Born-Projected Leptons in Drell–Yan Final State

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Synergies Towards the Future Standard Model
DESY Theory Workshop 2025

September 24, 2025

European Research Council
Established by the European Commission

The Drell–Yan Process

Definition

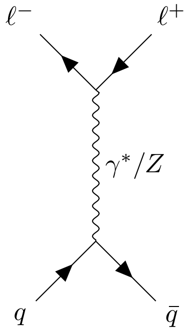
The **Drell–Yan process** describes the production of a lepton–antilepton pair via quark–antiquark annihilation in high-energy hadron–hadron collisions:

$$q + \bar{q} \to \gamma^*/Z \to \ell^+ + \ell^-$$

- The **high precision** of the DY process enables accurate extraction of key SM parameters $(M_W, M_Z, \sin^2\theta_W, \alpha_s, \ldots)$.
- **Hadronic–Leptonic Factorisation:**
  - The cross section can be factorised as:

$$\frac{d\sigma}{d^4q} = \sum_{V,V'} W^{\mu\nu}_{VV'}(q)\, L_{VV'\mu\nu}(q)$$

  - $W^{\mu\nu}$: Hadronic tensor for $pp \to V/V' + X \to \cdots$
  - $L_{\mu\nu}$: Leptonic tensor for $V/V' \to \ell^+\ell^-$



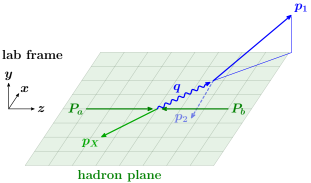
Angular Decomposition in Vector Boson Decays

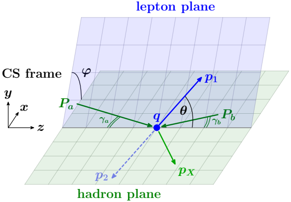
Leading Order Angular Decomposition

The differential cross section at LO can be decomposed as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\cos(\theta)\,\mathrm{d}\varphi} = \frac{3}{16\pi}\frac{\mathrm{d}\sigma}{\mathrm{d}^4q}\sum_{i=-1}^{7} A_i g_i(\theta,\varphi)$$

- Is a **general result** of a vector boson decaying into **two particles**.
- θ and φ are the spherical coordinates of p_1 in the **Collins-Soper frame**.
- Is only valid for **LO decay**.





[J.C. Collins & D.E. Soper, Phys.Rev.D 16 (1977) 2219]

Forward–Backward Asymmetry A_{FB}

Definition

The forward–backward asymmetry probes parity violation in the Drell–Yan process:

$$A_{\mathrm{FB}} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)}$$
$$= \frac{3}{8} A_4 = \frac{3}{2} \frac{v_\ell a_\ell}{v_\ell^2 + a_\ell^2} \frac{L_- W_4}{L_+ \left(2W_{-1} + W_0\right)}$$

- A_{FB} arises due to the **interference between vector and axial-vector currents**, F_-, sensitive to electroweak couplings.
- Particularly useful for extracting $\sin^2\theta_W$ from data.
- Measured as the **asymmetry in the lepton angular distribution** in the dilepton rest frame.

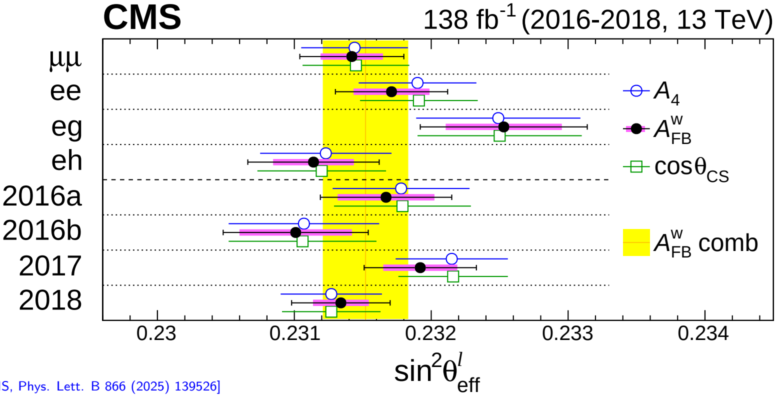
[Ebert et al., JHEP 04 (2021) 102]

Determining $\sin^2\theta_W$ from Angular Coefficient A_4

- The vector couplings depend on the weak mixing angle:

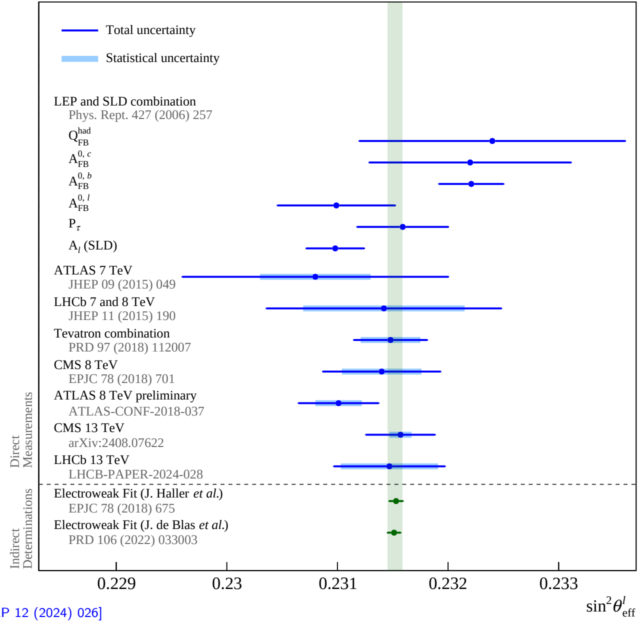
$$v_\ell = \frac{1}{2} - 2\sin^2\left(\theta^\ell_{\text{eff}}\right)$$

- Measuring A_4 gives direct sensitivity to $\sin^2\theta_W$.
- Used by CMS, ATLAS, and LHCb in $pp \to Z/\gamma^* \to \ell^+\ell^-$.
- Complementary to traditional A_{FB} methods.



[CMS, Phys. Lett. B 866 (2025) 139526]

Experimental measurements of $\sin^2(\theta_W)$

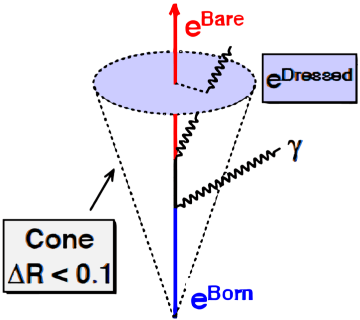


[LHCb, JHEP 12 (2024) 026]

Limits of Angular Decomposition

- Angular decomposition is strictly valid only at **LO**.
- **Final-State Radiation (FSR)** breaks the validity of the decomposition at NLO.
- Additional radiation changes the lepton kinematics and invalidates the angular decomposition.

- Experiments reconstruct a **Born-level lepton** by recombining FSR using Monte Carlo techniques.
- This helps **restore the angular structure**, but the **physical meaning is ambiguous**.
- No clear theoretical framework exists to interpret this Born-level observable rigorously.



Born-Projected Leptons

Solution: Project the full lepton final state (including arbitrary final state radiation) onto a Born-like 2-particle phase space configuration.

Projected Leptonic Tensor

$$F^{\mu\nu}_{VV'}(P_1, P_2) = (2\pi)^2 \int L^{\mu\nu}_{VV'}(\Phi_L)\delta^{(4)}(P_1 - \hat{P}_1(\Phi_L))\,\mathrm{d}\Phi_L$$
$$= \mathcal{F}^{\mu\nu}_{+}(\theta,\varphi)F_+ + \mathcal{F}^{\mu\nu}_{0}(\theta,\varphi)F_0 + \mathcal{F}^{\mu\nu}_{-}(\theta,\varphi)F_-$$

$\hat{P}_1(\Phi_L)$ defines the momentum P_1 in terms of the actual multi-body final state momenta.

The coefficients F_i are generalisations of the LO leptonic coefficients $L_\pm$:

$$F_\pm = L_\pm\,(1 + \mathcal{O}(\alpha_{\mathrm{em}}))$$
$$F_0 = \mathcal{O}(\alpha_{\mathrm{em}})$$

[Ebert et al., JHEP 04 (2021) 102]

Generalised Angular Decomposition

- Helicity decomposition is a consequence of decay of spin-1 particle:

$$F^{\mu\nu} = \sum_{i=-1}^{7} P_i^{\mu\nu} F_i \Longrightarrow F_{\mu\nu} W^{\mu\nu} = \sum_{i=-1}^{7} W_i F_i$$

- Having a Born phase space, allows us to write $F_i = \tilde{A}_i g_i(\theta, \varphi)$.

- The angular decomposition takes the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q \, \mathrm{d}\cos(\theta) \, \mathrm{d}\varphi} = \frac{3}{16\pi} \frac{\mathrm{d}\sigma}{\mathrm{d}^4 q} \sum_{i=-1}^{7} \tilde{A}_i g_i(\theta, \varphi)$$

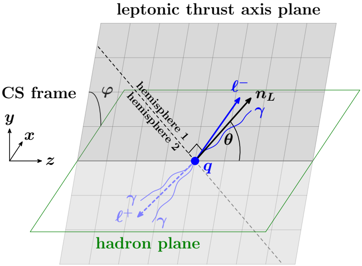
With angular coefficients valid including arbitrary final-state radiation.

$$\tilde{A}_0 = \frac{F_+ W_0 + F_0 W_{\text{incl}}}{\left(F_+ + \frac{3}{2} F_0\right) W_{\text{incl}}}, \qquad \tilde{A}_{i>0} = \frac{F_{\pm(i)} W_i}{\left(F_+ + \frac{3}{2} F_0\right) W_{\text{incl}}}$$

[Ebert et al., JHEP 04 (2021) 102]

Born-Projections via the Thrust Axis

The projection $\hat{P}_1^\mu(\Phi_L)$ is, in principle, arbitrary. A particular choice is to cluster the FSR using the thrust axis:



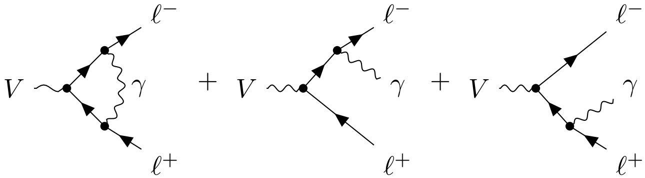
$$\hat{P}_1^\mu(\Phi_L) = \frac{Q}{2}\left(t^\mu \pm n_L^\mu\right),$$

$$t^\mu = \frac{1}{\sqrt{q^2}} q^\mu,$$

$$\vec{n}_L(\Phi_L) = \underset{\vec{n}\in S^2}{\arg\max} \sum_{i\in L} |\vec{n}\cdot\vec{p}_i|$$

- The sign must be chosen such that, when no radiation is present we recover $\hat{P}_1(\Phi_L) = p_1$.
- This gives a **concrete definition** of Born leptons, which allows for **calculable** theoretical predictions.

Results for $Z/\gamma^* \to \ell^-\ell^+$



$$F_{+VV'} = \frac{P_V^* P_{V'}}{Q^2} \alpha_{\rm em}(v_V v_{V'} + a_V a_{V'}) \left(\frac{2}{3} + \alpha_{\rm em} Q_\ell^2 \frac{16\log(2) - 11}{2\pi} \right)$$

$$F_{0VV'} = \frac{P_V^* P_{V'}}{Q^2} \alpha_{\rm em}(v_V v_{V'} + a_V a_{V'}) \left(\alpha_{\rm em} Q_\ell^2 \frac{12 - 16\log(2)}{3\pi} \right)$$

$$F_{-VV'} = \frac{P_V^* P_{V'}}{Q^2} \alpha_{\rm em}(v_V a_{V'} + a_V v_{V'}) \left(\frac{2}{3} + \alpha_{\rm em} Q_\ell^2 \frac{5 + 4\log(2)}{6\pi} \right)$$

$$P_V = \frac{Q^2}{Q^2 - m_V^2 + i\Gamma_V m_V}$$

Note: Only IR divergent diagrams were included in this calculation. Additional finite EW contributions should also be considered.

Conclusion & Outlook

- Angular coefficients A_i encode dynamics and are sensitive to key physical parameters.
- At **higher orders**, FSR leads to **ambiguities** in defining angular coefficients.
- **Born-projected leptons** are defined by projecting the final state to a **Born 2-particle state**.
- A **generalised angular decomposition** using Born-projected leptons is possible at **all perturbative orders**.

Future Work:

- Apply the projection method to the process $W \to \ell\nu$.
- Study different choices for the projection $\hat{P}_1(\Phi_L)$.
- Implement fiducial cuts and differential distributions.

Backup: Helicity Decomposition

Any rank-2 tensor orthogonal to the vector boson momentum q^μ, such as $L^{\mu\nu}$ or $W^{\mu\nu}$, can be decomposed in the helicity basis:

$$L^{\mu\nu} = \sum_{\lambda,\lambda'} L_{\lambda\lambda'}\, \epsilon^\mu_\lambda\, \epsilon^{\nu*}_{\lambda'} = \sum_{i=-1}^{7} L_i P_i^{\mu\nu}$$

Here:

- $\lambda, \lambda' \in \{+, 0, -\}$ denote the boson helicity states.
- The polarisation vectors ϵ^μ_λ satisfy:

$$q_\mu \epsilon^\mu_\lambda = 0, \qquad \epsilon_\lambda \cdot \epsilon^*_{\lambda'} = -\delta_{\lambda\lambda'}.$$

- The projectors $P_i^{\mu\nu}$ are linear combinations of $\epsilon^\mu_\lambda\, \epsilon^{\nu*}_{\lambda'}$ and form a basis of the space orthogonal to q^μ.

These projectors define the angular functions $g_i(\theta, \varphi)$ as

$$L_i = L_i(q^2) g_i(\theta, \varphi)$$

In the factorised cross-section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\, \mathrm{d}\cos(\theta)\, \mathrm{d}\varphi} = \sum_{V,V'} W^{\mu\nu}_{VV'}(q)\, L_{VV'\mu\nu}(q, \theta, \varphi) \sim \sum_i A_i g_i(\theta, \varphi)$$

Acknowledgments

European Research Council

Established by the European Commission

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant agreement 101002090 COLORFREE).