# Gravitational Waves from Inverse Phase Transitions


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w/ Giulio Barni, Simone Blasi, and Miguel Vanvlasselaer

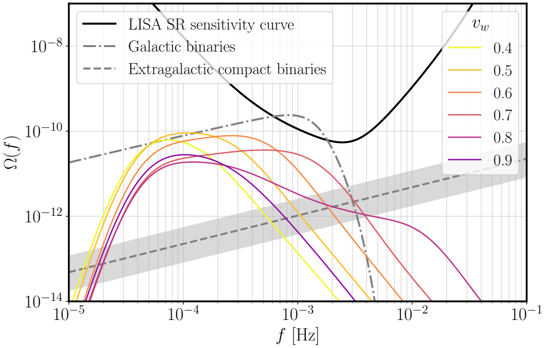


DESY Theory Workshop 2025: Synergies Towards the Future Standard Model

September 24, 2025

# Introduction

- inverse phase transitions have different fluid profiles than direct ones
- sound wave contribution to GW spectrum depends on fluid profiles




[Gowling, Hindmarsh (2021)]

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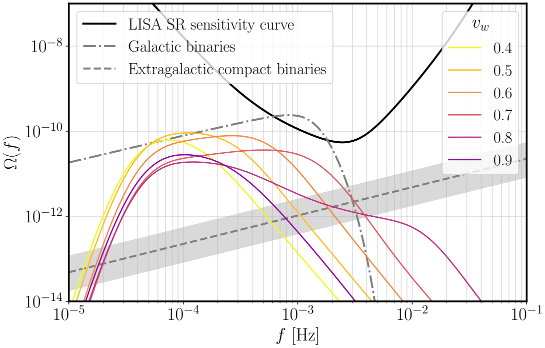


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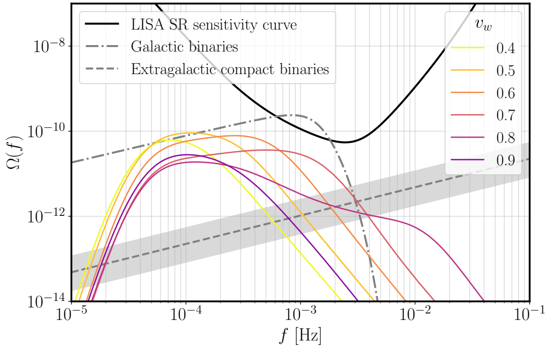


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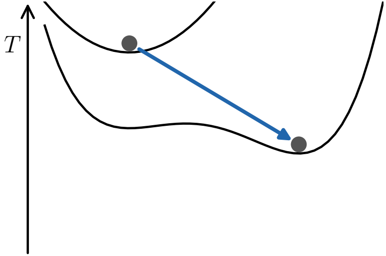
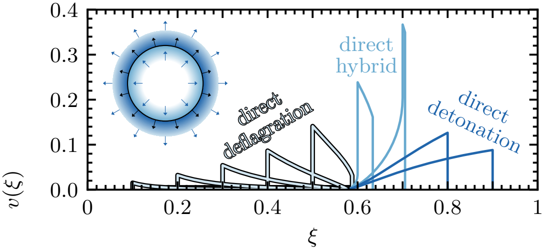
⚠ WARNING: work in progress

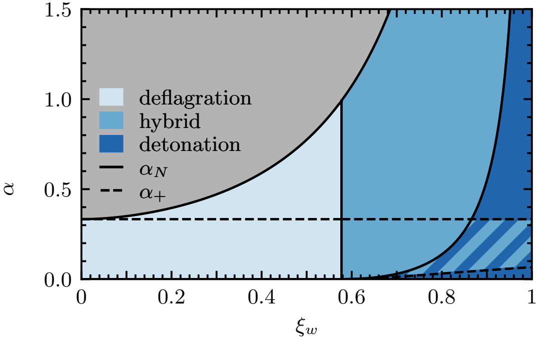
# Direct vs. Inverse Phase Transtitions

**direct transitions:** (exothermic)

plasma fluid pushed in front / dragged behind



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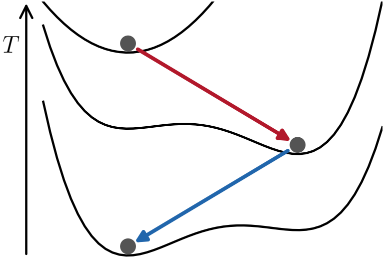


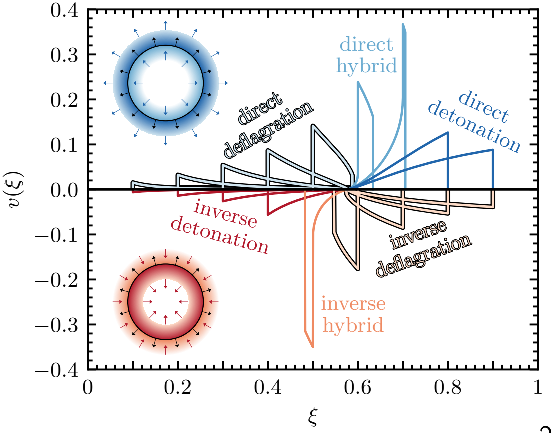

**direct transitions:** (exothermic)

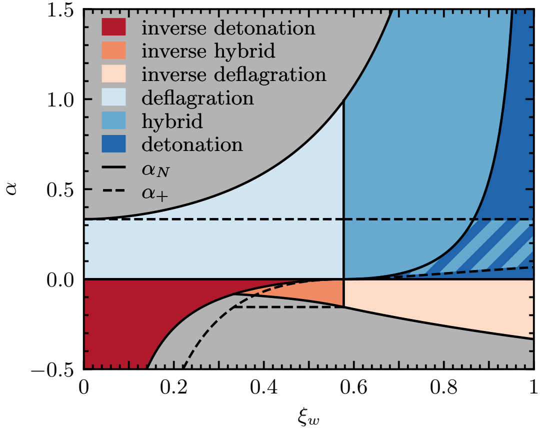
plasma fluid pushed in front / dragged behind

**inverse transitions:** (endothermic)

plasma fluid sucked/pushed invards

see: G. Barni's talk;
Barni, Blasi, Vanvlasselaer (2024);
arXiv:2503.01951 [hep-ph]



# Sound Shell Model

semi-analytic model for sound wave component of GW spectrum in weak - moderate transitions ($\alpha \ll 1$)

[Hindmarsh (2018); Hindmarsh, Hijazi (2019); Roper Pol, Procacci, Caprini (2024)]

- fluid shells propagate as sound waves

  $\Longrightarrow$ superposition of self-similar fluid profiles

$$\Omega_{\rm GW}(k) \sim \int \frac{d\tau_1}{\tau_1} \int \frac{d\tau_2}{\tau_2} \langle \Pi_{ij}(\tau_1, k) \Pi_{ij}(\tau_2, k) \rangle \cos k(\tau_1 - \tau_2)$$

$$\Pi_{ij} \sim \Lambda_{ijkl}\, v_l\, v_m$$

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$$\sim \int dp \int d\tilde{p}\, f(p, \tilde{p})\, E_{\rm kin}(p)\, E_{\rm kin}(\tilde{p})\, \Delta(k, p, \tilde{p}, \delta\tau_{\rm GW})$$

$E_{\rm kin} \sim \langle vv \rangle$

time dependence

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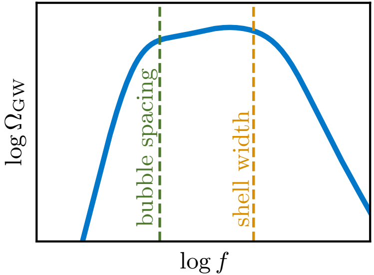


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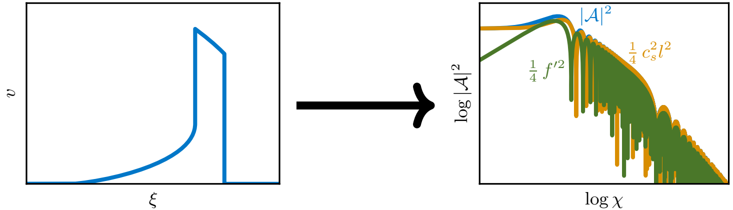
time dependence

# Gravitational Waves in the Sound Shell Model

$$f(\chi) = \frac{4\pi}{\chi} \int d\xi \, v(\xi) \sin(\chi\xi)$$

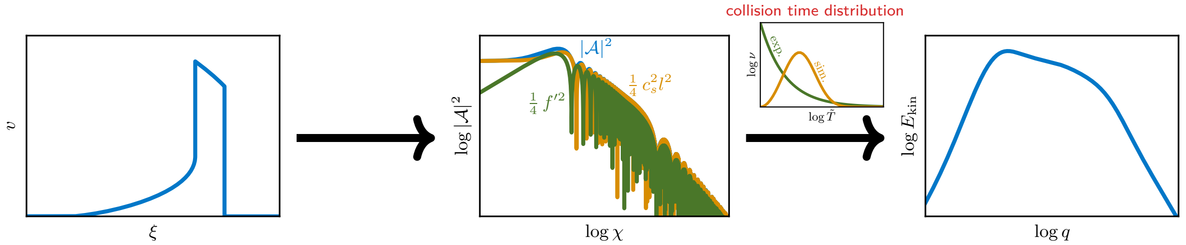
$$l(\chi) = \frac{4\pi}{\chi} \int d\xi \, \lambda(\xi) \sin(\chi\xi)$$



# Gravitational Waves in the Sound Shell Model

$$\langle v_i(\tau_1, \vec{k})\, v_j^*(\tau_2, \vec{p})\rangle = (2\pi)^5 \delta(\vec{k} - \vec{p})\, \frac{k_i k_j}{k^4}\, E_{\text{kin}}(k) \cos\left[k c_s (\tau_1 - \tau_2)\right]$$

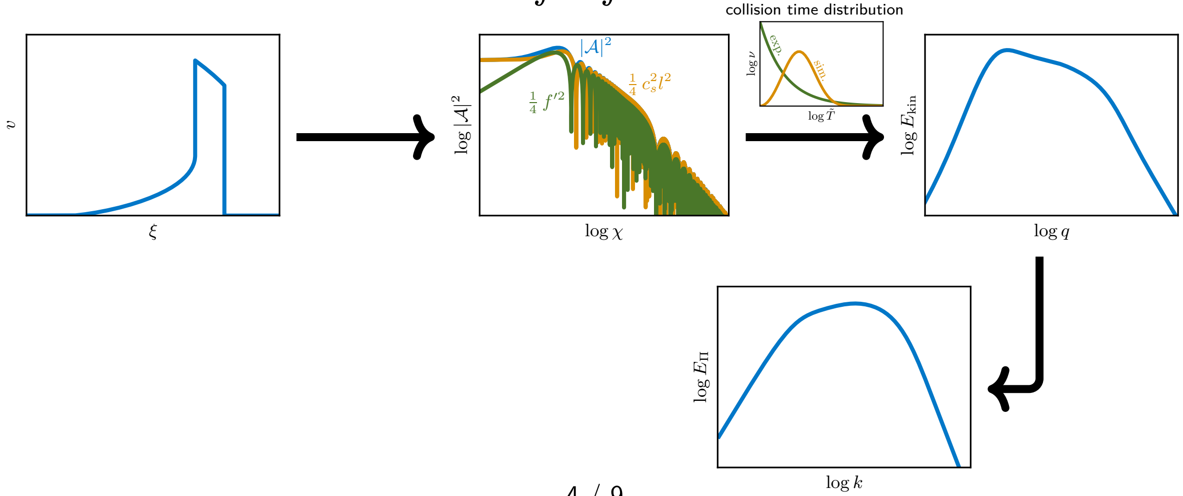
$$E_{\text{kin}}(k) \sim \int d\tilde{T}\, \nu(\tilde{T})\, \tilde{T}^6 |\mathscr{A}(\tilde{T} k/\beta)|^2$$



# Gravitational Waves in the Sound Shell Model

$$\langle \Pi_{ij}(\tau_1,\vec{k})\Pi^*_{ij}(\tau_2,\vec{p})\rangle = (2\pi)^6\delta(\vec{k}-\vec{p})\,\frac{E_\Pi(\tau_1,\tau_2,k)}{4\pi k^2}$$

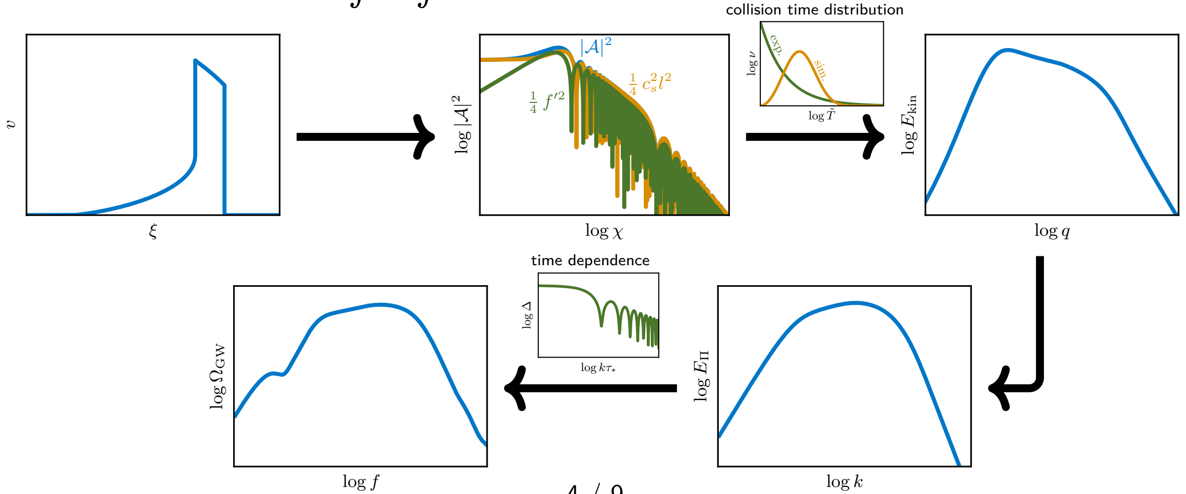
$$E_\Pi(k) \sim \int dp \int d\tilde{p}\, f(p,\tilde{p})\, E_{\rm kin}(p) E_{\rm kin}(\tilde{p})$$



# Gravitational Waves in the Sound Shell Model

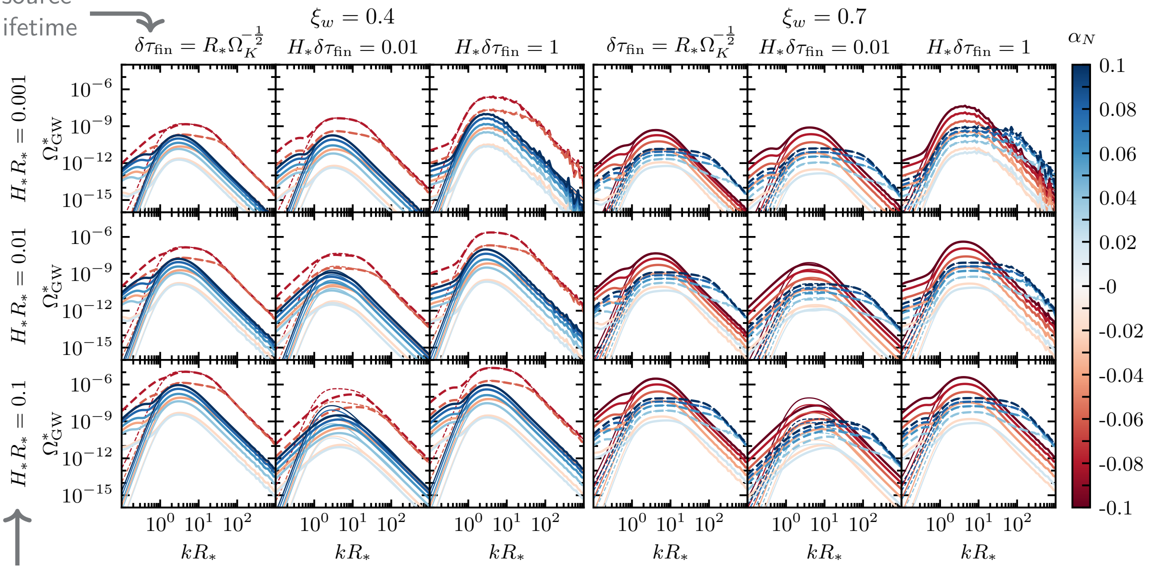
$$\Omega_{\rm GW}(k) \sim \int \frac{d\tau_1}{\tau_1} \int \frac{d\tau_2}{\tau_2}\, E_\Pi(k,\tau_1,\tau_2)\cos k(\tau_1-\tau_2)$$

$$\sim \int dp \int d\tilde{p}\, f(p,\tilde{p})\, E_{\rm kin}(p) E_{\rm kin}(\tilde{p})\, \Delta(\delta\tau_{\rm fin}, k, p, \tilde{p})$$



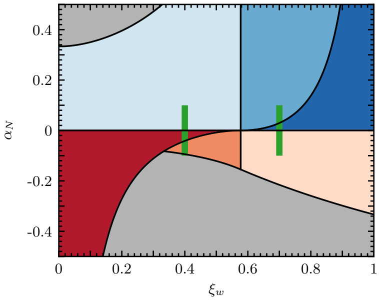
# Gravitational Wave Spectra

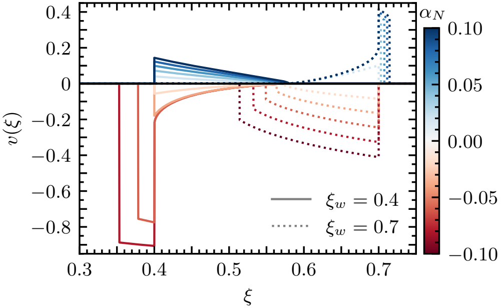
source lifetime




mean bubble spacing

# Gravitational Wave Amplitude



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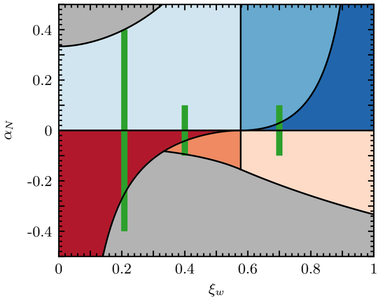
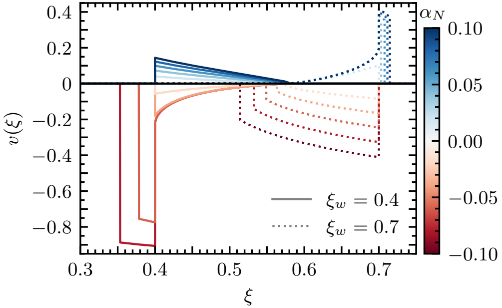
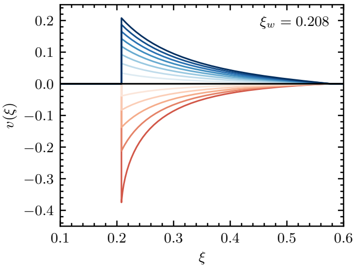
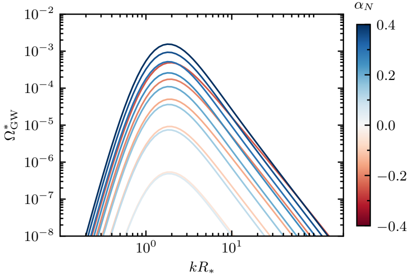


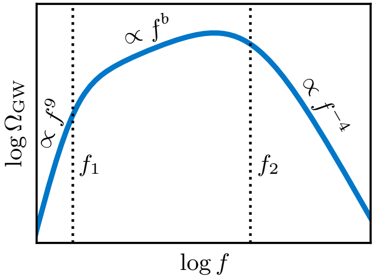



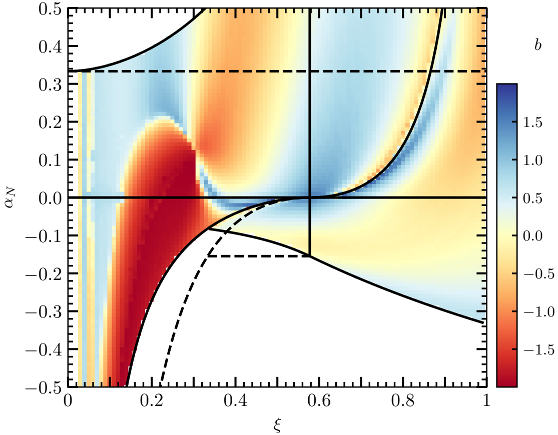



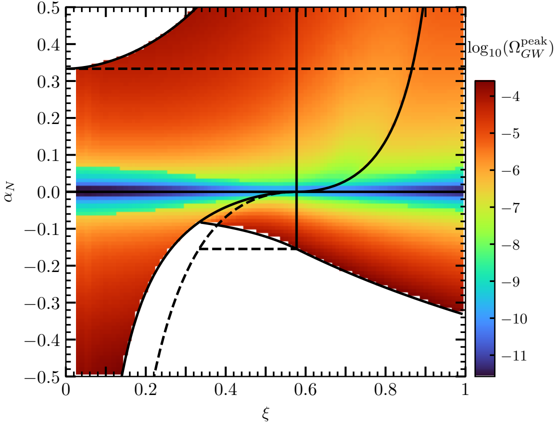



# Double-Broken Power Law Fit

$$\Omega_{\rm GW}(f) = \Omega_{\rm GW}^{\rm peak}\,\mathcal{N}\left(\frac{f}{f_1}\right)^9\left[1+\left(\frac{f}{f_1}\right)^4\right]^{\frac{b-9}{4}}\left[1+\left(\frac{f}{f_2}\right)^2\right]^{-\frac{b+4}{2}}$$

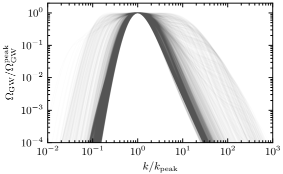


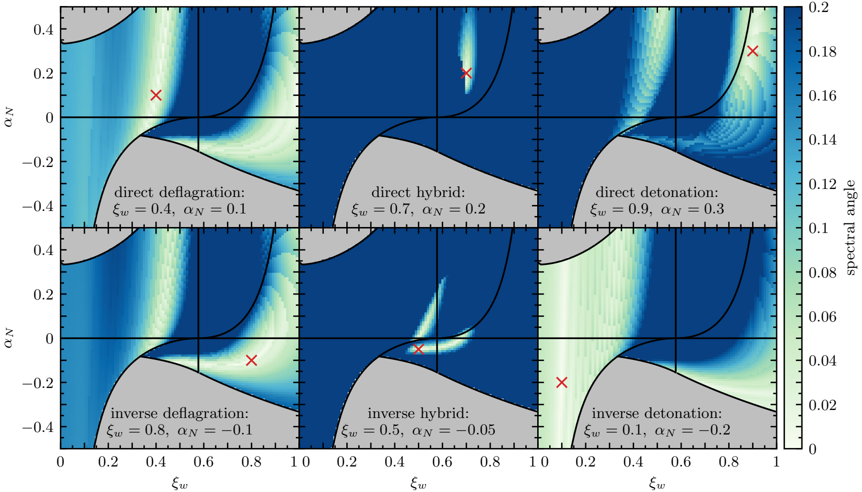



# Spectral discrimination

spectral angle: $$\theta_{12} = \arccos \int \log x \; \bar{\Omega}_1(x)\bar{\Omega}_2(x) \quad \text{with} \quad \bar{\Omega}(x) = \frac{\Omega_{\rm GW}(x f_{\rm peak})}{\sqrt{\int d\log f \; \Omega^2_{\rm GW}(f)}}$$



# Conclusion

- Tunneling away from the zero-temperature vacuum can lead to inverse PTs
- Inverse PTs can produce observable SGWB
- GW signal from inverse PTs similar to direct transitions

  $\Longrightarrow$ should to be taken into account when analyzing data

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Thank you for your attention!