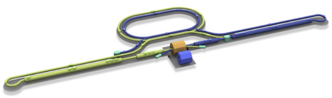
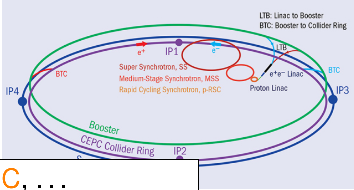
Theory challenges at future e^+e^- colliders

DESY Theory Workshop — Hamburg, September 23-26, 2025

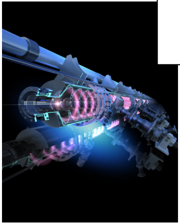
Matthias Steinhauser | TTP KIT

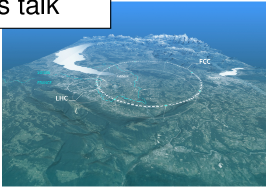
Future e^+e^- collider





- ILC, FCC-ee, CEPC, CLIC, ...
- circular or linear
- differences in energy, luminosity, ...
- not of primary importance for this talk

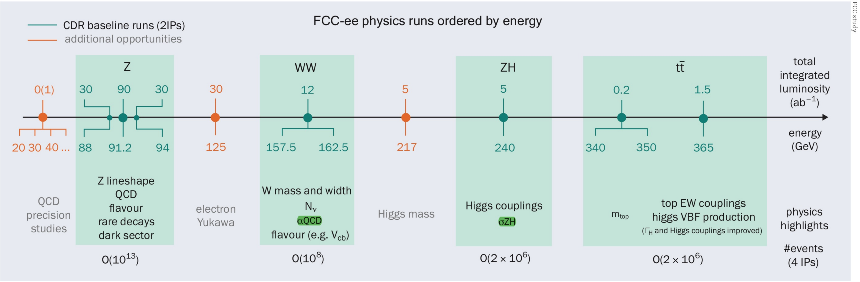




[linearcollider.org, CERN]

Energies

Z	WW	ZH	$t\bar{t}$
90	160	240	350



[Janot,Grojean CERN courier]

Physics questions — what can a future e^+e^- collider provide

- *Hcc* coupling
- Access to *Hss* and *Hee* couplings
- Higgs self coupling λ
- Significant improvement in New Physics reach (SMEFT fits, dark matter, light particles with small couplings, ...)
- Much more knowledge about known particles;
  enormous gain in precision (M_W, Γ_Z, Γ_H, α_s, ...)
  Test consistency of the SM at high loop order
  ➪ main topic of this talk

Experimental precision

SM parameters

similar for ILC,CEPC,...

	Γ_Z[MeV]	M_W[MeV]	Γ_W[MeV]	$\sin^2\theta^\ell_{\rm eff}\cdot 10^6$	$\alpha_S(M_Z)\cdot 10^4$
HL-LHC	2.3	9	42	160	10
FCC-ee	0.025	0.3	1	1.4	2
current TH	0.4	4	$\sim$ 0.3	45	–

Higgs couplings

	W	Z	g	b	c
HL-LHC	1.5%	1.3%	2%	2.5%	–
FCC-ee	0.3%	0.1%	0.8%	0.6%	1%

[Auchmann et al. '24]

- Theory challenge: match this precision
- Idealy: theory uncertainty should be (much) smaller than the experimental uncertainty

Z pole

ρ parameter

Relation between SM parameters: M_W, M_Z, M_H, G_F, α ...

$$\sqrt{2}G_F = \frac{\pi\alpha}{M_W^2 c_w^2}(1+\Delta r) \qquad \Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\delta\rho + \ldots$$

$$\rho = 1 + \delta\rho \qquad \delta\rho = \frac{\Pi_{ZZ}(0)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2} \sim \frac{m_t^2}{m_W^2}$$

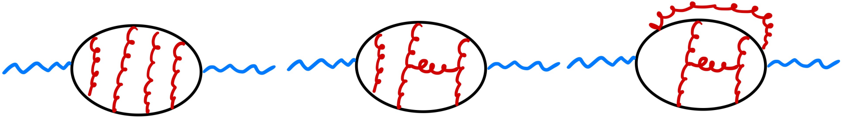
Know to 4 loops in QCD + partial 3 loop EW results [Veltman'77; Kniehl,Kühn,Stuart'88; ...; Schöder,Steinhauser'05; Chetyrkin,Faisst,Kühn,Maierhofer,Sturm'06; Boughezal,Czakon'06; ...]

Fix all parameters but M_W. What is the effect of $\delta\rho$ on ΔM_W?

	1 loop	2 loops	3 loops	4 loops	5 loops
order	α_s^0	α_s^1	α_s^2	α_s^3	α_s^4
ΔM_W	600 MeV	-60 MeV	-10.9 MeV	2 MeV	needed!

FCC-ee: $\Delta M_W^{\rm exp} = 0.3$ MeV

ρ parameter at 5 loops



Needed: 5-loop vacuum integrals

History: 3l: [Broadhurst'92;...] 4l: [Chetyrkin,Kühn,Mastrolia,Sturm'05; ...; Schöder'05; ...; Maier,Marquard'07]

Challenges at 5 loops:

- huge expressions $\mathcal{O}(100)$ GB ... TB
- reduction of Millions of integrals to "master integrals"
- compute the master integrals

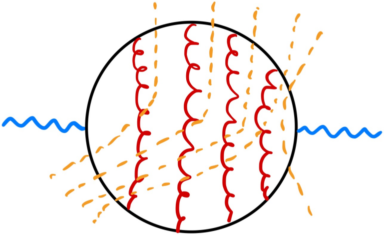
First step: [Maier,Marquard,Schöder]: 5-loop β function

Benefits (from 5-loop vacuum integrals):

- more precise m_c and m_b ➪ needed for H decay
- 5-loop decoupling relation for α_s
- ...

α_s

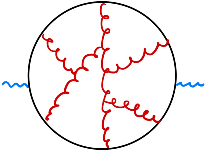
$\mathcal{O}(10^{12})$ Z decays ➪ precise α_s from Z decays: $\Delta\alpha_s = 0.0002$



- $\Gamma(Z \to \text{hadrons})$ known to $\mathcal{O}(\alpha_s^4)$ [Baikov,Chetyrkin,Kühn,Rittinger'12]

  ➪ $\Delta\alpha_s = 0.005$

- Needed: (imaginary part of) massless 6-loop photon propagator

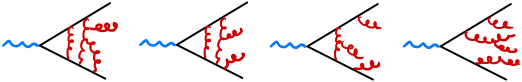


- Challenges: reduction to MIs, computation of MIs + many more

[3l'79; 4l'97; 5l'08-'12] always: Chetyrkin et al.

Event shapes

- α_s from jet observables, event shapes
- $e^+e^- \to 3$ jets needed to N^{3}LO, massless partons

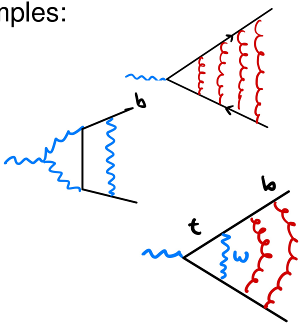


- NNLO: [Gehrmann-De Ridder,Gehrmann,Glover,Heinrich'08; Weinzierl'08; Del Duca et al.'16]
- First steps to N^{3}LO: direct calculation of fully differential 2-jet rate [Chen,Jakubcik,Marcoli,Stagnitto'25]
- Needed: (stable) subtraction scheme at N^{3}LO
- Hot topic at LHC (NNLO): Development of subtraction schemes
  "Sector-improved residue subtraction", "Nested Soft-Collinear subtraction scheme", "Local analytic subtraction", "Geometric", "Antenna subtraction", "Projection-to-Born"

  many groups: [Caola,Cacciari,Czakon,Gehrmann,Magnea,Melnikov,Troscanyi,. . .]

Z boson decay width

- FCC-ee: $\Delta\Gamma_Z = 0.025$ MeV ➪ 0.001%
- Some examples:



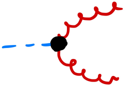
$\mathcal{O}(\alpha_s^4)$	0.016%	[Baikov,Chetyrkin,Kühn'08-'12]
$\mathcal{O}(\alpha^2)$	0.02%	[Dubovyk,Freitas,et al.'18]
$\mathcal{O}(G_F m_t^2 \alpha_s^2)$	0.02%	[Chetyrkin,Steinhauser'99]

Further challenges:

- technical: massive multi-loop integrals
- conceptual: γ_5 [Stöckinger,Kühler,Weißwange]
- Renormalization of the SM (at and beyond 2 loops)
- ISR at and beyond NNLO

Higgs decays

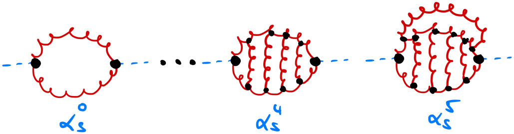
$H \to gg$



$\Rightarrow$ $$\boxed{\Gamma(H \to gg) = |C_1|^2 \mathrm{Im}\left[\Pi_{GG}\right]}$$

C_1 known to α_s^5 [Schröder,Steinhauser'05; Chetyrkin,Kühn,Sturm'05]

Π_{GG}:



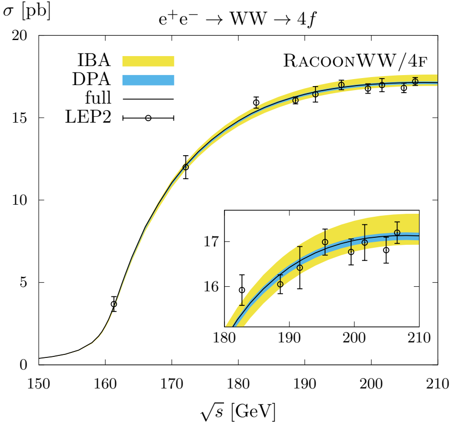
α_s^4: -0.6% [Davies et al.'17; Herzog et al.'17]

FCC-ee: 0.8%

α_s^5: unknown

WW threshold

WW threshold



[Denner,Dittmaier'19]

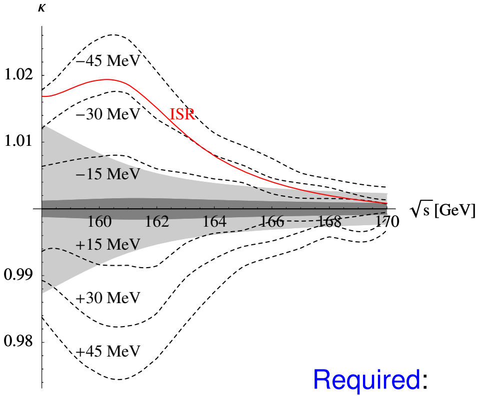
$\Rightarrow$

$\Delta M_W^{\exp}$ (MeV)		
LEP2	ILC	FCC-ee
200	3 – 6	0.3

- full NLO for $e^+e^- \to 4f$ $\Rightarrow$ "a few" 0.1% [Denner,Dittmaier,Roth,Wieders'05]
- EFT for *WW* threshold [Beneke,Falgari,Schwinn,Signer,Zanderighi'07; Actis,Beneke,Falgari,Schwinn'09]

WW threshold

[Beneke,Falgari,Schwinn,Signer,Zanderighi'07]



$$\kappa = \frac{\sigma_{WW}(s, M_W + \Delta M_W)}{\sigma_{WW}(s, M_W)}$$

Example: $\sqrt{s} = 161$ GeV:

$\Delta\kappa = 0.02\%$

$\leftrightarrow \quad \Delta M_W = 0.3$ MeV

➪ Challenge:

theory uncertainty at the level of 0.02% in σ_{WW} needed!

Required:

- full NNLO in threshold EFT
- NNLO to $e^+e^- \to f\bar{f}$
- ISR to higher order

$$e^+e^- \to ZH$$

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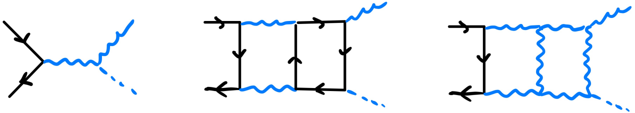
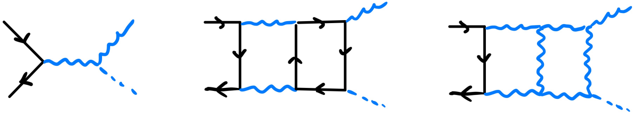


- For $\sqrt{s} = 240 - 250$ GeV: dominant Higgs production channel
- extract *HZZ* coupling
- expected experimental precision $\mathcal{O}(1\%)$ (or better)
- 2-loop ew corrections:
  - [Chen,Guan,He,Li,Liu,Ma'22]: purely numerical calculation of 2-loop diagrams; not phenomenological results
  - semi-analytic calculation, only (dominant?) fermionic contributions [Freitas,Song(,Xie)'23] ➪ estimated uncertainties from missing higher order corrections: $0.1\% - 0.3\% < 1\%$
  - still missing: bosonic corrections

$e^+e^- \to ZH$

Karlsruhe Institute of Technology



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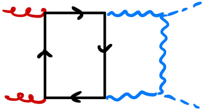
- Similar challenges as for $gg \to HH$ at the LHC:
  - 2 loop ew corrections
  - many mass scales



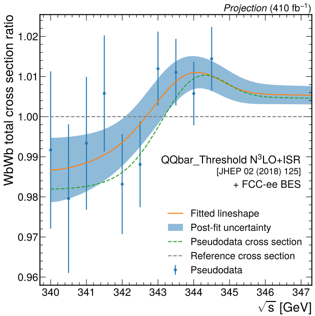
several groups: [Bi,Davies,Heinrich,Huang,Jones,Ma,Mühlleitner,. . .]

$t\bar{t}$ threshold production

Threshold scan

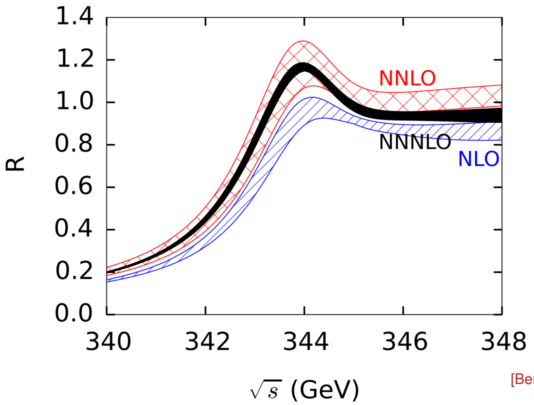
- scan of $e^+e^- \to t\bar{t}$ around threshold ➪ precise m_t (also: Γ_t, y_t, α_s)

[Defranchis,de Blas,Mehta,Selvaggi,Vos'25]



➪ $\Delta m_t^{\mathrm{PS}}\Big|_{\mathrm{exp}} = 6.8$ MeV $\qquad \Delta m_t^{\mathrm{PS}}\Big|_{\mathrm{N^3LO}} = 35$ MeV

$\sigma(e^+e^- \to t\bar{t})$



$$R = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

50 GeV $\leq \mu \leq$ 350 GeV

➪ 3% uncertainty

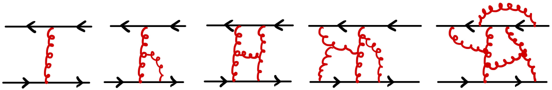
[Beneke,Kiyo,Marquard,Penin,Piclum,Steinhauser'15]

Beyond-QCD (Higgs, non-resonant, QED, *P* wave): [Eiras,Steinhauser'06; Hoang,Reißer'06;Beneke,Jantzen,Ruiz-Femenia'10; Penin,Piclum'12; Beneke,Maier,Piclum,Rauh'15; . . .]

(A few) Challenges towards N^{4}LO

$\sigma(e^+e^- \to t\bar{t})$ has many ingredients ...

- Static potential to 4 loops [1l:'80; 2l:'99; 3l:'10]



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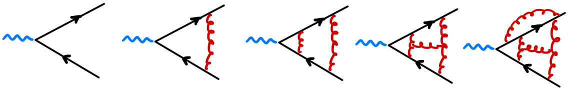

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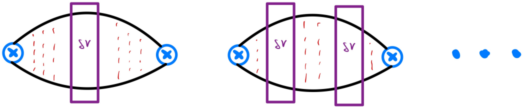

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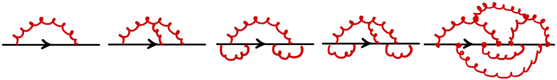
  Example: PS mass

  $$m_t^{\overline{\rm MS}} = 168.204 - 3.893 - 0.598 - 0.088 + 0.018 \text{ GeV}$$

  ➪ 5-loop relation needed!



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Technical developments

- Integration-by-parts

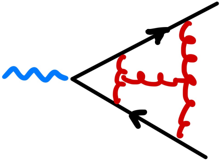
[Chetyrkin,Tkachov'81] ➪ [Laporta'00] ➪

FIRE [Smirnov et al.'08-23-. . .]
Kira [Usovitsch et al. '18-'25]
Blade [Guan et al. '24]
...

Example: massless 3-loop form factor

2009: reduction was barely possible

2025: reduction takes a few minutes



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. . .

- Differential equations

[Kotikov'91; Gehrmann,Remiddi'99] ➪

analytic solutions: [Henn'13,Lee'14]
numeric solutions: [Hidding'20; Fael et al.'21; . . .]

- Numerical integration

FIESTA [Smirnov et al.'08-'16-. . .]
pySecDec [Heinrich et al. '10-'23-. . .]
. . .

Contineous developments — will continue

Technical deleopments

Karlsruhe Institute of Technology

Computeralgebra program FORM [Vermaseren,Davies,Ueda,…'89…'25]

- Most (all?) precision calculations for LHC use FORM
- FROM will be essential to perform the calculations for future e^+e^- colliders
- Important: secure future of FORM!

Monte Carlo generators

[talk by Silvia Ferrario Ravasio on Tuesday]

- MG5_aMC [Frederix et al.]; Whizard [Kilian,Ohl,Reuter]; SHERPA [Bothmann,Krauss et al.]; ...
- Parton shower: NLL ➪ NNLL $\leftrightarrow$ LHC

  [Dasgupta,Höche,Nagy,Plätzer,Prestel,Salam,Soper,. . .]
- Automated hard matrix elements: NLO ➪ NNLO
- Matching of parton showers to higher order calculations for the hard scattering $\leftrightarrow$ LHC
- Non-perturbative aspects (hadronization, . . .)
- QED:
  Resummation of soft and collinear photon radiation in initial state
  Yennie, Frautschi and Suura (YFS) exponentiation
  collinear factorization, NLL [Frixione et al.]

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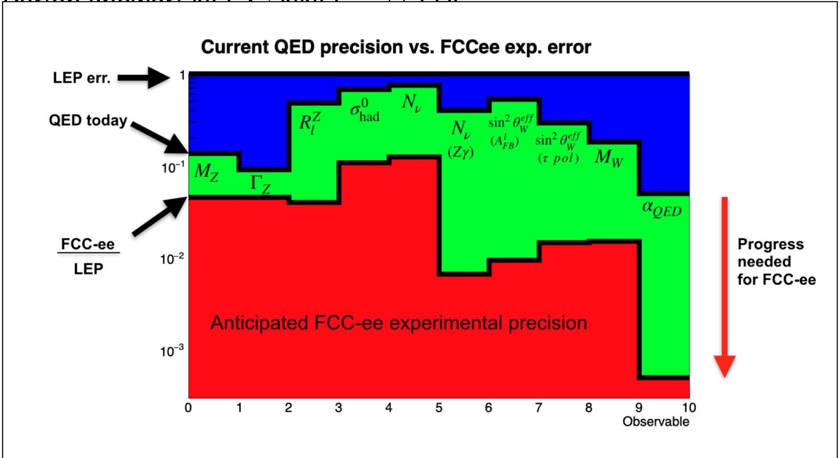

[Jadach,Skrzypek'19]

Further topics

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- Flavour physics

Particle count (10^9)	$B^0(\bar{B}^0)$	$B^\pm$	$B_s^0(\bar{B}_s^0)$	$B_c^\pm$	$\Lambda_b(\bar{\Lambda}_b)$	$c(\bar{c})$	$\tau^\pm$
Belle-II	55	55	0.6	N.A.	N.A.	130	90
FCC-ee	770	770	170	7	150	1400	400

[Zuo'23]

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[Zuo’23]

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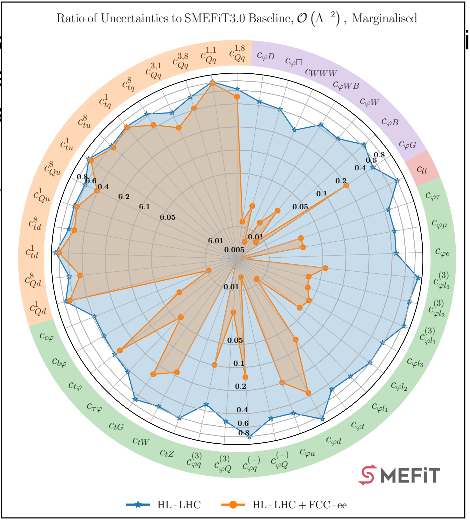

[Celada,Giani,ter Hoeve,Mantani,Rojo,Rossia,Thomas,Vryonidou’24]

[talk by Eleni Vryonidou on Tuesday]

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- “Pseudo observables” (couplings, masses, partial widths, asymmetries, . . .)
- Flavour physics
- Tau physics
- Rare Processes
- Lattice
- SMEFT
- Bhabha scattering
- Renormalize the SM to 2 (maybe 3) loops
- . . .

Conclusions

- 1 – 2 more orders in perturbation theory
- Enormous challenges!
- Can this be done?

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- 1 – 2 more orders in perturbation theory
- Enormous challenges!
- Can this be done? YES!
  - If needed, theory will provide results
  - Important: We have to train the young generation well