Non-Abelian aspects of chiral gauge theories in the BMHV scheme at two-loop order

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What is "the γ_5 problem" in DReg?

Chiral Gauge Theories are a fact of life, yet trusty dimensional continuation clashes with such **genuinely** 4-**dimensional** identities of γ_5 and $\epsilon^{\mu\nu\rho\sigma}$.

Contradictions immediately arise,

- $\{\gamma_\mu, \gamma_5\} = 0$,
- $\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = -4i\varepsilon_{\mu\nu\rho\sigma}$,
- $\mathrm{Tr}(\gamma_\mu\gamma_\nu) = \mathrm{Tr}(\gamma_\nu\gamma_\mu)$,

conspiring to give,

$$2(D-4)\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 0.$$

Several propositions for treating γ_5 in DReg exist: Naive scheme, Reading-Point, Larin scheme, BMHV (P. Breitenlohner, D. Maison, G. 't Hooft, M. Veltman), . . .

[PB,DM,Commun.Math.Phys.52,(1977)] [GH,MV,Nucl.Phys.B44,(1972)]

Brief(est) Exposition of BMHV [HB,AI,PK,MB,DS,MW,2411.02543]

The space of DReg decomposes as $\mathbb{M}_D = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}$ with formal symbols,

$$\bar{g}^{\mu\nu}\bar{g}_{\mu\nu} = 4, \qquad \hat{g}^{\mu\nu}\hat{g}_{\mu\nu} = -2\epsilon, \qquad \bar{g}^{\mu\rho}\hat{g}_{\rho\nu} = 0, \qquad \hat{p}^{\mu} = \hat{g}^{\mu\rho}p_{\rho}.$$

In this approach γ_5 **is non-anticommuting** and we have,

$$\{\bar{\gamma}^{\mu}, \gamma_5\} = 0, \qquad [\hat{\gamma}^{\mu}, \gamma_5] = 0, \qquad \{\hat{\gamma}^{\mu}, \gamma_5\} = 2\hat{\gamma}^{\mu}\gamma_5.$$

CONS:

loss of *D*-dimensional Lorentz covariance
spuriously breaks gauge invariance

PROS:

proven mathematically consistent at arbitrary loop order

A Simple Non-Abelian Toy Model

$SU(2)$ **gauge theory**: N_f gauged, right-handed fermions in some representation $\mathcal{R}_i$ and sterile, left-handed partners (corresponds to **Option 2b** in [PLE,DS,MW,PK,2411.02543],

$$S_0 = \int d^D x \, \mathcal{L}_{\text{kin}}^{\text{fermion}} + \mathcal{L}_{\text{int}}^{\text{fermion}} - \frac{1}{4} F^2 - \bar{c} D_{\mathcal{A}} c + \mathcal{L}_{\text{g-fix}} + \mathcal{L}_{\text{ext}},$$

with covariant derivative,

$$D_{\mathcal{R}}^{\mu\, ab} = \partial_\mu \delta^{ab} - ig (T_{\mathcal{R}})^c_{ab} G^{c\mu}$$

$$\text{where} \quad T^a_{\mathcal{R}} = \text{diag}(T^a_{\mathcal{R}_1} \ldots T^a_{\mathcal{R}_M}) \quad \text{and} \quad (T_{\mathcal{A}})^c_{ab} = (-i)\varepsilon^{abc}.$$

Anomaly cancellation requirement: $\sum_{\mathcal{R}_i} A(\mathcal{R}_i) d^{abc} = 0$ (trivial in $SU(2)$)

The interaction term admits several choices equivalent in 4 but not D dimensions,

$$\overline{\psi}\gamma^\mu P_R \psi \neq \overline{\psi} P_L \gamma^\mu \psi \neq \overline{\psi} P_L \gamma^\mu P_R \psi.$$

We opt for the **purely 4-dimensional** coupling,

$$\mathcal{L}_{\text{int}}^{\text{fermion}} = g T^a_{\mathcal{R}\, ij} \overline{\psi_{Ri}} \overline{\not{G}}^a \psi_{Rj}.$$

Regularization-induced Symmetry Breaking

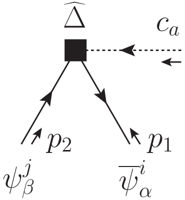
Consistency with DReg requires a fully D-dimensional kinetic term, hence,

$$\mathcal{L}_{\text{fermion}} = i\overline{\psi}_i \widehat{\not{\partial}} \psi_i + i\overline{\psi}_{Ri} \overline{\not{D}} \psi_{Ri} + i\overline{\psi}_{Li} \overline{\not{\partial}} \psi_{Li}.$$

The covariant term exhibits full 4-dimensional gauge invariance, whilst the breaking originates form the evanescent piece.

This can be quantified by the (D-dimensional) BRST transformations,

$$s_D(\psi_R)_i = igc^a T^a_{ij} \psi_j \qquad s_D(\psi_L)_i = 0 \qquad s_D(G^a_\mu) = (D_A)^{ab}_\mu c^b.$$



$$\hat{\Delta} = s_D \int d^D x \, \mathcal{L}_{\text{fermion}}$$

$$= \int d^D x \, g \, T^a_{ij} \, c^a \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\not{\partial}}} \mathbb{P}_R + \overrightarrow{\widehat{\not{\partial}}} \mathbb{P}_L \right) \psi_j \right\}$$

... and its Restoration

The ultimate goal: Slavnov-Taylor Identity

$$\underset{D\to4}{\mathrm{LIM}}\,\mathcal{S}_D(\Gamma_{\mathrm{DRen}}) = 0 \quad \text{with} \quad \mathcal{S}(\Gamma_{\mathrm{ren}}) = \int \mathrm{d}^4\mathrm{x}\, \frac{\delta\Gamma_{\mathrm{ren}}}{\delta\phi_i(x)} \frac{\delta\Gamma_{\mathrm{ren}}}{K_{\phi_i}(x)}.$$

$$K_{\phi_i} \supset \mathcal{L}_{\mathsf{ext}} = \rho_a^\mu s_D G_\mu^a + \zeta_a s_D c^a + \bar{R}^i s_D \psi_{Ri} + R^i s_D \overline{\psi_{Ri}}$$

The main tool: Quantum Action Principle in DReg

$$\mathcal{S}_D(\Gamma_{\mathrm{DReg}}) = \Delta \cdot \Gamma_{\mathrm{DReg}} \qquad \Delta = \mathcal{S}_D(S_0 + S_{\mathrm{ct}}).$$

the (subren.) breaking is local at any order!

At two-loop order the symmetry breaking evaluates to

$$\Delta \cdot \Gamma^{2\mathrm{L}}_{\mathsf{DReg}} = \widehat{\Delta} \cdot \Gamma^{2\mathrm{L}}_{\mathsf{DReg}} + \Delta^{1\mathrm{L}}_{\mathrm{ct}} \cdot \Gamma^{2\mathrm{L}}_{\mathsf{DReg}} + \mathcal{S}_D(S^{1\mathrm{L}}_{\mathrm{ct}}) + \Delta^{2\mathrm{L}}_{\mathrm{ct}}$$

$$= \Delta^{2\mathsf{L}}_{\mathsf{sct}} + \Delta^{2\mathsf{L}}_{\mathsf{fct}}$$

$$\Delta^{nL}_{\mathrm{ct}} = b_D(S^{nL}_{\mathrm{ct}}) \quad \text{and} \quad b_D = s_D + \int \mathrm{d}^{\mathrm{D}}\mathrm{x}\, \frac{\delta S_0}{\delta\phi_i} \frac{\delta}{\delta K_{\phi_i}}.$$

Counterterms@1Loop

Practical Procedure

find 4-dimensional, mass dimension ≤ 4, ghost number 0 CT's X such that,

$$b_4(X) = -\mathrm{LIM}_{D\to 4}\Delta \cdot \Gamma.$$

Result@1Loop [HB,AI,MB,DS, 2004.14398]

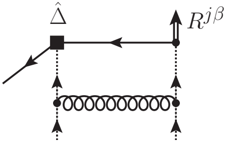
Relatively compact and simple structure (here $D = 4!$),

$$S^{(1)}_{\text{fct, restore}} = \frac{1}{16\pi^2}\Bigg\{ g^2 \frac{S_2(R)}{6}\left(5 S_{GG} - \int d^4x\, G^{a,\mu}\partial^2 G^a_\mu\right) + g^2 \frac{(T_R)^{abcd}}{3}\int d^4x\, \frac{g^2}{4} G^a_\mu G^{b,\mu} G^c_\nu G^{d,\nu} + g^2\left(1 + \frac{\xi - 1}{6}\right) C_2(R) S_{\overline{\psi}\psi} + g^2 \frac{S_2(R)}{6} S_{GGG} - g^2 \frac{\xi C_2(G)}{4}\left(S_{\bar{R}c\psi_R} + S_{Rc\overline{\psi_R}}\right)\Bigg\},$$

Counterterms@1Loop cont'd/@2loop

N.B. Finite external field CT's for the $R/\overline{R}$-sources from diagrams,

e.g. $- g^2 \frac{\xi C_2(G)}{4} \left(S_{\bar{R} c \psi_R} + S_{R c \overline{\psi_R}} \right)$



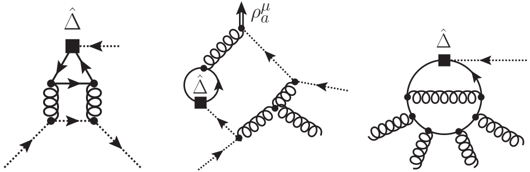
They give rise to additional evanescent contributions at ≥ 2 loop,

$$b_D(S_{\bar{R}c\psi_R}) = \int d^D x \, \frac{\delta S_0}{\delta \psi_{i\alpha}} \frac{S_{\bar{R}c\psi_R}}{\delta \bar{R}_{i\alpha}} = \bar{\Delta}^{1L, c\bar{\psi}\psi}_{\text{fct}} + \hat{\Delta}^{1L, c\bar{\psi}\psi}_{\text{fct}} + \ldots$$

$$\frac{\delta S_0}{\delta \psi_{i\alpha}} = \frac{\delta}{\delta \psi_{i\alpha}} (i \bar{\psi}_i \partial\!\!\!/ \psi_i) + \ldots$$

@2Loop

Breaking GF's with sources $\rho^{a\mu}$, ζ^a and $\overline{c}^a$; 5-point function(s)

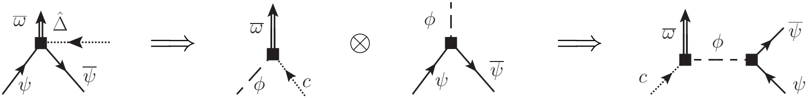


Implementation in FeynArts

Couple $\hat{\Delta}$ to an auxiliary anti ghost,

$$\widehat{\Delta} \cdot \Gamma_{\phi_1 \dots \phi_n} = \left. \frac{\delta \Gamma_{\phi_1 \dots \phi_n}}{\delta \overline{\omega}_\Delta} \right|_{\overline{\omega}_\Delta = 0},$$

To avoid 4-
point fermion vertices, a fictitious scalar separates ghosts and spinors.



$$\mathcal{L}_{\phi-\mathrm{aux}} = -\frac{\phi^2}{2} + \overline{\omega}_\Delta c^a \phi^a + g T^a_{\mathcal{R}ij} \left(\overline{\psi}_i \overleftarrow{\widehat{\partial}} \mathbb{P}_\mathrm{R} \psi_j + \overline{\psi}_i \overrightarrow{\widehat{\partial}} \mathbb{P}_\mathrm{L} \psi_j \right) \phi^a.$$

Treat BRST sources as composite fields,

$$\rho^{a\mu} \longrightarrow \bar{\omega}^a_\rho A^\mu$$

$$R_{j\beta} \longrightarrow \bar{\omega}_R \chi_{j\beta}.$$

External Field	Statistics	Dimension	Ghost Number	Lorentz Tr.
ρ^μ_a	Fermion	3	-1	Four-Vector
ζ^a	Boson	4	-2	Scalar
$\overline{R}^i_\alpha / R^j_\beta$	Boson	$\frac{5}{2}$	-1	Spinor
χ^a	Boson	2	0	Scalar

Difficulties compound for $\Delta^{1\mathrm{L,ct}}_{cc\overline{R}\psi}$!

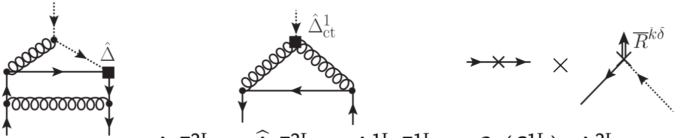
Consistency Checks

Keep a check on pitfalls incl. $\Delta_{\rm ct}$'s, the model setup, integrals...,

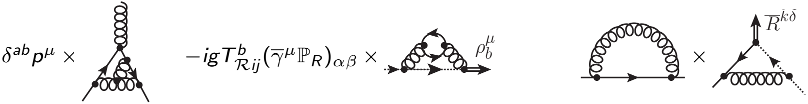
Quantum Action principle $\Longrightarrow$ $\mathcal{S}_D(\Gamma_{\rm DReg})=\Delta\cdot\Gamma_{\rm DReg}$

$$\Gamma^{\rm DReg}_{c^a\rho^{b\mu}}\Gamma^{\rm DReg}_{\psi_{j\beta}\overline{\psi}_{i\alpha}G^{b\mu}}+\Gamma^{\rm DReg}_{\psi_{j\beta}c^a\overline{R}_{k\delta}}\Gamma^{\rm DReg}_{\psi_{k\delta}(-p_1)\overline{\psi}_{i\alpha}(p_1)}-\Gamma^{\rm DReg}_{R_{k\delta}c^a\overline{\psi}_{i\alpha}}\Gamma^{\rm DReg}_{\psi_{j\beta}(p_2)\overline{\psi}_{k\delta}(-p_2)}=(\Delta\cdot\Gamma^{\rm DReg})_{\psi_{j\beta}(p_2)\overline{\psi}_{i\alpha}(p_1)c^a(p)}$$

Δ-insertions vs. full LHS



$$\supset\Delta\cdot\Gamma^{\rm 2L}_{\rm DReg}=\widehat{\Delta}\cdot\Gamma^{\rm 2L}_{\rm DReg}+\Delta^{\rm 1L}_{\rm ct}\cdot\Gamma^{\rm 1L}_{\rm DReg}+\mathcal{S}_D(S^{\rm 1L}_{\rm ct})+\Delta^{\rm 2L}_{\rm ct},$$



loop integrals: **All Massive Tadpoles** (only poles, [MM,MM,9409454] [KC,MM,MM,9711266]) and TARCER (full, [RM,RS,9801383]);

Two-loop Results for S^2_{fct}

Finite counterterm Lagrangian@2loop [PK, DS, 2504.06080],

$$\mathcal{L}^2_{\mathsf{fct}} = \sum_{\mathsf{M}} \chi^{(2)}_{\mathsf{M}} \mathcal{O}_{\mathsf{M}} \quad \text{where} \quad \mathsf{M} = \{\overline{G \,\Box\, G}, ((\widetilde{\overline{\partial G})\overline{G}})\overline{G}, \overline{G}^4, (\overline{GG})(\overline{GG}),$$

$$\overline{\psi}\overline{\partial\!\!\!/}\mathbb{P}_{\mathrm{R}}\psi, \overline{R}cT_{\mathcal{R}}\mathbb{P}_{\mathrm{R}}\psi, \overline{\psi}cT_{\mathcal{R}}\mathbb{P}_{\mathrm{L}}cR, \widetilde{\zeta cc}\}$$

Compact result with known monomials from the 1-loop CT's plus one new term to the source ζ^a,

$$(\frac{i}{16\pi^2})^2 g^5 \frac{35}{432} \sum_{\mathcal{R}} S_2(\mathcal{R}) \varepsilon^{abc} \zeta^a c^b c^c.$$

@>2-loop: No new terms are needed in M!
Operators missing in M are accommodated by symmetric CT's like $\overline{F}^2$ and invariants from finite renormalizations, e.g.

$$L_c = -b_4 \int d^4x\, \zeta_a c^a = \int d^4x\, \mathcal{L}_{\mathrm{ghost}} + \mathcal{L}_{\mathrm{ext}} - \chi_a B^a \quad \text{with} \quad \begin{matrix} c^a \to \sqrt{Z_c} c^a \\ \zeta^a \to \sqrt{Z_c}^{-1} \zeta^a \end{matrix}$$