Super-Leading Logarithms in t-$\bar{t}$ Production: Derivation

Josua Scholze | DESY Theory Workshop
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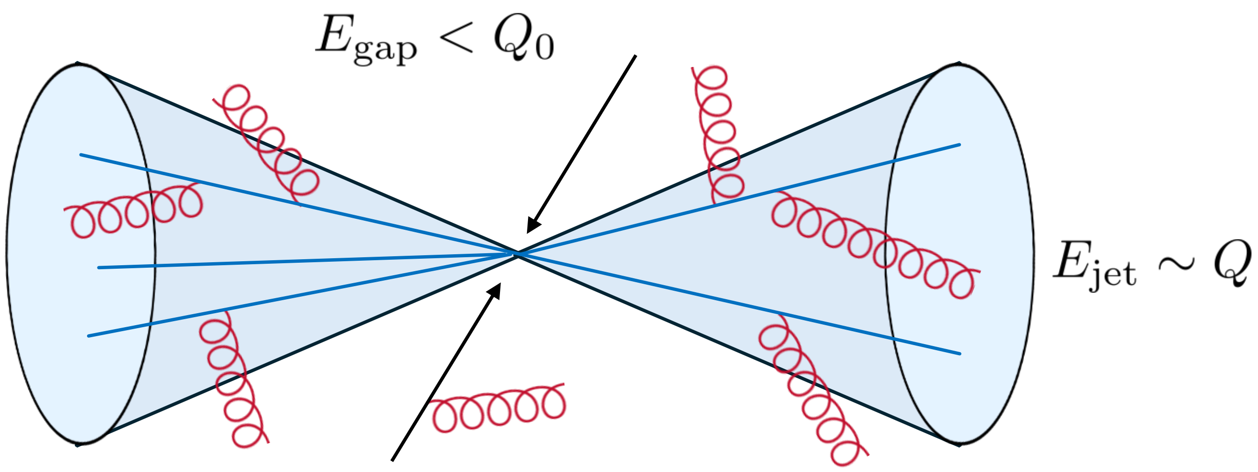
Super-Leading Logarithms



- Large logarithms in jet processes at hardon colliders (pp → jets):

$$\sigma \sim \sigma_{\text{Born}} \times \{1 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 L^5 + \alpha_s^5 L^7 + \ldots}_{\text{“super-leading logarithms”}}\} \qquad L = \ln(Q/Q_0) \gg 1$$

[J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)]

Super-Leading Logarithms

- SCET factorization theorem: [T. Becher, M. Neubert, D. Shao (2021); + M. Stillger (2023)]

$$\sigma_{2\to M}(Q_0) = \int d\xi_1 \int d\xi_2 \sum_{m=2+M}^{\infty} \langle \underbrace{\boldsymbol{\mathcal{H}}_m(\{\underline{n},\underline{v}\},\{\underline{m}\},s,\xi_1,\xi_2,\mu)}_{\text{Hard functions}} \otimes \underbrace{\boldsymbol{\mathcal{W}}_m(\{\underline{n},\underline{v}\},Q_0,\xi_1,\xi_2,\mu)}_{\text{Low-energy matrix elements}} \rangle$$

- Massless partons: $n_i = p_i/E_i\,, \quad n_i^2 = 0$
- Massive quarks: $v_I = p_I/m_I\,, \quad v_I^2 = 1$

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- Massless partons: $n_i = p_i/E_i\,, \quad n_i^2 = 0$
- Massive quarks: $v_I = p_I/m_I\,, \quad v_I^2 = 1$
- Hard functions:

$$\boldsymbol{\mathcal{H}}_m = \frac{1}{2\xi_1\xi_2 s}\prod_{i\neq 1,2}\int \frac{dE_i\,E_i^{d-3}}{\tilde{c}^{\varepsilon}(2\pi)^2}|\mathcal{M}_m(\{\underline{p}\},\{\underline{m}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\},\{\underline{m}\})| \times (\text{phase-space constraints})$$

multiplicity

Momentum conservation + $\Theta_{\text{hard}}(\{\underline{n},\underline{v}\})$

only over massless final states, remaining in $\otimes$

Super-Leading Logarithms

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- For resummation, set $\mu = \mu_s \sim Q_0$

PDFs

$$\boldsymbol{\mathcal{W}}_m(\xi_1,\xi_2,\mu_s) = f_1(\xi_1,\mu_s) f_2(\xi_2,\mu_s)\mathbf{1} + \mathcal{O}(\alpha_s)$$

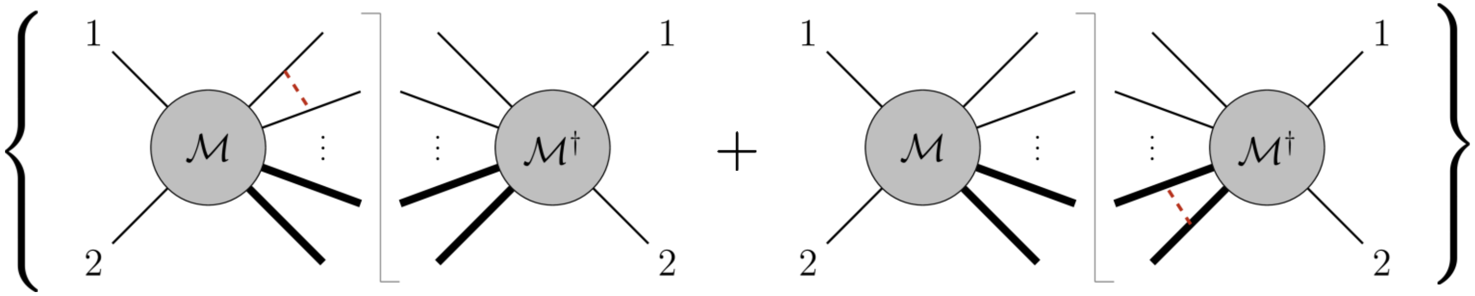
$$\boldsymbol{\mathcal{H}}_m(\mu_s) = \sum_{l\le m} \boldsymbol{\mathcal{H}}_l(\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu)\right]_{lm}$$

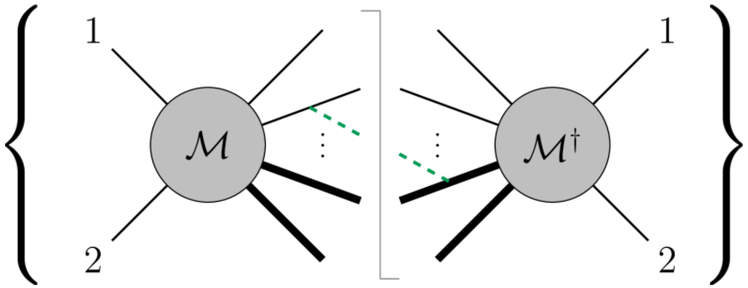
Anomalous dimension matrix:

Need to include **massive** final states

Anomalous dimension

- Divergences in the hard functions $\boldsymbol{\mathcal{H}}_m \propto \int d\mathcal{E}_m |\mathcal{M}_m(\{\underline{p}\}, \{\underline{m}\}, \mu)\rangle\langle\mathcal{M}_m(\{\underline{p}\}, \{\underline{m}\}, \mu)|$

- Virtual contributions: $\sum_{\substack{\text{all}\\\text{contributions}}}$ 

- Real contributions: $\sum_{\substack{\text{all}\\\text{contributions}}}$ 

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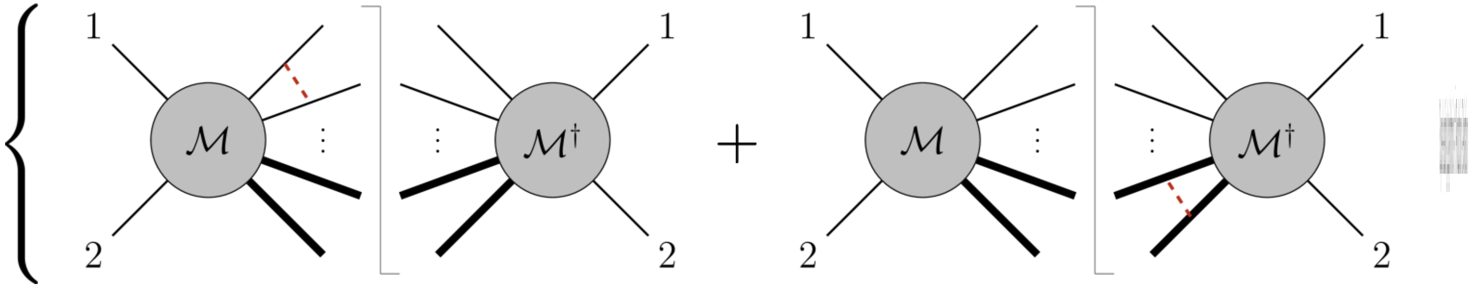

- Real contributions: $\sum\limits_{\substack{\text{all}\\\text{contributions}}}$

Anomalous dimension: Virtual corrections

- Anomalous dimension of the QCD amplitude: [T. Becher, M. Neubert (2009)]

$$\mathbf{\Gamma}^{\mathcal{M}}(\{\underline{p}\},\mu) = \frac{1}{2}\sum_{(ij)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln\left(\frac{\mu^2}{-s_{ij}}\right) + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s)$$

Distinct pairs

Color generators

$$s_{ij} = \begin{cases} +2p_i \cdot p_j + i\varepsilon & \text{partons both in- or outgoing} \\ -2p_i \cdot p_j + i\varepsilon & \text{otherwise} \end{cases}$$

Anomalous dimension: Virtual corrections

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$i, j:$ massless
$I, J:$ massive

$$\begin{aligned}\mathbf{\Gamma}^{\mathcal{M}}(\{\underline{p}\},\{\underline{m}\},\mu) = &\frac{1}{2}\sum_{(ij)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln\left(\frac{\mu^2}{-s_{ij}}\right) + \sum_i \gamma^i(\alpha_s) \\ &\left.\begin{aligned}&+\sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln\left(\frac{m_I \mu}{-s_{Ij}}\right) \\ &-\frac{1}{2}\sum_{(IJ)} \boldsymbol{T}_I \cdot \boldsymbol{T}_J \gamma_{\text{cusp}}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) + \mathcal{O}(\alpha_s^2)\end{aligned}\right\} \text{Additional terms for massive states}\end{aligned}$$

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$$\gamma_{\text{cusp}}(\beta_{IJ}, \alpha_s) = \gamma_{\text{cusp}}(\alpha_s)\, \beta_{IJ} \coth(\beta_{IJ}) + \mathcal{O}(\alpha_s^2)$$

$$\beta_{IJ} = \operatorname{arcosh}\left(\frac{-s_{IJ}}{2 m_I m_J}\right)$$

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$$+ \sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln\left(\frac{m_I \mu}{-s_{Ij}}\right)$$

$$- \frac{1}{2}\sum_{(IJ)} \boldsymbol{T}_I \cdot \boldsymbol{T}_J \gamma_{\text{cusp}}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) + \mathcal{O}(\alpha_s^2)$$

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Anomalous dimension: Virtual corrections

Real part:

$$\ln\left(\frac{\mu^2}{-s_{ij}}\right) = \ln\left(\frac{\mu}{2E_i}\right) + \ln\left(\frac{\mu}{2E_j}\right) - \int [\mathrm{d}\Omega_k] \overline{W}_{ij}^k$$

$$\ln\left(\frac{m_I \mu}{-s_{Ij}}\right) = \ln\left(\frac{\mu}{2E_j}\right) - \int [\mathrm{d}\Omega_k] \overline{W}_{Ij}^k$$

$$-\beta_{IJ}\coth(\beta_{IJ}) = -\int [\mathrm{d}\Omega_k] \overline{W}_{IJ}^k$$

Soft dipole radiators

$$\overline{W}_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k n_j \cdot n_k} \underbrace{- \frac{1}{n_i \cdot n_k}\delta(n_i - n_k) - \frac{1}{n_j \cdot n_k}\delta(n_i - n_k)}_{\text{subtract collinear limits } n_i || n_k,\ n_j || n_k}$$

$$\overline{W}_{Ij}^k = \frac{v_I \cdot n_j}{v_I \cdot n_k n_j \cdot n_k} - \frac{1}{n_j \cdot n_k}\delta(n_j - n_k)$$

$$\overline{W}_{IJ}^k = \frac{v_I \cdot v_J}{v_I \cdot n_k \, v_J \cdot n_k}$$

Anomalous dimension: Virtual corrections

Real part:

Imaginary part:

$$\ln\left(\frac{\mu^2}{-s_{ij}}\right) = \ln\left(\frac{\mu}{2E_i}\right) + \ln\left(\frac{\mu}{2E_j}\right) - \int [\mathrm{d}\Omega_k]\overline{W}^k_{ij} \quad + i\pi\Pi_{ij}$$

$$\ln\left(\frac{m_I\mu}{-s_{Ij}}\right) = \ln\left(\frac{\mu}{2E_j}\right) - \int [\mathrm{d}\Omega_k]\overline{W}^k_{Ij} \quad + i\pi\Pi_{Ij}$$

$$-\beta_{IJ}\coth(\beta_{IJ}) = -\int [\mathrm{d}\Omega_k]\overline{W}^k_{IJ} \quad + i\pi\frac{(v_I\cdot v_J)}{\sqrt{(v_I\cdot v_J)^2-1}}$$

$$\Pi_{\alpha\beta} = \begin{cases} 1 & \text{partons both in- or outgoing} \\ 0 & \text{otherwise} \end{cases}$$

Anomalous dimension: Virtual corrections

- For the hard functions:

Includes massive monopoles

$$\begin{aligned}\boldsymbol{\mathcal{H}}_m(\{\underline{n},\underline{v}\},\{\underline{m}\},\varepsilon) \supset \frac{\alpha_s}{4\pi}\Bigg\{ &-\frac{\gamma_0}{4\varepsilon}\sum_{\alpha,\beta}(\boldsymbol{T}_{\alpha,L}\cdot\boldsymbol{T}_{\beta,L}+\boldsymbol{T}_{\alpha,R}\cdot\boldsymbol{T}_{\beta,R})\int\frac{d^2\Omega_k}{4\pi}\overline{W}^k_{\alpha\beta}\boldsymbol{\mathcal{H}}_m(\{\underline{n},\underline{v}\},\{\underline{m}\},\mu) \\ &+\frac{\gamma_0}{\varepsilon}i\pi\Bigg[(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L}-\boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}) \\ &\qquad+\frac{1}{4}\sum_{(IJ)}(\boldsymbol{T}_{I,L}\cdot\boldsymbol{T}_{J,L}-\boldsymbol{T}_{I,R}\cdot\boldsymbol{T}_{J,R})\,v_{IJ}\Bigg]\boldsymbol{\mathcal{H}}_m(\{\underline{n},\underline{v}\},\{\underline{m}\},\mu) \\ &-\int d\mathcal{E}_m\sum_i\left[\gamma_0 C_i\left(\frac{1}{2\varepsilon^2}+\frac{1}{\varepsilon}\ln\left(\frac{\mu}{2E_i}\right)\right)-\frac{\gamma_0^i}{\varepsilon}\right]\widetilde{\boldsymbol{\mathcal{H}}}_m(\{\underline{p}\},\{\underline{m}\},\mu)\Bigg\}\end{aligned}$$

Purely soft

Glauber

Coulomb

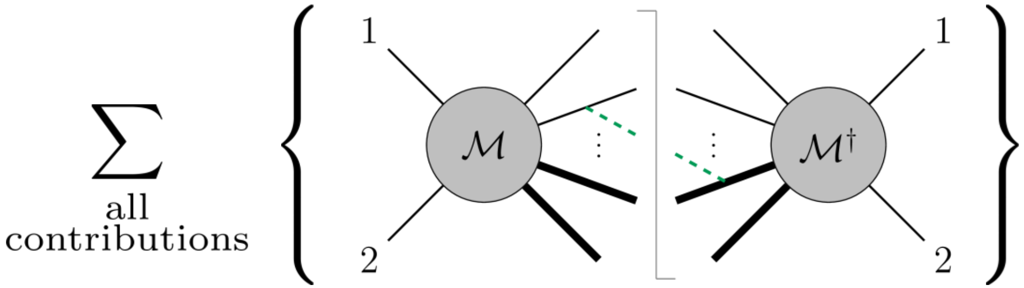
Soft+collinear

- With kinematical factor $v_{IJ} \equiv \dfrac{(v_I\cdot v_J)}{\sqrt{(v_I\cdot v_J)^2-1}}-1$

Anomalous dimension

- Divergencies in the hard functions $\boldsymbol{\mathcal{H}}_m \propto \int d\mathcal{E}_m |\mathcal{M}_m(\{\underline{p}\},\{\underline{m}\},\mu)\rangle\langle\mathcal{M}_m(\{\underline{p}\},\{\underline{m}\},\mu)|$

- Virtual contributions:

- Real contributions: 

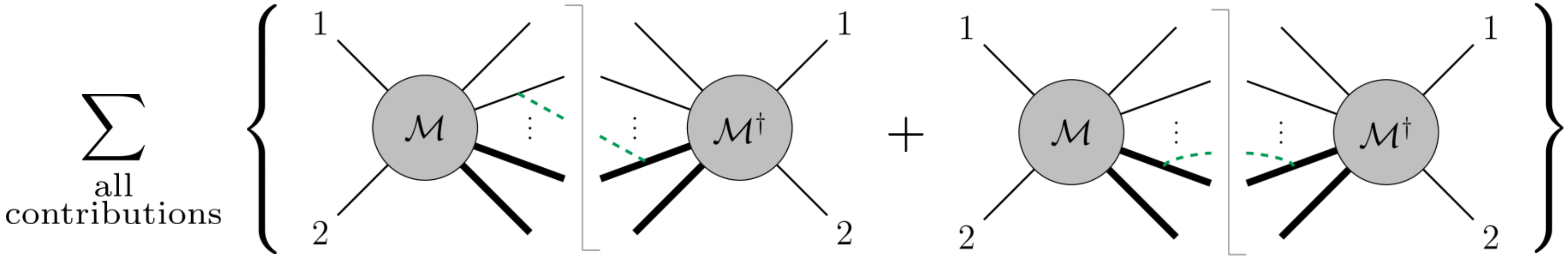
Anomalous dimension: Real emissions

- In the **soft** limit, the Soft theorem holds
- Separating the soft divergence:

$$\sum_{s_q} \boldsymbol{\mathcal{H}}_{m+1}(\{\underline{p},q\},\{\underline{m}\},\varepsilon) \supset \frac{\alpha_s}{\pi}\frac{1}{2\varepsilon}\sum_{\alpha,\beta}\theta_{\text{hard}}(n_q)\overline{W}^{q}_{\alpha\beta}\boldsymbol{T}_{\alpha,L}\circ\boldsymbol{T}_{\beta,R}\boldsymbol{\mathcal{H}}_m(\{\underline{p}\},\{\underline{m}\},\mu)$$

Color space of
emitted gluon

Monopoles for
massive quarks



Anomalous dimension: Real emissions

- **Collinear** limits: No additional divergences from massive quarks
- Unchanged compared to massless case:

$$\begin{aligned}
&\langle \boldsymbol{\mathcal{H}}_{m+1}(\{\underline{n},\underline{v}\},\{\underline{m}\},\varepsilon)\otimes \boldsymbol{\mathcal{W}}_{m+1}(\{\underline{n},\underline{v}\},\varepsilon)\rangle \\
&\quad\supset \frac{\alpha_s}{4\pi}\int d\mathcal{E}_m \sum_{i\neq 1,2}\left[C_i\gamma_0\left(\frac{1}{2\varepsilon^2}+\frac{1}{\varepsilon}\ln\left(\frac{\mu}{2E_i}\right)\right)-\frac{\gamma_0^i}{\varepsilon}\right] \\
&\qquad\times \langle \widetilde{\boldsymbol{\mathcal{H}}}_m(\{\underline{\hat{n}},\underline{\hat{v}}\},\{\underline{\hat{m}}\},\mu)\otimes \boldsymbol{\mathcal{W}}_m(\{\underline{\hat{n}},\underline{\hat{v}}\},\mu)\rangle+\mathcal{O}(\varepsilon^0)
\end{aligned}$$

- $\{\underline{\hat{n}},\underline{\hat{v}}\}$ and $\{\underline{\hat{m}}\}$ include parent parton P instead of collinear partons α and β

- Cancellations with virtual corrections

Anomalous dimension

- RG equation: $\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(\{\underline{n},\underline{v}\},\{\underline{m}\},s,\mu) = -\sum_{l=2+M}^{m}\boldsymbol{\mathcal{H}}_l(\{\underline{n},\underline{v}\},\{\underline{m}\},s,\mu) * \boldsymbol{\Gamma}^{\mathcal{H}}_{lm}(\{\underline{n},\underline{v}\},s,\mu)$

- Infinite matrix in multiplicity space:

$$\boldsymbol{\Gamma}^{\mathcal{H}}(\{\underline{n},\underline{v}\},s,\mu) = \frac{\alpha_s}{4\pi}\begin{pmatrix} \boldsymbol{V}_{2+M} & \boldsymbol{R}_{2+M} & 0 & 0 & \cdots \\ 0 & \boldsymbol{V}_{2+M+1} & \boldsymbol{R}_{2+M+1} & 0 & \cdots \\ 0 & 0 & \boldsymbol{V}_{2+M+2} & \boldsymbol{R}_{2+M+2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}\left(\alpha_s^2\right)$$

- Decomposition:

$$\boldsymbol{\Gamma}^{\mathcal{H}}(\xi_1,\xi_2) = \delta(1-\xi_1)\delta(1-\xi_2)\boldsymbol{\Gamma}^S + \underbrace{\delta(1-\xi_2)\boldsymbol{\Gamma}^C_1(\xi_1) + \delta(1-\xi_1)\boldsymbol{\Gamma}^C_2(\xi_2)}_{\text{Purely collinear parts: subleading}}$$

Soft part: derived

Soft anomalous dimension including massive final states

$$\boldsymbol{\Gamma}^S = \gamma_{\text{cusp}}(\alpha_s)\left(\boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \boldsymbol{V}^G + \boldsymbol{V}^{\text{Coul}}\right) + \frac{\alpha_s}{4\pi}\overline{\boldsymbol{\Gamma}} + \mathcal{O}\left(\alpha_s^2\right)$$

Soft+collinear part — Glauber and **Coulomb** phase — Purely soft part

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2}\left[C_i \mathbf{1} - \delta(n_i - n_k)\boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R}\right]$$

$$\boldsymbol{V}^G = -2\pi i\left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

$$\boldsymbol{V}^{\text{Coul}} = -\frac{1}{2}\pi i \sum_{(IJ)}\left(\boldsymbol{T}_{I,L}\cdot\boldsymbol{T}_{J,L} - \boldsymbol{T}_{I,R}\cdot\boldsymbol{T}_{J,R}\right)\left[\frac{(v_I\cdot v_J)}{\sqrt{(v_I\cdot v_J)^2 - 1}} - 1\right]$$

$$\overline{\boldsymbol{\Gamma}} = \frac{1}{2}\gamma_0 \sum_{\alpha,\beta}(\boldsymbol{T}_{\alpha,L}\cdot\boldsymbol{T}_{\beta,L} + \boldsymbol{T}_{\alpha,R}\cdot\boldsymbol{T}_{\beta,R})\int\frac{d^2\Omega_k}{4\pi}\overline{W}_{\alpha\beta}^k - 4\sum_{\alpha,\beta}\vartheta_{\text{hard}}(n_k)\overline{W}_{\alpha\beta}^k\boldsymbol{T}_{\alpha,L}\circ\boldsymbol{T}_{\beta,R}$$

Soft anomalous dimension including massive final states

$$\Gamma^S = \gamma_{\text{cusp}}(\alpha_s)\left(\Gamma^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + V^G + V^{\text{Coul}}\right) + \frac{\alpha_s}{4\pi}\overline{\Gamma} + \mathcal{O}\left(\alpha_s^2\right)$$

Soft+collinear part — Glauber and **Coulomb** phase — Purely soft part

$$\Gamma^c = \sum_{i=1,2}\left[C_i \mathbf{1} - \delta(n_i - n_k)\, T_{i,L} \circ T_{i,R}\right]$$

$$V^G = -2\pi i\left(T_{1,L}\cdot T_{2,L} - T_{1,R}\cdot T_{2,R}\right)$$

Vanishes for $m \to 0$

Close to threshold $\to \infty$: Sommerfeld enhancement

$$V^{\text{Coul}} = -\frac{1}{2}\pi i \sum_{(IJ)}\left(T_{I,L}\cdot T_{J,L} - T_{I,R}\cdot T_{J,R}\right)\left[\frac{(v_I\cdot v_J)}{\sqrt{(v_I\cdot v_J)^2 - 1}} - 1\right]$$

$$\overline{\Gamma} = \frac{1}{2}\gamma_0 \sum_{\alpha,\beta}\left(T_{\alpha,L}\cdot T_{\beta,L} + T_{\alpha,R}\cdot T_{\beta,R}\right)\int \frac{d^2\Omega_k}{4\pi}\overline{W}^k_{\alpha\beta} - 4\sum_{\alpha,\beta}\vartheta_{\text{hard}}(n_k)\overline{W}^k_{\alpha\beta}\, T_{\alpha,L}\circ T_{\beta,R}$$

Conclusion

- Anomalous dimension for massive final states $\longleftrightarrow$ Divergences in the hard function
  - Virtual and real contributions
  - Soft and collinear limits
- $\boldsymbol{\Gamma}^S = \gamma_{\text{cusp}}(\alpha_s)\left(\boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right) + \boldsymbol{V}^G + \boldsymbol{V}^{\text{Coul}}\right) + \frac{\alpha_s}{4\pi}\overline{\boldsymbol{\Gamma}} + \mathcal{O}\left(\alpha_s^2\right)$
- New Coulomb phase for massive quarks
- Contains now massive monopoles $\overline{W}_{II}^{k}$

Conclusion

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- New Coulomb phase for massive quarks
- Contains now massive monopoles $\overline{W}_{II}^k$

➢ Resummation of Coulomb SLLs + Numerical results:

Stay tuned for the next talk by Romy Grünhofer