

Gravitational production after inflation : Boltzmann and Bogoliubov approaches

Simon Cléry, TUM
DESY Theory Workshop 2025

* *Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production*, Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

See also

* *Boltzmann or Bogoliubov? Approaches Compared in Gravitational Particle Production*, Kaneta, Lee, Oda, **2206.10929**

* *A New Window into Gravitationally Produced Scalar Dark Matter*, Garcia, Pierre, Verner, **2305.14446**

* *Cosmological gravitational particle production and its implications for cosmological relics*, Kolb and Long, **2312.09042**

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Universität
München

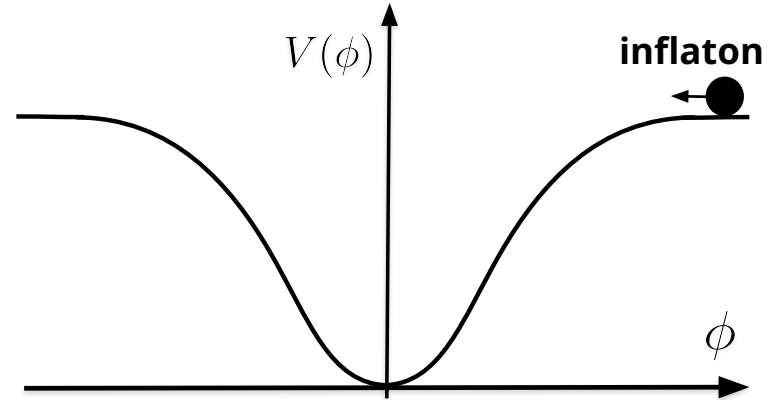
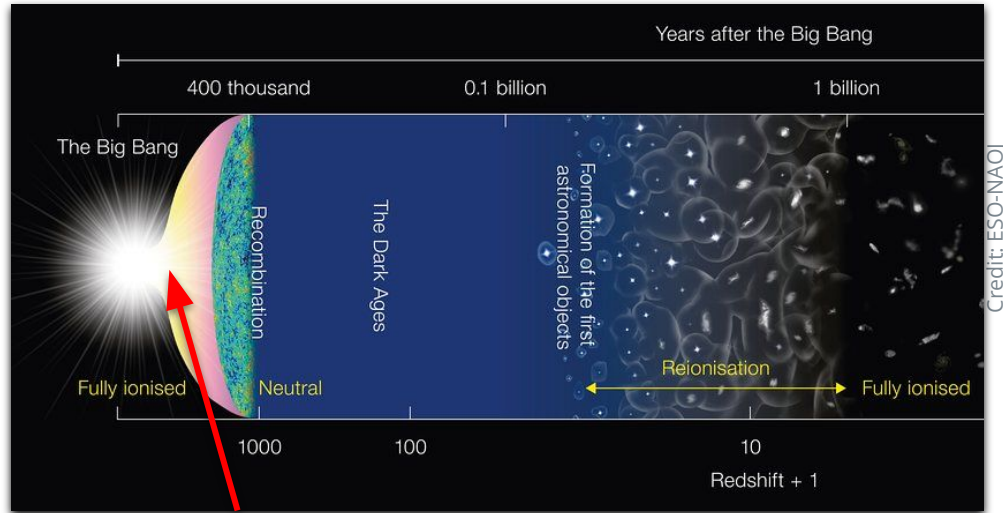


SFB 1258

Neutrinos
Dark Matter
Messengers

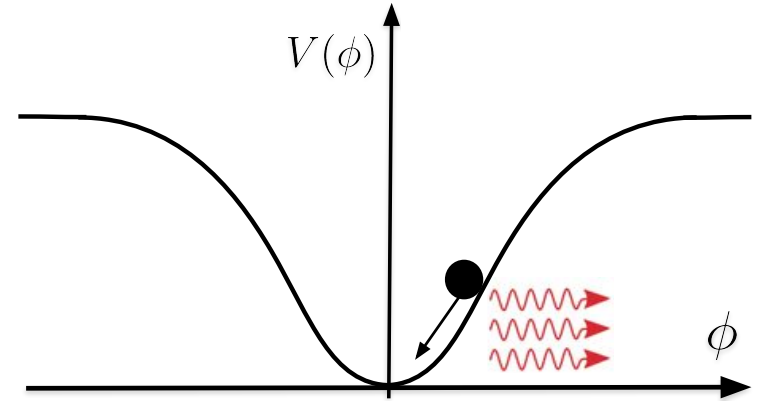
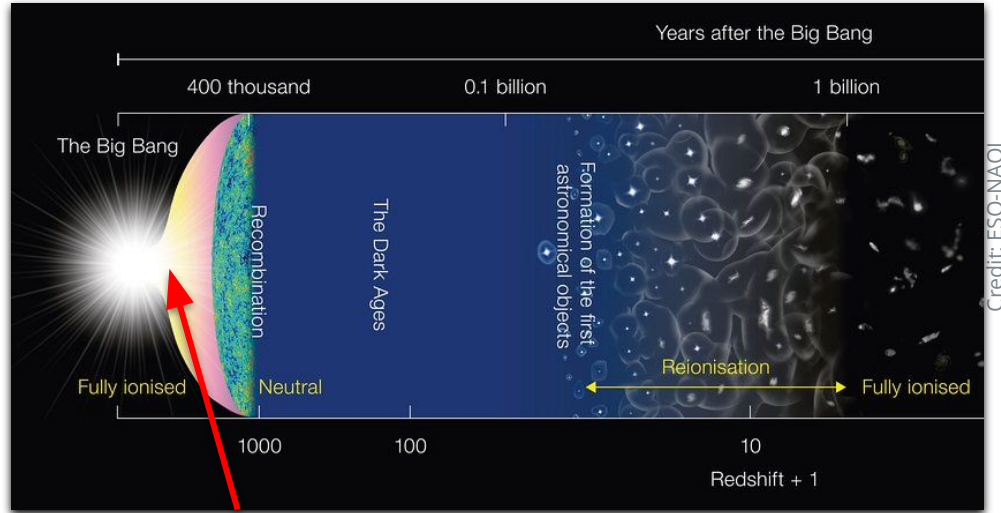


Reheating after inflation



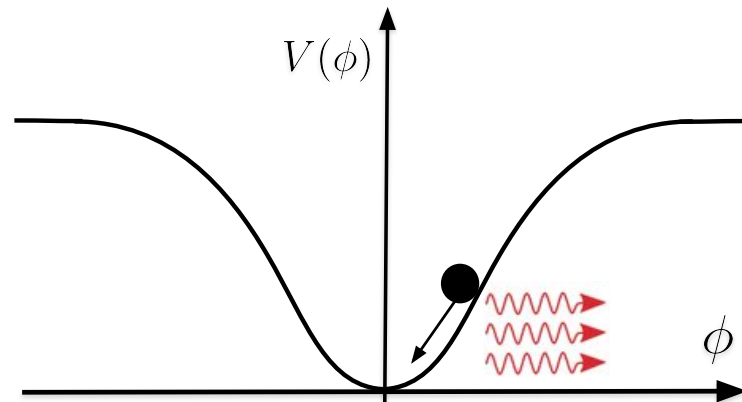
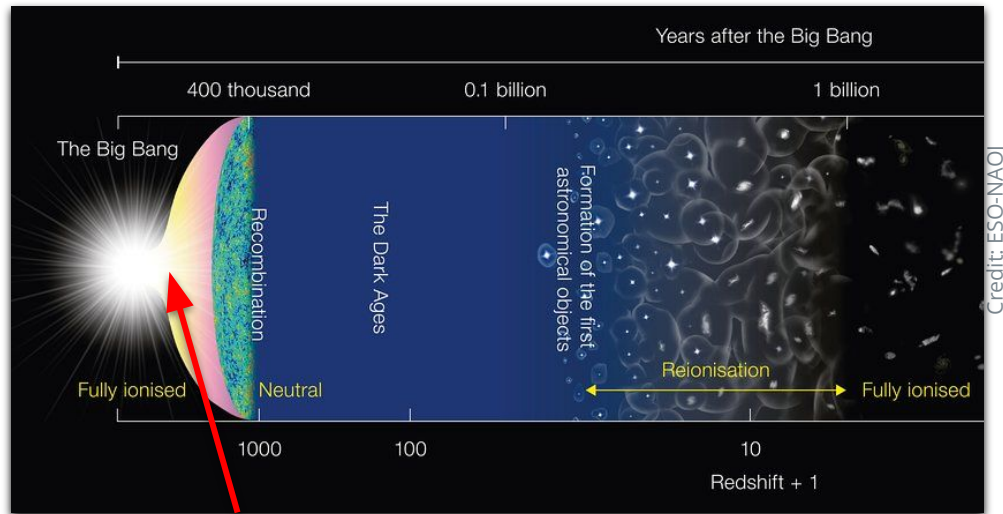
?

Reheating after inflation



Transition from cold inflaton condensate to a hot thermal plasma : **Reheating**

Reheating after inflation



What if inflaton has no other coupling than **gravitational** ones ?



Scalar field in curved space-time

Consider a spectator scalar field minimally coupled, in a classical gravitational background

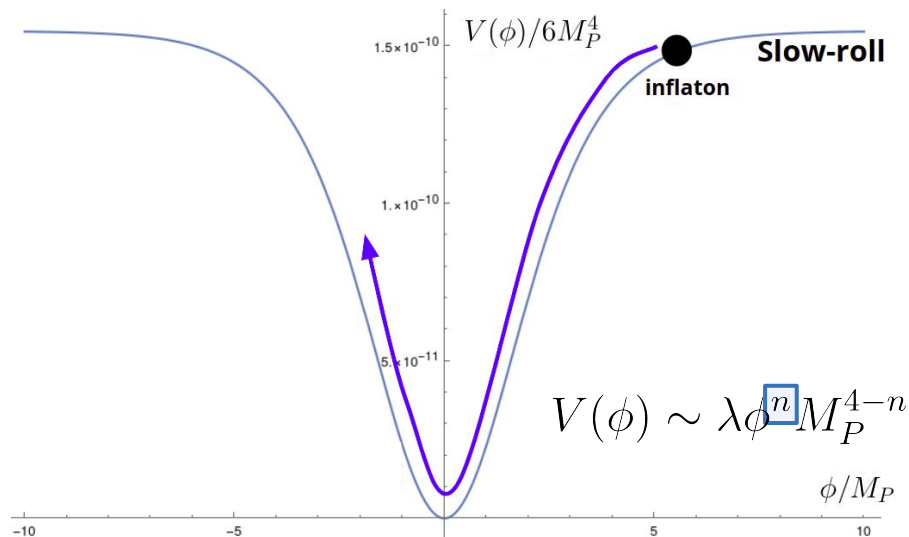
$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} m_\chi^2 \chi^2 + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

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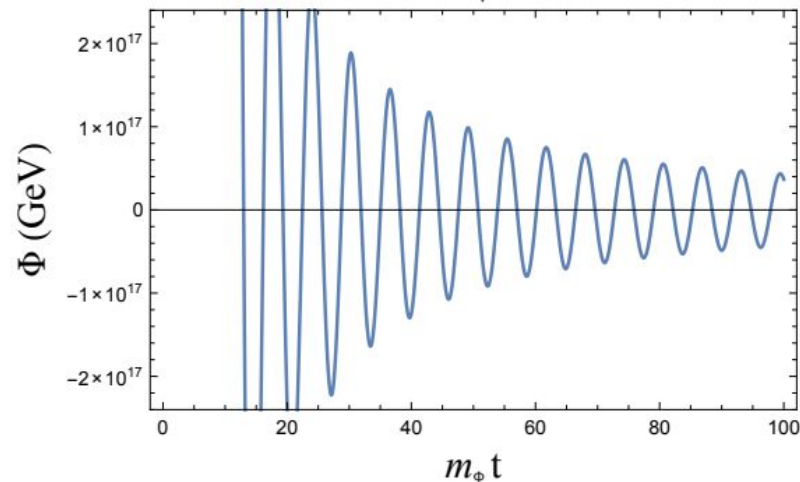
$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} m_\chi^2 \chi^2 + \underbrace{\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)}_{\text{inflaton (background)}} \right]$$

Inflaton oscillations during Reheating



$$\text{EOM: } \ddot{\phi}(t) + 3H\dot{\phi}(t) + V'(\phi(t)) = 0$$

Couplings of the inflaton induce friction and transfer of energy during the oscillations



Redshifted envelop and frequency of the oscillations depend on the shape of the potential near the minimum

$$w = \frac{P_\phi}{\rho_\phi} = \frac{\frac{1}{2}\langle\dot{\phi}^2\rangle - \langle V(\phi)\rangle}{\frac{1}{2}\langle\dot{\phi}^2\rangle + \langle V(\phi)\rangle} = \frac{n-2}{n+2}$$

Scalar field in curved space-time

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$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R + \underbrace{\frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} m_\chi^2 \chi^2}_{\text{spectator scalar}} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

Scalar field in curved space-time

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Cosmological background $g_{\mu\nu}^{\text{FLRW}}(x) = a^2(\eta) \text{diag}(1, -1, -1, -1)$

$$R(\eta) = -6a''/a^3 \quad \text{gravitational "potential" term}$$

Scalar field in curved space-time

Consider a spectator scalar field minimally coupled, in a classical gravitational background

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Cosmological background $g_{\mu\nu}^{\text{FLRW}}(x) = a^2(\eta) \text{diag}(1, -1, -1, -1)$

$R(\eta) = -6a''/a^3$ gravitational “potential” term

$$X_{\vec{k}} = a\chi_{\vec{k}} \longrightarrow X_{\vec{k}}'' + \left[k^2 + a^2 m_\chi^2 + \frac{a^2 R}{6} \right] X_{\vec{k}} = 0$$

$$m_{\text{eff}}^2(\eta) = m_\chi^2 + \frac{1}{6} R(\eta)$$

→ Time-dependent effective mass which sources gravitational effects

Modes equation in cosmological background

$$X''_{\vec{k}} + \underbrace{\left[k^2 + a^2 m_\chi^2 + \frac{a^2 R}{6} \right]}_{\omega_k(\eta)} X_{\vec{k}} = 0$$

Time-dependent frequency through the background evolution

- observers at different times may decompose operators onto different bases of mode functions and ladder operators
- vacuum can be further populated by scalar excitations throughout background evolution

Modes equation in cosmological background

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Time-dependent frequency through the background evolution

- observers at different times may decompose operators onto different bases of mode functions and ladder operators
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Already outlined by Schrödinger in 1939 as an
“alarming phenomenon” for expanding Universe
The Proper Vibrations of the Expanding Universe, Physica (1939)

Bogoliubov transformation



SU(1,1) Bogoliubov
transformation from
in to **out** base

$$X_{\vec{k}}^{\text{IN}}(\eta) = \alpha_k X_{\vec{k}}^{\text{OUT}}(\eta) + \beta_k X_{\vec{k}}^{\text{OUT}*}(\eta)$$

$$a_{\mathbf{k}}^{\text{IN}} = \alpha_k^* a_{\mathbf{k}}^{\text{OUT}} - \beta_k^* a_{-\mathbf{k}}^{\text{OUT}\dagger},$$

$$\eta \rightarrow +\infty$$

→ find “in” mode functions at late asymptotic times and project on the “out” base to “count” excitations

$$\langle 0|N|0\rangle = a^3 n_{\chi} = \int \frac{d^3 k}{(2\pi)^3} \langle 0|\tilde{a}_{-\vec{k}}^{\dagger} \tilde{a}_{\vec{k}}|0\rangle = \int \frac{d^3 k}{(2\pi)^3} |\beta_k|^2 \quad \text{spectrum for scalar modes sourced by gravity}$$

$$\beta_k = i \left(X_k^{\text{OUT}'} X_k^{\text{IN}} - X_k^{\text{IN}'} X_k^{\text{OUT}} \right) \simeq \lim_{\eta \rightarrow +\infty} \tilde{\beta}_k(\eta)$$

Bogoliubov transformation



SU(1,1) Bogoliubov transformation from **in** to **out** base

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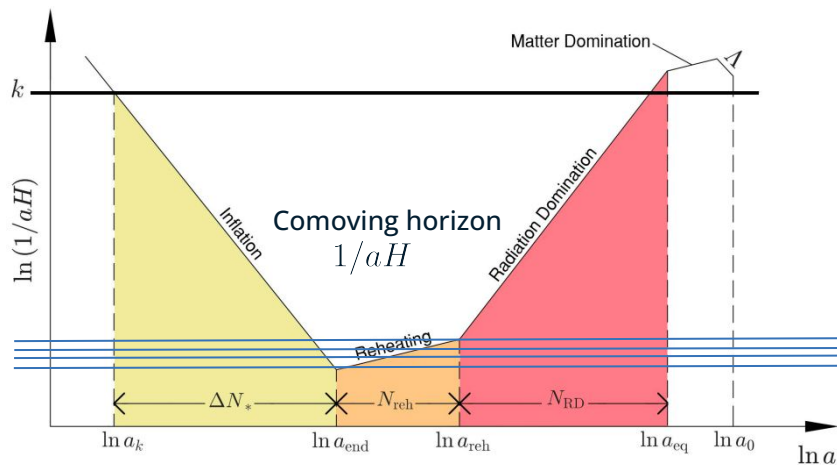
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$$n_k = |\beta_{\vec{k}}|^2 = \frac{1}{2\omega_k} |\omega_k X_{\vec{k}}^{\text{IN}} - i X_{\vec{k}}^{\text{IN}'}|^2 \quad \left| \begin{array}{l} \tilde{\alpha}'_k(\eta) = \tilde{\beta}_k(\eta) \frac{\omega'_k}{2\omega_k} e^{2i \int^\eta \omega_k(\tau) d\tau} \\ \tilde{\beta}'_k(\eta) = \tilde{\alpha}_k(\eta) \frac{\omega'_k}{2\omega_k} e^{-2i \int^\eta \omega_k(\tau) d\tau} \end{array} \right. \quad \eta \rightarrow +\infty$$

Long wavelength modes during Reheating



Super-horizon and reentry during reheating

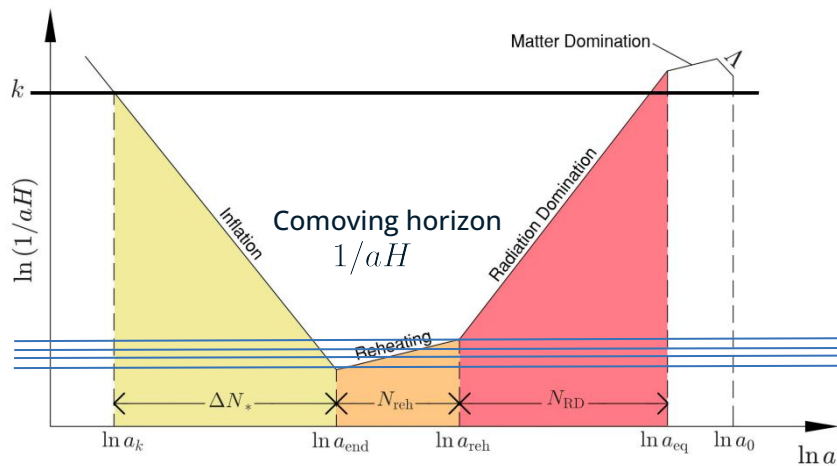
$$k^2 < \left(\frac{a''}{a} - a^2 m_\chi^2\right) |_{a_e} = a_e^2 (2H_e^2 - m_\chi^2)$$

From (P)reheating Effects of the Kähler Moduli Inflation I Model, Islam Khan, Aaron C. Vincent and Guy Worthey, 2111.11050

→ occupation number is affected by the equation of state during reheating w_ϕ

→ fast inflaton oscillations do not affect the occupation number of the long-wavelength modes

Long wavelength modes during Reheating



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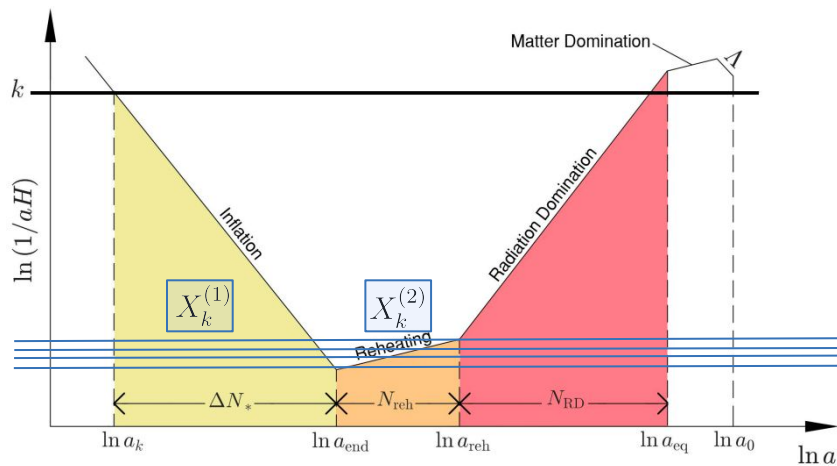
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$$n_k = |\beta_{\vec{k}}|^2 = \frac{1}{2\omega_k} |\omega_k X_{\vec{k}} - iX'_{\vec{k}}|^2$$

Solve numerically mode equations

Long wavelength modes during Reheating



Super-horizon and reentry during reheating

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Solve numerically mode equations

$$\beta_k = i \left(X_k^{(2)'} X_k^{(1)} - X_k^{(1)'} X_k^{(2)} \right)$$

Analytical derivation of the spectrum

Long wavelength modes during Reheating

$$X''_{\vec{k}} + \left[k^2 + a^2 \cancel{m_\chi^2} + \frac{a^2 R}{6} \right] X_{\vec{k}} = 0 \quad \text{consider first a **massless scalar field**}$$

Long wavelength modes during Reheating

$$X_{\vec{k}}'' + \left[k^2 + a^2 \cancel{m_\chi^2} + \frac{a^2 R}{6} \right] X_{\vec{k}} = 0 \quad \text{consider first a **massless scalar field**}$$

$$|\beta_k|_{\text{IR}}^2 = \frac{\mathcal{D}}{2\pi} \left(\frac{k_e}{k} \right)^{(2\bar{\nu}+3)},$$

$$a^3 \left. \frac{dn_\chi}{d \ln k} \right|_{\text{IR}} = 2\mathcal{D} \frac{k_e^3}{(2\pi)^3} \left(\frac{k_e}{k} \right)^{2\bar{\nu}}$$

Long wavelength modes during Reheating

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w_ϕ	$\bar{\nu}$
0	3/2
1/3	1/2
1/2	3/10
3/5	3/14
2/3	1/6
5/7	3/22
3/4	3/26
4/5	3/34
9/10	3/74

$$\bar{\nu} = \frac{3}{2} \frac{(1 - w_\phi)}{(1 + 3w_\phi)}$$

*Spectral behavior of gravitationally produced **massless** scalar perturbations (IR)*

Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production, Chakraborty, SC, Haque, Maity, Mambrini, 2503.21877

→ spectral behavior in the IR varies from k^{-3} for $w_\phi = 0$ to a flat spectrum in the limit $w_\phi \rightarrow 1$

Long wavelength modes during Reheating

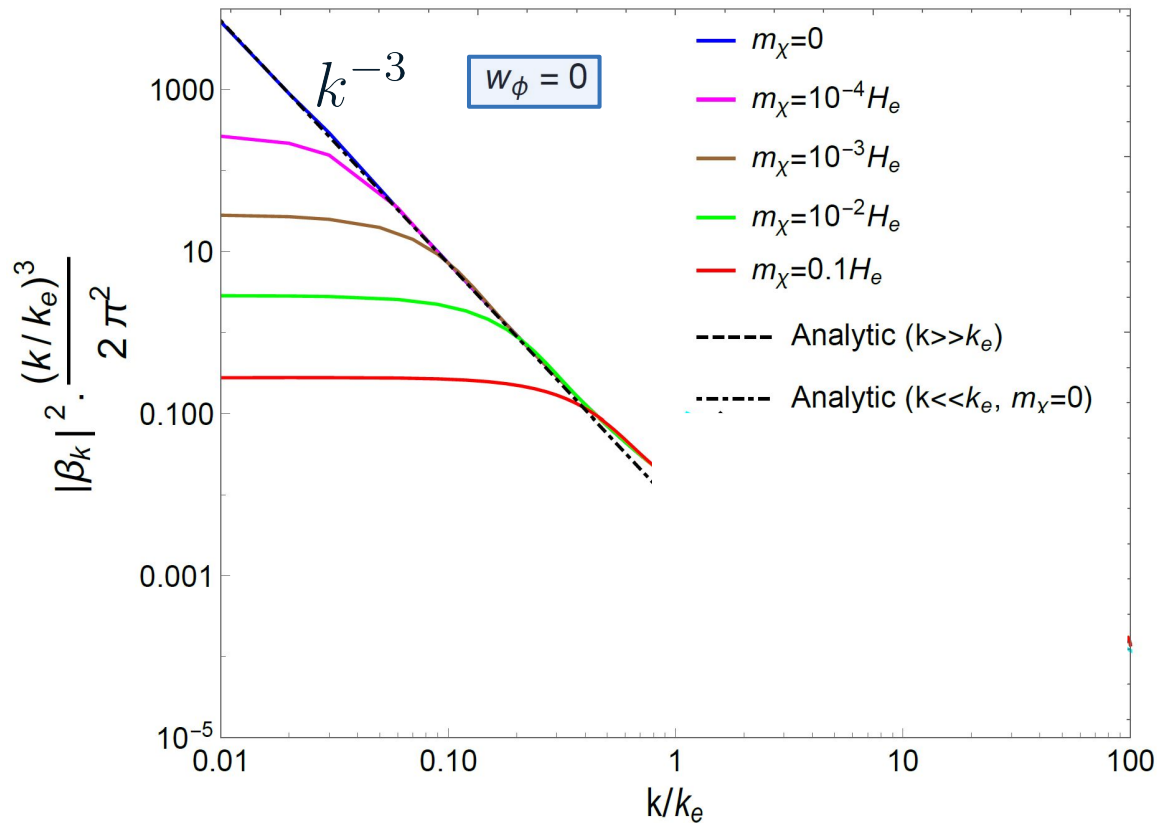
$$X''_{\vec{k}} + \left[k^2 + \boxed{a^2 m_\chi^2} + \frac{a^2 R}{6} \right] X_{\vec{k}} = 0 \quad \text{determine the effect of a **mass term**}$$

$$\frac{k_m}{k_e} = \left(\sqrt{\frac{2}{|3w_\phi - 1|}} \frac{m_\chi}{H_e} \right)^{\frac{1+3w_\phi}{3(1+w_\phi)}} \quad \text{below this comoving scale, mass term dominates at horizon reentry}$$

→ at smaller comoving scales, we obtain a flat spectrum $|\beta_k|^2 \propto (k_e/k)^3$
for massive scalar modes whatever the EoS w_ϕ

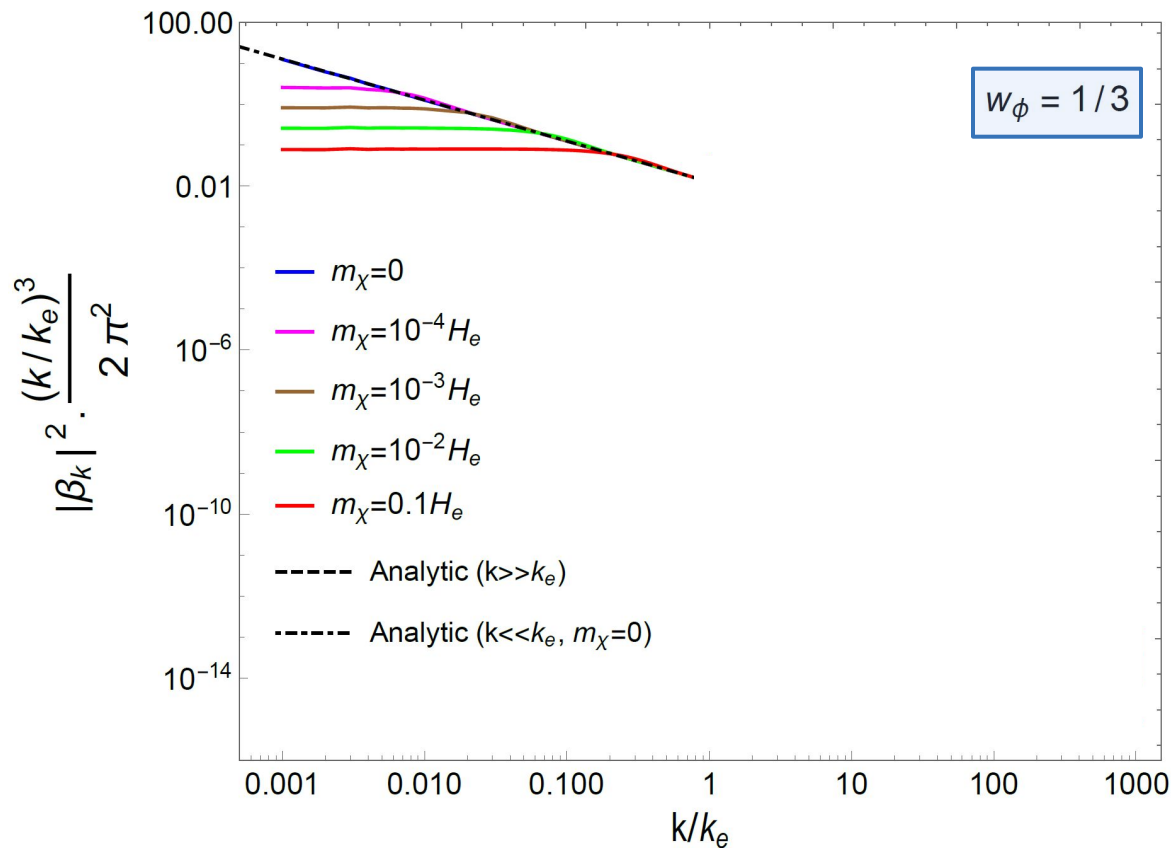
→ at large mass, $(m_\chi/H_e) > 3/2$, exponentially suppressed spectrum $\beta_k \propto e^{-\frac{m_\chi}{H_e} \frac{k^2}{(a_e H_e)^2}}$

Long wavelength modes during Reheating



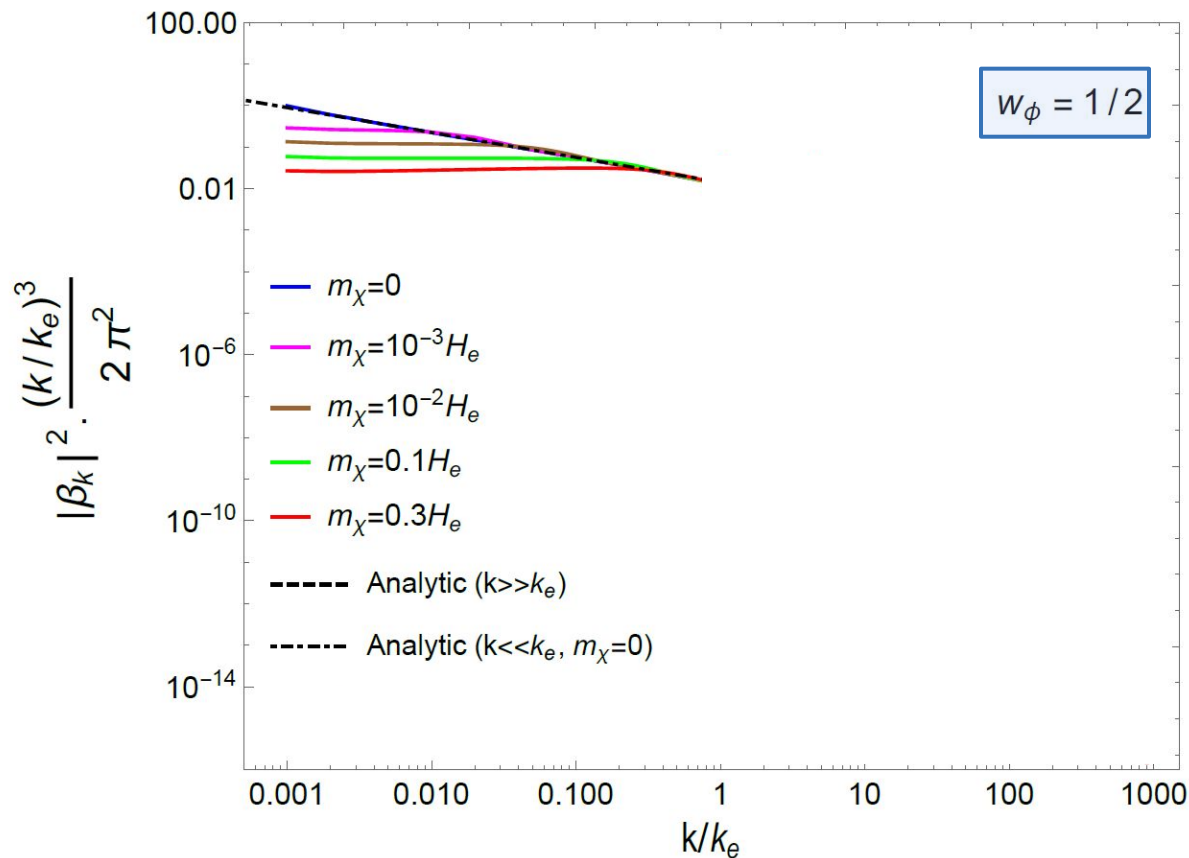
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Long wavelength modes during Reheating



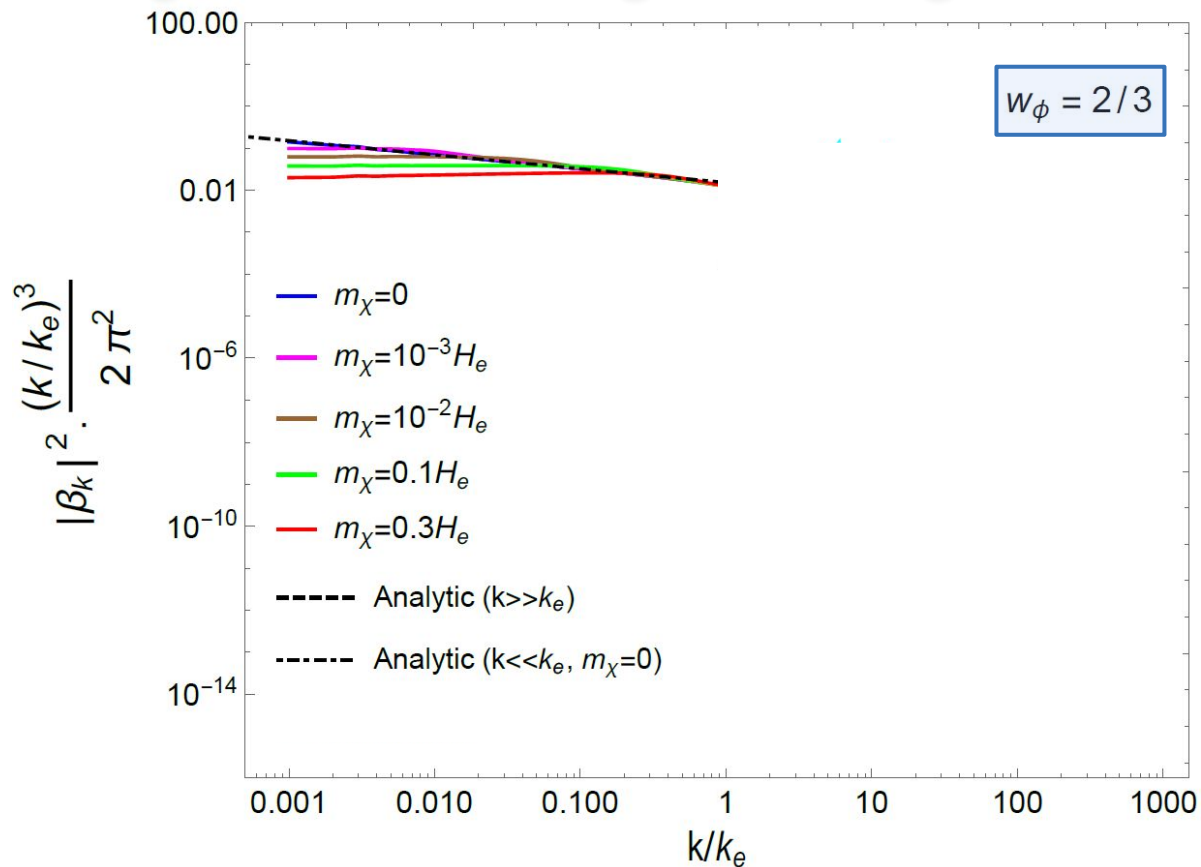
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Long wavelength modes during Reheating



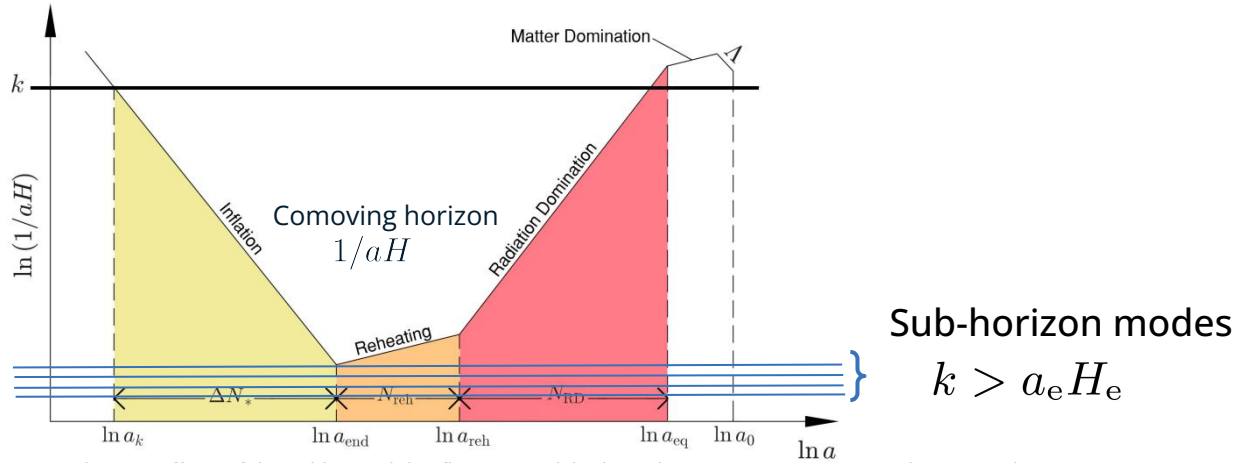
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Long wavelength modes during Reheating



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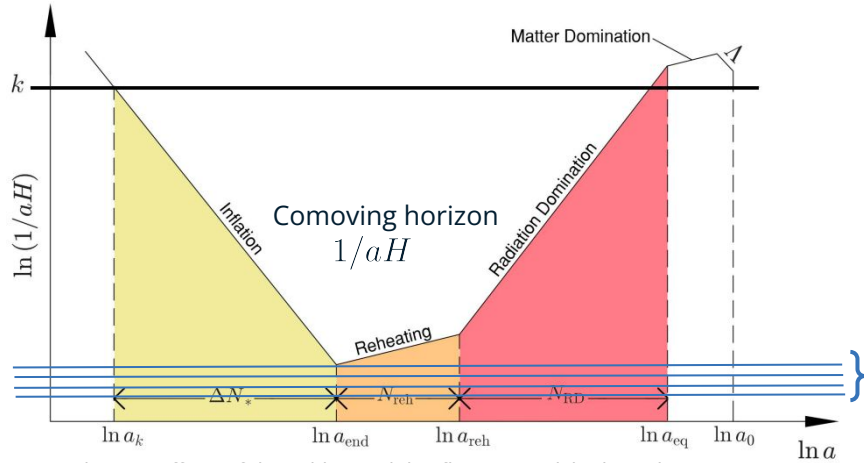
Short wavelength modes during Reheating



From (P)reheating Effects of the Kähler Moduli Inflation I Model, Islam Khan, Aaron C. Vincent and Guy Worthey, **2111.11050**

- no violation of adiabaticity in the evolution of mode frequency during inflation and reheating
- expect small occupation number in the UV spectrum induced by gravity
- fast inflaton oscillations affect the occupation number of the short-wavelength modes

Short wavelength modes during Reheating



Sub-horizon modes
 $k > a_e H_e$

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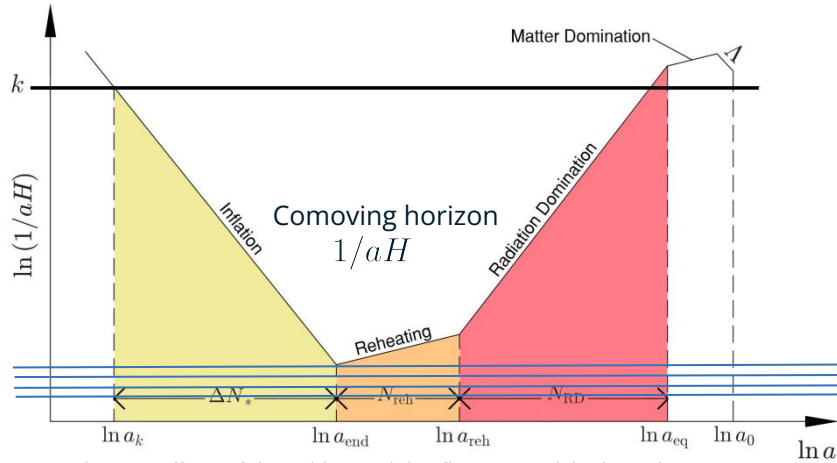
$$\begin{aligned}\tilde{\alpha}'_k(\eta) &= \tilde{\beta}_k(\eta) \frac{\omega'_k}{2\omega_k} e^{2i \int^\eta \omega_k(\tau) d\tau} \\ \tilde{\beta}'_k(\eta) &= \tilde{\alpha}_k(\eta) \frac{\omega'_k}{2\omega_k} e^{-2i \int^\eta \omega_k(\tau) d\tau}\end{aligned}$$

$|\tilde{\beta}_k(\eta)| \ll 1$
 $\xrightarrow{\frac{\omega'_k(\eta)}{\omega_k^2(\eta)} \ll 1}$

$$\begin{aligned}\beta_k(\eta) &\simeq \int_{\eta_e}^{\eta} d\eta' \frac{\omega'_k}{2\omega_k} e^{-2i\Omega_k(\eta')} \\ \Omega_k(\eta') &= \int_{\eta_e}^{\eta'} \omega_k(\eta) d\eta\end{aligned}$$

→ advantage to compute the Bogoliubov coefficient without solving exactly the mode equations

Short wavelength modes during Reheating



Sub-horizon modes

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→ advantage to compute the Bogoliubov coefficient without solving exactly the mode equations

Inflaton Fourier modes

Post-inflation background oscillations

$$\phi(t) = \Phi(t)\mathcal{P}(t) = \Phi(t) \sum_{\nu \neq 0} \mathcal{P}_\nu e^{i\nu\omega t}$$

→ a decaying amplitude $\Phi(t)$ and a quasi-periodic part $\mathcal{P}(t)$ both depends on w_ϕ

→ develop the oscillating part in Fourier modes \mathcal{P}_ν

Inflaton Fourier modes

Post-inflation background oscillations

$$\phi(t) = \Phi(t)\mathcal{P}(t) = \Phi(t) \sum_{\nu \neq 0} \mathcal{P}_\nu e^{i\nu\omega t}$$

→ a decaying amplitude $\Phi(t)$ and a quasi-periodic part $\mathcal{P}(t)$ both depends on w_ϕ

→ develop the oscillating part in Fourier modes \mathcal{P}_ν

$$H(a) \simeq \bar{H} \left(1 + \frac{\mathcal{P} \sqrt{6(1 - \mathcal{P}^{2n})}}{2(n+1)} \left(\frac{\phi_e}{M_P} \right) \left(\frac{a}{a_e} \right)^{-\frac{3}{n+1}} \right) \quad \bar{H} = H_e (a/a_e)^{-\frac{3n}{n+1}}$$

$$\frac{\dot{\omega}_k(t)}{\omega_k(t)} = \frac{a^2}{\omega_k^2} \left[H m_\chi^2 - 2H^3 - 3H\dot{H} - \frac{1}{2}\ddot{H} \right]$$

→ adiabatic variation of modes frequency with inflaton slowly decaying amplitude and fast oscillation

Short wavelength modes during Reheating

Case $w_\phi < 1/3$

→ Use the stationary phase approximation in the Bogoliubov integral and extract leading term

$$|\beta_k|_{\text{UV}, w_\phi < \frac{1}{3}}^2 = \begin{cases} (\bar{\mathcal{N}}_0)^2 \boxed{k^{\frac{9(w_\phi - 1)}{2 - 6w_\phi}}} + \underbrace{\bar{\mathcal{N}}_0 \bar{\mathcal{N}}_2 k^{\frac{45w_\phi - 21}{4(1 - 3w_\phi)}} \cos \psi}_{\text{interference term}} & w_\phi \leq 1/9 \\ (\bar{\mathcal{N}}_2)^2 \boxed{k^{-6}} + \underbrace{\bar{\mathcal{N}}_0 \bar{\mathcal{N}}_2 k^{\frac{45w_\phi - 21}{4(1 - 3w_\phi)}} \cos \psi}_{\text{interference term}} & w_\phi > 1/9 \end{cases}$$

→ Recover the known result $k^{-3/2}$ spectral behavior for $w_\phi = 0$

Short wavelength modes during Reheating

Case $w_\phi \geq 1/3$

→ No stationary phase within integration range: can extract only the large momentum contribution

$$|\beta_k|_{\text{UV}, w_\phi \geq \frac{1}{3}}^2 \simeq \frac{1}{16f^2(w_\phi)} \left(\frac{a_e}{k} \right)^6 \times \sum \sum \left[\mathcal{N}_0 + \mathcal{N}_1 + \mathcal{N}_2 + \mathcal{N}_3 \right]^2$$

→ On the whole range $1/9 < w_\phi \leq 1$ spectrum independent of w_ϕ in the UV and $\propto k^{-3}$

Effective graviton portal from perturbative computation

Graviton portal from effective gravitational interaction to small metric perturbations

$$g_{\mu\nu} \simeq \eta_{\mu\nu} + 2h_{\mu\nu}/M_P$$

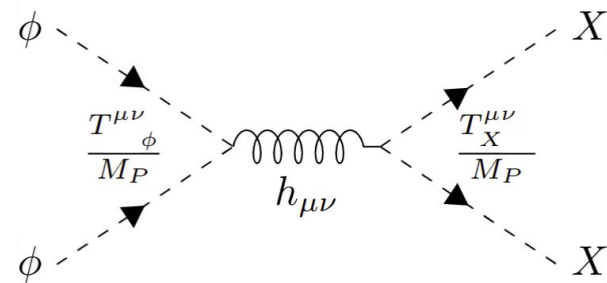
$$\mathcal{L}_{\text{min.}} = -\frac{1}{M_P} h_{\mu\nu} \left(T_{\phi}^{\mu\nu} + T_X^{\mu\nu} \right)$$

→ consider massless gravitons coupled to stress-energy and compute the amplitude of the process

Spin-2 Portal Dark Matter, Bernal, Dutra, Mambrini, Olive, Peloso, Pierre, **1803.01866**

Gravitational Production of Dark Matter during Reheating, Mambrini, Olive, **2102.06214**

Gravitational portals in the early Universe, SC, Mambrini, Olive, Verner, **2112.15214**

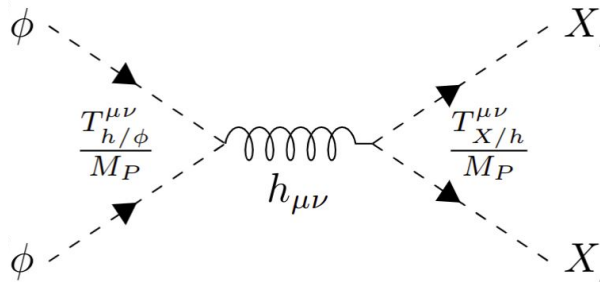


$$T_0^{\mu\nu} = \partial^\mu S \partial^\nu S - g^{\mu\nu} \left[\frac{1}{2} \partial^\alpha S \partial_\alpha S - V(S) \right]$$

$$\sum_{\nu=1}^{\infty} |\overline{\mathcal{M}}_\nu|^2 = \frac{1}{2} \times \sum_{\nu=1}^{\infty} \frac{\rho_\phi^2}{M_P^4} |\mathcal{P}_\nu^{2n}|^2$$

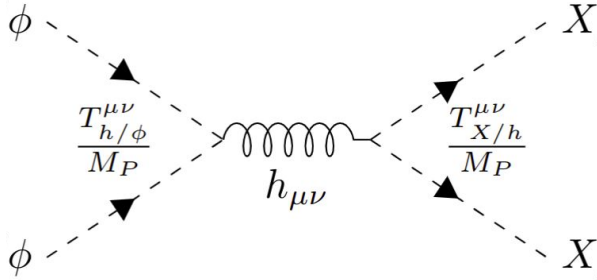
→ each Fourier mode contribute to the transition amplitude

Short wavelength modes from Boltzmann approach



$$\frac{\partial f_X}{\partial t} - H|\vec{p}| \frac{\partial f_X}{\partial |\vec{p}|} = C[f_X(|\vec{p}|, t)]$$

Short wavelength modes from Boltzmann approach

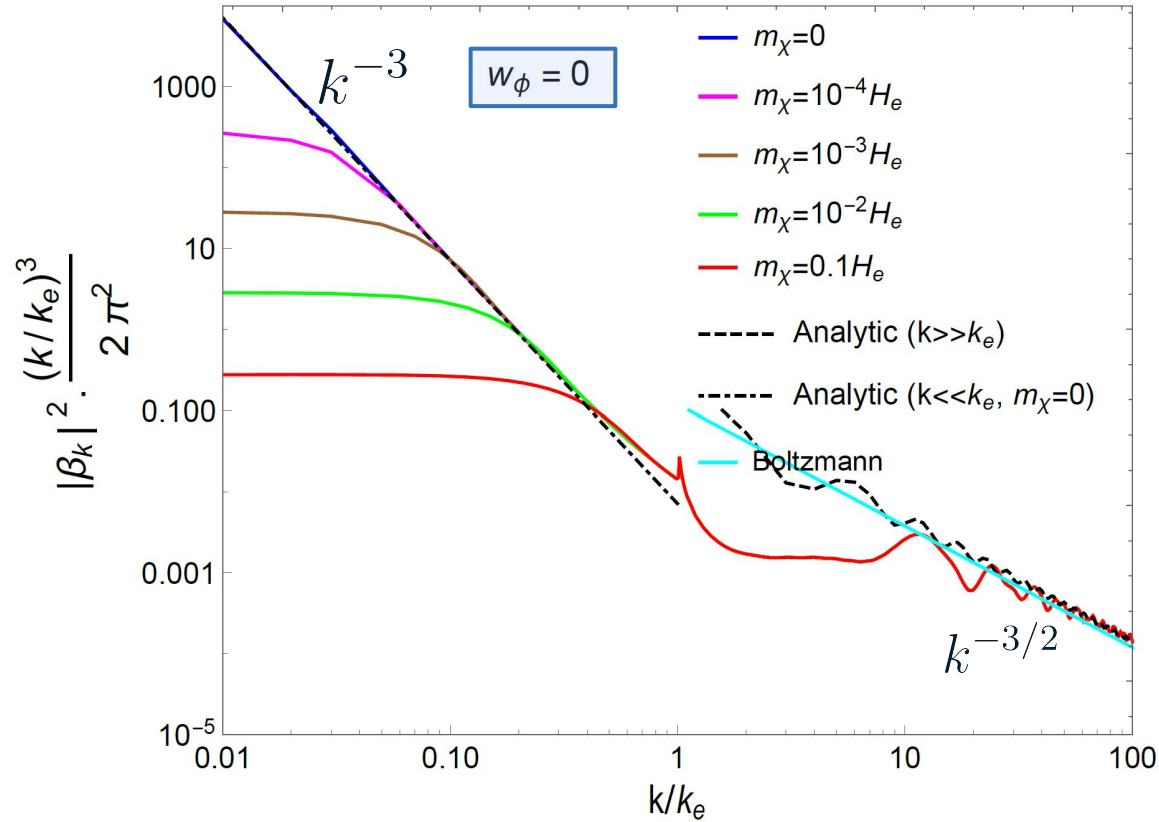


$$\frac{\partial f_\chi}{\partial t} - H|\vec{p}| \frac{\partial f_\chi}{\partial |\vec{p}|} = C[f_\chi(|\vec{p}|, t)]$$

$$f_\chi(k, a) = \frac{9\pi}{4|3w_\phi - 1|} \left(\frac{\bar{\alpha} m_\phi^e}{H_e} \right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}} \left(\frac{k_e}{k} \right)^{\frac{9(1-w_\phi)}{2(1-3w_\phi)}} \sum_{\nu=1}^{\infty} |\mathcal{P}_\nu^{2n}|^2 \left(\frac{\nu}{2} \right)^{\frac{3(1+3w_\phi)}{2(1-3w_\phi)}} \theta \left(\left(\frac{2k}{\nu a_e m_\phi^e \bar{\alpha}} \right)^{\frac{1}{1-3w_\phi}} - 1 \right) \\ \times \theta \left(\left(\frac{a}{a_e} \right) \left(\frac{2k}{\nu a_e m_\phi^e \bar{\alpha}} \right)^{\frac{1}{3w_\phi - 1}} - 1 \right)$$

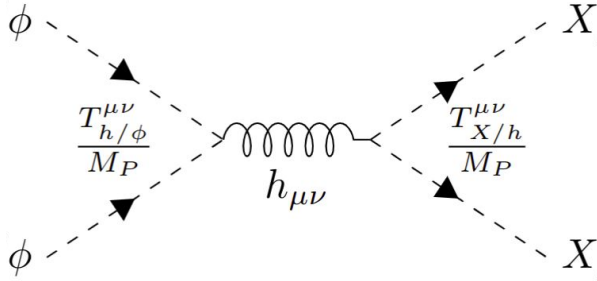
→ for $w_\phi < 1/3$ same stationary phase contribution as in the Bogoliubov approach

Spectrum for $w = 0$



Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production, Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

Short wavelength modes from Boltzmann approach



$$\frac{\partial f_\chi}{\partial t} - H|\vec{p}| \frac{\partial f_\chi}{\partial |\vec{p}|} = C[f_\chi(|\vec{p}|, t)]$$

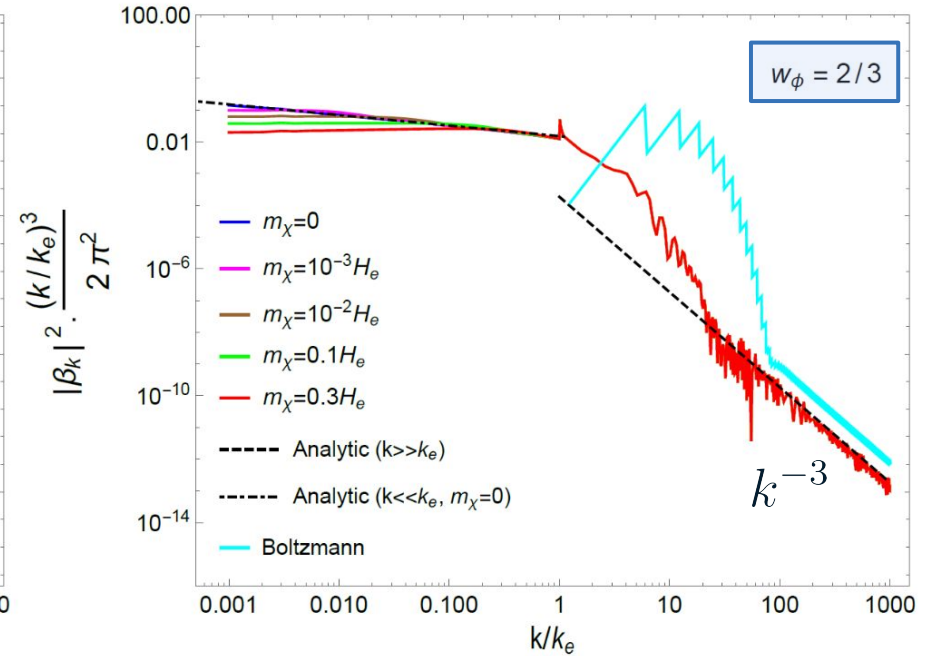
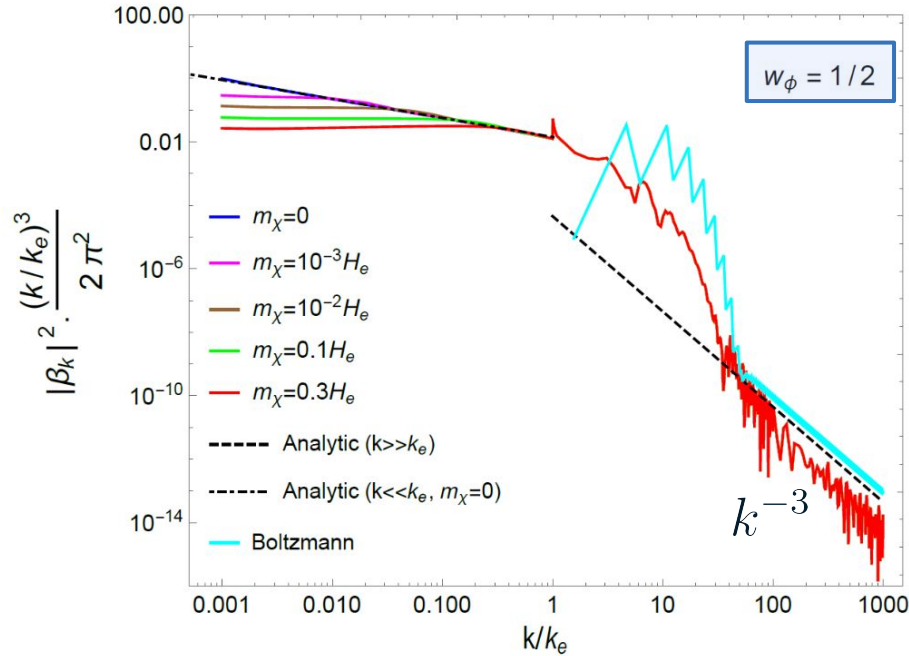
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$$\times \theta \left(\left(\frac{a}{a_e} \right) \left(\frac{2k}{\nu a_e m_\phi^e \bar{\alpha}} \right)^{\frac{1}{3w_\phi - 1}} - 1 \right)$$

$$\nu \bar{\alpha} / 2 \geq (k / a_e m_\phi^e)$$

→ for $w_\phi > 1/3$ negative spectral index but higher Fourier modes are heavily suppressed

Spectrum for higher EoS

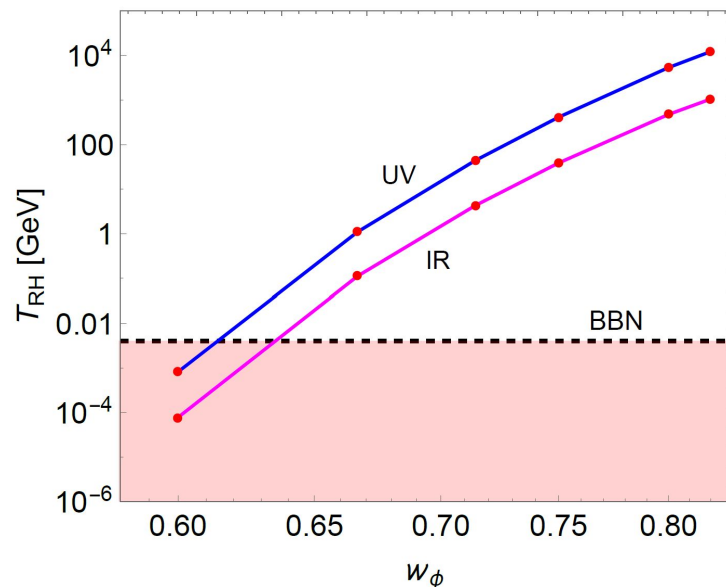


Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production, Chakraborty, SC, Haque, Maity, Mambrini, **2503.21877**

UV and IR modes contributions to Reheating

Determine the contribution to radiation bath from gravitational production at the end of reheating

$$\rho_R a^4 = \int_{k_{\text{RH}}}^{k_e} \frac{k^3}{2\pi^2} |\beta_k|_{\text{IR}}^2 dk + \int_{k_e}^{k_{\text{Planck}}} \frac{k^3}{2\pi^2} |\beta_k|_{\text{UV}}^2 dk$$

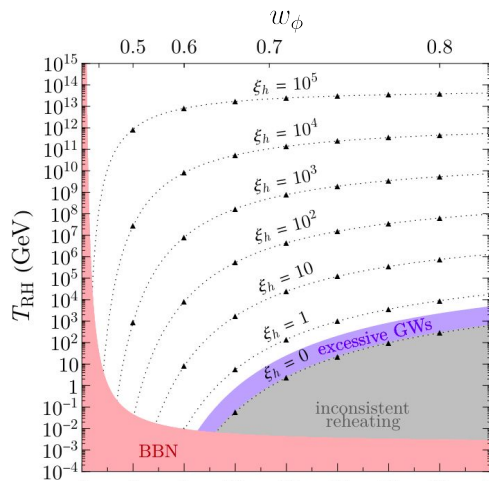
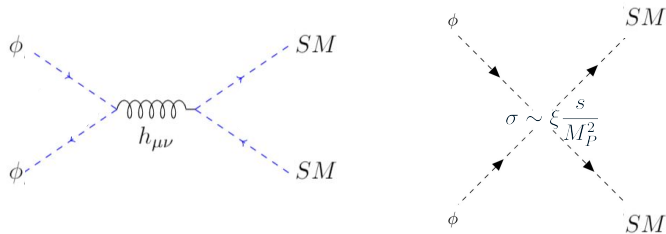


Bogoliubov		Boltzmann	
w_ϕ	T_{RH} (GeV)	w_ϕ	T_{RH} (GeV)
3/5	1.12×10^{-3}	3/5	9.40×10^{-4}
2/3	2.17	2/3	0.97
5/7	58.26	5/7	11.76
3/4	3.54×10^2	3/4	2.42×10^2
4/5	5.84×10^3	4/5	2.54×10^3

Generalizing the Bogoliubov vs Boltzmann approaches in gravitational production,
Chakraborty, SC, Haque, Maity, Mambrini,
2503.21877

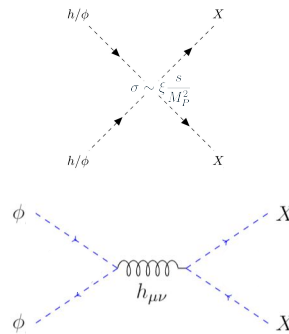
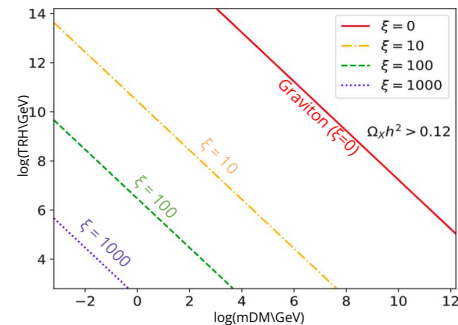
Perspectives

Gravitational reheating

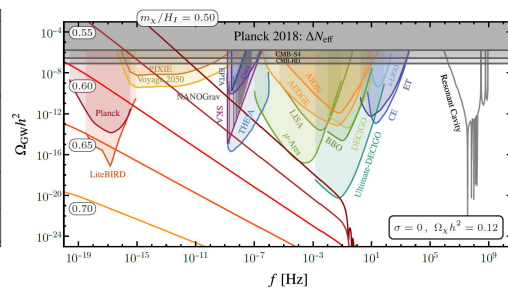
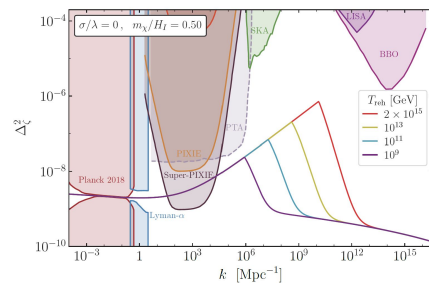


*Gravity as a Portal to
Reheating, Leptogenesis and
Dark Matter*, Barman, SC,
Co, Mambrini, Olive,
2210.05716

Gravitational DM and Isocurvature



Gravitational Portals with Non-Minimal Couplings, SC,
Mambrini, Olive, Shkerin, Verner, **2203.02004**

Gravitational Waves from Spectator Scalar Fields, Garcia, Verner, **2506.12126**

Conclusions

- Generalize results of **scalar gravitational production** by including inflaton **dynamics after inflation**
- **IR spectrum flatter for increasing EoS** and **UV tail is independent of the EoS** for $1/9 < w_\phi \leq 1$
- **Same UV tail power-law** with the **non-perturbative Bogoliubov approach** and the solution to the **Boltzmann equation** from **perturbative gravitational portal**

Perspectives :

- **Gravitational reheating**
- **Gravitational DM production**
- **SIGW from spectator scalar field**



Conclusions

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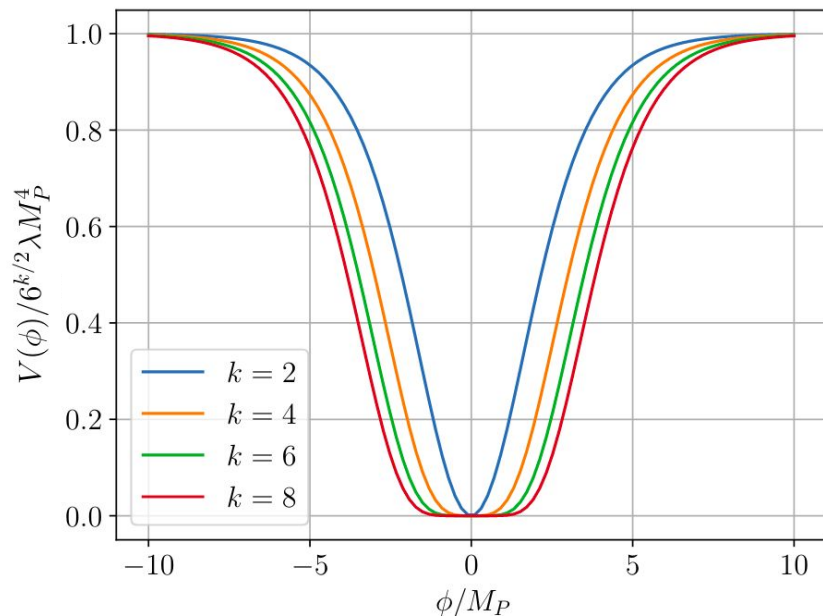
Thank you !

Backup

α -attractor Models

Inflation driven by an homogeneous scalar field ϕ in the potential

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6} \tanh \left(\frac{\phi}{\sqrt{6} M_P} \right) \right]^k$$



Inflaton potential for T-models and for different values of k .

$$\lambda \simeq \frac{18\pi^2 A_{S*}}{6^{k/2} N_*^2}$$

→ determined by the CMB scalar power spectrum amplitude A_S

→ PLANCK measurements give $\lambda \sim 10^{-11}$ for $k = 2$

Reheating and Post-inflationary Production of Dark Matter, Garcia, Kaneta, Mambrini, Olive, **2004.08404**

Universality Class in Conformal Inflation, Kallosh and Linde, **1306.5220**

Conformal invariance and gravitational production

Cosmological spacetimes (FLRW) are related to Minkowski by a time-dependent conformal transformation

$$g_{\mu\nu}^{\text{FLRW}}(\eta) = a^2(\eta)\eta_{\mu\nu}$$

Under a generic conformal transformation of the metric

$$g_{\mu\nu}(x) \rightarrow e^{2\Omega(x)} g_{\mu\nu}(x)$$

$$\delta\mathcal{S} = \frac{1}{2} \int d^4x \sqrt{-g} T_{\mu\nu} \delta g^{\mu\nu} = \int d^4x \sqrt{-g} \boxed{T^\mu{}_\mu} \delta\Omega(x)$$

For non-minimally
coupled scalar

$$T^\mu{}_\mu = (6\xi - 1) (g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + \chi \square \chi) + m_\chi^2 \chi^2$$

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$$T^\mu{}_{\mu, A} = 0$$

For a massless spin-1 vector field

$$T^\mu{}_{\mu, 1/2} \propto m$$

For a massive spin-1/2 fermion field

Asymptotic adiabatic modes and mixing of frequencies

How to track the excitations of the fields due to expansion ?

→ consider asymptotic early and late times, for which the comoving frequency is slowly varying

$$\begin{aligned} X_{\vec{k}}^{\text{IN}}(\eta) &\sim \frac{e^{-i \int^{\eta} d\eta' \omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}} \\ X_{\vec{k}}^{\text{OUT}}(\eta) &\sim \frac{e^{-i \int^{\eta} d\eta' \omega_k(\eta')}}{\sqrt{2\omega_k(\eta)}} \end{aligned} \quad \boxed{\frac{\omega'_k(\eta)}{\omega_k^2(\eta)} \ll 1} \quad (\eta \rightarrow \pm\infty) \quad \text{asymptotic adiabatic condition}$$

→ at intermediate times, write the mixing of positive and negative frequency modes

$$X_{\vec{k}}^{\text{IN}}(\eta) = \boxed{\alpha_k(\eta)} \frac{1}{\sqrt{2\omega_k(\eta)}} e^{-i \int^{\eta} \omega_k(\tau) d\tau} + \boxed{\beta_k(\eta)} \frac{1}{\sqrt{2\omega_k(\eta)}} e^{i \int^{\eta} \omega_k(\tau) d\tau}$$

$$\boxed{|\alpha_k(\eta)|^2 - |\beta_k(\eta)|^2 = 1} \quad \text{CCR preserved by EOM}$$

Mode functions during and after inflation (IR)

Massless scalar mode in de Sitter

$$X_k^{(1)}(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left[1 - \frac{i}{k\eta} \right] \simeq -\frac{i}{\sqrt{2}k^{\frac{3}{2}}\eta} e^{-ik\eta}$$



Massless scalar mode during Reheating

$$X_k^{(2)}(\eta) = \sqrt{\frac{\bar{\eta}}{\pi}} e^{i(3\bar{\mu}\frac{k}{k_e} + \frac{\pi}{4})} \times K_{\bar{\nu}}(ik\bar{\eta})$$

K are modified Bessel function with

$$\bar{\nu} = \frac{3}{2} \frac{(1 - \omega_\phi)}{(1 + 3w_\phi)} \quad \bar{\mu} = \frac{(1 + w_\phi)}{(1 + 3w_\phi)}$$

Massive scalar mode in de Sitter

$$X_k^{(1)}(\eta) = \frac{\sqrt{-\pi\eta}}{2} e^{i(\pi/4 + \pi\bar{\nu}_1/2)} H_{\bar{\nu}_1}^{(1)}(k|\eta|)$$



H are modified Hankel function with $\bar{\nu}_1 = \sqrt{\frac{9}{4} - \frac{m_\chi^2}{H_e^2}}$

No generic solution for arbitrary EoS and for massive scalar

→ Use a WKB approximation :

$$X_k^{(2)}(\eta) \simeq \frac{e^{-i\Omega_k(\eta)}}{\sqrt{2\omega_k(\eta)}}$$

Short wavelength modes during Reheating

Case $w_\phi < 1/3$

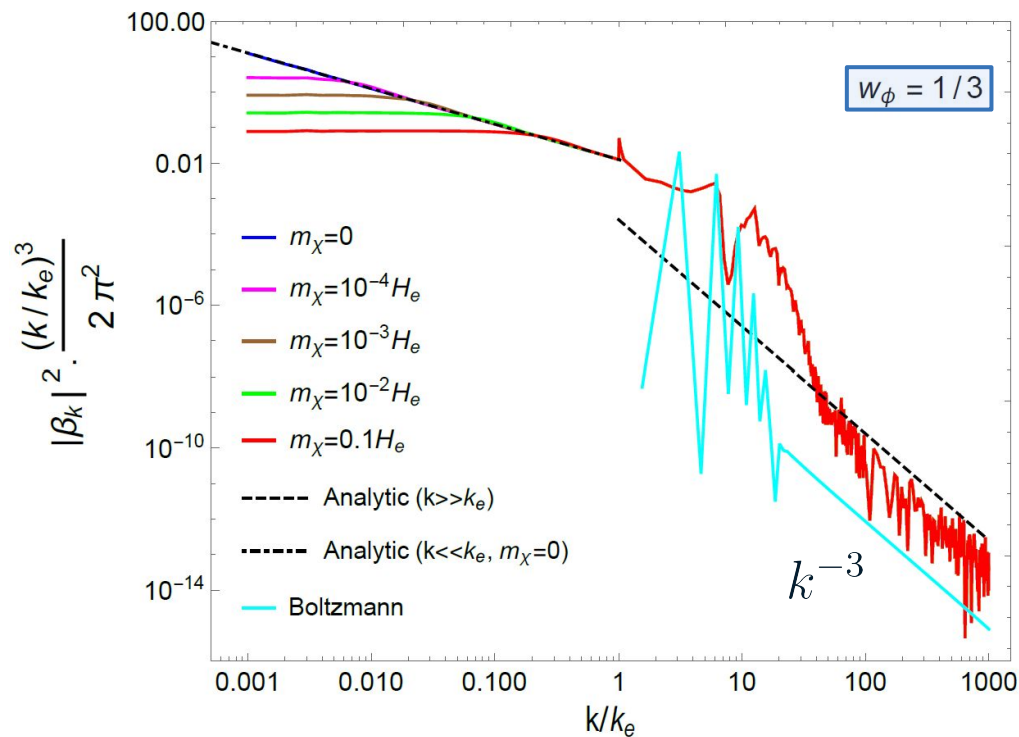
$$\beta_k \simeq \frac{1}{2} \sum_{\nu, l \neq 0} \int_{t_e}^t dt' \left(\frac{t_e}{t'} \right)^3 \left[\cancel{\mathcal{N}_0} e^{i(\nu+l)\omega t'} \left(\frac{t'}{t_e} \right)^{\frac{1}{n}} + \cancel{\mathcal{N}_1} e^{i\nu\omega t'} + \mathcal{N}_2 + \cancel{\mathcal{N}_3} e^{i(\nu+l)\omega t'} \left(\frac{t_e}{t'} \right)^{\frac{1}{n}} \right] \times \frac{e^{-2i\Omega_k(t')}}{\left(\frac{k^2}{a^2} + m_\chi^2 \right)}$$

use stationary phase approximation

integrate by parts and take the large momentum contribution

$$|\beta_k|_{\text{UV}, w_\phi < \frac{1}{3}}^2 = \begin{cases} (\bar{\mathcal{N}}_0)^2 \boxed{k^{\frac{9(w_\phi - 1)}{2 - 6w_\phi}}} + \underbrace{\bar{\mathcal{N}}_0 \bar{\mathcal{N}}_2 k^{\frac{45w_\phi - 21}{4(1 - 3w_\phi)}} \cos \psi}_{\text{interference term}} & w_\phi \leq 1/9 \\ (\bar{\mathcal{N}}_2)^2 \boxed{k^{-6}} + \underbrace{\bar{\mathcal{N}}_0 \bar{\mathcal{N}}_2 k^{\frac{45w_\phi - 21}{4(1 - 3w_\phi)}} \cos \psi}_{\text{interference term}} & w_\phi > 1/9 \end{cases}$$

Short wavelengths spectrum for $w = 1/3$



$$f_\chi^{w_\phi=1/3}(k, a) = 3\pi \left(\frac{H_e}{m_\phi^e \bar{\alpha}} \right)^2 \left[1 - \left(\frac{a_e}{a} \right)^3 \right] \sum_{\nu=1}^{+\infty} \frac{|\mathcal{P}_\nu^{2n}|^2}{\nu^2} \delta \left(\frac{k}{k_e} - \frac{\nu \bar{\alpha} m_\phi^e}{2H_e} \right)$$

Bogoliubov approach with non-minimal coupling

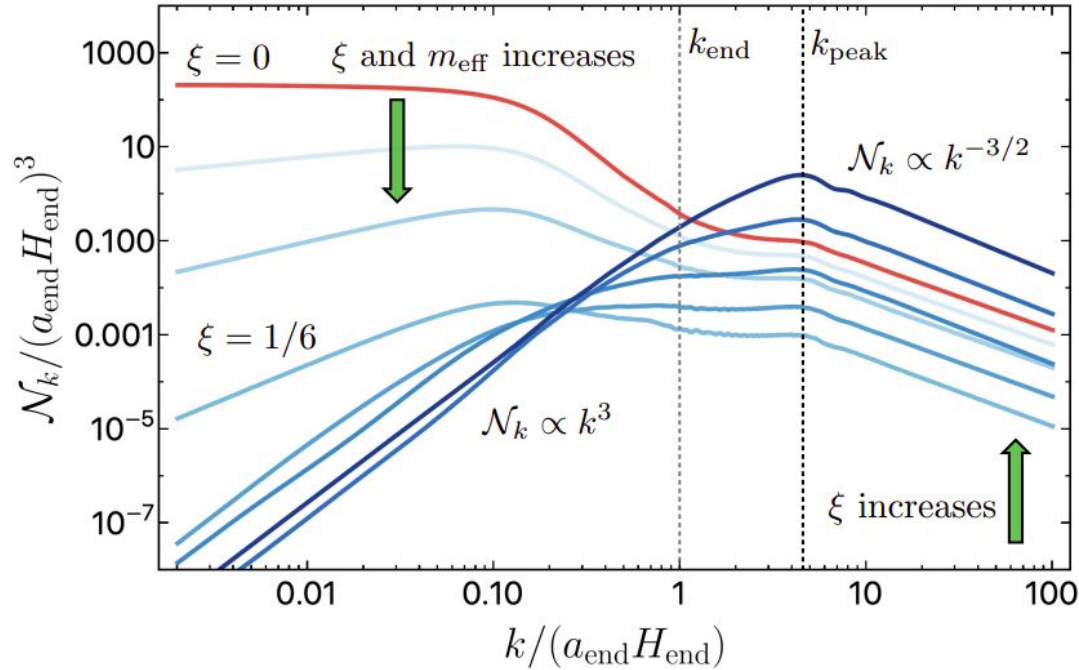
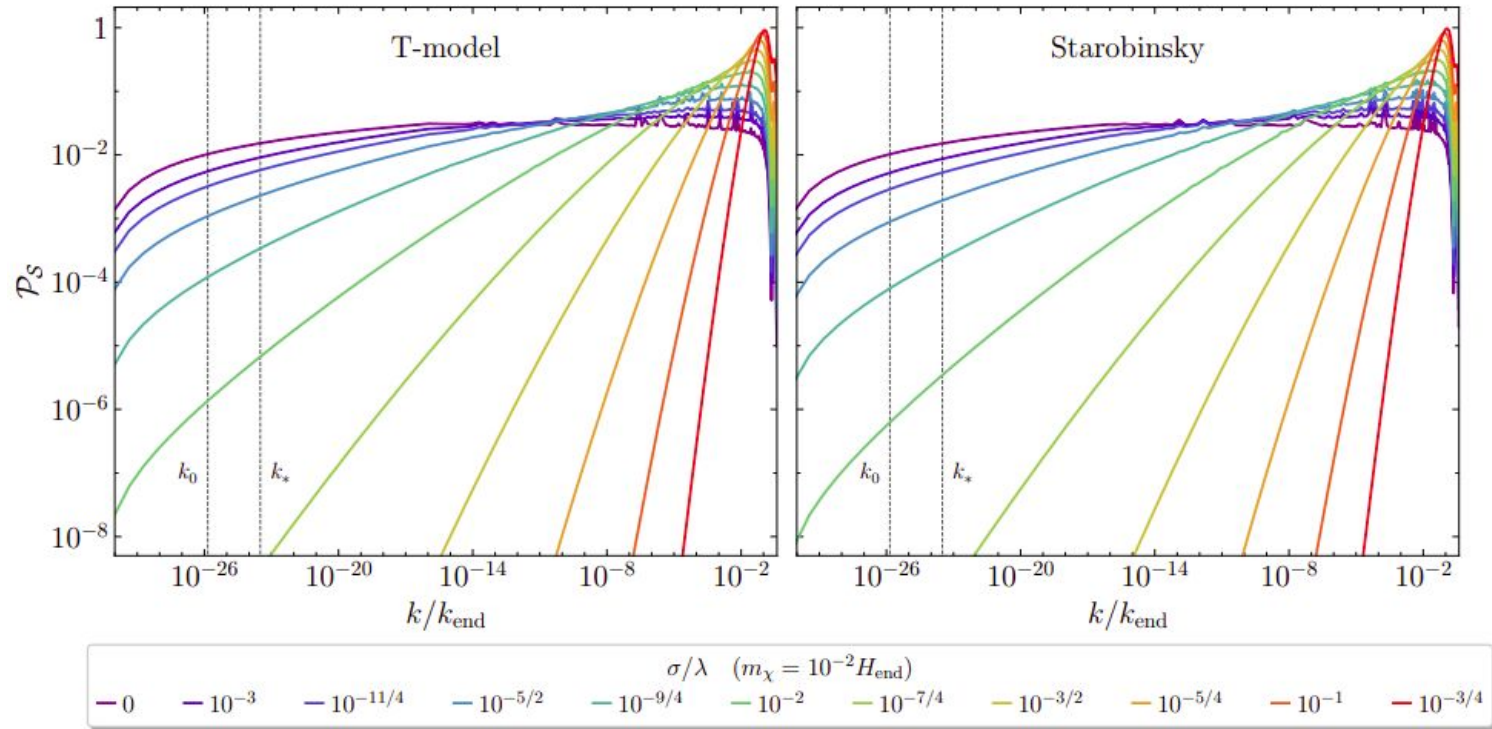


Diagram illustrating the dependence of the produced comoving number density spectrum \mathcal{N}_k on non-minimal coupling ξ as a function of rescaled horizon modes momenta

From A New Window into Gravitationally Produced Scalar Dark Matter, Garcia, Pierre, Verner, **2305.14446**

Isocurvature perturbations



DM isocurvature power spectrum for different inflaton-DM couplings with $m_\chi/H_{\text{end}} = 10^{-2}$

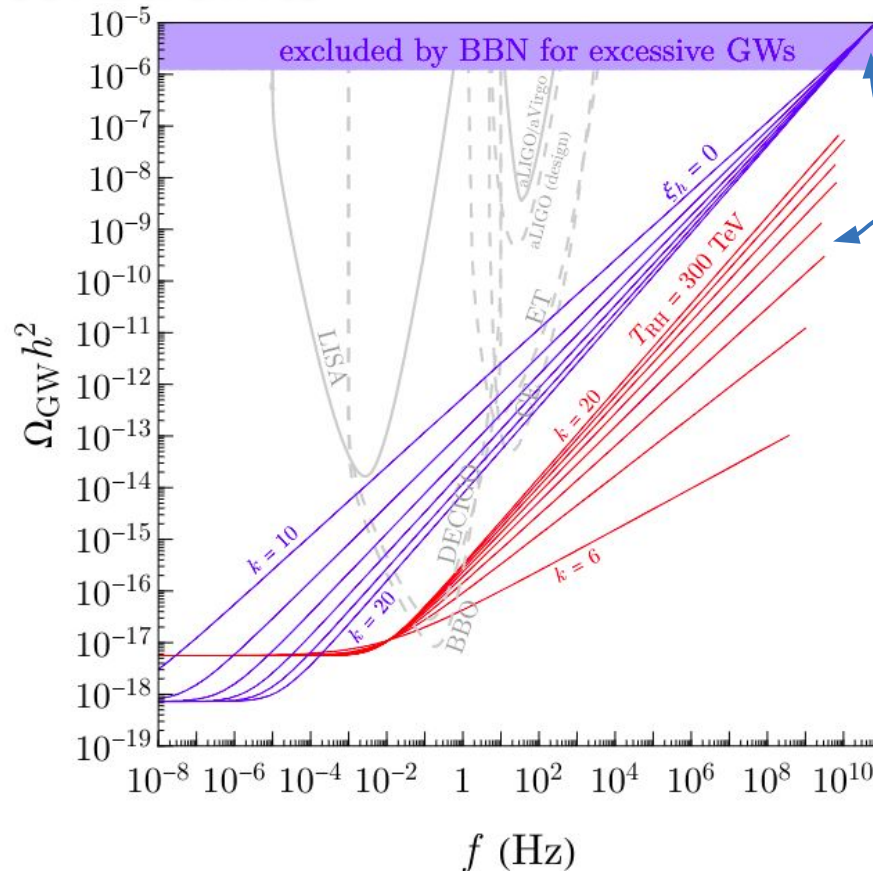
From *Isocurvature Constraints on Scalar Dark Matter Production from the Inflaton*, Garcia, Pierre, Verner, **2303.07459**

Primordial GWs constraints

→ Primordial GWs re-entering the horizon during reheating, can be enhanced.

$$\Omega_{\text{GW}}^0 h^2 \propto f^{\frac{k-4}{k-1}}$$

→ The slope of this spectrum depends on inflaton potential

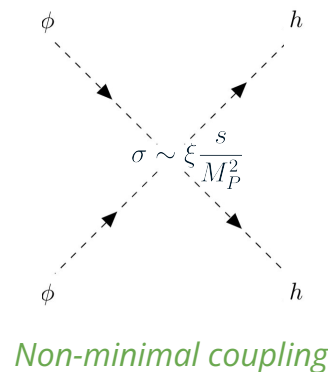
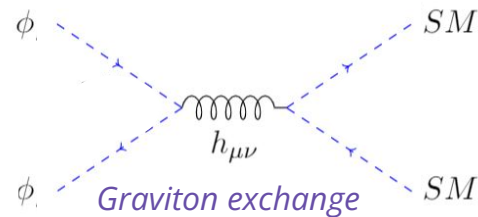
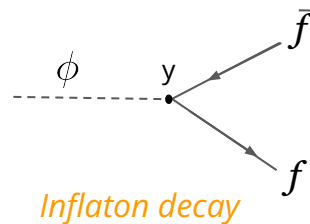
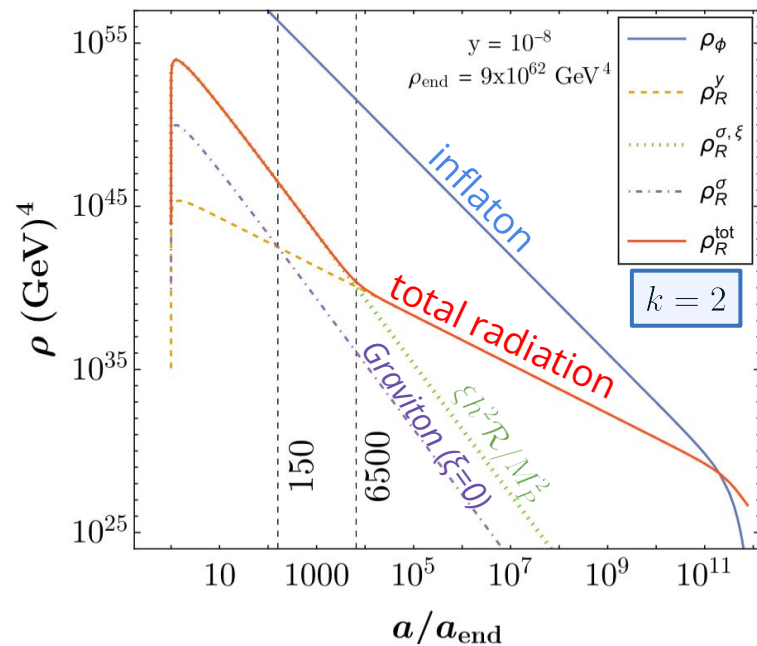


Largest enhancement for modes re-entering the horizon right after inflation

Gravity as a Portal to Reheating, Leptogenesis and Dark Matter, Barman, SC, Co, Mambrini, Olive, 2210.05716

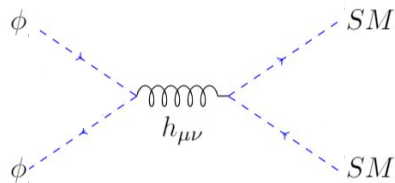
Primordial GWs strength as function of its frequency f . Blue curves fix $\xi_h = 0$ and Red curves fix $T_{\text{RH}} = 300 \text{ TeV}$ for k in $[6, 20]$. The sensitivity of several future GWs experiments are shown.

Radiation perturbative production



Energy densities of inflaton (blue), total radiation (red), radiation from inflaton decay (orange), from scattering mediated by graviton (purple) and from non-minimal coupling (green), with $\xi = 2$

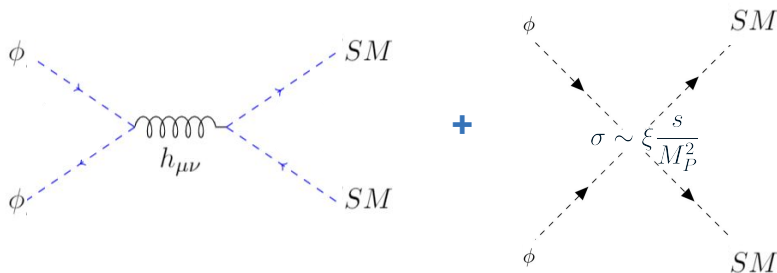
Gravitational reheating



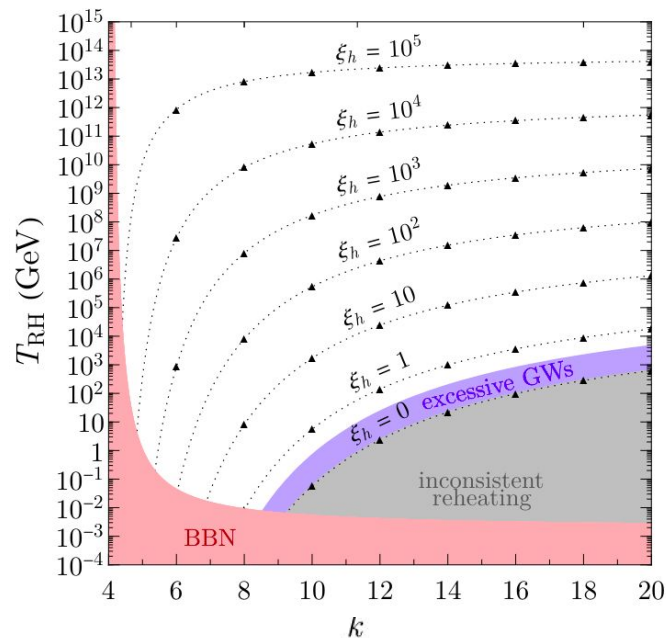
→ Graviton exchange processes can be sufficient to reheat entirely, for sufficiently steep inflaton potential : $k > 9$

Gravity as a Portal to Reheating, Leptogenesis and Dark Matter, Barman, SC, Co, Mambrini, Olive, 2210.05716

$$\rho_\phi(a) \propto \left(\frac{a_{\text{end}}}{a}\right)^{\frac{6k}{k+2}} \quad \rho_R(a) \propto \left(\frac{a_{\text{end}}}{a}\right)^4$$

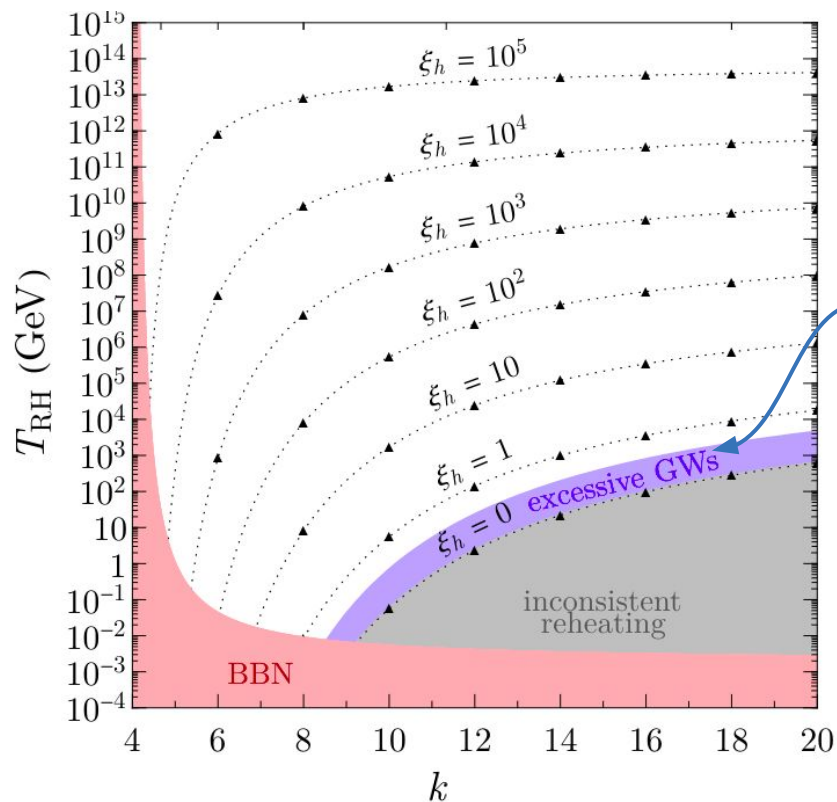


→ Requirement of large k can be relaxed adding the non-minimal contribution to radiation production (but still need $k > 4$)



Reheating temperature from gravitational portals as function of k , for different ξ_h

Primordial GWs constraints



Reheating temperature from gravitational portals as
function of k , for different ξ_h

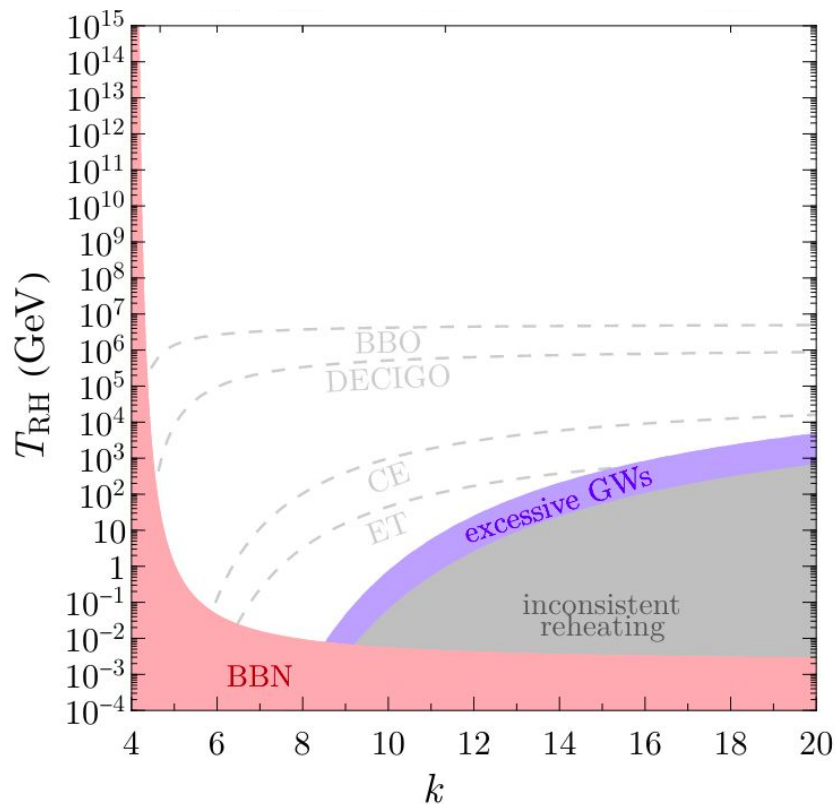
→ GWs leave the same imprint as free-streaming dark radiation on CMB

→ The case of minimal gravitational reheating is excluded by the CMB + BBN bound of $\Omega_{\text{GW}}^0 h^2 \lesssim 10^{-6}$, from excessive GWs as dark radiation

→ The constraint is relaxed when radiation production is increased by non-minimal gravitational interactions $\xi_h > 0$

Gravity as a Portal to Reheating, Leptogenesis and Dark Matter, Barman, SC, Co, Mambrini, Olive, 2210.05716

Primordial GW constraints



Reheating temperature from gravitational portals as function of k , for different ξ_h

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→ The case of minimal gravitational reheating is excluded by the CMB + BBN bound of $\Omega_{\text{GW}}^0 h^2 \lesssim 10^{-6}$, from excessive GWs as dark radiation

→ The constraint is relaxed when radiation production is increased by non-minimal gravitational interactions $\xi_h > 0$

→ An important part of the parameter space for reheating could be probed by future GWs experiments

Done more generically in *Measuring Inflaton Couplings via Primordial Gravitational Waves*, Barman, Ghoshal, Grzadkowski, Socha, 2305.00027

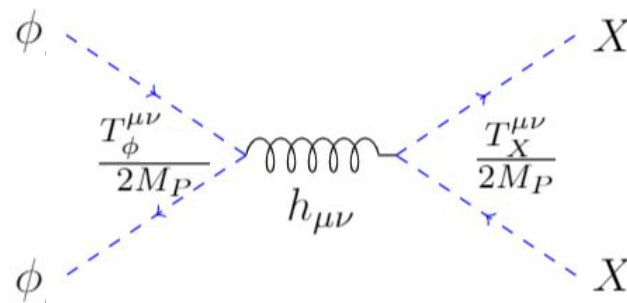
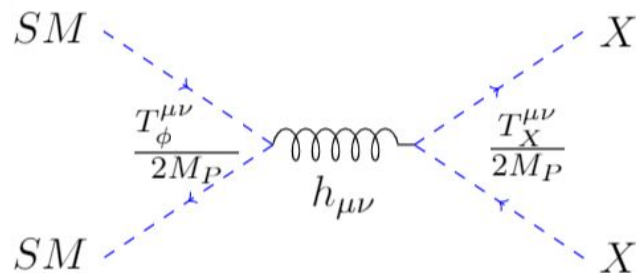
Gravitational portals

$$R_j(T) = \beta_j \frac{T^8}{M_P^4} \quad \text{for spin } j = 0, \frac{1}{2} \text{ DM final state}$$

See *Spin-2 Portal Dark Matter*, Nicolás Bernal, Maíra Dutra, Yann Mambrini, Keith Olive, Marco Peloso, **1803.01866**

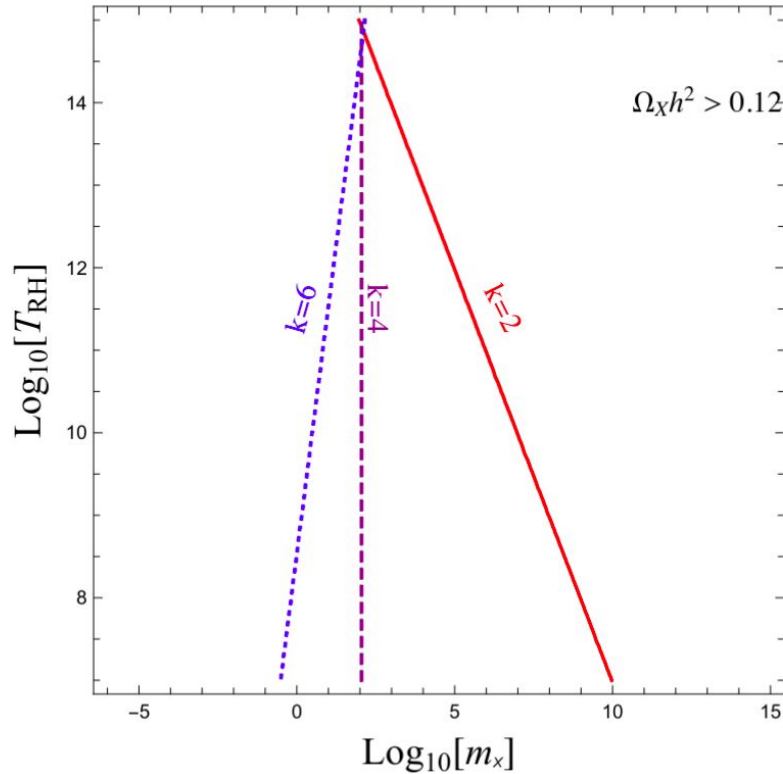
$$R_0^{\phi^k} = \frac{2 \times \rho_\phi^2}{16\pi M_P^4} \sum_{n=1}^{\infty} |\mathcal{P}_n^k|^2 \left[1 + \frac{2m_X^2}{E_n^2} \right]^2 \sqrt{1 - \frac{4m_X^2}{E_n^2}} \quad \text{spin 0}$$

$$R_{1/2}^{\phi^k} = \frac{2 \times \rho_\phi^2}{4\pi M_P^4} \frac{m_X^2}{m_\phi^2} \sum_{n=1}^{+\infty} |\mathcal{P}_n^k|^2 \frac{m_\phi^2}{E_n^2} \left[1 - \frac{4m_X^2}{E_n^2} \right]^{3/2} \quad \text{spin } \frac{1}{2}$$

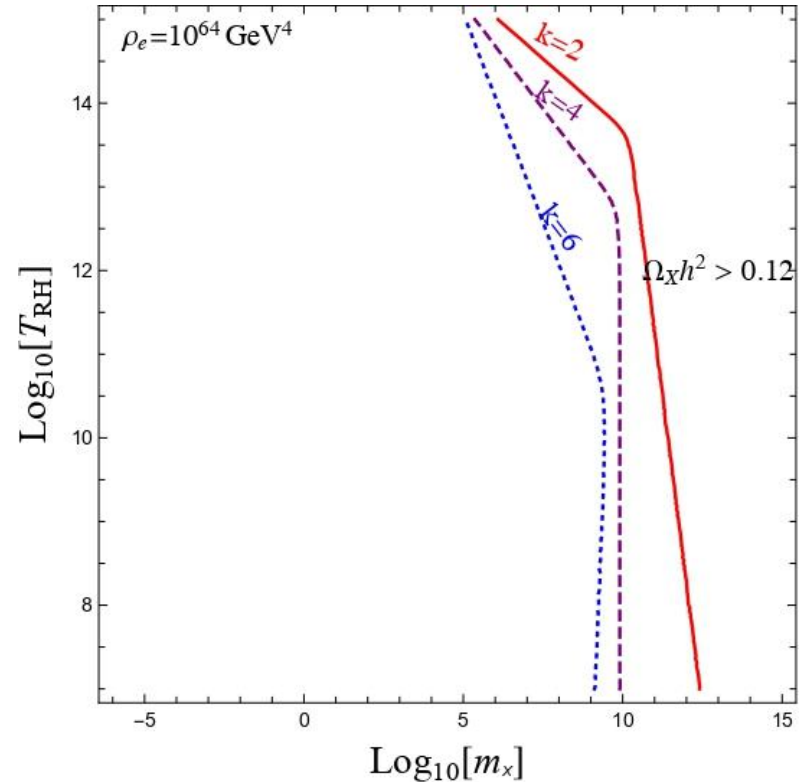


Gravitational portals in the early Universe, **SC**, Mambrini, Olive, Verner, **2112.15214**

Dark Matter gravitational production during reheating

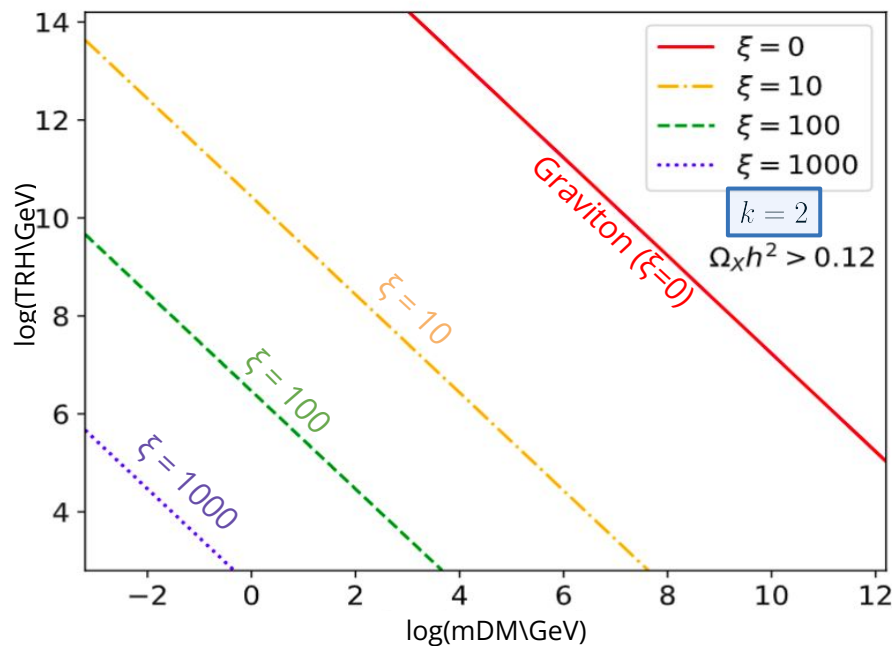


$\Omega_X h^2 = 0.12$ in the case of a spin 0 DM
all contributions added

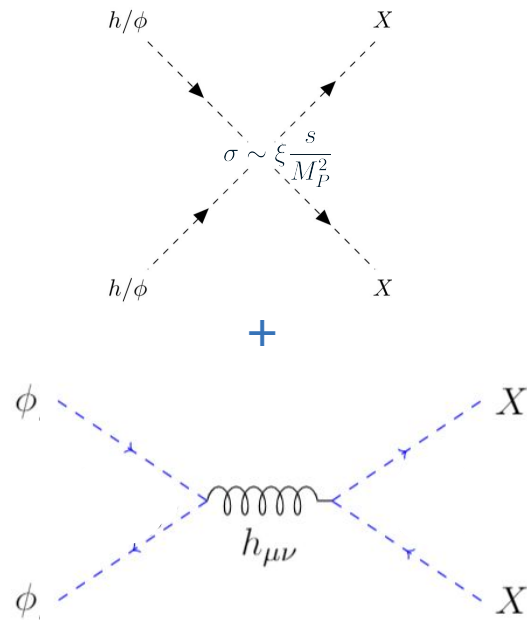


$\Omega_X h^2 = 0.12$ in the case of a spin $\frac{1}{2}$ DM, all
contributions added

Non-minimal production of Dark Matter



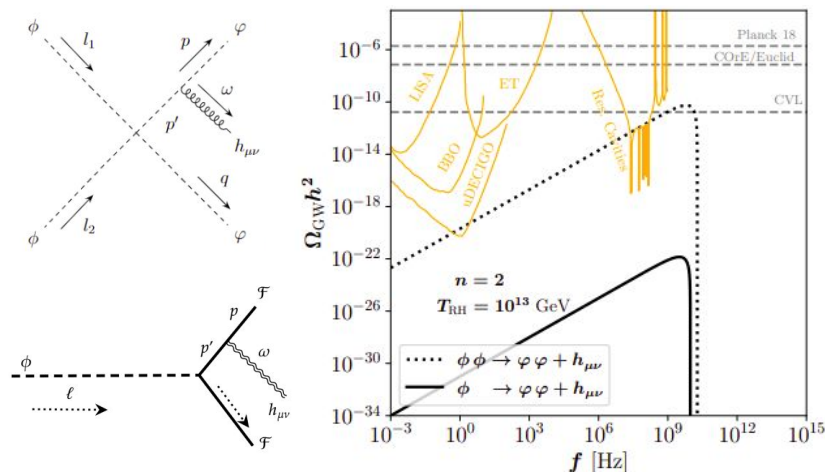
Contours respecting $\Omega_X h^2 = 0.12$ for spin 0 DM, for different values of $\xi_h = \xi_X = \xi$. Both minimal and non-minimal contributions are added.



Gravitational Portals with Non-Minimal Couplings, **SC**, Mambrini, Olive, Shkerin, Verner, **2203.02004**

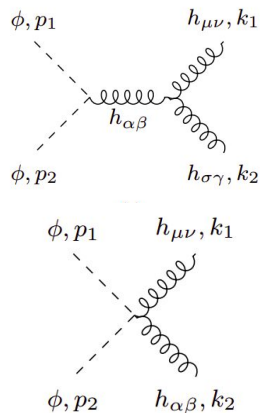
Observable signal of effective gravitational interaction

→ Look at particle origin for stochastic GWs background that generates a spectrum at high frequencies, and depends on the details of reheating



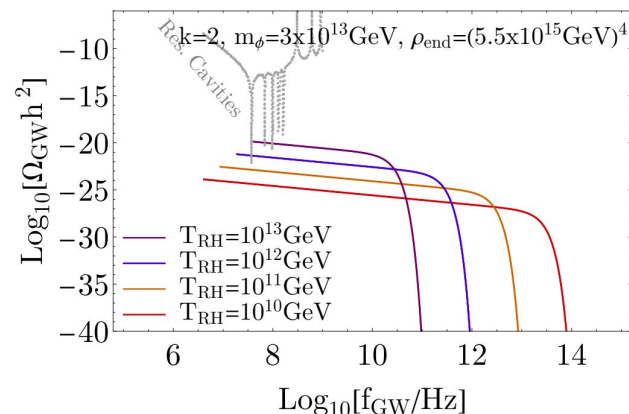
Graviton bremsstrahlung

Probing Reheating with Graviton Bremsstrahlung,
Bernal, **SC**, Mambrini and Xu, **2311.12694**



Direct gravitons production

Minimal production of prompt gravitational waves
during reheating, Choi, Ke, Olive, **2402.04310**



Non minimal coupling to gravity

The natural generalization of this minimal interaction is to introduce non-minimal couplings to gravity of the form :

$$\mathcal{L}_{\text{non-min.}} = \boxed{-\frac{M_P^2}{2}\Omega^2\tilde{R}} + \mathcal{L}_\phi + \mathcal{L}_h + \mathcal{L}_X$$

in the Jordan frame

$$g_{\mu\nu} = \Omega^2 \tilde{g}_{\mu\nu}$$

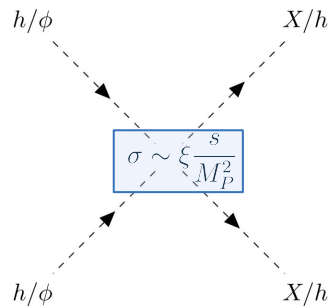
$$\mathcal{L}_{\text{non-min.}} = -\sigma_{hX}^\xi h^2 X^2 - \sigma_{\phi X}^\xi \phi^2 X^2 - \sigma_{\phi h}^\xi \phi^2 h^2$$

in the Einstein frame

with

$$\Omega^2 \equiv 1 + \underbrace{\frac{\xi_\phi \phi^2}{M_P^2}}_{\text{inflaton}} + \underbrace{\frac{\xi_h h^2}{M_P^2}}_{\text{SM}} + \underbrace{\frac{\xi_X X^2}{M_P^2}}_{\text{DM}}$$

Non-minimal couplings induce leading-order interactions in the small fields limit, involved in radiation and DM production



Reheating and Dark Matter Freeze-in in the Higgs- R^2 Inflation Model, Aoki, Lee, Menkara, Yamashita, **2202.13063**

Gravitational Portals with Non-Minimal Couplings, **SC**, Mambrini, Olive, Shkerin, Verner, **2203.02004**

Non-minimal coupling : the small-field limit

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[-\frac{M_P^2}{2} R + \frac{1}{2} K^{ij} g^{\mu\nu} \partial_\mu S_i \partial_\nu S_j - \frac{V_\phi + V_h + V_X}{\Omega^4} \right] \quad \text{in Einstein frame}$$

with

$$\Omega^2 \equiv 1 + \frac{\xi_\phi \phi^2}{M_P^2} + \frac{\xi_h h^2}{M_P^2} + \frac{\xi_X X^2}{M_P^2} \quad \text{and} \quad K^{ij} = 6 \frac{\partial \log \Omega}{\partial S_i} \frac{\partial \log \Omega}{\partial S_j} + \frac{\delta^{ij}}{\Omega^2} \quad \text{non-canonical kinetic term}$$

No field redefinition to the canonical form, unless all three non-minimal couplings vanish

$$\frac{|\xi_\phi| \phi^2}{M_P^2}, \quad \frac{|\xi_h| h^2}{M_P^2}, \quad \frac{|\xi_X| X^2}{M_P^2} \ll 1$$

Small-field limit: expand the action in powers of M_P^{-2} obtain canonical kinetic term and leading-order interactions induced by the non-minimal couplings

At the end of inflation we have $\phi_{\text{end}} \sim M_P$ and the inflaton field is decreasing during the reheating

$$|\xi_\phi| \lesssim 1$$

Parametric resonances

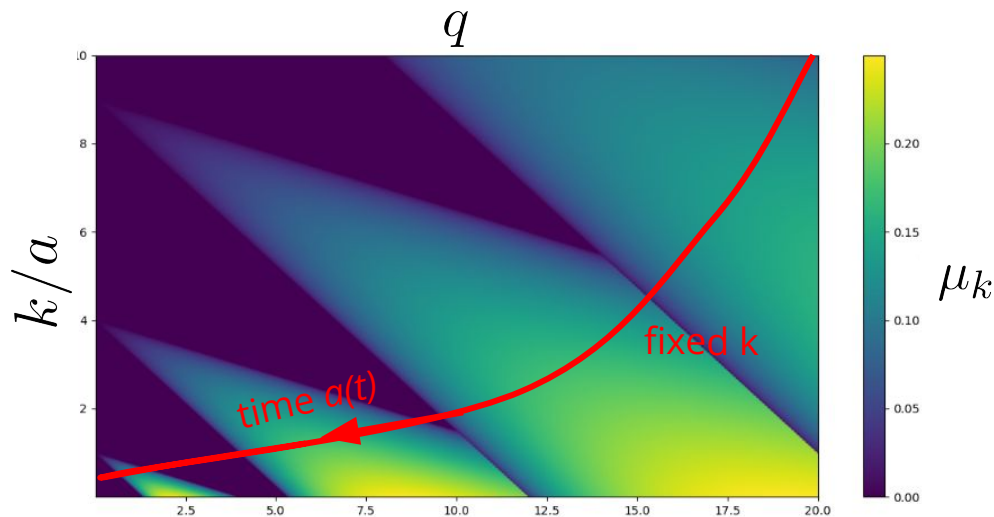
Time dependent background coupled to fields can lead to parametric resonance or tachyonic instability

$$\mathcal{L} \supset \sigma \phi^2 \chi^2 + \lambda \phi^k M_P^{4-k}$$

$$\chi_k'' + \left(\frac{k^2}{m_\phi^2 a^2} + 2q - 2q \cos(2z) \right) \chi_k = 0$$

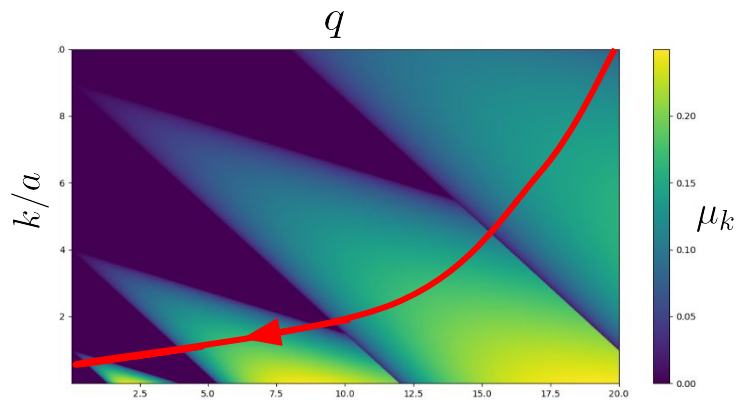
EOM for Fourier modes in the oscillating background

$$q \equiv \frac{\sigma \phi_0^2}{2m_\phi^2} \sim \frac{\sigma}{\lambda}$$



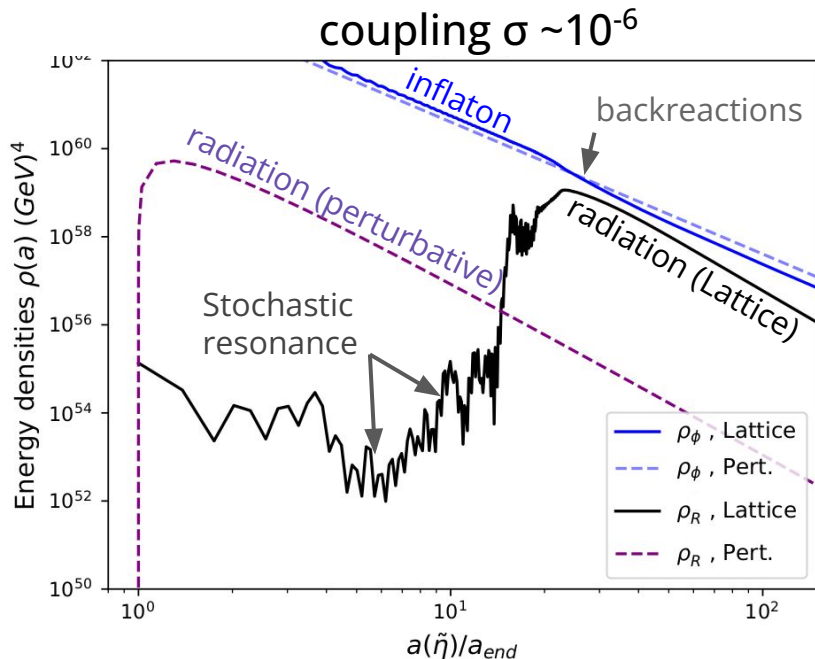
Instabilities in the colored regions
→ increasing occupation number of the modes

Preheating through non-perturbative processes



Instabilities in the colored regions
 → number of occupation increasing

$$\chi_k \propto \exp[\mu_k t]$$

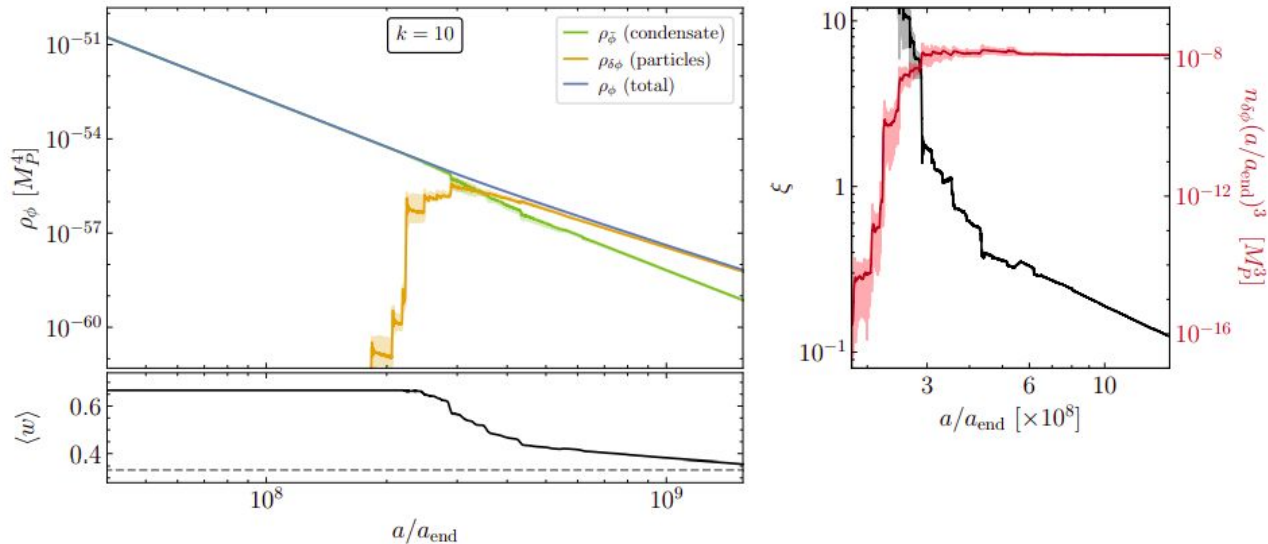


Preheating corresponds to resonances and exponential production

For large couplings, reach a regime of large backreactions of the fields on the background → Lattice

Freeze-in from preheating, Garcia, Kaneta, Mambrini, Olive, Verner, **2109.13280**

Inflation self-fragmentation



k	y_{eff}	μ_{eff}	σ_{eff}	T_{RH}
4	1.61×10^{-1}	3.57×10^{10} GeV	3.57×10^{-6}	1.14×10^{13} GeV
6	1.58×10^{-2}	1.84×10^5 GeV	5.37×10^{-10}	1.19×10^{10} GeV
8	1.32×10^{-3}	6.33×10^{-1} GeV	9.59×10^{-15}	1.50×10^7 GeV
10	3.62×10^{-5}	1.49×10^{-6} GeV	6.47×10^{-20}	1.80×10^4 GeV

From Garcia , Gross, Mambrini , Olive, Pierre, and Yoon, **2308.16231**