Effects of Super-Leading Logarithms in $t\bar{t}$-Production

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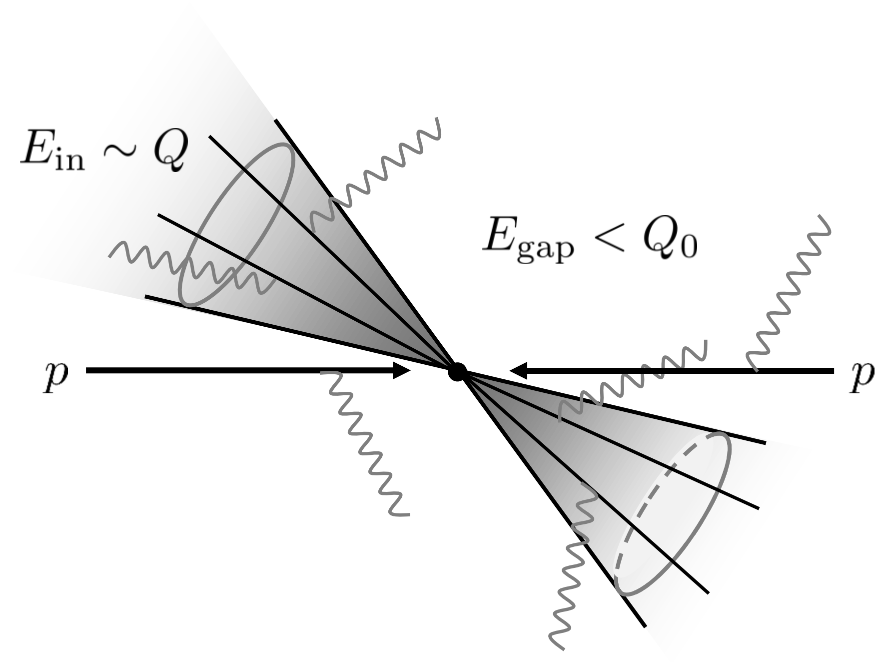
Super-Leading Logarithms for Massive Final States

Origin of SLLs

Large Logarithms in $pp \to \text{jets}$ processes: $\ln\left(\frac{Q}{Q_0}\right) \gg 1$



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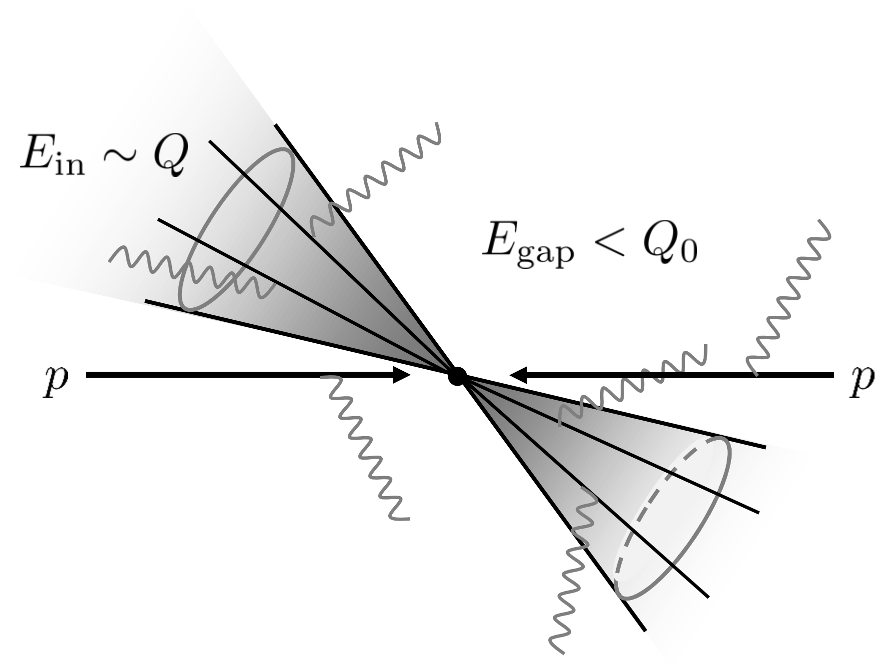
Partonic cross section evaluated at $\mu = \mu_s \sim Q_0$:

Hard function

Trivial low energy matrix element

$$\hat{\sigma}^{\text{SLL}}_{2\to M} = \langle \boldsymbol{\mathcal{H}}(Q, \mu = \mu_s) \otimes \mathbf{1} \rangle$$



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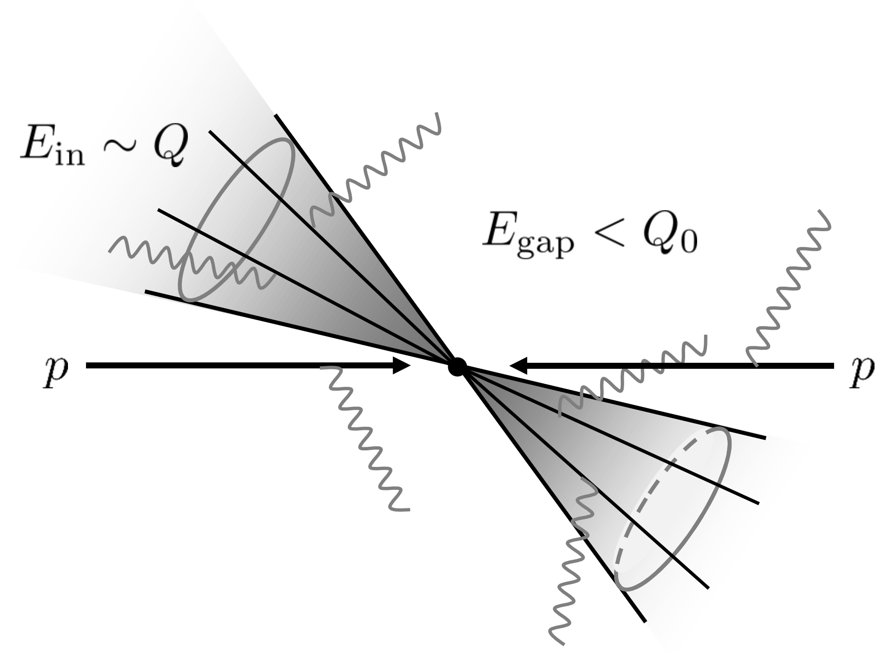

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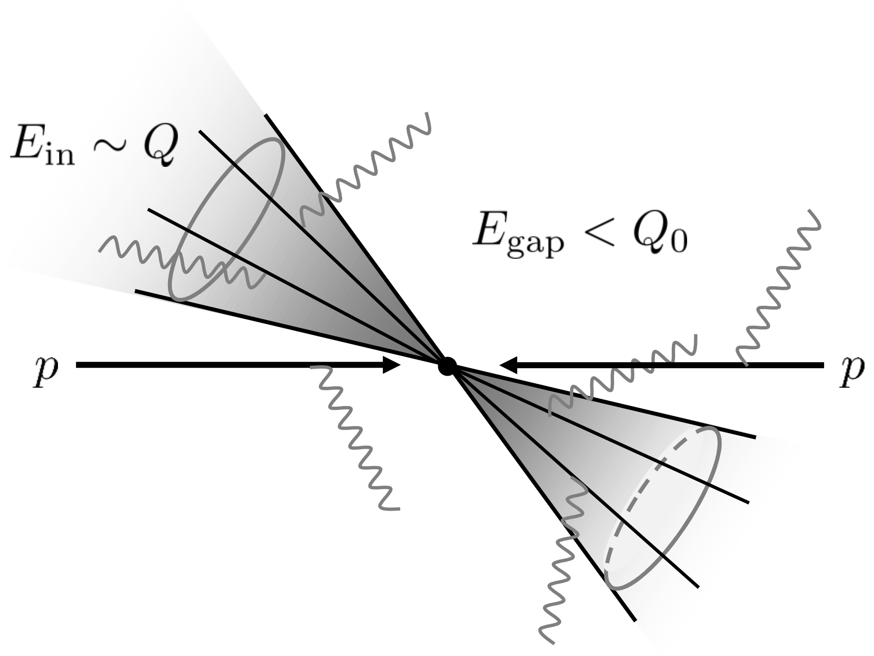

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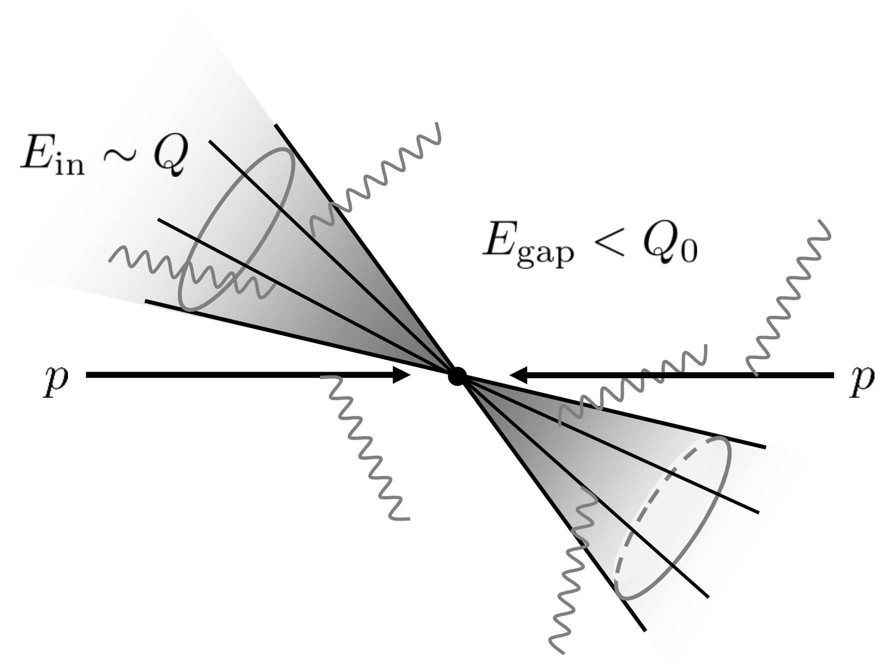

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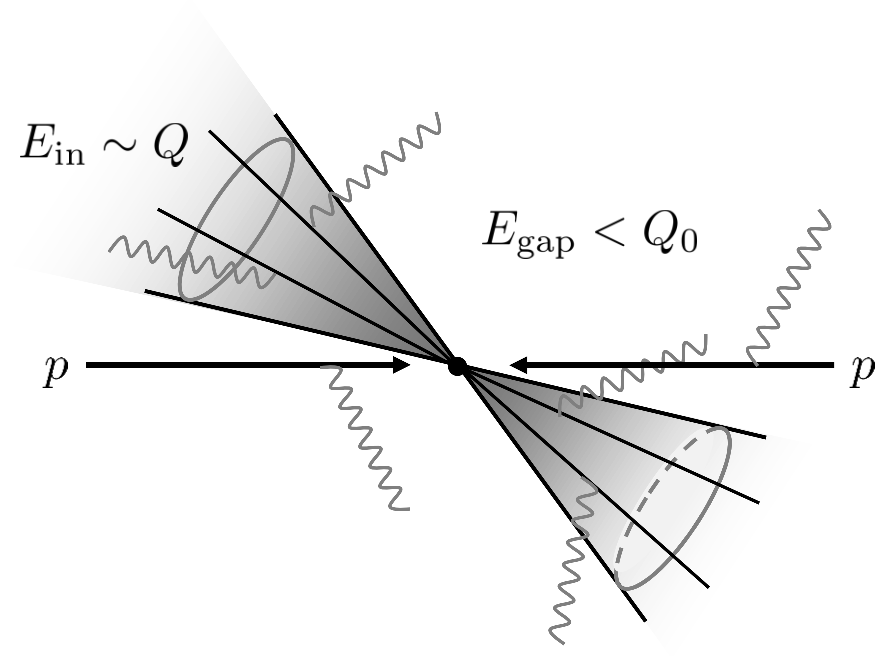

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Anomalous dimension matrix

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Anomalous Dimension

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$$\boldsymbol{\Gamma}^{\mathcal{H}}(\xi_1,\xi_2) = \delta(1-\xi_1)\delta(1-\xi_2)\boldsymbol{\Gamma}^S + \delta(1-\xi_2)\boldsymbol{\Gamma}_1^C + \delta(1-\xi_1)\boldsymbol{\Gamma}_2^C$$

⟹ Josua's talk

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Collinear parts $\Longrightarrow$ subleading effects

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Collinear parts $\Longrightarrow$ subleading effects

Soft and soft-collinear part:

$$\boldsymbol{\Gamma}^S(\mu) = \frac{\alpha_s(\mu)}{4\pi}\left[\overline{\boldsymbol{\Gamma}} + \gamma_0 \boldsymbol{V}^G + \gamma_0 \boldsymbol{V}^{\mathrm{Coul}} + \gamma_0 \boldsymbol{\Gamma}^c \ln\left(\frac{\mu^2}{\mu_h^2}\right)\right] + \mathcal{O}(\alpha_s^2)$$

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Purely soft emissions

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Coulomb phase $\Longrightarrow$ only for massive partons!

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Cusp contribution

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Colour Traces

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- Want as many $\boldsymbol{\Gamma}^c$ as possible $\overset{?}{\Longrightarrow}$ $\langle \boldsymbol{\mathcal{H}}(\mu_h)\,(\boldsymbol{\Gamma}^c)^n \otimes \mathbf{1}\rangle$

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Super-Leading Logarithms for Massive Final States

Coulomb SLLs

Resummed cross section:

$$\hat{\sigma}_{2\to M}^{\text{SLL}} = \left\langle \mathcal{H}(Q,\mu_h) \times \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h} \frac{\mathrm{d}\mu}{\mu} \boldsymbol{\Gamma}^{\mathcal{H}}(\mu)\right] \otimes \mathbf{1} \right\rangle$$

$$= \mathcal{H}(Q,\mu_h) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \mathcal{H}(Q,\mu_h) * \boldsymbol{\Gamma}^{\mathcal{H}}(\mu_1)$$
$$+ \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \mathcal{H}(Q,\mu_h) * \boldsymbol{\Gamma}^{\mathcal{H}}(\mu_1) * \boldsymbol{\Gamma}^{\mathcal{H}}(\mu_2)$$
$$+ \dots$$

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- Scale integrals

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- Scale integrals
- Colour traces:

$$\langle \boldsymbol{\mathcal{H}}(\mu_h) \mathbf{V}^{\mathrm{Coul}} \left(\mathbf{\Gamma}^c\right)^n \mathbf{V}^G \overline{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle \; + \; \text{different orderings}$$
$$\langle \boldsymbol{\mathcal{H}}(\mu_h) \mathbf{V}^{\mathrm{Coul}} \mathbf{V}^{\mathrm{Coul}} \overline{\mathbf{\Gamma}} \otimes \mathbf{1} \rangle$$

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- Scale integrals
- Colour traces: $\langle \boldsymbol{\mathcal{H}}(\mu_h) \mathbf{V}^{\text{Coul}} (\boldsymbol{\Gamma}^c)^n \mathbf{V}^G \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$ + different orderings
  $\langle \boldsymbol{\mathcal{H}}(\mu_h) \mathbf{V}^{\text{Coul}} \mathbf{V}^{\text{Coul}} \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$

Can be simplified in general

Now: consider specific process

Super-Leading Logarithms for Massive Final States

$2 \to t\bar{t}$ Processes

For $t\bar{t}$-production:

- $q\bar{q} \to t\bar{t}$
- $gg \to t\bar{t}$

Super-Leading Logarithms for Massive Final States

$2 \to t\bar{t}$ Processes

For $t\bar{t}$-production:

- $q\bar{q} \to t\bar{t}$ vanishes: colour trace $= 0$
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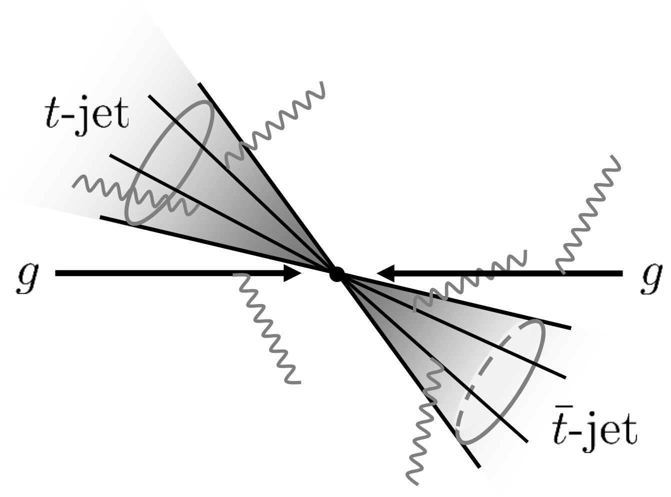

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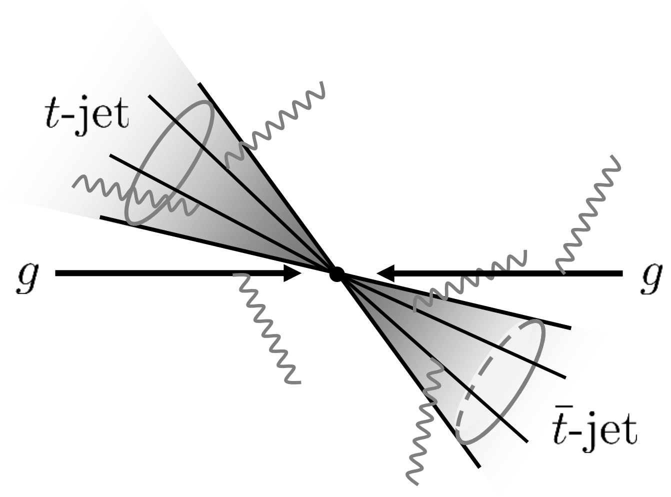

Center-of-mass frame: kinematics encoded in

- $\beta \equiv \beta_t = \beta_{\bar{t}}$ where $\beta_I = \sqrt{1 - \frac{m_I^2}{E_I^2}}$
- $\eta \equiv \eta_t = -\eta_{\bar{t}}$ where $\eta_I = \mathrm{artanh}\left(\cos(\theta_I)\right)$

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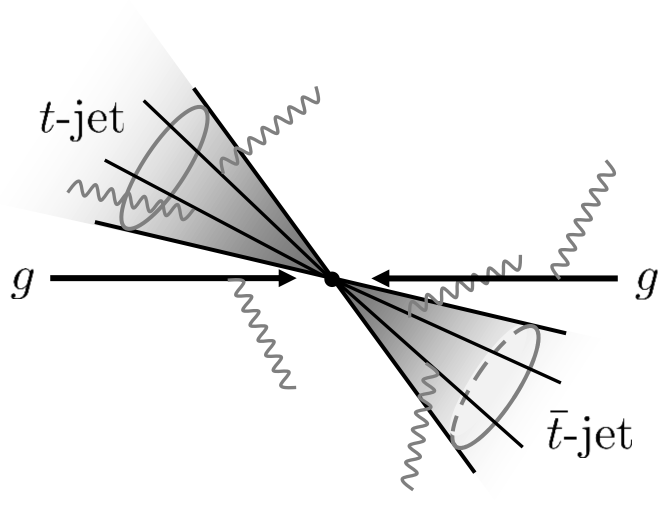

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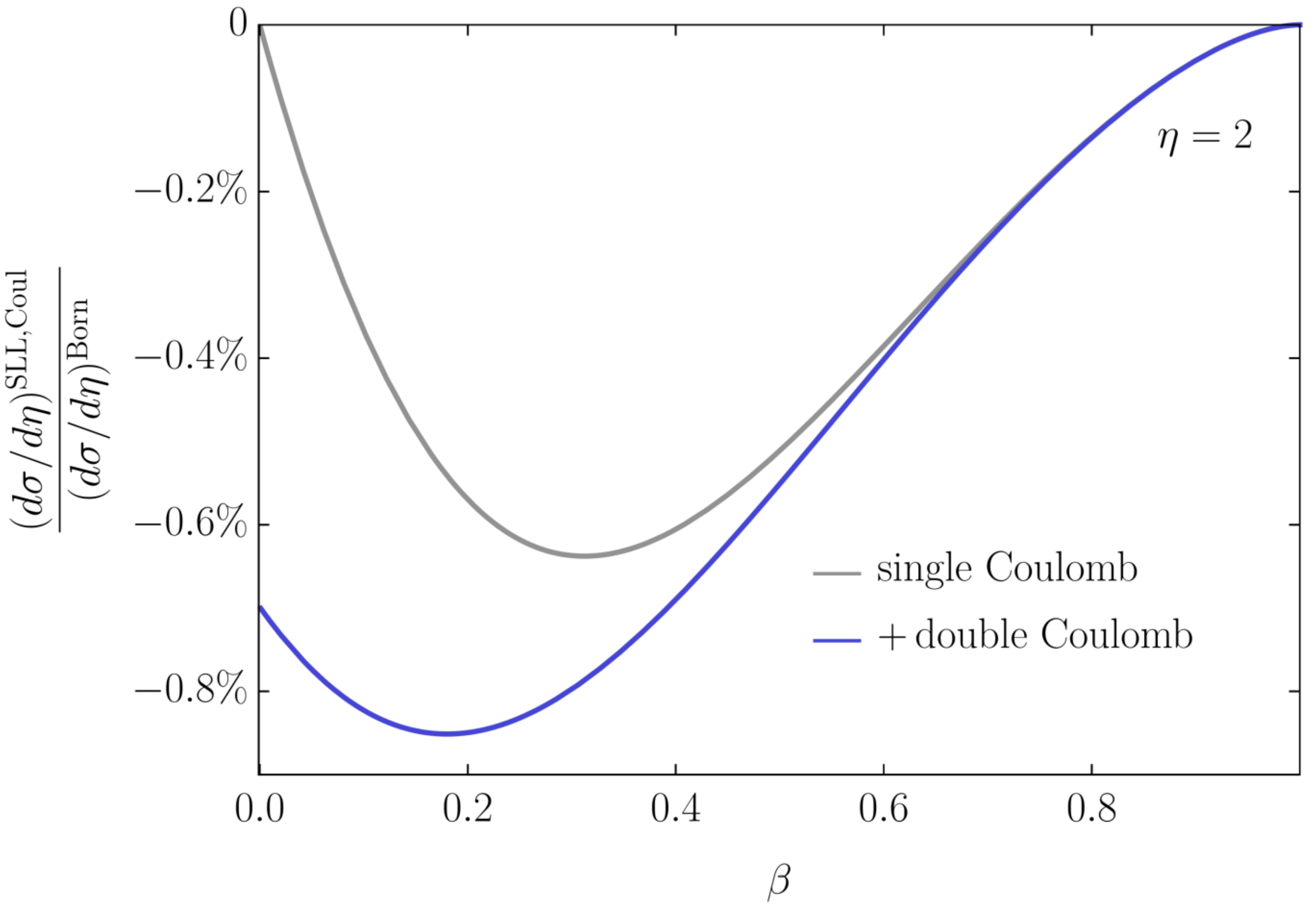
Veto region between the jets: $\eta \in [-1, 1]$

Soft scale: $\mu_s = 20\ \mathrm{GeV}$

Hard scale: $\mu_h = \frac{2m_t}{\sqrt{1-\beta^2}}$

Numerical Effects

$gg \to t\bar{t}$



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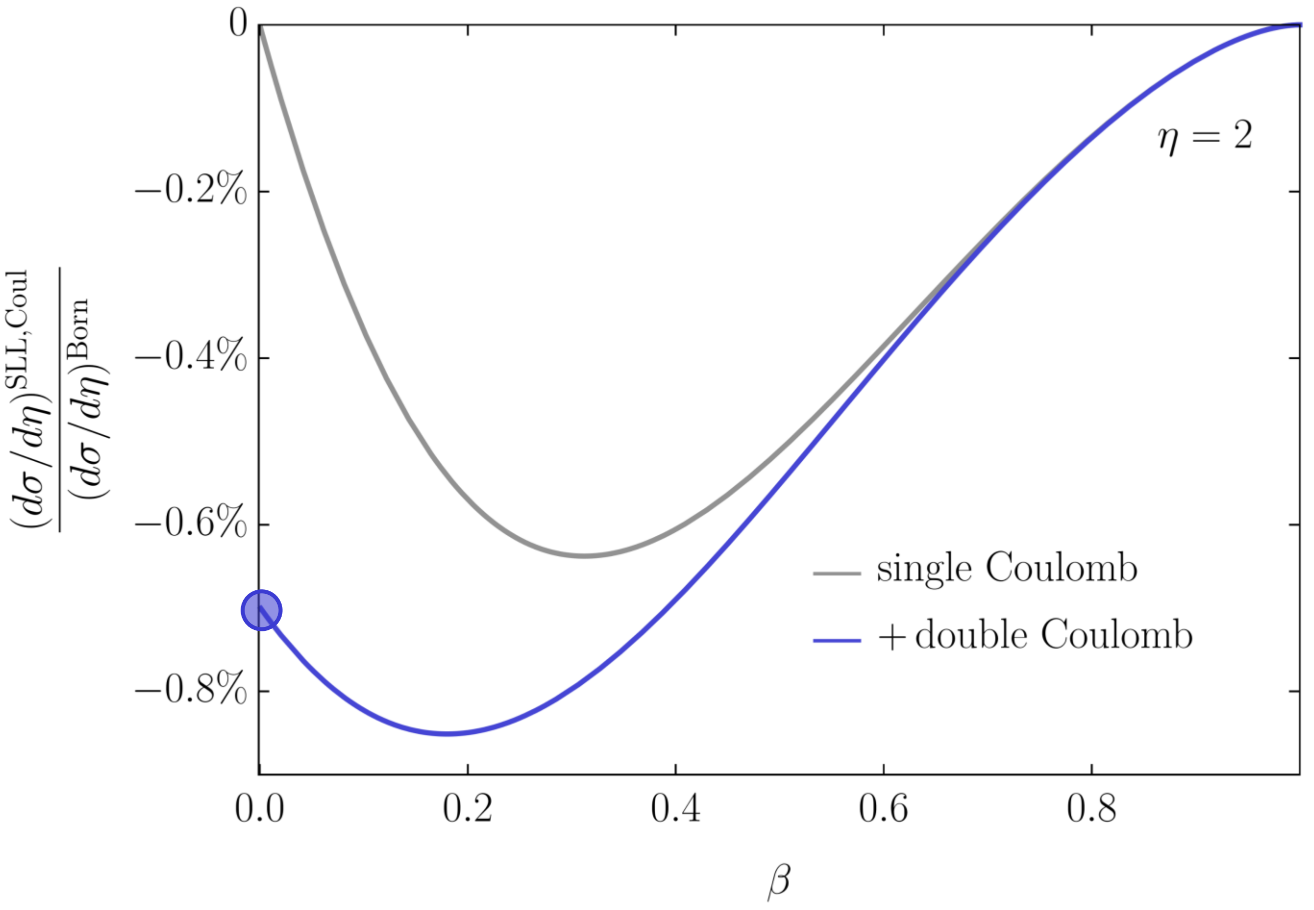
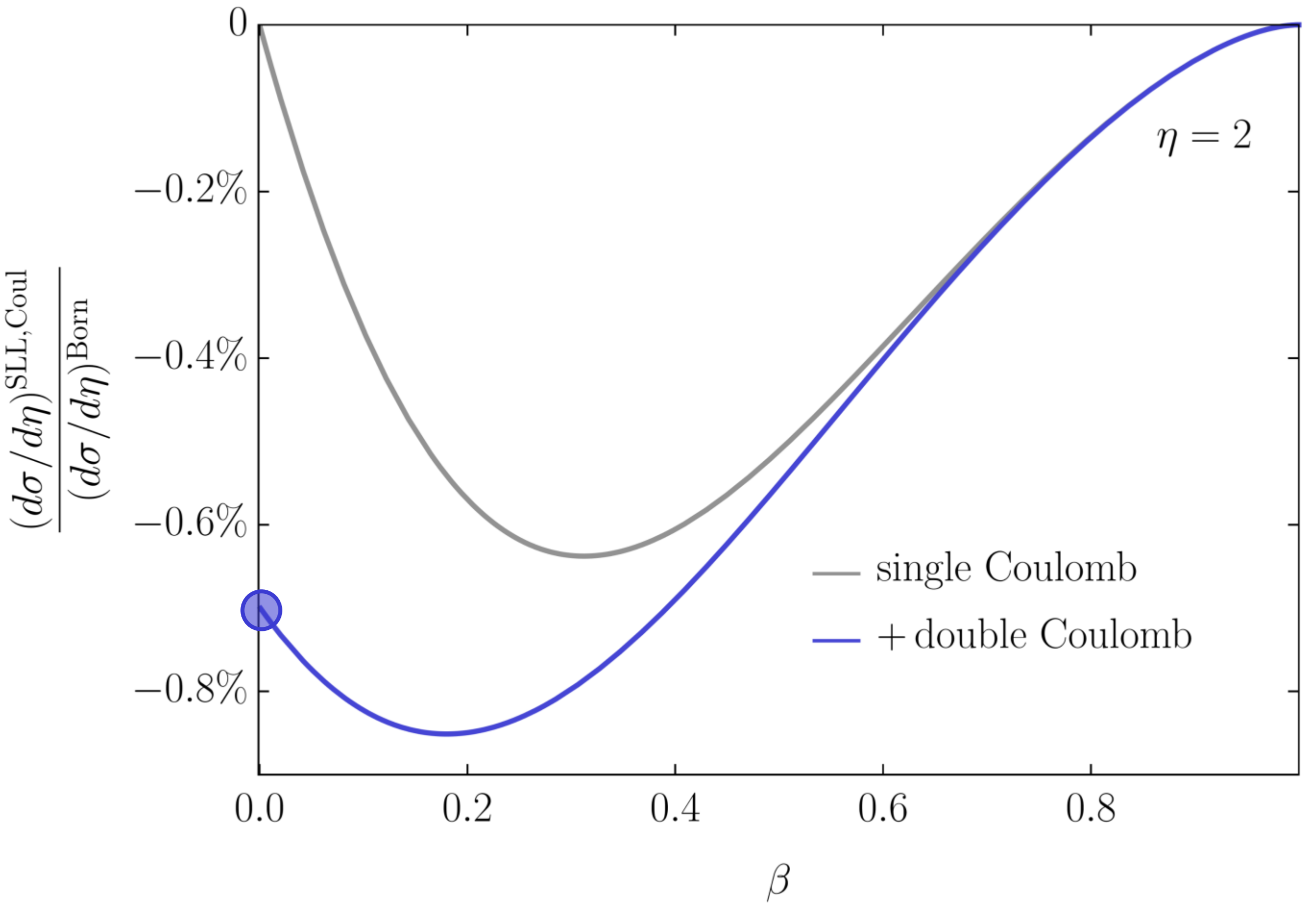


$$\mathbf{V}^{\text{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

Numerical Effects

$gg \to t\bar{t}$



Colour generators of t and $\bar{t}$

$$\mathbf{V}^{\mathrm{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R}\right) v_{t\bar{t}}$$

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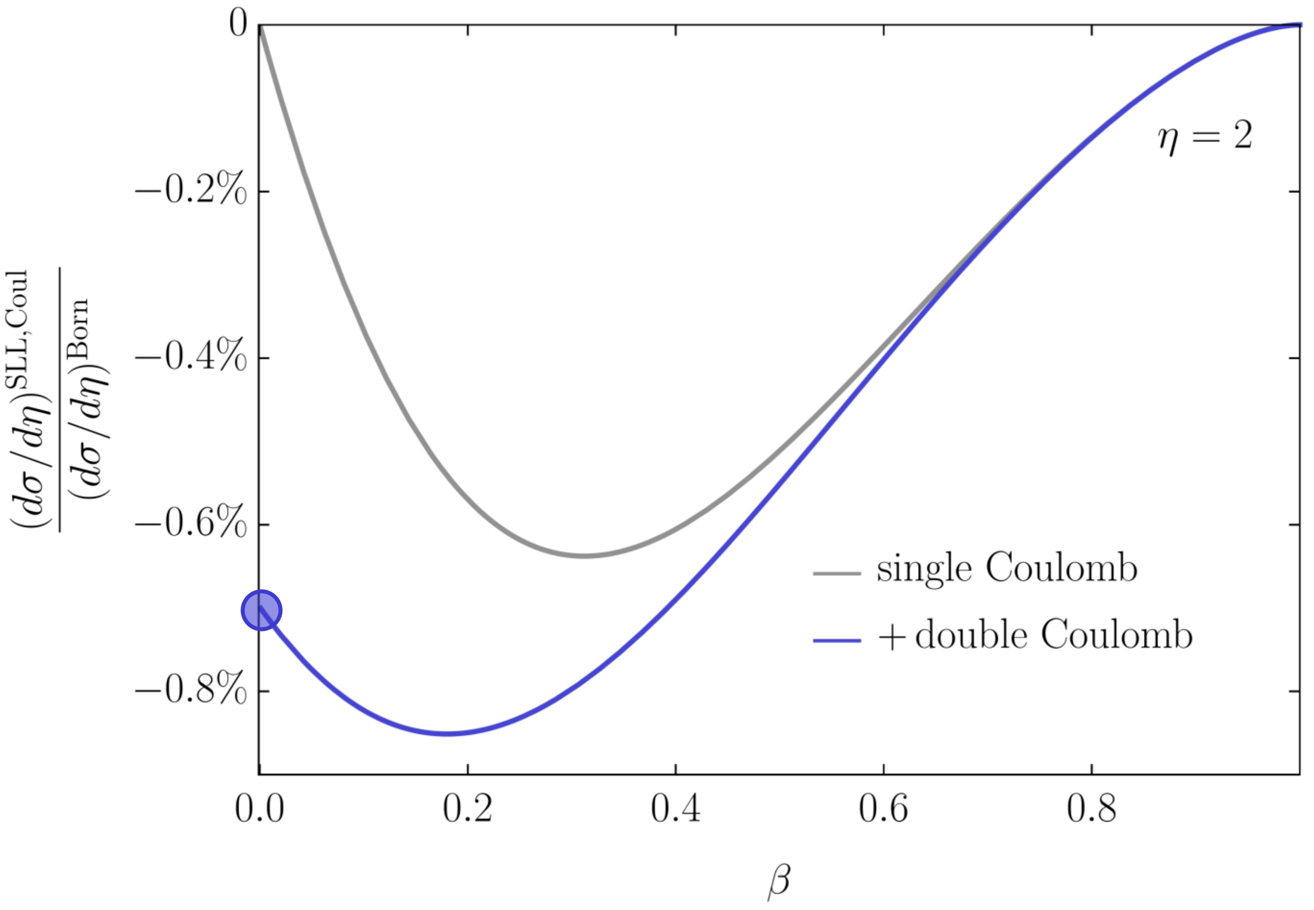

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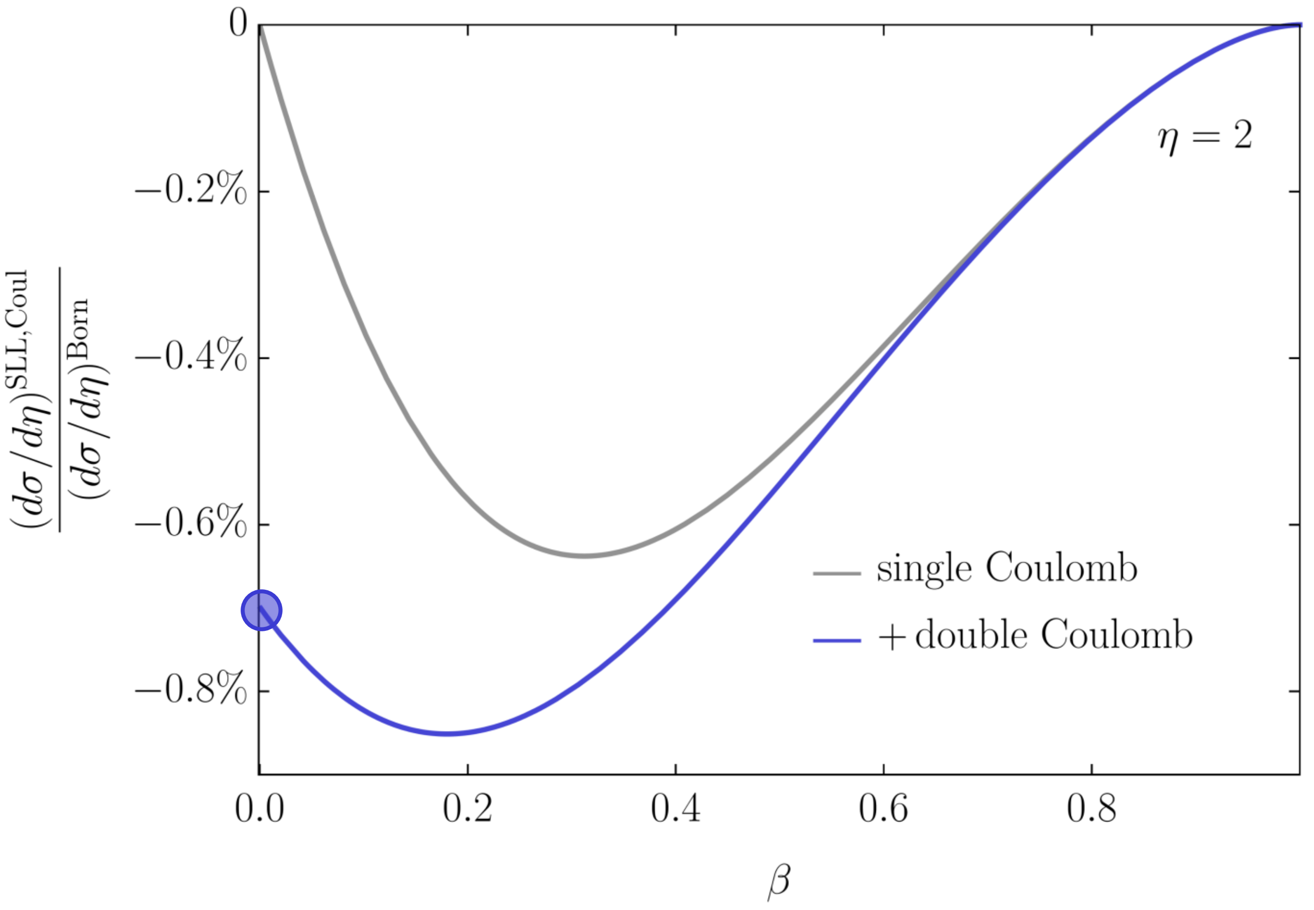
Kinematical factor $v_{t\bar{t}} = \frac{(1-\beta)^2}{2\beta}$

$\Longrightarrow$ diverges for $\beta \to 0$

Numerical Effects

$gg \to t\bar{t}$



Colour generators of t and $\bar{t}$

$$\mathbf{V}^{\mathrm{Coul}} = -i\pi \left(\boldsymbol{T}_{t,L} \cdot \boldsymbol{T}_{\bar{t},L} - \boldsymbol{T}_{t,R} \cdot \boldsymbol{T}_{\bar{t},R} \right) v_{t\bar{t}}$$

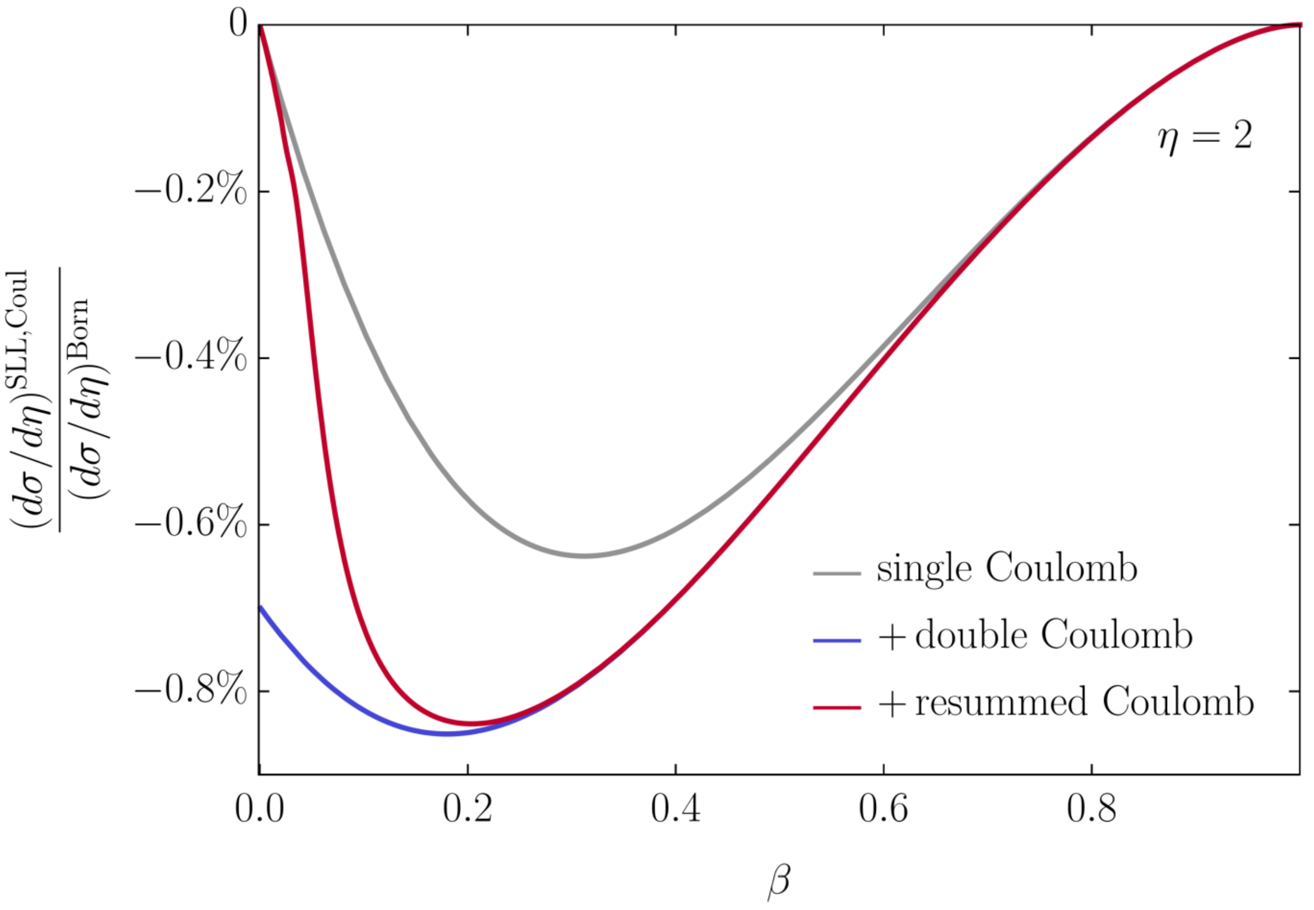
Kinematical factor $v_{t\bar{t}} = \frac{(1-\beta)^2}{2\beta}$

$\Longrightarrow$ diverges for $\beta \to 0$

- $\left(\mathbf{V}^{\mathrm{Coul}}\right)^1$: $\mathcal{O}(\beta^1)$ $\Longrightarrow$ 0 for $\beta \to 0$
- $\left(\mathbf{V}^{\mathrm{Coul}}\right)^2$: $\mathcal{O}(\beta^0)$ $\Longrightarrow$ constant for $\beta \to 0$
- $\left(\mathbf{V}^{\mathrm{Coul}}\right)^3$: $\mathcal{O}(\beta^{-1})$ $\Longrightarrow$ diverges for $\beta \to 0$
- $\dots$

Numerical Effects

$gg \to t\bar{t}$



Colour generators of t and $\bar{t}$

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⟹ diverges for $\beta \to 0$

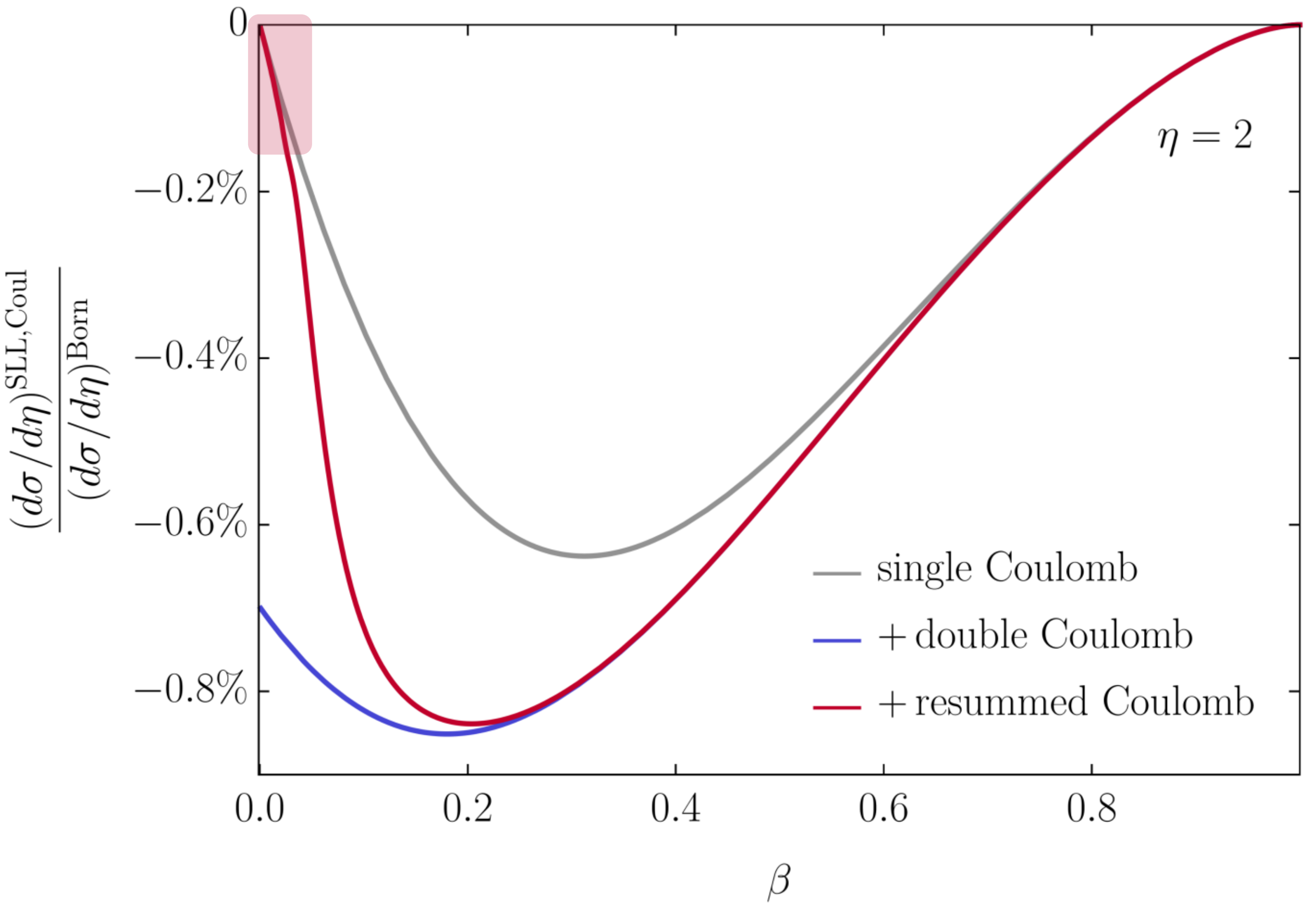
⟹ Sommerfeld effect

⟹ Resummation of arbitrary (even) number of Coulomb insertions!

$$\langle \boldsymbol{\mathcal{H}}(\mu_h) \left(\mathbf{V}^{\text{Coul}} \right)^{2n} \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle$$

Numerical Effects

$gg \to t\bar{t}$



Colour generators of t and $\bar{t}$

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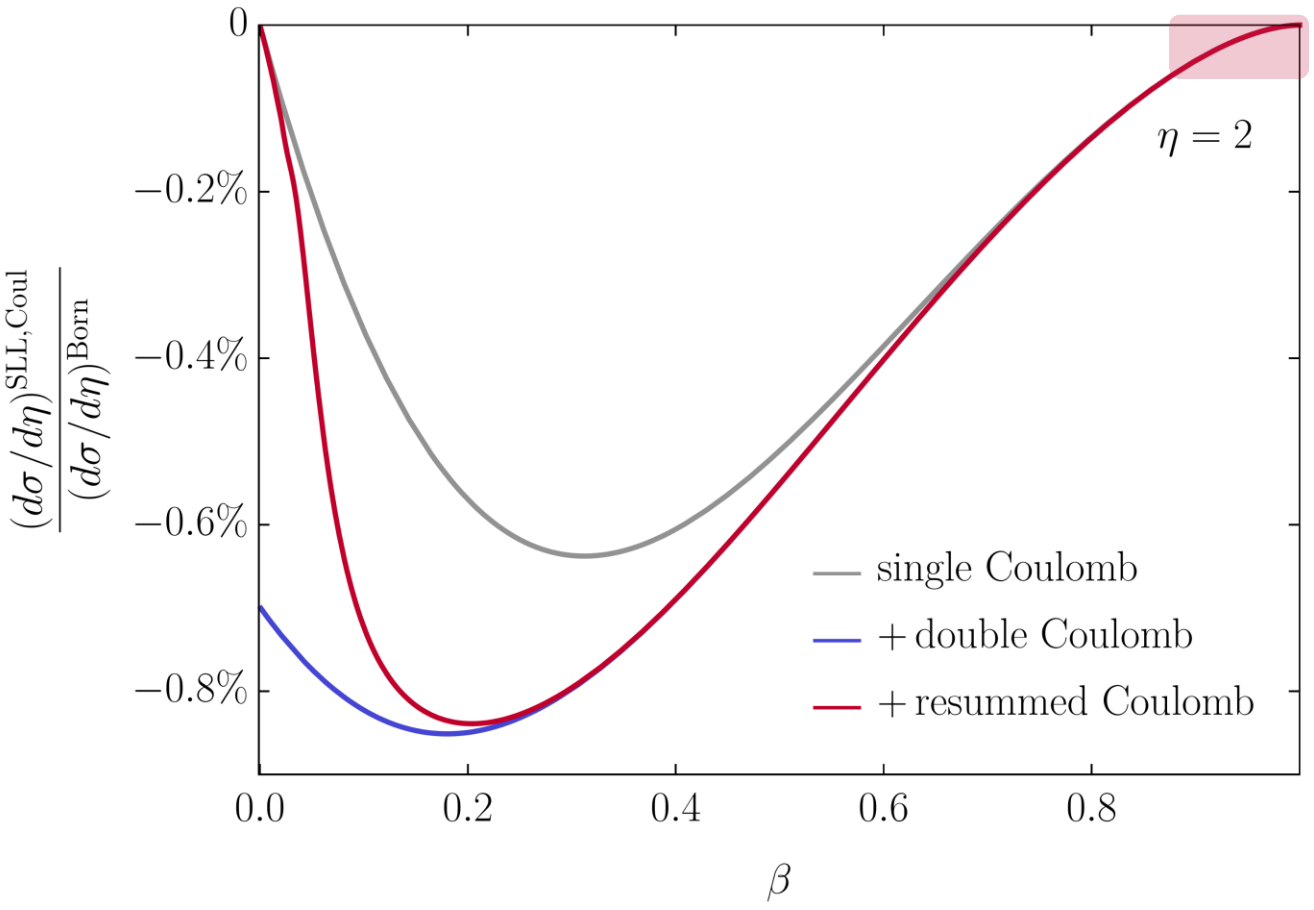

Colour generators of t and $\bar{t}$

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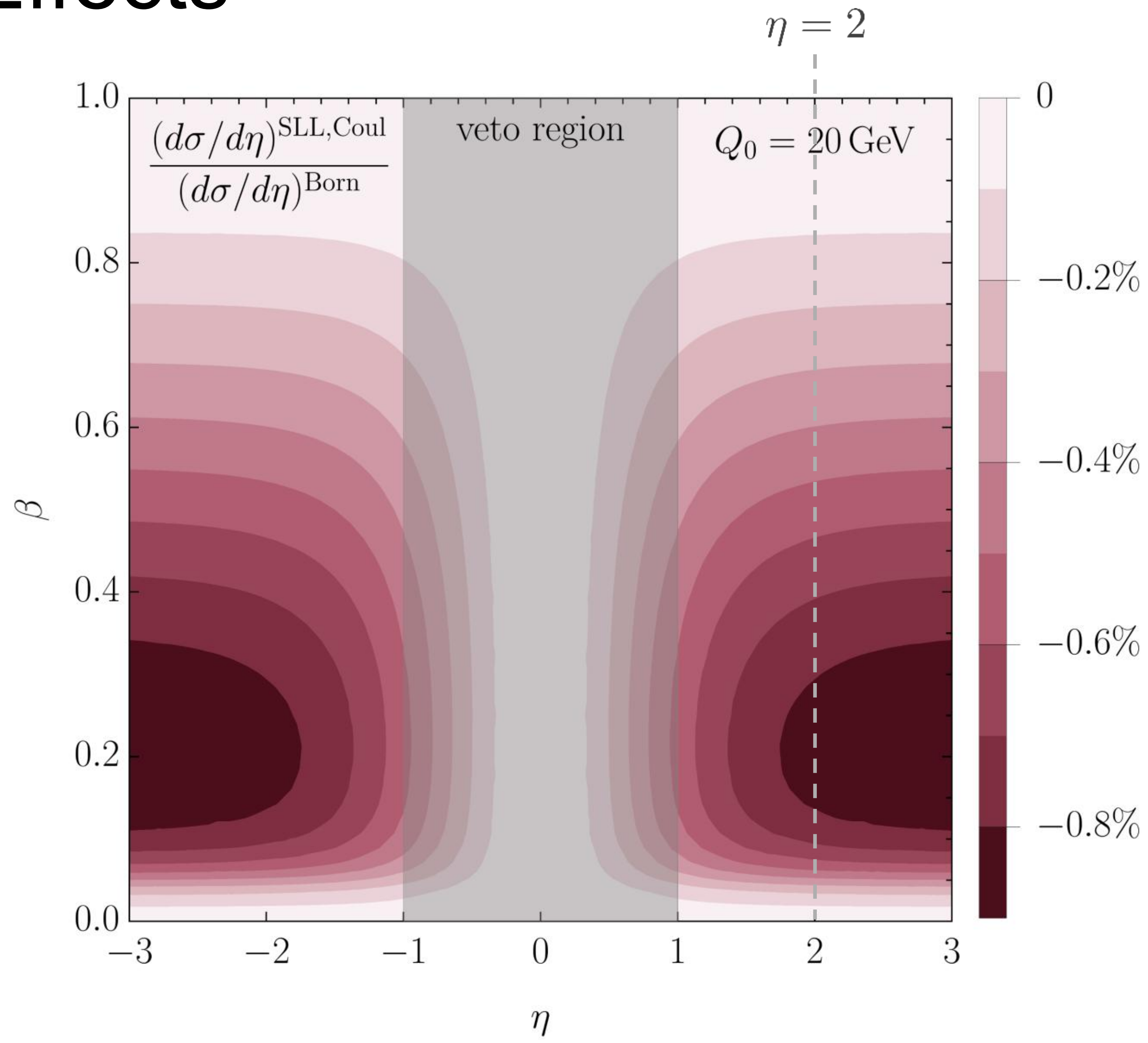
Numerical Effects

$gg \to t\bar{t}$

Coulomb SLLs



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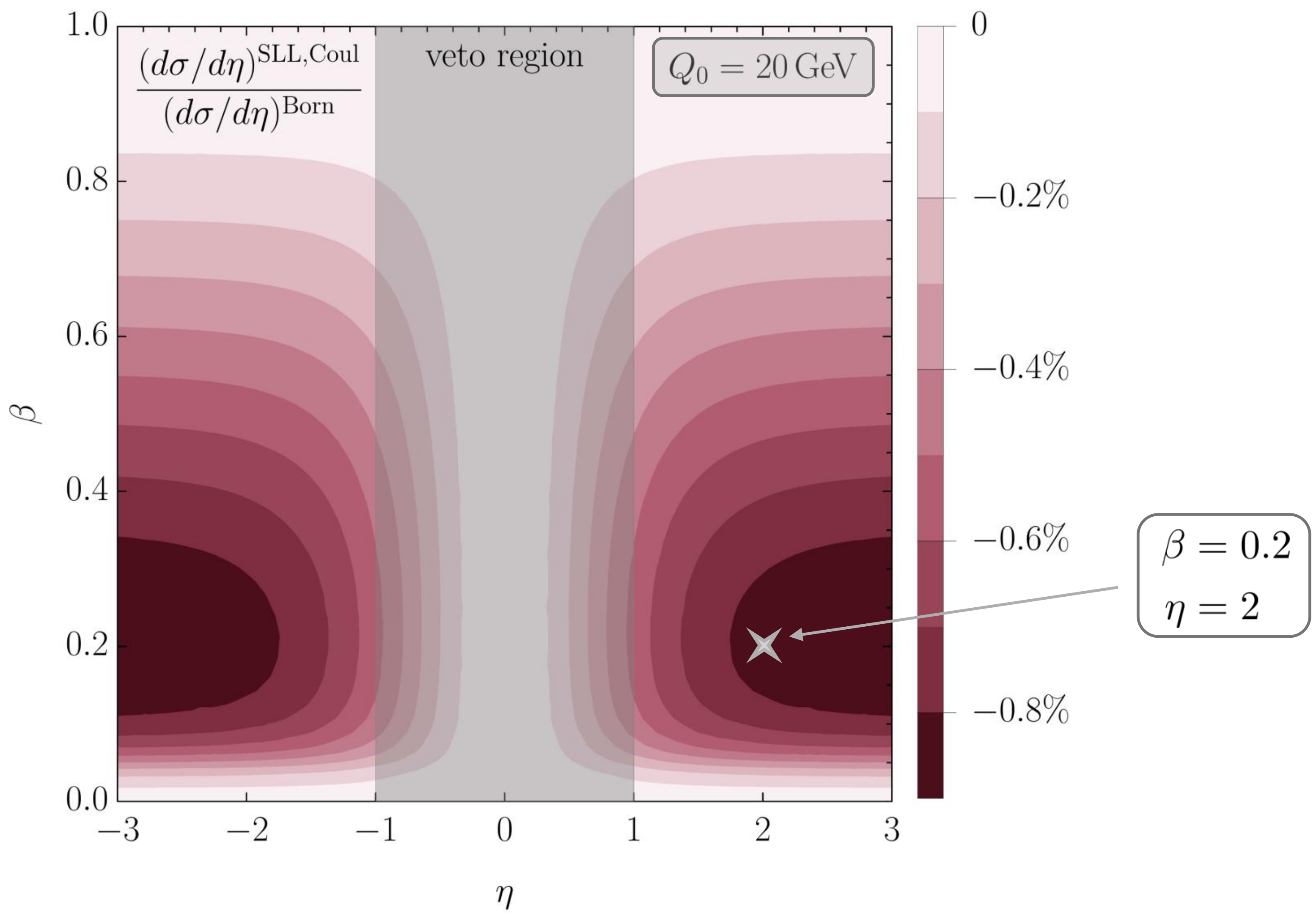
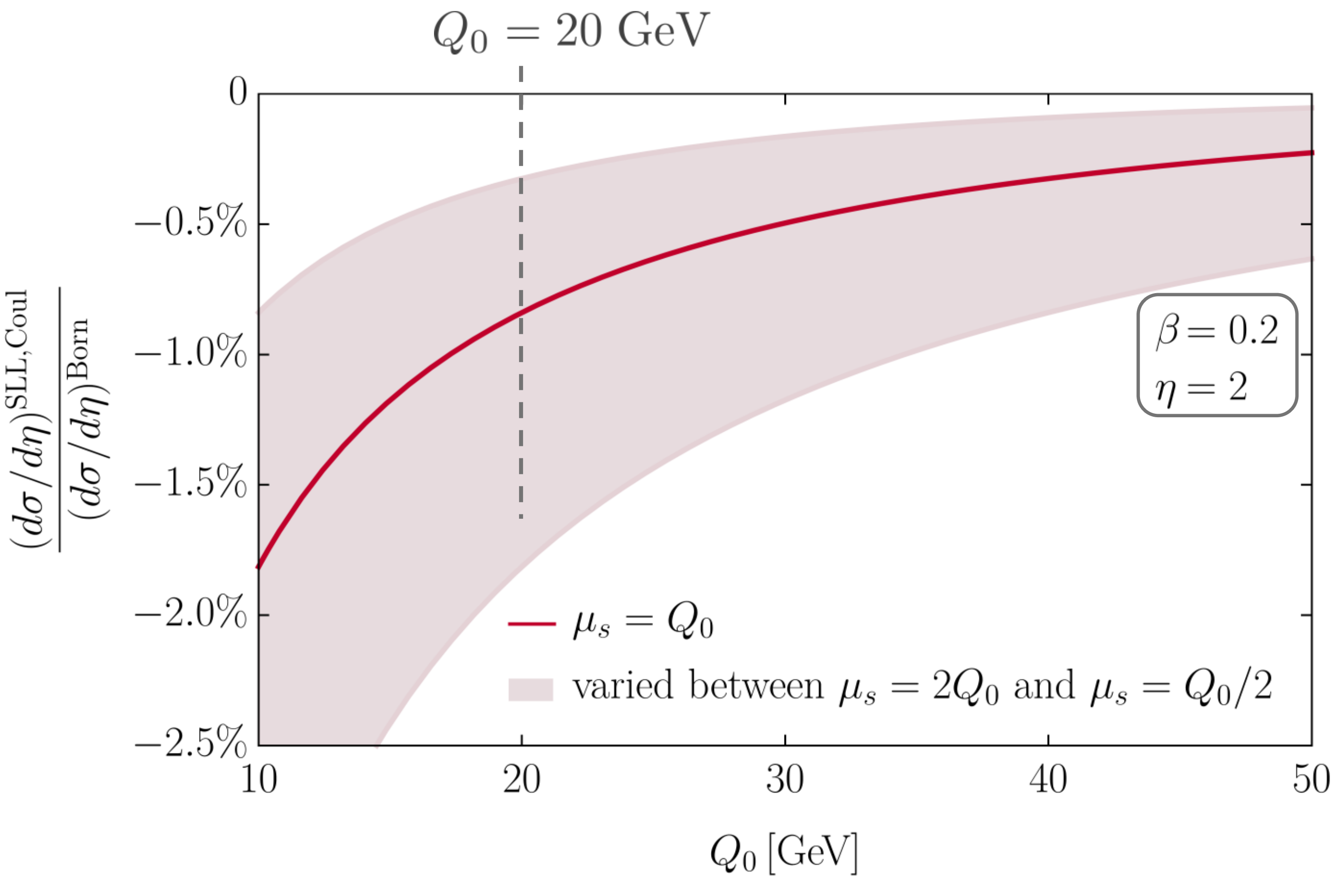


Numerical Effects

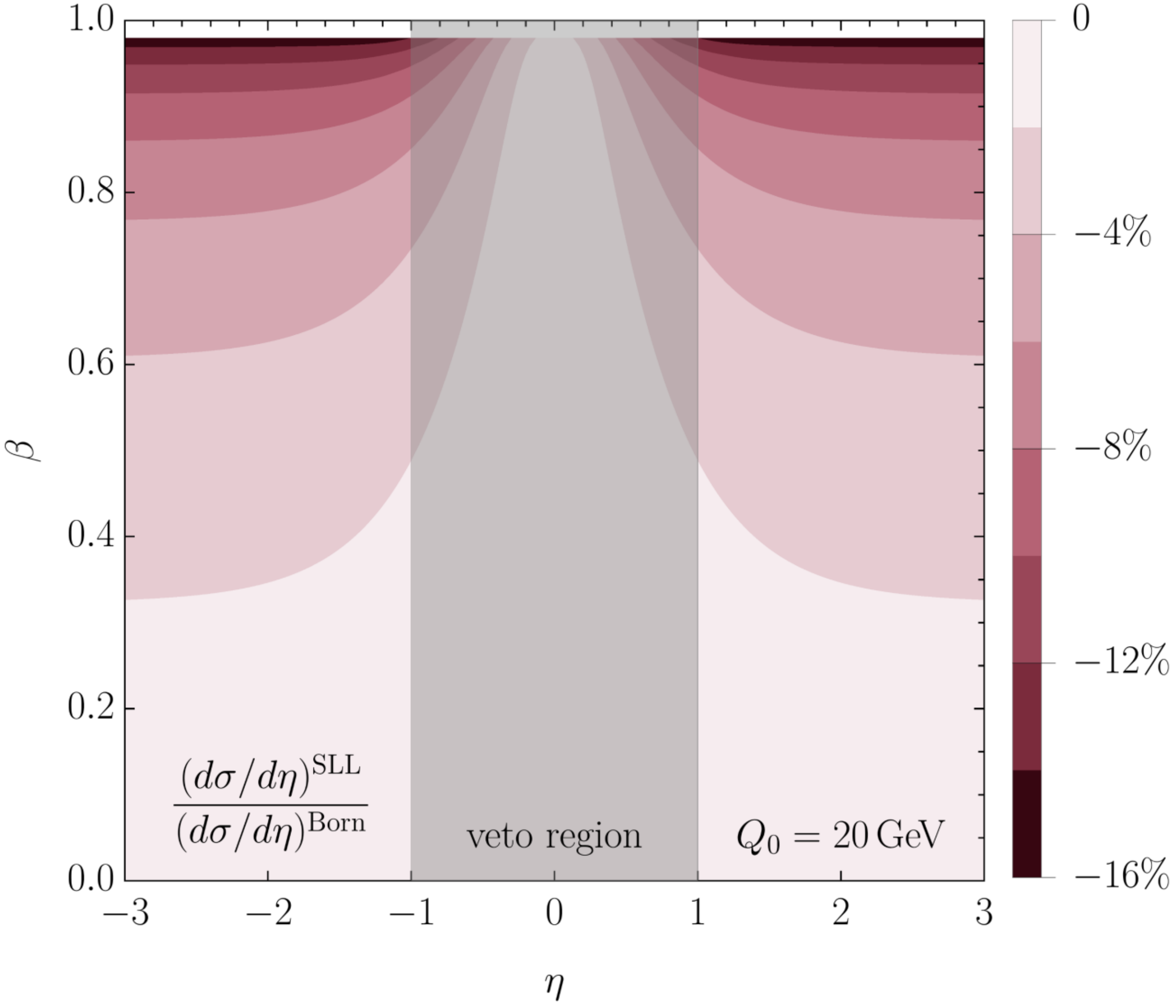
$gg \to t\bar{t}$



Numerical Effects

$gg \to t\bar{t}$

Coulomb and
Glauber SLLs



Conclusion

New source of super-leading logarithms for massive final states:

- Vanish for $\beta \to 1$ (masslesss limit)
- Enhanced for $\beta \to 0$ (threshold limit): Sommerfeld effect

  $\Longrightarrow$ Requires resummation close to threshold

Numerical impact:

- $q\bar{q} \to t\bar{t}$: no contribution
- $gg \to t\bar{t}$: up to $\sim 1\%$ effects in the differential cross section

Conclusion

New source of super-leading logarithms for massive final states:

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Thank you for your attention!

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- $q\bar{q} \to t\bar{t}$: no contribution
- $gg \to t\bar{t}$: up to $\sim 1\%$ effects in the differential cross section

Backup-Slides

Anomalous Dimension

$$\overline{\boldsymbol{\Gamma}} = \frac{1}{2}\gamma_0 \sum_{\alpha,\beta} (\boldsymbol{T}_{\alpha,L} \cdot \boldsymbol{T}_{\beta,L} + \boldsymbol{T}_{\alpha,R} \cdot \boldsymbol{T}_{\beta,R}) \int \frac{d^2\Omega_k}{4\pi} \overline{W}^k_{\alpha\beta} - 4 \sum_{\alpha,\beta} \theta_{\mathrm{hard}}(n_k) \overline{W}^k_{\alpha\beta} \boldsymbol{T}_{\alpha,L} \circ \boldsymbol{T}_{\beta,R}$$

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2} \left[C_i \, \mathbf{1} - \delta(n_i - n_k) \, \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \right]$$

$$\boldsymbol{V}^G = -2\pi i \, (\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R})$$

$$\boldsymbol{V}^{\mathrm{Coul}} = -\frac{1}{2}\pi i \sum_{(IJ)} (\boldsymbol{T}_{I,L} \cdot \boldsymbol{T}_{J,L} - \boldsymbol{T}_{I,R} \cdot \boldsymbol{T}_{J,R}) \, v_{IJ}$$

Cross section

$$U^c(1;\mu_i,\mu_j) = \exp\left[N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\gamma_{\rm cusp}(\alpha_s(\mu))\ln\left(\frac{\mu^2}{\mu_h^2}\right)\right]$$

$$\begin{aligned}\left(\frac{d\sigma}{d\eta}\right)^{\rm SLL,Coul} &= -\frac{1}{\cosh^2(\eta)}\frac{\beta}{32\pi M^2}\frac{1}{\mathcal{N}_1\mathcal{N}_2} \\ &\quad\times\Bigg\{16\pi^2\,{\rm Tr}\big(\boldsymbol{\mathcal{H}}_{2\to 2}(\mu_h)\boldsymbol{X}^{\rm Coul}\big)\int_1^{x_s}\frac{dx}{x}\frac{1}{\beta_0^3}U^c(1;\mu_h,\mu)\left(\ln^2(x_s)-\ln^2(x)\right) \\ &\qquad + \frac{3}{2}\pi^2\,{\rm Tr}\big(\boldsymbol{\mathcal{H}}_{2\to 2}(\mu_h)\boldsymbol{X}^{\rm 2Coul}\big)\frac{1}{\beta_0^3}\ln^3(x_s)\Bigg\}\end{aligned}$$

where $\boldsymbol{X}^{\rm Coul}_{2\to t\bar{t}} = J_{43}v_{t\bar{t}}f^{abe}f^{cde}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\boldsymbol{T}_3^c\boldsymbol{T}_4^d$

$$\boldsymbol{X}^{\rm 2Coul}_{2\to t\bar{t}} = v_{t\bar{t}}^2 f^{abe}f^{cde}\left(\boldsymbol{T}_3^c\{\boldsymbol{T}_4^b,\boldsymbol{T}_4^d\} - \boldsymbol{T}_4^c\{\boldsymbol{T}_3^b,\boldsymbol{T}_3^d\}\right)\left(\tilde{J}_1^{34}\boldsymbol{T}_1^a + \tilde{J}_2^{34}\boldsymbol{T}_2^a\right)$$