Fermion (P)reheating in a Quartic Inflaton Potential

Nabeen Bhusal

Based on hep-ph/2510.xxxxx in collaboration with
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DESY Theory Workshop 2025:
Synergies towards the future Standard Model

CLUSTER OF EXCELLENCE QUANTUM UNIVERSE
DESY THEORY WORKSHOP
SYNERGIES TOWARDS THE FUTURE STANDARD MODEL
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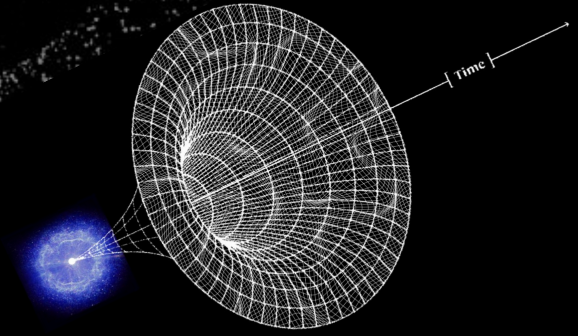

Credit: Big Think / Ben Gibson

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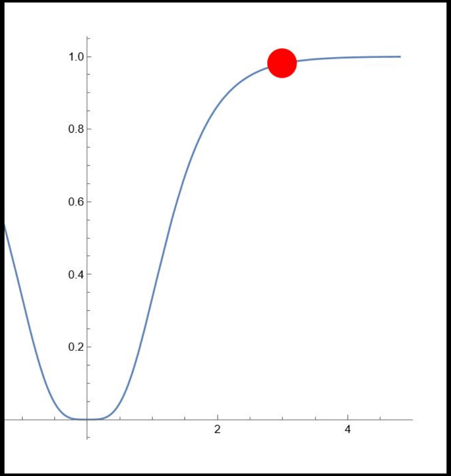
Content

Our setup

Consider $\mathcal{S} = \int d^4x \sqrt{-g}\left[-\frac{M_P^2}{2}R + \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) + \mathcal{L}_{\rm int}\right]$

With $V(\phi) = \lambda M_P^4 \left(\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right)^k$ and $\mathcal{L}_{int} = y\phi\bar{\psi}\psi$, where we set k=4



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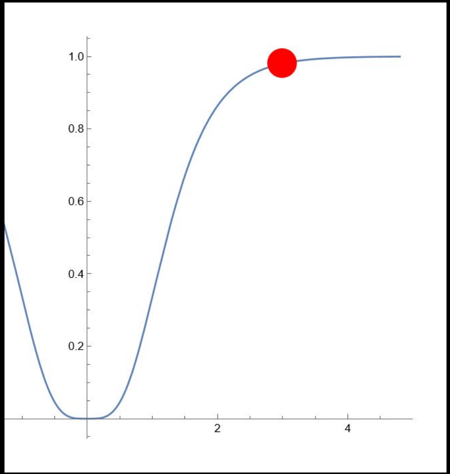
The inflaton mass is given then by: $m_\phi^2(t) = \lambda k (k-1) \phi_{\rm end}^{k-2} \left(\frac{a}{a_{\rm end}} \right)^{-6(k-2)/(k+2)}$

And the inflationary coupling $\lambda = \frac{18 \pi^2 A_s}{6^{k/2} N_*^2}$



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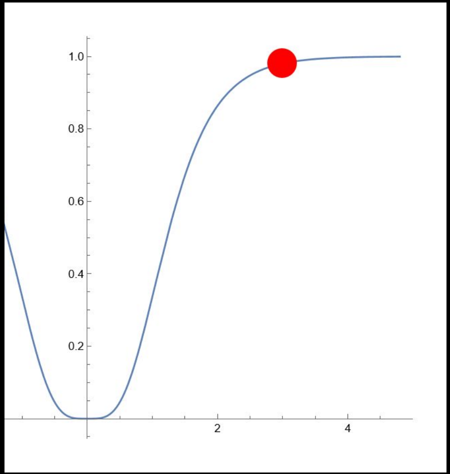
The inflation oscillates as $\phi(t) = \phi_0(t)\,\mathcal{P}(t) \simeq \phi_{\rm end}\left(\frac{a}{a_{\rm end}}\right)^{-6/(k+2)}\mathcal{P}(t)$

Where $\mathcal{P}(t) = \mathrm{sn}\,(t,-1)$ is the Jacobi sine function

In such a set up the inflaton fragments at ~ 180 $a/a_{\rm end}$

See JCAP 11 (2024) 004



Boltzmann vs. Bogoliubov

Boltzmann

Perturbative particle production from the oscillating inflation :

→ can only account for sub-horizon modes.

→ difficult to account for Pauli-blocking correctly.

→ subject to kinematics.

Bogoliubov

Non-perturbative gravitational production of fermion quanta out of the background, accounting for Pauli-blocking and all wavelengths :

→ can account for super-horizon modes.

→ Pauli-blocking is inherited from the fermion statistics.

→ can produce fermions out of equilibrium.

[See Simon's talk]

Motivation and Goals

Why fermions?

Reheating into bosons is well understood in all regimes and theoretical constraints on reheat temperature are solid. This is not the case for fermions.

Why Quartic?

It is conformal and inflaton fragments relatively early. Post-fragmentation production was expected to continue without suppression.

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- Determine the range of perturbative validity
- Perform a complete non-perturbative analysis to comment on whether reheating is realistically achievable before fragmentation
- Discuss post-fragmentation particle production in this setup

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In general, provide a "**more complete**" description of reheating with fermions.

Perturbative Fermion Production

Need for Non-perturbativity

The transition amplitude of the nth fourier mode of the coherently oscillating condensate is given by:

$$\left|\overline{\mathcal{M}_n}\right|^2 = \frac{2n^2\omega_\phi^2}{g_\psi}\bar{y}_n^2\beta_n^2\phi_0^2\left|\mathcal{P}_n\right|^2$$

where ,

$$\beta_n = \sqrt{1 - \frac{\mathcal{R}\mathcal{P}^2}{n^2}} \qquad \mathcal{R} \equiv \left.\frac{4m_\psi^2(t)}{\omega_\phi^2(t)}\right|_{\phi\to\phi_0} = \frac{4y^2\phi_{\rm end}^2}{\omega_{\rm end}^2}\left(\frac{a}{a_{\rm end}}\right)^{\frac{6(k-4)}{2+k}}$$

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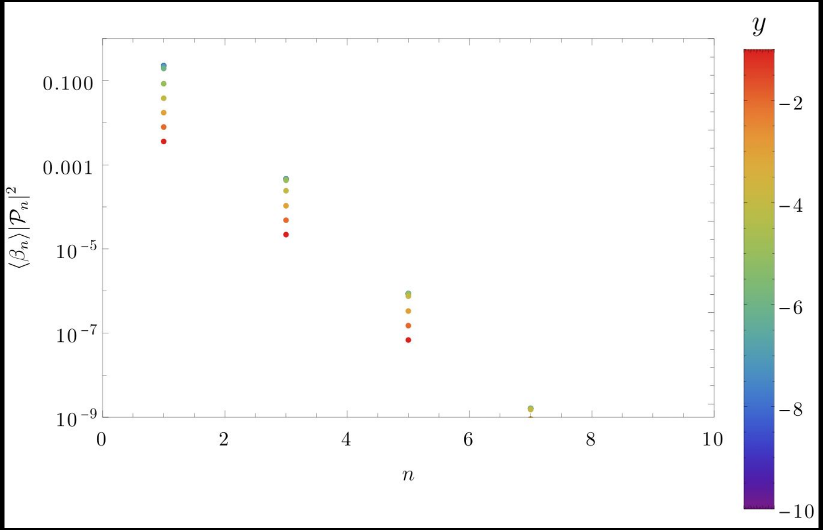

- For any Yukawa, the first fourier coefficient is the most dominant.

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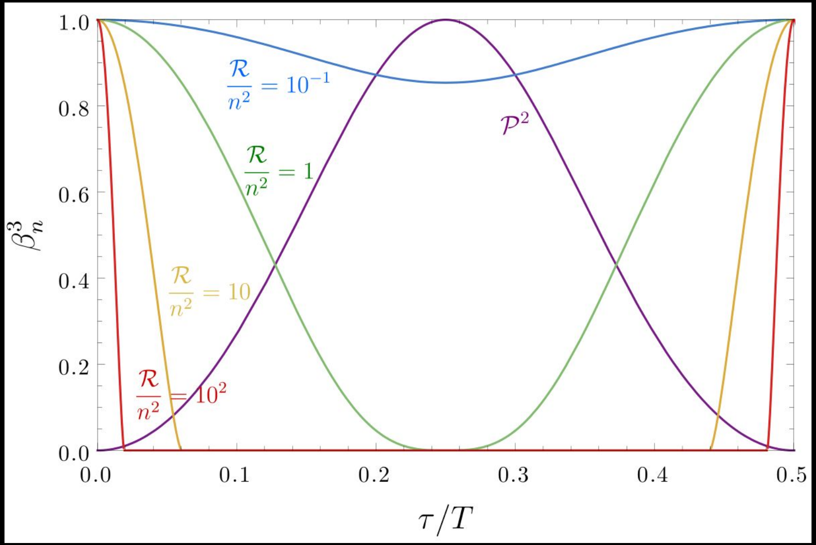


→ kinematic suppression for large fermion mass (large couplings)

The idea: Plug this into the collision term of a boltzmann equation

Non-Perturbative Fermion Production

Need for Non-perturbativity

We solve for the energy density of the fermions produced from the inflation as

$$\rho_\psi = \frac{2}{(2\pi)^3 a^4} \int d^3\boldsymbol{p}\, \omega_p n_p$$

Where the occupation number is given by $n_p = \frac{1}{2}\left|\left(1+\frac{am_\psi}{\omega_p}\right)^{1/2} U_2 - \left(1-\frac{am_\psi}{\omega_p}\right)^{1/2} U_1\right|^2$

In terms of the recast spinor mode equations $U_1'(\eta) = -ipU_2(\eta) + iam_\psi U_1(\eta).$ which are derived from dirac eqn.

$$U_2'(\eta) = -ipU_1(\eta) - iam_\psi U_2(\eta).$$

And the initialization is done in the Bunch-Davies vacuum.

→ This is the 'Non-Perturbative' or 'Bogoliubov' approach
→ Valid until inflaton fragmentation

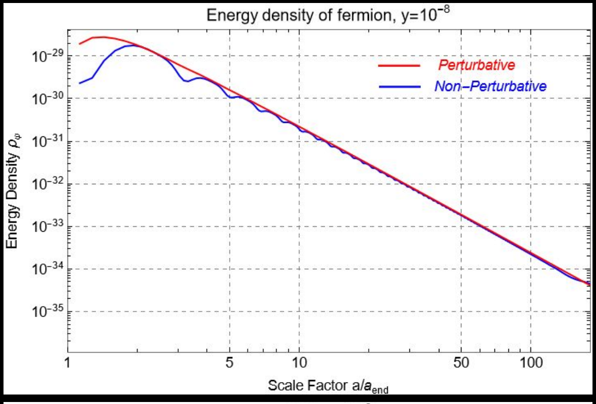
Comparing the two

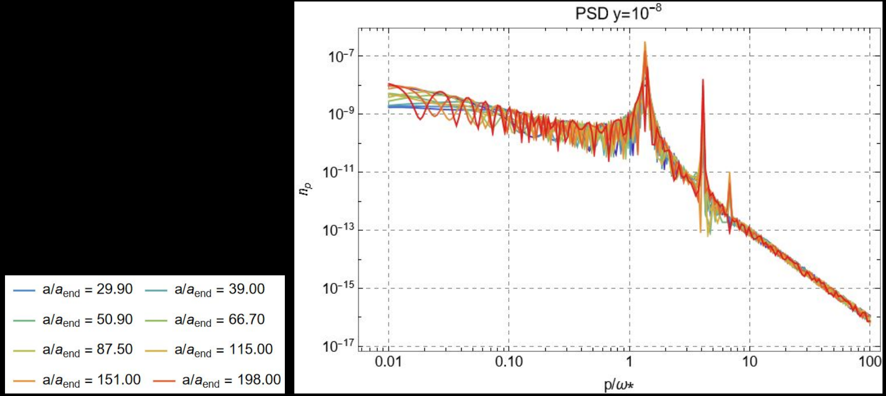
In the **small coupling** regime fermion production is unsuppressed [$y \lesssim 10^{-8}$]

- PSD is unaware of Pauli-statistics but occupation numbers are small → *Good approximation.*

The inflaton energy density scales as radiation i.e a^{-4} and at $a/a_{end} = 1$, inflaton energy density ~ 10^{-11}.

Perturbative energy density of fermions scales as a^{-3}. Naively, reheating will occur .... But





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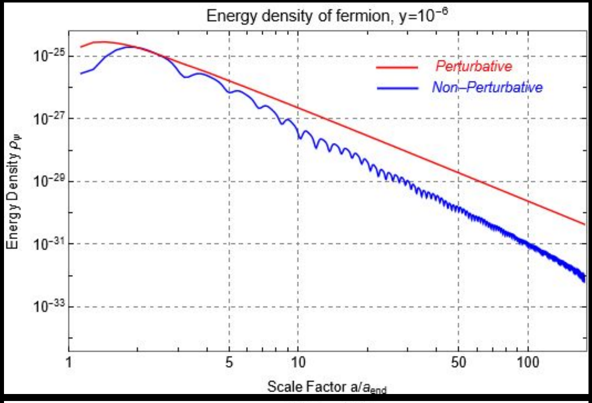
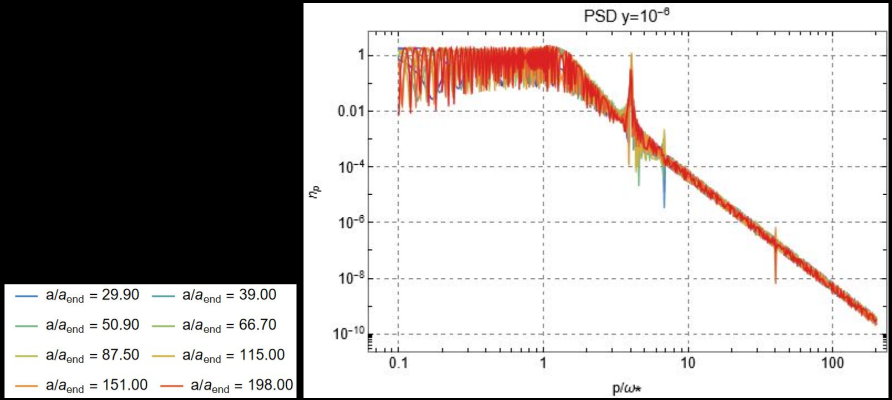
For **moderate couplings** production occurs when inflaton is close to 0

- PSD saturates due to Pauli-statics
  → Perturbative calculations *overestimate fermion production (approximation starts to break down)*





Comparing the two

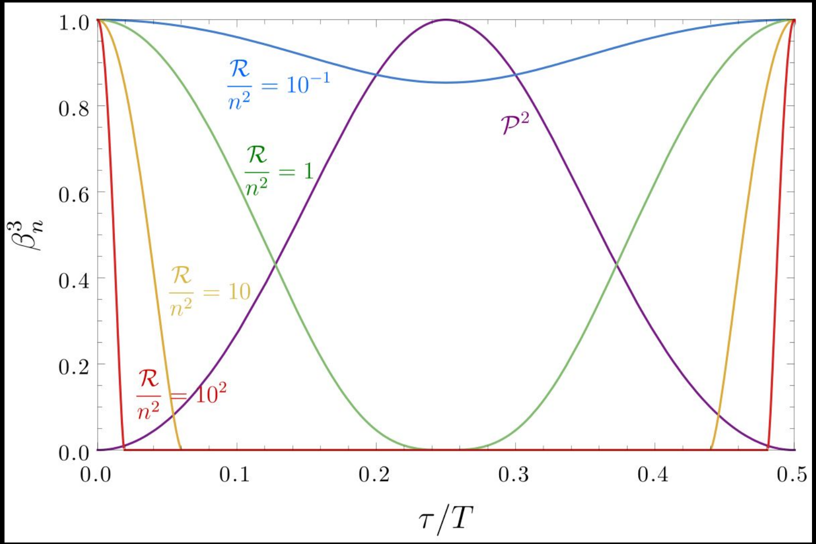
In the **small coupling** regime fermion production is unsuppressed

- PSD is unaware of Pauli-statistics but occupation numbers are small → *Good approximation*

For **moderate couplings** production occurs when inflaton is close to 0 (kinematics)

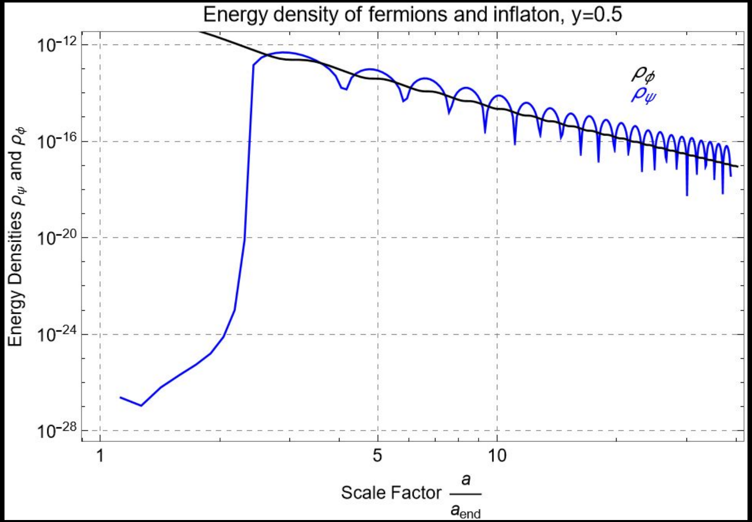
- PSD would saturate if aware of Pauli-statics → *overestimates fermion production (approximation starts to break down)*

For **large couplings**, the entire perturbative approach is invalid
→ Non-perturbative production



So, is reheating even possible before fragmentation?

- For couplings $y \gtrsim 0.3$, seems to be possible.
- Occurs in first few oscillations.
- Large yukawa can potentially spoil flatness.
  → need to handle this carefully



❑ When the energy densities are of the same order backreaction effects could become important to consider.
We do not consider it in this work.

→ Could this affect the ability of inflaton to reheat via fermions prior to fragmentation ?
Perhaps even prevent it completely in quartic models ?

Post-fragmentation production

- Up until fragmentation, the non-perturbative production is the 'correct' description.

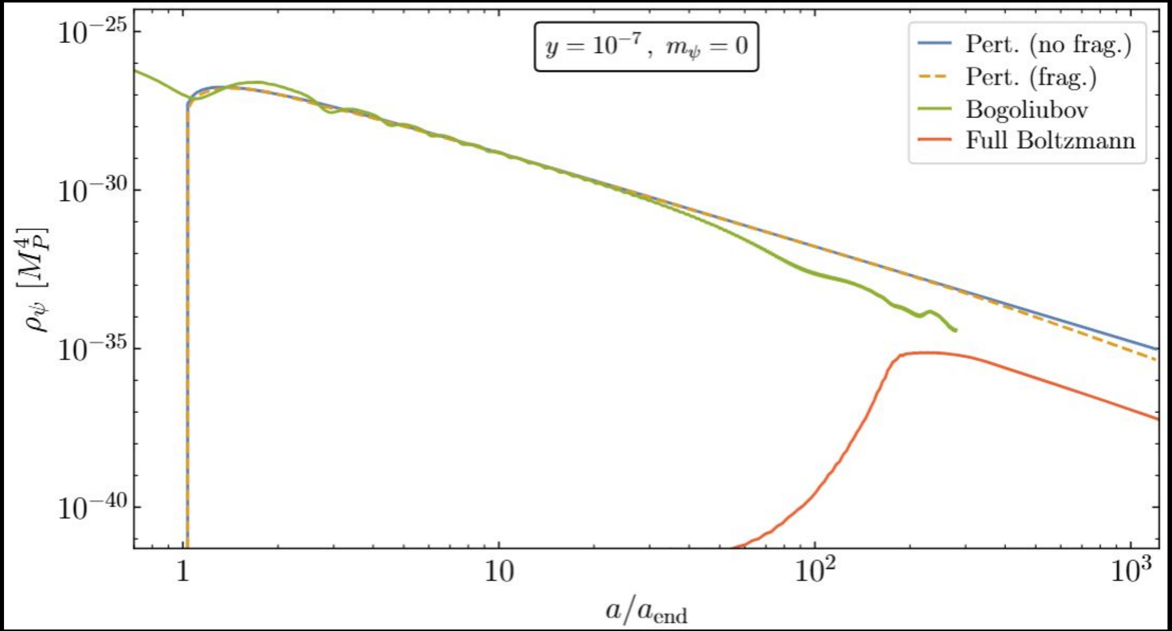
- Post-fragmentation, we implement **perturbative** inflaton fluctuation decays to fermions.

- For sufficiently large couplings [$y \gtrsim 10^{-8}$], the post-fragmentation decays start to be kinematically blocked and Pauli suppressed (since pre-fragmentation production saturates the IR) leading to difficulty reheating.
  → Must include effects of Pauli-blocking and kinematic suppression.

- Previous work has ignored suppression effects in the post-fragmentation production, thus overestimating reheat temperatures.

Post-fragmentation production

An example of a full analysis including post-fragmentation production from inflaton quanta.



Disclaimer: the Full Boltzmann line is initialized here assuming that the PSD is empty → not right.
Correctly done, the post fragmentation energy density is even more suppressed than in the plot.

Conclusion

In quartic-minimum inflaton potentials,

Prior to fragmentation:

- Perturbative particle production or the 'Boltzmann approach' is valid for very small couplings [$y \lesssim 10^{-8}$].
  → These couplings do not lead to reheating (BBN bounds).
- To really even speak of fermion reheating, it is necessary to be in the 'non-perturbative' or large yukawa limit.
  → Bogoliubov approach is necessary.
  → Possible reheating before fragmentation but the effect of backreaction needs to be checked.

Post-fragmentation:

- Suppression effects (kinematic or Pauli-blocking) are important in the range of viable Yukawa couplings.
  → Reheat temperatures in previous work will be suppressed and couplings will be constrained.

Further Questions:

- Will backraction further suppress the ability to reheat ?
- Does axial coupling or a bare mass help?

THANK YOU

Backup slides

Need for Non-Perturbativity

PSD for **large** Yukawas show exponential tails in the UV where the 'perturbative' calculations show a power law behavior:

→ Smaller coupling showed power law UV tails.

→ Indicative of the breakdown of the perturbative approach since even UV modes are 'non-perturbative'.

- Notice saturation of occupation number upto large momenta.