Post-inflationary enhancement of adiabatic perturbations in modular cosmology

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Modular symmetry

- UV complete theories usually employs the modular symmetry
- Plethora of scalar fields are generated when compactifying extra dimensions.

The kinetic term of a complex field τ

$$\frac{\partial\tau\partial\bar{\tau}}{(\mathrm{Im}\tau)^2} \tag{1}$$

is invariant under $SL(2,\mathbb{R})$ transformations

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

with $ad - cb = 1$ and $[a, b, c, d] \in \mathbb{R}$.

- Continuous symmetry broken down to its discrete version $[a, b, c, d] \in \mathbb{Z}$ by the modular potential

$$\frac{\mathcal{L}(\tau,\bar{\tau})}{\sqrt{-g}} = \frac{M_{Pl}^2}{2}R - \frac{3\alpha}{4}\frac{\partial\tau\partial\bar{\tau}}{(\mathrm{Im}\tau)^2} - V(\tau,\bar{\tau})\,. \tag{2}$$

Modular model

- In hyperbolic coordinates, we have the modular invariant model

$$\frac{\mathcal{L}(\varphi,\theta)}{\sqrt{-g}} = \frac{M_{Pl}^2}{2} R - \frac{1}{2}(\partial\varphi)^2 - \frac{3\alpha}{4} e^{-2\sqrt{\frac{2}{3\alpha}}\varphi} (\partial\theta)^2 - V_{\rm mod}(\varphi,\theta)\,. \tag{3}$$

Single-field limit

$$\lim_{\varphi\to\infty} V_{\rm mod}(\varphi,\theta) \sim V_0 \left(1 - \frac{6}{\pi} e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + \frac{72}{\pi^2} e^{-2\sqrt{\frac{2}{3\alpha}}\varphi} e^{-2\pi e^{\sqrt{\frac{2}{3\alpha}}\varphi}} \cos(2\pi\theta)\right).$$

- $\alpha-$attractor like expansion.
- Universal observables

$$A_s \simeq \frac{V_0 N^2}{18\pi^2\alpha}\,, \quad n_s \simeq 1 - \frac{2}{N}\,, \quad r \simeq \frac{12\alpha}{N^2}\,.$$

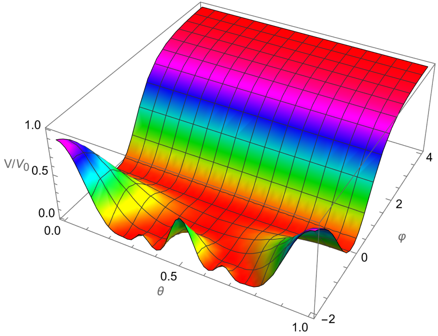


Figure 1: Plot of $V_{\rm mod}$.

Single vs multi-field

Single-field

- Field space is 1D thus, perturbations $\delta\varphi$ lie along the background trajectory.

Multi-field

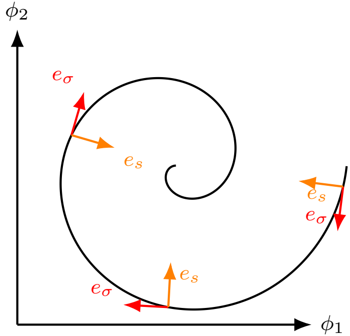
- Field space is higher-dimensional, thus perturbations are align with the background trajectory.

Convenient to define a basis as

$$G_{IJ} e^I_\sigma e^J_s = 0\,, \quad G_{IJ} e^I_s e^J_s = 1 \quad \text{and} \quad G_{IJ} e^I_\sigma e^J_\sigma = 1\,, \tag{4}$$

with G_{IJ} the field metric

- e^I_σ is the adiabatic unit vector pointing along the background trajectory.
- e^I_s is the entropic unit vector pointing orthogonal to the background trajectory.



Curvature and isocurvature perturbations

- Adiabatic modes → Curvature perturbation

$$\mathcal{R} \equiv \frac{1}{\sqrt{2\epsilon}} G_{IJ} e^I_\sigma \delta\phi^J \,.$$

- Entropic modes → Isocurvature perturbation

$$\mathcal{S} \equiv \frac{1}{\sqrt{2\epsilon}} G_{IJ} e^I_s \delta\phi^J \,, \tag{5}$$

The equations of motion of these perturbations read

$$\ddot{\mathcal{R}} + (3+\eta_H) H \dot{\mathcal{R}} + \frac{k^2}{a^2}\mathcal{R} = \frac{1}{a^3\epsilon}\frac{d}{dt}\left(2\epsilon a^3 H \eta_\perp \mathcal{S}\right) \,, \tag{6}$$

$$\ddot{\mathcal{S}} + (3+\eta_H) H \dot{\mathcal{S}} + \left(\frac{k^2}{a^2} + m^2_{\mathcal{S}}\right)\mathcal{S} = -2H\eta_\perp \dot{\mathcal{R}} \,. \tag{7}$$

Super-horizon limit

$$\dot{\mathcal{R}} = 2H\eta_\perp \mathcal{S} \,, \quad \ddot{\mathcal{S}} + (3+\eta_H) H \dot{\mathcal{S}} + m^2_{\mathcal{S},\mathrm{eff}}\mathcal{S} = 0 \,, \tag{8}$$

The turn rate is defined as:

$$\eta_\perp = \frac{1}{H} e_{Is} D_t e^I_\sigma = -\frac{1}{H} e_{I\sigma} D_t e^I_s \,. \tag{9}$$

Inflationary dynamics

Suppressed turn rate

$$\eta_\perp \sim e^{-e^{\varphi}}$$

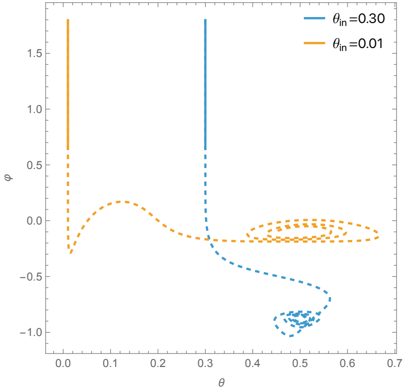


Figure 2: Background evolution.

Quasi massless isocurvature mode

$$m^2_{\mathcal{S},\mathrm{eff}} \sim \frac{\log[N]}{N^2}$$

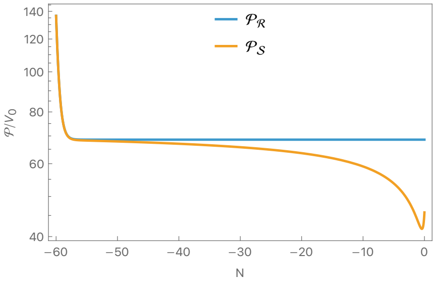


Figure 3: Curvature and isocurvature power spectra.

Post-inflationary dynamics

- No assumption on ϵ on superhorizon equations of motion.
- The power spectrum can be split as

$$\mathcal{P}_\mathcal{R} = \mathcal{P}^0_\mathcal{R}(\eta_\perp = 0) + \mathcal{P}^s_\mathcal{R}(\eta_\perp \neq 0)\,.$$

- The relative enhancement is defined as

$$\mathcal{E} \equiv \frac{\mathcal{P}^s_\mathcal{R}}{\mathcal{P}^0_\mathcal{R}}\,.$$

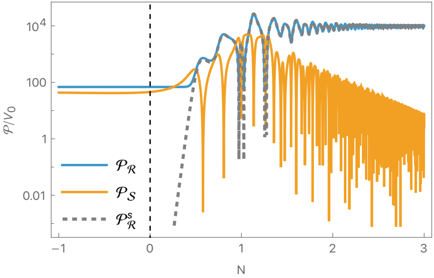


Figure 4: Post-inflationary curvature and isocurvature power spectra.

Enhancement effect on observables

Amplitude of scalar perturbations:

- The $N-$scaling remains the same as before the enhancement

$$A_s \propto N^2 \,. \tag{10}$$

Spectral index:

- The spectral index before and after the enhancement have the same leading order

$$n_s^{\text{post}} = n_s^{\text{end}} \approx 1 - \frac{2}{N} \,. \tag{11}$$

Tensor-to-scalar ratio:

- Suppression due to enhancement

$$r^{\text{post}} = \frac{r^{\text{end}}}{\mathcal{E}} \approx \frac{12\alpha}{\mathcal{E}N^2} \,. \tag{12}$$

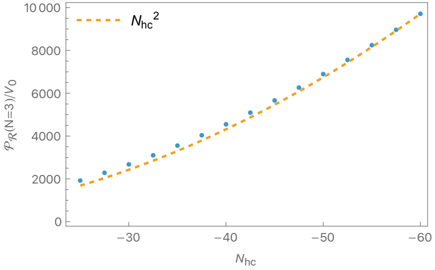


Figure 5: N-scaling of the amplitude of scalar perturbations.

Summary and conclusions

- · Generically, important post-inflationary multi-field effects may take place if
  · Isocurvature modes are massless or very light.
  · A strong turn after the end of inflation is present
- α-attractors-like expansion of the potential+ hyperbolic geometry $\rightarrow$ universal N-scaling for $A_s^{\rm post}$.