DESY Theory Workshop
26 September 2025

Towards High Precision at the (HL-)LHC

Stephen Jones
IPPP Durham / Royal Society URF

Durham University

THE ROYAL SOCIETY

Outline

Recent Progress

Review of Fixed Order

Selected Highlights

Challenges

Overview

Example 1: Higgs Production in ggF

Example 2: Higgs Boson Self-Coupling

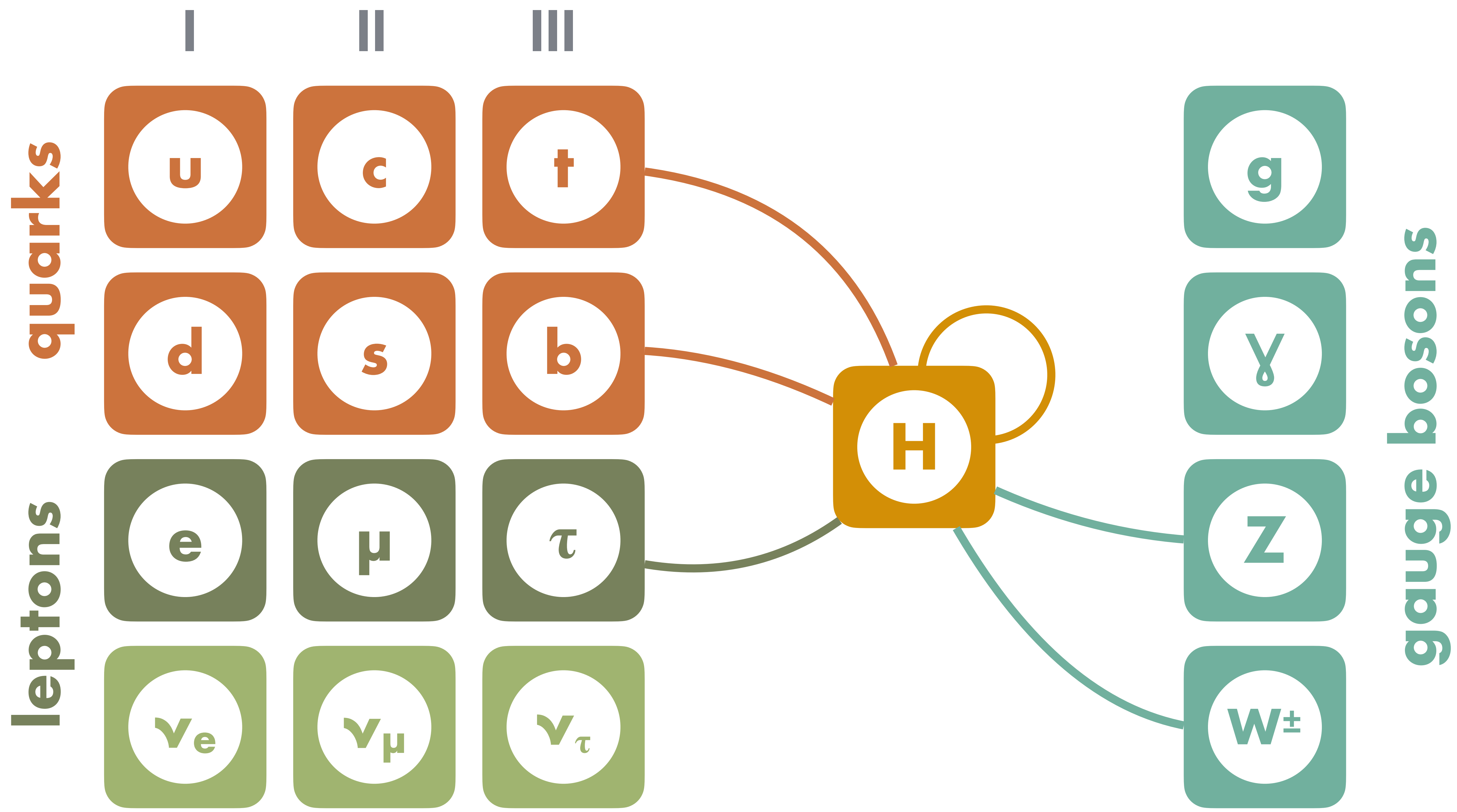
Synergies

Amplitudes

Precision Interpretation & EFTs

Gravity and beyond

Future Directions & Outlook



``Precision''

Goal

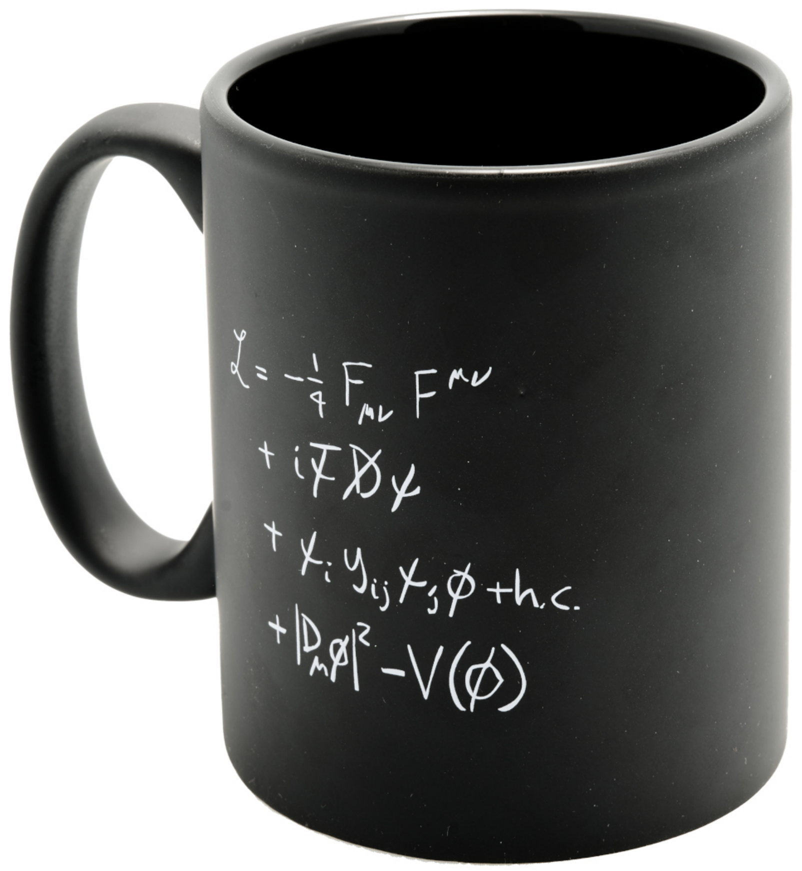
Theory → concrete, testable predictions

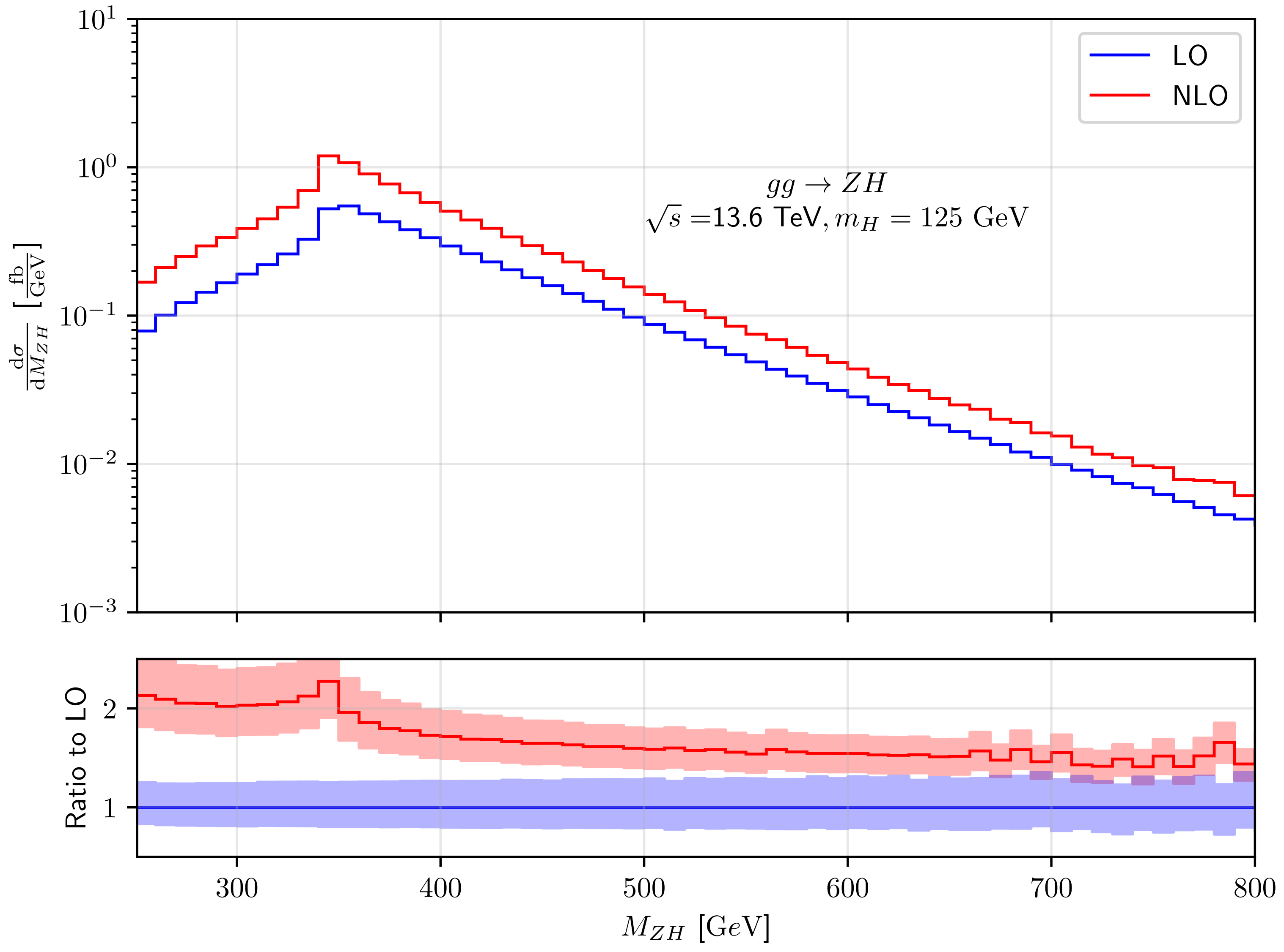
Experimental observations → parameters, interactions, dynamics

Precise predictions require a detailed understanding of the theory

- Fixed order amplitudes and subtraction schemes (QCD & EW)
- Parton Distribution Functions
- Jets/Flavour
- Parton Showers/Resummations
- Non-perturbative aspects

For production process & backgrounds





LHCHWG: Updated $gg \to ZH$ Prediction

Aveleira, Chen, Davies, Degrassi, Giardino, Gröber, Heinrich, SPJ, Kerner, Schlenk, Steinhauser, Vitti 25

Synergy: Precision is the melting pot of our best description of nature and the interface between theory and experiment

Introduction to Precision

Fixed Order Calculations

Integral Reduction

Computing Integrals

Sampling Phase-space

Subtraction

Process Simulation & Modelling

Resummation

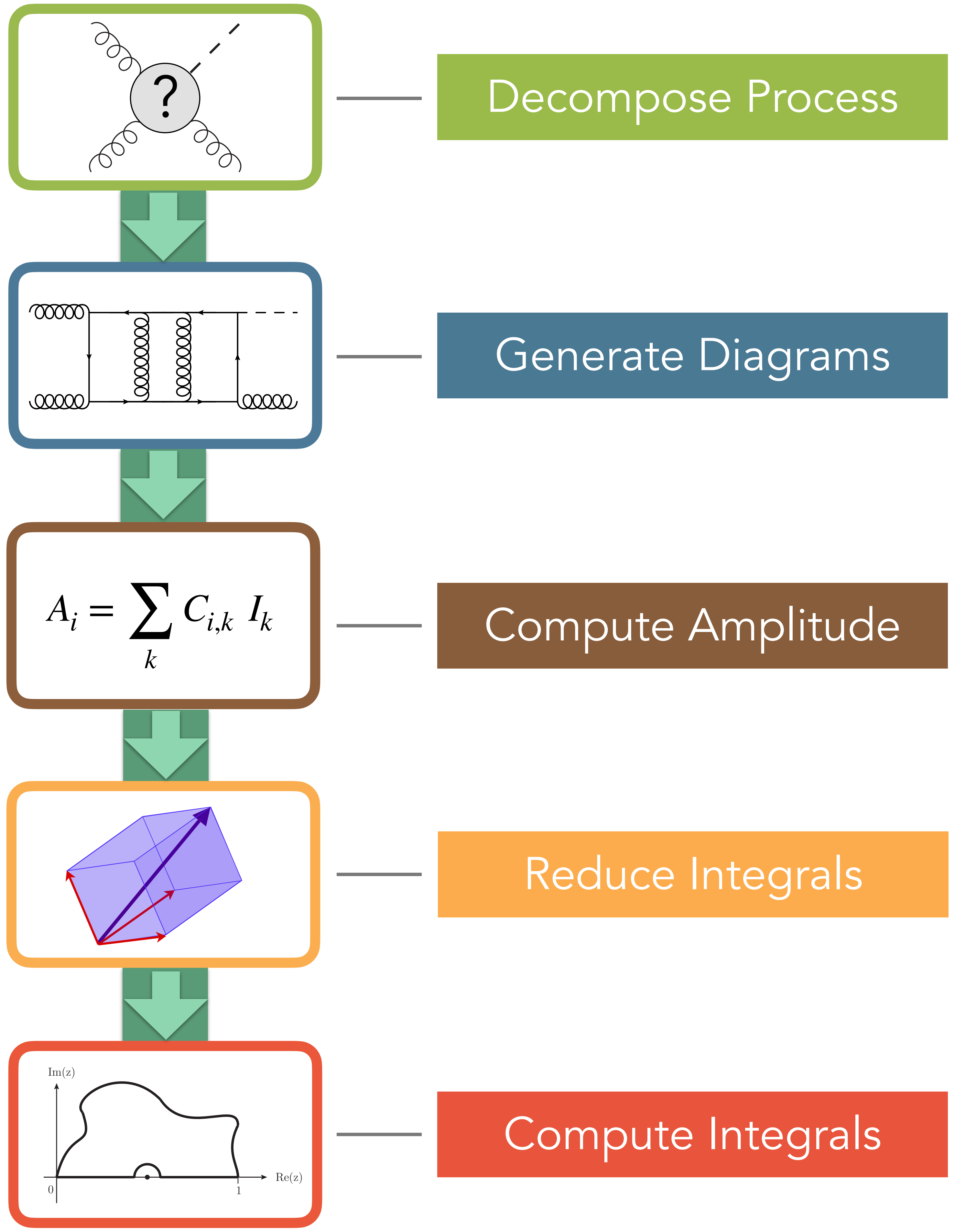
Parton Showers → **Silvia (Tuesday)**

Negative Events

Building Predictions

Parton Distribution Functions (PDFs) → **Oleksandr (Today)**

Estimating Uncertainties



Recent Progress

Fixed Order Progress

Impressive progress made in recent years venturing into very difficult territory (beyond MPLs, internal masses, 2→3)

Higgs

process	known	desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO_{QCD}^{(t,t\times b)}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}^{(HTL)}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.)
$pp \to H+j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL}\otimes NLO_{QCD}+NLO_{EW}}$ $\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO_{QCD}}$
$pp \to H+2j$	$\mathrm{NLO_{HTL}\otimes LO_{QCD}}$ $\mathrm{N^3LO_{QCD}^{(VBF^*)}}$ (incl.) $\mathrm{NNLO_{QCD}^{(VBF^*)}}$ $\mathrm{NLO_{EW}^{(VBF)}}$	$\mathrm{NNLO_{HTL}\otimes NLO_{QCD}+NLO_{EW}}$ $\mathrm{N^3LO_{QCD}^{(VBF^*)}}$ $\mathrm{NNLO_{QCD}^{(VBF)}}$ $\mathrm{NLO_{QCD}}$
$pp \to H+3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO_{QCD}^{(VBF)}}$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ $\mathrm{NNLO_{QCD}^{(VBF^*)}}$
$pp \to VH$	$\mathrm{N^3LO_{QCD}}$ (incl.)$+\mathrm{NLO_{EW}}$ $\mathrm{NLO_{gg\to HZ}^{(t,b)}}$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to VH+j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD}+NLO_{EW}}$	
$pp \to HH$	$\mathrm{N^3LO_{HTL}\otimes NLO_{QCD}}$ $\mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to HH+2j$	$\mathrm{N^3LO_{QCD}^{(VBF^*)}}$ (incl.) $\mathrm{NNLO_{QCD}^{(VBF^*)}}$ $\mathrm{NLO_{EW}^{(VBF)}}$	$\mathrm{NLO_{QCD}}$
$pp \to HHH$	$\mathrm{NNLO_{HTL}}$	$\mathrm{NLO_{QCD}}$
$pp \to H+t\bar{t}$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ $\mathrm{NNLO_{QCD}}$ (approx.)	$\mathrm{NNLO_{QCD}}$
$pp \to H+t/\bar{t}$	$\mathrm{NLO_{QCD}+NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$

Vector Bosons

process	known	desired
$pp \to V$	$\mathrm{N^3LO_{QCD}+N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD}+NLO_{EW}}$ + Full $\mathrm{NLO_{QCD}}$ $(gg \to ZZ)$, approx. $\mathrm{NLO_{QCD}}$ $(gg \to WW)$	Full $\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V+j$	$\mathrm{NNLO_{QCD}+NLO_{EW}}$	hadronic decays
$pp \to V+2j$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD}+NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$
$pp \to V+b\bar{b}$	$\mathrm{NLO_{QCD}}$	$\mathrm{NNLO_{QCD}+NLO_{EW}}$
$pp \to W+b\bar{b}$	$\mathrm{NNLO_{QCD}}$	
$pp \to VV'+1j$	$\mathrm{NLO_{QCD}+NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to VV'+2j$	$\mathrm{NLO_{QCD}}$ (QCD component) $\mathrm{NLO_{QCD}+NLO_{EW}}$ (EW component)	Full $\mathrm{NLO_{QCD}+NLO_{EW}}$
$pp \to W^+W^+ +2j$	Full $\mathrm{NLO_{QCD}+NLO_{EW}}$	
$pp \to W^+W^- +2j$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (EW component)	
$pp \to W^+Z+2j$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (EW component)	
$pp \to ZZ+2j$	Full $\mathrm{NLO_{QCD}+NLO_{EW}}$	
$pp \to VV'V''$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (w/ decays)	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell)
$pp \to WWW$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell)	
$pp \to W^+W^+(V\to jj)$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell)	
$pp \to WZ(V\to jj)$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell)	
$pp \to \gamma\gamma$	$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma+j$	$\mathrm{NNLO_{QCD}+NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma\gamma+j$	$\mathrm{NNLO_{QCD}+NLO_{EW}}$ $+\mathrm{NLO_{QCD}}$ (gg channel)	
$pp \to \gamma\gamma\gamma$	$\mathrm{NNLO_{QCD}}$	$\mathrm{NLO_{EW}}$

Top

process	known	desired
$pp \to t\bar{t}$	$\mathrm{NNLO_{QCD}+NLO_{EW}}$ (w/o decays) $\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell) $\mathrm{NNLO_{QCD}}$ (w/ decays)	$\mathrm{N^3LO_{QCD}}$
$pp \to t\bar{t}+j$	$\mathrm{NLO_{QCD}}$ (off-shell effects) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD}+NLO_{EW}}$ (w decays)
$pp \to t\bar{t}+2j$	$\mathrm{NLO_{QCD}}$ (w/o decays)	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (w decays)
$pp \to t\bar{t}+V'$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (w decays)	$\mathrm{NNLO_{QCD}+NLO_{EW}}$ (w decays)
$pp \to t\bar{t}+\gamma$	$\mathrm{NLO_{QCD}}$ (off-shell)	
$pp \to t\bar{t}+Z$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell)	
$pp \to t\bar{t}+W$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell)	
$pp \to t/\bar{t}$	$\mathrm{NNLO_{QCD}}$*(w decays) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD}+NLO_{EW}}$ (w decays)
$pp \to tZj$	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off shell)	$\mathrm{NNLO_{QCD}+NLO_{EW}}$ (w/o decays)
$pp \to t\bar{t}t\bar{t}$	$\mathrm{NLO_{QCD}}$ (w decay) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NLO_{QCD}+NLO_{EW}}$ (off-shell) $\mathrm{NNLO_{QCD}}$

Jets

process	known	desired
$pp \to 2$ jets	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD}+NLO_{EW}}$	$\mathrm{N^3LO_{QCD}+NLO_{EW}}$
$pp \to 3$ jets	$\mathrm{NNLO_{QCD}+NLO_{EW}}$	

Les Houches Wishlist [2504.06689]

Fixed Order Progress

Impressive progress made in recent years venturing into very difficult territory (beyond MPLs, internal masses, 2→3)

Higgs

process	known	desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO^{(t,t\times b)}_{QCD}}$ $\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.)
$pp \to H + j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ $\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO_{QCD}}$
$pp \to H + 2j$	$\mathrm{NLO_{HTL} \otimes LO_{QCD}}$ $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF^*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	$\mathrm{NNLO_{HTL} \otimes NLO_{QCD} + NLO_{EW}}$ $\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ $\mathrm{NNLO^{(VBF)}_{QCD}}$ $\mathrm{NLO_{QCD}}$
$pp \to H + 3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO^{(VBF)}_{QCD}}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ $\mathrm{NNLO^{(VBF^*)}_{QCD}}$
$pp \to VH$	$\mathrm{N^3LO_{QCD}}$ (incl.)$\mathrm{+ NLO_{EW}}$ $\mathrm{NLO^{(t,b)}_{gg\to HZ}}$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to VH + j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to HH$	$\mathrm{N^3LO_{HTL} \otimes NLO_{QCD}}$ $\mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to HH + 2j$	$\mathrm{N^3LO^{(VBF^*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF^*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	$\mathrm{NLO_{QCD}}$
$pp \to HHH$	$\mathrm{NNLO_{HTL}}$	$\mathrm{NLO_{QCD}}$
$pp \to H + t\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ $\mathrm{NNLO_{QCD}}$ (approx.)	$\mathrm{NNLO_{QCD}}$
$pp \to H + t/\bar{t}$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$

Vector Bosons

process	known	desired
$pp \to V$	$\mathrm{N^3LO_{QCD} + N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ + Full $\mathrm{NLO_{QCD}}$ ($gg \to ZZ$), approx. $\mathrm{NLO_{QCD}}$ ($gg \to WW$)	Full $\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	hadronic decays
$pp \to V + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$
$pp \to V + b\bar{b}$	$\mathrm{NLO_{QCD}}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$
$pp \to W + b\bar{b}$	$\mathrm{NNLO_{QCD}}$	
$pp \to VV' + 1j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{NNLO_{QCD}}$
$pp \to VV' + 2j$	$\mathrm{NLO_{QCD}}$ (QCD component) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$
$pp \to W^+W^+ + 2j$	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to W^+W^- + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	
$pp \to W^+Z + 2j$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (EW component)	
$pp \to ZZ + 2j$	Full $\mathrm{NLO_{QCD} + NLO_{EW}}$	
$pp \to VV'V''$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w/ decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)
$pp \to WWW$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to W^+W^+(V \to jj)$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to WZ(V \to jj)$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to \gamma\gamma$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD}}$
$pp \to \gamma\gamma + j$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ $+\mathrm{NLO_{QCD}}$ (gg channel)	
$pp \to \gamma\gamma\gamma$	$\mathrm{NNLO_{QCD}}$	$\mathrm{NLO_{EW}}$

Top

process	known	desired
$pp \to t\bar{t}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/o decays) $\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell) $\mathrm{NNLO_{QCD}}$ (w/ decays)	$\mathrm{N^3LO_{QCD}}$
$pp \to t\bar{t} + j$	$\mathrm{NLO_{QCD}}$ (off-shell effects) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to t\bar{t} + 2j$	$\mathrm{NLO_{QCD}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to t\bar{t} + V'$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (w decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to t\bar{t} + \gamma$	$\mathrm{NLO_{QCD}}$ (off-shell)	
$pp \to t\bar{t} + Z$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to t\bar{t} + W$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell)	
$pp \to t/\bar{t}$	$\mathrm{NNLO_{QCD}}$*(w decays) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w decays)
$pp \to tZj$	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off shell)	$\mathrm{NNLO_{QCD} + NLO_{EW}}$ (w/o decays)
$pp \to t\bar{t}t\bar{t}$	$\mathrm{NLO_{QCD}}$ (w decay) $\mathrm{NLO_{EW}}$ (w/o decays)	$\mathrm{NLO_{QCD} + NLO_{EW}}$ (off-shell) $\mathrm{NNLO_{QCD}}$

Jets

process	known	desired
$pp \to 2\,\mathrm{jets}$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD} + NLO_{EW}}$	$\mathrm{N^3LO_{QCD} + NLO_{EW}}$
$pp \to 3\,\mathrm{jets}$	$\mathrm{NNLO_{QCD} + NLO_{EW}}$	

- completed since 2021 (green)

- progress since 2021 (yellow)

Les Houches Wishlist [2504.06689]

Fixed Order Progress

Update

LH2025 wishlist will also include systematic summaries of status of resummation & PS

Also needed

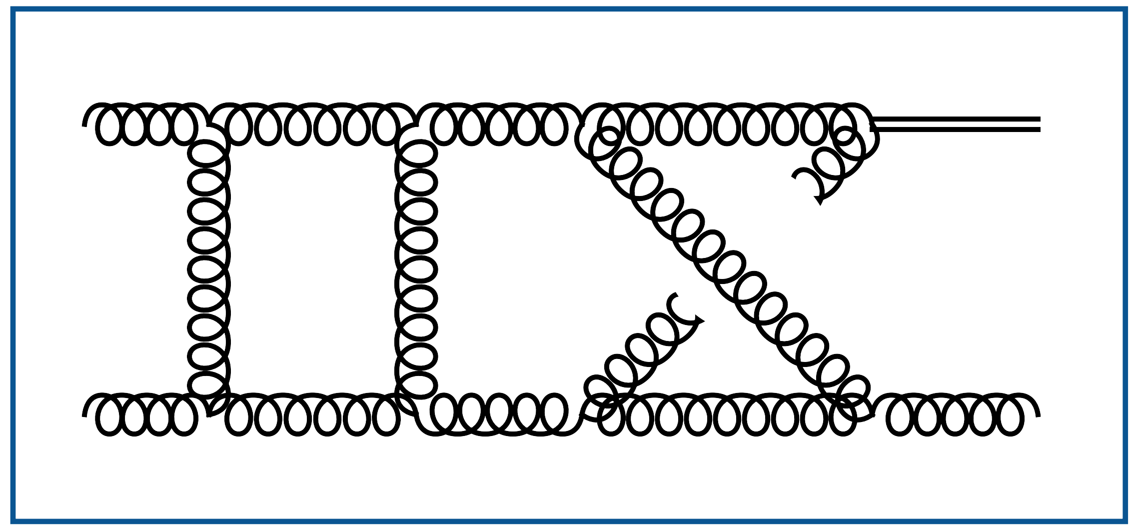
1. Get processes/precision for Electron-Ion Collider @ Brookhaven under control
2. Get a serious handle on Electroweak precision (for HL-LHC but esp. for FCC-ee, FCC-hh)

Highlight: Higgs + Jet @ $\mathrm{N^3LO_{HTL}}$

Work in the Heavy Top Limit (HTL)

$$\mathcal{L}_{eff} = \mathcal{L}_{SM,5} - \frac{C^0}{4} H G_a^{\mu\nu} G_{a,\mu\nu}$$

HTL valid for $\sqrt{s_{12}} \ll 2m_T$



Chen, Guan, Mistlberger 25

Virtual amplitudes for Higgs + Jet @ 3 Loop

Generalised leading colour $\{N_c^6, N_c^5 n_f, N_c^4 n_f^2, N_c^3 n_f^3\}$

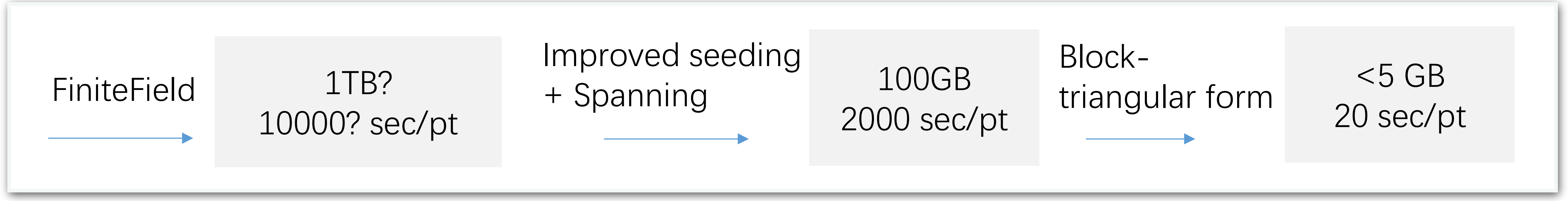
Reduction

Very challenging calculation involving difficult IBP reduction

Non-planar, Rank 6, 305 master integrals per family

Blade (+improved seeding → seed with different ranks in different sectors)

Guan, Liu, Ma, Wu 24; Liu, Ma 18; Guan, Liu, Ma 20; Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch 24



Guan (QCD@LHC 2025)

Master Integrals

3-loop massless + one off-shell leg

Planar + non-planar result expressed in terms of GPLs

Many sectors in literature, remaining 2 computed

Further topologies required to obtain sub-leading colour

Vita, Mastrolia, Schubert, Yundin 14
Canko, Syrrakos 21, 23
Henn, Lim, Torres Bobadilla 23
Gehrmann, Henn, Jakubcik, Lim, Mella, Syrrakos, Tancredi, Torres Bobadilla 24

Highlight: Higgs + Jet @ NLO_{EW}

Two independent calculations of EW corrections to HJ

Kira + Fire + Blade → 3600 Master Integrals

Klappert, Lange, Maierhöfer, Usovitsch 08; Smirnov 15; Guan, Liu, Ma, Wu 24;

Find basis with d and invariants factorising & at most linear in d

Boundary conditions for master integrals using AMFlow Liu, Ma, Wang 18; Liu, Ma 23;

Solve differential equations numerically

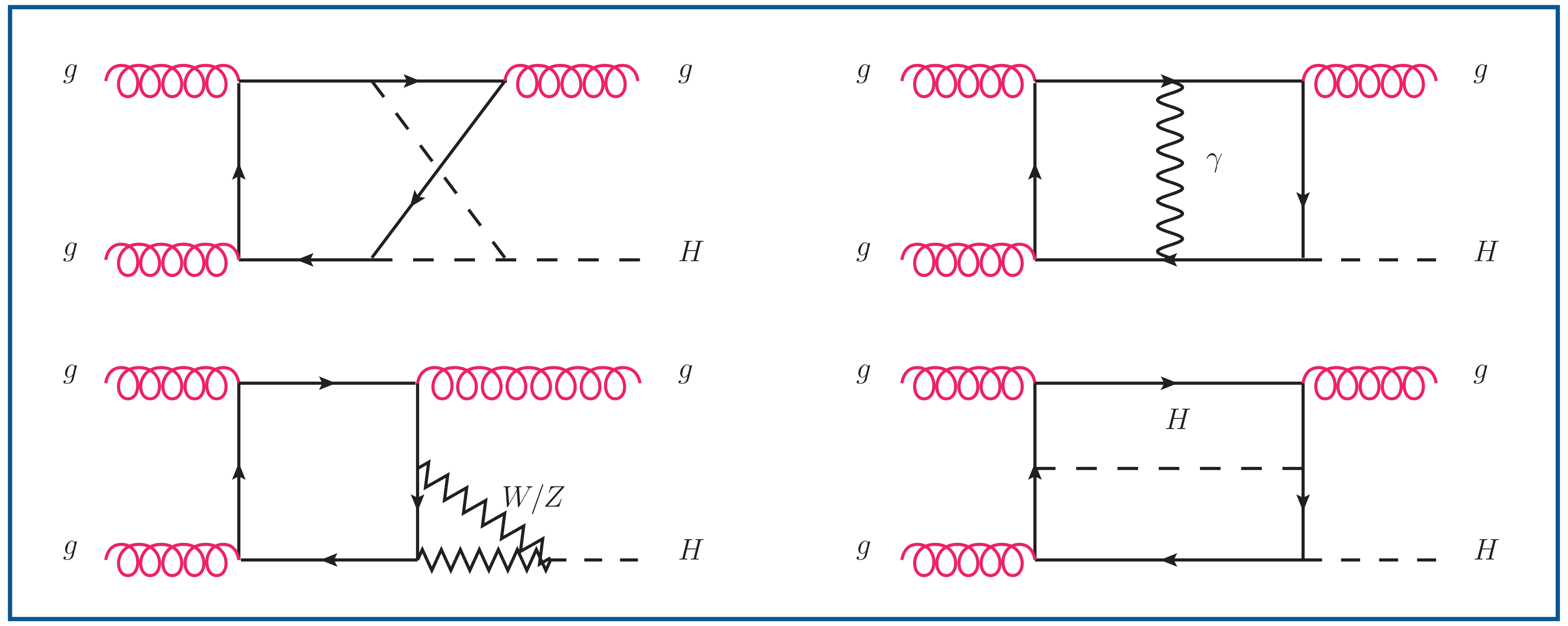
Chen, Li, Sang 25

Blade + FiniteFlow fixing ϵ numerically Peraro 19

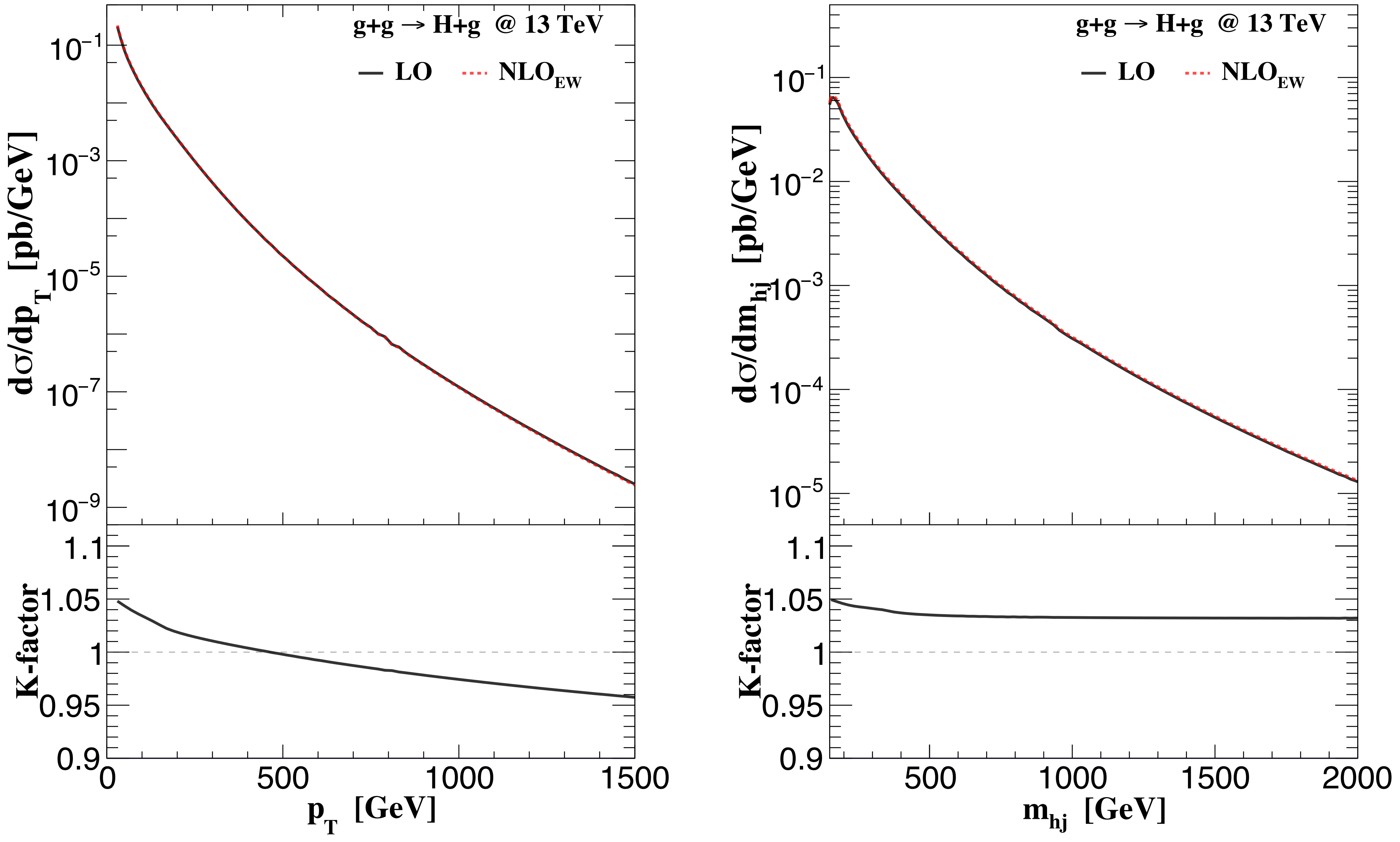
Compute master integrals using AMFlow

Bi, Ma, Mu 25

Both groups find that the EW corrections enhance the cross-section by around 4-5%, K-factor depends on p_T



Bi, Ma, Mu 25; Chen, Li, Sang 25;

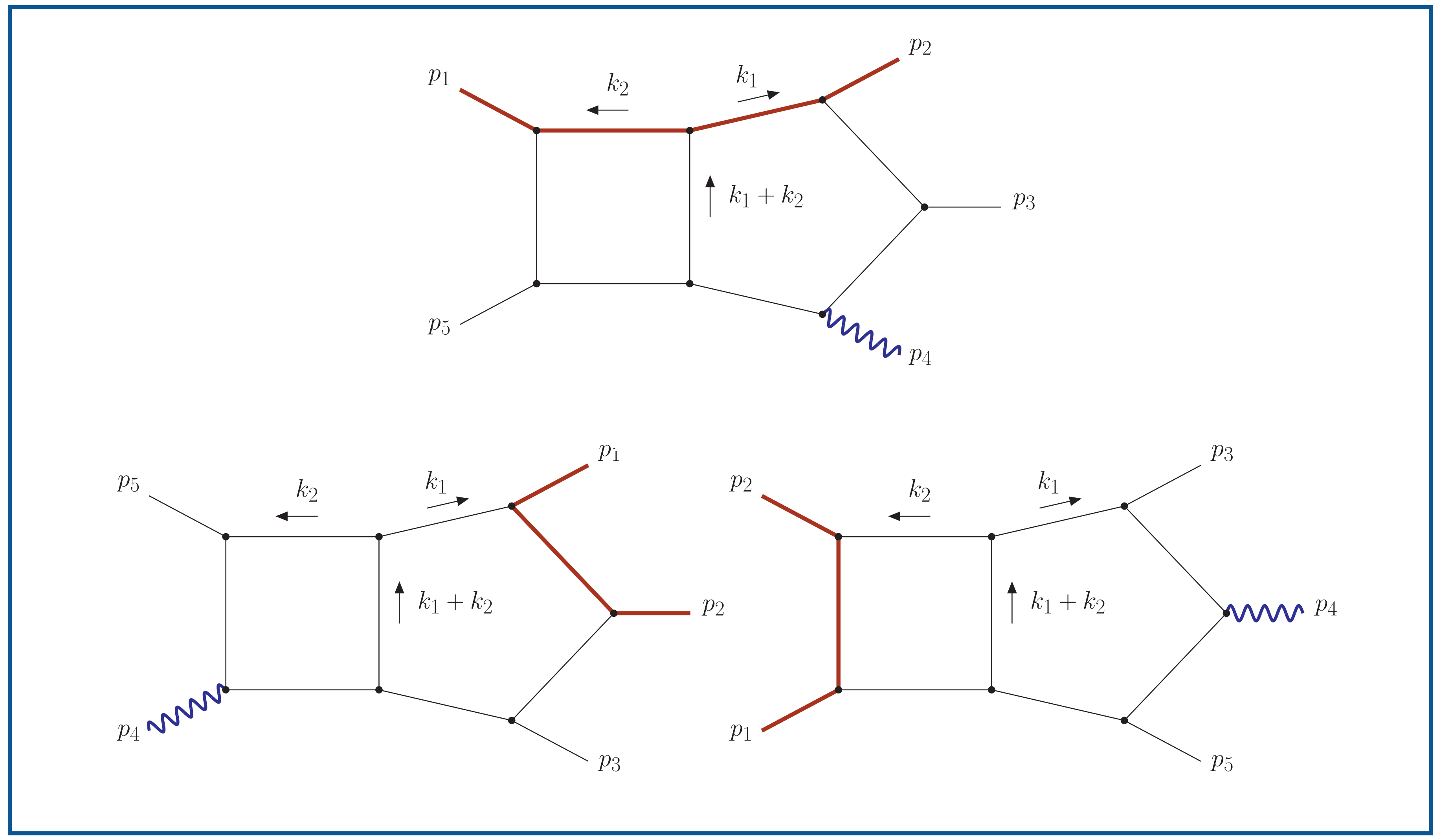


Chen, Li, Sang 25;

Highlight: Leading-Colour $t\bar{t}W$ @ $\mathrm{NNLO}_{\mathrm{QCD}}$

Complete set of master integrals for NLO $t\bar{t}W$ production

$2 \to 3$ process with internal + external top mass, external W mass



Becchetti, Canko, Chestnov, Peraro, Pozzoli, Zoia 25

Master Integrals

Very challenging, nested square-roots and elliptic curves

Detailed study of structure of MIs and appearance of elliptic curves

Obtain ϵ-factorised form for sectors without nested sq-roots, elliptic curves

$\mathrm{d}A$ matrix free of spurious denominator factors*

Numerical evaluation via generalised series expansions DiffExp

Moriello 20; Hidding 21

family	basis size	elliptic curves	nested square roots	entries	letters	non-d log one-forms
F_1	141	2 (figs. 3a and 3b)	1 (fig. 2)	2339	101	119
F_2	122	1 (fig. 3c)	0	2027	122	84
F_3	131	1 (fig. 3c)	0	2333	137	96

Challenges

~~Overview~~ Challenges

Fixed Order Calculations

Integral Reduction

Computing Integrals

II.

Sampling Phase-space

Subtraction

Process Simulation & Modelling

Resummation

Parton Showers

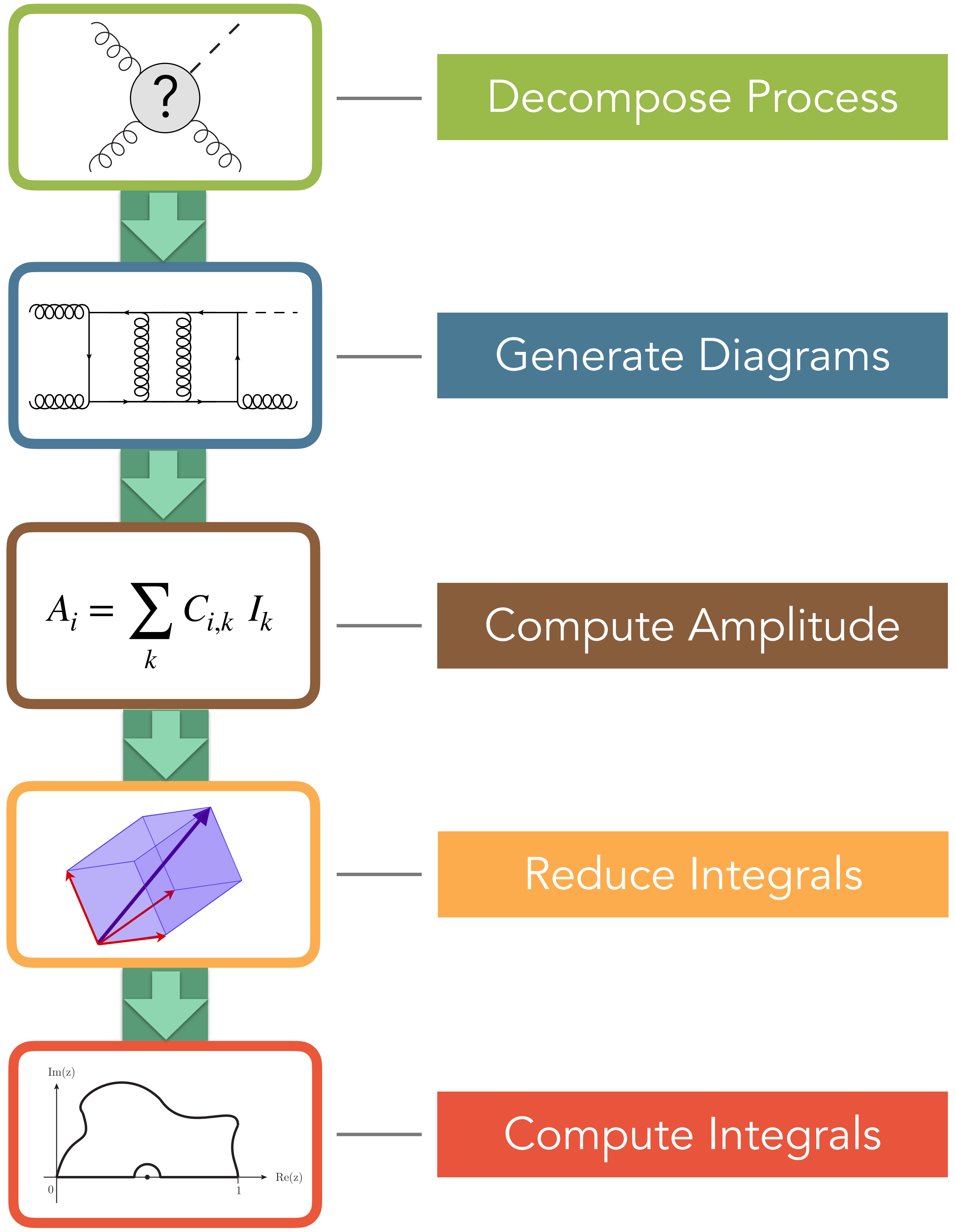
Negative Events

Building Predictions

Parton Distribution Functions (PDFs)

Estimating Uncertainties

I.



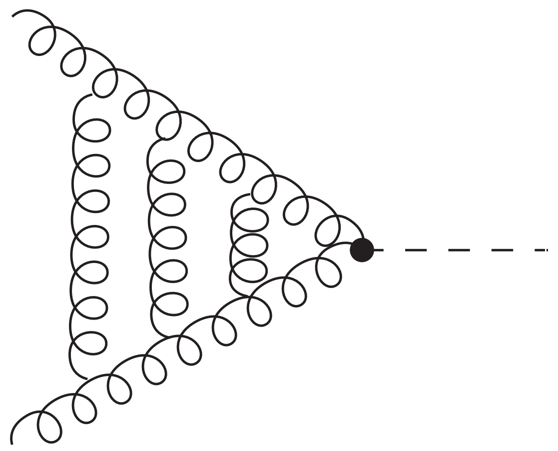
Example 1: Higgs Production in ggF

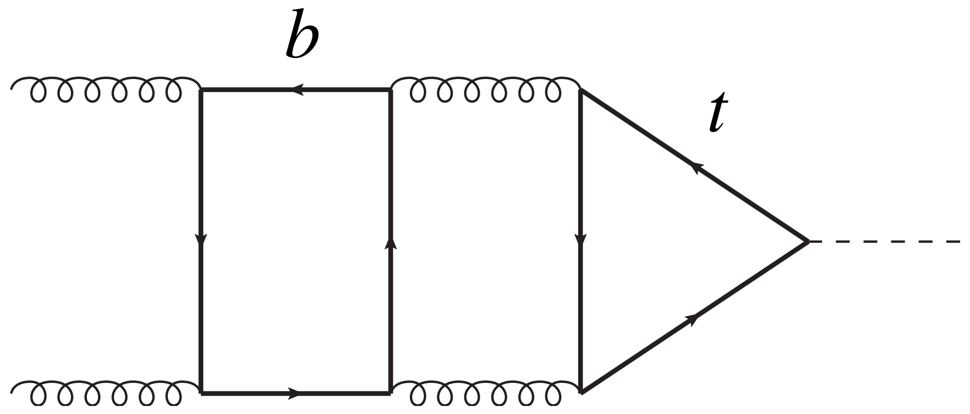
LHCHWG ggF (Yellow Report 5 Preparation)

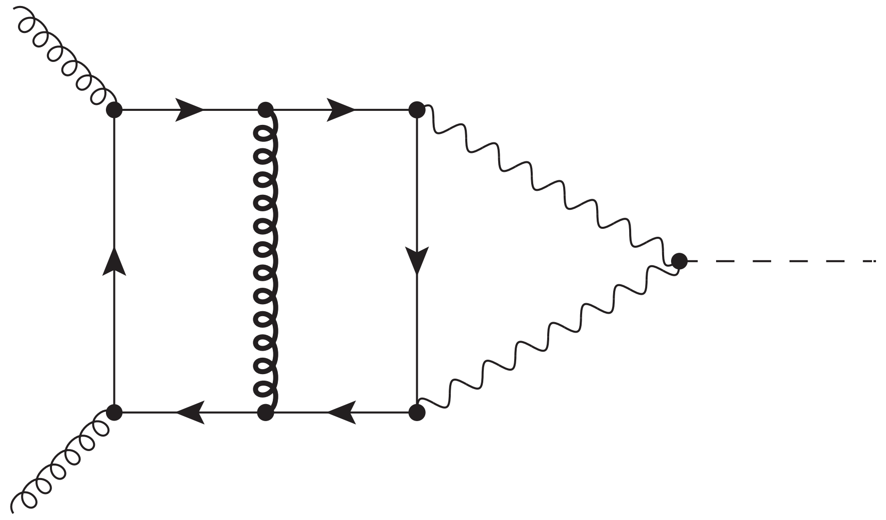
Theoretical uncertainties dominate experimental sys/stat even for σ_{tot}, **it is critically important that we improve theory precision**

ATLAS-CONF-2025-006; CMS-PAS-HIG-21-018

Calculations	**References**
Approximate N^4LO HTL QCD	[1]
N^3LO HTL QCD	[2,3,4]
NNLO HTL QCD	[5,6,7]
NLO HTL QCD	[8,9]
NNLO QCD	[10,11,12,13]
NLO QCD	[14,15]
EW & Mixed QCD-EW Corrections	[16,17,18,19]
N^3LL Threshold Resummation	[20,21]
PDFs	**References**
Approximate N^3LO PDFs	[22,23,24,25]
QED Evolution PDFs	[26,27,28,29,30,31,32]
NNLO PDFs	[33,34,35,36,37]
Codes	**References**
N^3LO QCD	[38]







[1] Das, Moch, Vogt 20; [2,3] Anastasiou, Duhr, Dulat, (Furlan, Gehrmann), Herzog, (Lazopoulos), Mistlberger 15, (16); [4] Mistlberger 18; [5] Anastasiou, Melnikov 02; [6] Harlander, Kilgore 02; [7] Ravindran, Smith, van Neerven 03; [8] Dawson 91; [9] Djouadi, Spira, Zerwas 91; [10] Czakon, Niggetiedt 20; [11] Czakon, Harlander, Klappert, Niggetiedt 21; [12,13] Czakon Eschment, Niggetiedt (Poncelet, Schellenberger) 24 (24);[14] Graudenz, Spira, Zerwas 93; [15] Spira Djouadi, Graudenz, Zerwas 95; [16,17] Actis, Passarino, Sturm, Uccirati 08, 09; [18] Aglietti, Bonciani, Degrassi, Vicini 04; [19] Becchetti, Bonciani, Del Duca, Hirschi, Moriello, Schweitzer 21; [20] Bonvini, Marzani 14; [21] Catani, Cieri, de Florian, Ferrera, Grazzini 14; [22] MSHT 22; [23] NNPDF 24; [24] Benchmarking 24; [25] MSHT+NNPDF 25; [26,27] Manohar, Nason, Salam Zanderighi 16, 17; [28,29] NNPDF 18, 24; [30,31] MSHT 19, 22, 24; [32] CTEQ 22; [33] PDF4LHC21; [34] CTEQ 21; [35] MSHT 21; [36] ABM 17; [37] NNPDF 17; [38] iHixs 2 18

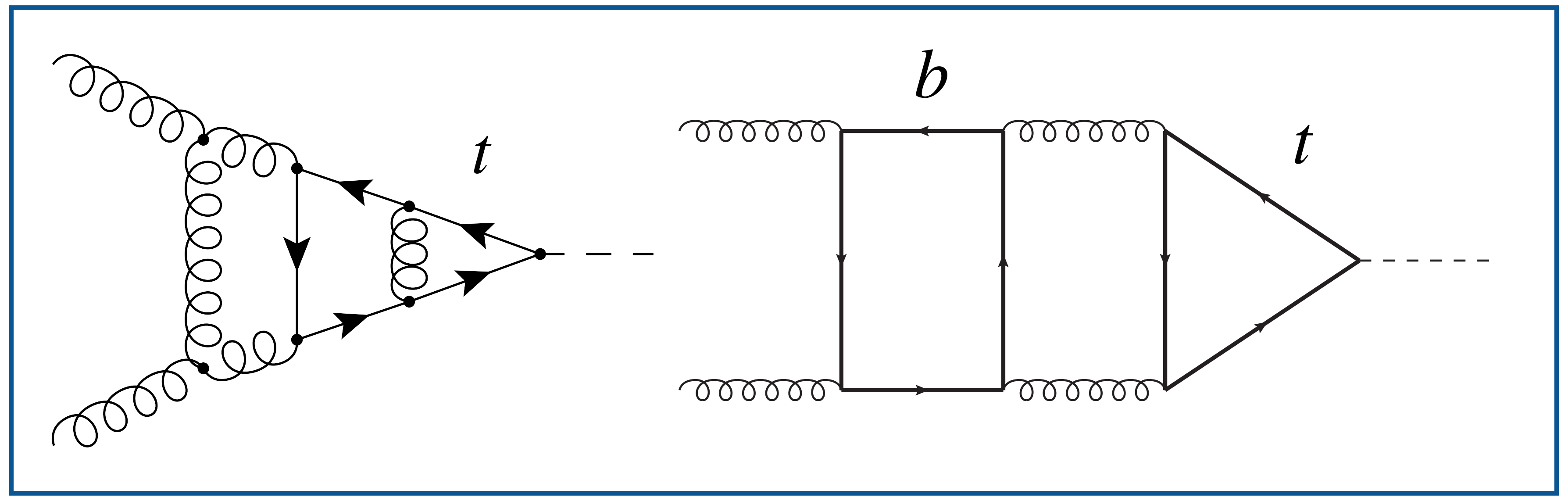
Example 1: NNLO Mass Corrections

Top-Mass Corrections

Requires H @ 3-loop & H+j @ 2-loop

Computed using numerical solution of differential equations

Czakon, Niggetiedt 20;
Czakon, Harlander, Klappert, Niggetiedt 21



Decreases σ_{tot} by $-0.26\,\%$ @ 13 TeV compared to heavy top limit (HTL)

Top-Bottom Interferences

H @ 3-loop with two different quark masses (m_t, m_b) in the on-shell and $\overline{\text{MS}}$ quark mass scheme

Niggetiedt, Usovitsch 23; Czakon, Eschment, Niggetiedt, Poncelet, Schellenberger 23, 24

290 MIs computed by combining asymptotic expansion $m_b^2 \ll m_H^2 \ll m_t^2$ + AMFlow

Liu, Ma, Wang 17; Liu, Ma 22, 22

Order	$(\sigma_t^{\overline{\text{MS}}} - \sigma_t^{\text{OS}})$ [pb]
$\sqrt{s} = 13$ TeV	
$\mathcal{O}(\alpha_s^2)$	-0.04
LO	$-0.04^{+0.12}_{-0.17}$
$\mathcal{O}(\alpha_s^3)$	$+0.02$
NLO	$-0.02^{+0.14}_{-0.30}$
$\mathcal{O}(\alpha_s^4)$	$+0.01$
NNLO	$-0.01^{+0.12}_{-0.24}$

Order	$\sigma_{t\times b}$ [pb]			
$\sqrt{s} = 13$ TeV				
	5FS $m_t = 173.06$ GeV $\overline{m}_b(\overline{m}_b) = 4.18$ GeV	5FS $m_t = 173.06$ GeV $m_b = 4.78$ GeV	5FS $m_t(m_t) = 162.7$ GeV $\overline{m}_b(\overline{m}_b) = 4.18$ GeV	4FS $m_t = 173.06$ GeV $\overline{m}_b(\overline{m}_b) = 4.18$ GeV
$\mathcal{O}(\alpha_s^2)$	-1.11	-1.98	-1.12	-1.15
LO	$-1.11^{+0.28}_{-0.43}$	$-1.98^{+0.38}_{-0.53}$	$-1.12^{+0.28}_{-0.42}$	$-1.15^{+0.29}_{-0.45}$
$\mathcal{O}(\alpha_s^3)$	-0.65	-0.44	-0.64	-0.66
NLO	$-1.76^{+0.27}_{-0.28}$	$-2.42^{+0.19}_{-0.12}$	$-1.76^{+0.27}_{-0.28}$	$-1.81^{+0.28}_{-0.30}$
$\mathcal{O}(\alpha_s^4)$	$+0.02$	$+0.43$	-0.02	-0.02
NNLO	$-1.74(2)^{+0.13}_{-0.03}$	$-1.99(2)^{+0.29}_{-0.15}$	$-1.78(1)^{+0.15}_{-0.03}$	$-1.83(2)^{+0.14}_{-0.03}$

Example 1: Approximate N^3LO PDFs

Would like to match the N^3LO matrix elements with N^3LO Parton Distribution Functions

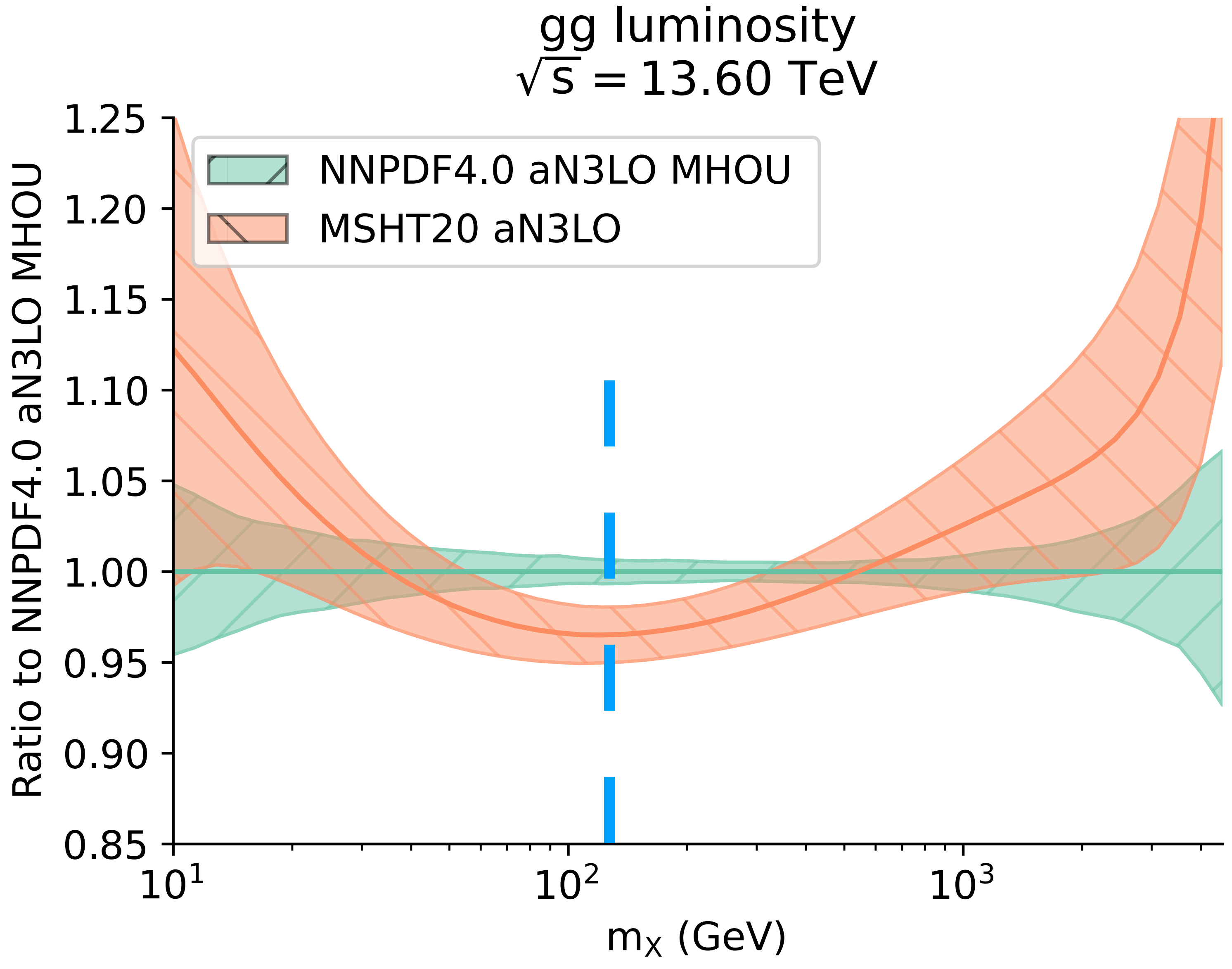
Fitting higher-order PDFs relies on many fixed-order/multi-loop ingredients:

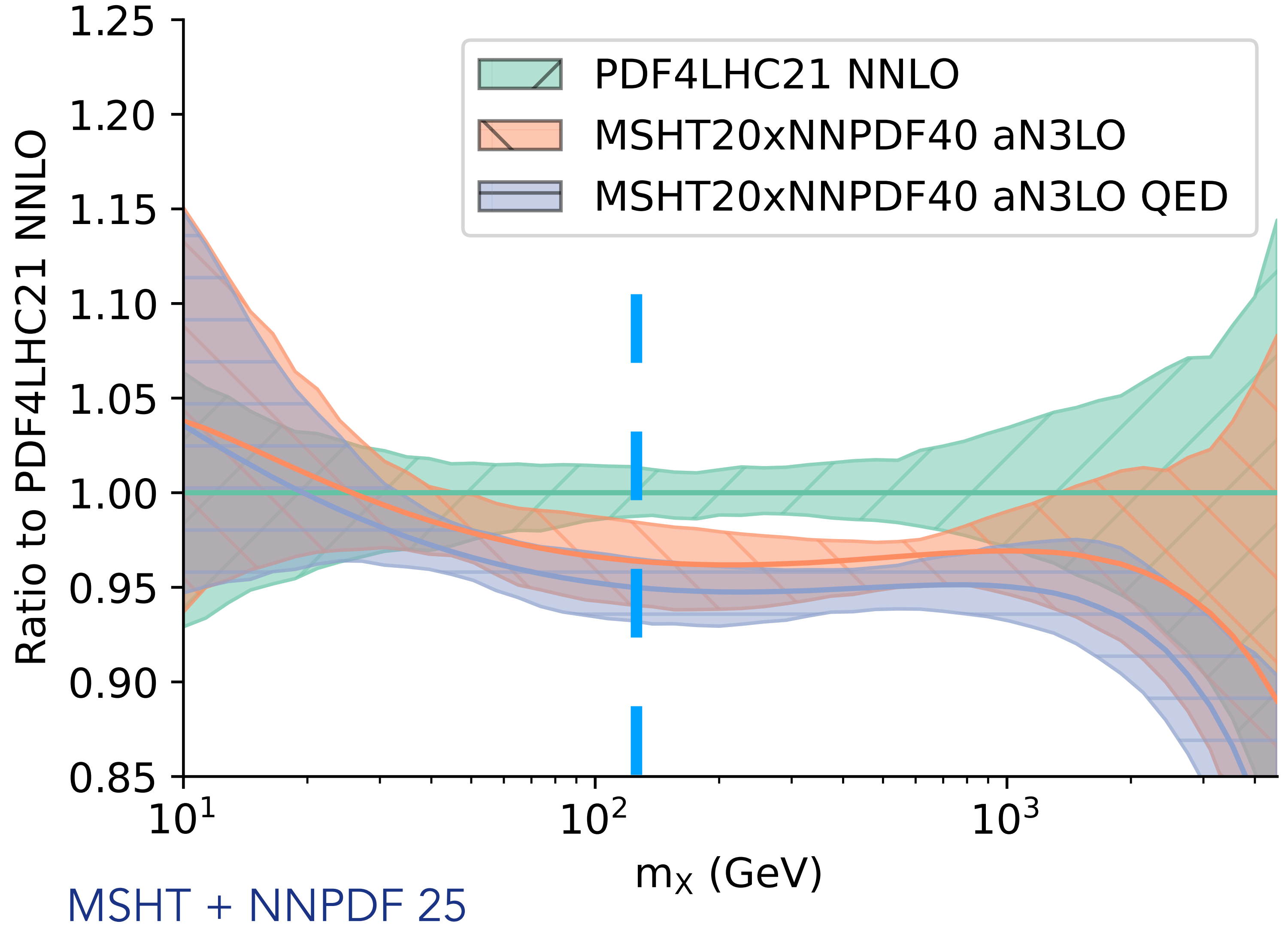
- **DIS Coefficient Functions Massless (KNOWN) / Massive (APPROX)**
- **4-loop Splitting Functions (APPROX) (Mellin moments + large/small-x limits)**
- **Transition matrix elements (KNOWN)**
- **N^3LO hadronic matrix elements (UNKNOWN)**

Fits enhance their uncertainties to account for this:
NNPDF use scale variations
MSHT use extra nuisance terms

Approximate N^3LO PDFs have been produced and combined by MSHT & NNPDF using the partial theory input available at the time of the fits

MSHT 22; NNPDF 24; Benchmarking 24; MSHT+NNPDF 25;





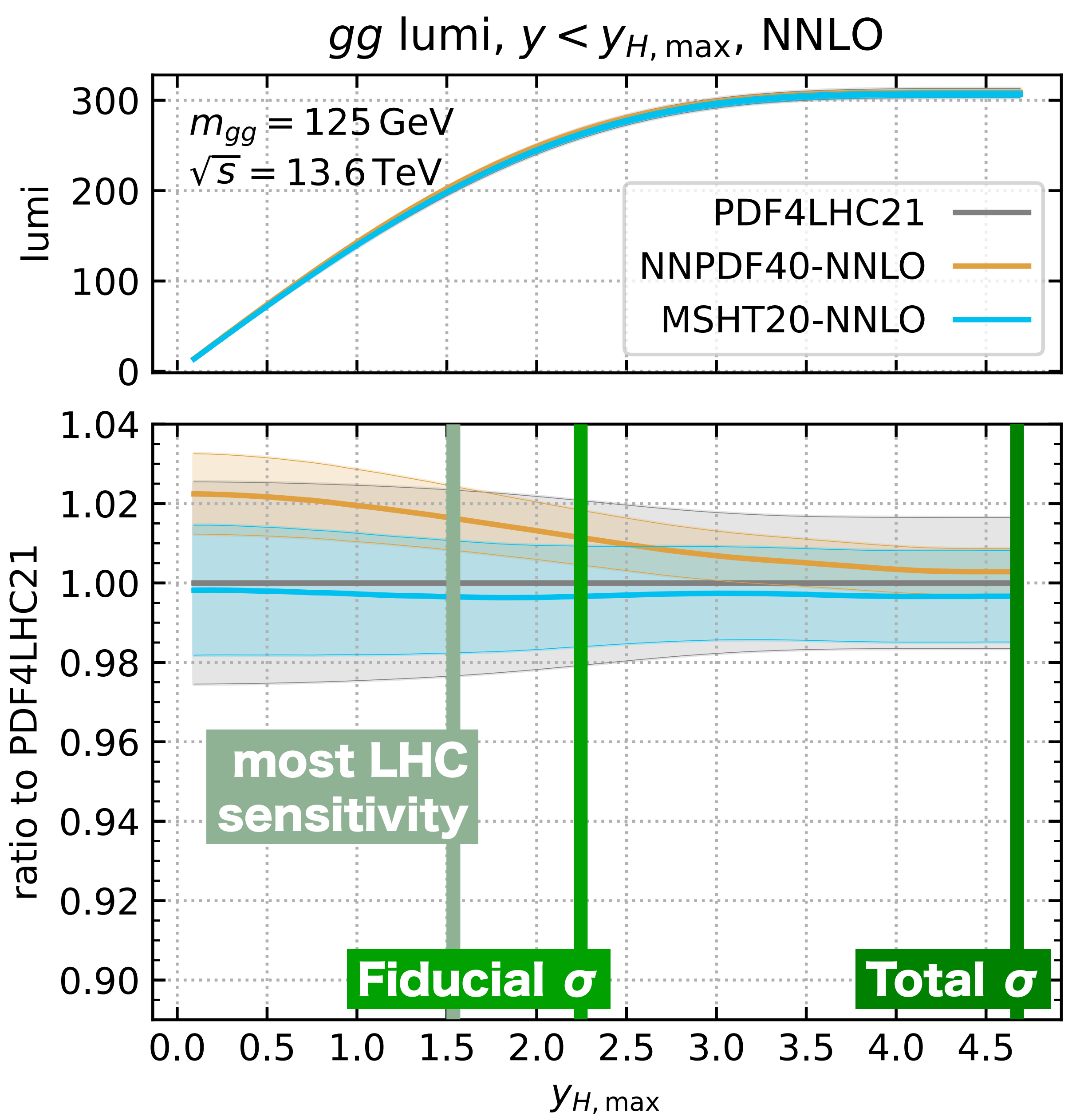
MSHT + NNPDF 25

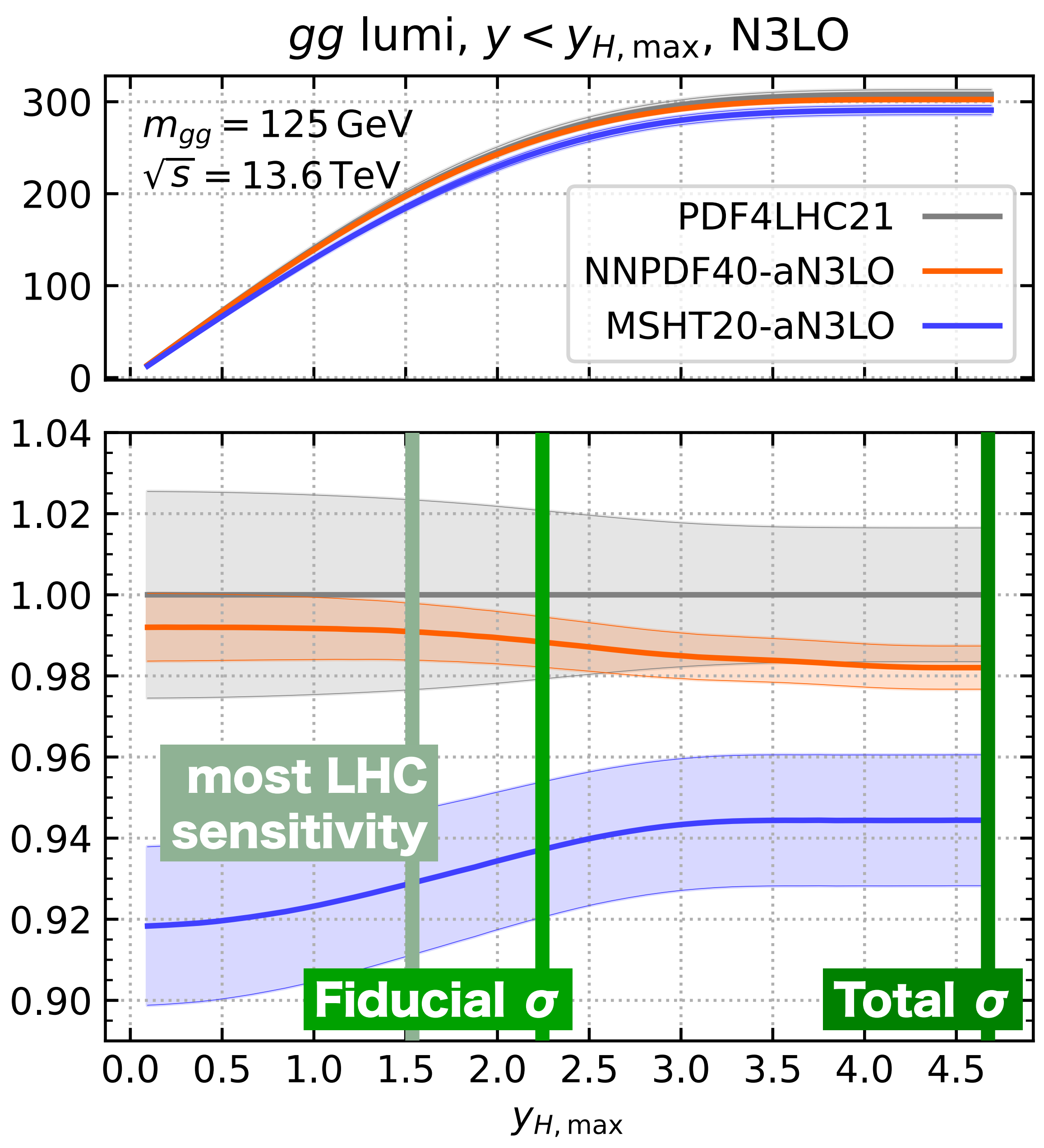
Example 1: Approximate $\mathrm{N^3LO}$ PDFs

	(aN3LO PDF ⊗ N3LO HTL) vs (NNLO PDF ⊗ N3LO HTL)					
	aN3LO vs NNLO			aN3LO vs PDF4LHC21 NNLO		
$\sqrt{s}$ [TeV]	MSHT20xNNPDF40	MSHT20	NNPDF40	MSHT20xNNPDF40	MSHT20	NNPDF40
7	−4.2%	−6.0%	−2.0%	−5.2%	−6.8%	−3.2%
13.6	−3.8%	−5.2%	−2.1%	−4.2%	−5.9%	−2.2%
100	−0.7%	+0.6%	−1.6%	−1.0%	−0.7%	−1.0%
	(aN3LO PDF ⊗ N3LO HTL) vs (NNLO PDF ⊗ NNLO HTL)					
	aN3LO vs NNLO			aN3LO vs PDF4LHC21 NNLO		
$\sqrt{s}$ [TeV]	MSHT20xNNPDF40	MSHT20	NNPDF40	MSHT20xNNPDF40	MSHT20	NNPDF40
7	−0.8%	−2.6%	+1.5%	−1.8%	−3.5%	+0.3%
13.6	−0.5%	−1.9%	+1.3%	−0.9%	−2.6%	+1.2%
100	+3.1%	+4.4%	+2.1%	+2.8%	+3.1%	+2.8%

$\mathrm{aN^3LO}$ vs NNLO PDFs has a large impact on the total cross-section around LHC energies

Using matched order for PDF and matrix element gives smaller shifts except at higher collider energies





Salam (Higgs Hunting 2025)

Differences between sets larger at $\mathrm{aN^3LO}$ (3-4%) than at NNLO (1-2%), adding rapidity cuts further increases the size of the differences (~6%)

What is the right thing to do?

How should we assign a PDF-TH uncertainty to this?

Example 1: ggF Summary

Final result receives contributions from many sources

Uncertainties related to finite masses greatly reduced, QED evolution effects should now be included in central recommendation

$$\delta(\text{theory}) = \delta(\text{scale}) + \delta(\text{EWK}) + \delta(t,b,c)$$

$$\delta(t,b,c) = \delta^{\text{scheme}}(t) + \delta^{\text{MHOU}}(t) + \delta^{\text{scheme}}(t\times b) + \delta^{\text{MHOU}}(t\times b) + \delta^{\text{MHOU}}(b,c,t\times c,b\times c)$$

Negligible $\pm 0.17\,\%$ $\pm 0.26\,\%$ $\pm 0.17\,\%$ Negligible

$$\delta(\text{PDF}+\alpha_s) = \sqrt{\delta(\text{PDF})^2 + \delta(\alpha_s)^2}$$

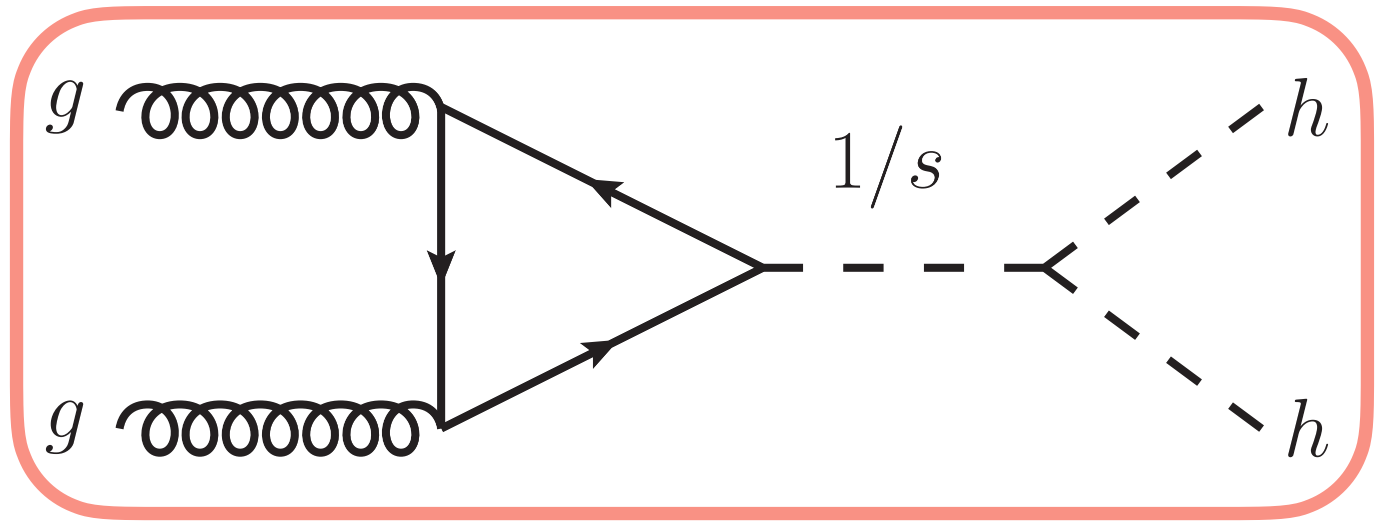
Estimated using NLO vs NNLO PDFs
Use NNLO vs aN3LO? (will be ~1.5x larger)

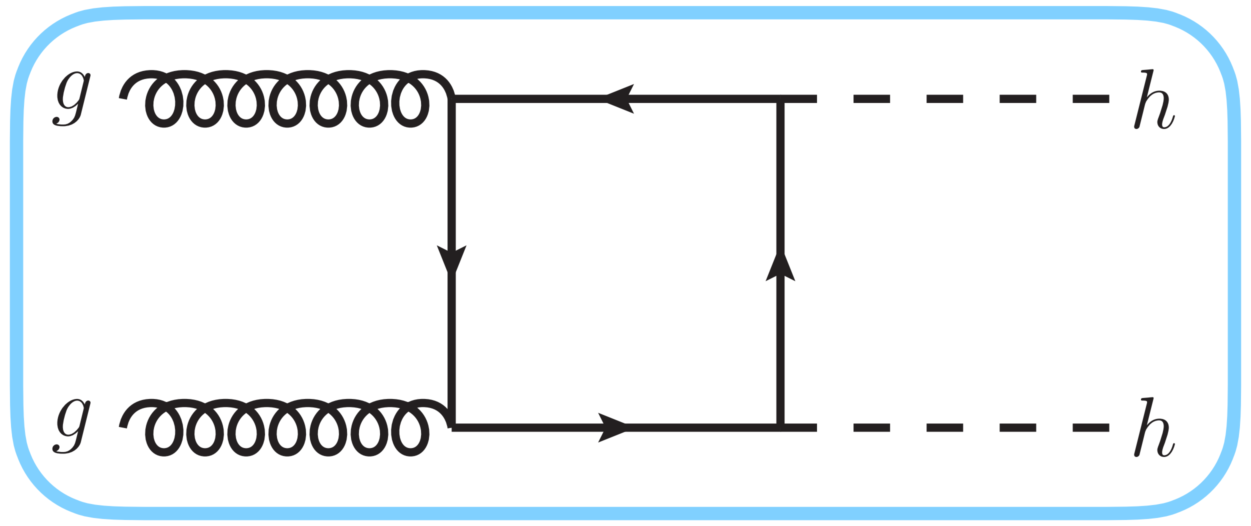
$\sqrt{s} = 13.6\,\text{TeV}$						NNLO PDF						aN³LO PDF	
M_{H} [GeV]	σ [pb]	δ(theory)	δ(scale)	δ(EWK)	δ(t, b, c)	$\delta(\text{PDF}+\alpha_s)$	δ(PDF)	$\delta(\alpha_s)$	δ(PDF–TH)	Δ(QED)	Δ [pb]	$\delta(\text{PDF}+\alpha_s)$	Δ(QED)
120.00	55.89	$^{+1.91}_{-4.92}\%$	$^{+0.31}_{-3.32}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.68}_{-2.27}\%$	$^{+1.65}_{-1.65}\%$	$^{+2.12}_{-1.57}\%$	$\pm2.42\%$	-1.09%	-2.13	$^{+2.07}_{-2.07}\%$	-1.45%
122.00	54.31	$^{+1.92}_{-4.91}\%$	$^{+0.32}_{-3.31}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.68}_{-2.27}\%$	$^{+1.65}_{-1.65}\%$	$^{+2.11}_{-1.56}\%$	$\pm2.40\%$	-1.10%	-2.08	$^{+2.07}_{-2.07}\%$	-1.45%
124.00	52.79	$^{+1.91}_{-4.91}\%$	$^{+0.31}_{-3.31}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.68}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.56}\%$	$\pm2.37\%$	-1.10%	-2.03	$^{+2.07}_{-2.07}\%$	-1.45%
124.60	52.34	$^{+1.91}_{-4.91}\%$	$^{+0.31}_{-3.31}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.56}\%$	$\pm2.36\%$	-1.11%	-2.01	$^{+2.07}_{-2.07}\%$	-1.45%
124.80	52.20	$^{+1.91}_{-4.91}\%$	$^{+0.31}_{-3.31}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.36\%$	-1.11%	-2.01	$^{+2.07}_{-2.07}\%$	-1.45%
125.00	52.05	$^{+1.91}_{-4.91}\%$	$^{+0.31}_{-3.31}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.36\%$	-1.11%	-2.00	$^{+2.07}_{-2.07}\%$	-1.45%
125.09	51.98	$^{+1.91}_{-4.91}\%$	$^{+0.31}_{-3.31}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.36\%$	-1.11%	-2.00	$^{+2.07}_{-2.07}\%$	-1.45%
125.20	51.90	$^{+1.91}_{-4.91}\%$	$^{+0.31}_{-3.31}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.35\%$	-1.11%	-2.00	$^{+2.07}_{-2.07}\%$	-1.46%
125.30	51.83	$^{+1.91}_{-4.90}\%$	$^{+0.31}_{-3.30}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.35\%$	-1.11%	-1.99	$^{+2.08}_{-2.08}\%$	-1.46%
125.38	51.77	$^{+1.91}_{-4.90}\%$	$^{+0.31}_{-3.30}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.35\%$	-1.11%	-1.99	$^{+2.08}_{-2.08}\%$	-1.46%
125.60	51.62	$^{+1.91}_{-4.90}\%$	$^{+0.31}_{-3.30}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.35\%$	-1.11%	-1.99	$^{+2.08}_{-2.08}\%$	-1.46%
126.00	51.33	$^{+1.91}_{-4.90}\%$	$^{+0.31}_{-3.30}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.11}_{-1.55}\%$	$\pm2.34\%$	-1.11%	-1.98	$^{+2.08}_{-2.08}\%$	-1.46%
128.00	49.93	$^{+1.90}_{-4.89}\%$	$^{+0.30}_{-3.29}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.26}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.10}_{-1.55}\%$	$\pm2.32\%$	-1.12%	-1.93	$^{+2.08}_{-2.08}\%$	-1.46%
130.00	48.59	$^{+1.90}_{-4.88}\%$	$^{+0.30}_{-3.28}\%$	$\pm1.00\%$	$\pm0.60\%$	$^{+2.67}_{-2.25}\%$	$^{+1.64}_{-1.64}\%$	$^{+2.10}_{-1.54}\%$	$\pm2.30\%$	-1.13%	-1.88	$^{+2.08}_{-2.08}\%$	-1.46%

Example 2: Higgs Boson Self-Coupling

$$\mathcal{L} \supset -V(\phi), \qquad V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2 \quad \xrightarrow{\text{EWSB}} \quad V(H) = \frac{1}{2}m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

Higgs boson pair production provides us with a direct experimental handle on this coupling



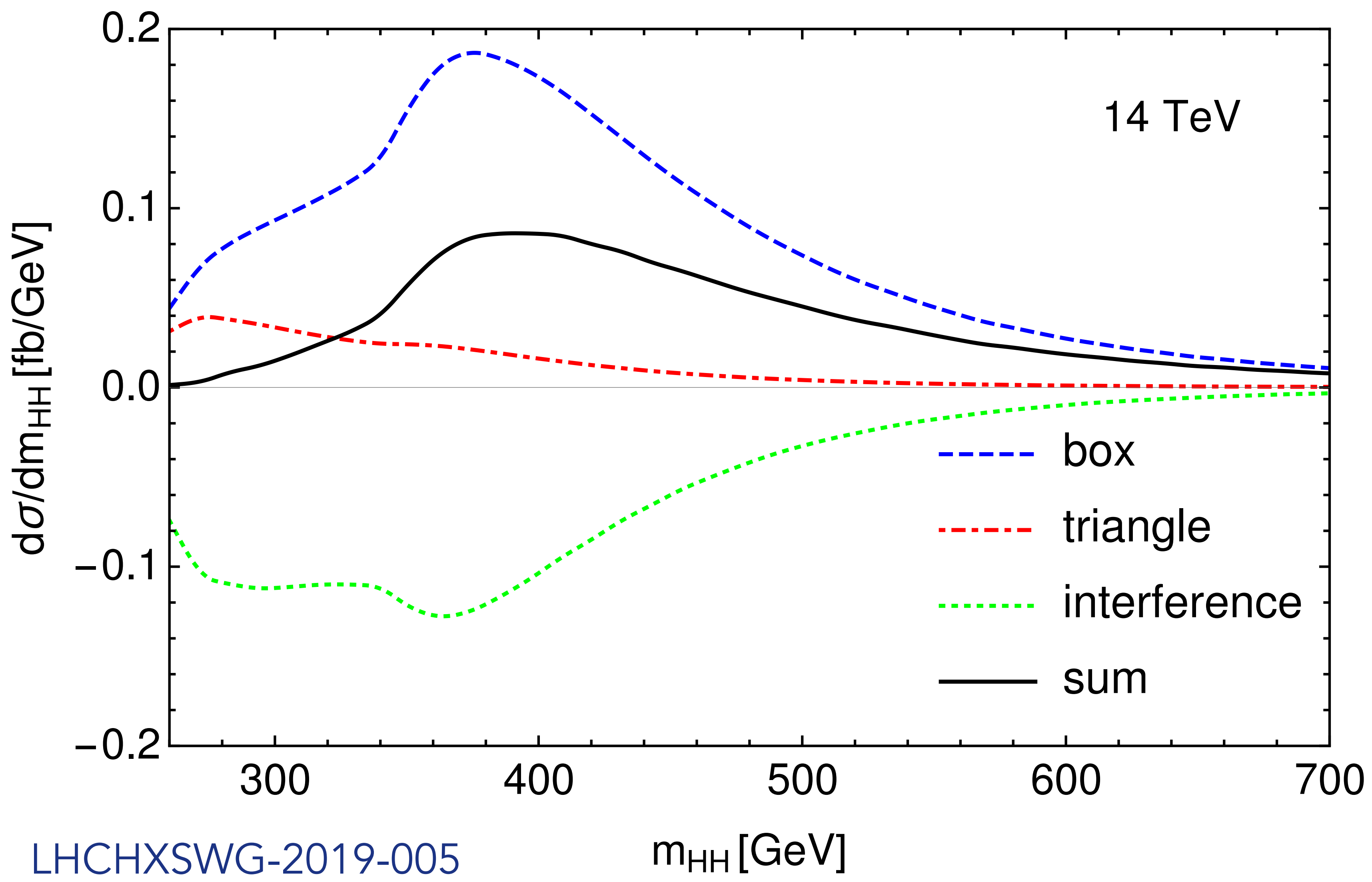


Structure of QCD corrections to the amplitude

$$\mathcal{M} = \varepsilon_{1,\mu}\,\varepsilon_{2,\nu}\,\delta^{AB}\left(A_1 P_1^{\mu\nu} + A_2 P_2^{\mu\nu}\right)$$

$$A_1 = T_F \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{2\pi} s \left[\frac{3m_H^2}{s - m_H^2} A_{1,y_t\lambda} + A_{1,y_t^2}\right]$$

$$A_2 = T_F \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{2\pi} s \left[A_{2,y_t^2}\right]$$



LHCHXSWG-2019-005

Example 2: Higgs Boson Self-Coupling

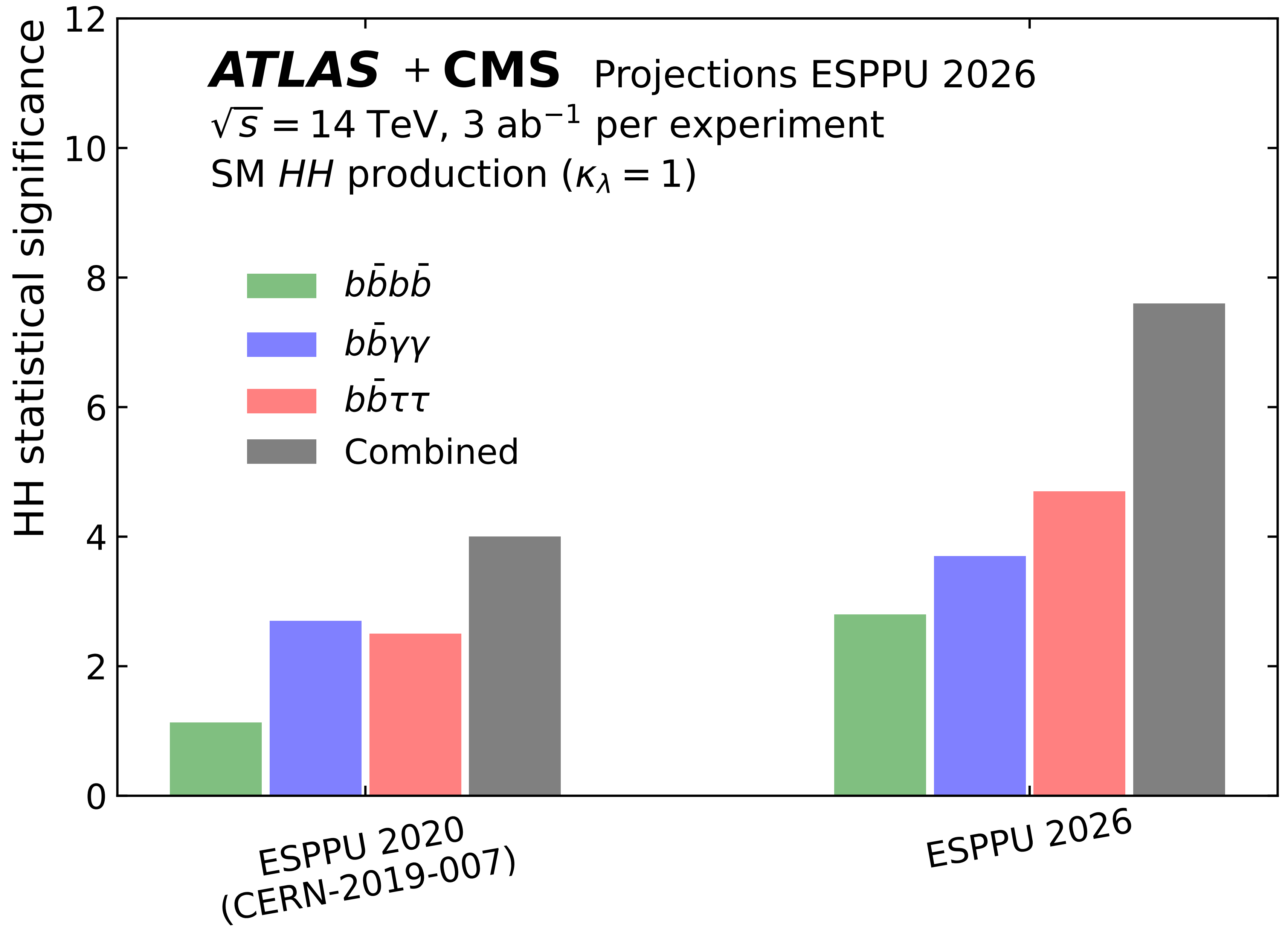
Latest HL-LHC projections:

Anticipate 7σ measurement of HH

Precision on $\kappa_3 < 30\,\%$

Potential for further analysis improvements

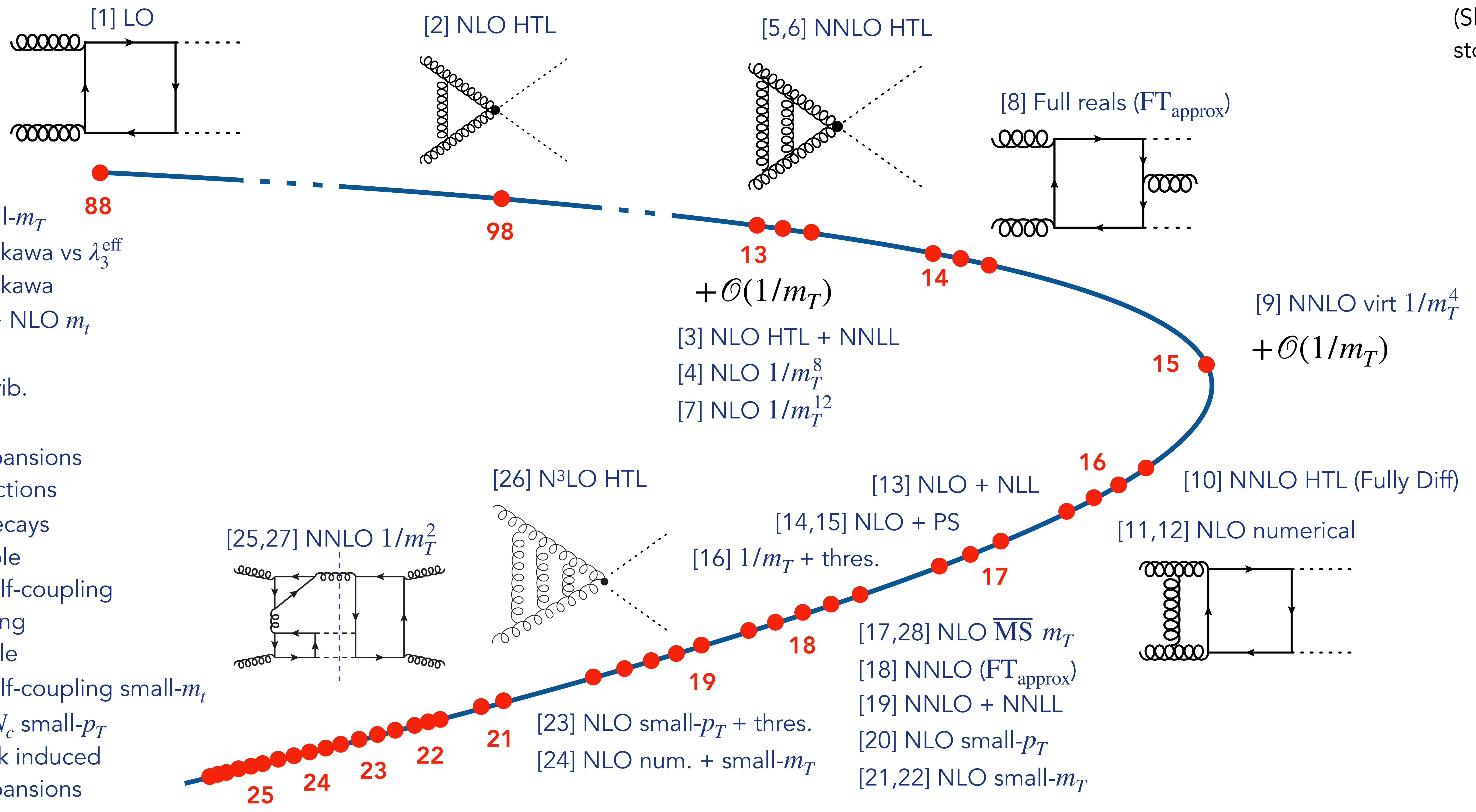
→ **Steven (Tuesday)**



ATL-PHYS-PUB-2025-018; CMS-HIG-25-002

Example 2: HH Production Overview

(Slide design shamelessly stolen from G. Salam)



[29] small-p_T + small-m_T
[30] EW: leading Yukawa vs λ_3^{eff}
[31] EW: leading Yukawa
[32] N³LO + N³LL + NLO m_t
[33] small-t
[34] NNLO: n_f contrib.
[35] EW: $1/m_T$
[36] NLO comb expansions
[37] NLO EW corrections
[38] NLO + $b\bar{b}\gamma\gamma$ decays
[39] NNLO: reducible
[40] EW: Yukawa/self-coupling
[41] EW: self-coupling
[42] EW: Factorizable
[43] EW: Yukawa/self-coupling small-m_t
[44] NNLO: Large-N_c small-p_T
[45] EW: Light quark induced
[46] NLO comb expansions

[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Mühlleitner, Schlenk, Spira 22; [31] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [32] Ajjath, Shao 22; [33] Davies, Mishima, Schönwald, Steinhauser 23; [34] Davies, Schönwald, Steinhauser 23; [35] Davies, Schönwald, Steinhauser, Zhang 23; [36] Bagnaschi, Degrassi, Gröber 23; [37] Bi, Huang, Huang, Ma Yu 23 [38] Li, Si, Wang, Zhang, Zhao 24; [39] Davies, Schönwald, Steinhauser, Vitti 24; [40] Heinrich, SPJ, Kerner, Stone, Vestner 24 [41] Li, Si, Wang, Zhang, Zhao 24; [42] Davies, Schönwald, Steinhauser, Zhang 24; [43] Davies, Schönwald, Steinhauser, Zhang 25; [44] Davies, Schönwald, Steinhauser 25; [45] Bonetti, Rendler, Bobadilla 25; [46] Davies, Schönwald, Stremmer 25;

Example 2: State-of-the Art

Known to ($\mathrm{NLO_{QCD}}$ + $\mathrm{NNLO_{FTapprox}}$ + $\mathrm{N^3LO_{HTL}}$ + $\mathrm{N^3LL}$) + $\mathrm{NLO_{EW}}$

Scale uncertainty $\sim 3\,\%$ (reweighted $\mathrm{N^3LO_{HTL}}$)

PDF+α_s uncertainty $\sim 2.5\,\%$

Can consider top quark mass in OS or $\overline{\mathrm{MS}}$ scheme

$$\frac{m(\mu)}{M} = \frac{Z_m^{\mathrm{OS}}}{Z_m^{\overline{\mathrm{MS}}}} \equiv \sum_{n\geq 0}\left(\frac{\alpha_s(\mu)}{2\pi}\right)^n \left(z_m^n(M) + z_m^{n,\log}(\mu)\right)$$

4-loop: Marquard, Smirnov, Smirnov, Steinhauser, Wellmann 16

Top quark mass scheme/scale is the **dominant theoretical uncertainty**

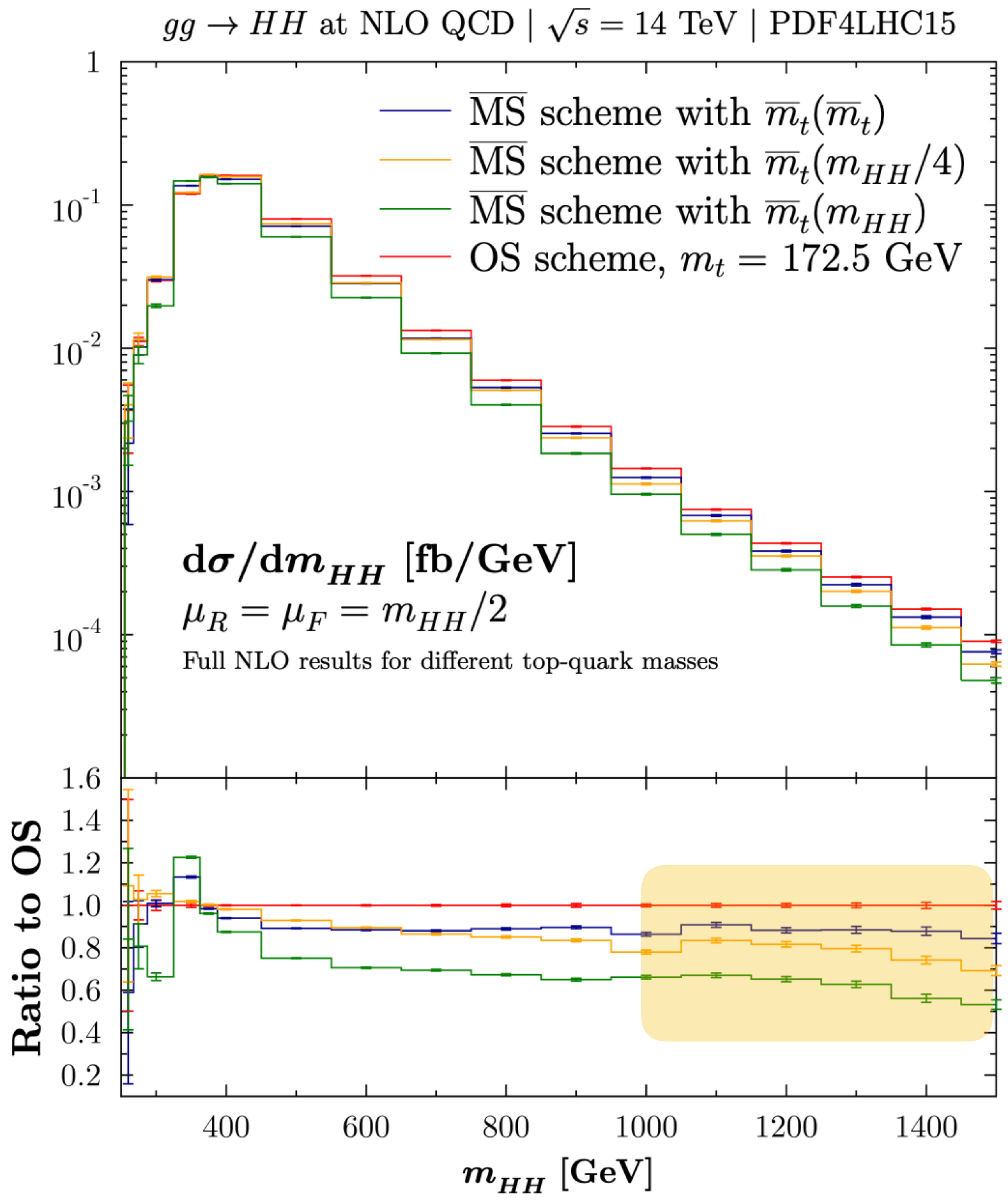
$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=300\ \mathrm{GeV}} = 0.02978(7)^{+6\%}_{-34\%}\ \mathrm{fb/GeV},$$

$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=400\ \mathrm{GeV}} = 0.1609(4)^{+0\%}_{-13\%}\ \mathrm{fb/GeV},$$

$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=600\ \mathrm{GeV}} = 0.03204(9)^{+0\%}_{-30\%}\ \mathrm{fb/GeV},$$

$$\left.\frac{d\sigma_{NLO}}{dQ}\right|_{Q=1200\ \mathrm{GeV}} = 0.000435(4)^{+0\%}_{-35\%}\ \mathrm{fb/GeV}$$

Large uncertainty comparing OS with $\overline{\mathrm{MS}}$ mass



Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 18, 20, 20

Example 2: Leading Power Expansion

Consider the LO and NLO finite virtual corrections

$$A^{\text{fin}}_{i,j} = \frac{\alpha_s}{2\pi} A^{(0)}_{i,j} + \left(\frac{\alpha_s}{2\pi}\right)^2 A^{(1)}_{i,j} + \mathcal{O}(\alpha_s^3)$$

Use ``SCET'' IR scheme for virtuals Becher, Neubert 09, 13;

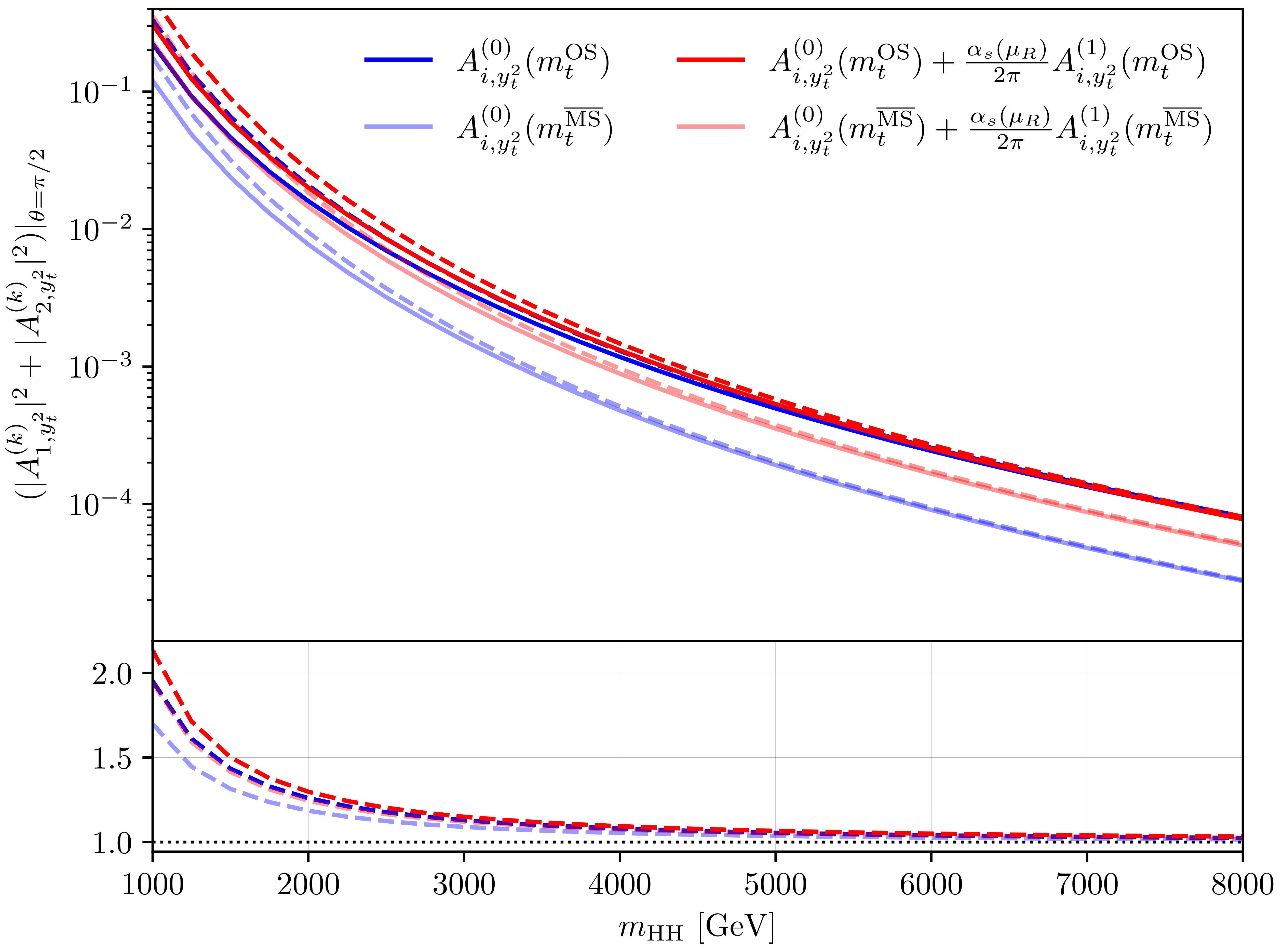
Neglecting real contributions

Solid Lines: Full TH result

Davies, Mishima, Steinhauser, Wellmann 18;

Dashed Lines: Leading power expansion

Leading power is a good approximation for $\sqrt{s} \gtrsim 1~\text{TeV}$, let us focus on the **very high energy** behaviour of the **amplitude**



Example 2: Structure of Box Amplitude @ High Energy

The $gg \to HH$ amplitude in the $\overline{\text{MS}}$ scheme has the following leading power structure

$l_\mu = \log(\mu_t^2/s)$

$l_m = \log(\mu_t^2/s), \log(m_t^2/s)$

$$\text{LO}: \alpha_s y_t^2 (c_0 + m_t n_0),$$

$$\text{NLO}: \alpha_s^2 y_t^2 (a_1 l_\mu + c_1 + m_t n_1),$$

Davies, Mishima, Steinhauser, Wellmann 18; Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 20

$$\text{NNLO}: \alpha_s^3 y_t^2 (a_2 l_\mu^2 + b_2 l_m + c_2 + m_t n_2),$$

Master integrals known

Caola, von Manteuffel, Tancredi 20; Bargiela, Caola, von Manteuffel, Tancredi 22;

$$\text{N}^3\text{LO}: \alpha_s^4 y_t^2 (a_3 l_\mu^3 + b_3 l_m^2 + d_3 l_m + c_3 + m_t n_3),$$

$$\text{N}^i\text{LO}: \alpha_s^{i-1} y_t^2 (a_i l_\mu^i + b_i l_m^{i-1} + d_i l_m^{i-2} + \ldots + c_i + m_t n_i)\,.$$

Leading log structure generated to all orders by RG running

$$m^{\text{LL}}(\mu) = M \exp\left[a_{\gamma_m}^{\text{LL}}(\mu)\right] z_m(M)$$

$$a_{\gamma_m}^{\text{LL}}(\mu) = \frac{3C_F}{2\beta_0} \ln\left(1 - \frac{\alpha_s(\mu)}{2\pi}\beta_0 \ln\left(\frac{\mu^2}{M^2}\right)\right)$$

LP LL

Known from RG running of top-quark mass

Jaskiewicz, SPJ, Szafron, Ulrich 25

LP NLL

RG running + massification

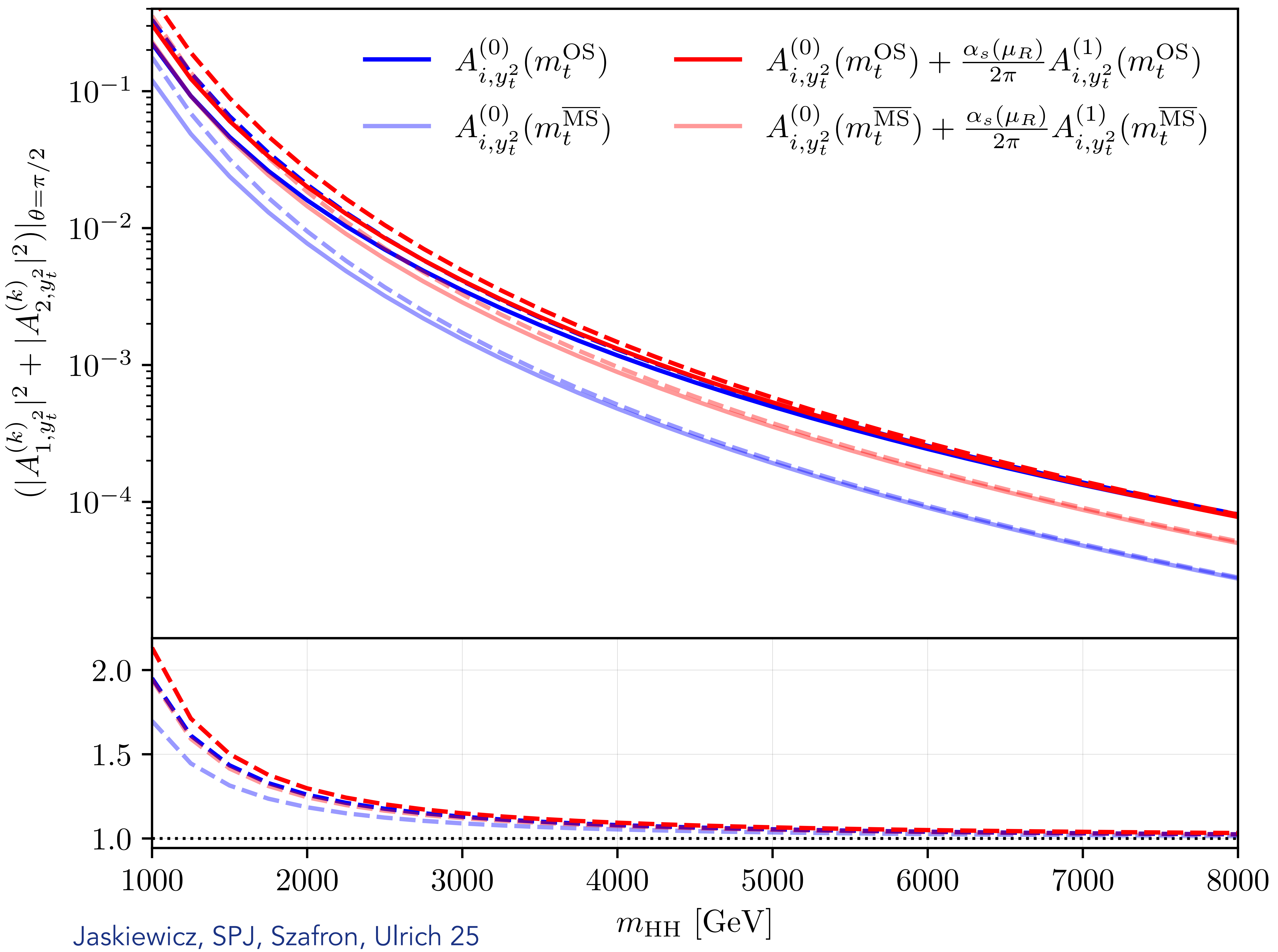
Penin 06; Moch, Mitov 07; Becher, Melnikov 07; Engel et al 19; Wang, Xia, Yang, Ye 23;

LP Constant

Hard region ($m_t = 0$) contribution only, known to NLO

Example 2: Mass Scheme Uncertainty

At high-energy the differences between OS and $\overline{\mathrm{MS}}$ in the box amplitudes is driven by the known LP LL terms, let's include them also for the OS



Jaskiewicz, SPJ, Szafron, Ulrich 25

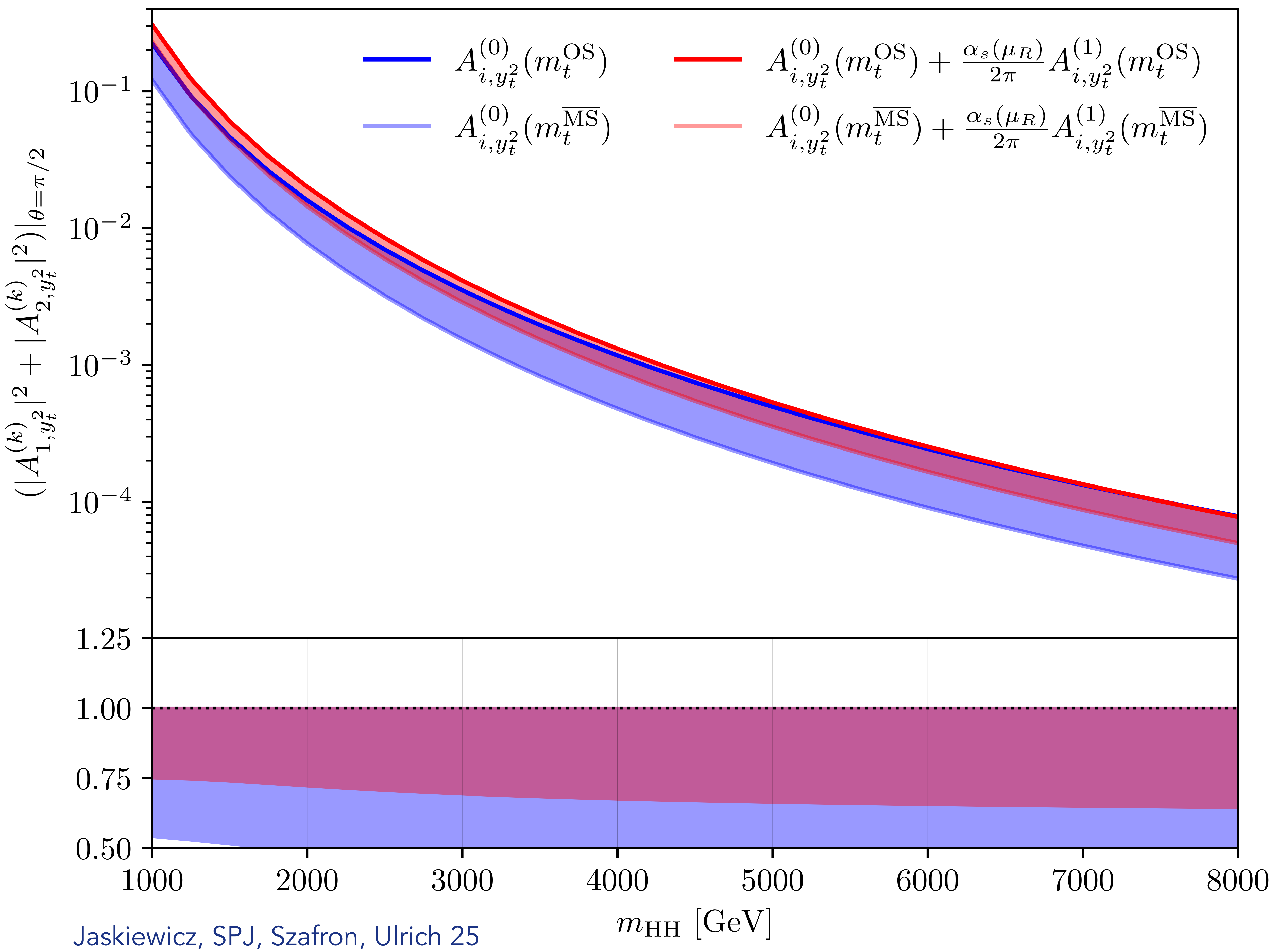
Example 2: Mass Scheme Uncertainty

At high-energy the differences between OS and $\overline{\mathrm{MS}}$ in the box amplitudes is driven by the known LP LL terms

Uncertainty

LO: ~70% (blue band) →

NLO: ~25% (red band) →



Jaskiewicz, SPJ, Szafron, Ulrich 25

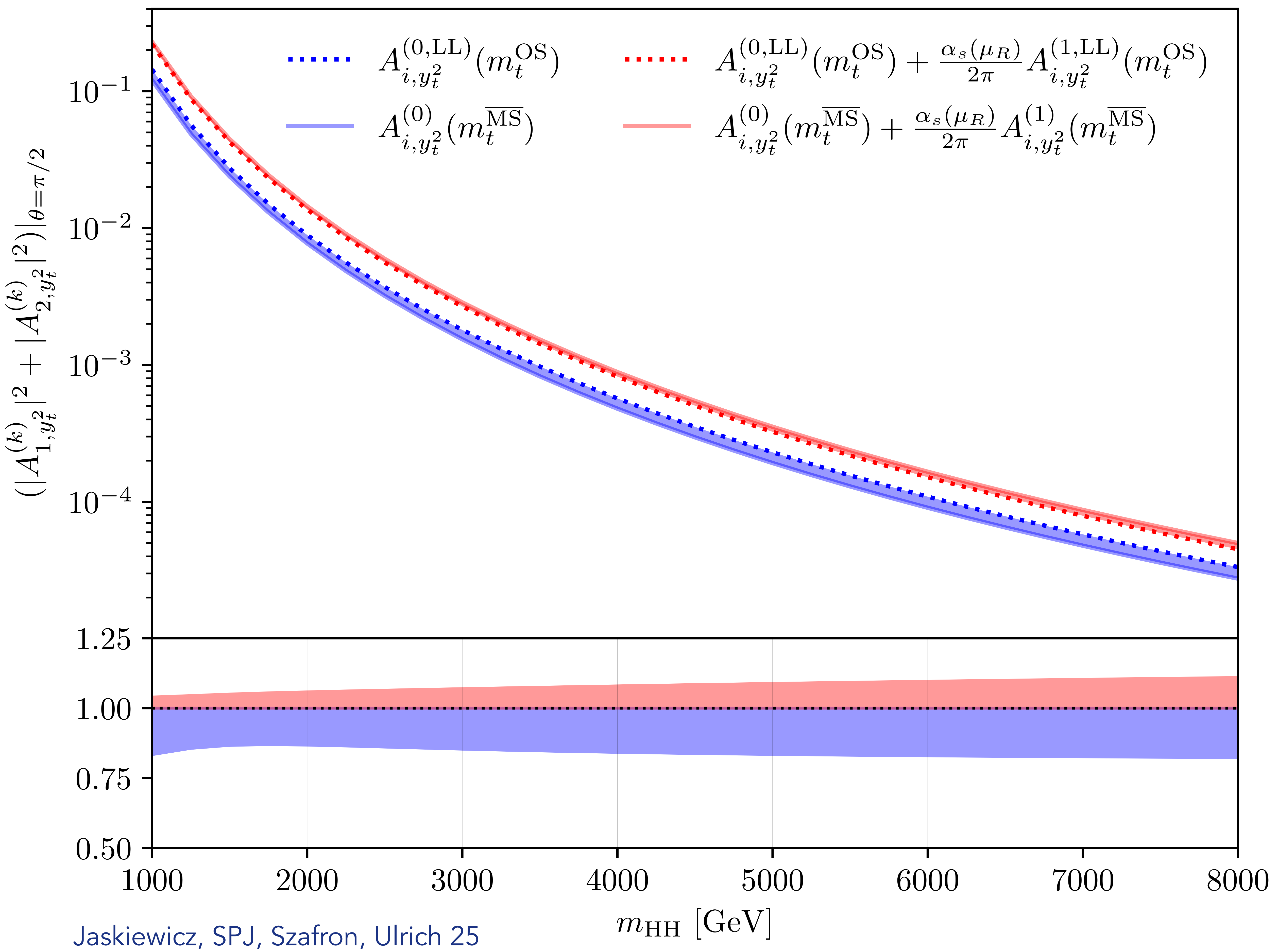
Example 2: Mass Scheme Uncertainty

At high-energy the differences between OS and $\overline{\mathrm{MS}}$ in the box amplitudes is driven by the known LP LL terms, let's include them also for the OS

Uncertainty

LO: ~70% → 15% (blue band)

NLO: ~25% → 7% (red band)



Jaskiewicz, SPJ, Szafron, Ulrich 25

Example 2: Mass Scheme Uncertainty

At high-energy the differences between OS and $\overline{\mathrm{MS}}$ in the box amplitudes is driven by the known LP LL terms, let's include them also for the OS

Uncertainty

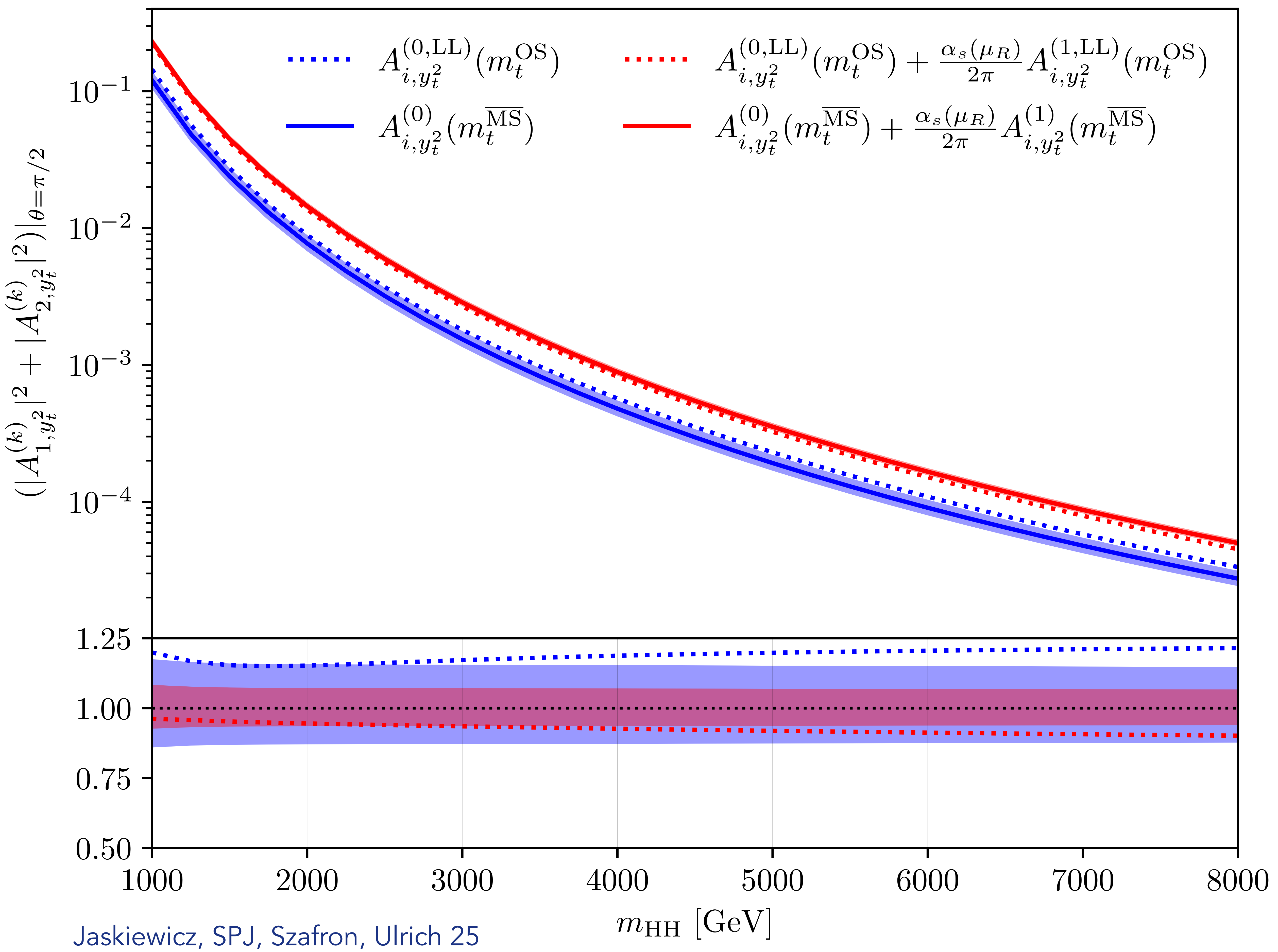
LO: ~70% → 15% (blue band)

NLO: ~25% → 7% (red band)

This uncertainty can be further reduced by computing the LP amplitude at NNLO

Nevertheless, all only valid for very high energies, so far only considered the virtual amplitude…

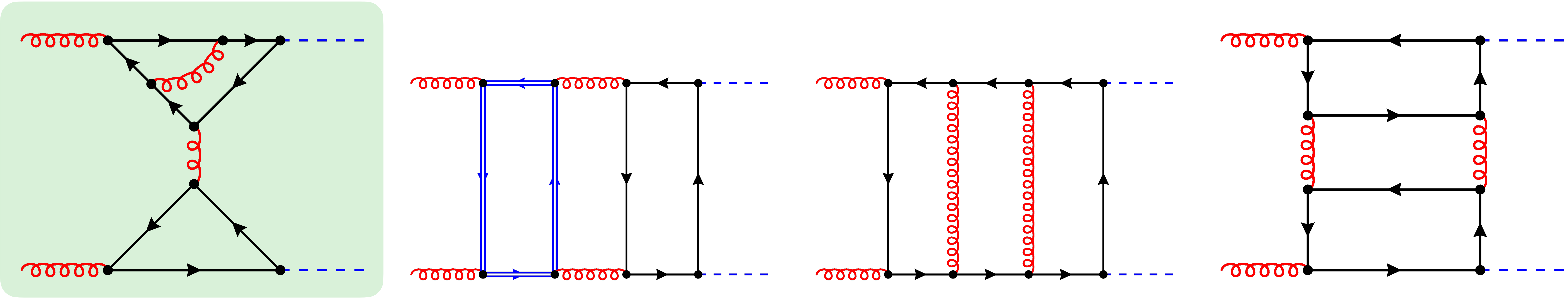
Need an NNLO calculation



Jaskiewicz, SPJ, Szafron, Ulrich 25

Example 2: Higgs Boson Pair Production @ NNLO

Figure: Davies, Schönwald, Steinhauser, Vitti (LHCHWG HH Report)



Very challenging computation

Reducible contribution computed recently ~(Higgs production with off-shell gluon leg)^2 Davies, Schönwald, Steinhauser, Vitti 24

General calculation requires **3-loop IBPs** with (s, t, m_T, m_H) including internal and external masses

Large number of very complicated (=beyond MPLs) massive master integrals

Real-virtual also very challenging ($2 \to 3$ @ 2-loops) Davies, Steinhauser 19; Davies, Herren, Mishima, Steinhauser 19, 21;

One promising approach (captures bulk of total cross-section at NLO) is to expand in the forward limit

Bonciani, Degrassi, Giardino, Gröber 18; Gröber, Maier, Rauh 19; Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; Bagnaschi, Degrassi, Gröber 23; Davies, Mishima, Schönwald, Steinhauser 23; Davies, Schönwald, Stremmer 25;

First significant steps: Large-N_c in the forward limit ($t = 0$, $m_H = 0$) Davies, Schönwald, Steinhauser 23, 25

Important to now develop robust methods for massive calculations beyond 2-loop

Synergies

Interpretation & EFTs

Precision theory is the meeting point of ideas from **amplitudes**, **phenomenology**, **EFTs**

``Precision'' can significantly feed into our interpretation at **colliders**

Example

Consider (partial) EW corrections to $gg \to HH$ with arbitrary 3-Higgs (g_3), 4-Higgs (g_4) and Yukawa vertices (g_t)

Encounter $1/\epsilon$ UV divergences which require renormalisation

Heinrich, SPJ, Kerner, Stone, Vestner 24

Fix vev counterterm using a vertex, but obtain different $1/\epsilon$ in renormalisation constants depending on the vertex

$$\delta_v^{g_t}(g_t, g_3, g_4)|_{\mathrm{UV}} = -\frac{g_3 g_t m_H^2 + 2 g_t^2 m_t \left(m_H^2 - 4 m_t^2\right) N_c}{32\pi^2 m_H^2 m_t \epsilon}$$

$$\delta_v^{g_4}(g_t, g_3, g_4)|_{\mathrm{UV}} = -\frac{2 g_t g_4 N_c (g_t(m_H^4 + 6 m_H^2 m_t^2) - 2 g_3 m_t^3) + g_4^2 m_H^4 - 24 g_t^4 m_H^4 N_c}{32\pi^2 g_4 m_H^4 \epsilon}$$

SM limit all ok →

$$\delta_v^{g_t}\left(\frac{m_t}{v}, \frac{3m_H^2}{v}, \frac{3m_H^2}{v^2}\right)\Bigg|_{\mathrm{UV}} = \delta_v^{g_3}\left(\frac{m_t}{v}, \frac{3m_H^2}{v}, \frac{3m_H^2}{v^2}\right)\Bigg|_{\mathrm{UV}} = \delta_v^{g_4}\left(\frac{m_t}{v}, \frac{3m_H^2}{v}, \frac{3m_H^2}{v^2}\right)\Bigg|_{\mathrm{UV}} \overset{!}{=} \delta_v|_{\mathrm{UV}}$$

Ok to talk about κ_3, κ_4 when considering QCD corrections, EW corrections need something more e.g. EFTs

Outlook

Current Status

Theory uncertainties have already started to limit our ability to explore the Higgs sector

It is critically important to have a robust fixed-order programme alongside precision developments in parton showers and analysis techniques

Future Community Goals

Lift standard candle processes and interesting final states to $\mathrm{N^3LO}$ making this the new standard

Refine tools/techniques for multi-loop electroweak corrections, with two-loop electroweak becoming a new standard (also in prep for FCC-ee) → Matthias (Today)

Continue to develop our understanding of formal limits of amplitudes, higher-point amplitudes, impact of masses, …

Thank you for listening!

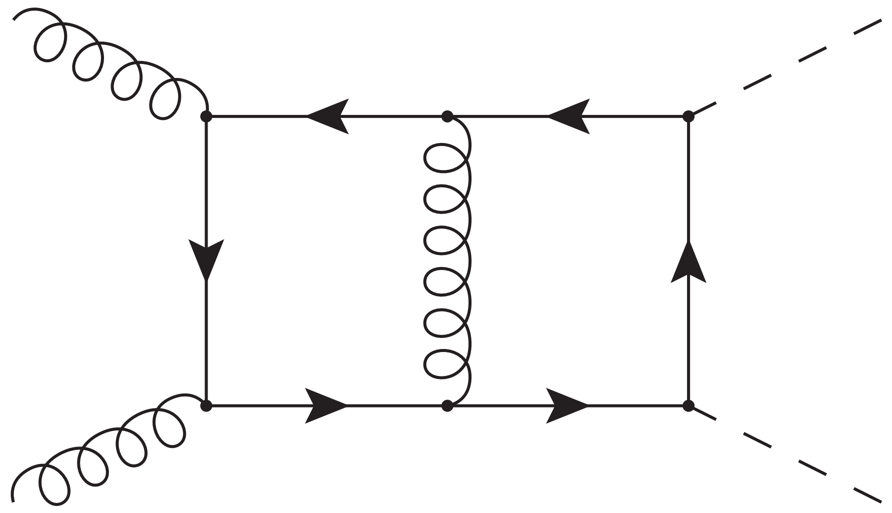
Backup

High-energy limit

Expanding amplitude perturbatively $A_i^{\text{fin}} = \frac{\alpha_s}{2\pi} A_i^{(0),\text{fin}} + \left(\frac{\alpha_s}{2\pi}\right)^2 A_i^{(1),\text{fin}} + \mathcal{O}(\alpha_s^3)$ and around $m_t \sim 0$

$gg \to HH$

Davies, Mishima, Steinhauser, Wellmann 18; Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 20



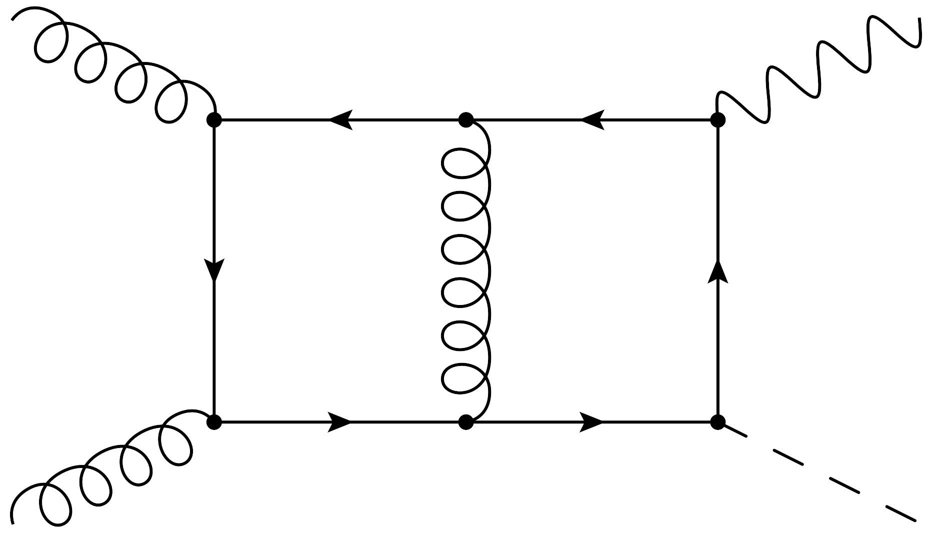
$$A_{i,y_t^2}^{(0)} \sim y_t^2 f_i(s,t) + y_t^2 \mathcal{O}(m_t^2)$$

$$A_{i,y_t^2}^{(1)} \sim 3 C_F A_i^{(0)} \log\left[\frac{m_t^2}{s}\right] + y_t^2 \mathcal{O}(m_t^2)$$

Leading $\log(m_t^2)$ from mass counter term, converting to $\overline{\text{MS}}$ gives $\log\left[\mu_t^2/s\right]$ → scale choice of $\mu_t^2 \sim s$

$gg \to ZH$

Davies, Mishima, Steinhauser 20; Chen, Davies, Heinrich,SPJ, Kerner, Mishima, Schlenk, Steinhauser 22



$$A_i^{(0)} \sim y_t m_t f_i(s,t) \ \log^2\left[\frac{m_t^2}{s}\right]$$

$$A_i^{(1)} \sim \frac{(C_A - C_F)}{12} A_i^{(0)} \ \log^2\left[\frac{m_t^2}{s}\right]$$

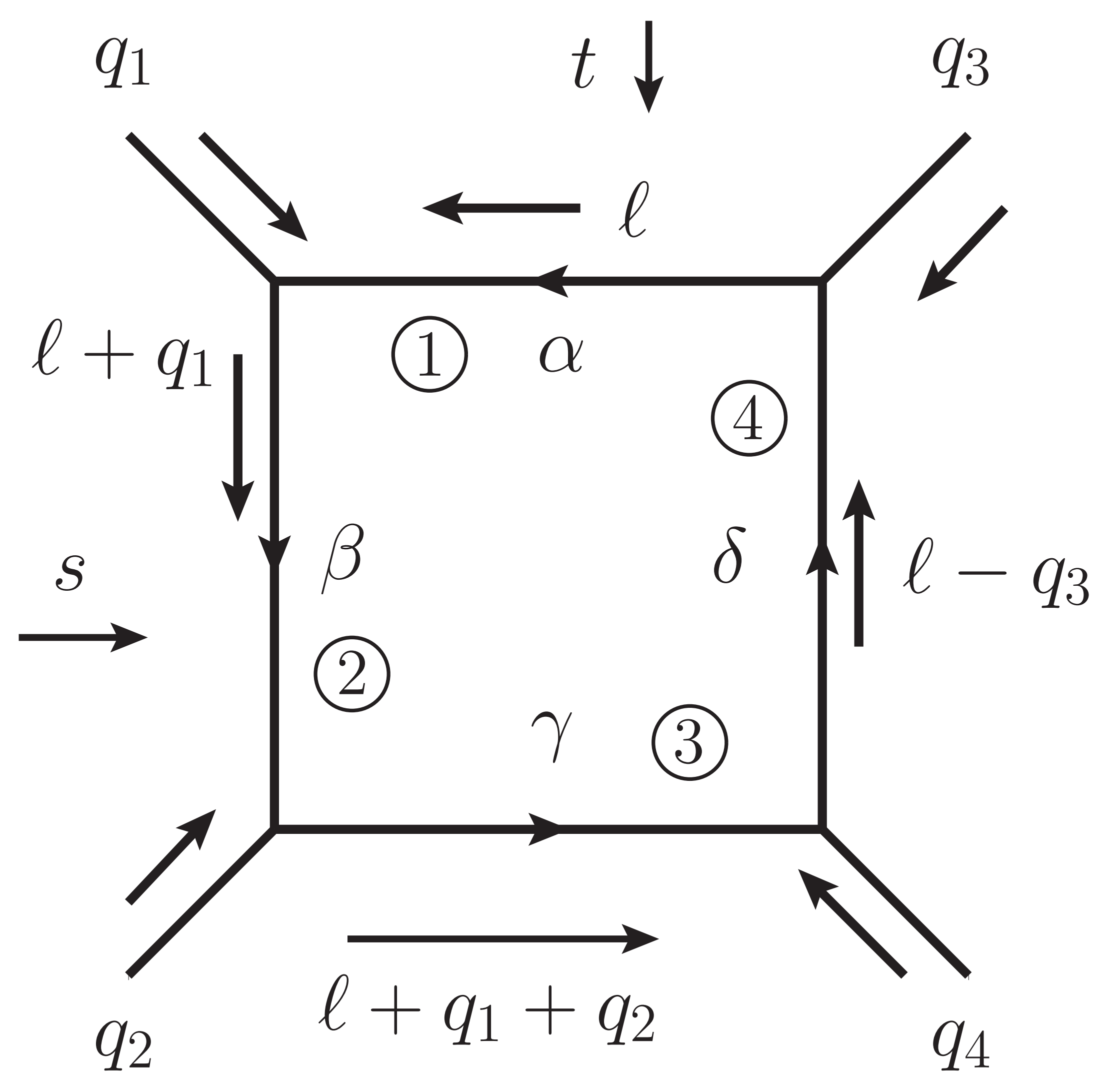
Leading $\log(m_t^2)$ not coming from mass counter term ($C_A - C_F$ structure)

Goal: How does the simple structure in $gg \to HH$ arise? Does it generalise to all orders in α_s? Can we resum these logarithms?

Application to $gg \to HH$:
Scalar Integral Level

High-Energy Expansion of $gg \to HH$ @ 1-loop

Limit: $s, |t|, |u| \gg m_t^2 \gg m_H^2\,,\quad m_H^2 \to 0$ and $\lambda \sim m_t/Q$



$$\int \frac{d^d\ell}{(2\pi)^d} \frac{1}{\ell^2 - m_t^2} \frac{1}{(\ell + q_1)^2 - m_t^2} \frac{1}{(\ell + q_1 + q_2)^2 - m_t^2} \frac{1}{(\ell - q_3)^2 - m_t^2}$$

↓

Hard region $\mathbf{u}^{(0)} = (0,0,0,0)$

↓

Every propagator scales as λ^0

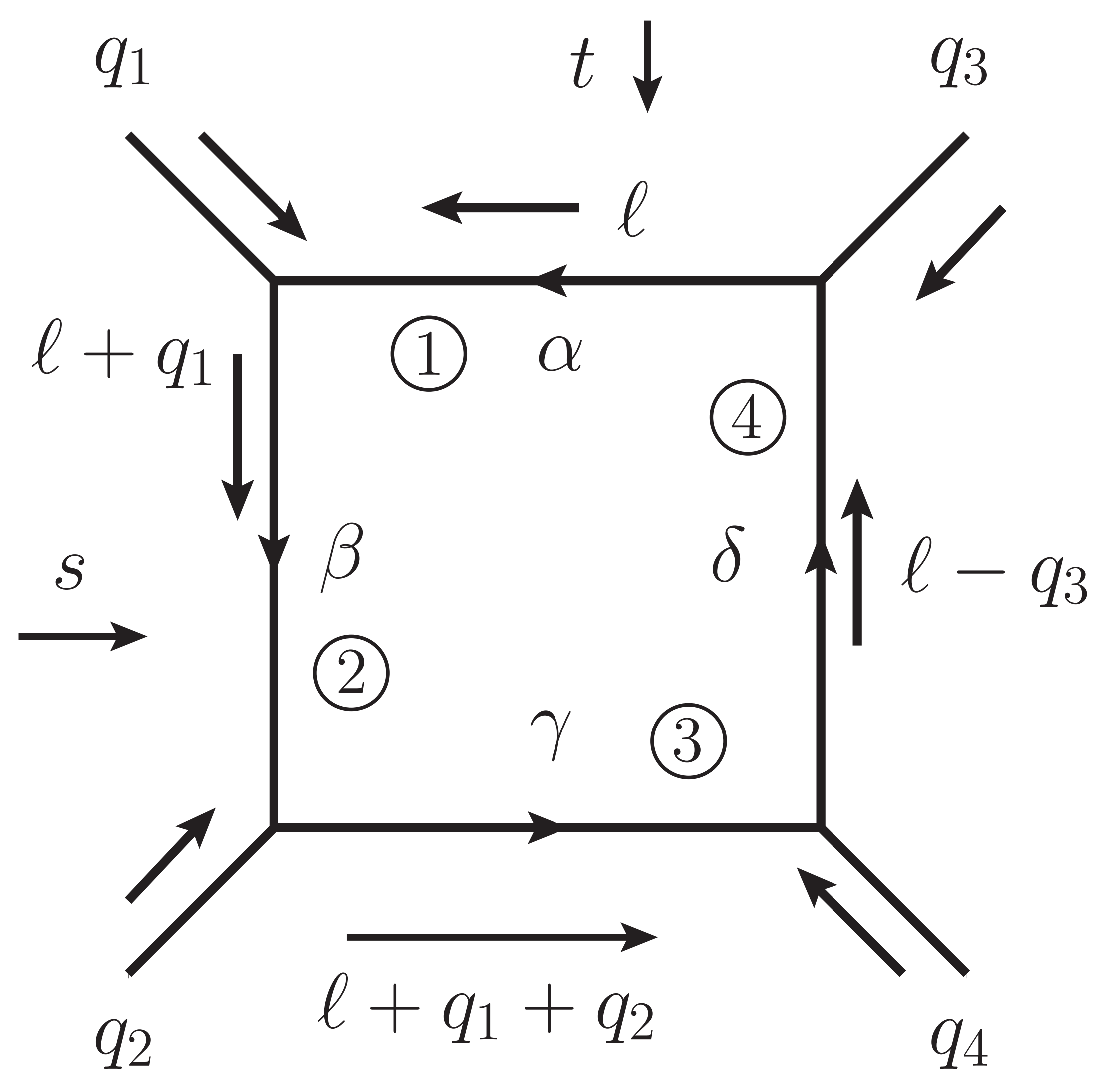
Achieved by **hard scaling** of the loop momenta $\ell^\mu = Q(1,1,1)$

↓

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{1}{\ell^2} \frac{1}{(\ell + q_1)^2} \frac{1}{(\ell + q_1 + q_2)^2} \frac{1}{(\ell - q_3)^2} \sim \lambda^0$$

High-Energy Expansion of $gg \to HH$ @ 1-loop

Limit: $s, |t|, |u| \gg m_t^2 \gg m_H^2\,, \quad m_H^2 \to 0$ and $\lambda \sim m_t/Q$



Automatically find remaining regions in parameter space

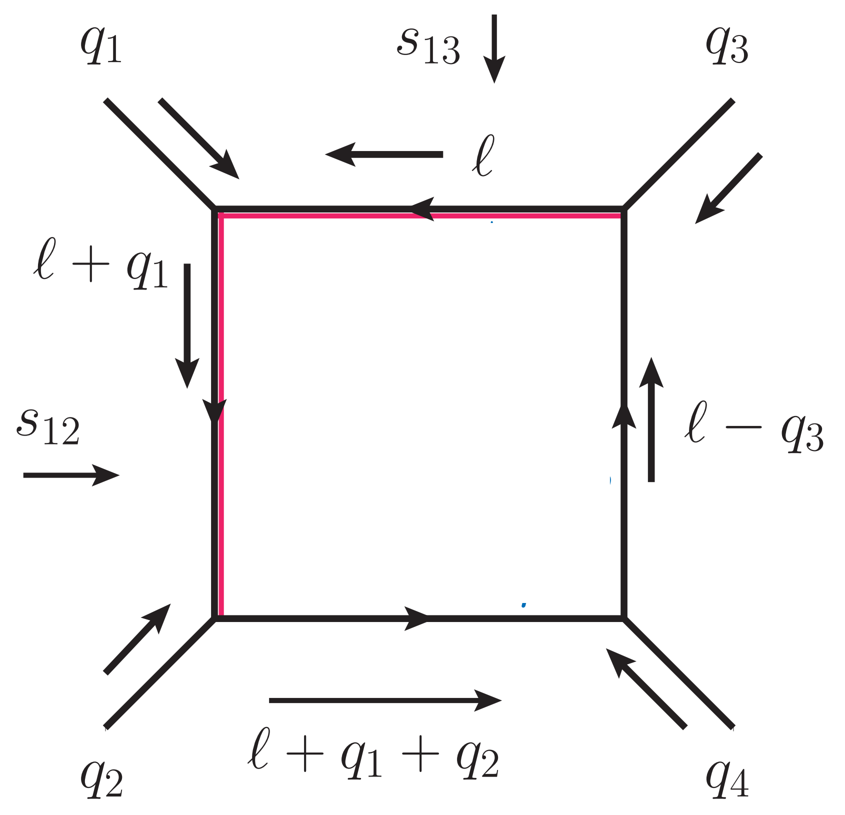
$\mathbf{u}^R$	order	interpretation	routing
(-2, -2, 0, 0)	$4 - 2(\epsilon + \alpha + \beta)$	c_1	ℓ
(0, -2, -2, 0)	$4 - 2(\epsilon + \beta + \gamma)$	c_2	$\ell - q_1$
(-2, 0, 0, -2)	$4 - 2(\epsilon + \alpha + \delta)$	c_3	$\ell + q_3$
(0, 0, -2, -2)	$4 - 2(\epsilon + \gamma + \delta)$	c_4	$\ell - q_1 - q_2$
(0, 0, 0, 0)	0	h	n/a

Using a set of possible loop momenta modes can systematically search for momentum routing to give a momentum space interpretation

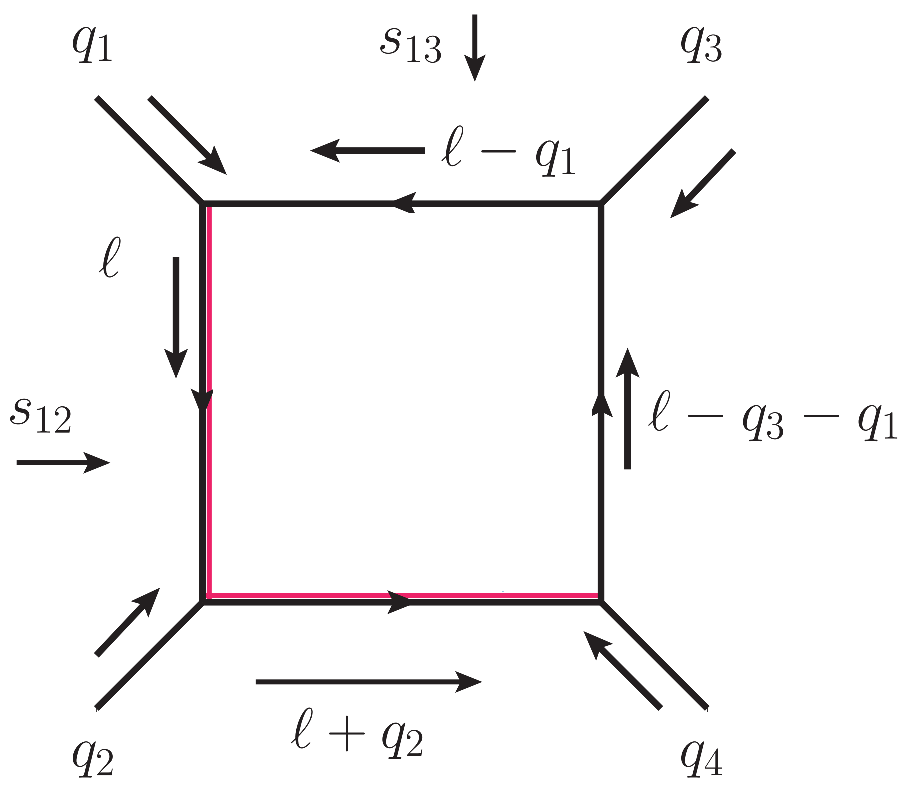
Implemented in pySecDec by Y. Ulrich (TBA)

High-Energy Expansion of $gg \to HH$ @ 1-loop

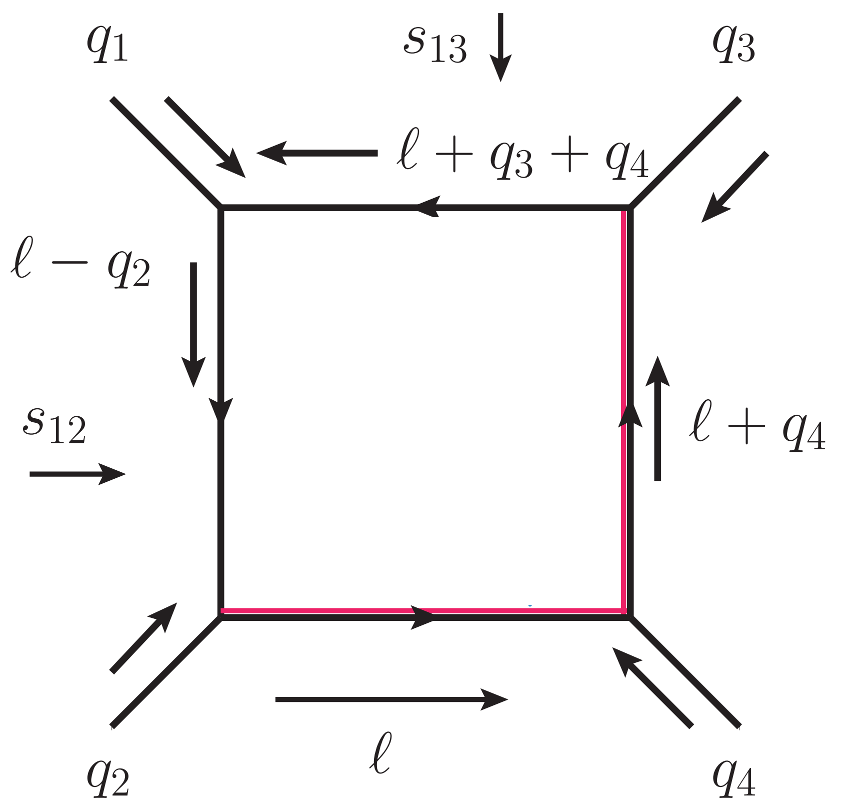
Collinear Regions



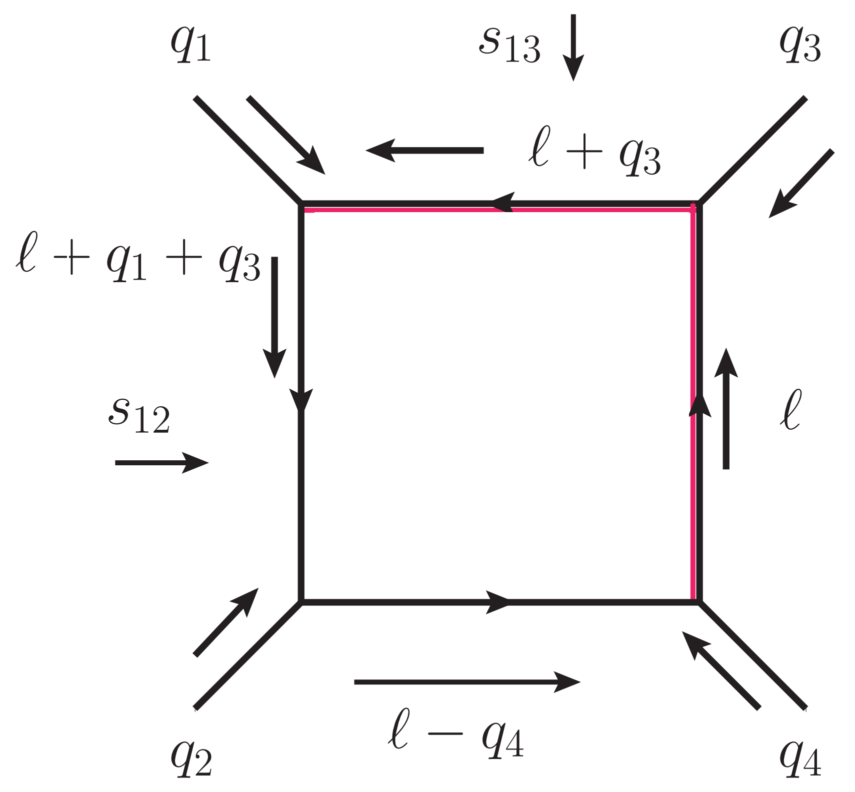
$\mathbf{u}^{(1)} = (-2,-2,0,0)$



$\mathbf{u}^{(2)} = (0,-2,-2,0)$



$\mathbf{u}^{(4)} = (0,0,-2,-2)$



$\mathbf{u}^{(3)} = (-2,0,0,-2)$

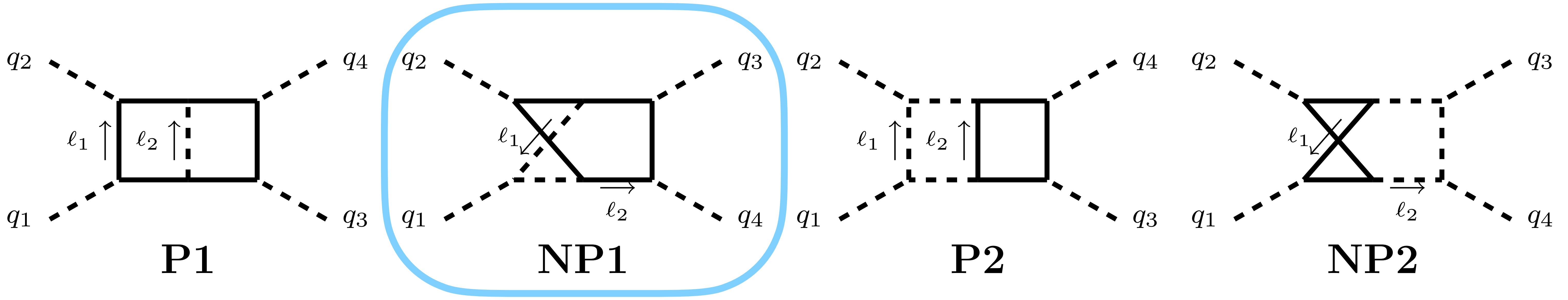
Collinear q_1 region $\quad \ell^\mu = Q(1,\lambda^2,\lambda)$

$$\underbrace{\int \frac{d^d\ell}{(2\pi)^d}}_{\lambda^4} \underbrace{\frac{1}{\ell^2 - m_t^2}}_{\frac{1}{\lambda^2}} \underbrace{\frac{1}{(\ell + q_1)^2 - m_t^2}}_{\frac{1}{\lambda^2}} \frac{1}{2(\ell + q_1)\cdot q_2} \frac{1}{-2\ell \cdot q_3}$$

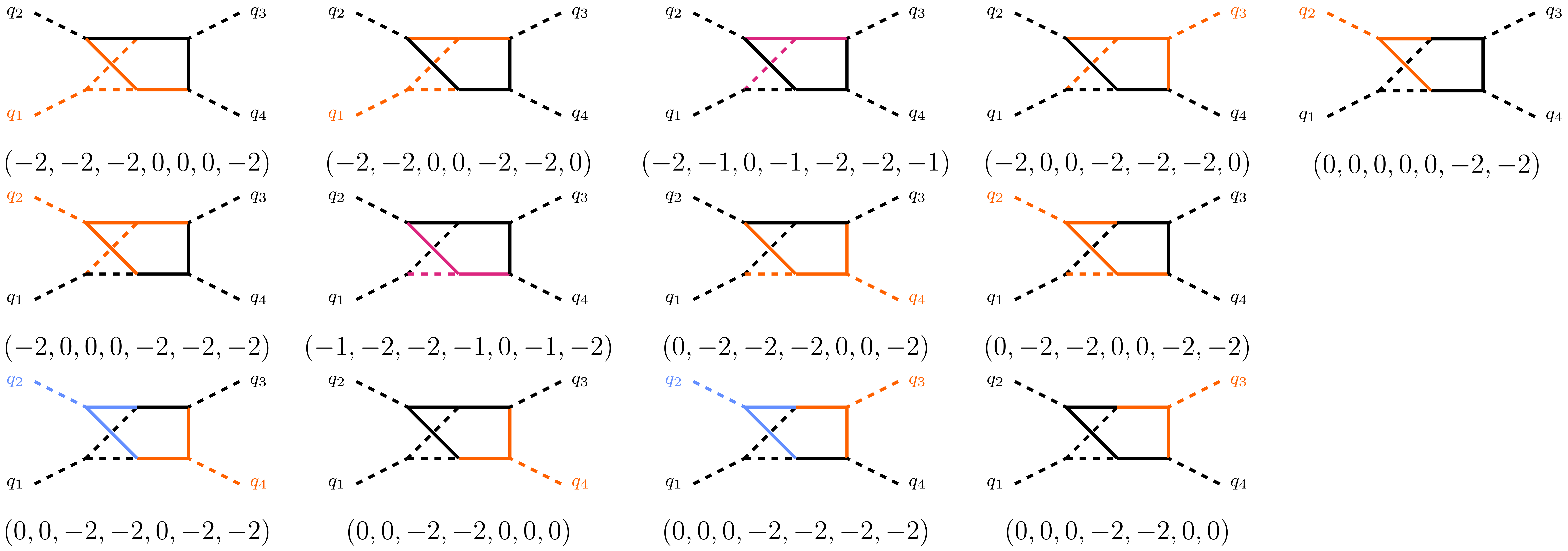
Collinear regions are also leading power at the level of scalar integrals!

High-Energy Expansion of $gg \to HH$ @ 2-loops

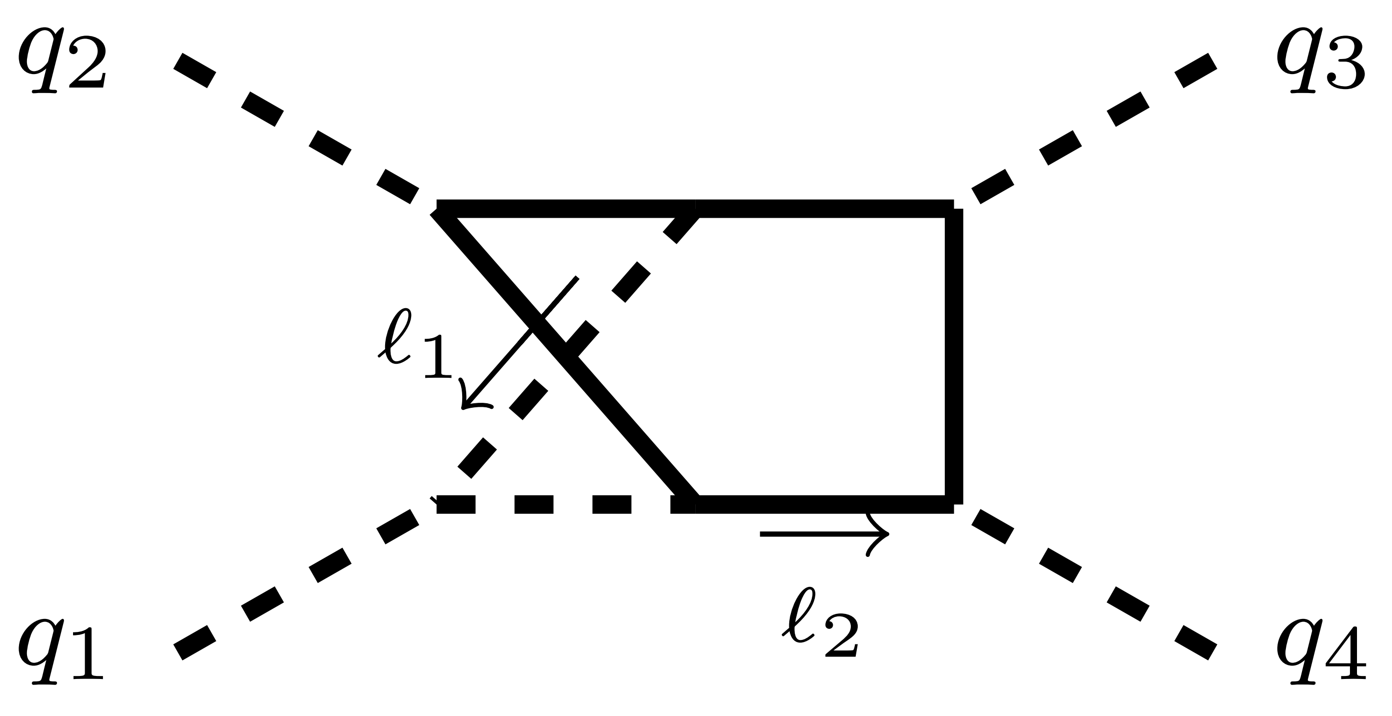
Topologies



Regions (Parameter Space)



High-Energy Expansion of $gg \to HH$ @ 2-loops



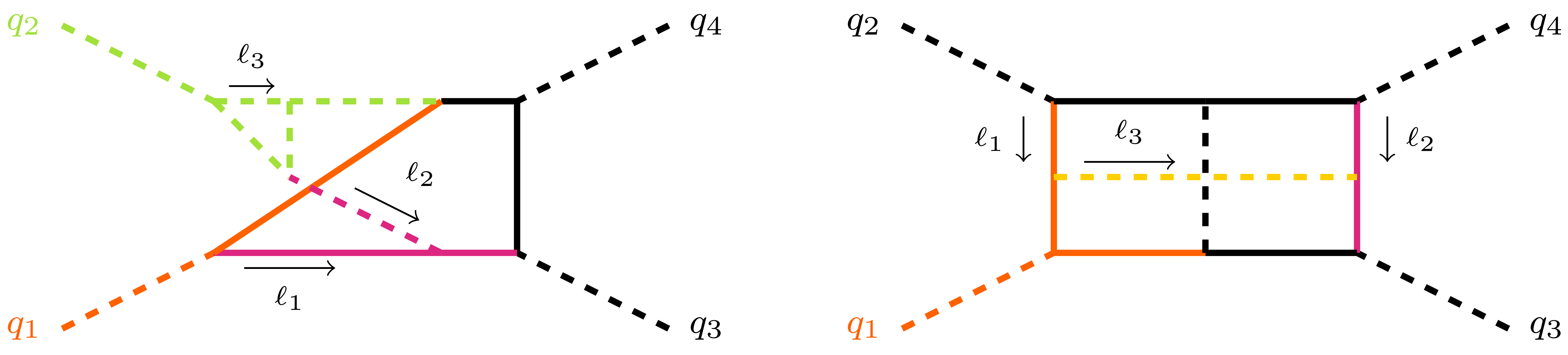
New features:

1. Soft modes appear $l_S^\mu = Q(\lambda, \lambda, \lambda)$
2. Soft regions are power enhanced at level of scalar integral

$\mathbf{u}^R$	order	interpretation	routing
(-2, -2, -2, 0, 0, 0, -2)	-4ϵ	c_1c_1	$\ell_1,\ \ell_2$
(-2, -2, 0, 0, -2, -2, 0)	-4ϵ	c_1c_1	$\ell_1,\ \ell_2 - q_3 - q_4$
(-2, -1, 0, -1, -2, -2, -1)	$-1 - 4\epsilon$	ss	$\ell_1,\ \ell_2 - q_3 - q_4$
(-2, 0, 0, -2, -2, -2, 0)	-4ϵ	c_3c_3	$\ell_1,\ \ell_2 - q_4$
(-2, 0, 0, 0, -2, -2, -2)	-4ϵ	c_2c_2	$\ell_1,\ \ell_2 - q_3 - q_4$
(-1, -2, -2, -1, 0, -1, -2)	$-1 - 4\epsilon$	ss	$\ell_1 - q_1,\ \ell_2$
(0, -2, -2, -2, 0, 0, -2)	-4ϵ	c_4c_4	$\ell_1 - q_1,\ \ell_2$
(0, -2, -2, 0, 0, -2, -2)	-4ϵ	c_2c_2	$\ell_1 - q_1,\ \ell_2$
(0, 0, -2, -2, 0, -2, -2)	-4ϵ	$c_4\bar{c}_2$	$\ell_1 - \ell_2 + q_3 + q_4,\ \ell_1$
(0, 0, -2, -2, 0, 0, 0)	-2ϵ	c_4h	$\ell_1 - \ell_2 + q_3 + q_4,\ \ell_1$
(0, 0, 0, -2, -2, -2, -2)	-4ϵ	$c_3\bar{c}_2$	$\ell_1 - \ell_2 + q_3,\ \ell_1 - q_4$
(0, 0, 0, -2, -2, 0, 0)	-2ϵ	c_3h	$\ell_1 - \ell_2 + q_3,\ \ell_1 - q_4$
(0, 0, 0, 0, 0, -2, -2)	-2ϵ	hc_2	$\ell_1,\ \ell_1 + \ell_2 - q_3 - q_4$
(0, 0, 0, 0, 0, 0, 0)	0	hh	n/a

Can again find momentum space interpretation

Considering $gg \to HH$ diagrams at 3-loops we systematically checked for new loop momenta modes



Indeed find new modes entering

Hard-collinear $l^{\mu}_{HC_i} = Q(1, \lambda, \lambda^{\frac{1}{2}})$

Soft-collinear $l^{\mu}_{SC_i} = Q(\lambda, \lambda^2, \lambda^{\frac{3}{2}})$

Ultra-soft $l^{\mu}_{US} = Q(\lambda^2, \lambda^2, \lambda^2)$

Expect new modes entering at each loop order, consistent with results in the literature

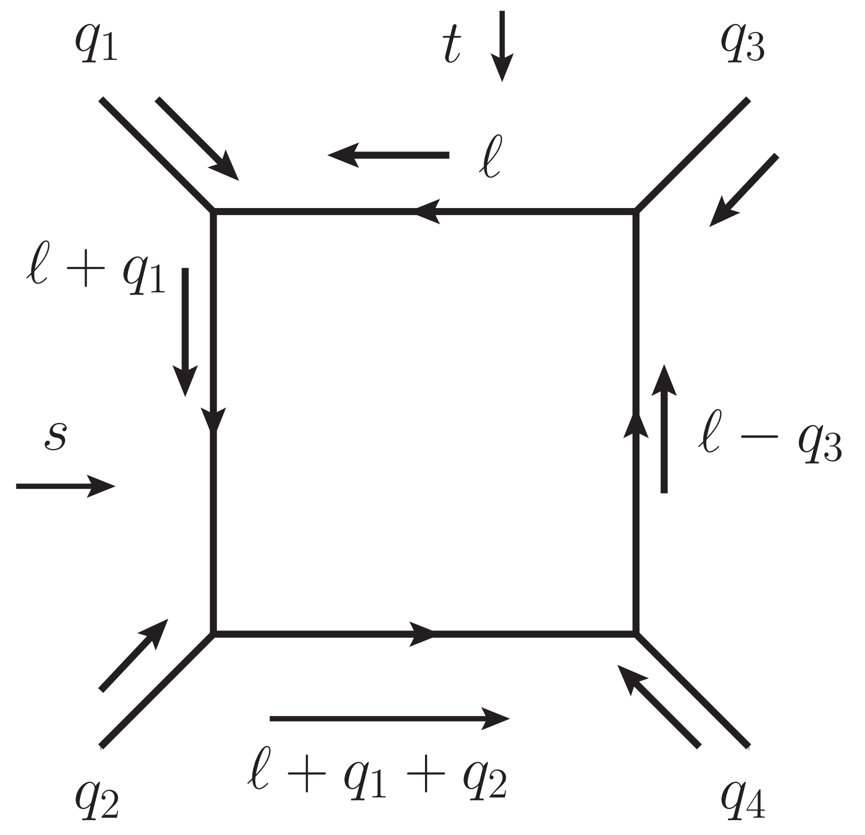
Ma 23

Application to $gg \to HH$: Amplitude Level

Amplitude Level Results @ 1-loop

Can compute amplitude level results for each region, at the 1-loop level:

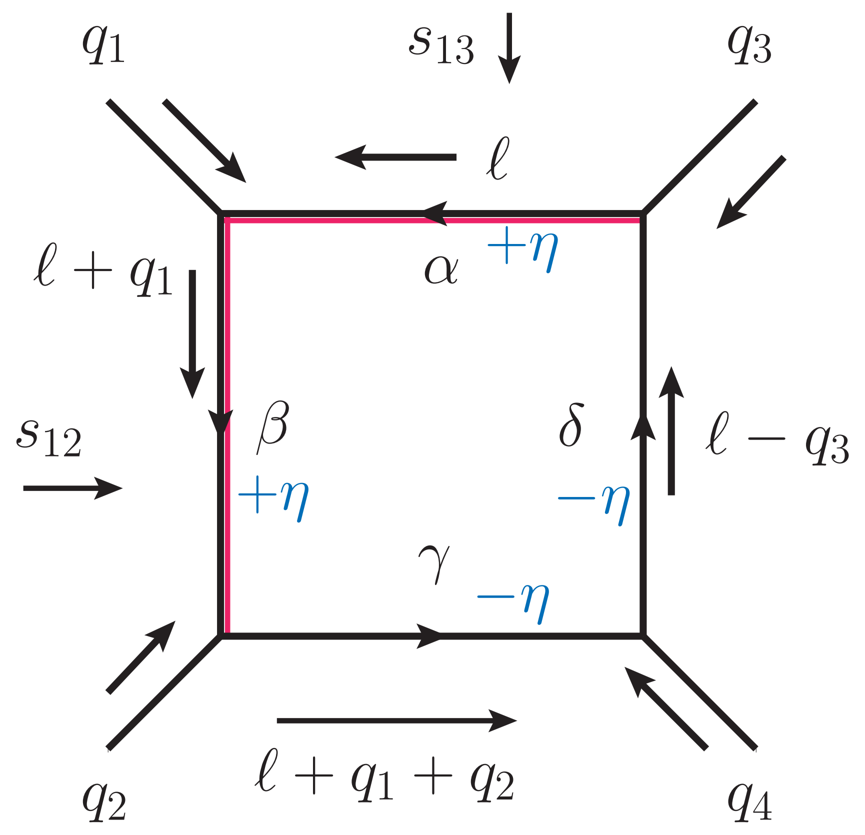
Hard region



Leading Power (LP)

$$A^{(h)}_{1,y_t^2} = \frac{4y_t^2}{s}\left\{2 - 2m_t^2\left[-\frac{2}{\epsilon^2 s} - \frac{1}{\epsilon}\left(\frac{s^2+2tu}{stu}\,l_s + \frac{l_t}{u} + \frac{l_u}{t}\right)\right.\right.$$
$$+\frac{-l_s^2+2l_t^2+2l_u^2}{s} + \frac{l_s\,l_t}{t} + \frac{l_s\,l_u}{u} + \frac{(t-u)^2\,l_t\,l_u}{stu}$$
$$\left.\left.-\left(\frac{2}{s}+\frac{1}{t}+\frac{1}{u}\right)l_s - \frac{t\,l_t}{su} - \frac{u\,l_u}{st} + \frac{60+13\pi^2}{6s}\right] + \mathcal{O}(m_t^4)\right\}$$

Collinear q_1 region



Next-to-Leading Power (NLP)

$$A^{(c_1)}_{1,y_t^2} = \frac{-4y_t^2 m_t^2\,\mu^{2\epsilon}e^{\gamma_E\epsilon}\Gamma(2\eta)^2\Gamma(\epsilon+2\eta)}{s(stu)(1-2\eta-\epsilon)\Gamma(4\eta)\Gamma(1+\eta)^2}\left(\frac{1}{m_t^2}\right)^{2\eta+\epsilon}$$
$$\times\left\{(-t)^\eta(-u)^\eta\left[s^2\left(1+2\epsilon^2-(2+\eta)\epsilon\right)-2\eta\,tu\right]\right.$$
$$+(-s)^\eta(-t)^\eta\left[su\left(1+2\epsilon^2-(2+\eta)\epsilon\right)-tu(1-3\eta-(2-5\eta)\epsilon)\right]$$
$$\left.+(-s)^\eta(-u)^\eta\left[st\left(1+2\epsilon^2-(2+\eta)\epsilon\right)-tu(1-3\eta-(2-5\eta)\epsilon)\right]\right\}$$

Generates $\log(m_t^2)$ at NLP

Amplitude Level Results @ 2-loop

Could compute each region at 2-loops (tedious), can instead examine numerator prior to reduction

Region $c_1 c_1$

$$\ell_1^\mu \sim \ell_2^\mu \sim Q(1, \lambda^2, \lambda)$$

$$l_1^2 \sim \lambda^2 Q^2, \quad l_2^2 \sim \lambda^2 Q^2, \quad l_1 \cdot l_2 \sim \lambda^2 Q^2,$$
$$l_1 \cdot q_1 \sim \lambda^2 Q^2, \quad l_2 \cdot q_1 \sim \lambda^2 Q^2,$$
$$l_1 \propto q_1, \quad l_2 \propto q_1 .$$

Region ss

$$\ell_1^\mu \sim \ell_2^\mu \sim Q(\lambda, \lambda, \lambda)$$

$$l_1^2 \sim \lambda^2 Q^2, \quad l_2^2 \sim \lambda^2 Q^2, \quad l_1 \cdot l_2 \sim \lambda^2 Q^2,$$
$$l_1 \cdot q_2 \sim \lambda^2 Q^2, \quad l_1 \cdot q_3 \sim \lambda^2 Q^2, \quad l_1 \cdot q_4 \sim \lambda^2 Q^2,$$
$$l_2 \cdot q_2 \sim \lambda^2 Q^2, \quad l_2 \cdot q_3 \sim \lambda^2 Q^2, \quad l_2 \cdot q_4 \sim \lambda^2 Q^2,$$

Inserting into amplitude, projecting form factors and computing traces
Numerator gives a λ^2 suppression for all soft/collinear regions

Consistent with the result of Steinhauser et al. for the $m_t \to 0$ limit

Davies, Mishima, Steinhauser, Wellmann 18;

Suggests that regions other than the hard region are **helicity suppressed** by at least $\lambda \sim m_t$

Effective field theory Analysis

Study amplitude using Soft Collinear Effective Theory (SCET)

[C. Bauer, S. Fleming, D. Pirjol and I. Stewart, hep-ph/0011336] [C. Bauer, D. Pirjol, I. Stewart, hep-ph/0109045]
[M. Beneke, A. Chapovsky, M. Diehl, T. Feldmann, hep-ph/0206152] [M. Beneke, T. Feldmann, hep-ph//0211358]

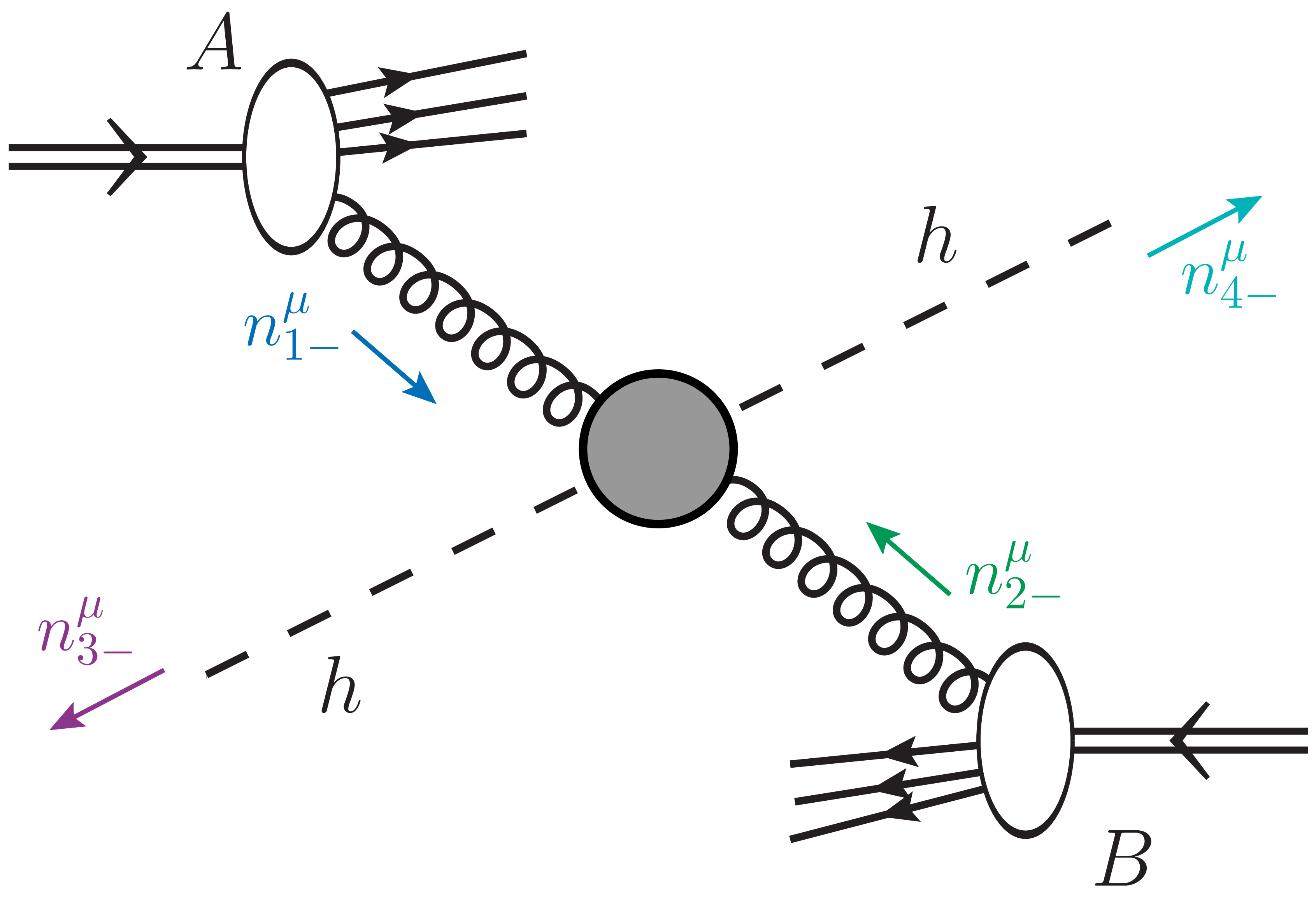
$$\psi(x) \rightarrow \underbrace{\psi_1(x) + \ldots + \psi_N(x)}_{N \text{ collinear fermion fields}} + q(x)$$

$$\mathscr{L}_{\text{SCET}} = \sum_{i=1}^{N} \mathscr{L}_{c_i} + \mathscr{L}_{\text{soft}}$$

Lagrangians belong to a specific collinear direction
Can be expanded in powers of the small parameter

$$\mathscr{L}_{c_i} = \underbrace{\mathscr{L}_{c_i}^{(0)}}_{\text{LP}} + \underbrace{\mathscr{L}_{c_i}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathscr{L}_{c_i}^{(2)}}_{\mathcal{O}(\lambda^2)} + \ldots$$

Keep collinear, anti-collinear, and soft degrees of freedom
Hard modes are integrated out



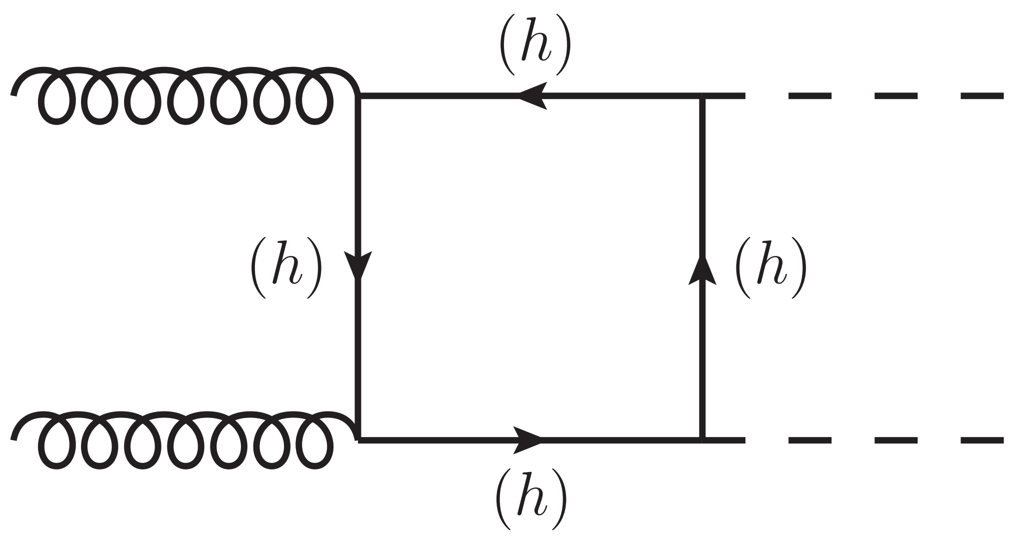
Generic N-jet operator has the form:

M. Beneke, M. Garny, R. Szafron, J. Wang, 17, 17, 18, 19

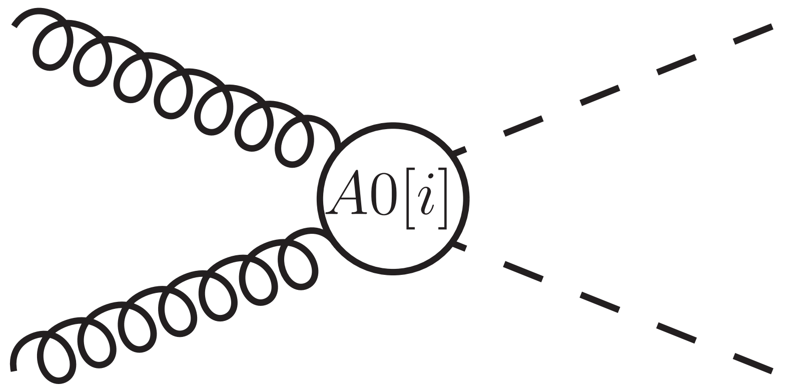
$$J = \int \left[\prod_{ik} dt_{i_k}\right] C(\{t_{i_k}\}) \prod_{i=1}^{N} J_{c_i}(t_{i_1}, t_{i_2} \ldots)$$

Leading Power Analysis

Leading power matching $J_{\rm LP}^{[i]}(t_1,t_2,t_3,t_4) = y_t^2\, P_i^{\mu\nu}\, \mathscr{A}_{c_1\perp_1\,\mu}(t_1 n_{1+})\, \mathscr{A}_{c_2\perp_2\,\nu}(t_2 n_{2+})\, h_{c_3}(t_3 n_{3+})\, h_{c_4}(t_4 n_{4+})$



$+ \ldots = C_{[i]}^{A0}(s,t)$



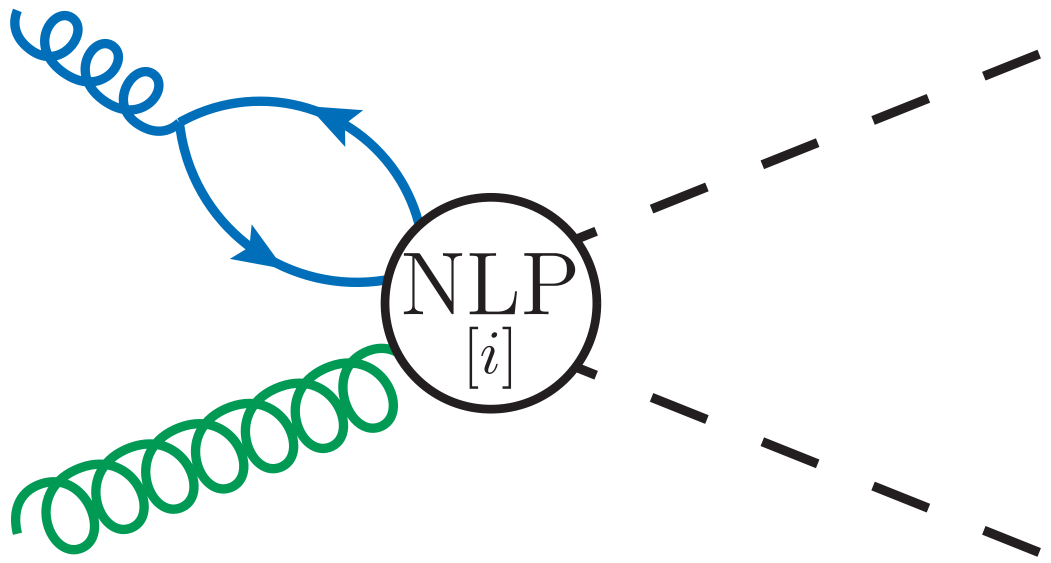
Collinear Regions c_1, c_2 $\mathcal{M}_{\rm LP}^{\rm QCD} \propto \left(\bar{v}_{c_1}(\bar{r}q_1)\frac{\not{n}_{1+}}{2}u_{c_1}(rq_1)n_{3-\nu}\varepsilon^{\nu}_{\perp_1}(q_2) + \bar{v}_{c_1}(\bar{r}q_1)\frac{\not{n}_{1+}}{2}\gamma_5 u_{c_1}(rq_1)n^{\mu}_{3-}i\epsilon^{\perp_1}_{\mu\nu}\varepsilon^{\nu}_{\perp_1}(q_2)\right)$

Relevant operator structures

$$J_{S_i}(\{t_{i_1},t_{i_2}\}) = \bar{\chi}_{c_i}(t_{i_2}n_{i+})\frac{\not{n}_{i+}}{2}\,\chi_{c_i}(t_{i_1}n_{i+}),$$

$$J_{P_i}(\{t_{i_1},t_{i_2}\}) = \bar{\chi}_{c_i}(t_{i_2}n_{i+})\frac{\not{n}_{i+}}{2}\,\gamma_5\,\chi_{c_i}(t_{i_1}n_{i+}),$$

Mixing with the external gluon is forbidden at LP



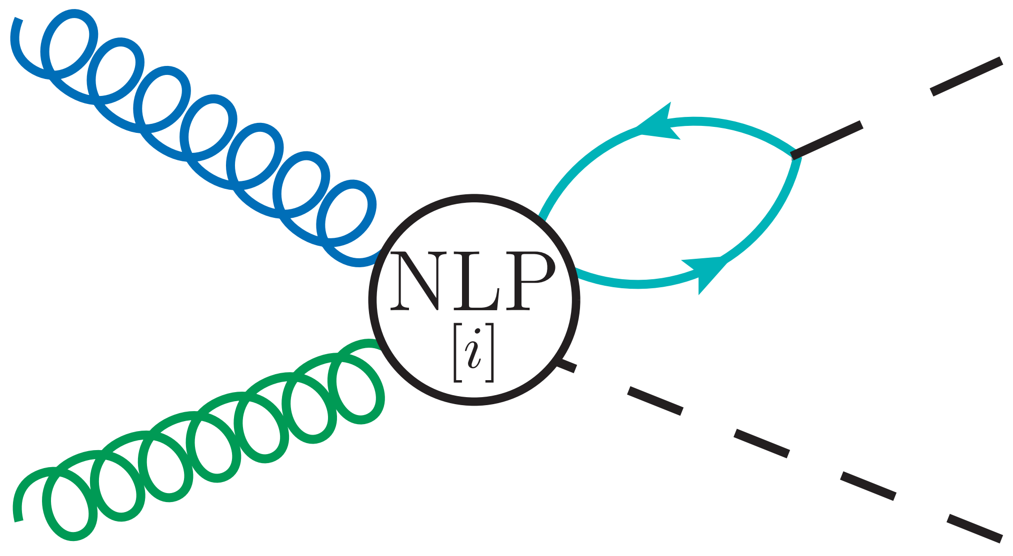
Leading Power Analysis

Collinear Regions c_3, c_4

$$\mathcal{M}_{\text{LP}}^{\text{QCD}} \sim i g_s^2 \mathbf{T}^B \mathbf{T}^A \left[\frac{g_W y_t}{2}\right] \bar{v}_{c_3}(q') \frac{1}{\bar{r}(n_{3+}q_3)} \left[\frac{2 n_{3-\mu}}{(n_{1+}q_1) n_{3-} \cdot n_{1-}} \frac{\not{n}_{3+}}{2} \gamma_{\nu\perp_3} u_{c_3}(q) + \frac{1}{r(n_{3+}q_3)} n_{3+\nu} \frac{\not{n}_{3+}}{2} \gamma_{\mu\perp_3} u_{c_3}(q)\right] \varepsilon^{\nu}_{\perp_2}(q_2)\ \varepsilon^{\mu}_{\perp_1}(q_1)$$

Situation reversed, structures appearing at LP are vector-like

Mixing with the external Higgs is forbidden at LP



Result holds to all orders in α_s due to helicity conservation for $m_t \to 0$

Next-to-Leading Power

Structure of the amplitude allows mixing with external gluon/Higgs

Expect contributions from collinear/soft regions