The Supercool Audible Axion

DESY THEORY WORKSHOP 2025
SYNERGIES TOWARDS THE FUTURE STANDARD MODEL
Hamburg, 25 September 2025

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Based on work with Daniel Schmitt and Pedro Schwaller

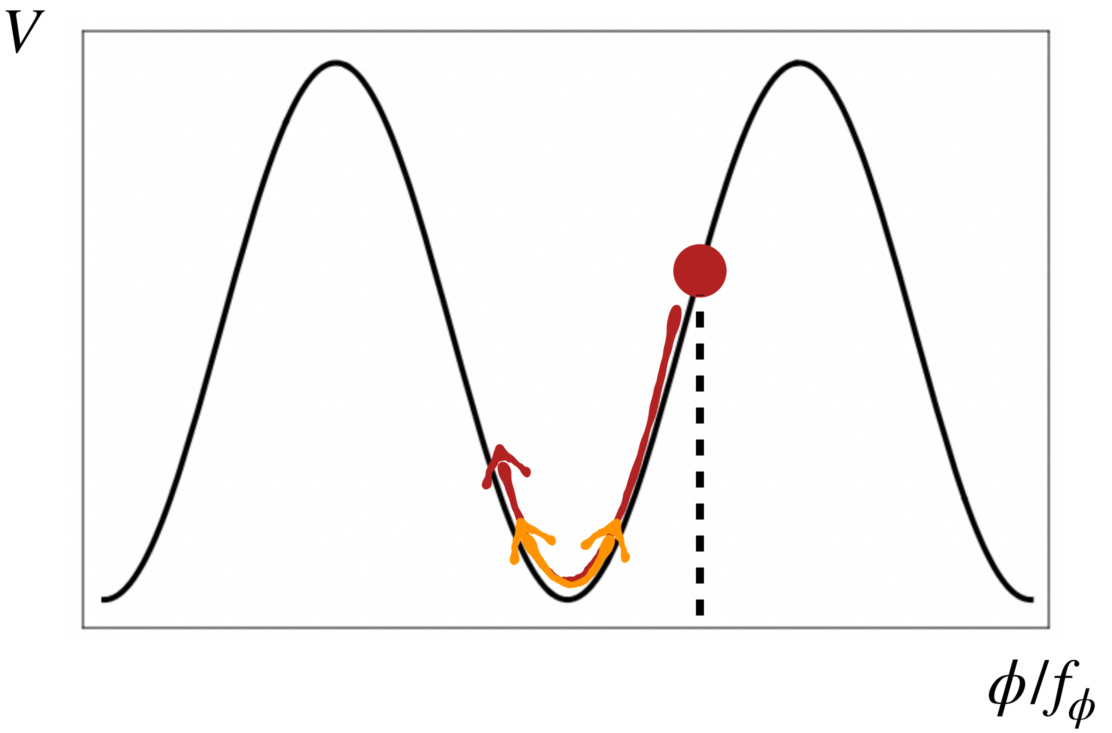
JHEP 09 (2025) 185 [ArXiv: 2504.05386]

Table of Content

The original mechanism

- ALP potential: $V(\phi) = m_\phi^2 f_\phi^2 \left(1 - \cos\frac{\phi}{f_\phi}\right)$
- EoM: $\phi'' + 2aH\phi' + a^2\frac{\partial V}{\partial \phi} = 0$
- Starts to role: $H \sim m_\phi$
- At oscillation: $T_{osc,aa} \sim \sqrt{m_\phi M_{\rm Pl}}$

$$\Omega_{\phi,osc} \sim \left(\frac{\theta f_\phi}{M_{\rm Pl}}\right)^2$$



non-exhaustive list:
[Machado, Ratzinger, Schwaller, Stefanek 1811.01950, 1912.01007]
[Ratzinger, Schwaller, Stefanek 2012.11584]
[Banerjee, Madge, Perez, Ratzinger, Schwaller 2105.12135]
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- EoM: $\phi'' + 2aH\phi' + a^2\frac{\partial V}{\partial \phi} = \boxed{\frac{\alpha}{f_\phi} a^2 \vec{E}\cdot\vec{B}}$

Couple to dark gauge boson

$$\mathcal{L} \supset -\frac{\alpha}{4f_\phi}\phi X_{\mu\nu}\tilde{X}^{\mu\nu}$$

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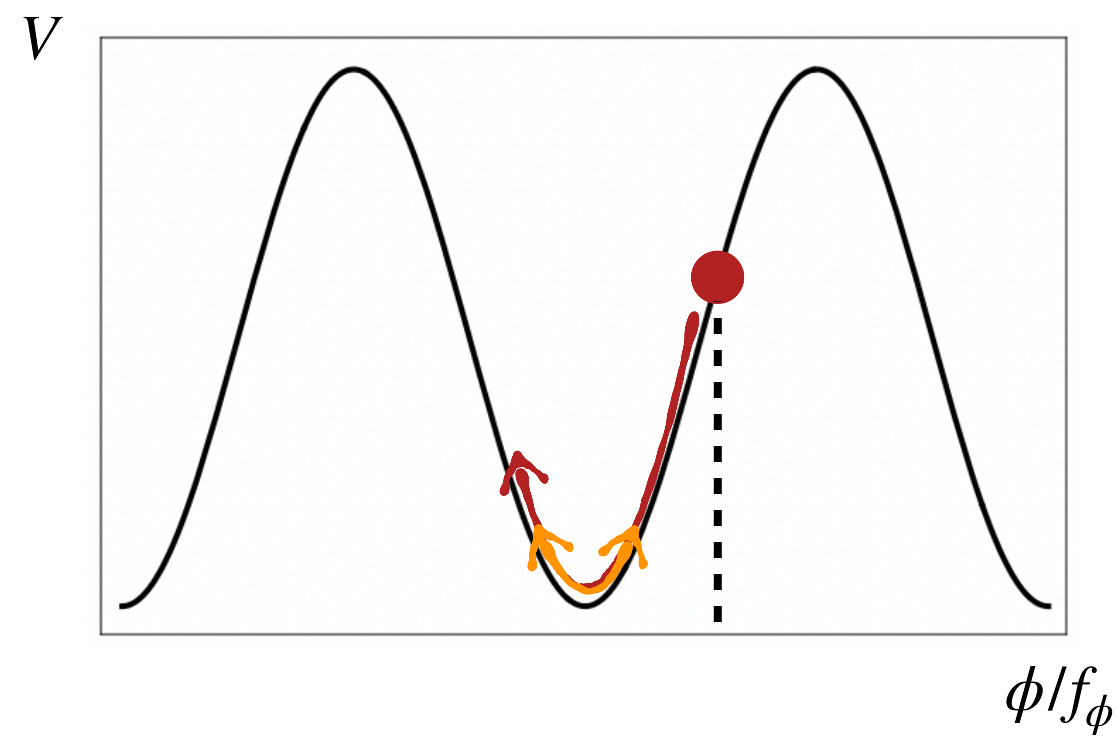

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1. The standard audible axion

[Hook, Marques-Tavares 1607.01786]

Gravitational waves from tachyonic instability

- **Photon modes:** $v''_\pm(k,\tau) + \underbrace{\omega^2_\pm(k,\tau)}_{k^2 \mp k\frac{\alpha}{f_\phi}\phi'(\tau)} v_\pm(k,\tau) = 0 \;\rightarrow\; v \propto \exp(-i\omega_\pm \tau)$

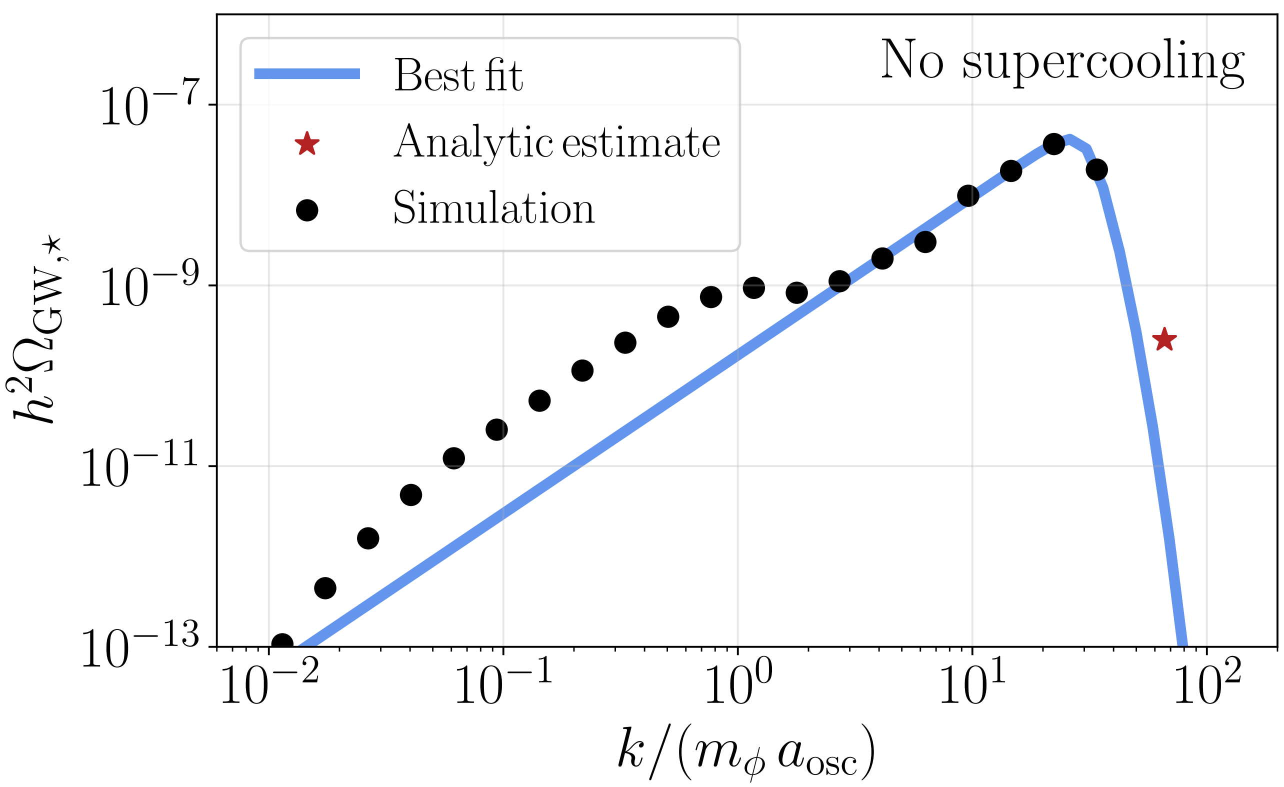
with frequency $k^2 \mp k\frac{\alpha}{f_\phi}\phi'(\tau)$

Unstable modes:

$$0 < k < \frac{\alpha}{f_\phi}|\phi'| \;\rightarrow\; \omega^2_\pm < 0$$

$$\Omega_{\rm GW} = c_{eff}^2 \Omega^2_{\phi,\star}\left(\frac{H_\star a_\star}{2\tilde{k}_\star}\right)^2 \sim \left(\frac{f_\phi}{M_{\rm Pl}}\right)^4$$

$m_\phi = 10^{-11}\,\mathrm{GeV},\ f_\phi = 10^{17}\,\mathrm{GeV},\ \alpha = 55,\ \theta = 1.2$



Motivation and effect of delaying oscillation

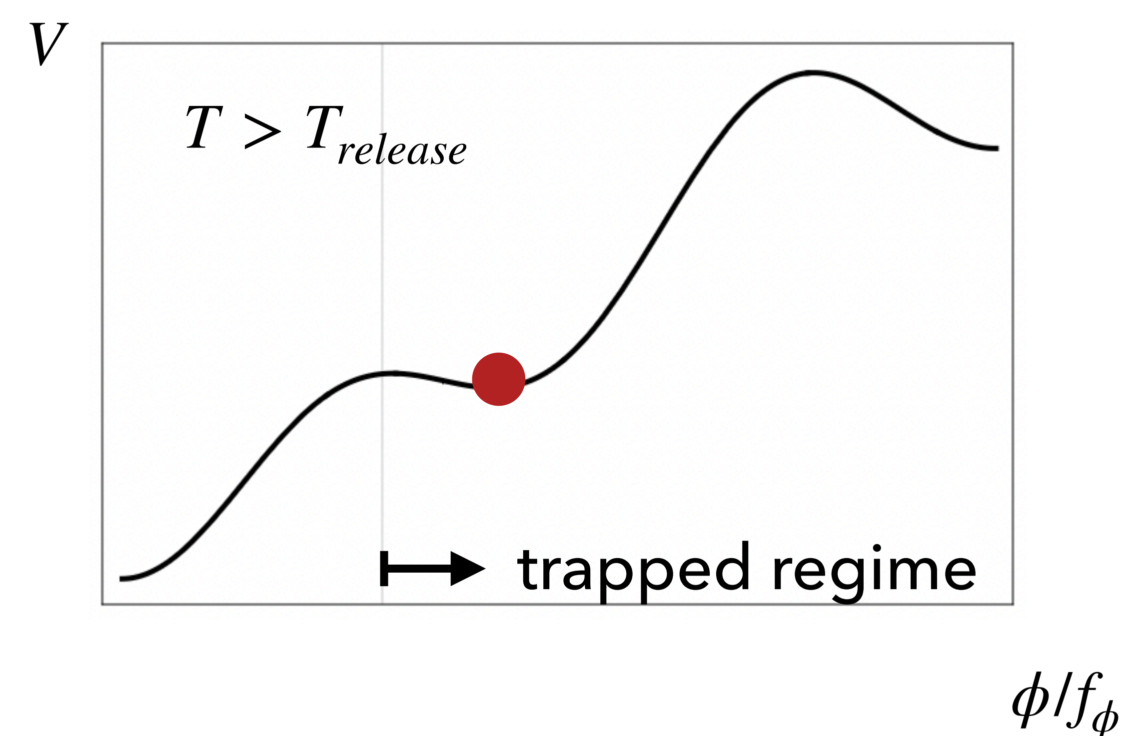
- Sizable GWs: need large coupling + large decay constant
- Delaying the evolution can help!
  - e.g. Trapped misalignment [Higaki, Jeong, Kitajima, Takahashi 1603.02090] [Di Luzio, Sørensen 2408.04623] + refs. therein
- Parametrize the supercooling:

$$r_{sc} = \frac{T_{osc}}{T_{osc,aa}}$$

T_{osc}: supercooled temperature

$T_{osc,aa}$: temp. in original audible axion

$$\Omega_{\phi,osc} = r_{sc}^{-4}\Omega_{\phi,osc}^{aa}\frac{g^{osc,aa}}{g^{osc}}$$



- Effect on the growth time between oscillation and GW emission:

$$\frac{a_\star}{a_{osc}} = 1 + \frac{\pi}{\alpha\theta}r_{sc}^2 \ln\left(\frac{128\pi^2}{\alpha^4\theta^2}\frac{f_\phi^2}{m_\phi^2}\right)$$

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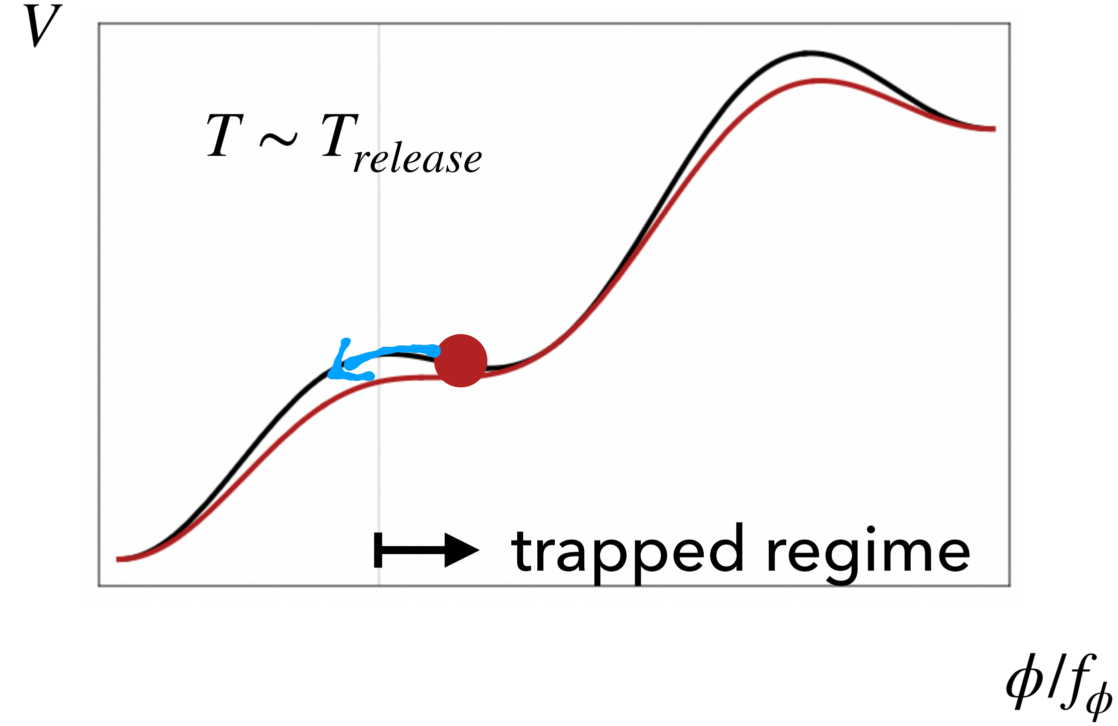

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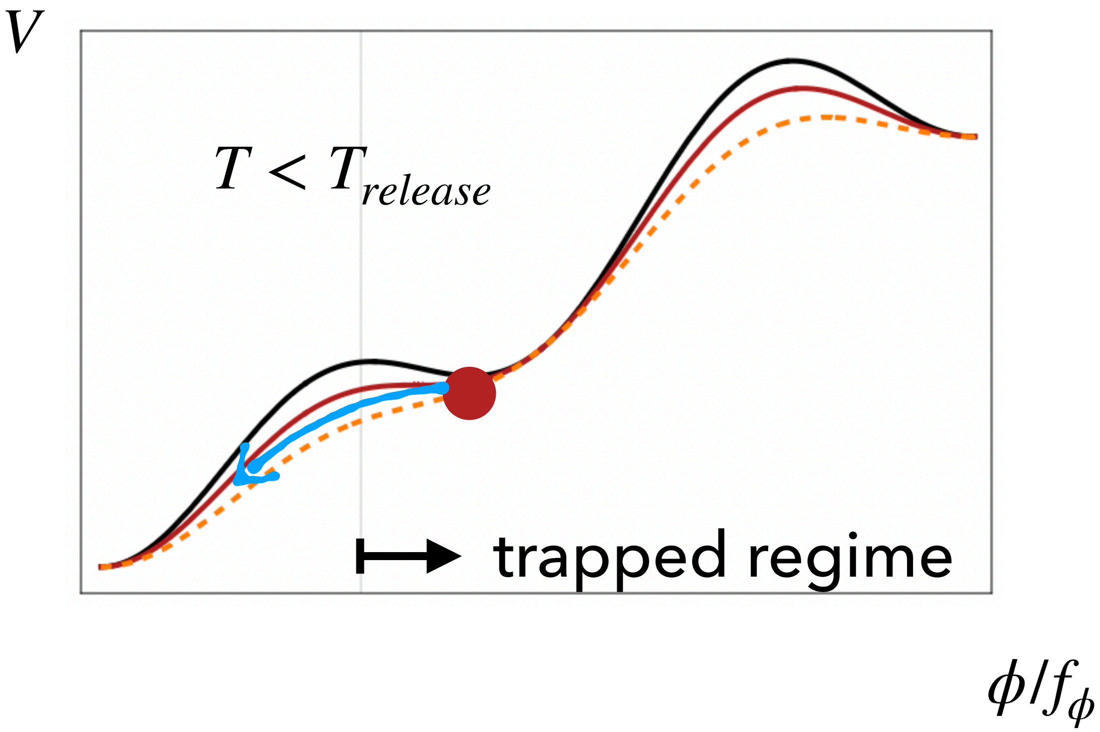
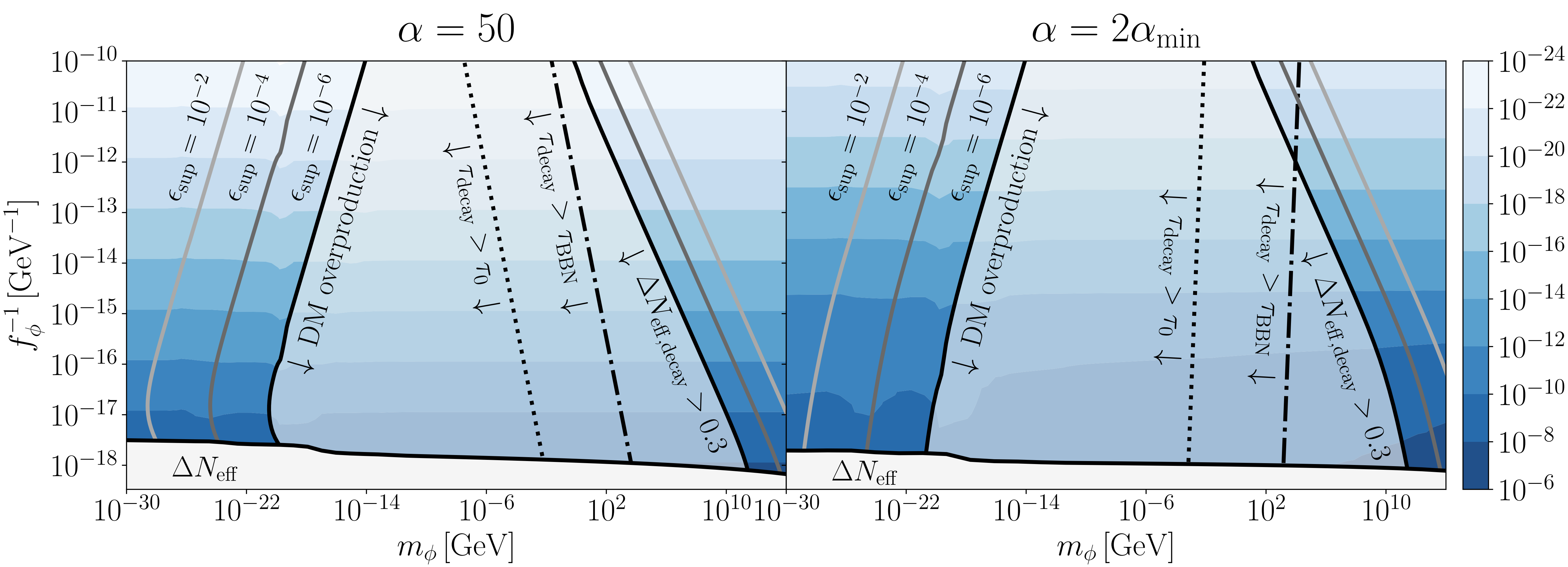


3. Dark photon scenario

How much supercooling for the dark photon case?

- More supercooling means less growth time means smaller α possible!
- ΔN_{eff} leads to maximum length of supercooling = lower limit on r_{sc}:

$$r_{sc}^{min} \sim \left(\frac{a_\star}{a_{osc}}\right)^{1/4} \left(\frac{\theta f_\phi}{M_{\rm Pl}}\right)^{1/2}$$



Gravitational waves (I)

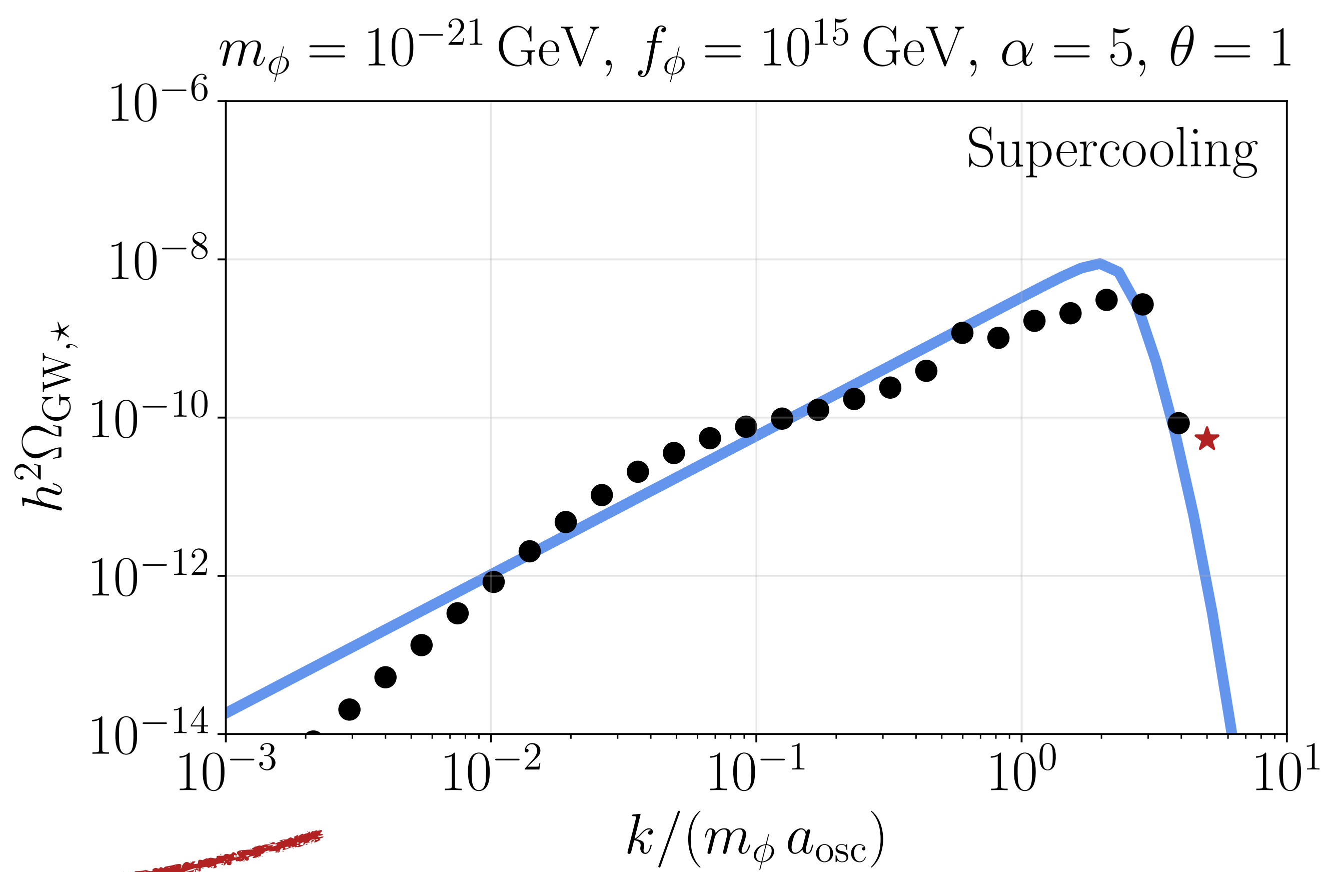
- Generally, for the peak of the GW spectrum:

$$\Omega_{GW,\star} = c_{eff}^2 \Omega_{\phi,\star}^2 \left(\frac{H_\star a_\star}{2\tilde{k}_\star} \right)^2 \qquad f_{GW,\star} = 2\frac{\tilde{k}_\star}{a_\star}$$

- We find:

$$\Omega_{\text{GW}} \sim \left(\frac{f_\phi}{M_{\text{Pl}}} \right)^2 \frac{1}{\alpha^2}$$

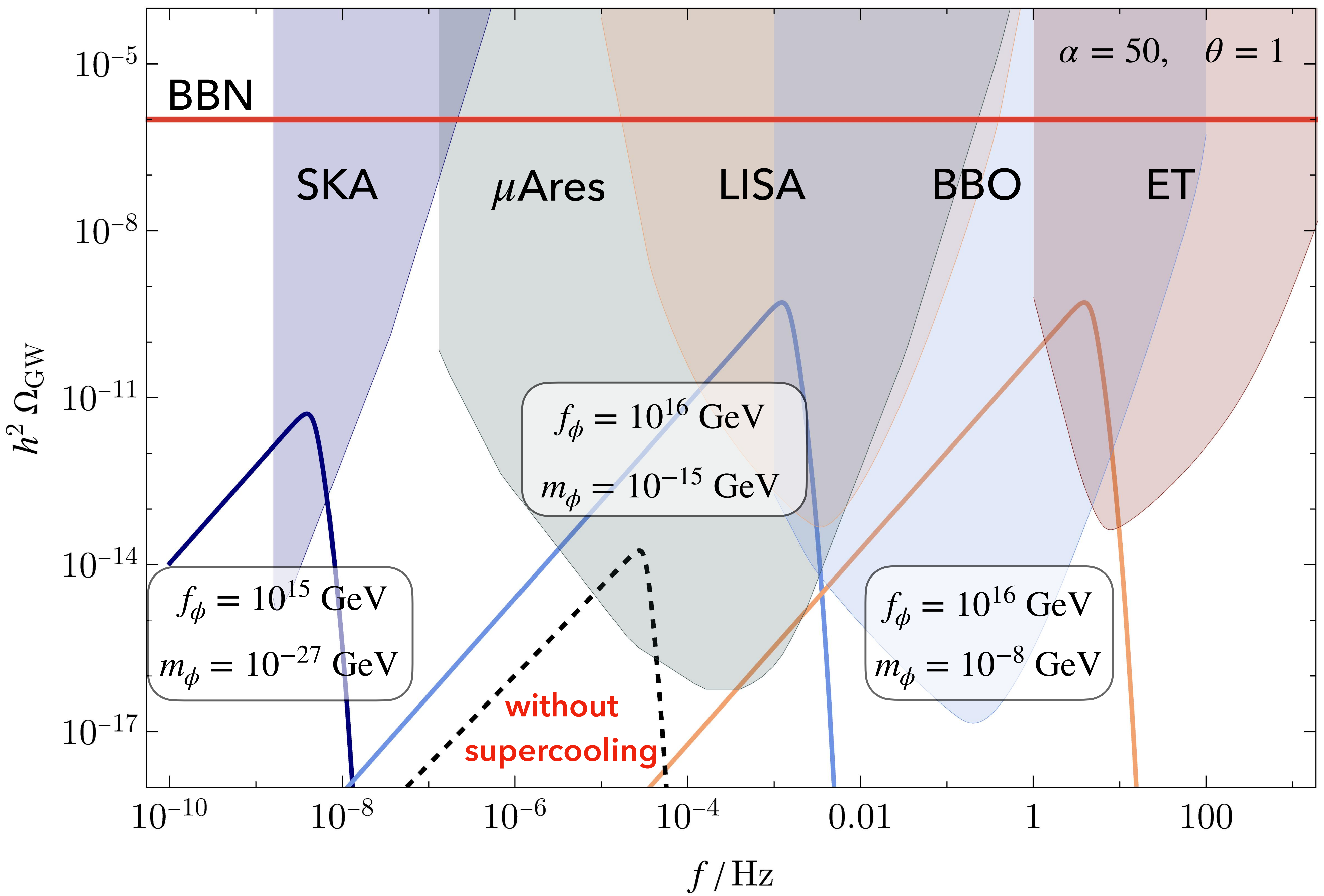
Linear numeric simulation



Extract fit parameters for exact result

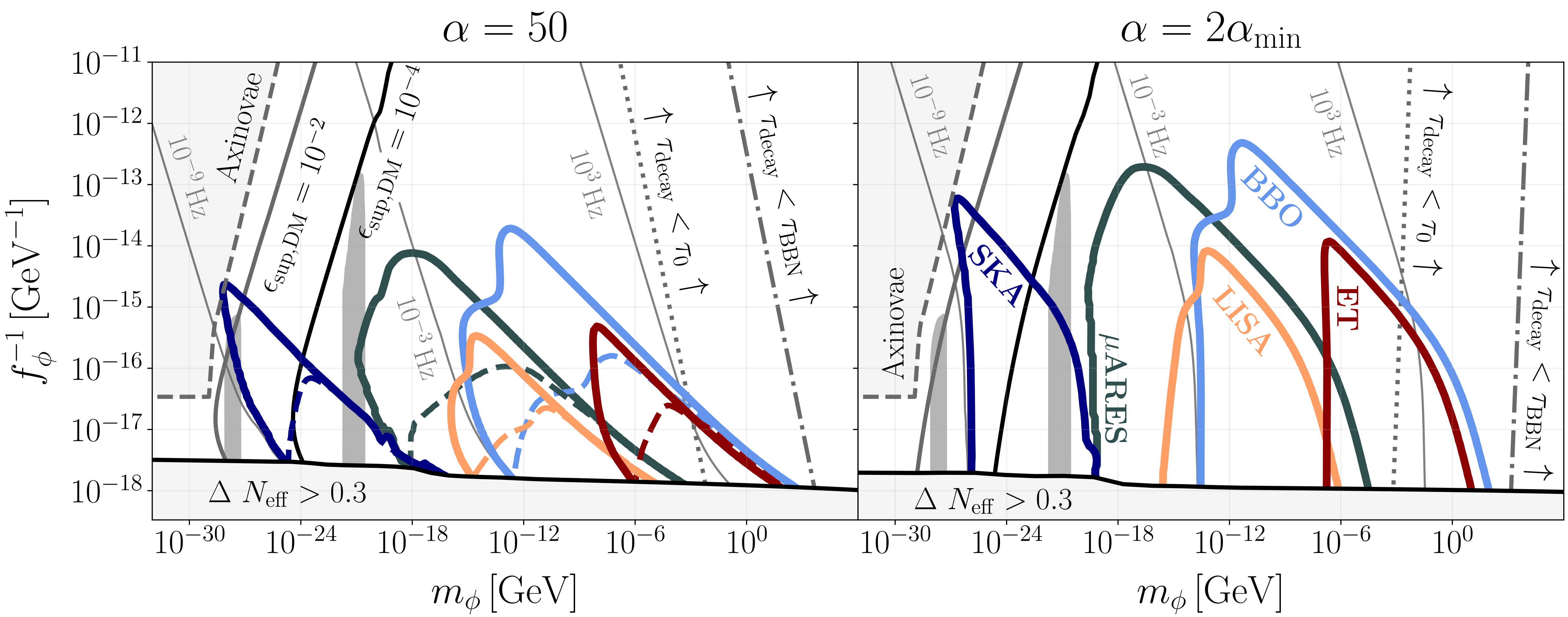
3. Dark photon scenario

Gravitational waves (II)



3. Dark photon scenario

Gravitational waves (III)



3. Dark photon scenario

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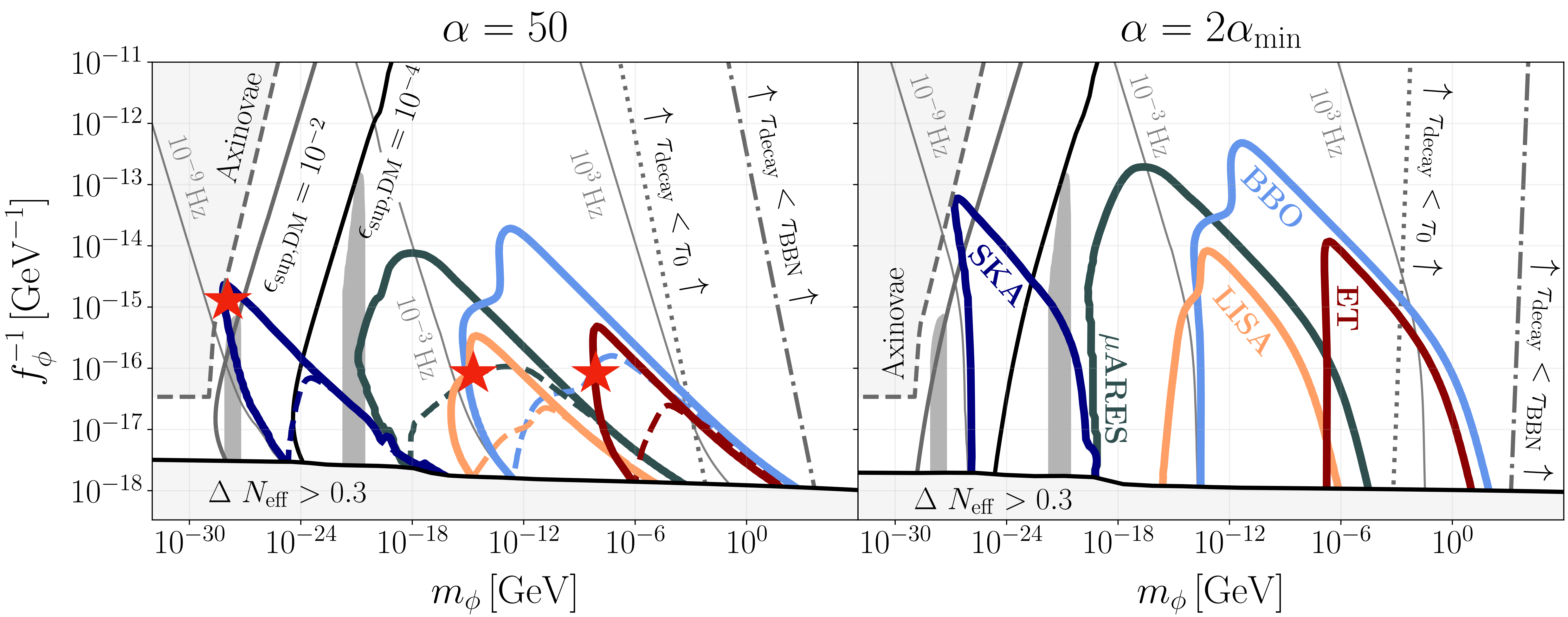

Overcoming the suppression

- A bit more complicated…

$$\omega^2 - k^2 \mp k\frac{\alpha}{f}\phi' = \frac{a^2 m_D^2}{2}\left(\frac{\omega}{2k}\ln\left(\frac{\omega+k}{\omega-k}\right) - \frac{\omega^3}{2k^3}\ln\left(\frac{\omega+k}{\omega-k}\right) + \frac{\omega^2}{k^2}\right)$$

Debye mass: $m_D \sim eT$

➞ peak frequency suppressed with $(eT)^2$

➞ need supercooling to open band:

$$r_{sc} \sim (m_\phi / M_{\rm Pl})^{1/2}$$

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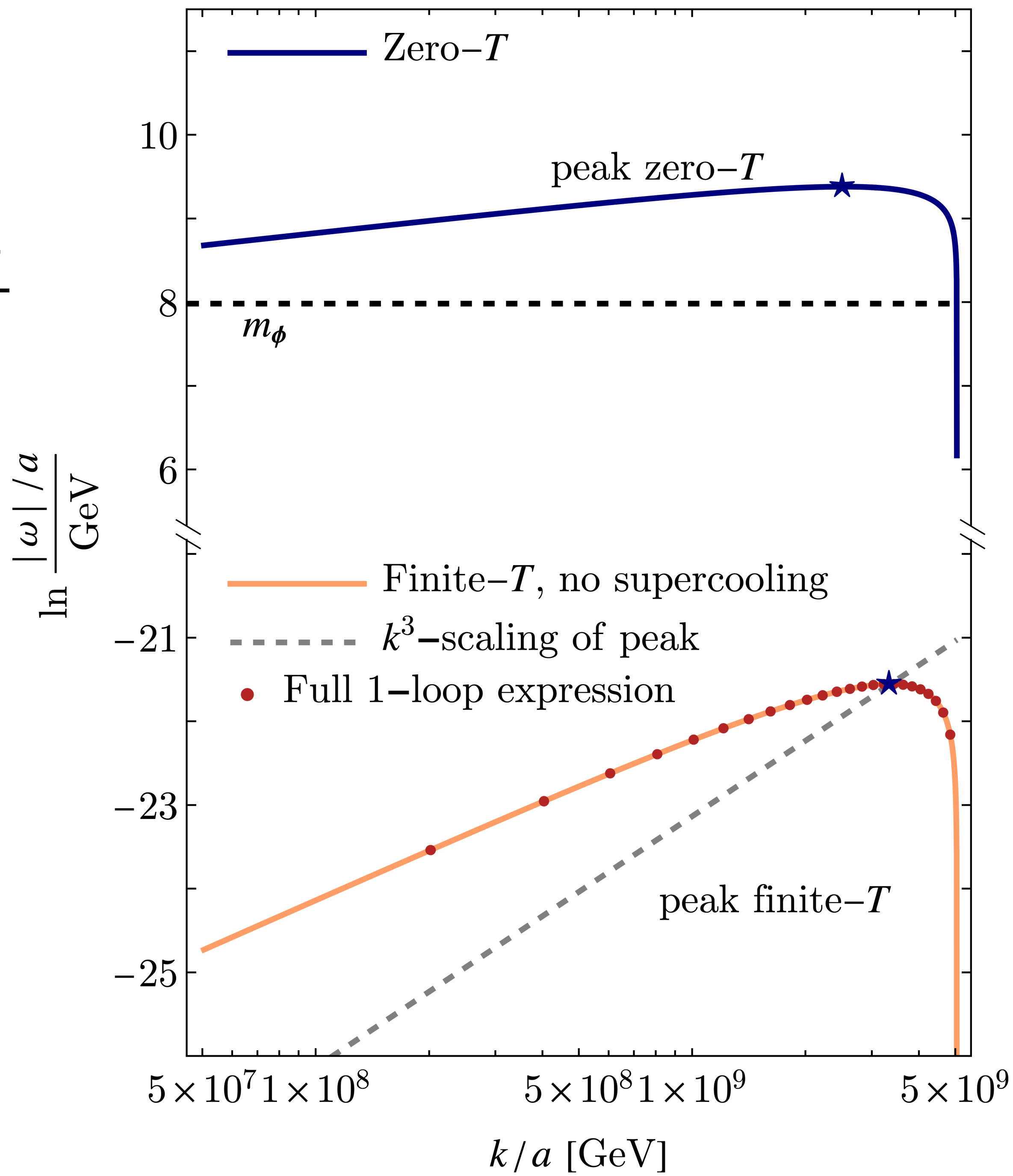

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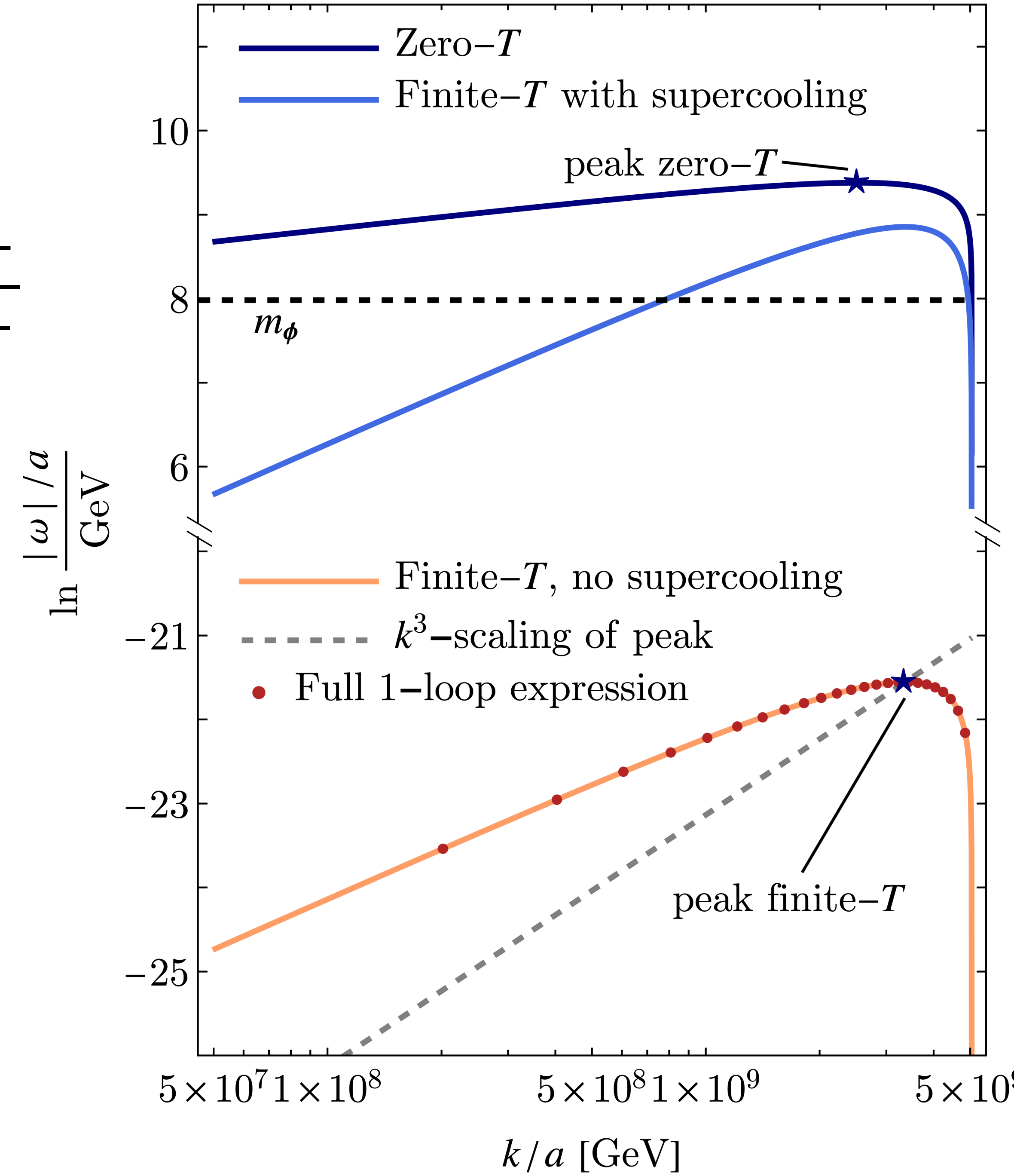


4. The SM photon scenario

Cosmology

- Extra degrees of freedom play no role
- Additional **thermal inflation** in most of parameter space (before photon production)
- **Schwinger pair production** limits the energy we can transfer to photons:

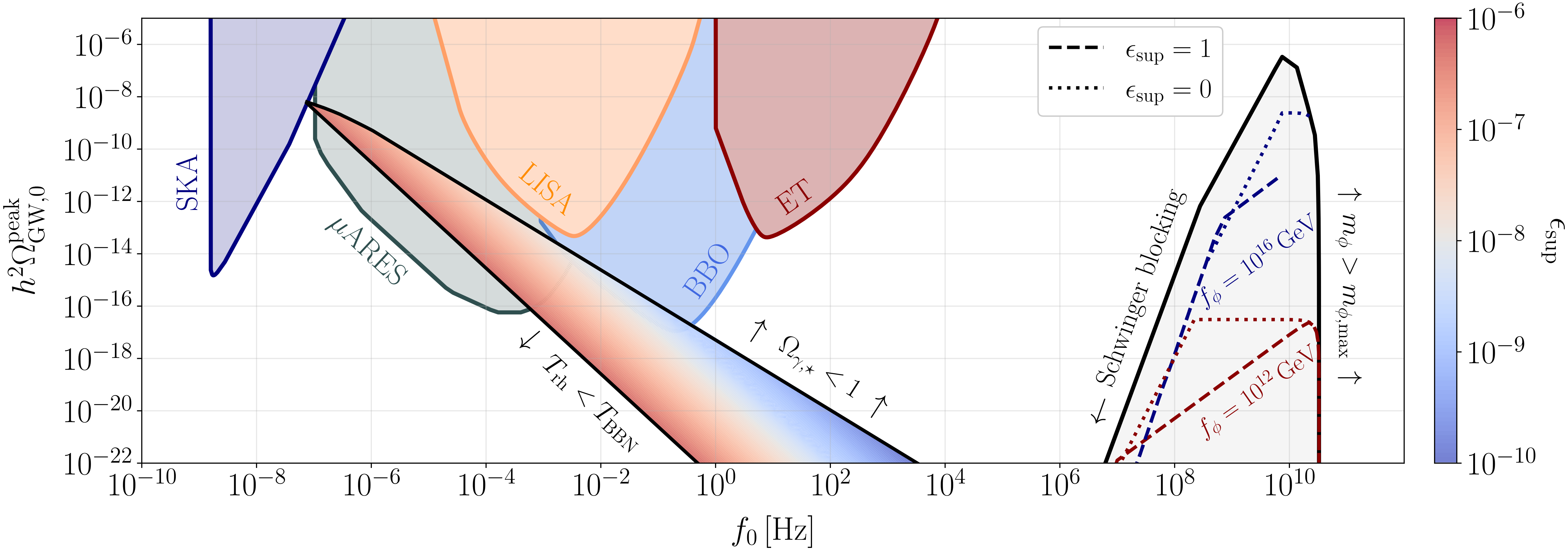
$$E^2 + B^2 - \zeta EB + \frac{eQ}{2}\frac{E}{H}J_{ind} = 0$$

$$J_{ind} \propto \exp\left(-\frac{\pi m_e^2}{eE}\right)$$

  - Suppressed by electron mass for small ALP masses
  - ALP domination once production unsuppressed
  - Suppressed by thermal electron mass for large masses

[Schwinger 1951]
[Domcke, Ema, Mukaida 1910.01205] + *many more*

Gravitational waves



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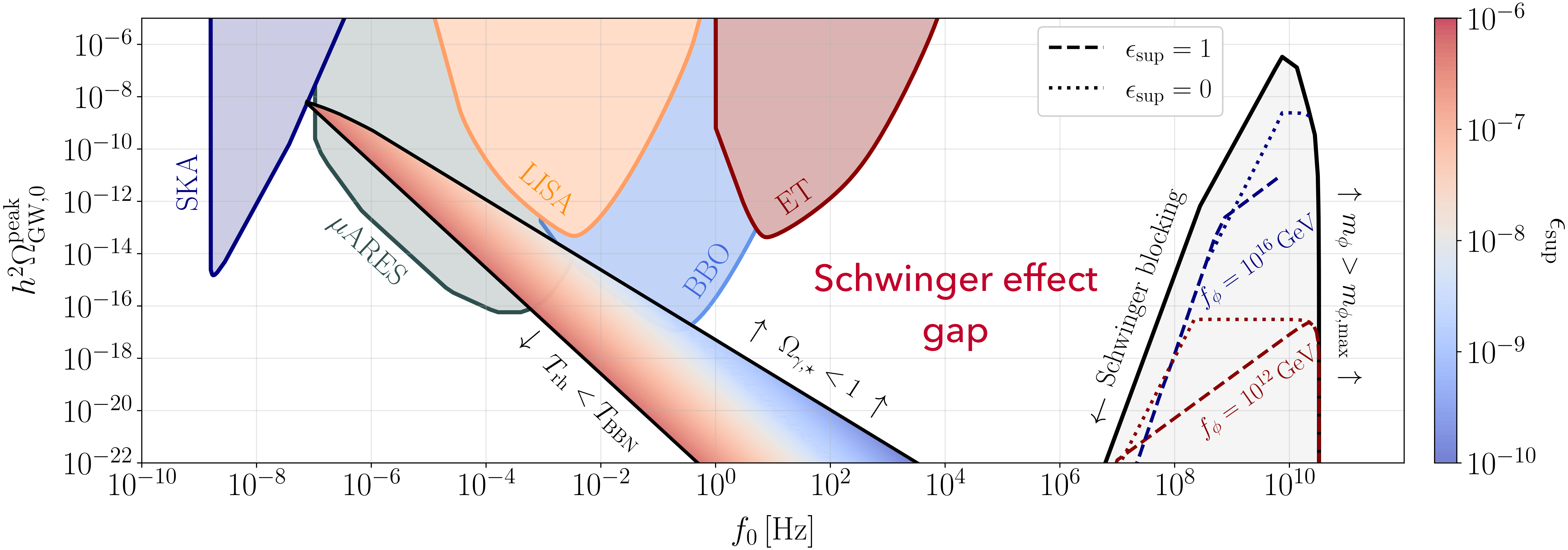

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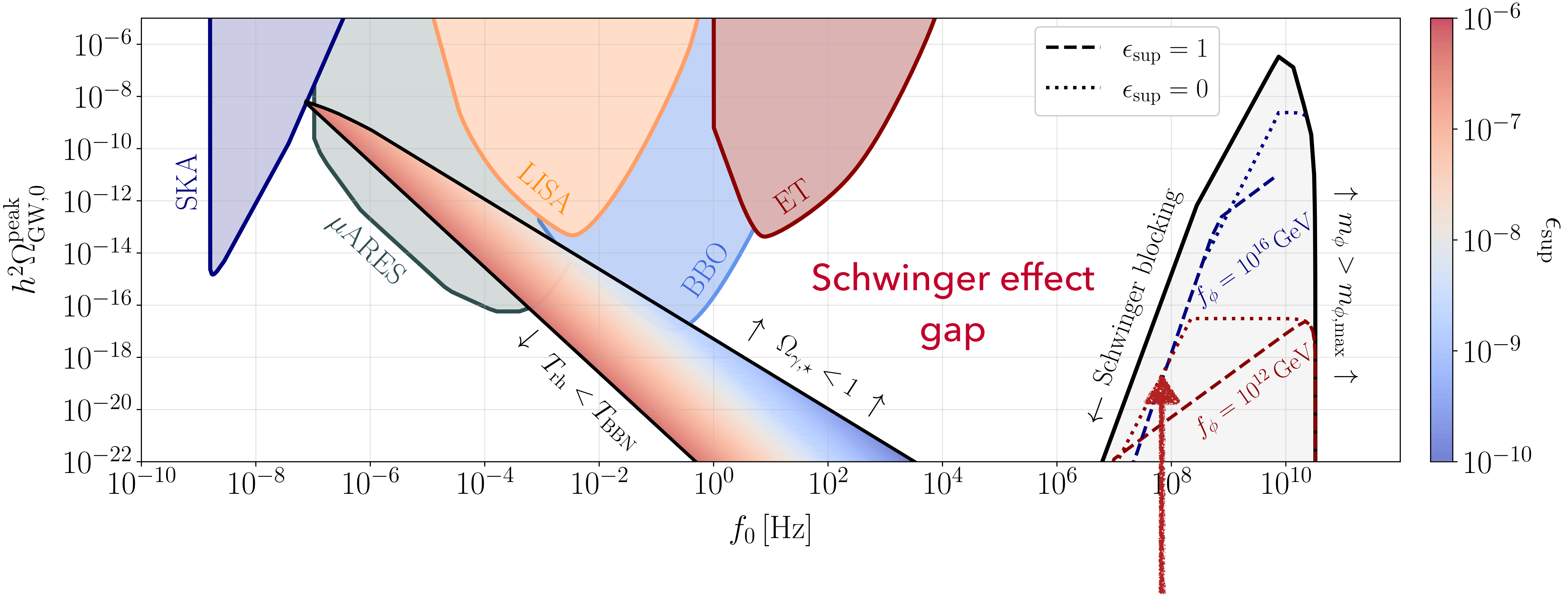
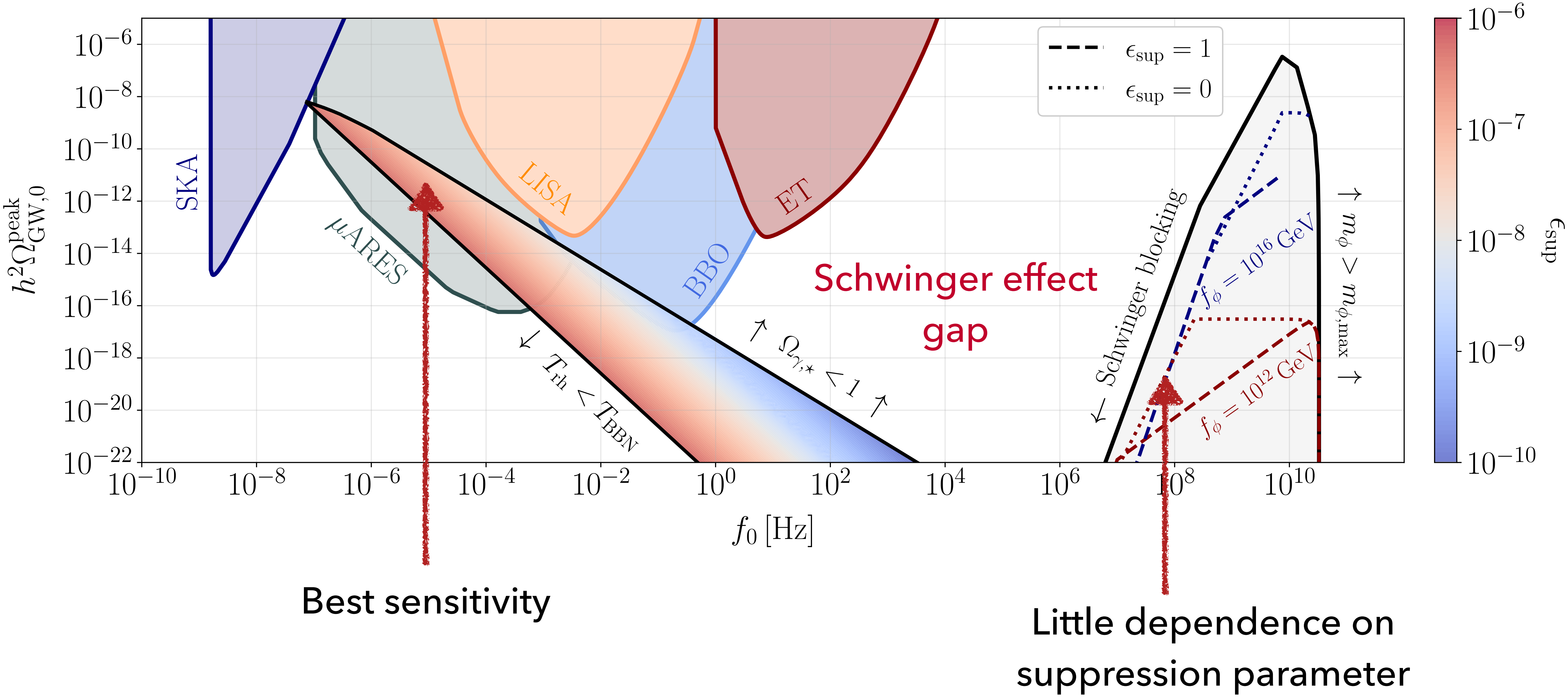


Little dependence on suppression parameter

4. The SM photon scenario

Gravitational waves



5. Outlook

Future directions of the model

- **Magnetic fields as signatures:**
  - SM coupling leads to strong helical magnetic field in early universe

$$E^2 + B^2 - \zeta EB + \frac{eQ}{2}\frac{E}{H}J_{ind} = 0$$

  - Redshift to today & compare to observation:
    lower bound on IGMF from absence of GeV photons from blazars

- **Lattice study**: so far, backscattering not covered – treat ϵ_{sup} as parameter

- **Thermal effects:** Schwinger equations with full fermionic dispersion relation

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Planned

- **Thermal effects**: Schwinger equations with full fermionic dispersion relation

What you should take away

Dark photon: Period of supercooling enhances SGWB

- Smaller decay constant f_ϕ and smaller photon-coupling α possible: $f_\phi \sim 10^{12}\,\mathrm{GeV}$ and $\alpha \sim 1$ can be sufficient

SM photon: supercooling can open the tachyonic band

- Pair production limits parameter space
- Small ALP mass + $f_\phi \sim 10^{16}\,\mathrm{GeV}$ could be in μARES and BBO range
- UHF range window

6. Conclusions

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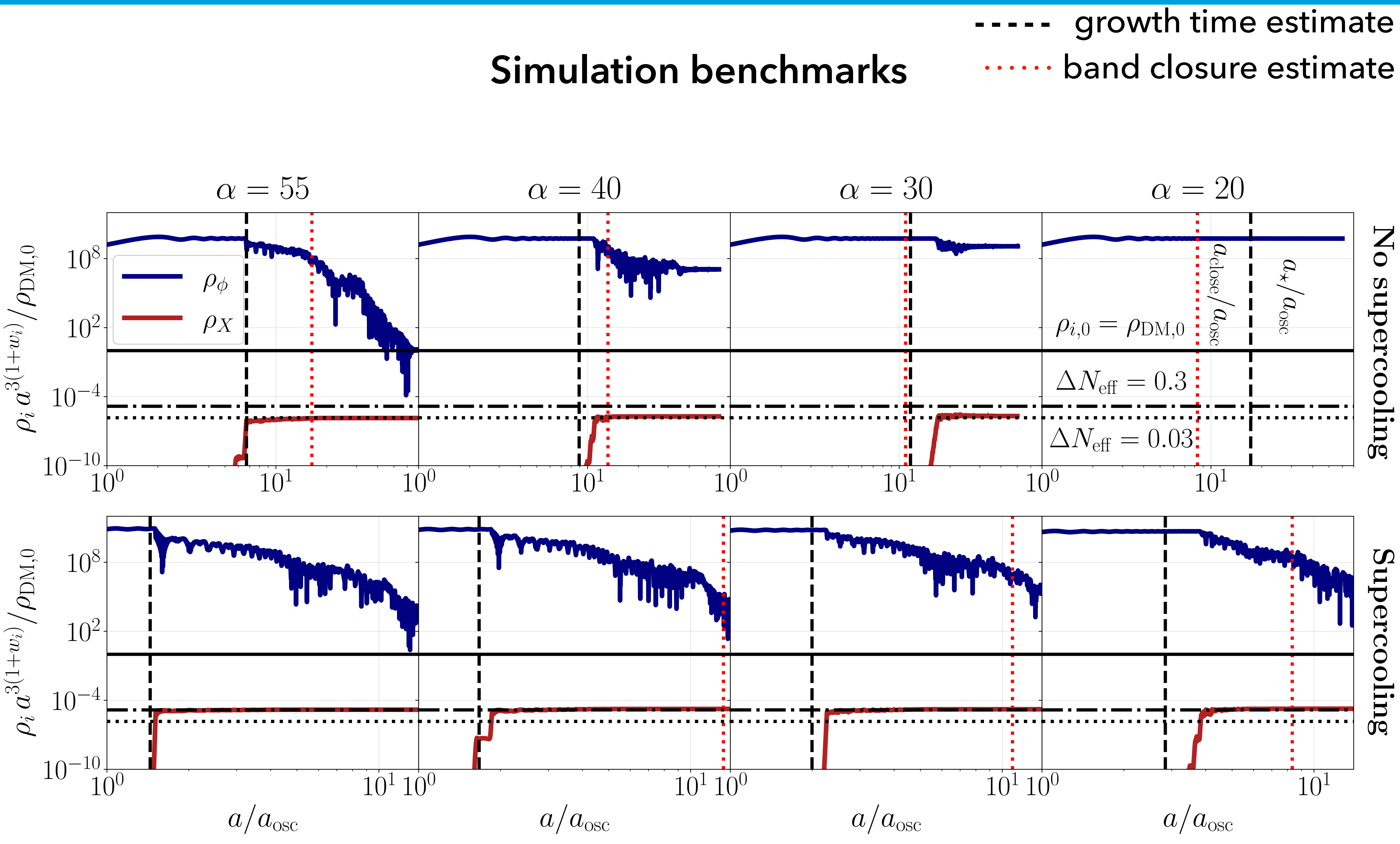
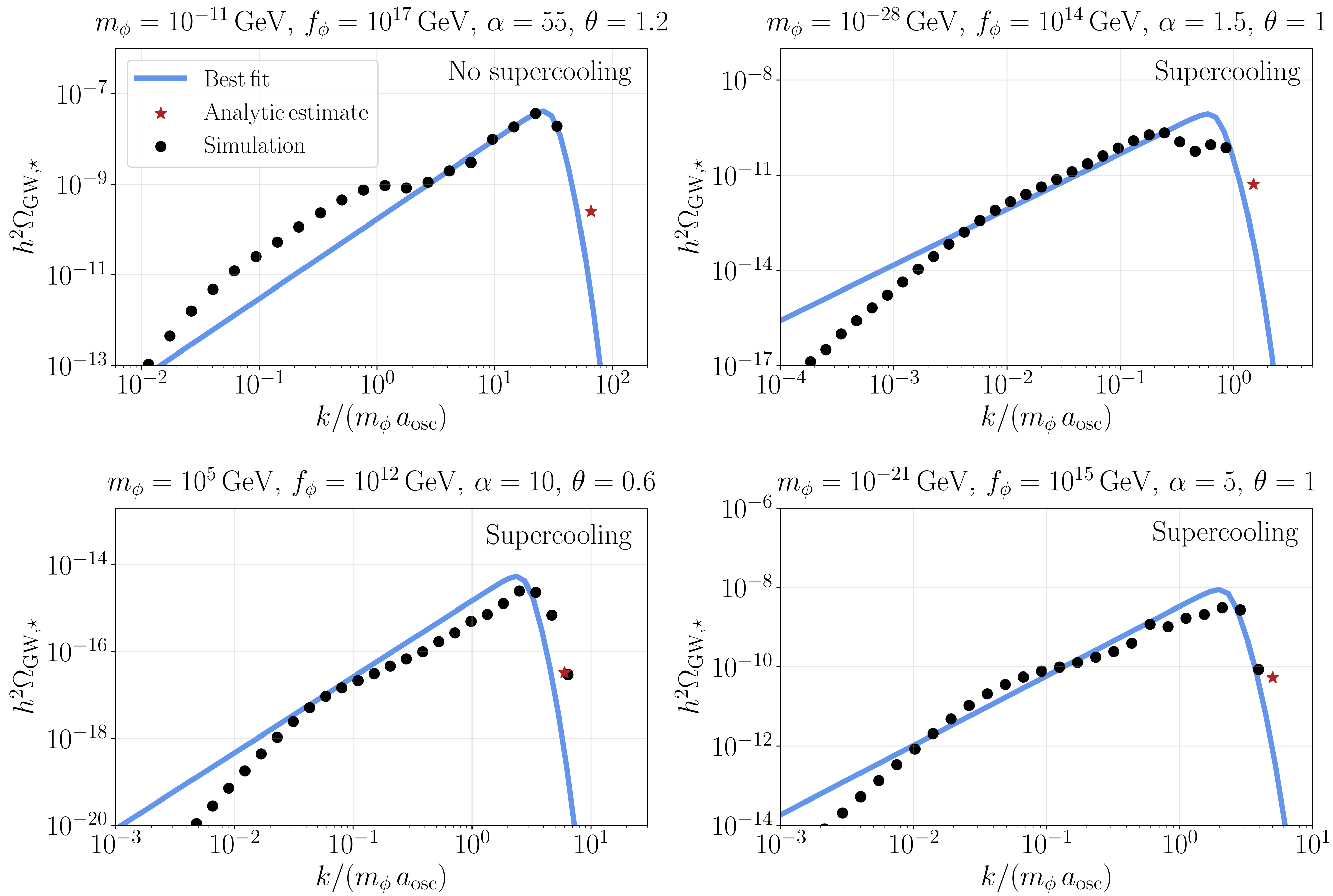
Thank you for your attention!

Simulation benchmarks



Backup

GW fits overview



GW fit functions

- Fit template: [Madge, Ratzinger, Schmitt, Schwaller 2111.12730]

$$\Omega_{\mathrm{GW},0}(f) = \mathcal{A}_s \tilde{\Omega}_{\mathrm{GW},0} \frac{\left(\tilde{f}/f_s\right)^p}{1 + \left(\tilde{f}/f_s\right)^p \exp\left[\gamma\left(\tilde{f}/f_s - 1\right)\right]}$$

- Dark photon result:

$$h^2\tilde{\Omega}_{\mathrm{GW},0} = 7.69 \times 10^{-5} \left(\frac{f_\phi}{M_{\mathrm{Pl}}}\right)^2 \frac{1}{\alpha^2},$$

$$\tilde{f}_0 = 28.53\,\mathrm{Hz} \left(\frac{a_{\mathrm{osc}}}{a_\star}\right)^{\frac{3}{4}} \alpha\theta^{\frac{1}{2}} \left(\frac{m_\phi}{\mathrm{eV}}\right)^{\frac{1}{2}} \left(\frac{10^{10}\,\mathrm{GeV}}{f_\phi}\right)^{\frac{1}{2}}$$

- SM photon result:

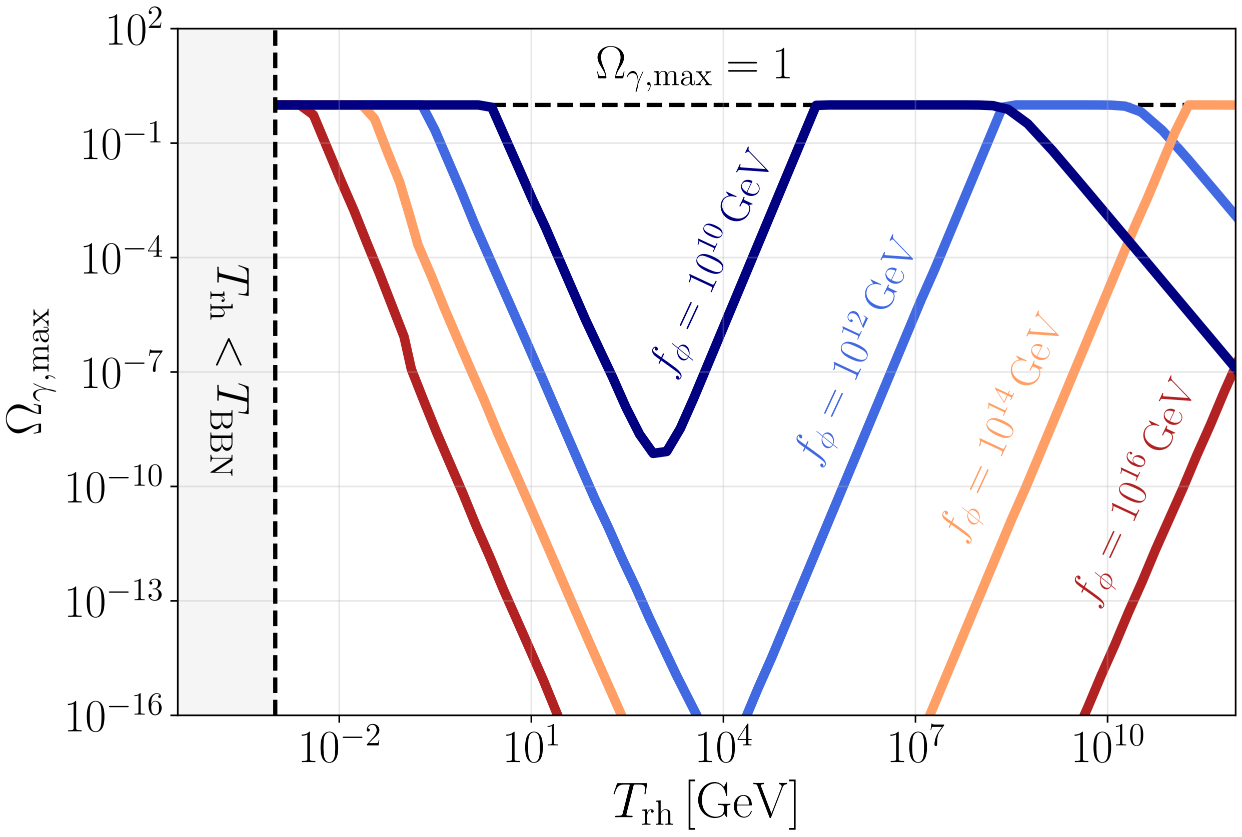
$$\tilde{f}_0 = 8.69 \times 10^{-8}\,\mathrm{Hz} \left(\frac{100}{g_\epsilon^{\mathrm{rh}}}\right)^{\frac{1}{12}} \alpha\theta \frac{m_\phi}{\mathrm{eV}} \left(\frac{a_{\mathrm{osc}}}{a_\star}\right)^{\frac{3}{2}} \left(\frac{\mathrm{GeV}}{H_{\mathrm{rh}}}\right)^{\frac{1}{2}} \min\left\{1, \frac{a_\star}{a_{\mathrm{rh}}}\right\},$$

$$h^2\tilde{\Omega}^{\mathrm{MD}}_{\mathrm{GW},0} = 4.20 \times 10^{-4} \left(\frac{100}{g_\epsilon^\star}\right)^{\frac{1}{3}} \chi^2_{\mathrm{sp}} \left(\frac{f_\phi}{\alpha M_{\mathrm{Pl}}}\right)^2 \min\left\{1, \frac{a_{\mathrm{md}}}{a_{\mathrm{rh}}}\right\},$$

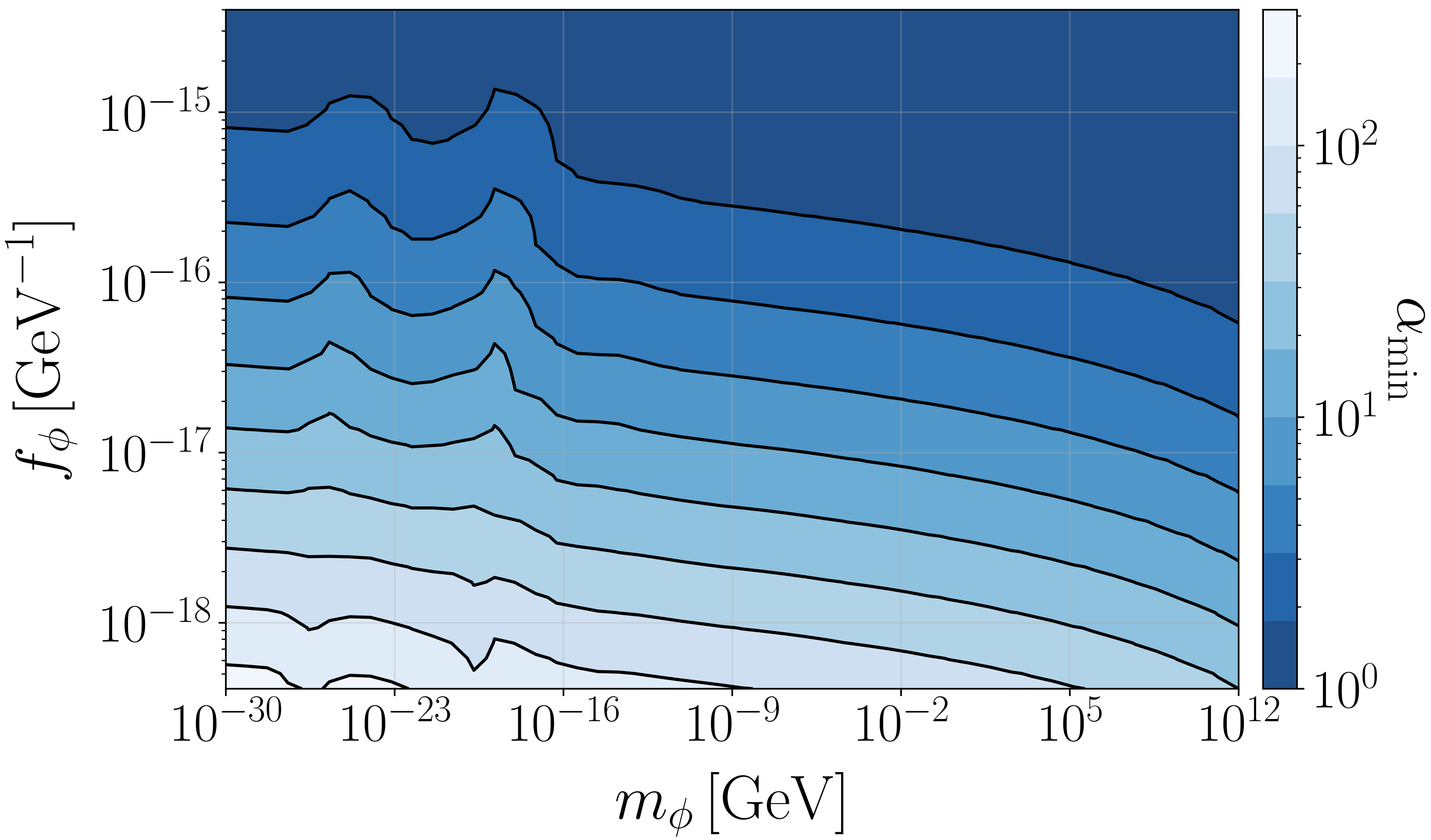
$$h^2\tilde{\Omega}^{\mathrm{RD}}_{\mathrm{GW},0} = 7.01 \times 10^{-5} \left(\frac{100}{g_\epsilon^\star}\right)^{\frac{1}{3}} \chi^2_{\mathrm{sp}} \left(\frac{\theta}{\alpha}\right)^2 \left(\frac{f_\phi}{r_{\mathrm{sc}} M_{\mathrm{Pl}}}\right)^4 \left(\frac{a_\star}{a_{\mathrm{osc}}}\right) \min\left\{1, \frac{a_{\mathrm{md}}}{a_{\mathrm{rh}}}\right\}$$

Schwinger pair production. Some more details

- Gauge field equation of motion: $\dot{\rho}_\gamma = -4H\rho_\gamma + 2\zeta HEB - eQEJ_{ind}$
- Induced current suppressed by electron mass: $J_{ind} \propto \exp\left(-\frac{\pi m_e^2}{eE}\right)$
- Assume dynamical equilibrium: $$E^2 + B^2 - \zeta EB + \frac{eQ}{2}\frac{E}{H}J_{ind} = 0$$
  - Maximize photon energy fraction on this contour
  - Plot as fcn. of would-be reheating temp.



The minimal coupling



$$\frac{a_{close}}{a_{osc}} = (\alpha\theta)^{2/3} \qquad \text{vs.} \qquad \frac{a_\star}{a_{osc}} = 1 + \frac{\pi}{\alpha\theta} r_{sc}^2 \ln\left(\frac{128\pi^2}{\alpha^4\theta^2}\frac{f_\phi^2}{m_\phi^2}\right)$$