OPENING THE WINDOW FOR ULTRA-LIGHT PBH

DESY 25

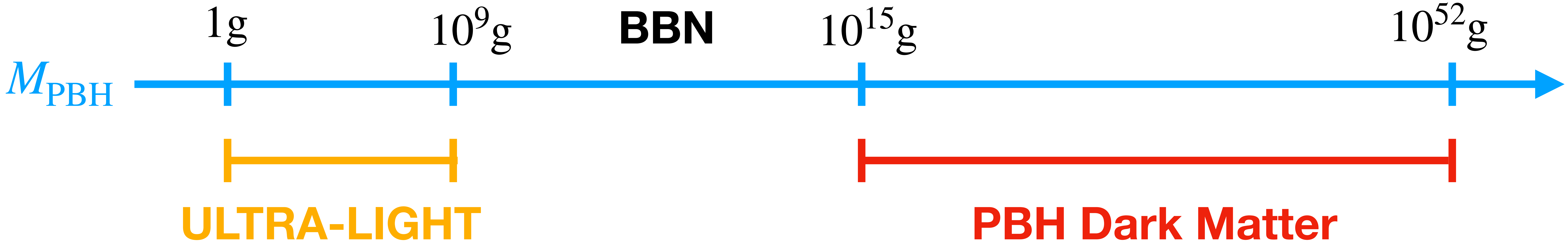
25.09.2025

Nicholas Leister

Based on work with Yann Gouttenoire and Pedro Schwaller

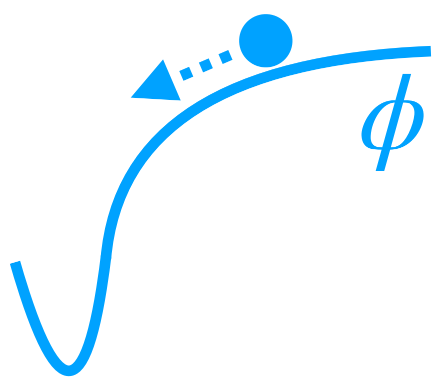
in preparation

ULTRA LIGHT PBHs



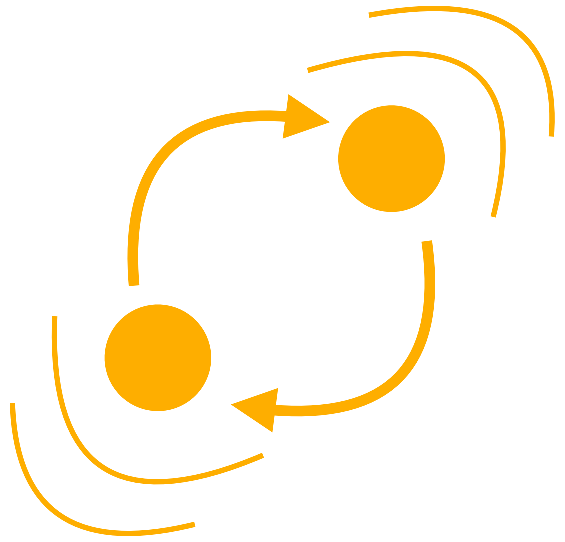
Also see talk by TaeHun Kim

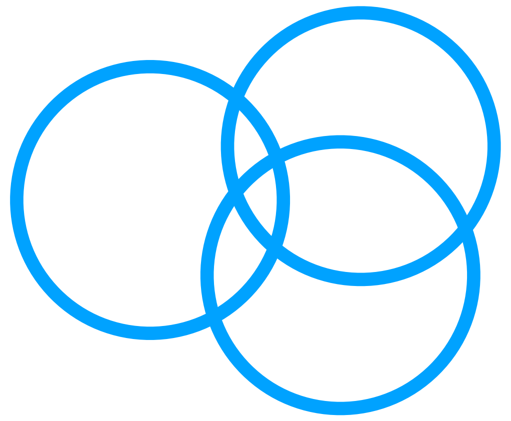
INTRODUCTION



INFLATION

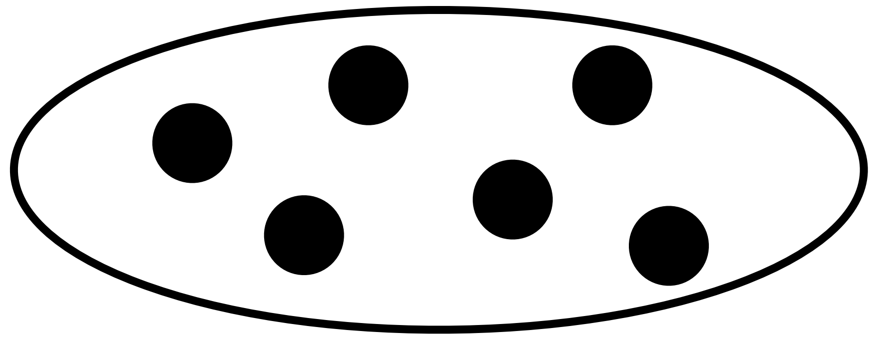
BINARIES



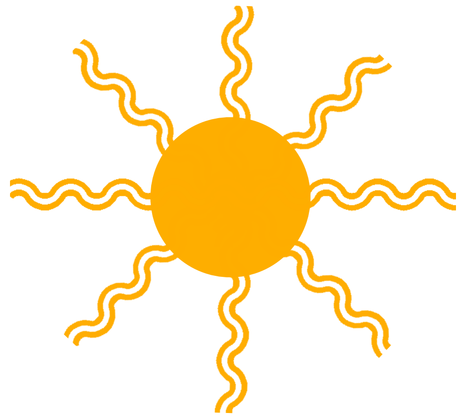


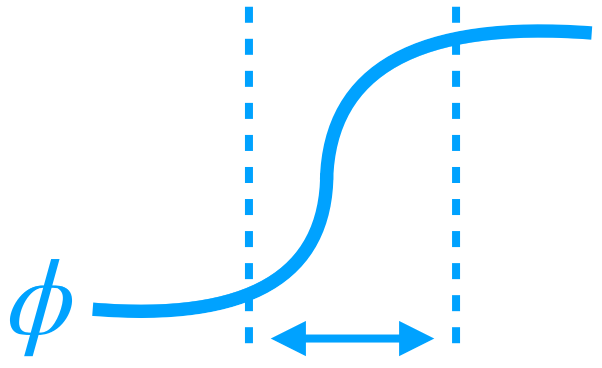
FOPT

PRIMORDIAL BLACK HOLES



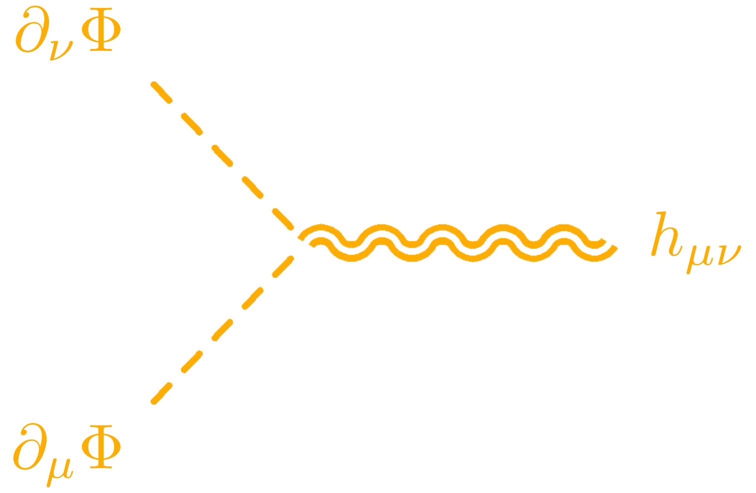
GRAVITONS



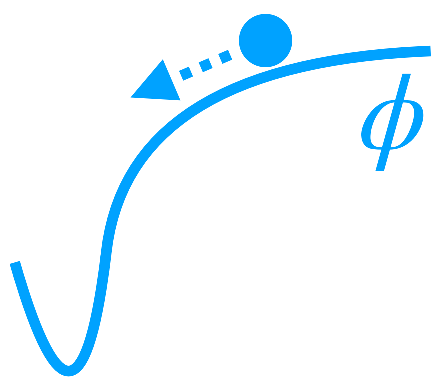


DWs

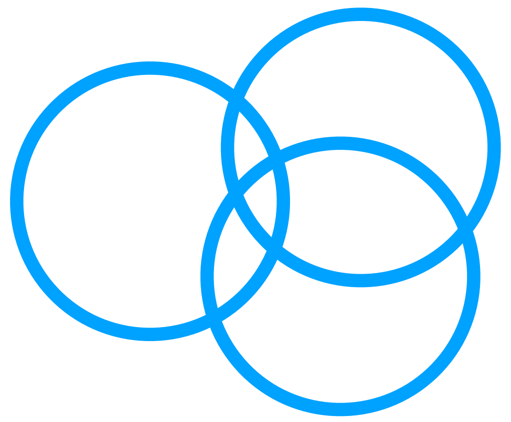
SCALAR-INDUCED



INTRODUCTION



INFLATION



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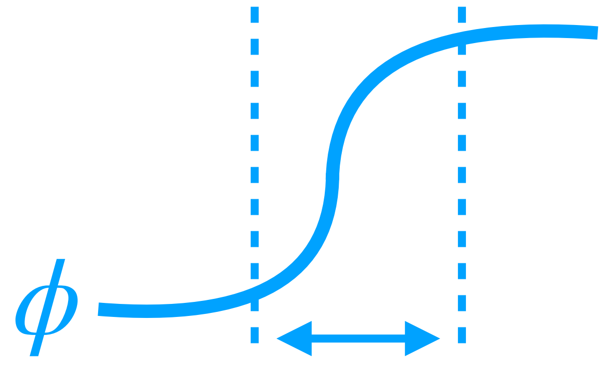
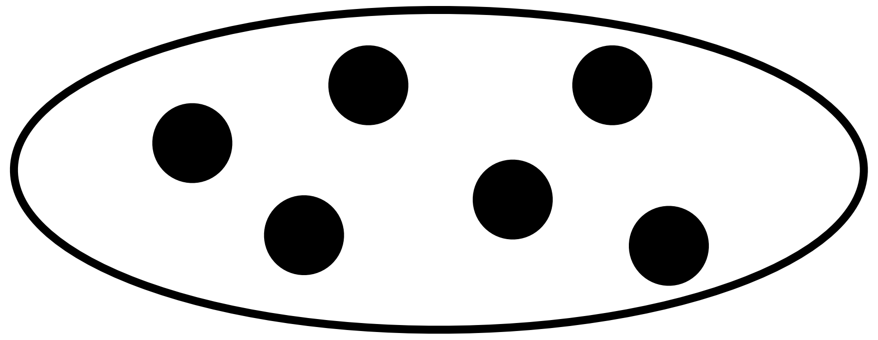


DWs

PRIMORDIAL BLACK HOLES



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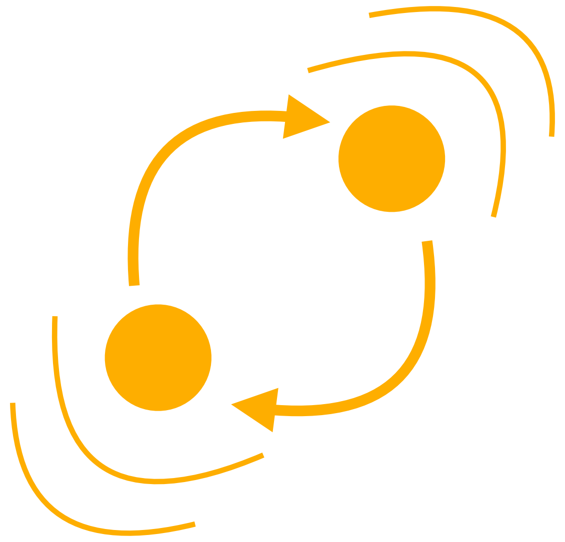
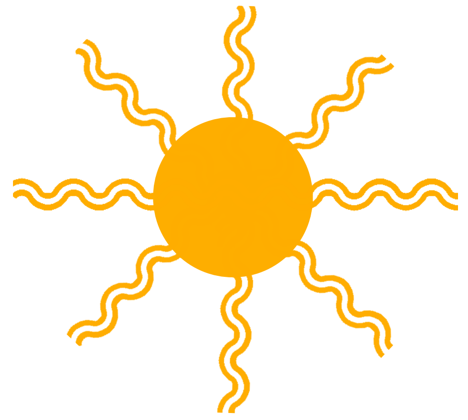


GRAVITONS



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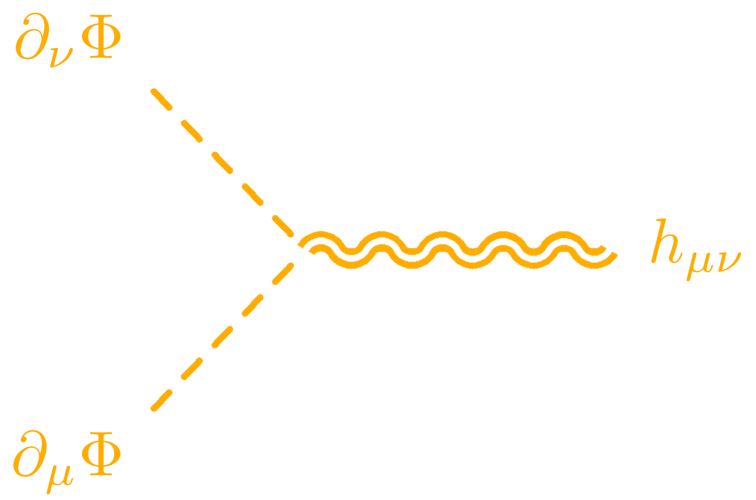

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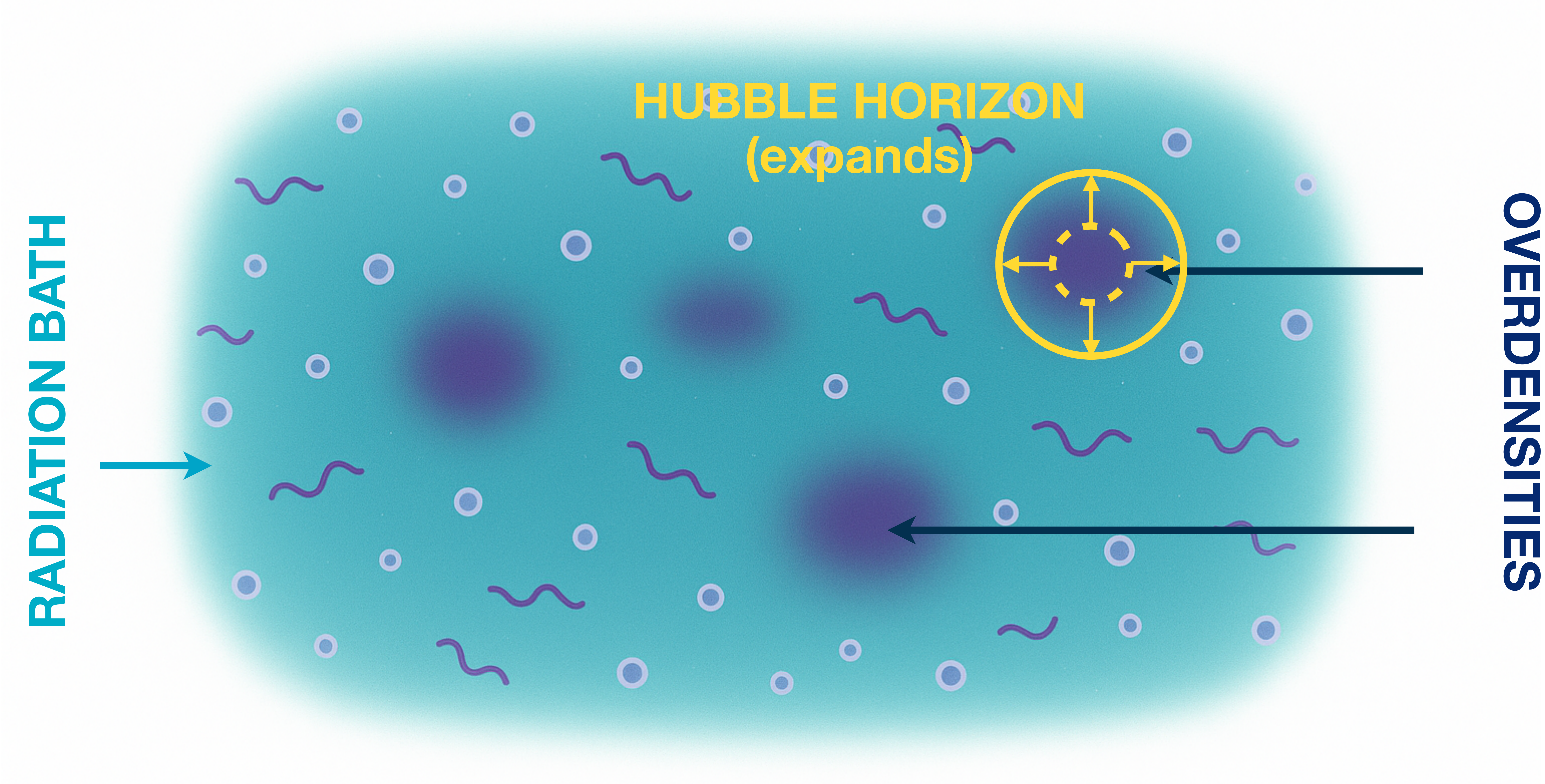

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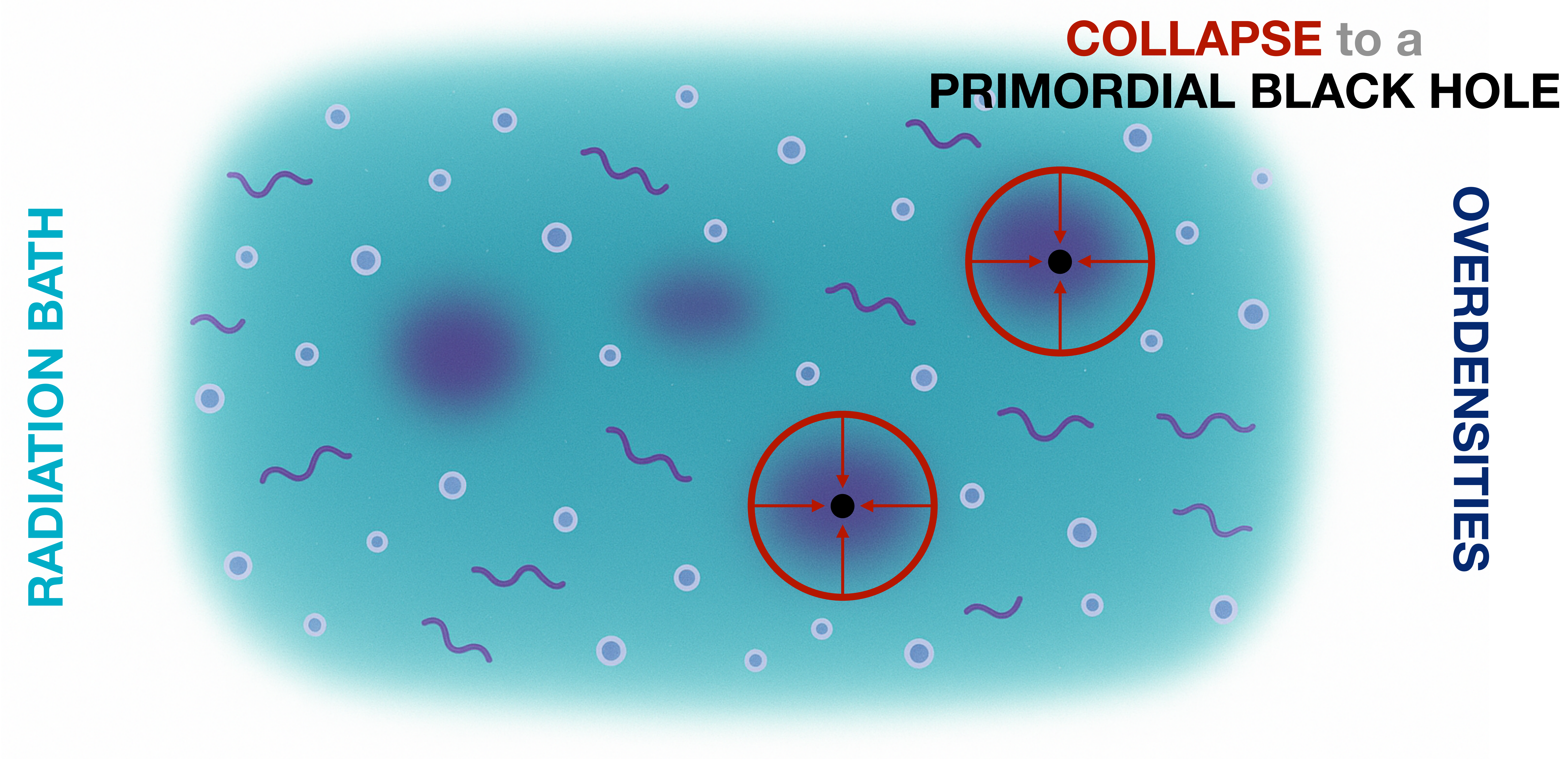

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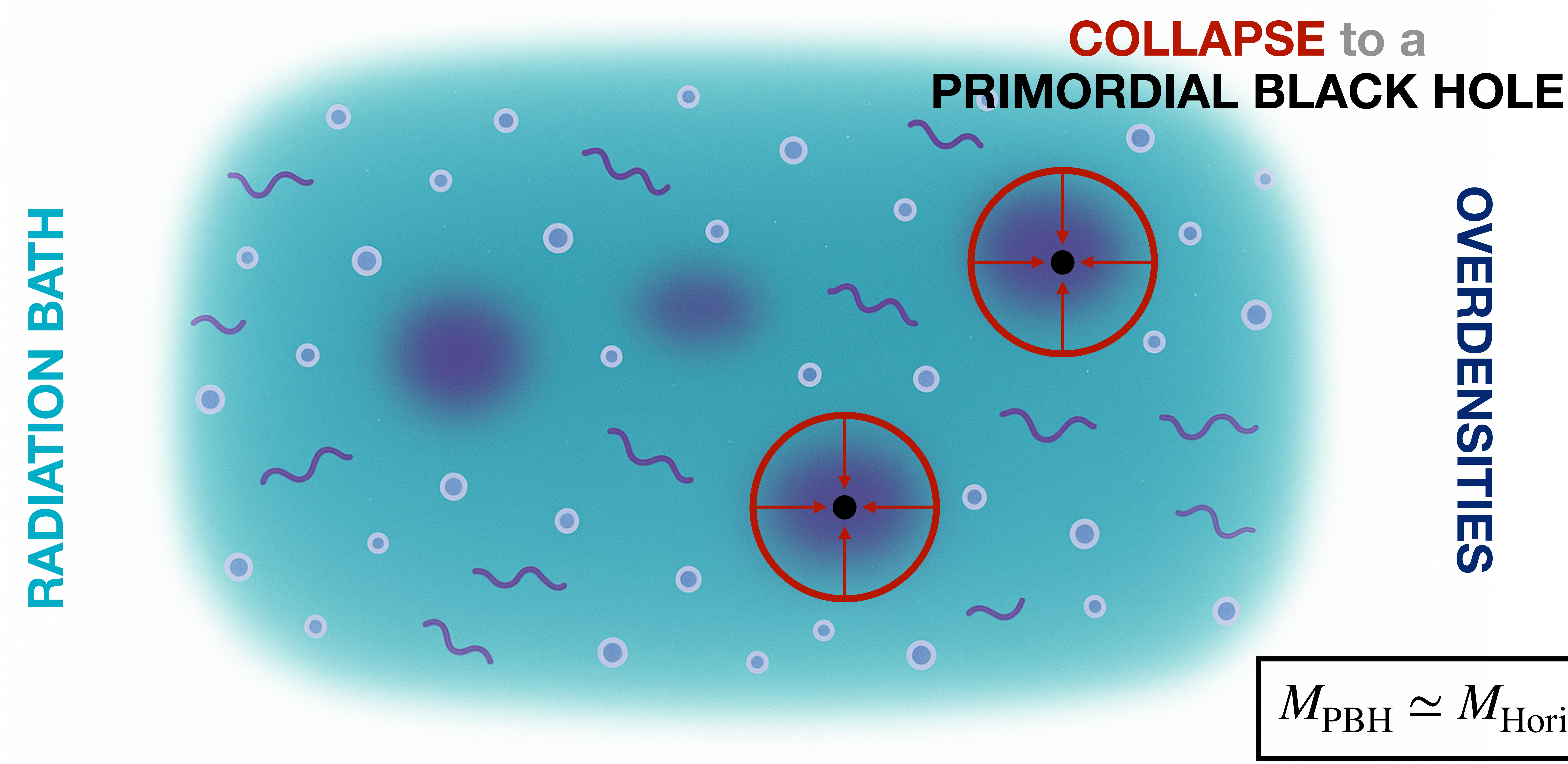
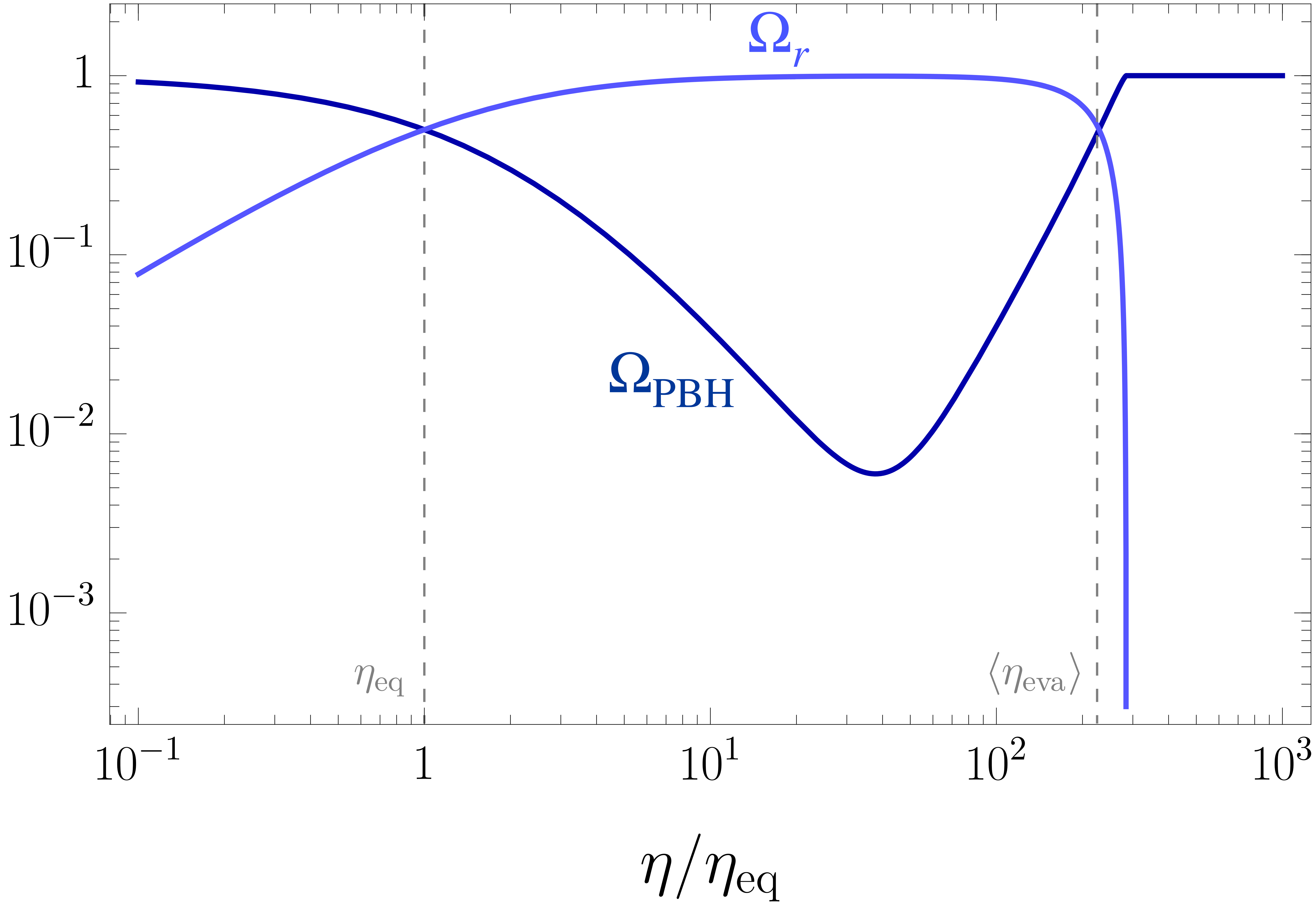


$$M_{\text{PBH}} \simeq M_{\text{Horizon}}$$

PART I:

What we know …

PRIMORDIAL BLACK HOLES

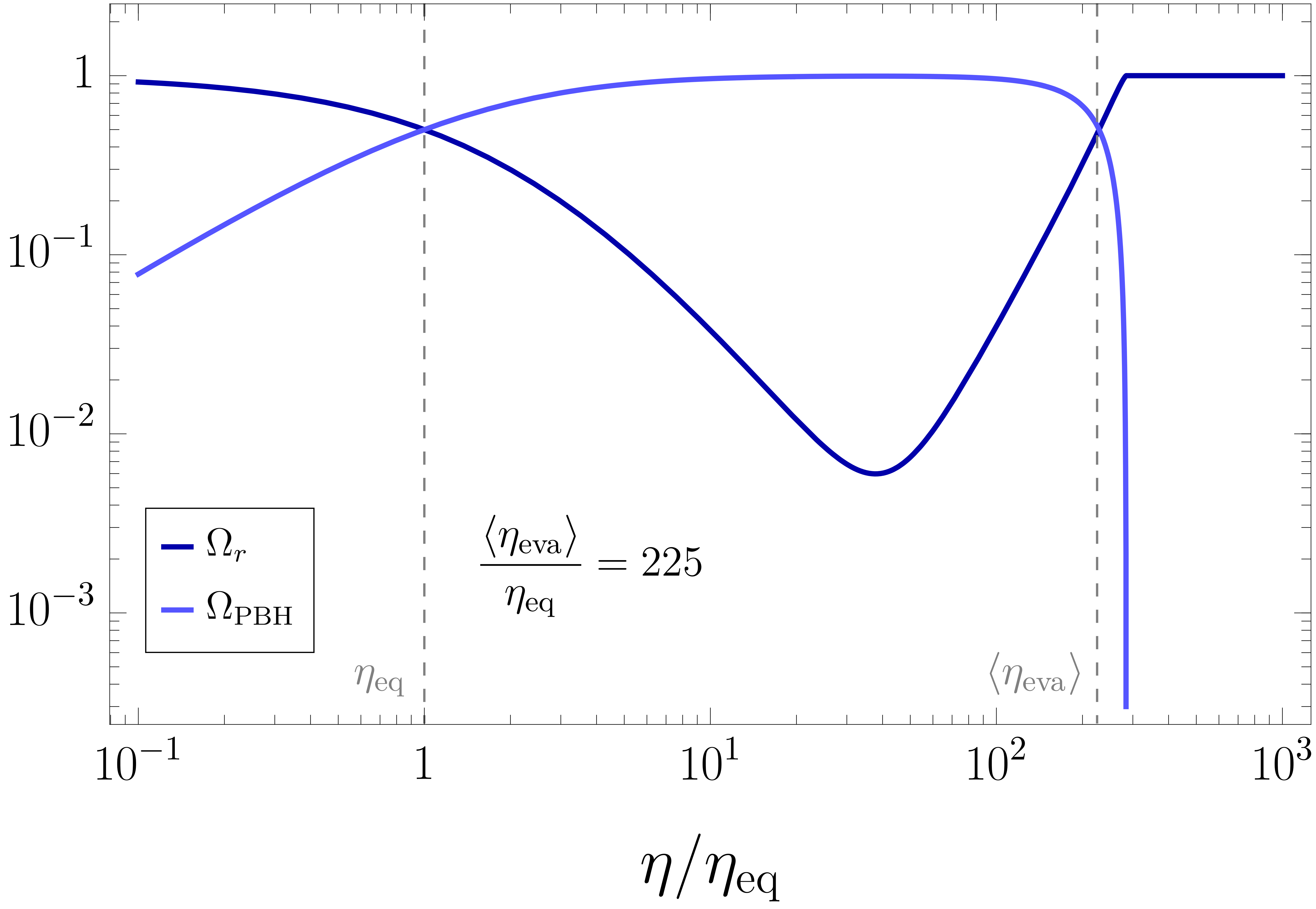


$$\rho_m' = -(3\mathscr{H} + \Gamma)\rho_m$$

$$\rho_r' = -4\mathscr{H}\rho_r + \Gamma\rho_m$$

$$\mathscr{H} = \frac{a}{\sqrt{3}M_{\rm pl}}\sqrt{\rho_{\rm tot}}$$

PRIMORDIAL BLACK HOLES



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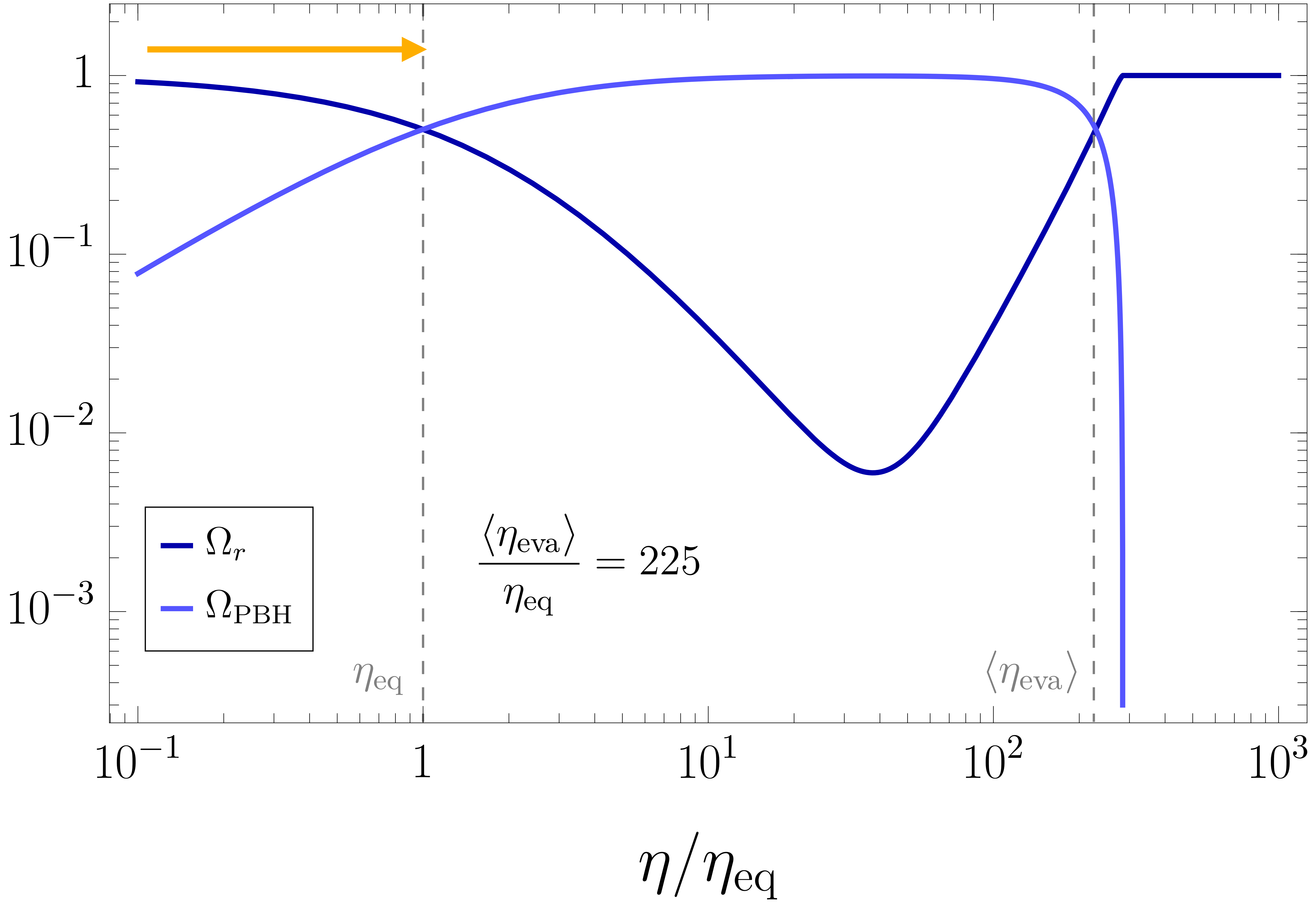
PRIMORDIAL BLACK HOLES

$\beta > \beta_c$

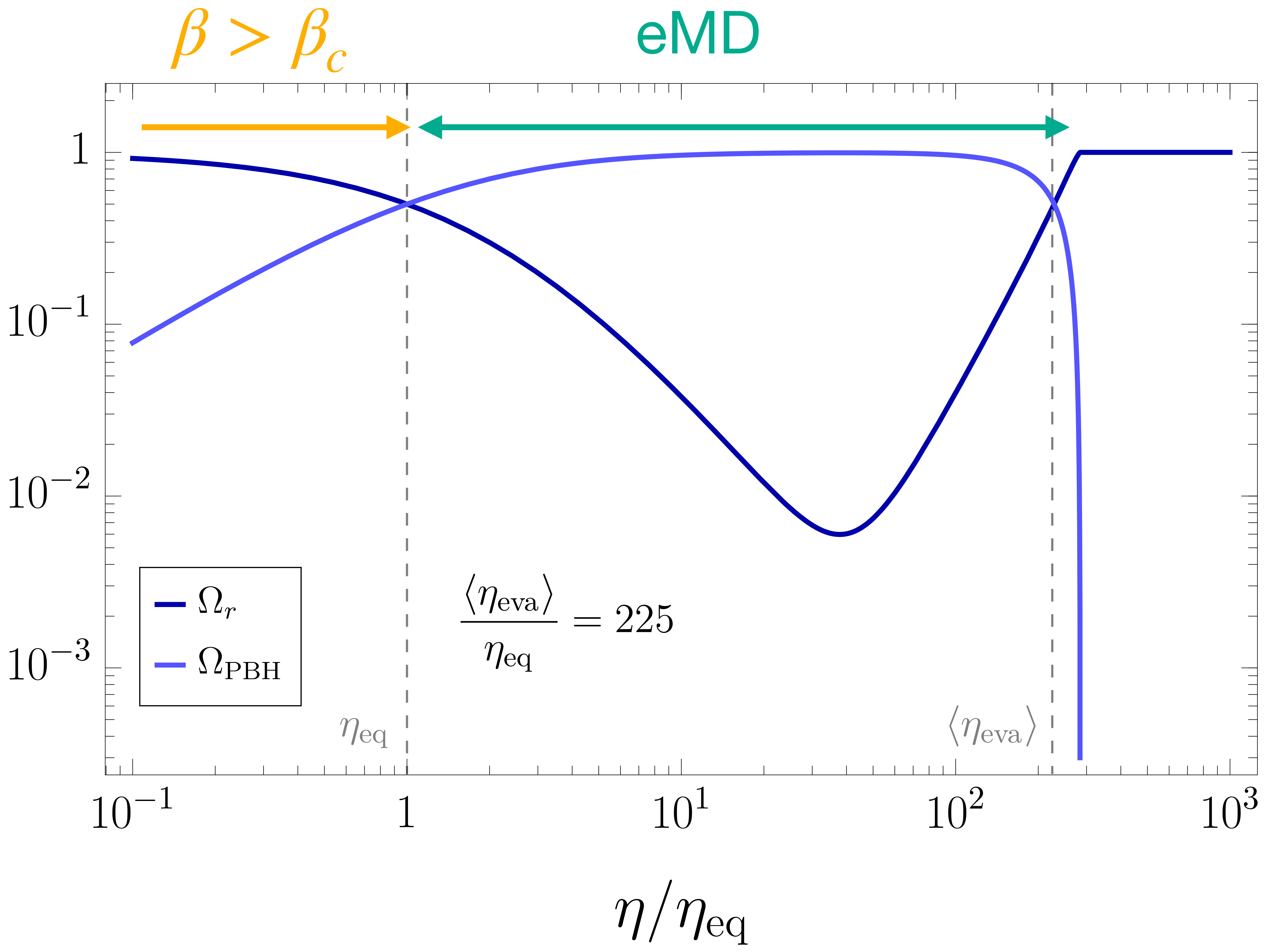


$$\rho'_m = -(3\mathcal{H} + \Gamma)\rho_m$$

$$\rho'_r = -4\mathcal{H}\rho_r + \Gamma\rho_m$$

$$\mathcal{H} = \frac{a}{\sqrt{3}M_{\rm pl}}\sqrt{\rho_{\rm tot}}$$

PRIMORDIAL BLACK HOLES



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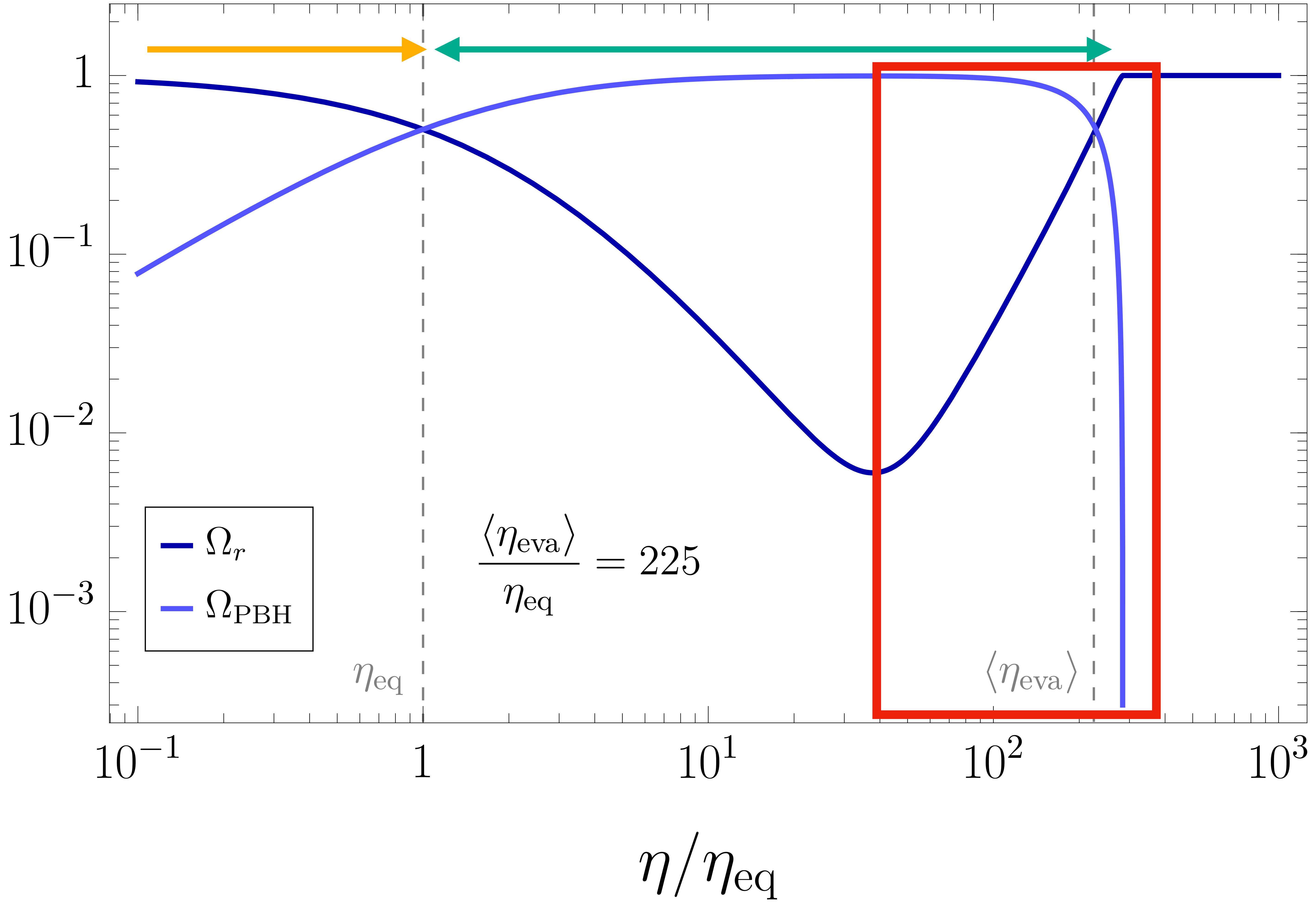
PRIMORDIAL BLACK HOLES

$\beta > \beta_c$ eMD Hawking Evaporation



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$$\mathscr{H} = \frac{a}{\sqrt{3}M_{\mathrm{pl}}}\sqrt{\rho_{\mathrm{tot}}}$$

SOLVING PERTURBATIONS

Isotropic + homogeneous universe

$$\mathrm{d}s^2 = a^2(\eta)\left[\mathrm{d}\eta^2 - \delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right]$$

SOLVING PERTURBATIONS

Cosmological perturbation theory

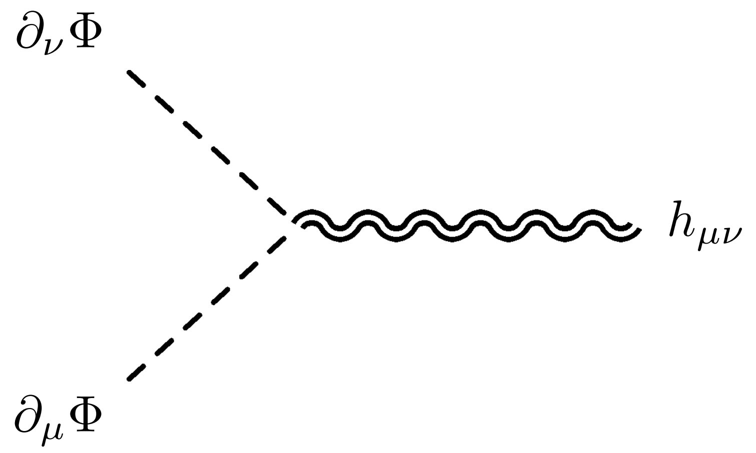
$$\mathrm{d}s^2 = a^2(\eta)\left[(1+2\Psi)\mathrm{d}\eta^2 - (1-2\Phi)\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j + h_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right]$$

In Newtonian gauge and with out anisotropic stress ($\Phi = -\Psi$)

At 1st order in PT: no scalar-tensor mixing

At 2nd order in PT: SIGWs

$$h''_{ij} + 2\mathscr{H}h'_{ij} - \partial_k\partial^k h_{ij} = \mathscr{P}^{ab}{}_{ij}\left\{T_{ab}\right\} \approx \mathscr{P}^{ab}{}_{ij}\left\{4\partial_a\Phi\partial_b\Phi + 2\partial_a\delta\varphi\partial_b\delta\varphi\right\}$$



$$\mathscr{P}_h \propto h^2 \propto \Phi^4$$

SOLVING PERTURBATIONS

Cosmological perturbation theory

$$\mathrm{d}s^2 = a^2(\eta)\left[(1+2\Psi)\mathrm{d}\eta^2 - (1-2\Phi)\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j + h_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right]$$

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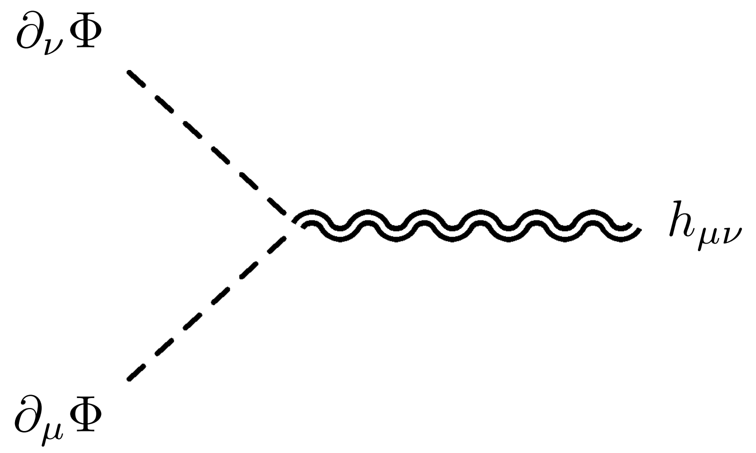


$$\mathscr{P}_h \propto h^2 \propto \Phi^4$$

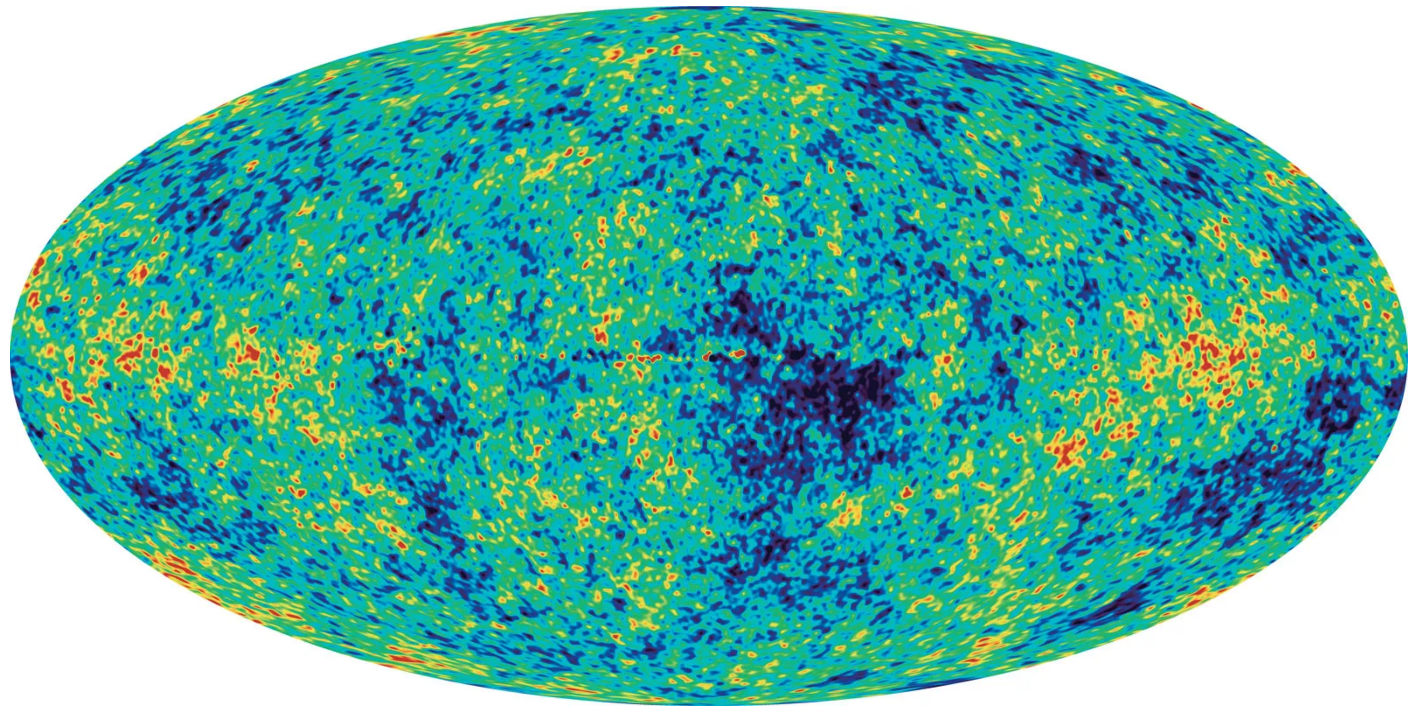
$$\delta\phi \sim \Phi + \frac{\Phi'}{\mathscr{H}}$$

SOLVING PERTURBATIONS

Cosmological perturbation theory

$$\mathrm{d}s^2 = a^2(\eta)\left[(1-2\Phi)\mathrm{d}\eta^2 - (1-2\Phi)\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j + h_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\right]$$

Primordial Scalar Perturbations Φ



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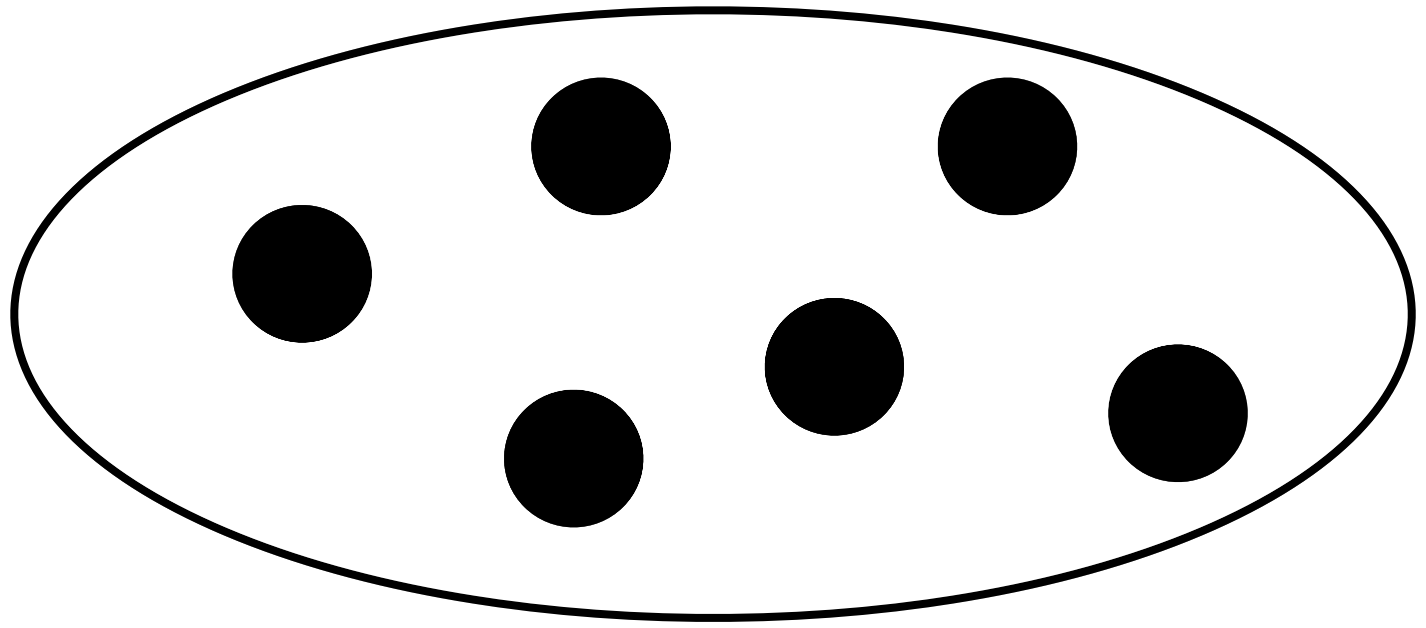


Adiabatic Perturbations (Inflation)

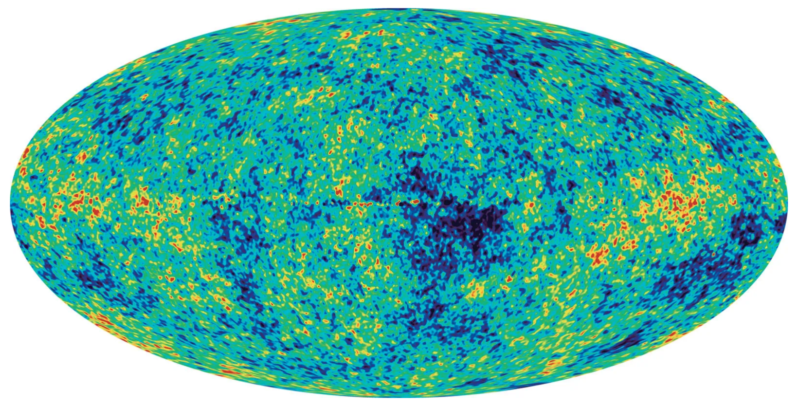
PBH Isocurvature

SOLVING PERTURBATIONS

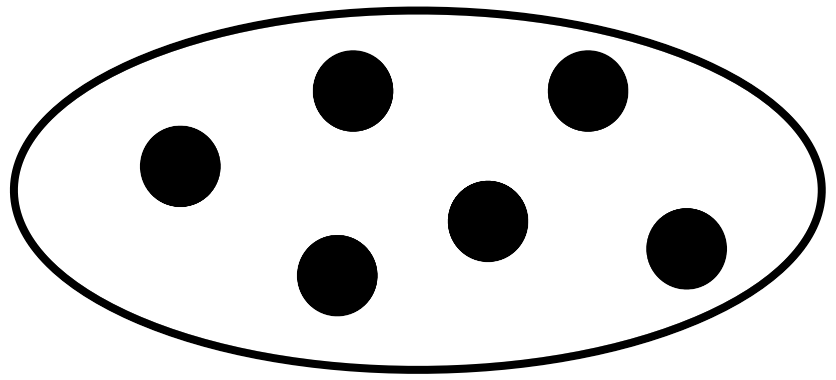
Cosmological perturbation theory

$$ds^2 = a^2(\eta)\left[(1-2\Phi)d\eta^2 - (1-2\Phi)\delta_{ij}\, dx^i\, dx^j + h_{ij}\, dx^i\, dx^j\right]$$

Primordial Scalar Perturbations Φ

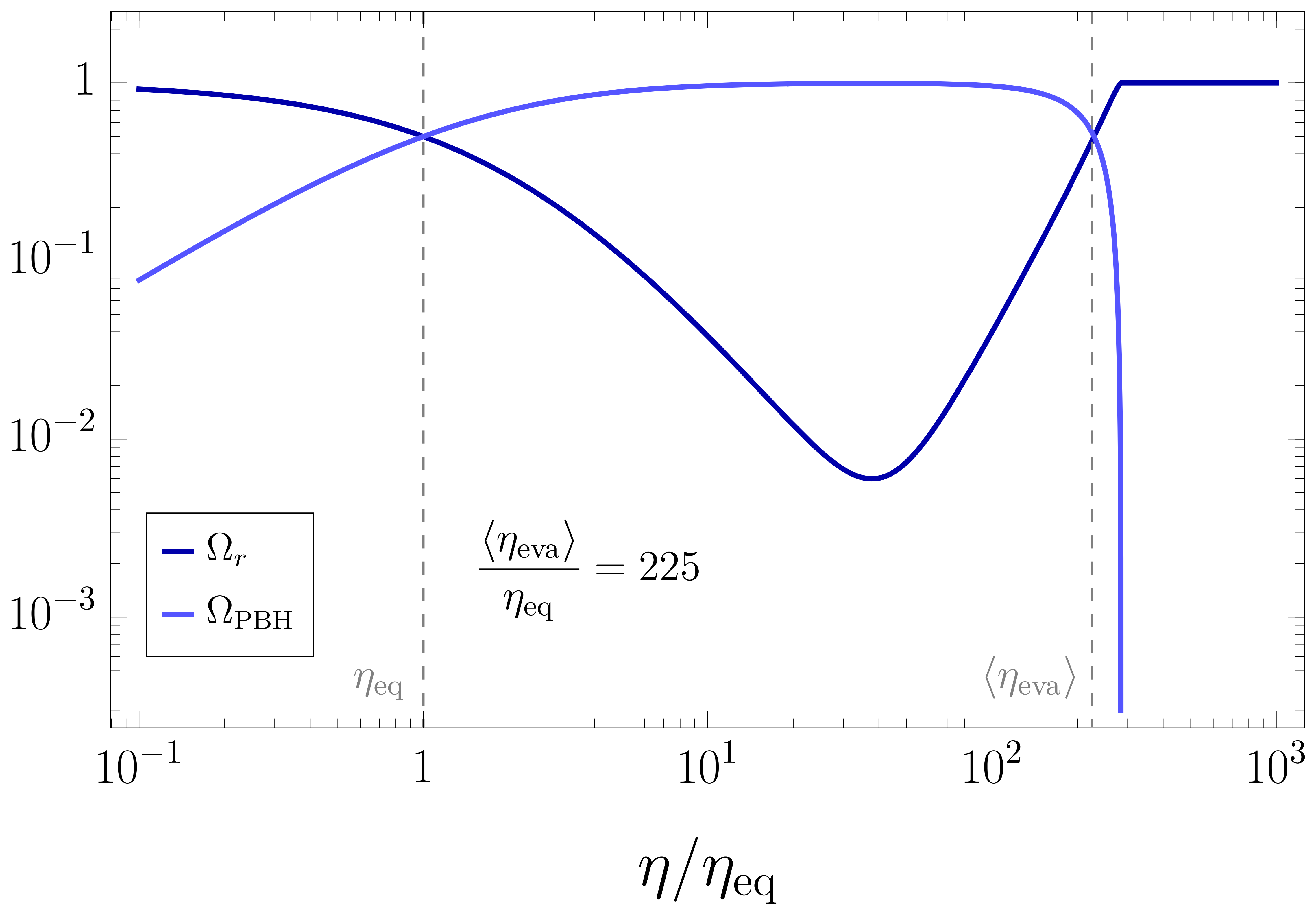


$$\mathscr{P}_\Phi \propto \langle \Phi_k \Phi_{k'} \rangle$$



Two point correlators can be predicted from inflation + Poisson statistics of PBH gas

SOLVING PERTURBATIONS



$$\boxed{\mathscr{P}_h \propto \Phi^4_{\mathrm{osc}}}$$

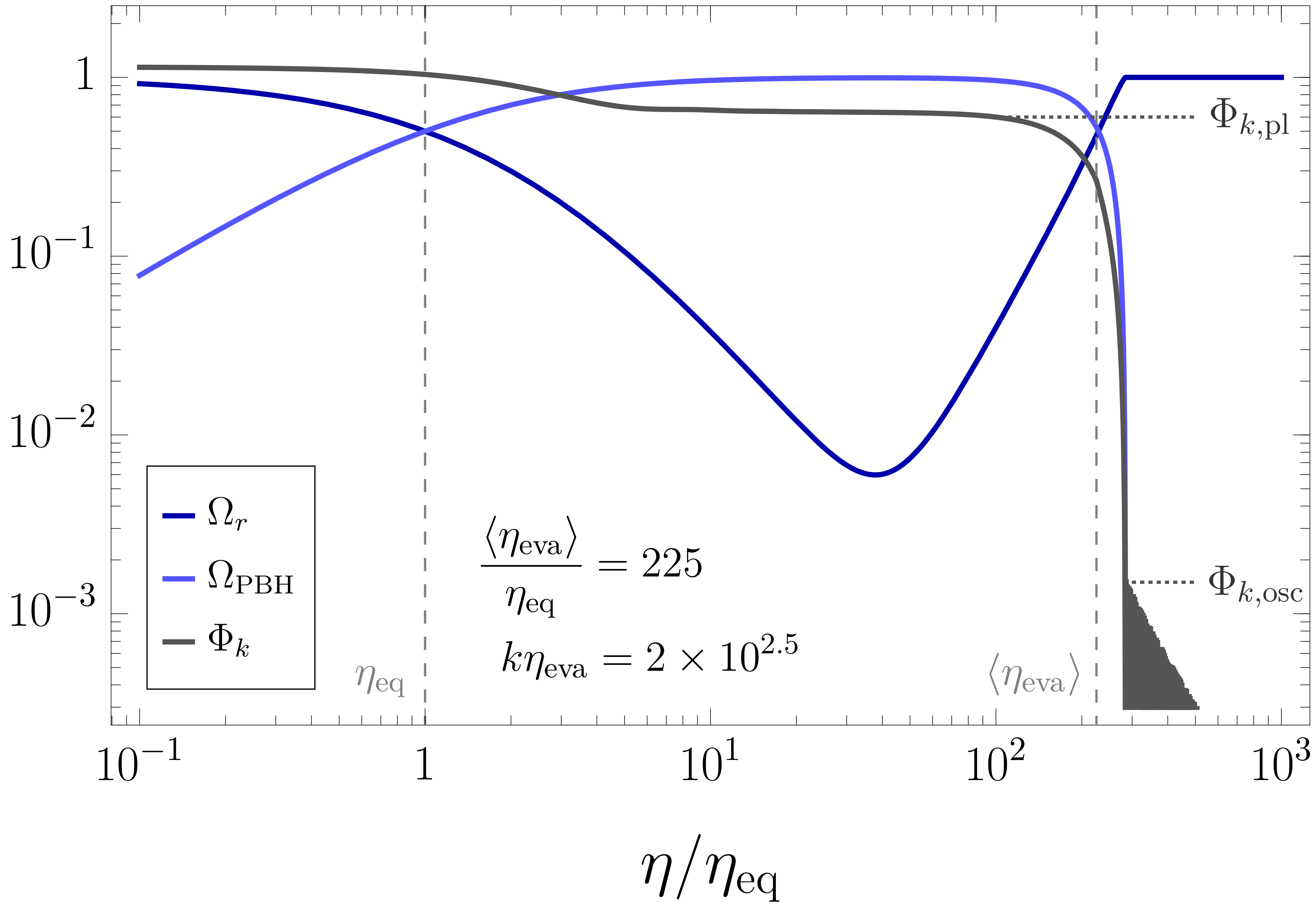
Combine:

1. **Transfer function** $\mathscr{T}(k)$:
   RD-to-MD transition

2. **Suppression factor** $\mathscr{S}(k)$
   MD-to-RD transition

Effects decouple

SOLVING PERTURBATIONS



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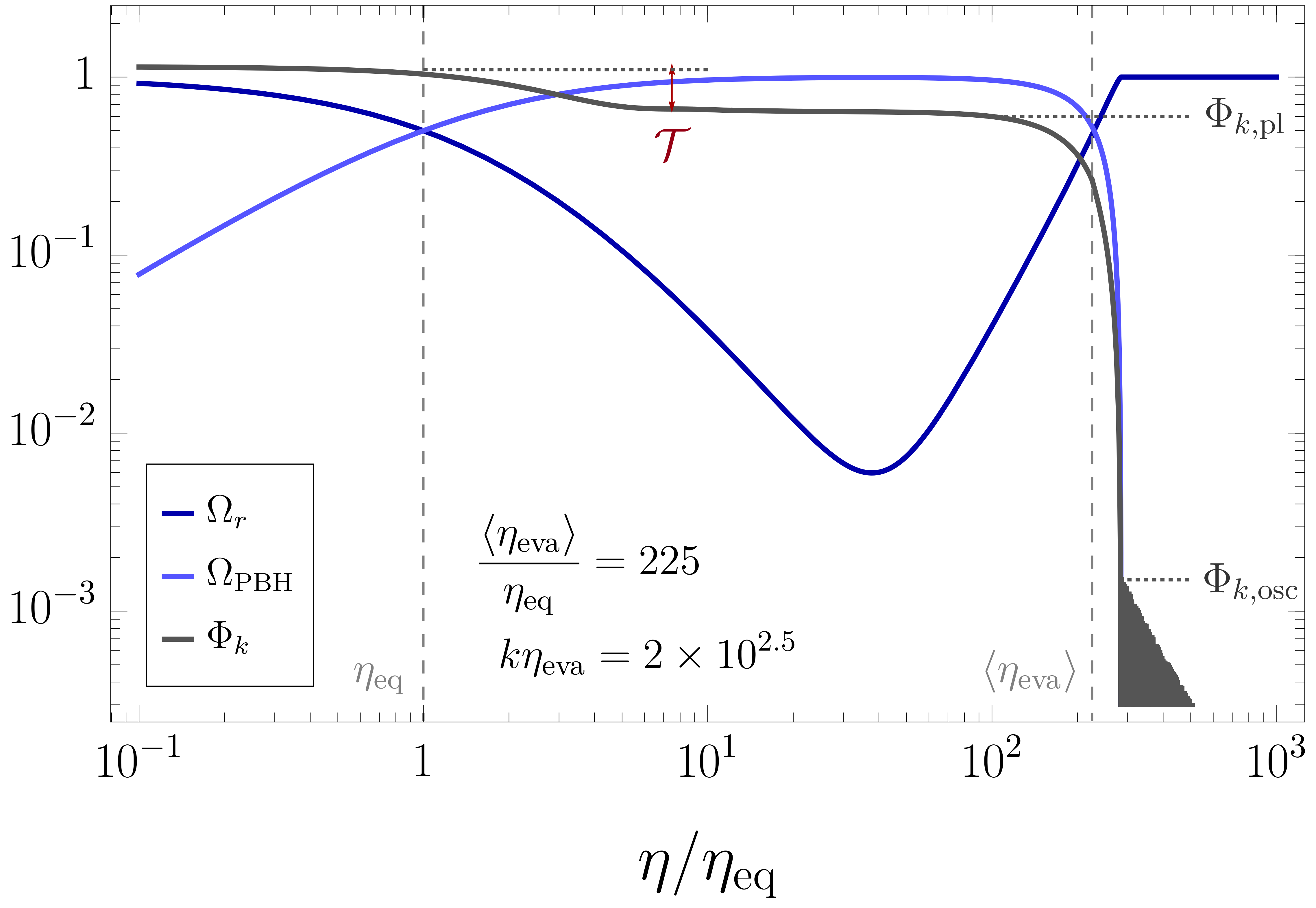

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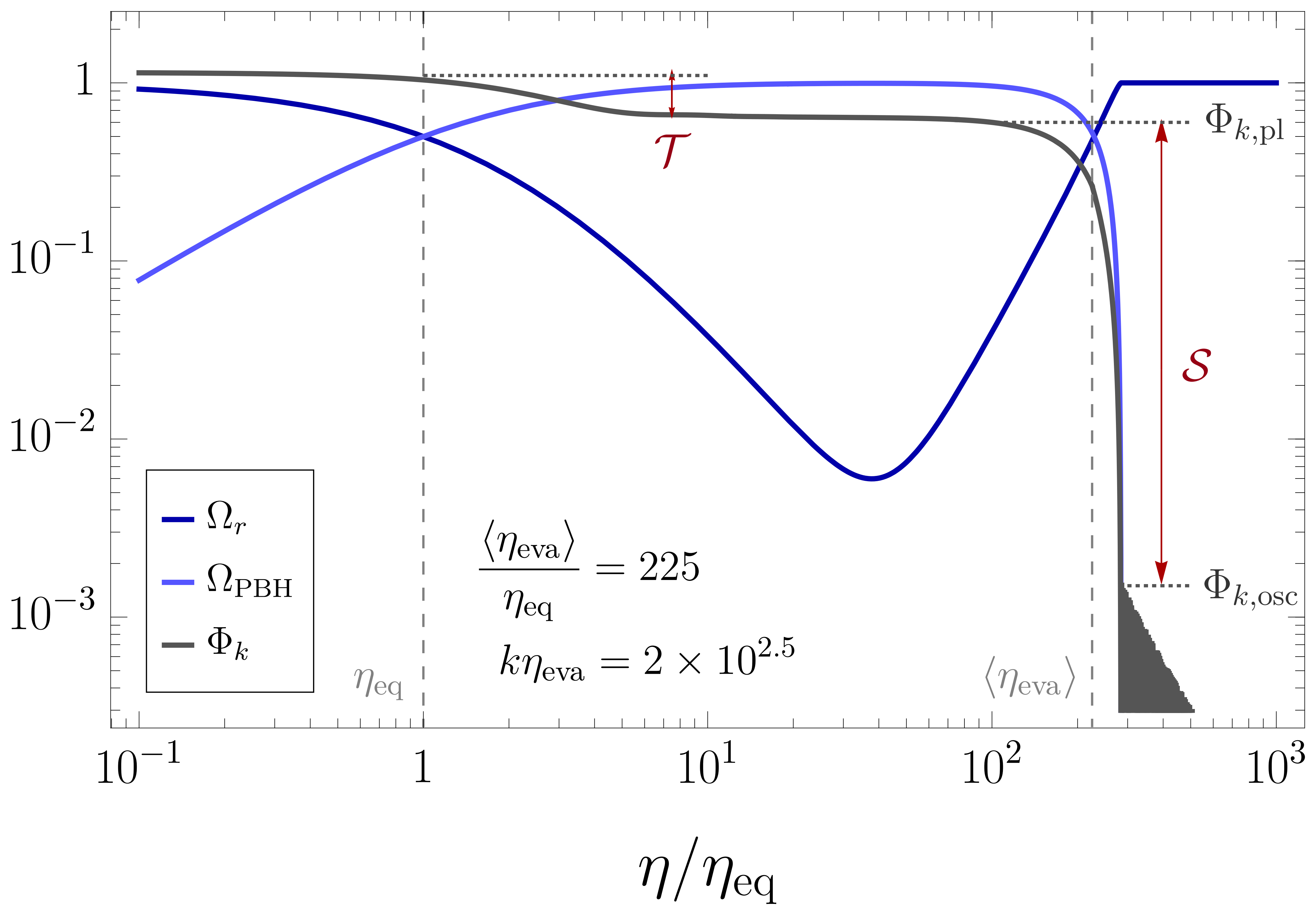

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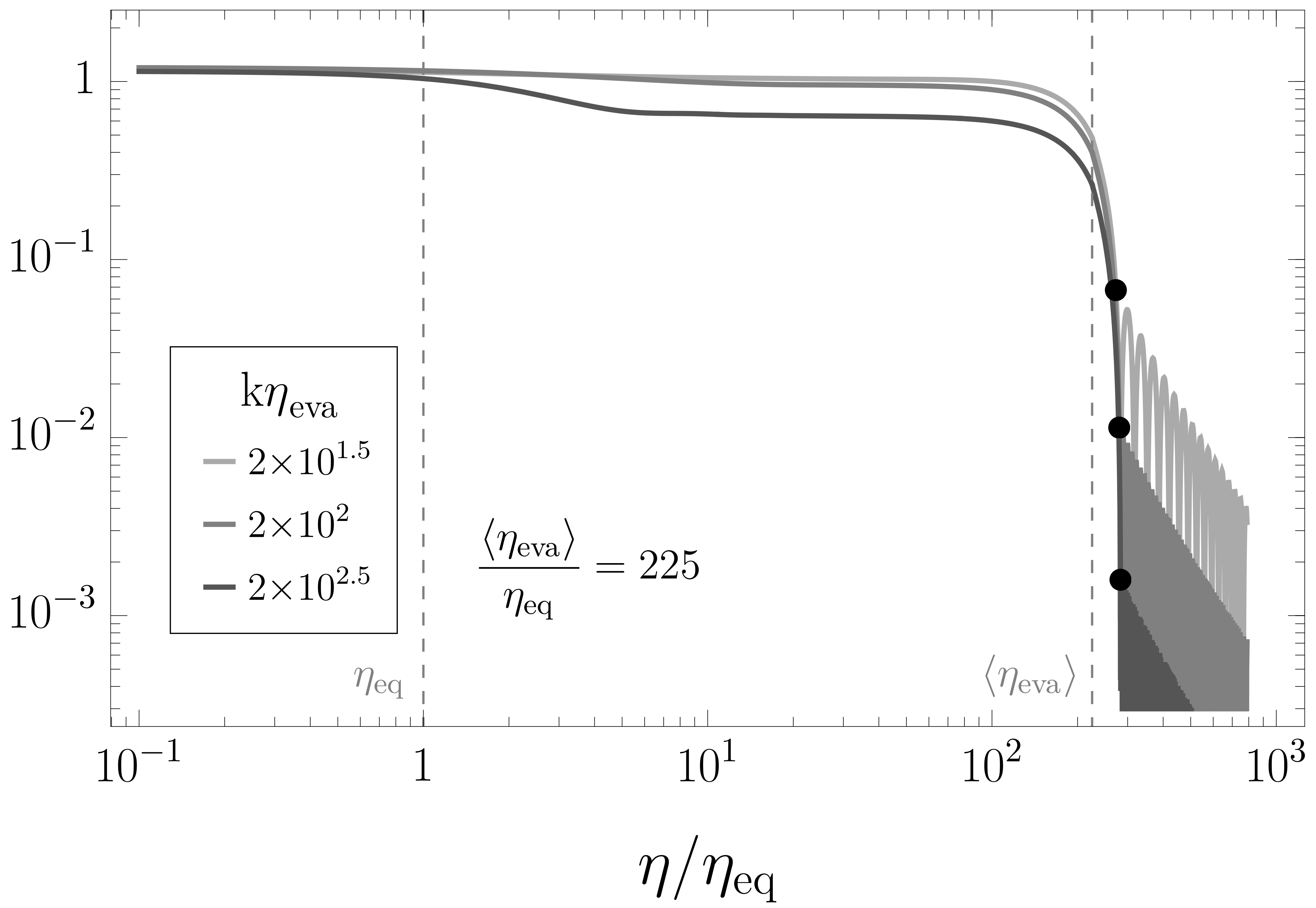

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Combine:

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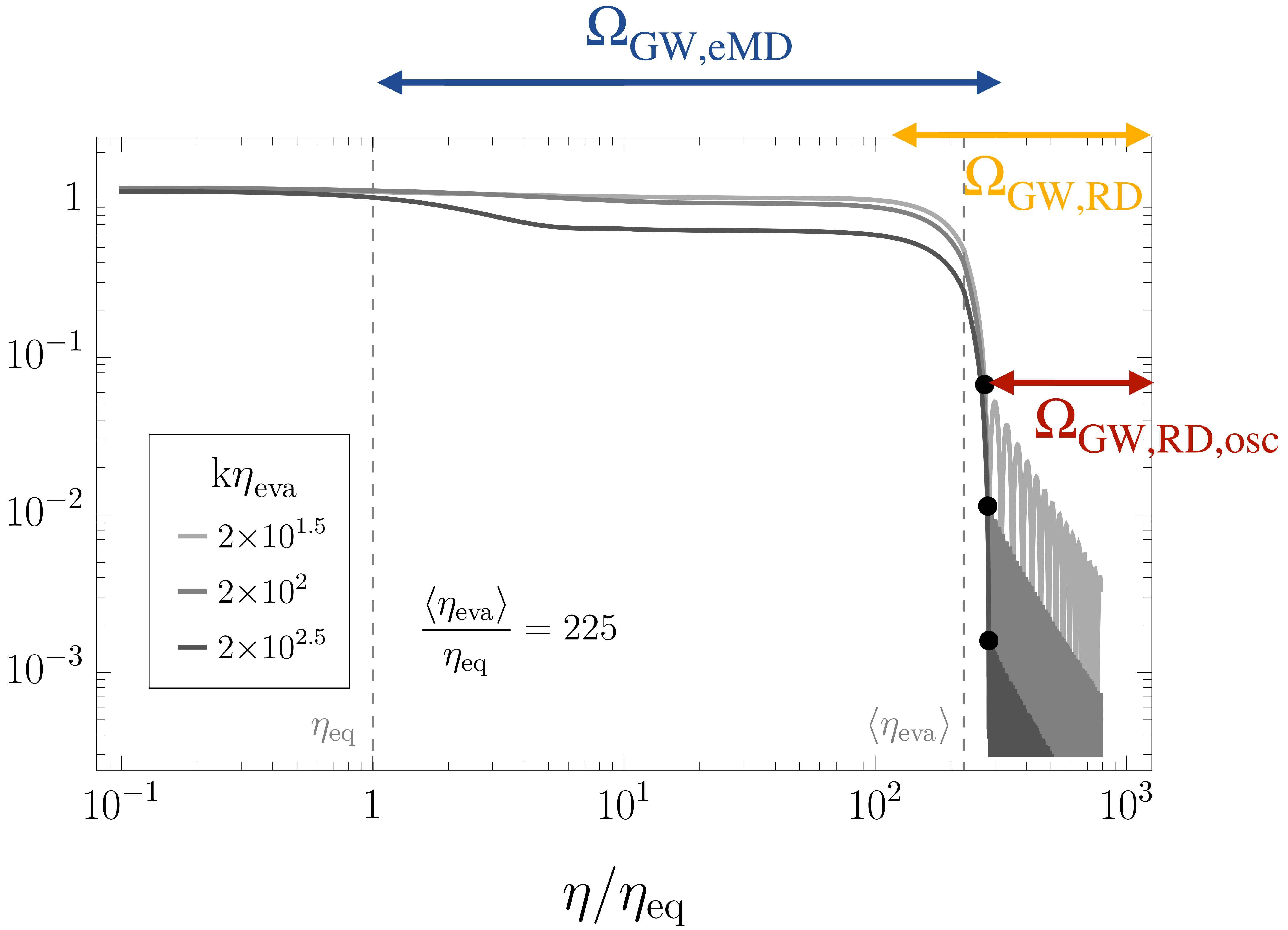
Effects decouple



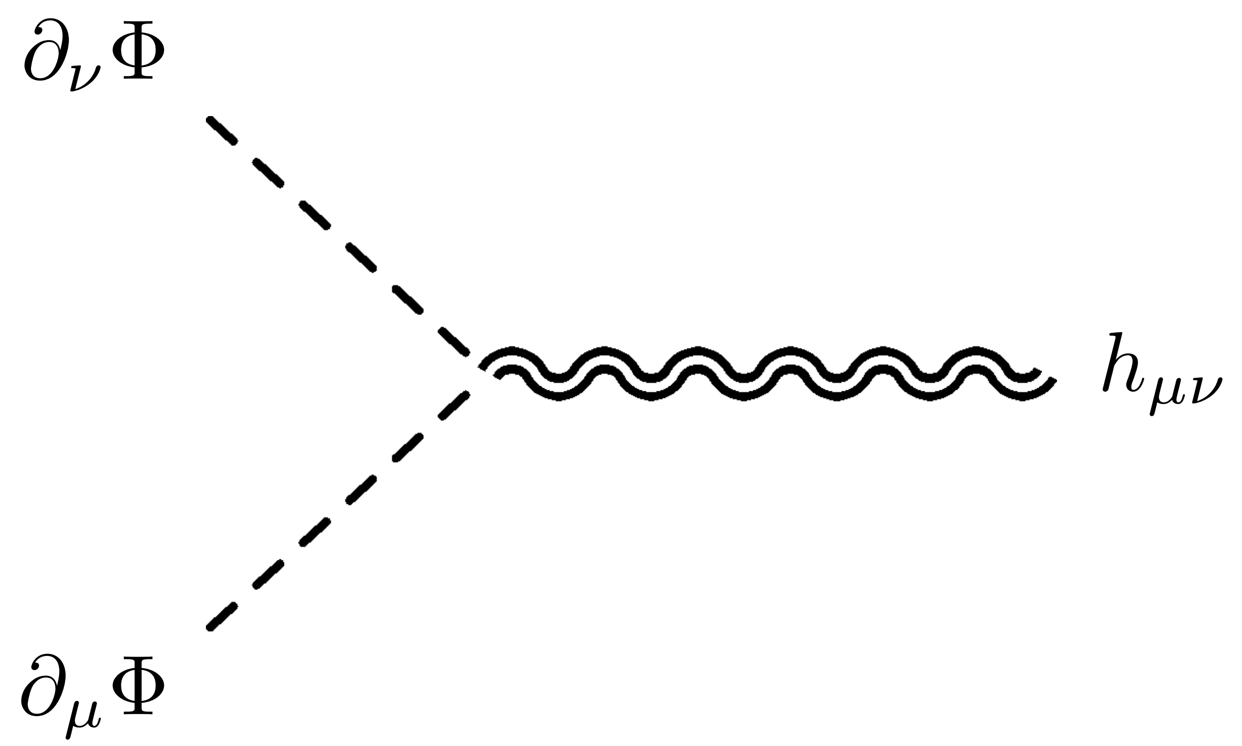
$$\mathscr{P}_h \subset \Phi^4, \quad \left(\frac{\Phi'}{\mathscr{H}}\right)^4$$

SIGW

$$\boxed{\Omega_{\mathrm{GW}}(k) = \frac{k^2}{12\mathscr{H}^2}\overline{\mathscr{P}_{h,\mathrm{RD}}}}$$

$$\overline{\mathscr{P}_{h,\mathrm{RD}}}(k, \eta, \bar{x} \gg 1) \sim \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \; (4v^2 - (1 + v^2 - u^2)^2)^2$$

$$\times\, \mathscr{P}(uk)\mathscr{P}(vk)\mathscr{T}^2(uk)\mathscr{T}^2(vk)\mathscr{S}^2(uk)\mathscr{S}^2(vk)\overline{\mathscr{I}^2_{\mathrm{osc}}}(\bar{x}, u, v),$$



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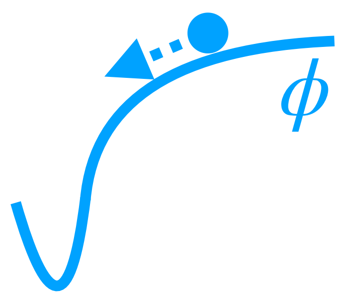
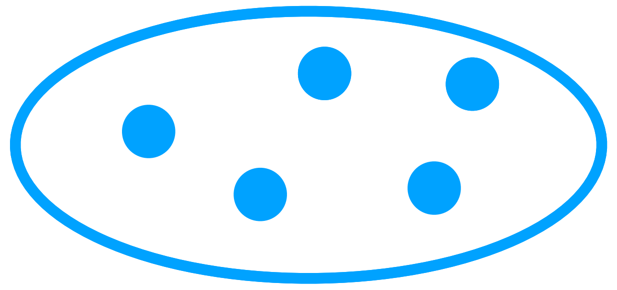
SEED

1. Adiabatic



2. Isocurvature



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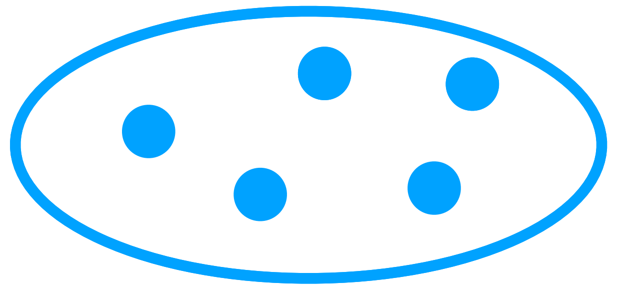

TRANSFER

RD-MD transition

SIGW

$$\overline{\mathscr{P}_{h,\mathrm{RD}}}(k,\eta,\bar{x}\gg 1)\sim\int_0^\infty \mathrm{d}v\int_{|1-v|}^{1+v}\mathrm{d}u\;(4v^2-(1+v^2-u^2)^2)^2(\bar{x},u,v)$$

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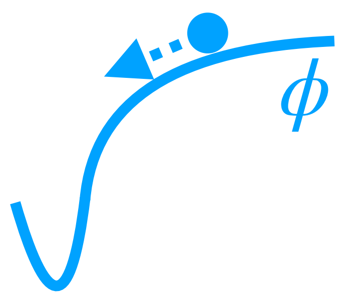

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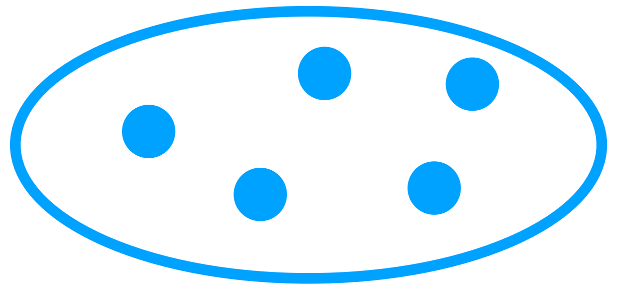

TRANSFER

RD-MD transition

SUPPRESSION

Evaporation with finite width

SIGW

$$\overline{\mathscr{P}_{h,\mathrm{RD}}}(k,\eta,\bar{x}\gg 1)\sim\int_0^\infty \mathrm{d}v\int_{|1-v|}^{1+v}\mathrm{d}u\;(4v^2-(1+v^2-u^2)^2)^2(\bar{x},u,v)$$

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SEED

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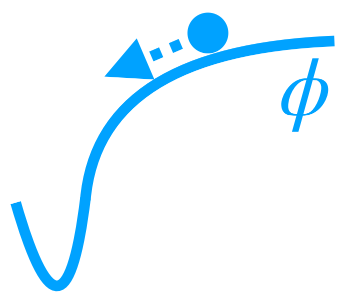

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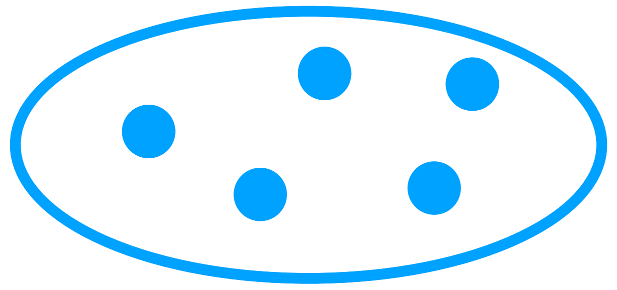


TRANSFER

RD-MD transition

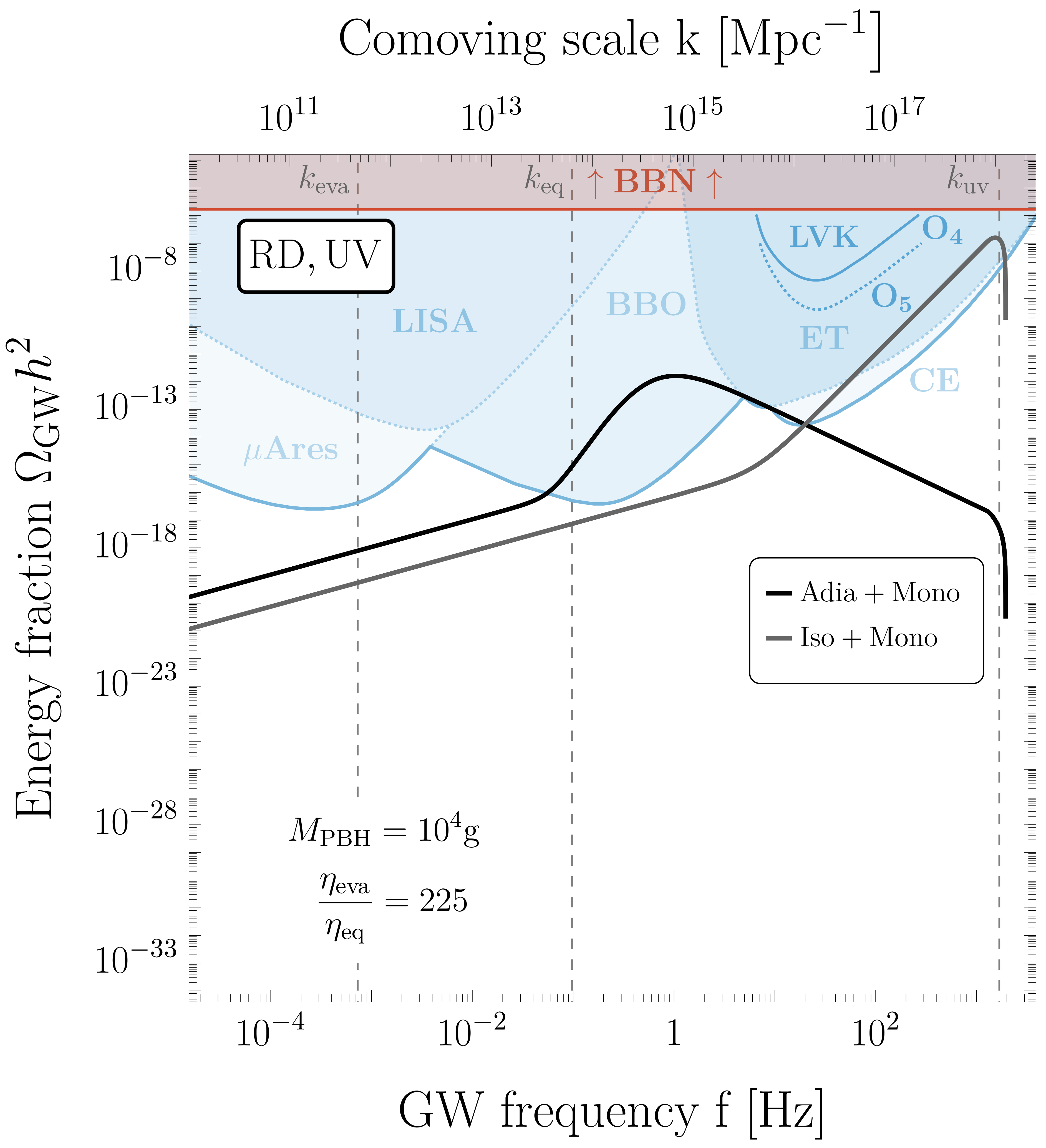
SUPPRESSION

Evaporation with finite width

KERNEL

GW propagation

SIGW



$$\Omega_{\rm GW}(k) = \frac{k^2}{12\mathcal{H}^2}\overline{\mathcal{P}_{h,\rm RD}}$$

$$\boxed{\begin{aligned}\overline{\mathcal{P}_{h,\rm RD}}(k,\eta,\bar{x}\gg 1) \sim \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u\ (4v^2-(1+v^2-u^2)^2)^2\overline{\mathcal{I}^2_{\rm osc}}(\bar{x},u,v) \\ \times \mathcal{P}(uk)\mathcal{P}(vk)\mathcal{S}^2(uk)\mathcal{S}^2(vk)\mathcal{T}^2(uk)\mathcal{T}^2(vk),\end{aligned}}$$

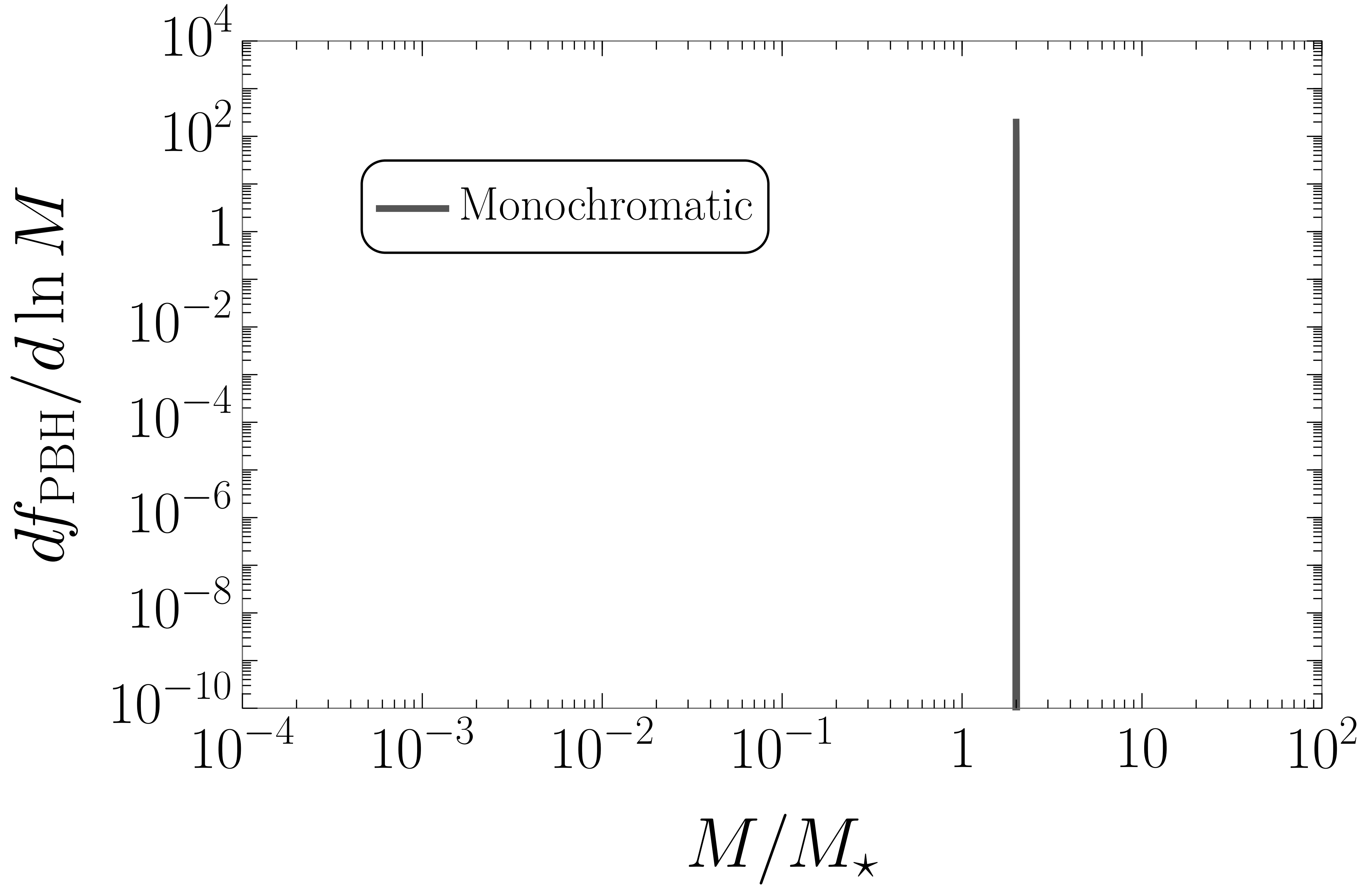
$$\mathcal{P}_{\rm Adia}(k) = A_s\left(\frac{k}{k_*}\right)^{n_s-1}\Theta(k_{\rm UV}-k)\Theta(k_{k-\rm IR})$$

$$\mathcal{P}_{\rm Iso}(k) = \frac{2}{3\pi}\left(\frac{k}{k_*}\right)^3\Theta(k_{\rm UV}-k)$$

PART II:

Our new results.

SUPERHORIZON COLLAPSE



So far:

1. Assumes monochromatic[1,2,3] or (narrow) log-normal[1] mass distribution

2. Fast reheating

[1]*Poltergeist* mechanism: Inomata et al. (2020)
[2]Domenech et. al (2021)
[3]Domènech and Tränkle (2025)

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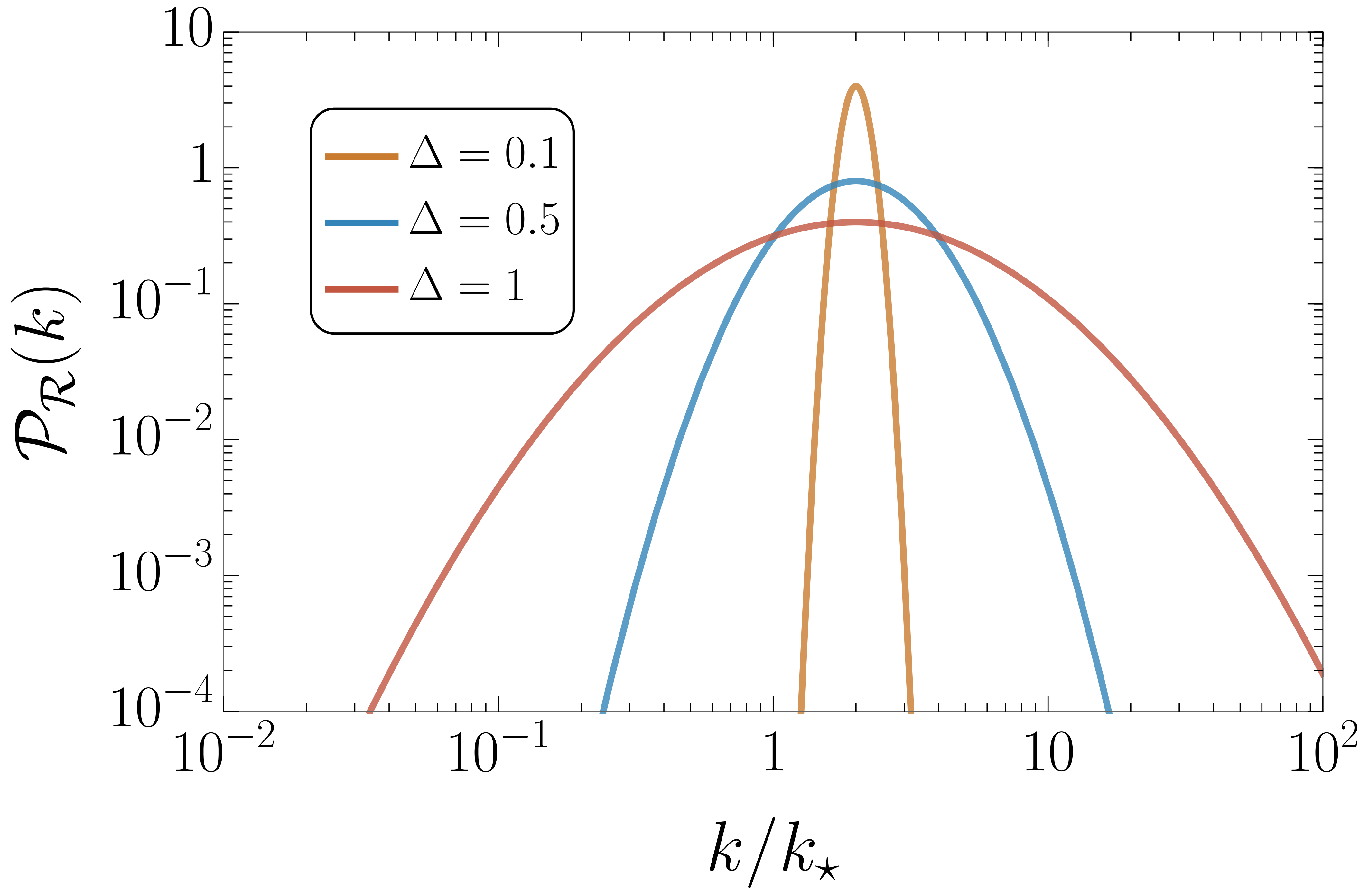

- Superhorizon gravitational collapse:

  1. **Critical scaling**[1]

$$M(\delta_m) = \kappa M_k(\delta_m - \delta_c)^\gamma$$

  2. **Press-Schechter** formalism[2]

$$\frac{\mathrm{d}f_{\mathrm{PBH}}}{\mathrm{d}\log M} \propto M^{1+1/\gamma} e^{-c_1 (M/\langle M \rangle)^{c_2}}$$

- c_1, c_2 depend on curvature power spectrum. Instead:

$$\boxed{\frac{\mathrm{d}f_{\mathrm{PBH}}}{\mathrm{d}\log M} \propto M^{1+1/\gamma} \theta(M_{\mathrm{cut}} - M)}$$

[1] Choptuik (1993)
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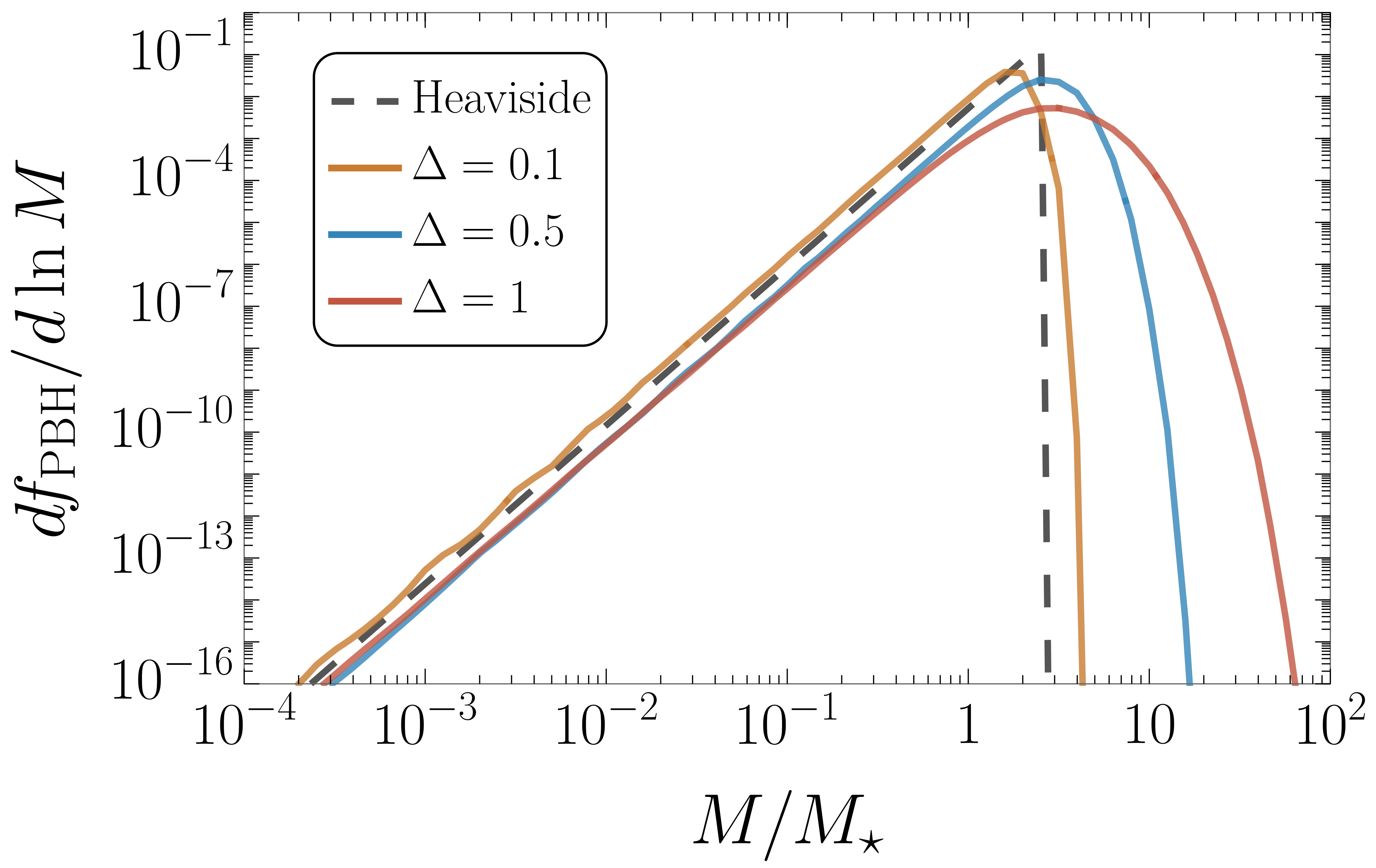

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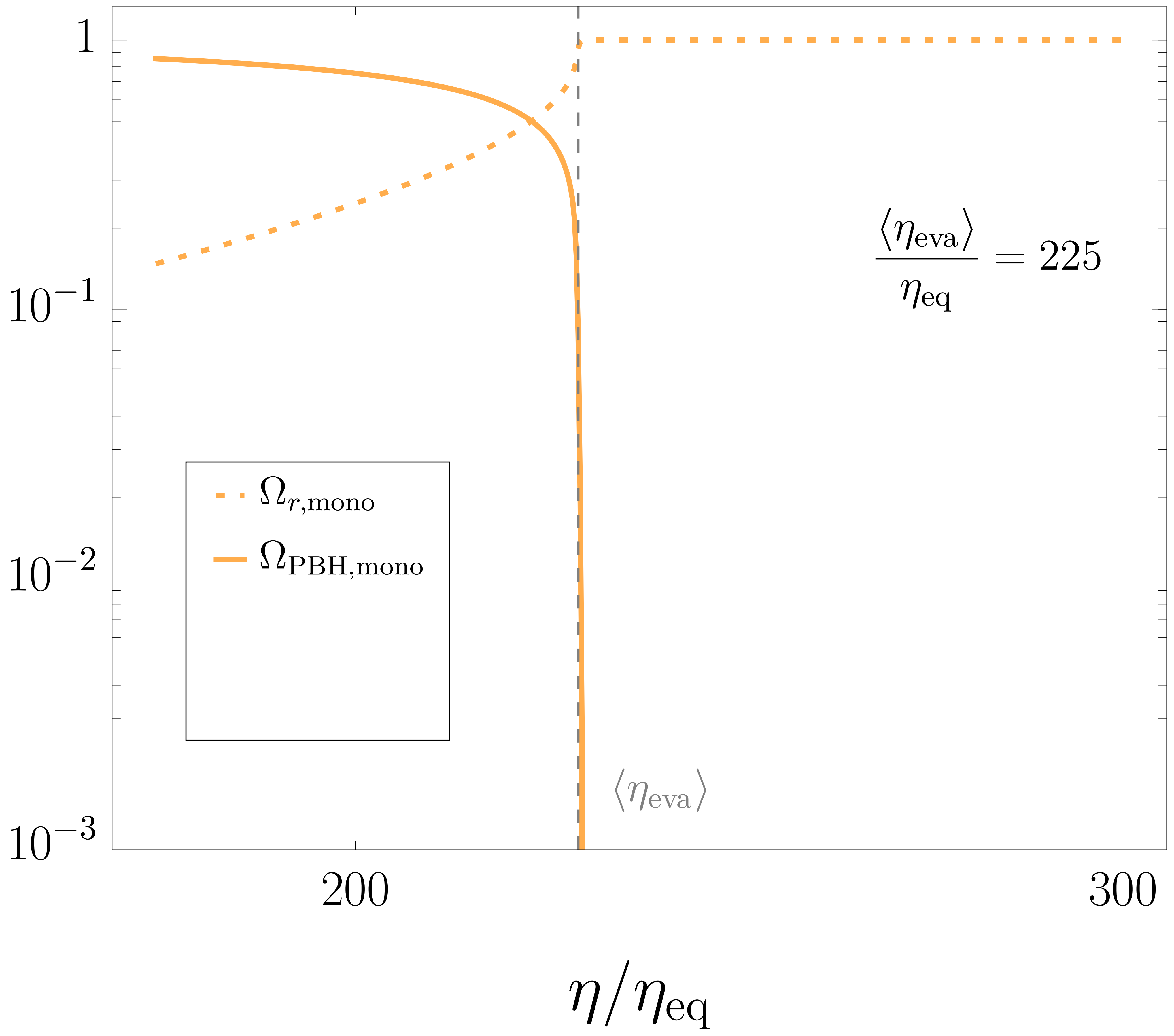

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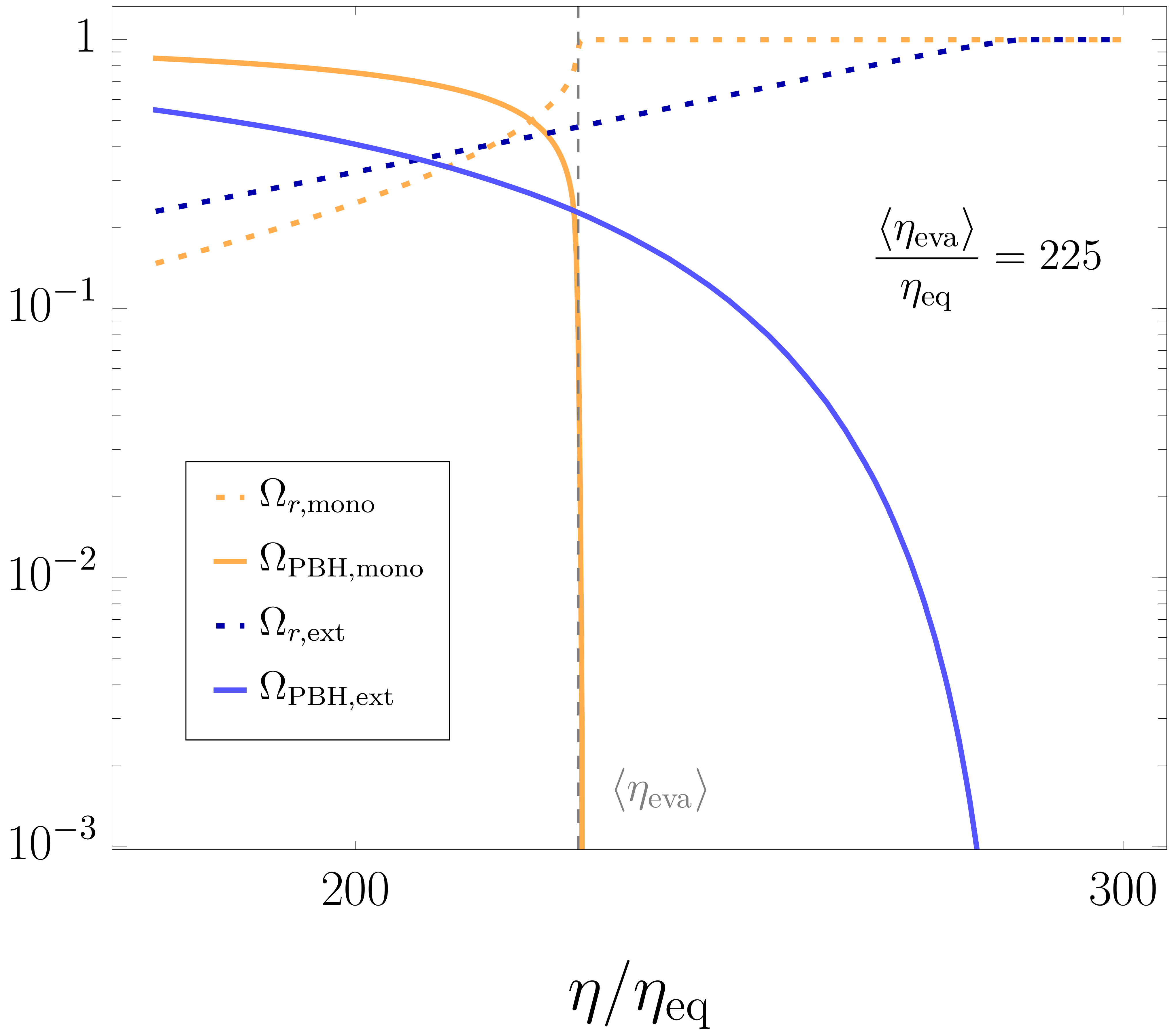

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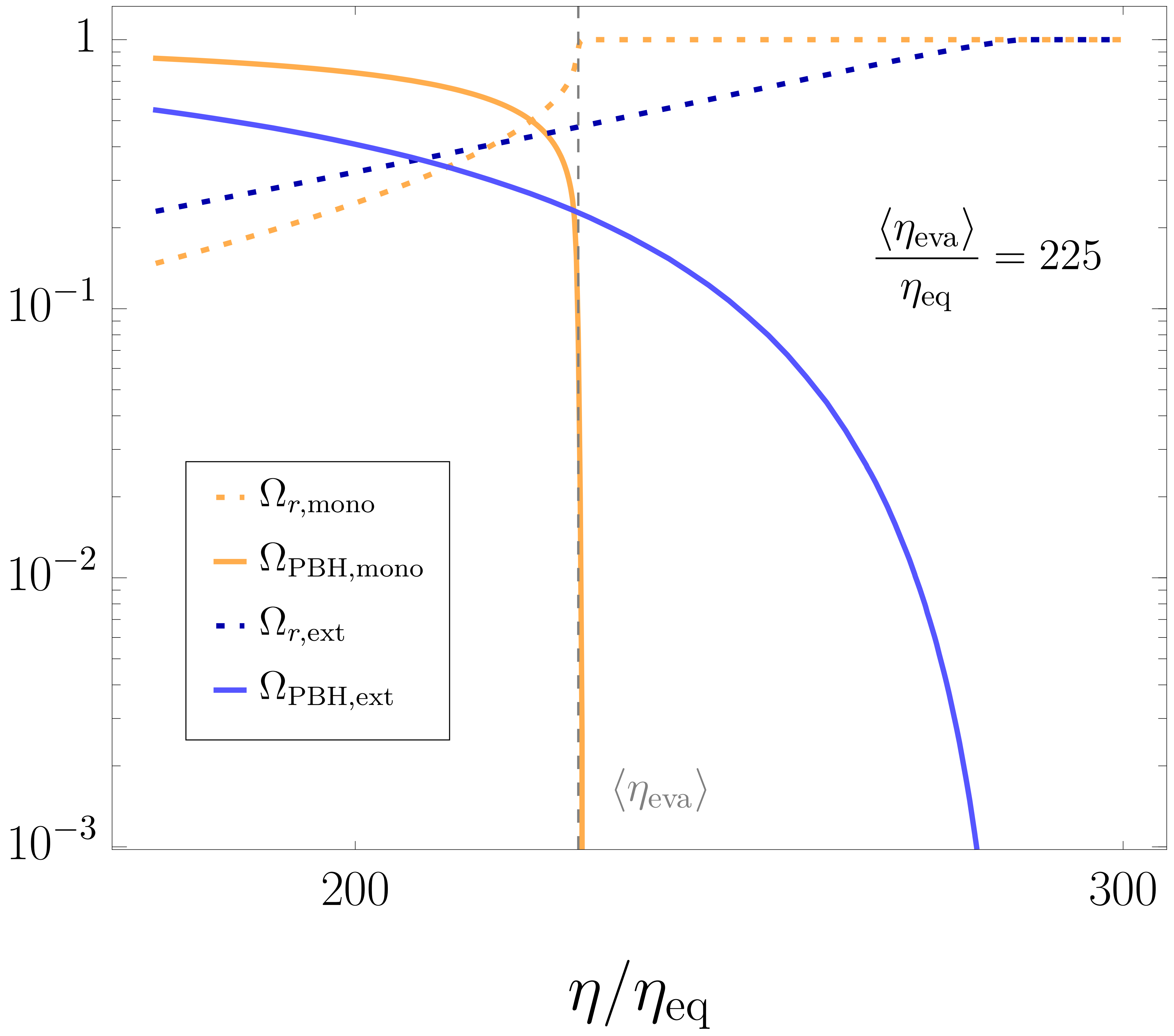
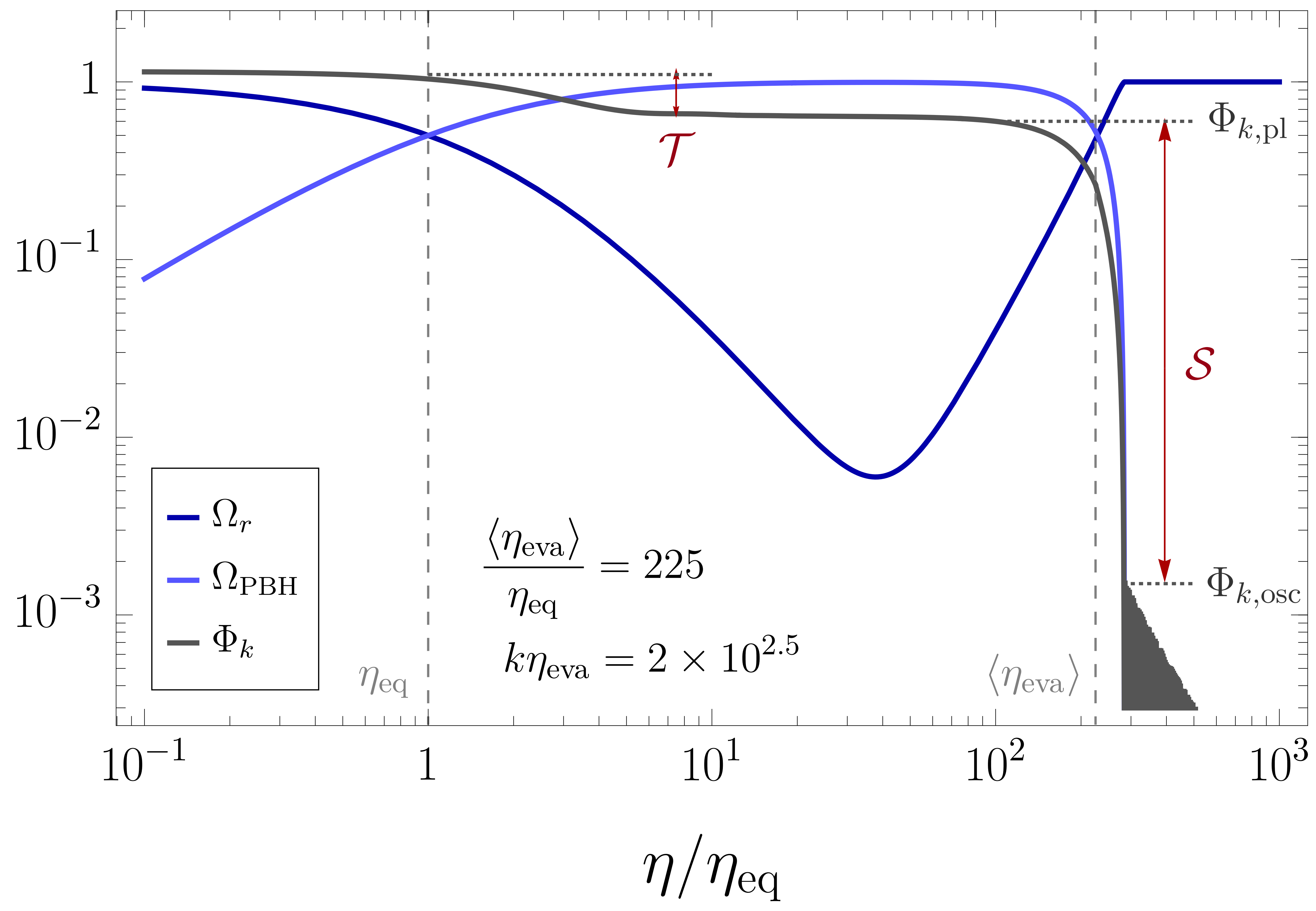


$$\rho_m \to \langle \rho_m \rangle = \int \rho_m(M) f_{\mathrm{PBH}}(M) \mathrm{d} \log M$$

$$\rho_r \to \langle \rho_r \rangle = \int \rho_r(M) f_{\mathrm{PBH}}(M) \mathrm{d} \log M$$

$$\Gamma \rho_m \to \langle \Gamma \rho_m \rangle = \int \Gamma(M) \rho_m(M) f_{\mathrm{PBH}}(M) \mathrm{d} \log M$$

SUPPRESSION FACTOR



Suppression factor: $\mathscr{P}_h \propto \Phi_{\rm osc}^4 \propto \mathcal{S}^4(k)$

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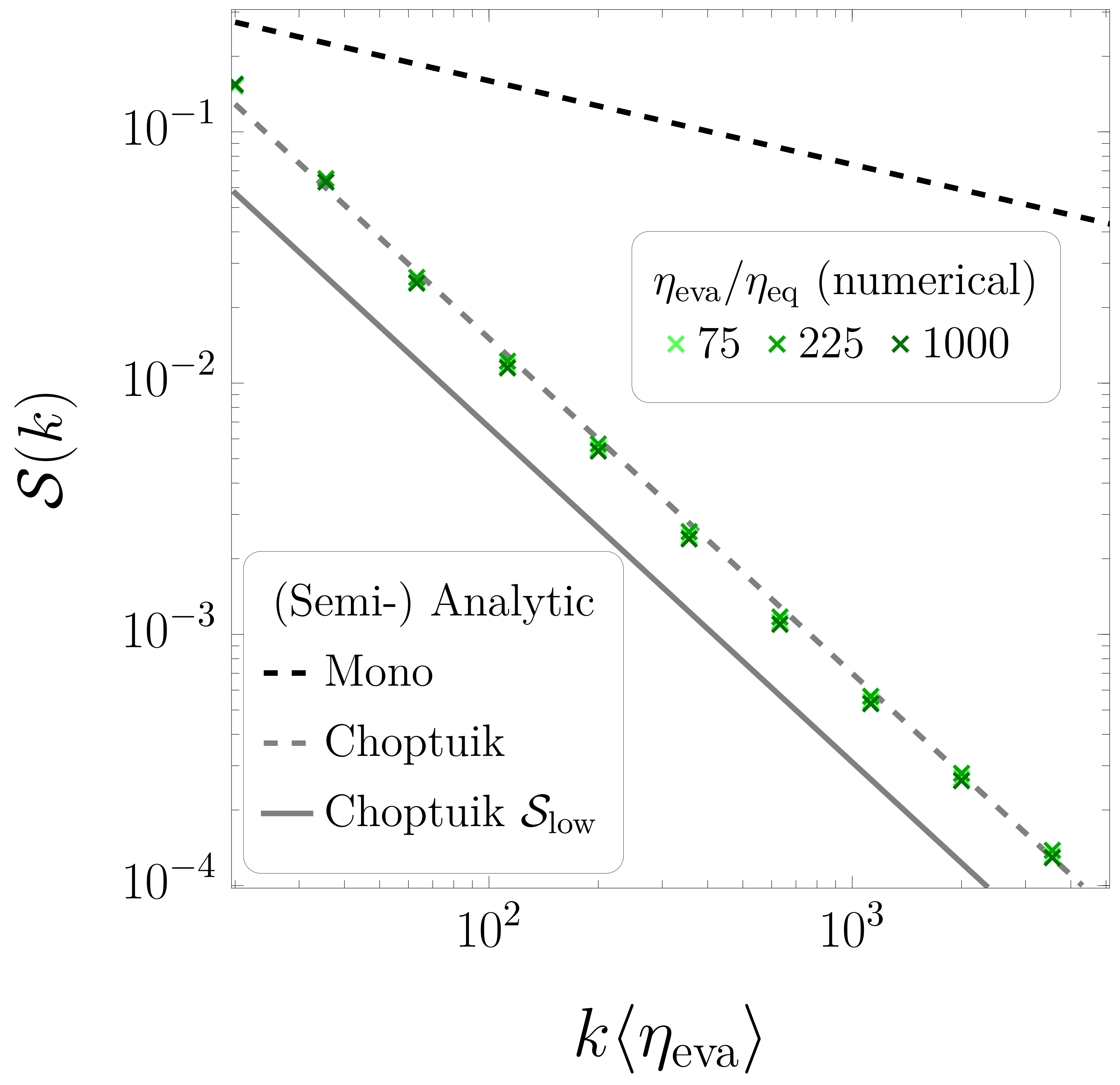


Suppression factor: $\mathscr{P}_h \propto \Phi^4_{\rm osc} \propto \mathcal{S}^4(k)$

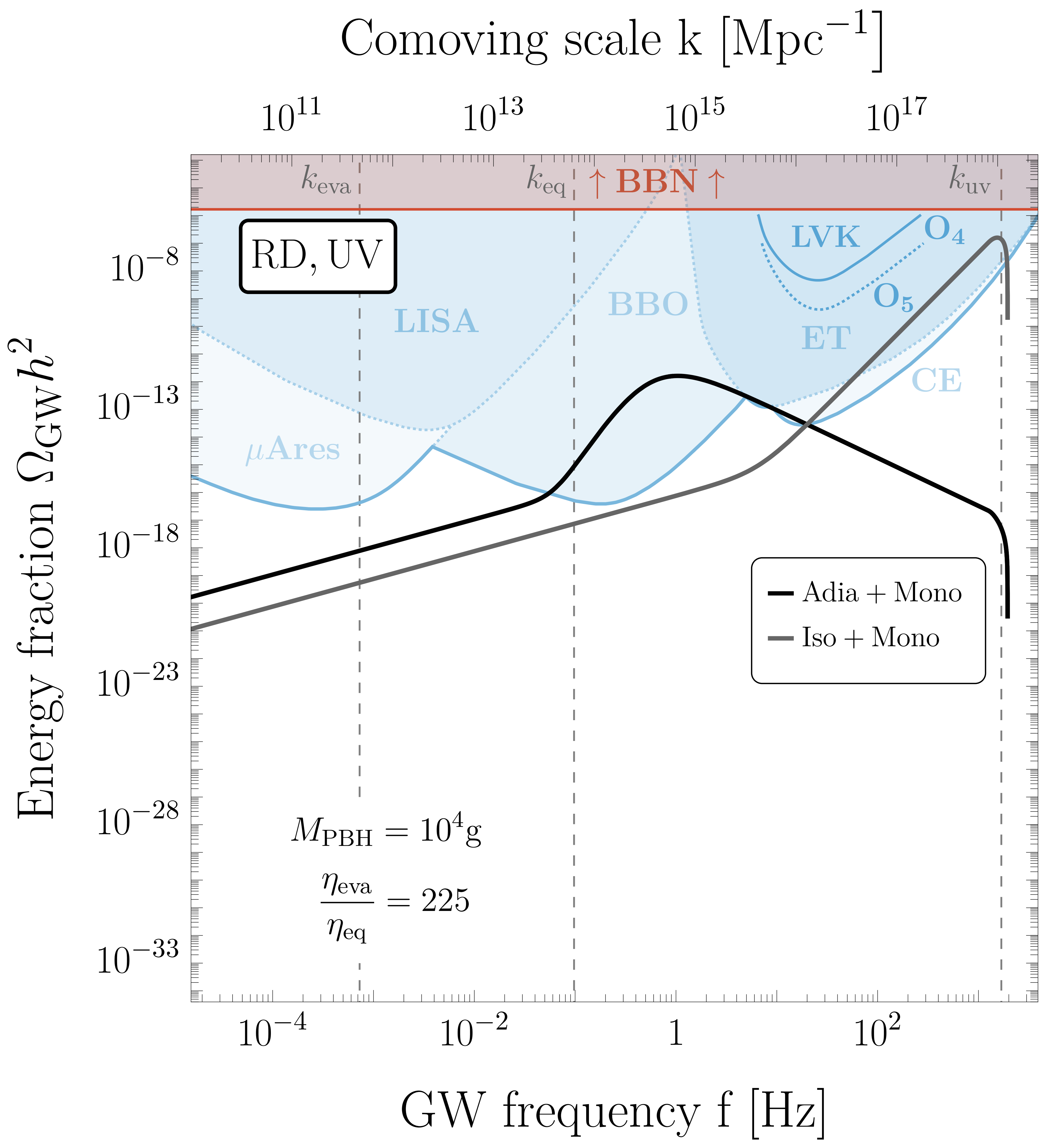
Good agreement numerical and analytical treatment:

$$\boxed{\mathcal{S}_{\rm ext}(k) \simeq 2.8\left(\frac{k}{k_{\rm eva}}\right)^{-4/3}}$$

Previously:

$$\boxed{\mathcal{S}_{\rm mono}(k) \simeq \left(\sqrt{\frac{2}{3}}\frac{k}{k_{\rm eva}}\right)^{-1/3}}$$

SIGW



$$\Omega_{\rm GW}(k) = \frac{k^2}{12\mathcal{H}^2}\overline{\mathcal{P}_{h,\rm RD}}$$

$$\boxed{\begin{aligned}\overline{\mathcal{P}_{h,\rm RD}}(k,\eta,\bar{x} \gg 1) \sim \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \ (4v^2 - (1+v^2-u^2)^2)^2\,\overline{\mathcal{I}^2_{\rm osc}}(\bar{x},u,v) \\ \times \mathcal{P}(uk)\mathcal{P}(vk)\mathcal{S}^2(uk)\mathcal{S}^2(vk)\mathcal{T}^2(uk)\mathcal{T}^2(vk),\end{aligned}}$$

$$\mathcal{P}_{\rm Adia}(k) = A_s\left(\frac{k}{k_*}\right)^{n_s-1}\Theta(k_{\rm UV}-k)\Theta(k_{k-\rm IR})$$

$$\mathcal{P}_{\rm Iso}(k) = \frac{2}{3\pi}\left(\frac{k}{k_*}\right)^{3}\Theta(k_{\rm UV}-k)$$

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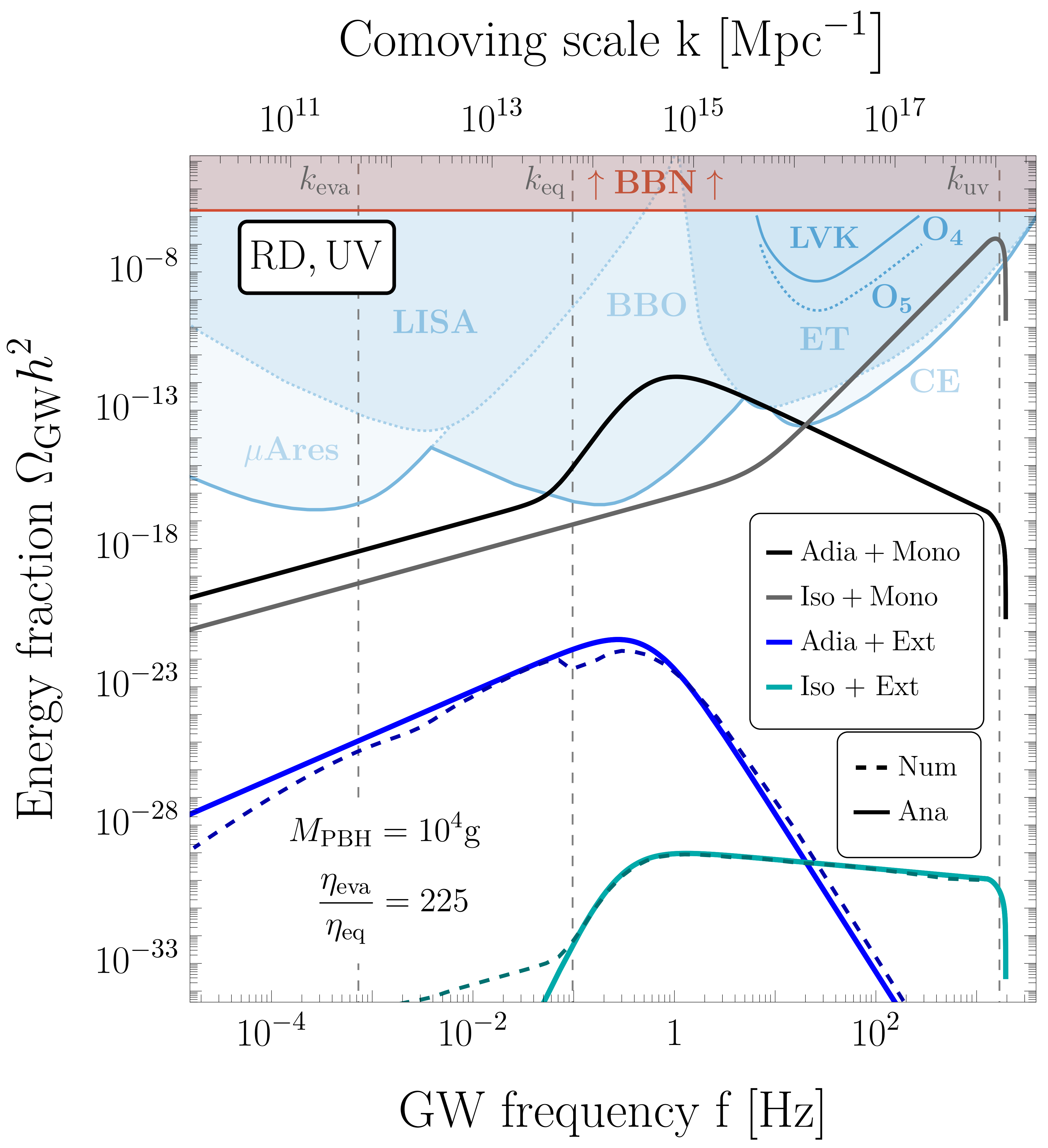

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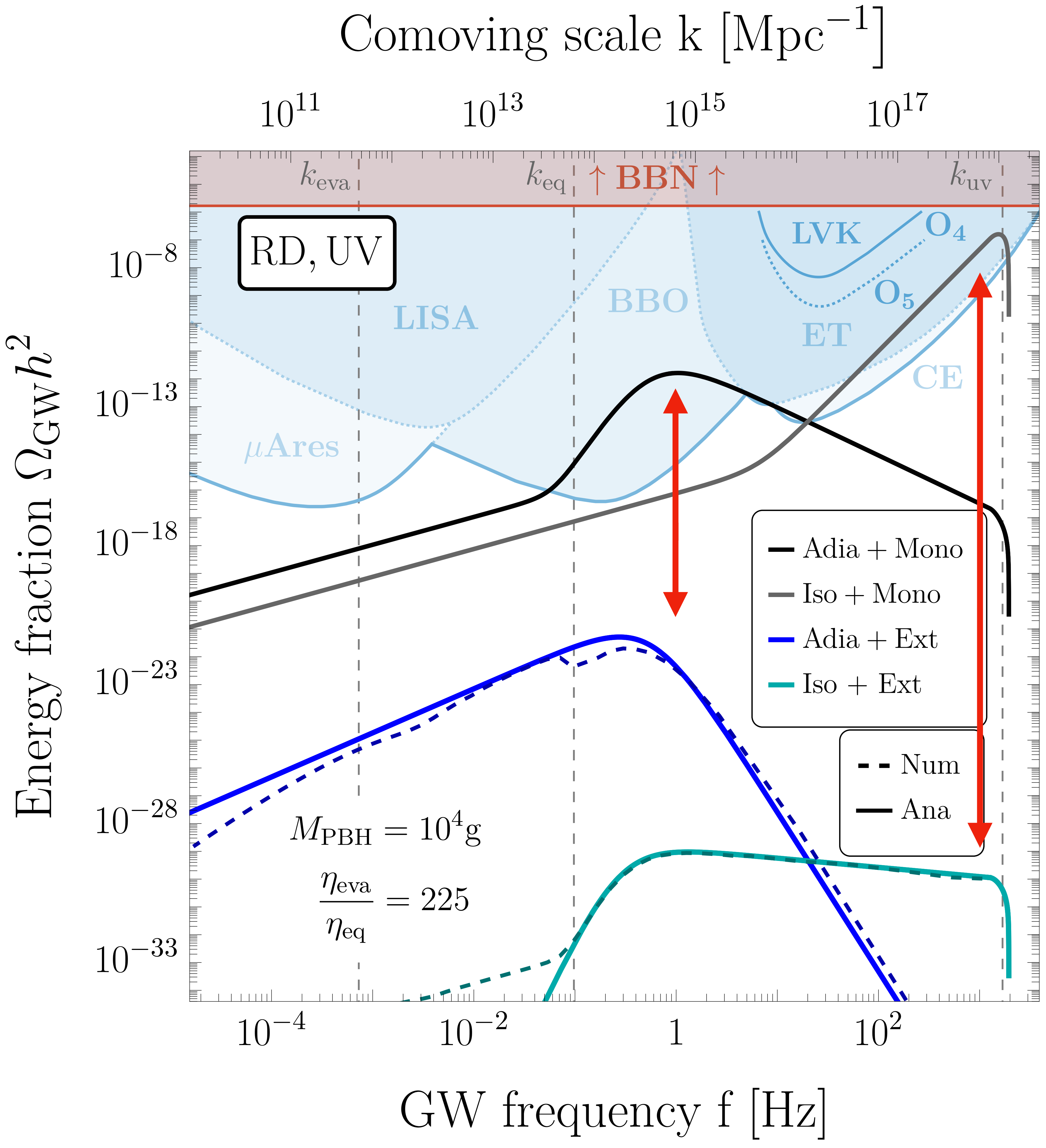

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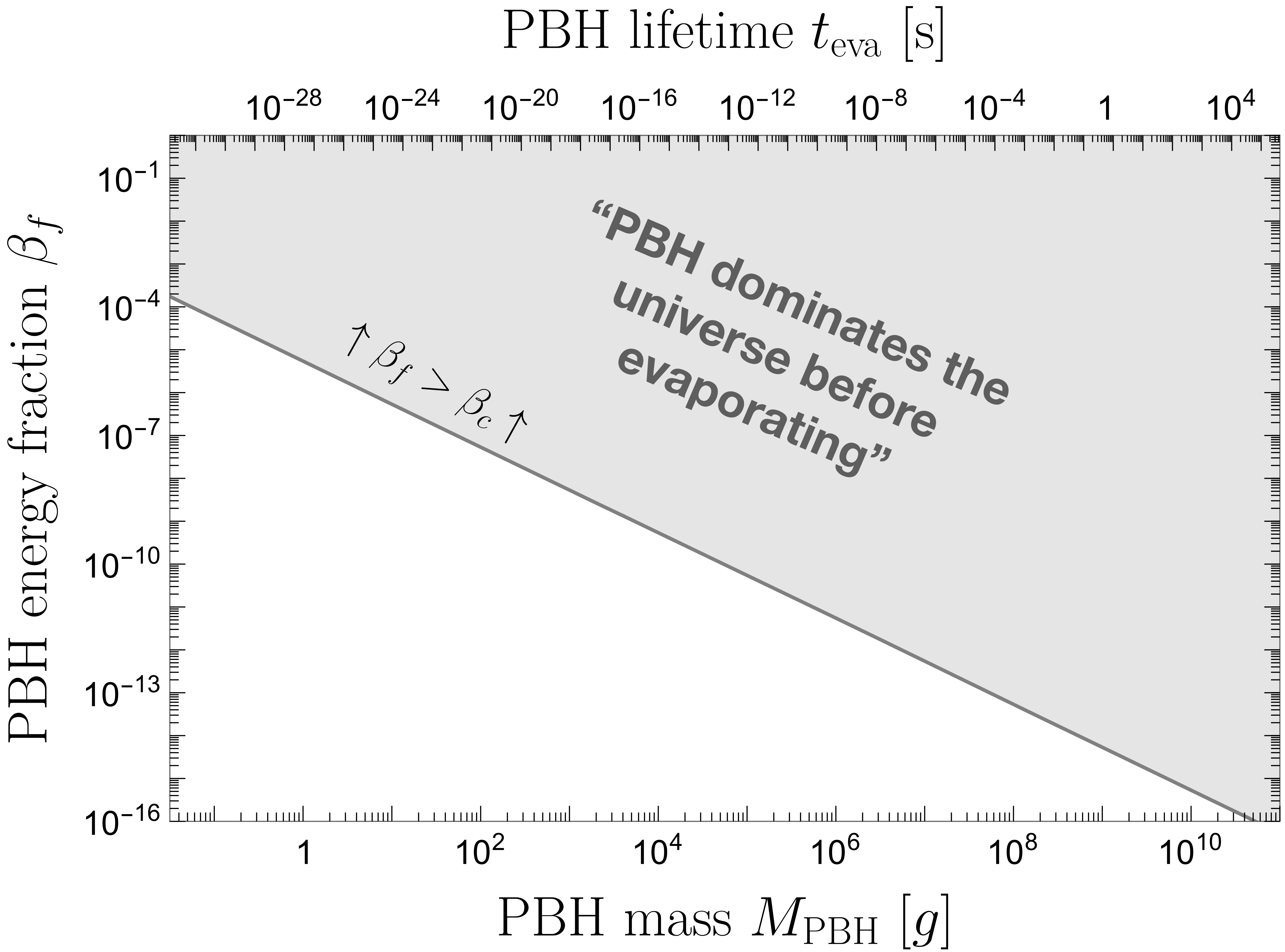
IMPLICATIONS FOR PBH



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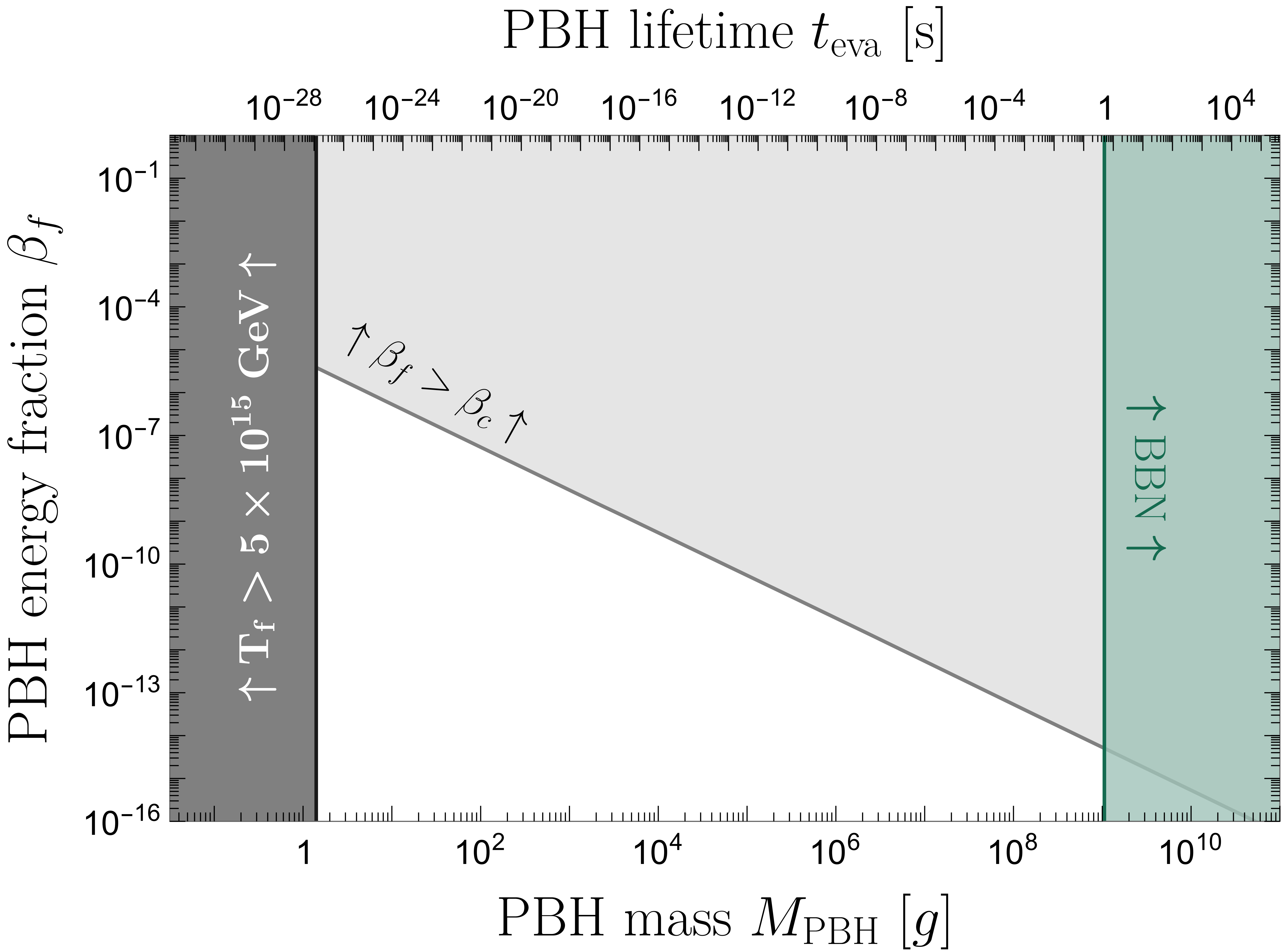

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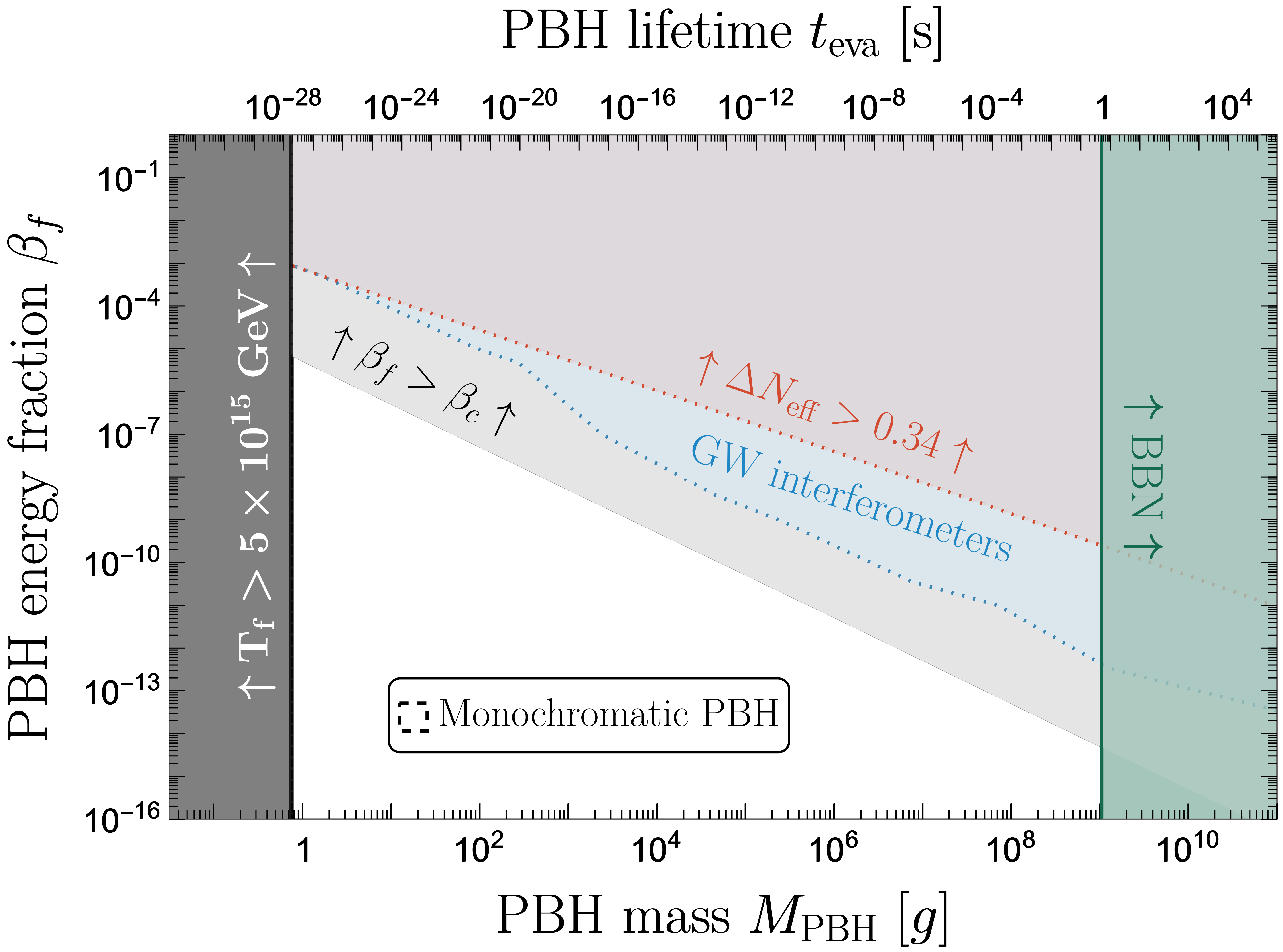

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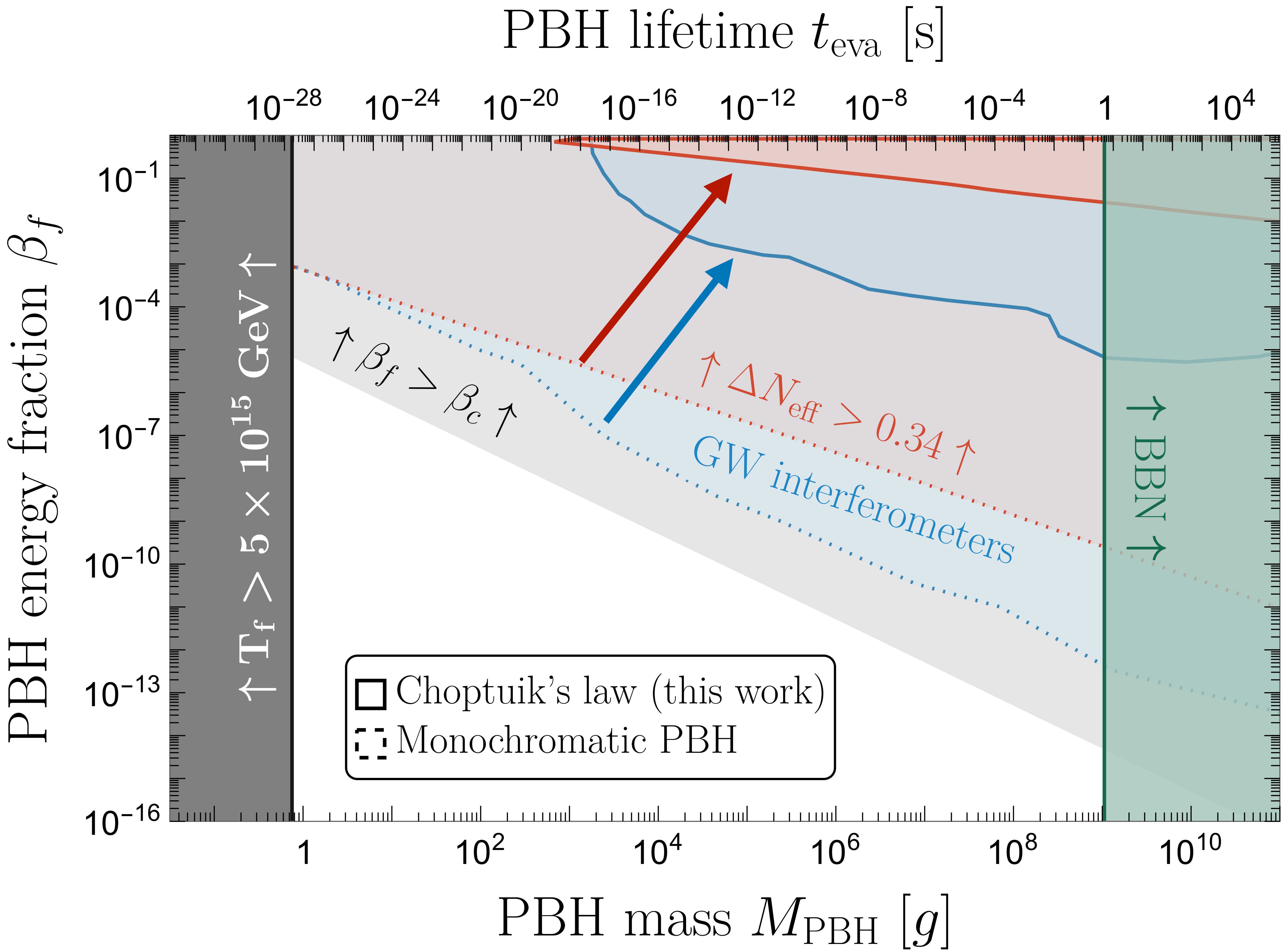

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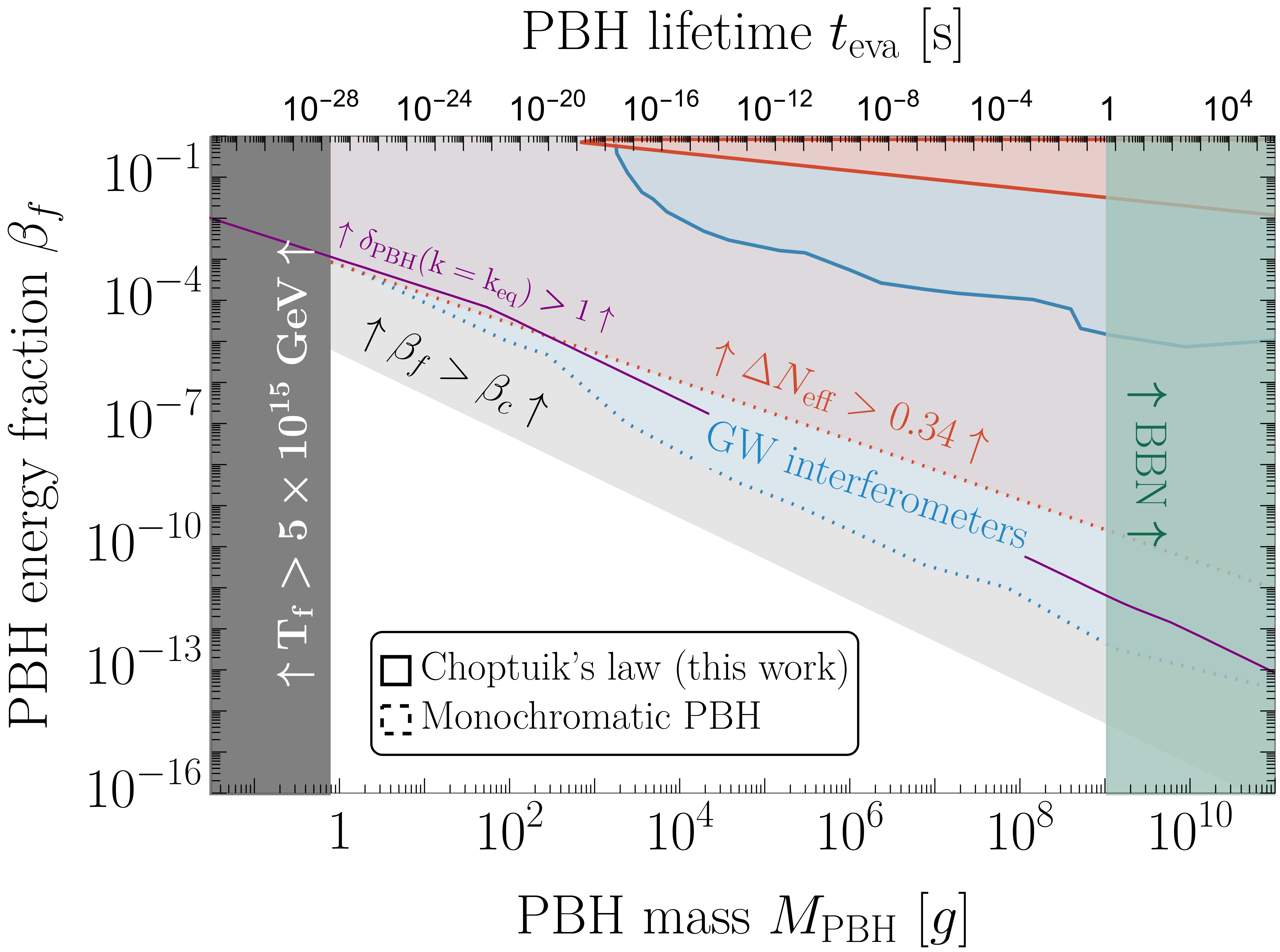
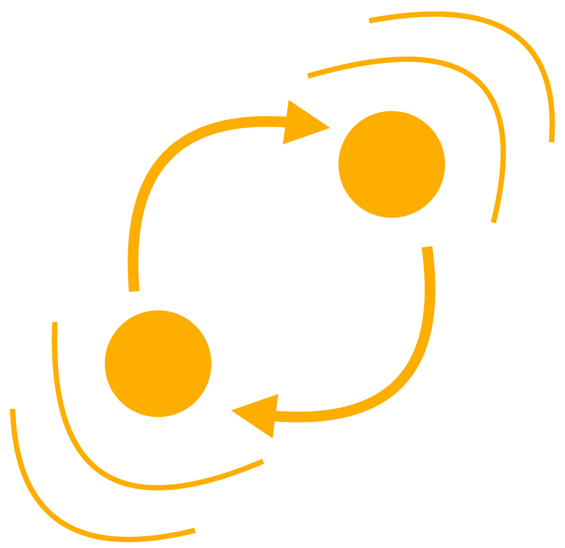


Non-linearities

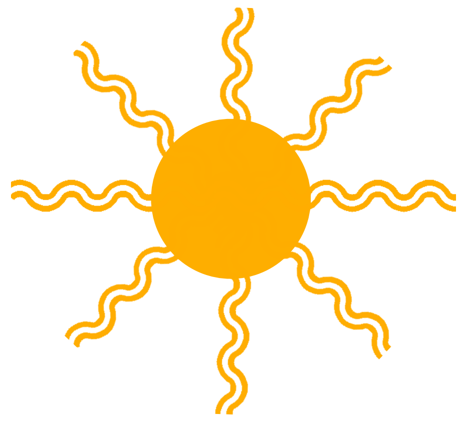
1. Breakdown of PT
2. Probably overestimates signal

ADDITIONAL SOURCES

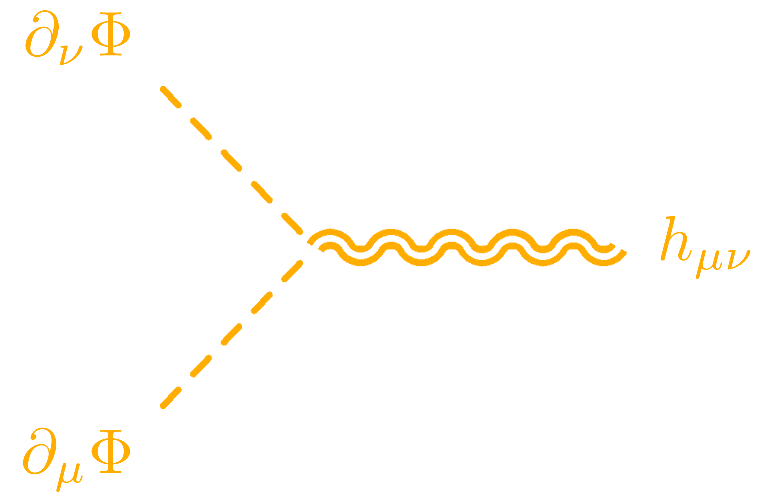
BINARIES



GRAVITONS



SCALAR-INDUCED

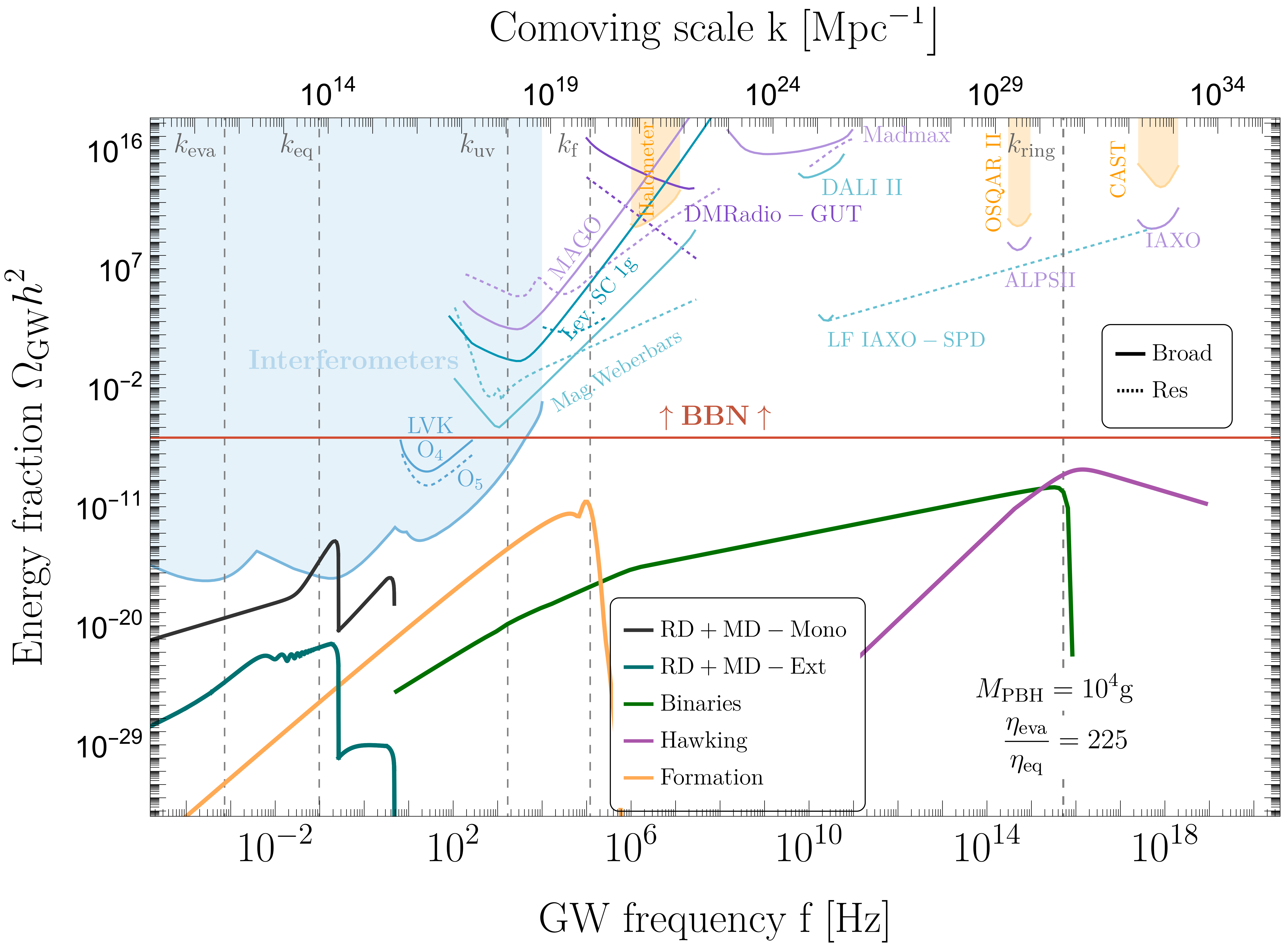


RD (Poltergeist)

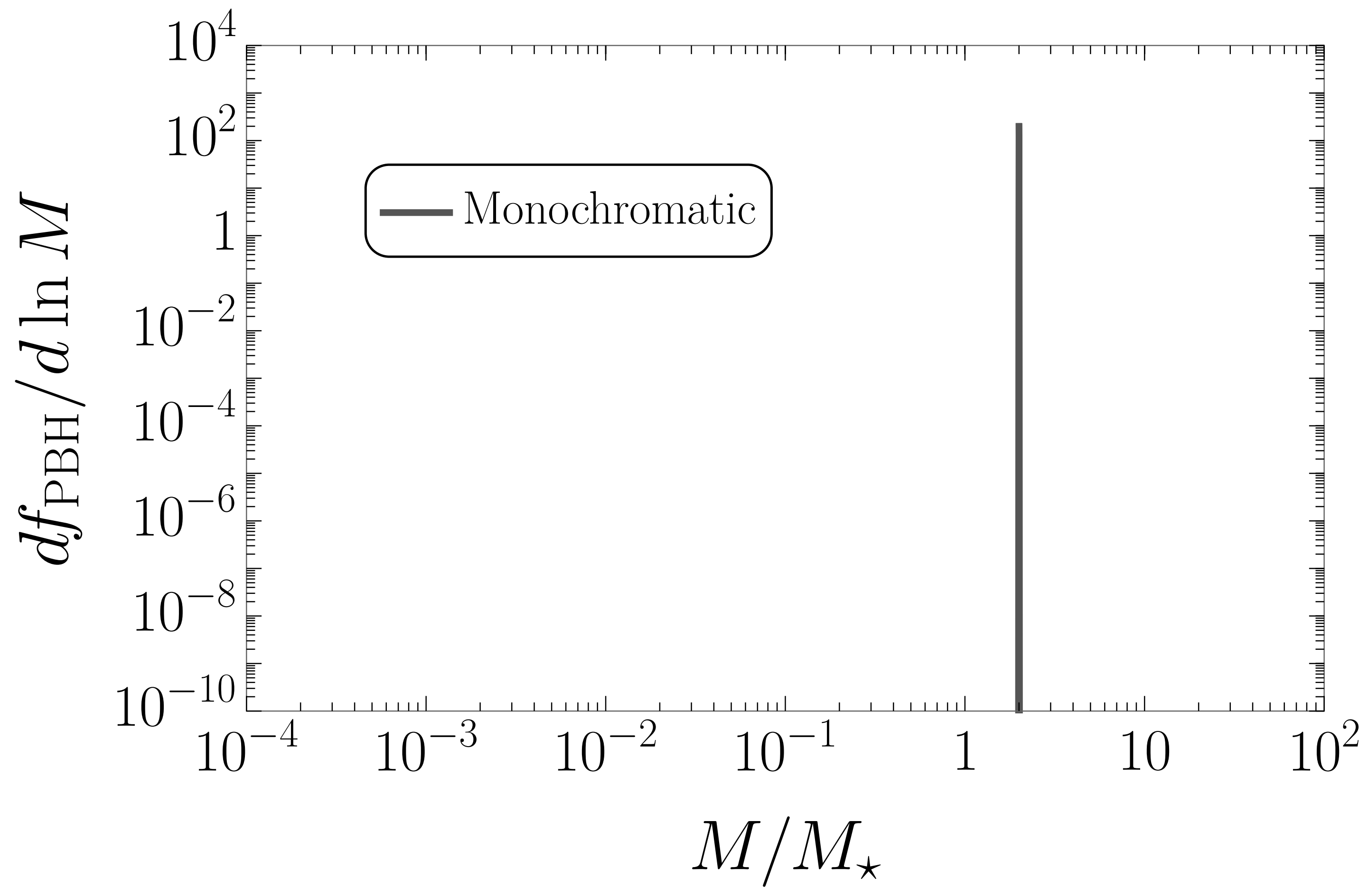
eMD

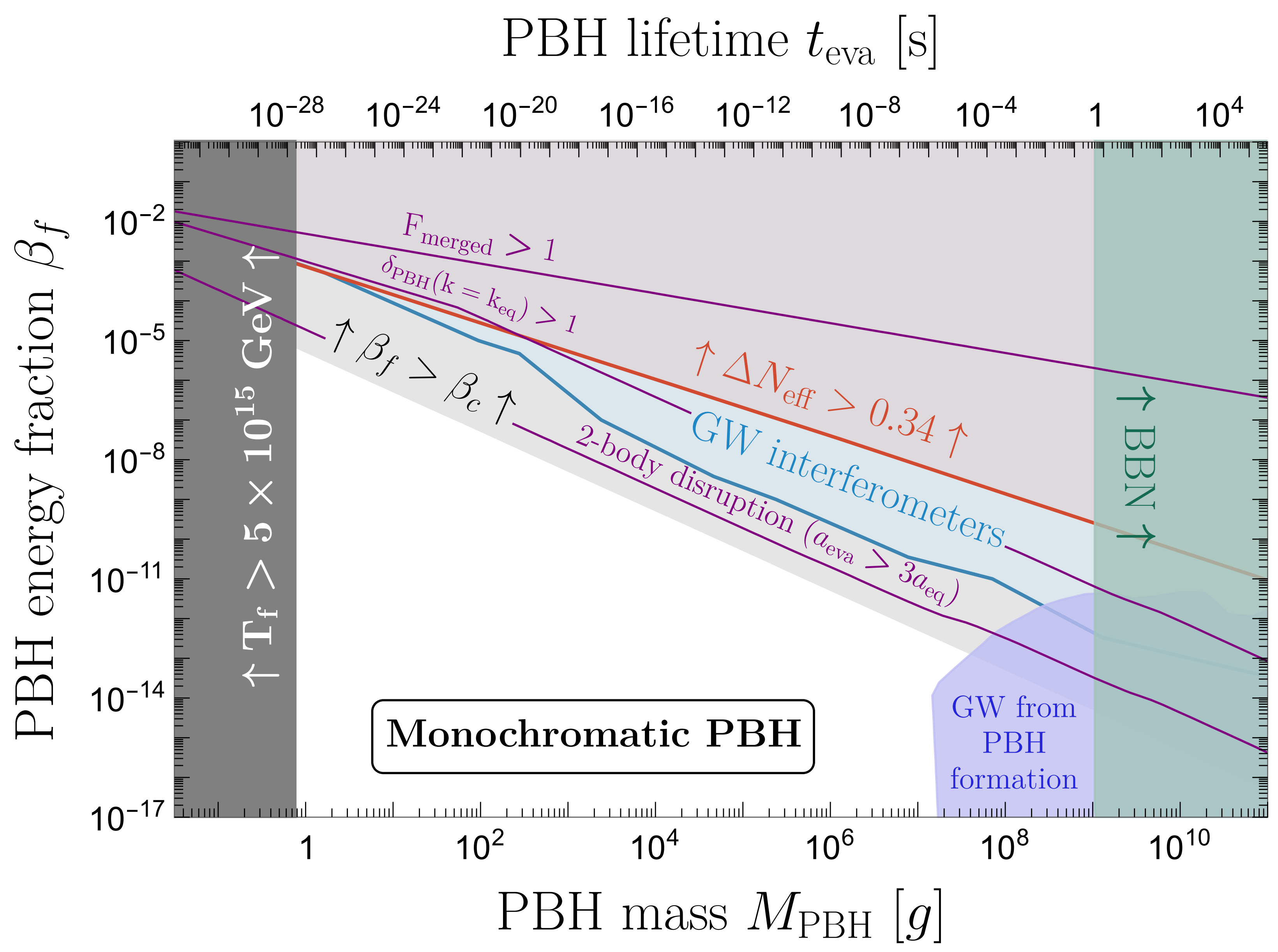
Formation

ADDITIONAL SOURCES

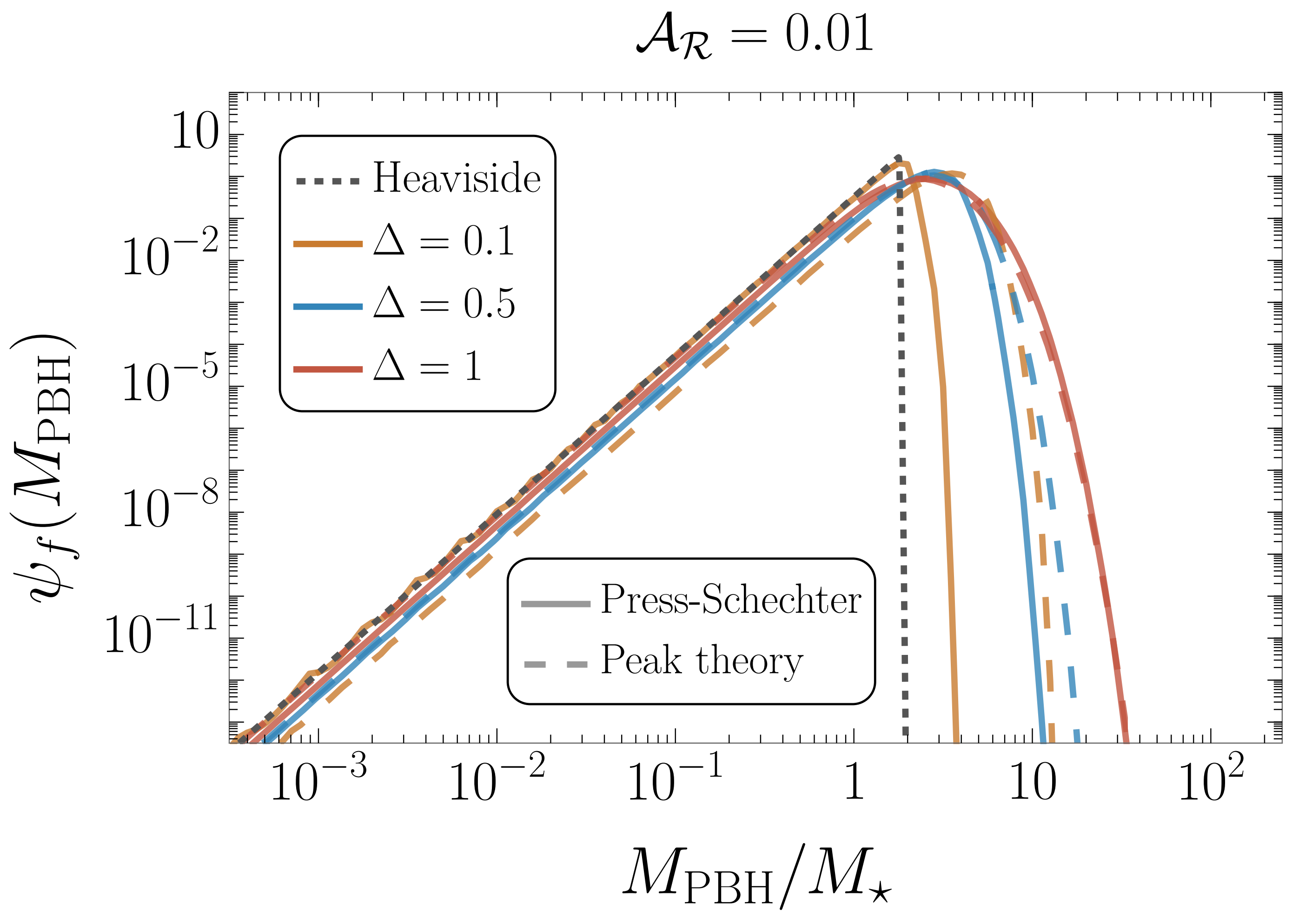


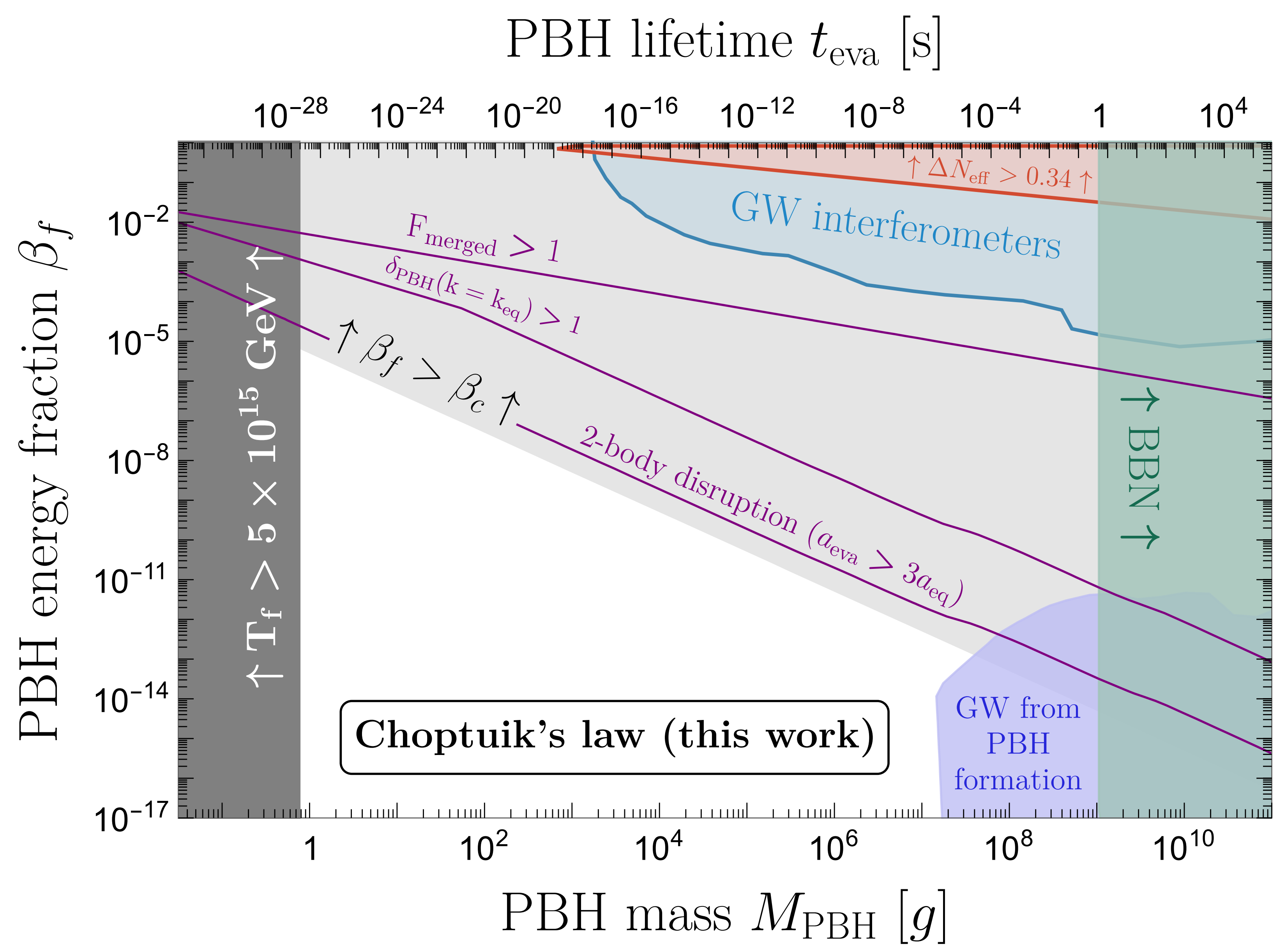
Monochromatic





Choptuik Scaling





BACK-UP

BACK-UP

Cosmological perturbation theory

$$\mathrm{d}s^2 = a^2(\eta)\Big[(1+2\Psi)\mathrm{d}\eta^2 - (1-2\Phi)\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j + h_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j\Big]$$

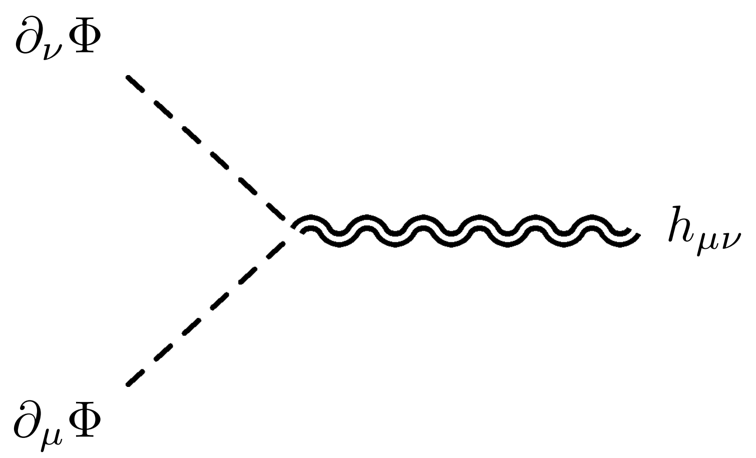
In Newtonian gauge and with out anisotropic stress ($\Phi = -\Psi$)

Switching to **synchronous gauge**

$$\mathrm{d}s^2 = a^2\left(\mathrm{d}\eta^2 - H_{ij}\mathrm{d}x^i\mathrm{d}x^j\right) \quad \text{with} \quad H_{ij} = \delta_{ij} + \hat{k}_i\hat{k}_j\gamma + \left(\hat{k}_i\hat{k}_j + \frac{1}{3}\delta_{ij}\right)\epsilon$$

Translating back

$$\Phi_k = \epsilon - \mathscr{H}\frac{(6\epsilon+\gamma)'}{2k^2}\,.$$



BACK-UP

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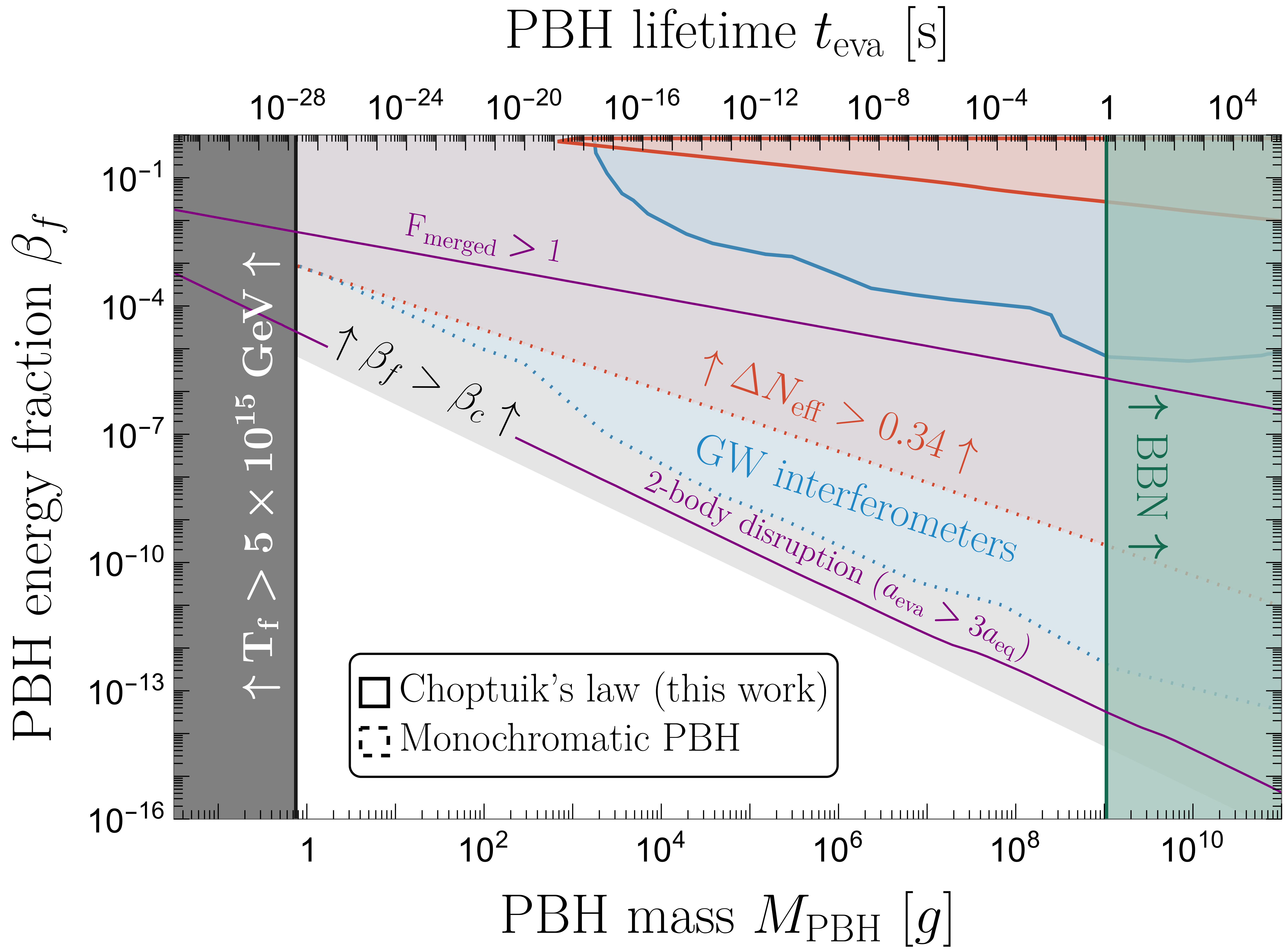
$$\boxed{\begin{aligned}\overline{\mathcal{P}_{h,RD}}(k,\eta,\bar{x}\gg 1) \simeq \frac{c_s^4 x_{\mathrm{eva}}^8}{\bar{x}^2}\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u\ (4v^2-(1+v^2-u^2)^2)^2\,\overline{\mathcal{I}^2_{\mathrm{osc}}}(\bar{x},u,v)\\ \times\,\mathcal{P}(uk)\mathcal{P}(vk)\mathcal{S}^2_{\mathrm{eva}}(uk)\mathcal{S}^2_{\mathrm{eva}}(vk)\mathcal{T}^2_{\mathrm{eva}}(uk)\mathcal{T}^2_{\mathrm{eva}}(vk),\end{aligned}}$$

$$\overline{\mathcal{I}^2_{\mathrm{osc,res}}}(\bar{x},u,v) = \frac{1}{32}\mathrm{Ci}(|1-c_s(u+v)x_{\mathrm{eva}}/2|)^2,$$

$$\overline{\mathcal{I}^2_{\mathrm{osc,LV}}}(\bar{x},u,v) = \frac{1}{8}\frac{k^2}{k^2_{\mathrm{eva}}}\,.$$

In the resonant case, one can treat $\mathrm{Ci}(|1-c_s(u+v)x_{\mathrm{eva}}/2|)^2$ as a delta distribution and reduce the double integral to a one-dimensional one for $s = u - v$.

BACK-UP



Binary formation threshold:

1. Cluster Formation
2. Major modification of mass distribution (elongation)

BACK-UP

Non-linearities

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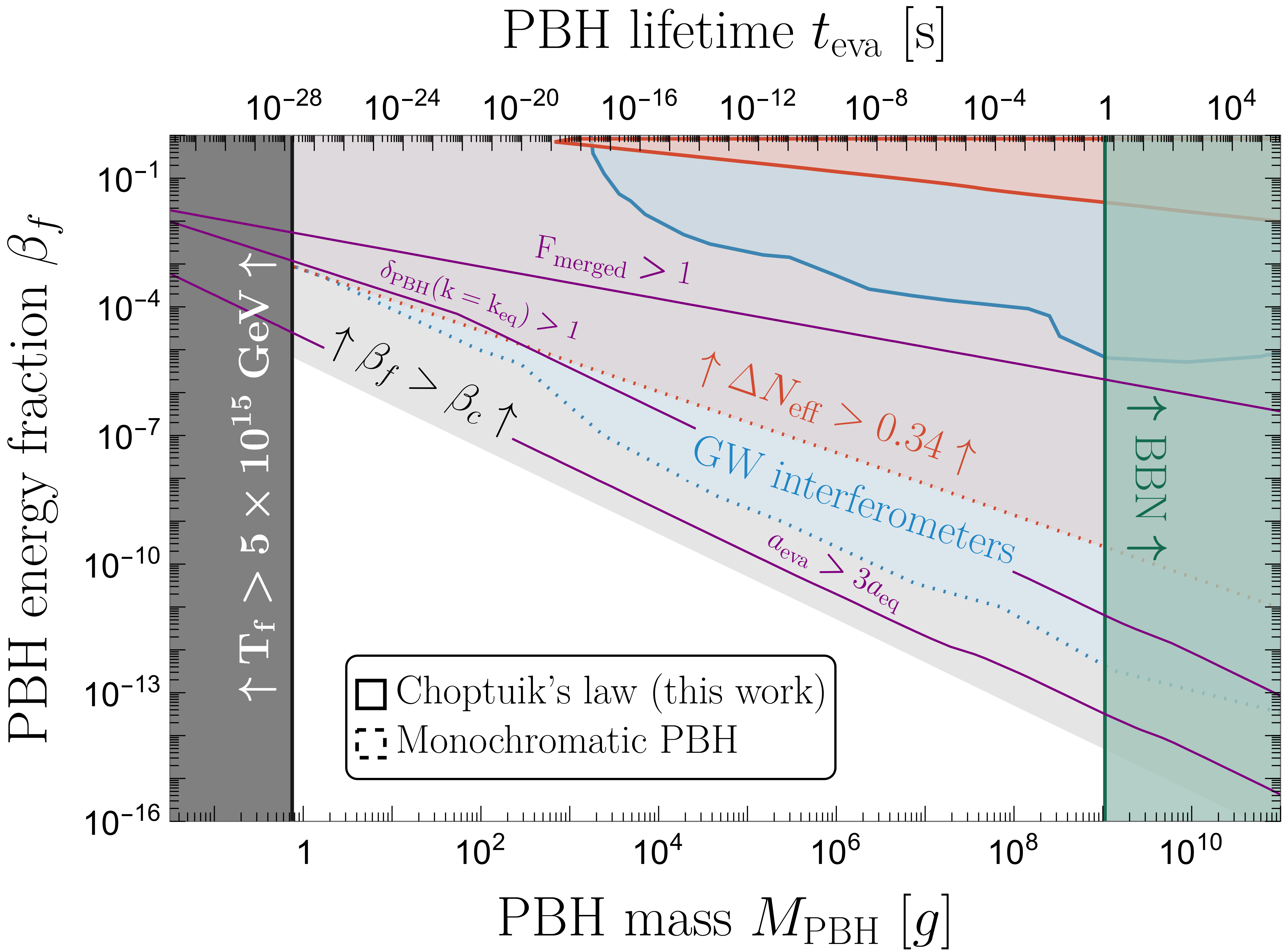

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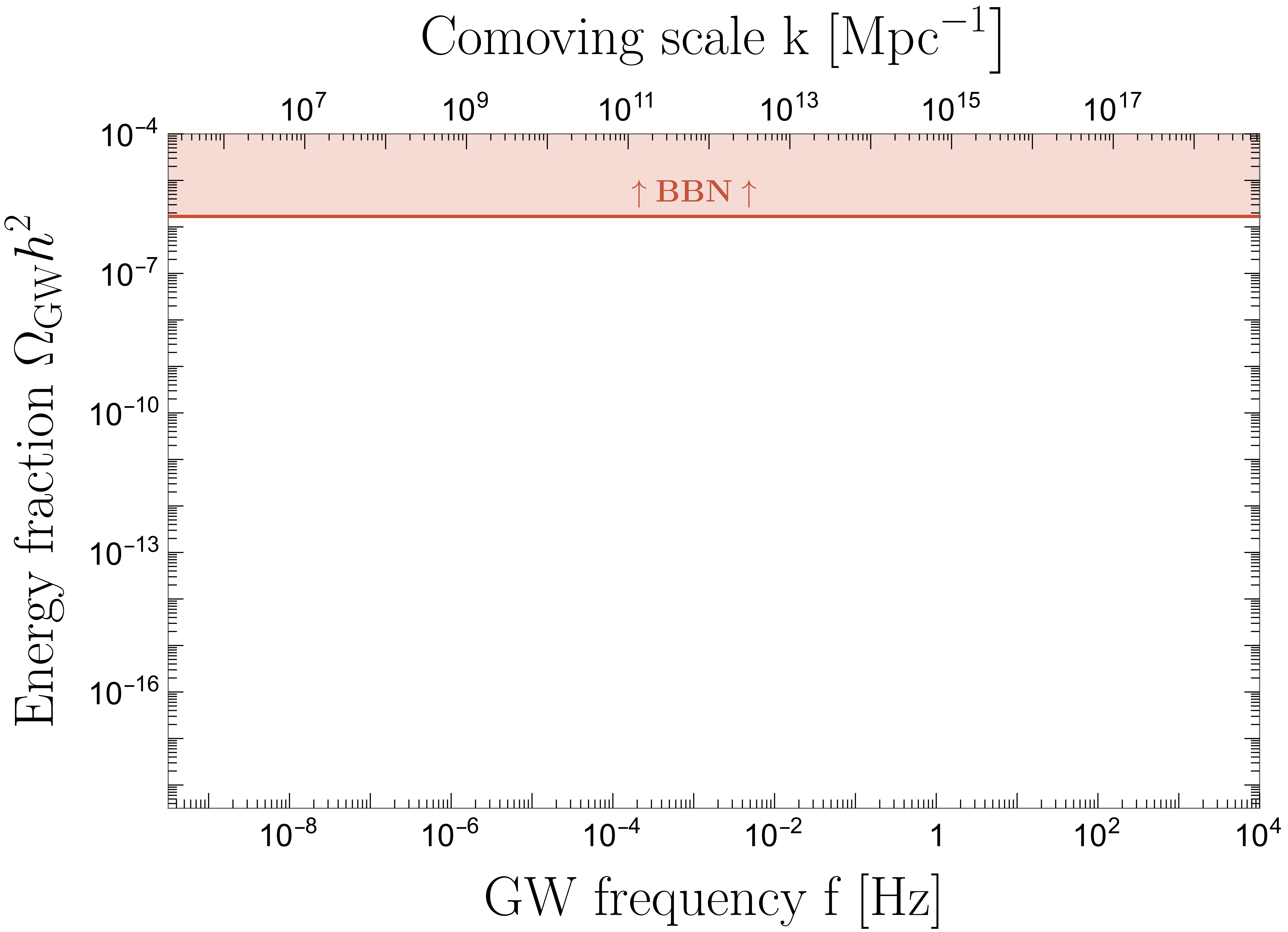
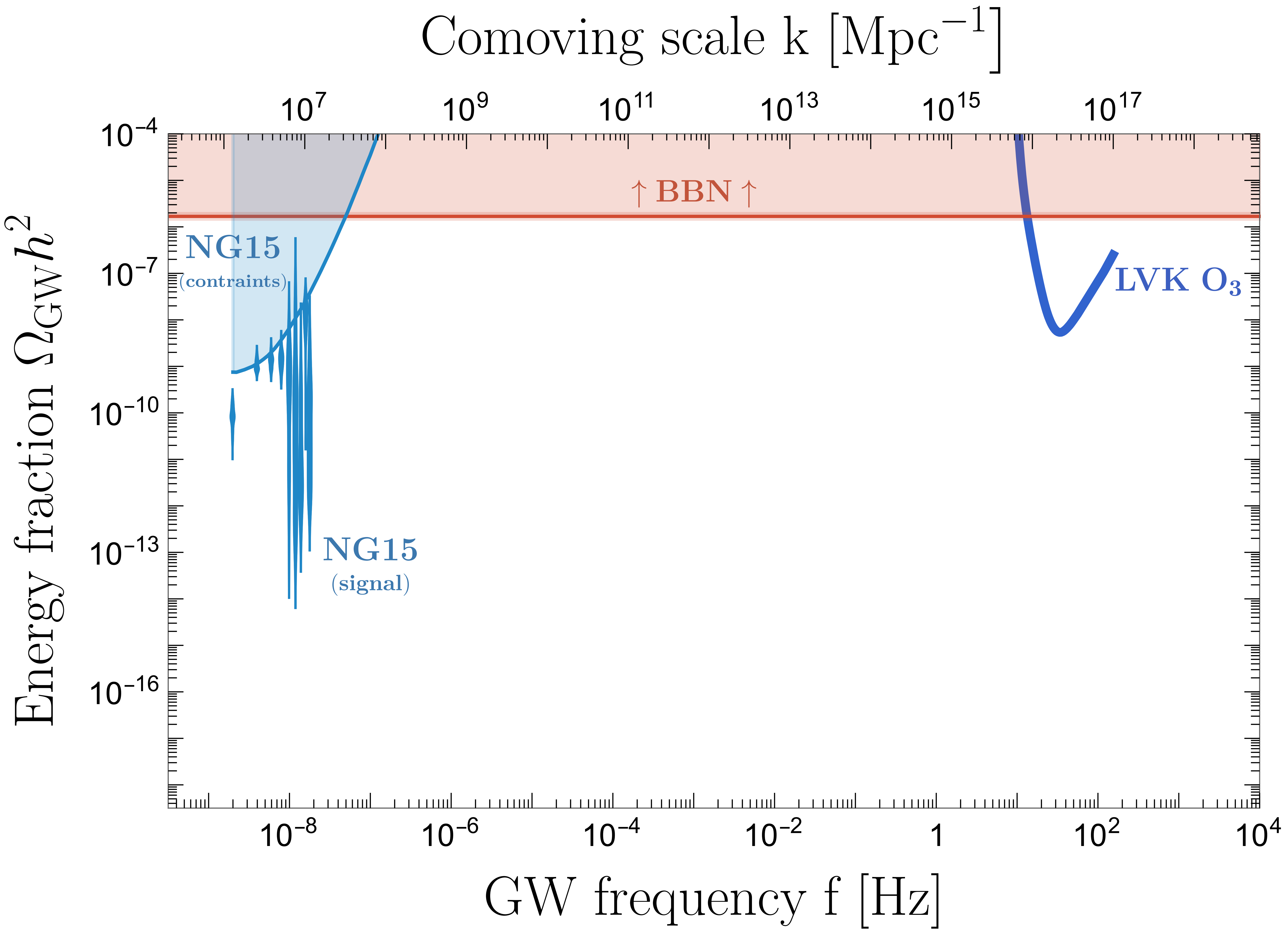


Comoving scale k $[\mathrm{Mpc}^{-1}]$
10^{7}
10^{9}
10^{11}
10^{13}
10^{15}
10^{17}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
↑ BBN ↑
Energy fraction $\Omega_{\mathrm{GW}} h^2$
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
10^{2}
10^{4}
GW frequency f [Hz]

BACK-UP



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Energy fraction $\Omega_{\mathrm{GW}} h^2$
↑ BBN ↑
NG15
(contraints)
NG15
(signal)
LVK O_3
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10^{-2}
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10^2
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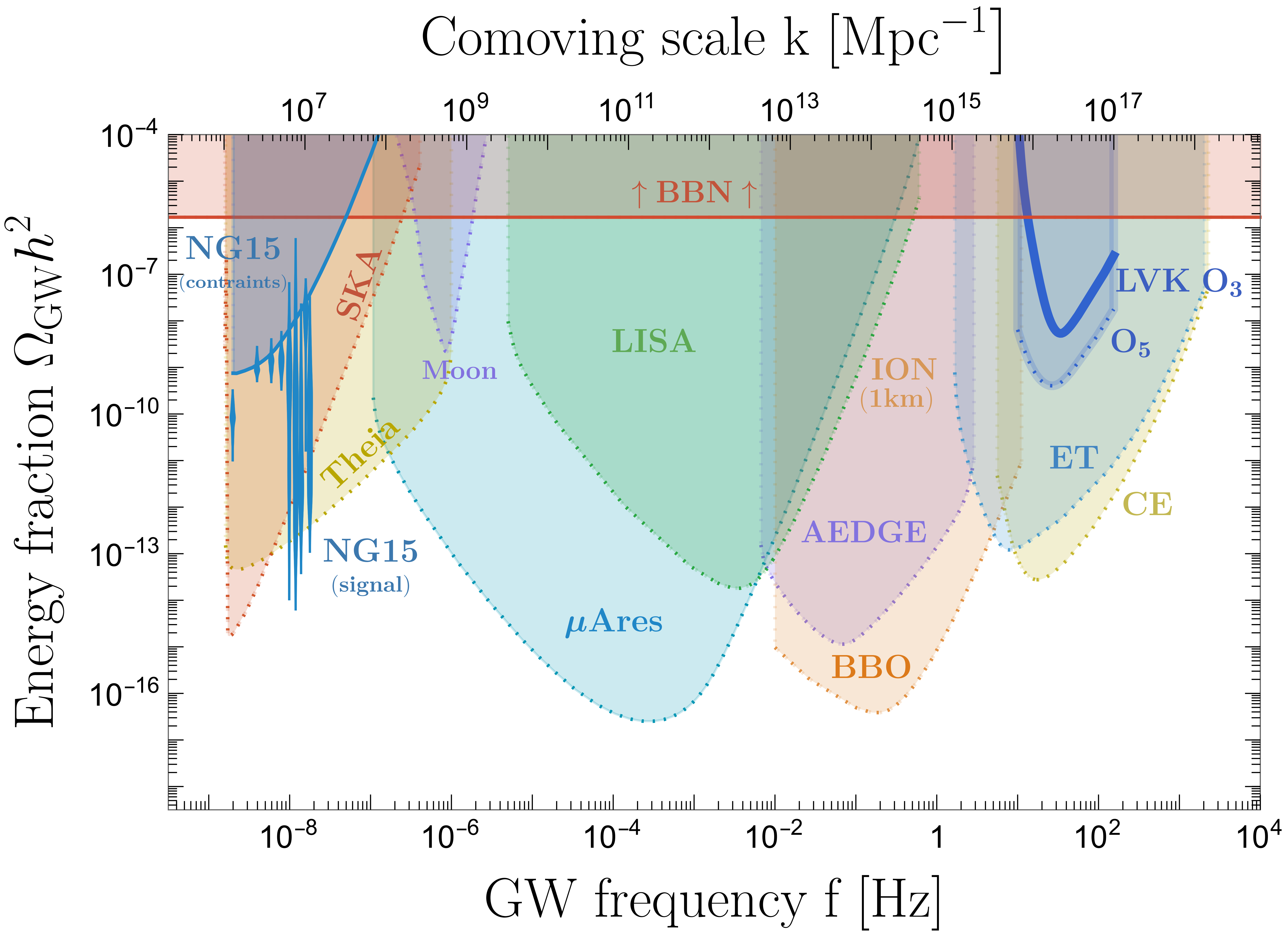
GW frequency f [Hz]

BACK-UP



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NG15
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SKA
LISA
Moon
ION
(1km)
LVK O_3
O_5
ET
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Theia
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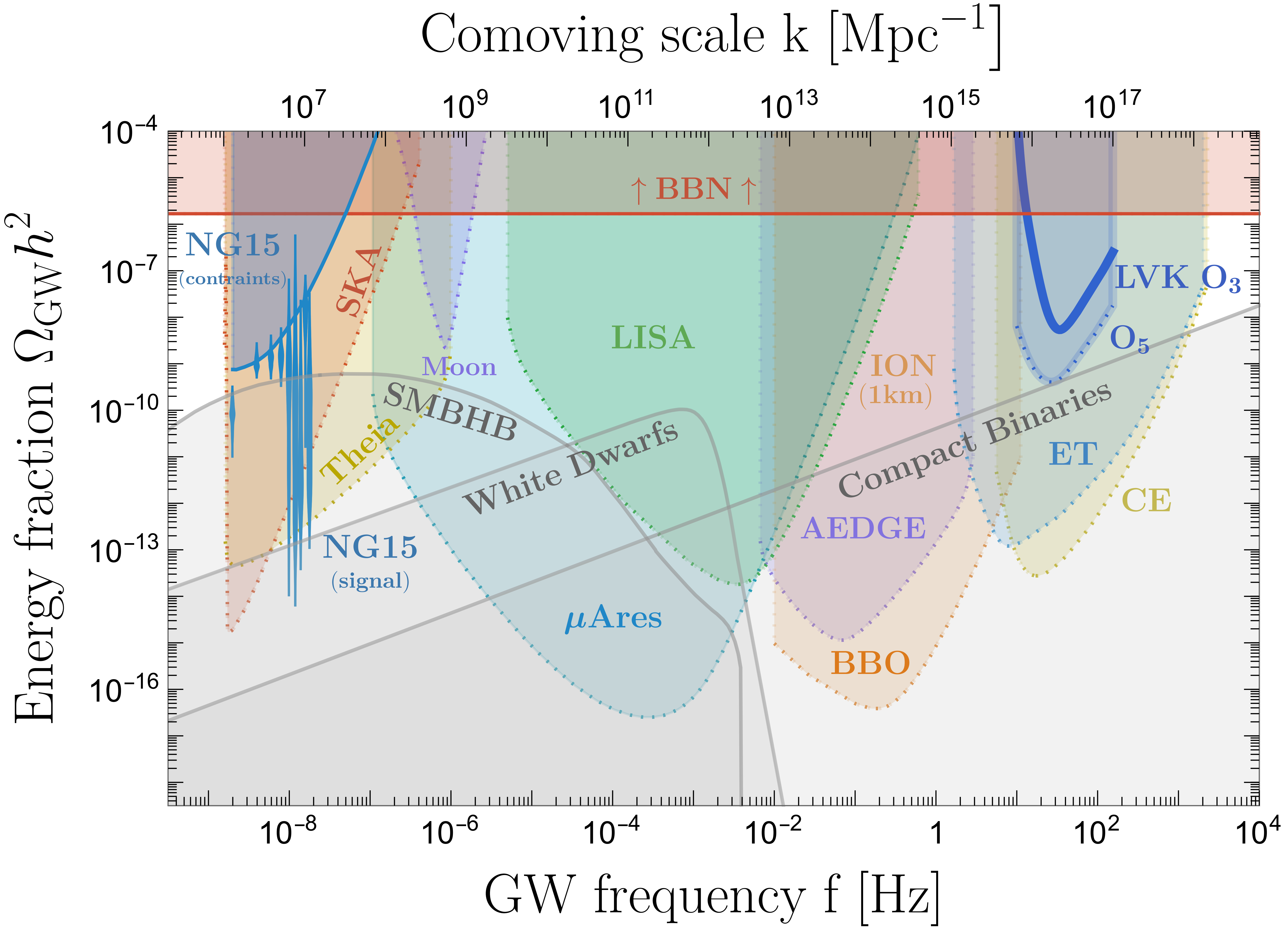
BACK-UP



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Compact Binaries
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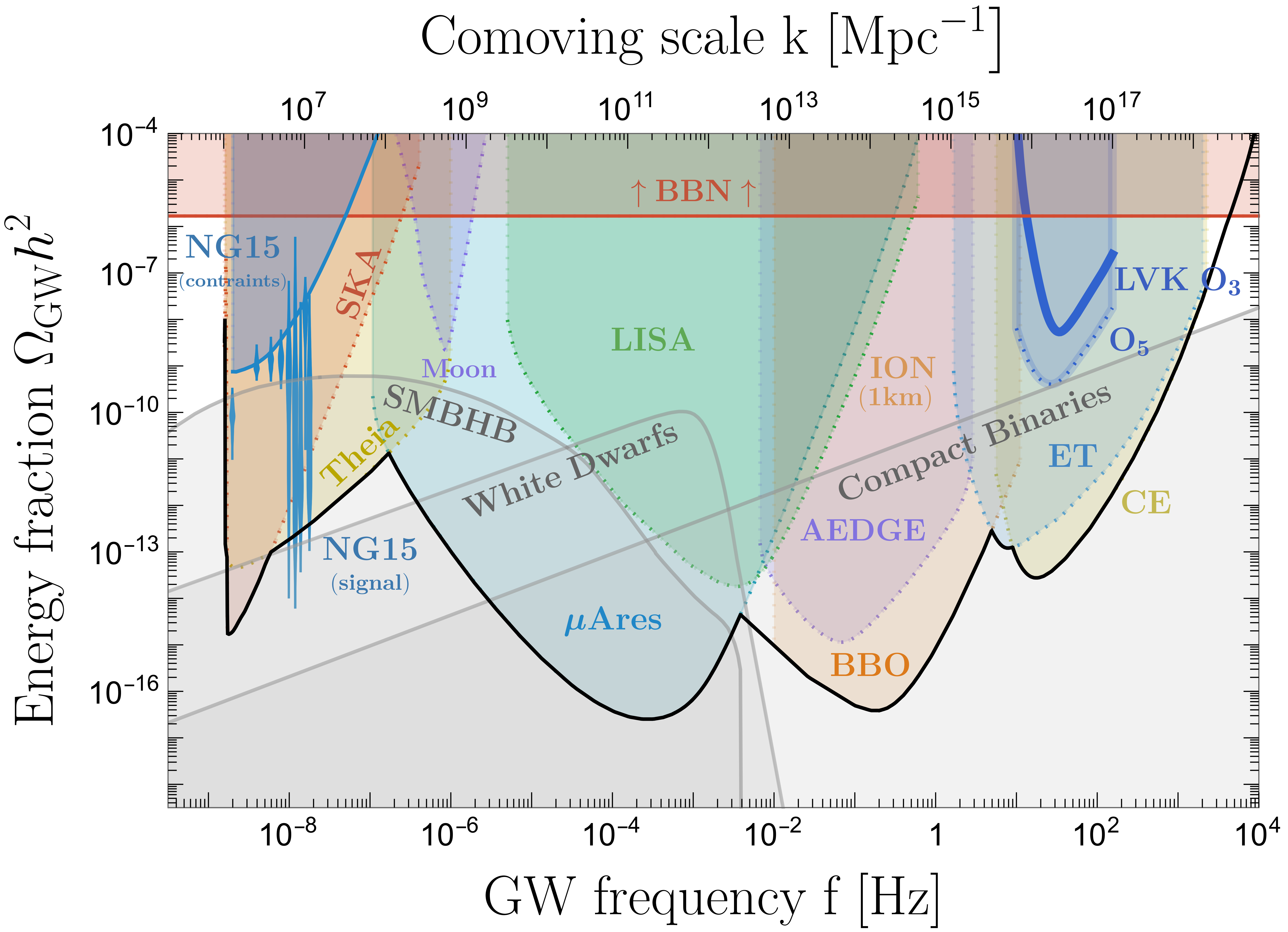
BACK-UP



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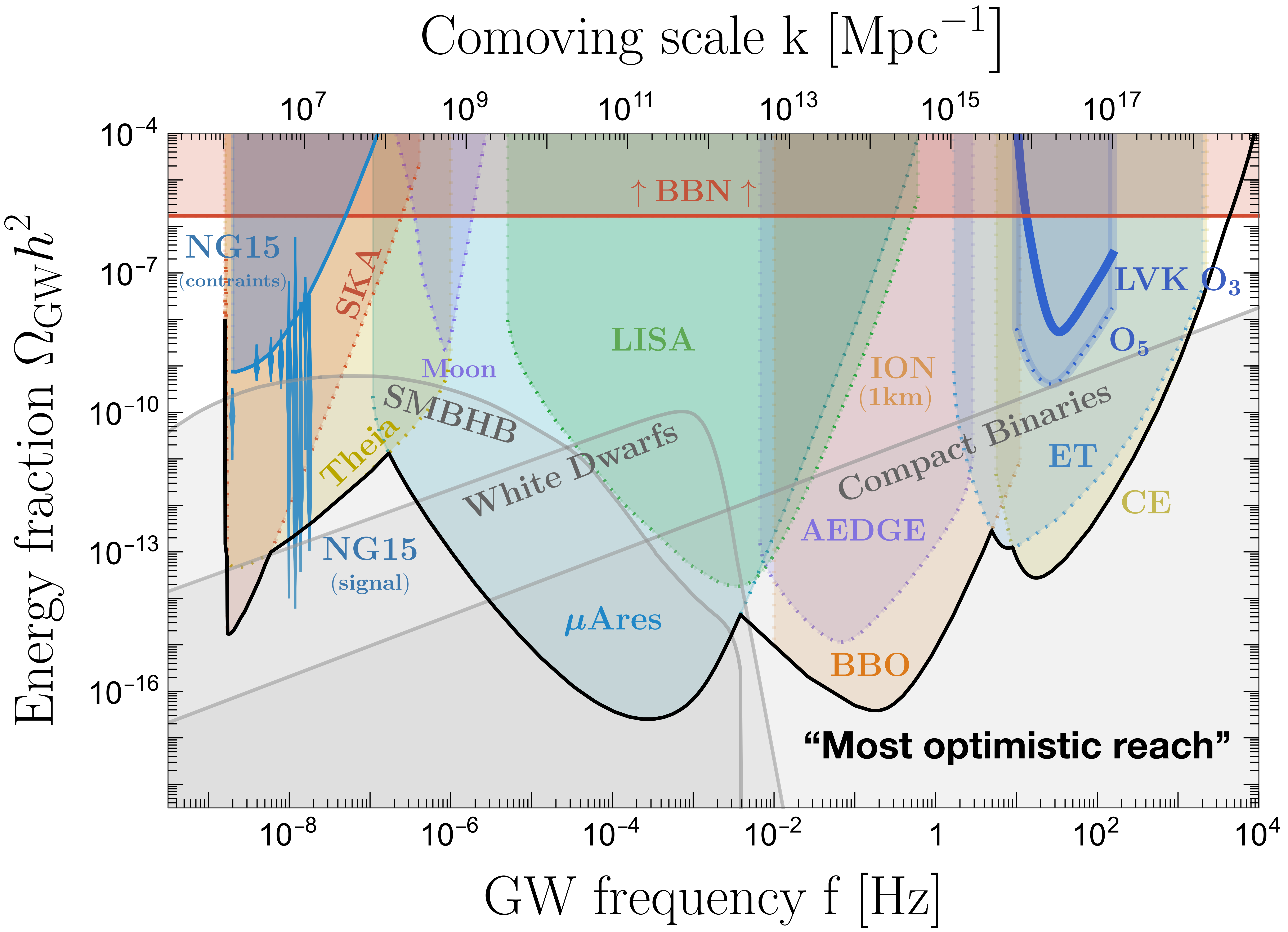
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"Most optimistic reach"
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GW frequency f [Hz]