + p_-}$

Latent heat

Manipulating the matching conditions lead to

$$\alpha_\vartheta = \frac{D\vartheta}{3w_+}$$

where ϑ is a generalisation of the

Trace anomaly : $T^\mu_\mu = e - 3p$

that is nothing but the **latent heat** (L)

$L > 0$ **exothermic** PT $L < 0$ **endothermic** PT

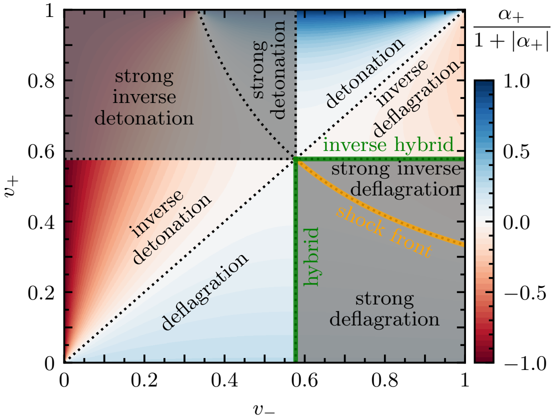
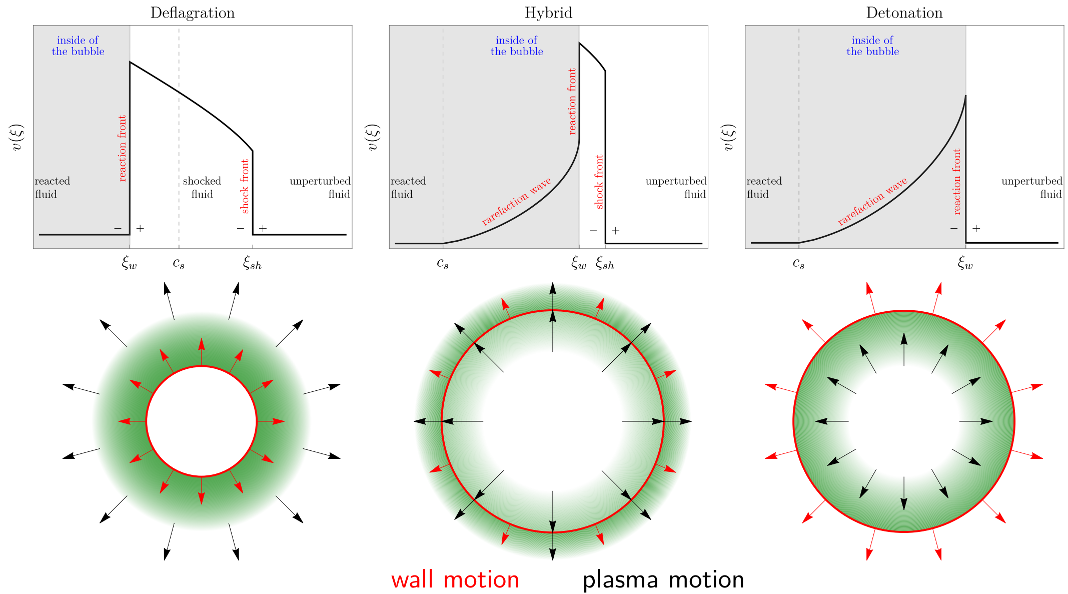
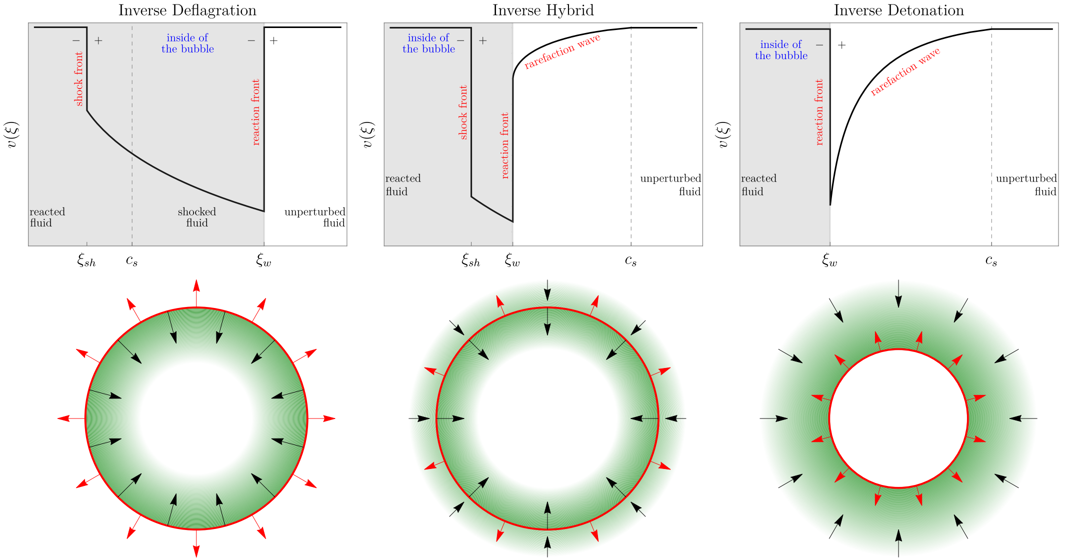


Figure: Using bag EoS $\alpha_\vartheta \equiv \alpha_+ = 4\epsilon/3w_+$

Hydrodynamic description $(L > 0)$



Hydrodynamic description $(L < 0)$



wall motion plasma motion

Supercooling vs Superheating

Inverse PTs while cooling? 

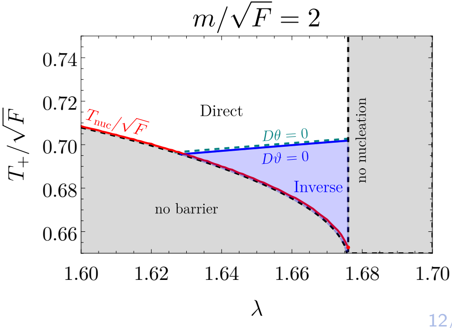
Yes ... but SUSY! (proof of principle)

O'Raifeartaigh Model: SUSY breaking field X + $\Phi_{1,2}$ and $\tilde{\Phi}_{1,2}$ mediator fields

$$W = -FX + \lambda X \Phi_1 \tilde{\Phi}_2 + m(\Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2)$$

where $\sqrt{F}$ SUSY breaking scale. The model has a $U(1)$ $R-$symmetry.

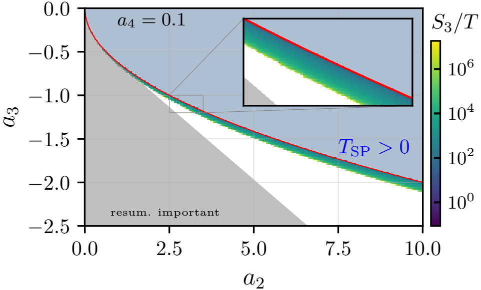
- Peculiar thermal history: origin is global minimum both at $T = 0$ and $T \to \infty$.
- There is a $R-$symmetry breaking PT while cooling that can be *inverse*.

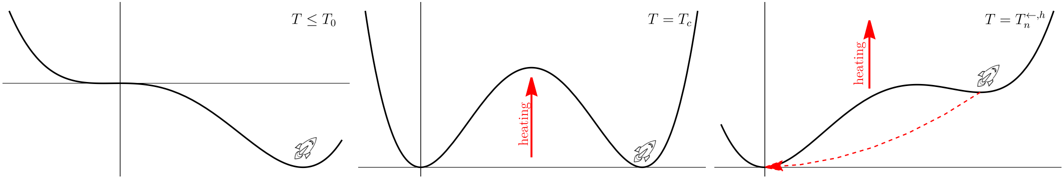


Inverse PTs while heating?

Toy Model: $V_T(\phi) = a_0 T^4 + a_1 \phi T^3 + \frac{a_2}{2}\phi^2 T^2 + \frac{a_3}{3}\phi^3 T + \frac{a_4}{4}\phi^4 \xrightarrow{T\to\infty} T^4 f(\varphi)$ (scale invariant)

- $\exists$ two minima $\Delta \equiv a_3^2 - 4a_2 a_4 > 0$
- origin is global minimum $2a_2 a_4 > \Delta - a_3\sqrt{\Delta}$

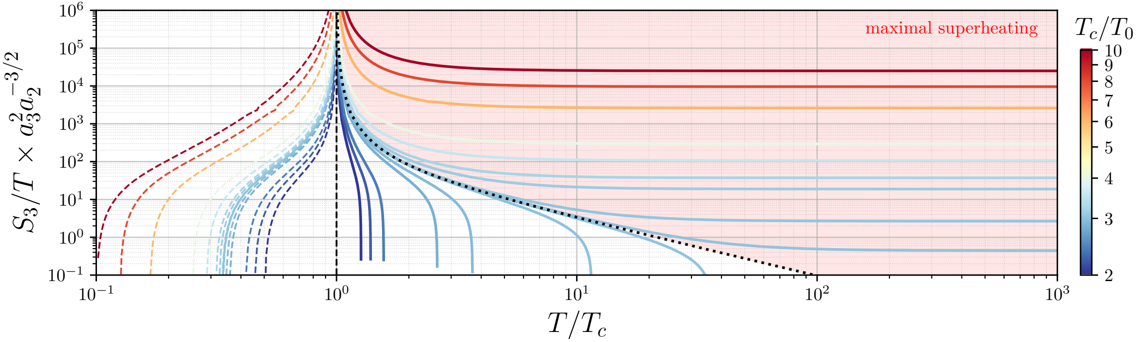




Inverse PTs while heating?

Instability at lower temperatures: $a_2 \to a_2(T) = a_2 - M^2/T^2$

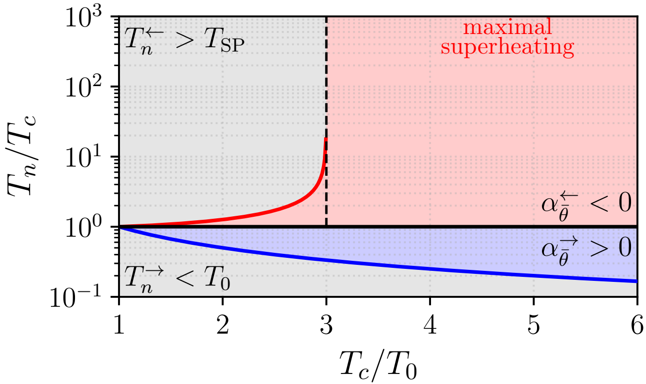
$$T_0 = \frac{M}{\sqrt{a_2}}, \qquad T_c = \frac{M}{\sqrt{a_2}}\left(1 - \frac{2}{9}\frac{a_3^2}{a_2 a_4}\right)^{-1/2}, \quad T_{\rm SP} = \frac{M}{\sqrt{a_2}}\left(1 - \frac{1}{4}\frac{a_3^2}{a_2 a_4}\right)^{-1/2}$$



Transitions $\to$ **Transitions** $\leftarrow$

Inverse PTs while heating?

Answer: Yes! Natural place for **inverse**...



but hard to reheat the **whole** Universe!

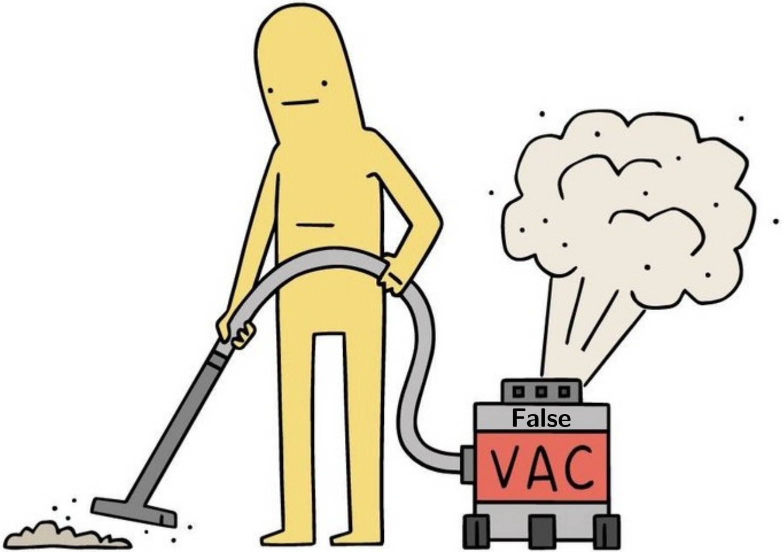
Conclusions

1. Difference between *direct* and *inverse* PTs from the hydrodynamical point of view.
2. In *direct* PTs the wall pushes the plasma and (part of) the **vacuum energy is converted in kinetic energy**.
3. In *inverse* PTs the bubble **sucks the plasma** into it consequently pushing the wall.
4. *Inverse* PTs with both supercooling or superheating of the Universe, but hard to realize.

Outlooks:

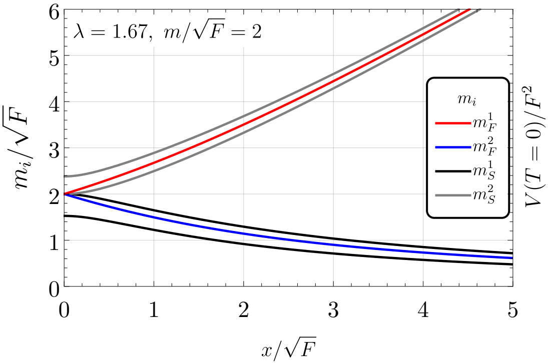
1. Distinguish Direct/Inverse from GWs spectra using **SoundShellModel** (see **Eric Madge**'s talk)
2. Hard to reheat the whole Universe ... what about a **compact system**?
3. What does change at **finite chemical potential**?

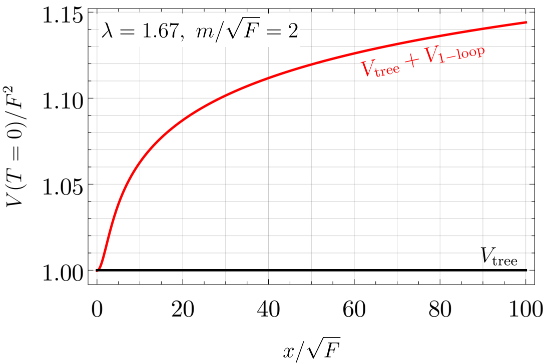
Thanks for
your attention!



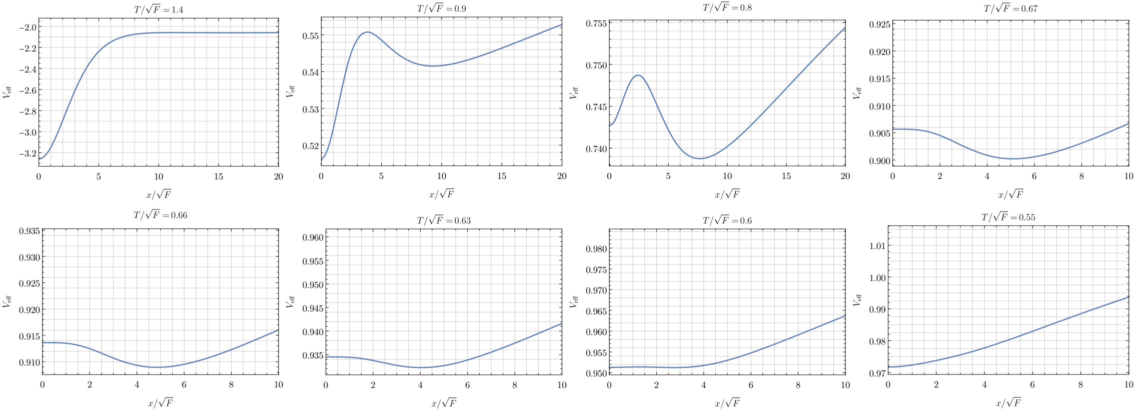
Backup

Sprectrum of the SUSY model





More on thermal history



Matching conditions and possible solutions

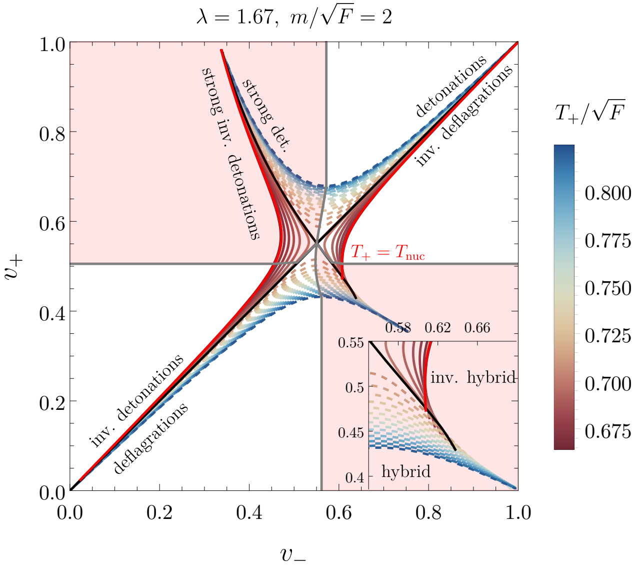
$$v_+ v_- = \frac{p_+ - p_-}{e_+ - e_-}\,, \qquad \frac{v_+}{v_-} = \frac{e_- + p_+}{e_+ + p_-}$$

where $p = -V_{\text{eff}}(T)$, $w = T\frac{\partial p}{\partial T}$ and $e = w - p$.

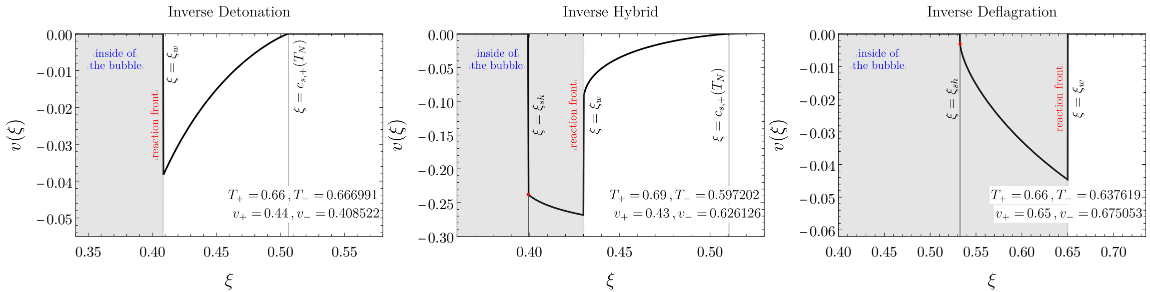
Direct/Inverse: $\alpha_\vartheta \gtrless 0$

$$\alpha_\vartheta = \frac{1}{3w_+(T_+)}\left(De(T_+) - \frac{\delta e}{\delta p}(T_+, T_-)Dp(T_+)\right)$$

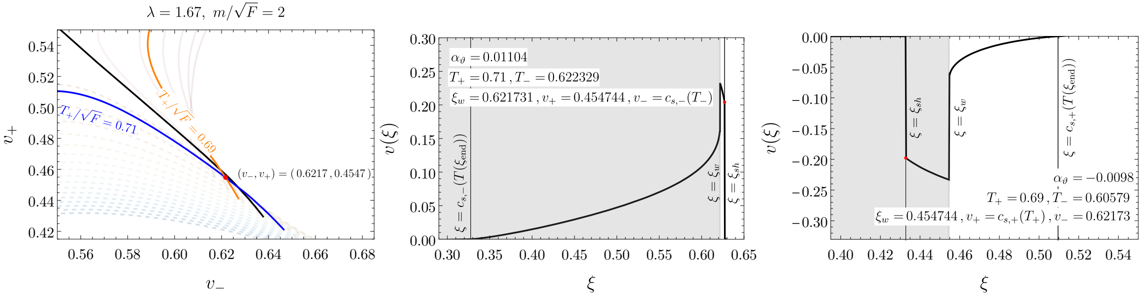
$Df = f_+(T_+) - f_-(T_+)$ and $\delta f = f_-(T_+) - f_-(T_-)$.



Full numerical fluid profiles



Overlap in the hybrid corner



Inverse PTs while heating?

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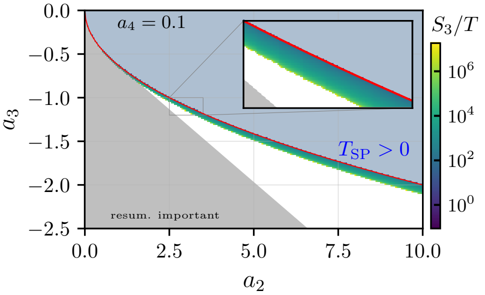
- Example for O(N) scale inv. sector:

$$V_S = \frac{\lambda_{\rm mix}}{2}\phi^2 \sum_{i=1}^{N} S_i S_i + \frac{\lambda_0}{4!}\phi^4 + \frac{\lambda_S}{4}\Big(\sum_i S_i S_i\Big)^2 ,$$

- Perturbativity: $\bar\lambda \equiv \frac{\lambda_{\rm mix}\sqrt{N}}{16\pi^2}$,

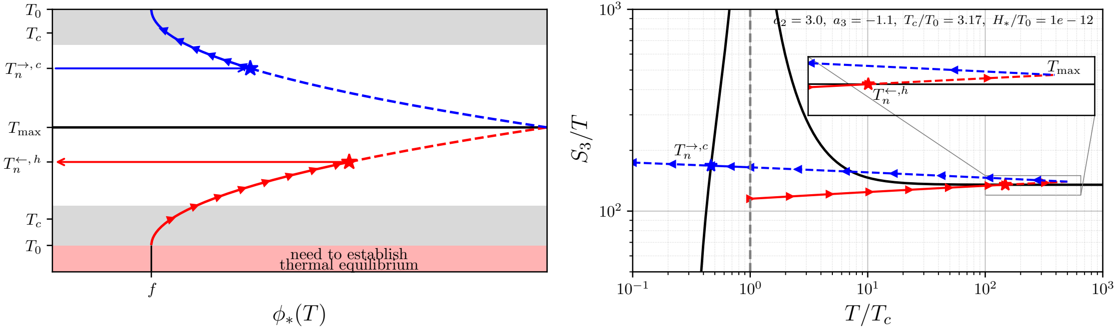


$$a_0 = -\frac{\pi^2}{90}N, \quad a_1 = 0\,, \quad a_2 = N\frac{\lambda_{\rm mix}}{12}, a_3 = -N\frac{\lambda_{\rm mix}^{3/2}}{4\pi}, \quad a_4 = \frac{\lambda_0}{6} - N\frac{\lambda_{\rm mix}^2}{16\pi^2}\ell\,,$$

where $\ell \equiv \log\big(\lambda_{\rm mix}\phi^2/(T^2 c_B)\big)$. Works for $\bar\lambda \approx 0.015$ and $N \approx 250$.

Inverse PTs while heating? 🔥

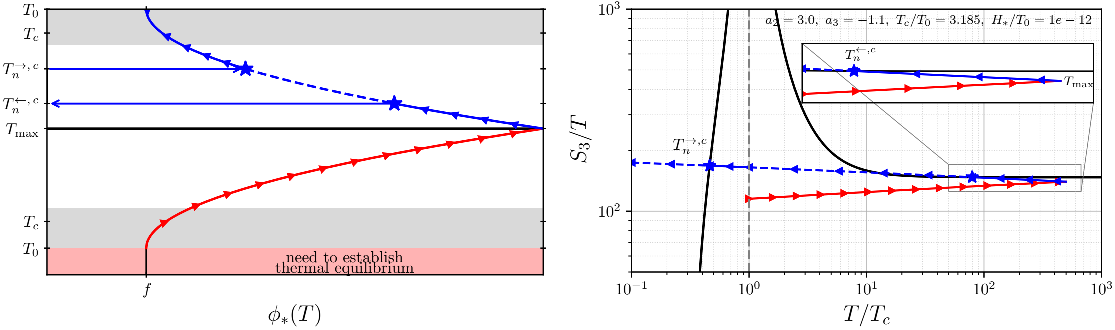
Transition while heating at $T = T_n^{\leftarrow,h}$



Note. $\leftarrow$: transition towards the origin $\rightarrow$: transition away from the origin

Inverse PTs while heating?

Transition while cooling at $T = T_n^{\leftarrow,c}$



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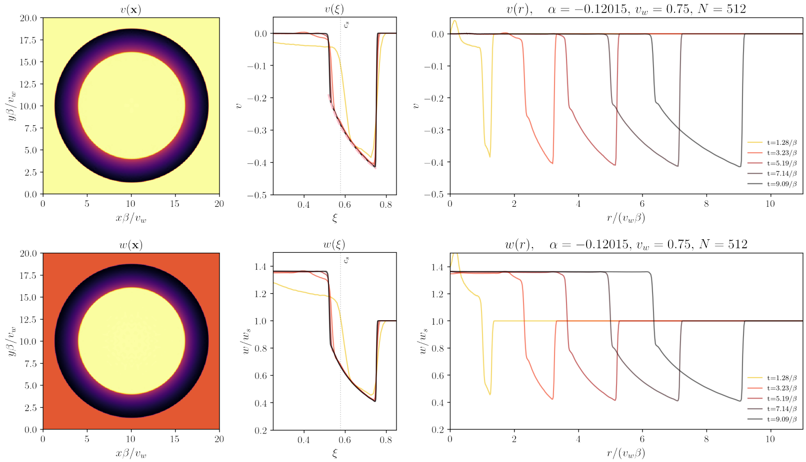
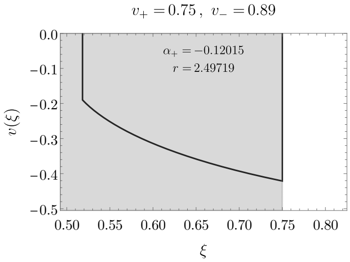
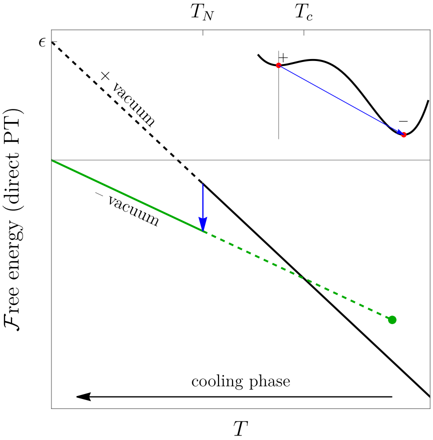
Self-similar solutions (dynamical evolution)



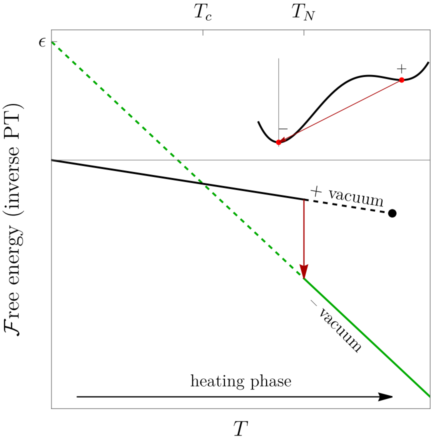


Thanks to Isak Stomberg!

BAG Equation of State (EoS)



$$\epsilon_+ = \epsilon, \quad \epsilon_- = 0$$



$$\epsilon_+ = 0, \quad \epsilon_- = \epsilon$$

Thermodynamic Quantities

Once the microphysics is specified (i.e., a model is chosen), we can compute the free energy, related to the pressure via:

$$p = -\mathcal{F} = -V_{\text{eff}} = -\left(V_0 + V_{\text{1-loop}} + V_T\right)$$

From the pressure, other thermodynamic quantities follow:

$$w = T\frac{\partial p}{\partial T}, \qquad e = w - p, \qquad c_s^2 = \frac{\partial p}{\partial e}$$

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Different levels of approximation can be used:

1. **Bag EOS**: $p_\pm = c_s^2 a_\pm T_\pm^4 - \epsilon_\pm$ with constant $c_s^2 = \dfrac{1}{3}$.

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2. $\mu\nu$-**model**: $p_\pm = c_{s,\pm}^2 a_\pm T_\pm^{\nu_\pm} - \epsilon_\pm$, where $\nu_\pm = 1 + 1/c_{s,\pm}^2$ and $\nu_- = \mu$, $\nu_+ = \nu$.

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$$w = T\frac{\partial p}{\partial T}, \qquad e = w - p, \qquad c_s^2 = \frac{\partial p}{\partial e}$$

Different levels of approximation can be used:

1. **Bag EOS**: $p_\pm = c_s^2 a_\pm T_\pm^4 - \epsilon_\pm$ with constant $c_s^2 = \dfrac{1}{3}$.
2. $\mu\nu$**-model**: $p_\pm = c_{s,\pm}^2 a_\pm T_\pm^{\nu_\pm} - \epsilon_\pm$, where $\nu_\pm = 1 + 1/c_{s,\pm}^2$ and $\nu_- = \mu$, $\nu_+ = \nu$.
3. **Full model**: $p_\pm = -\mathcal{F}(\phi_\pm)$, with $c_{s,\pm}(T)$ derived from the full free energy.

Energy budget & efficiency

Energy budget of PTs

$$w(\xi) = w(\xi_0)\,\exp\left[\int_{v(\xi_0)}^{v(\xi)} \left(\frac{1}{c_s^2}+1\right)\gamma^2(v)\mu(\xi(v),v)\;dv\right]$$

Energy budget (direct): $$\underbrace{\frac{\xi_w^3}{3}\epsilon}_{\text{vacuum energy}} + \underbrace{\frac{3}{4}\int w_N \xi^2 d\xi}_{\text{initial thermal energy}} = \underbrace{\int \gamma^2 v^2 w \xi^2 d\xi}_{\text{fluid motion}} + \underbrace{\frac{3}{4}\int w \xi^2 d\xi}_{\text{final thermal energy}}$$

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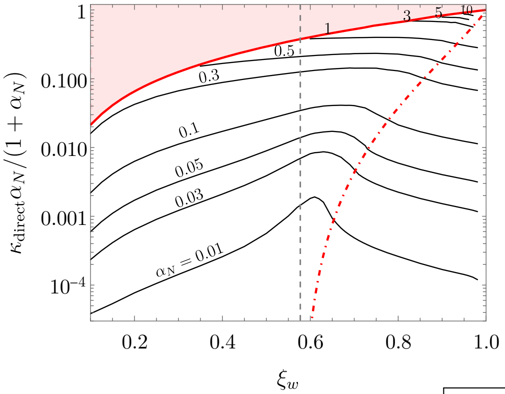
Energy budget (inverse): $$\underbrace{\frac{3}{4}\int w_N \xi^2 d\xi}_{\text{initial thermal energy}} = \underbrace{\frac{\xi_w^3}{3}\epsilon}_{\text{vacuum energy}} + \underbrace{\int \gamma^2 v^2 w \xi^2 d\xi}_{\text{fluid motion}} + \underbrace{\frac{3}{4}\int w \xi^2 d\xi}_{\text{final thermal energy}}$$

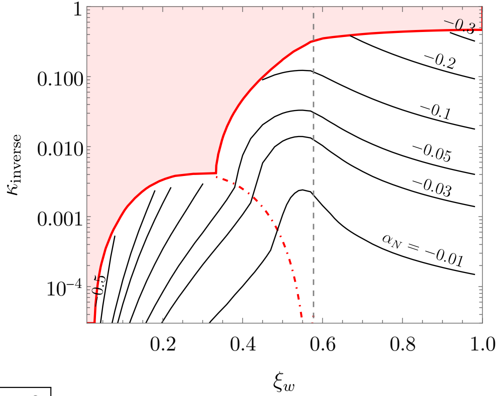
Initial energy will be in part converted in **kinetic bulk motion**!

Efficiency factors

$$\left.\frac{\rho_{\rm kin}}{\rho_{\rm tot}}\right|_{\rm direct} \equiv \kappa_{\rm direct}\frac{\alpha_N}{1+\alpha_N}\,, \qquad \kappa_{\rm direct} = \frac{3}{\epsilon \xi_w^3}\int \gamma^2 v^2 w \xi^2 d\xi$$



$$\left.\frac{\rho_{\rm kin}}{\rho_{\rm tot}}\right|_{\rm inverse} \equiv \kappa_{\rm inverse} \equiv \frac{4}{\bar{v}^3}\int \xi^2 d\xi\, v^2\gamma^2 \frac{w}{w_N}$$



$$\Omega_{\rm GW} \sim \left(\frac{\rho_{\rm kin}}{\rho_{\rm tot}}\right)^2$$

Types of solitions (detailed)

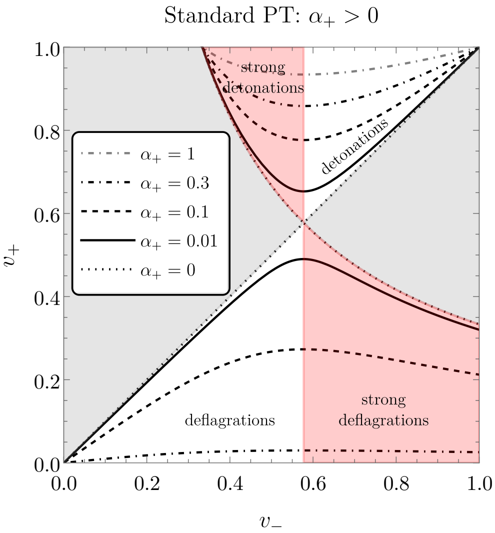
Types of discontinuities for cosmological *direct* phase transitions		
	Detonations $p_+ < p_-, v_+ > v_-$	Deflagrations $p_+ > p_-, v_+ < v_-$
Weak	$v_+ > c_s, v_- > c_s$ Physical	$v_+ < c_s, v_- < c_s$ Physical
Chapman-Jouguet	$v_+ > c_s, v_- = c_s$ Physical	$v_+ < c_s, v_- = c_s$ Physical
Strong	$v_+ > c_s, v_- < c_s$ Forbidden	$v_+ < c_s, v_- > c_s$ Unstable

Types of discontinuities for cosmological *inverse* phase transitions		
	Inverse Detonations $(p_+ < p_-, v_+ > v_-)$	Inverse Deflagrations $(p_+ > p_-, v_+ < v_-)$
Weak	$v_+ < c_s, v_- < c_s$ Physical	$v_+ > c_s, v_- > c_s$ Physical
Chapman-Jouguet	$v_+ = c_s, v_- < c_s$ Physical	$v_+ = c_s, v_- > c_s$ Physical
Strong	$v_+ > c_s, v_- < c_s$ Forbidden	$v_+ < c_s, v_- > c_s$ Unstable

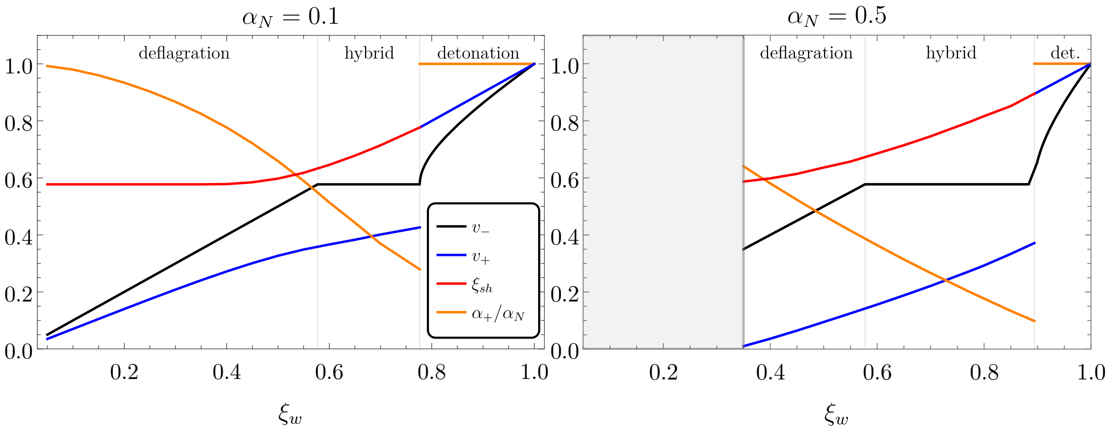
Impossibility of strong solutions

- **Strong detonations**: velocity has to be zero at the centre of the bubble and very far away from the wall, and having $v > 0$ translates into
$$\frac{\mu^2}{c_s^2} - 1 > 0 \ , \qquad v_- > c_s$$
so detonations with $v_- < c_s$ are fordibben.

- **Strong deflagration**:
  - unstable wrt perturbations
  - entropy decreases



Evolution of quantities across the wall (direct)



Evolution of quantities across the wall (inverse)