Multi-parton contributions to $\bar{B} \to X_s\gamma$ at NLO

Kevin Brune, Tobias Huber, Lars-Thorben Moos

based on arXiv 2509.xxxxx

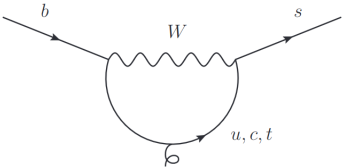
Universität Siegen

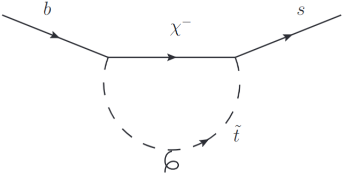
Center for Particle Physics Siegen

$\bar{B} \to X_s\gamma$

- At parton level $b \to s\gamma$ (FCNC)
  - $\Rightarrow$ Forbidden at tree-level
- Highly sensitive to virtual particles
  - $\Rightarrow$ New physics probe
- Experimental average
  - $\mathcal{B}_{s\gamma}^{\text{exp}} = (3.49 \pm 0.19) \times 10^{-4}$ [HFLAV;25]
  - $E_\gamma > E_0 = 1.6$ GeV
  - CP- and isospin-averaged BRs

$\Rightarrow$ $\pm 5.4\%$ accuracy





Theory status

- SM prediction
  - $\mathcal{B}_{s\gamma}^{\text{SM}} = (3.40 \pm 0.17) \times 10^{-4}$ [Misiak,Rehman,Steinhauser;20]
  - $E_0 = 1.6$ GeV

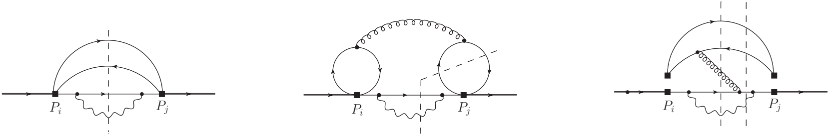
$\Rightarrow$ $\pm 5\%$ accuracy

- Upcoming data from B-factories [Belle II]

  $\Rightarrow$ $\pm 2.6\%$ experimental uncertainty
- Theory uncertainty
  - $\pm 3\%$ missing higher order
  - $\pm 3\%$ interpolation m_c
  - $\pm 2.5\%$ non-perturbative
- Lots of progress to reduce theory uncertainty [Fael,Lange,Schönwald,Steinhauser;23][Czaja, et al;23] [Gunawardana,Paz;19][Bartocci,Böer,Hurth;25]...many more
- Missing multi-parton final states at NLO

  $\Rightarrow$ This work

Current work



(a) (b) (c)

- $\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^{\text{parton}}\gamma)_{E_\gamma > E_0} + \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$
  - $\Gamma(b \to X_s^{\text{parton}}\gamma)_{E_\gamma > E_0} = \Gamma(b \to s\gamma) + \ldots + \Gamma(b \to sq\bar{q}\gamma) + \Gamma(b \to sq\bar{q}g\gamma) + \ldots$

$$\Gamma(b \to sq\bar{q}\gamma)_{E_\gamma > E_0} + \Gamma(b \to sq\bar{q}g\gamma)_{E_\gamma > E_0} = \Gamma_0 \sum_{i,j} C_i^{\text{eff}\,*}(\mu)\, C_j^{\text{eff}}(\mu)\, \widehat{G}_{ij}(\mu, z_c, \delta)$$

- $\widehat{G}_{ij}(\mu, z_c, \delta) = \widehat{G}_{ij}^{(0)}(\delta) + \frac{\alpha_s(\mu)}{4\pi}\left(G_{ij}^{(1)}(\mu, z_c, \delta) + G_{ij}^{(1)}(\mu, \delta)\right) + \mathcal{O}(\alpha_s^2)$

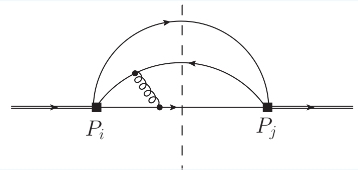
  [Kaminski,Misiak,Poradzinski;12][Huber,Poradzinski,Virto,;15]

$\Rightarrow$ $G_{ij}^{(1)}(\mu, \delta)$ formally completes NLO

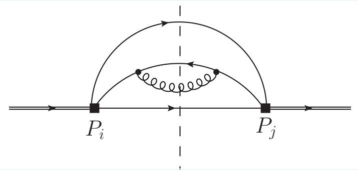
Master formula

$$G^{(1)}_{ij}(\mu,\delta) = V_{ij} + V^*_{ij} + R_{ij} + 2\,Z_\psi T_{ij} + M_{ij} + M^*_{ij} + \sum \{ Z_{jk} T_{ik} + Z_{im} T_{mj} \}$$

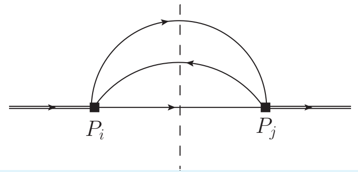
V_{ij}



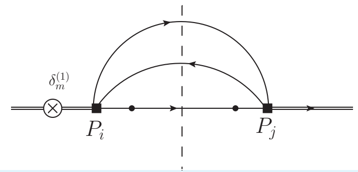
R_{ij}



T_{ij}



M_{ij}



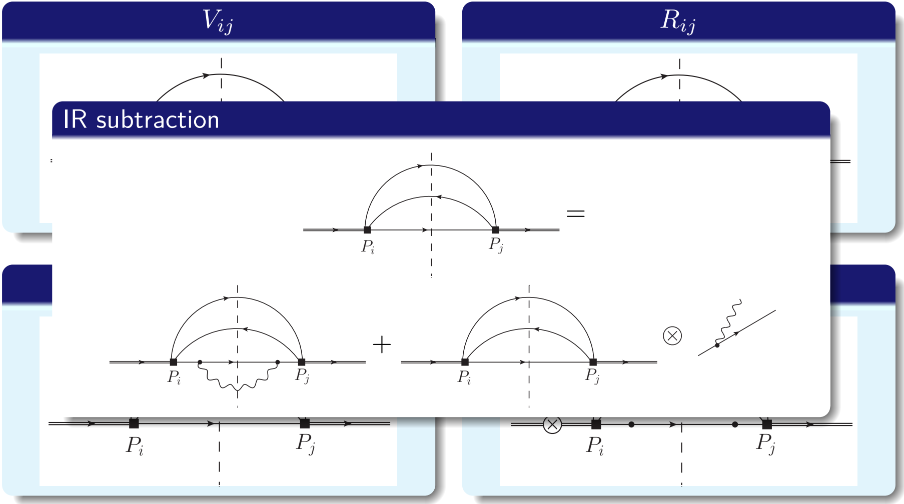
Master formula

$$G_{ij}^{(1)}(\mu,\delta) = V_{ij} + V_{ij}^* + R_{ij} + 2\,Z_\psi T_{ij} + M_{ij} + M_{ij}^* + \sum \{ Z_{jk} T_{ik} + Z_{im} T_{mj} \}$$

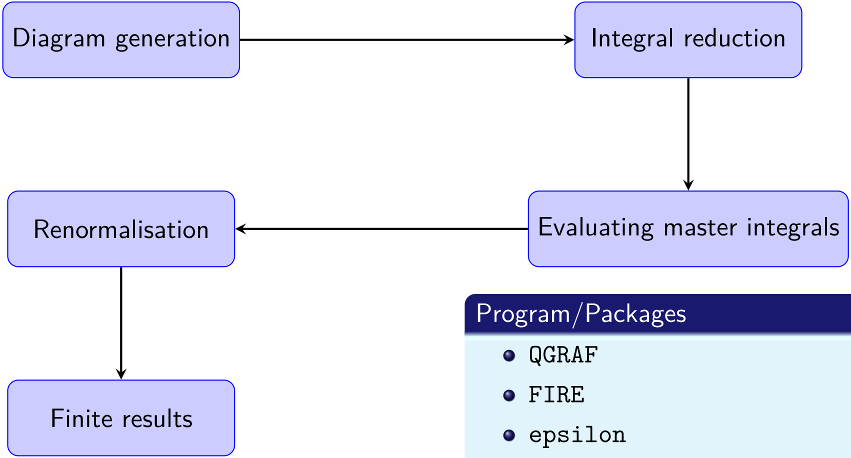
V_{ij}

R_{ij}

IR subtraction



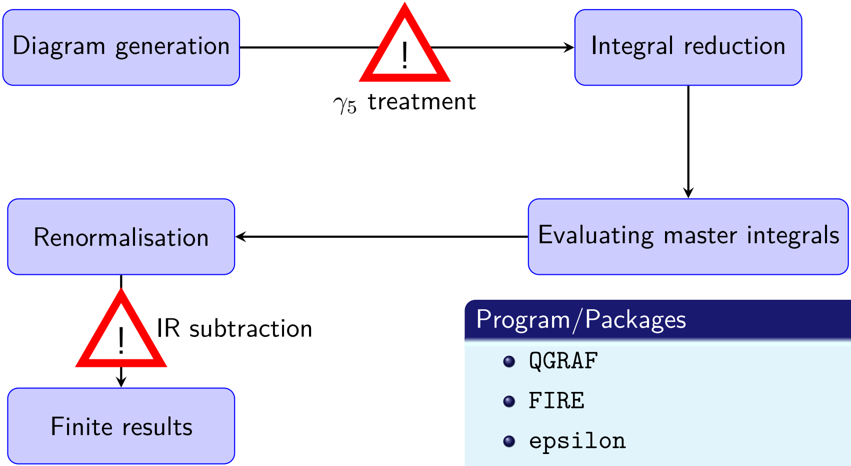
Workflow



Program/Packages

- `QGRAF`
- `FIRE`
- `epsilon`
- `MB`
- `HarmonicSums`

Workflow



Program/Packages

- `QGRAF`
- `FIRE`
- `epsilon`
- `MB`
- `HarmonicSums`

γ_5 treatment

γ_5 Problem

- $\{\gamma_5, \gamma_\mu\} = 0$
- Cyclicity of the trace

- γ_5 inherently four-dimensional object
  - $\Rightarrow$ Many schemes have been proposed to deal with this in D-dimensions
    - Naive dimensional regularisation [Chanowitz,Furman,Hinchliffe;79]
    - ' t Hooft-Veltmann scheme [' t Hooft,Veltman;72]
    - Larin scheme [Larin;93]
- We employ the KKS scheme

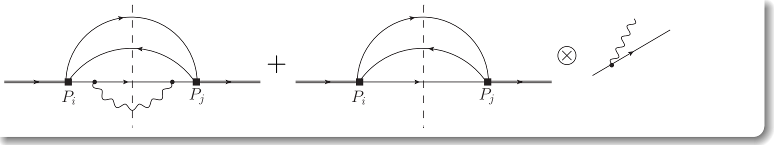
KKS scheme [Kreimer;90][Korner,Kreimer,Schilcher;92]

i) Anti-commutation relations holds
ii) Cyclicity of the trace forbidden
iii) Reading point
iv) Anticommute all occurring γ_5

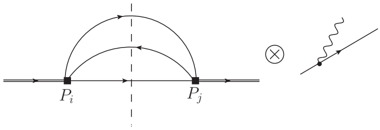
IR subtraction

- Why do we even need it ?
  - $\Rightarrow$ Collinear photon radiated from light quarks
- Translate the divergences into logarithms of the light quark mass
  [Kaminski,Misiak,Poradzinski;12]

$$\frac{\mathrm{d}\Gamma_m}{\mathrm{d}z} = \frac{\mathrm{d}\Gamma_\varepsilon}{\mathrm{d}z} + \frac{\mathrm{d}\Gamma_{\text{shift}}}{\mathrm{d}z}$$



$\frac{\mathrm{d}\Gamma_{\text{shift}}}{\mathrm{d}z}$



- IR Subtraction Kernels

$$S_0 = \frac{\alpha_e}{4\pi} \frac{2\pi e^{\gamma_E \varepsilon}(1-\varepsilon)(1+\bar{z}_\gamma^2)\csc(\pi\varepsilon)}{z_\gamma \Gamma(1-\varepsilon)} \left(\frac{m_q z_\gamma}{\mu}\right)^{-2\varepsilon}$$

$\Rightarrow$ Difference between massive and massless quark to photon splitting function

$$S_1 = \frac{\alpha_e \alpha_s}{(4\pi)^2} C_F \left(\frac{m_q}{\mu}\right)^{-4\varepsilon} \left[\frac{1}{2\varepsilon^2}\left(P^{(0)}_{q\to q} \otimes P^{(0)}_{q\to\gamma}\right)(z_\gamma) - \frac{1}{2\varepsilon} R^{(1)}_{q\to\gamma}(z_\gamma)\right]$$

- At tree level

$$\left(\frac{\mathrm{d}\Gamma^{T,0}_{\text{shift}}}{\mathrm{d}z}\right)_{ij} = \frac{1}{2m_b}\frac{1}{2N_c}\int dPS_3\, \mathcal{T}_{ij}(s_{kl})\frac{\alpha_e}{2\pi\bar{z}}\left\{Q_1^2\left[1+\frac{(z-s_{23})^2}{(1-s_{23})^2}\right]\right.$$
$$\left.\times\left[\frac{\pi e^{\gamma_E\varepsilon}(1-\varepsilon)\csc(\pi\varepsilon)}{\Gamma(1-\varepsilon)}\left(\frac{m_{q_1}(1-z)}{\mu(1-s_{23})}\right)^{-2\varepsilon}\right]\Theta(z-s_{23}) + \text{cyclic}\right\}$$

Numerical Results

$$\Delta\Gamma \equiv \Gamma_0 \sum_{i,j} \mathcal{C}_i^{\text{eff}\,*}(\mu)\, \mathcal{C}_j^{\text{eff}}(\mu)\, \widehat{G}_{ij}(\mu, z_c, \delta)$$

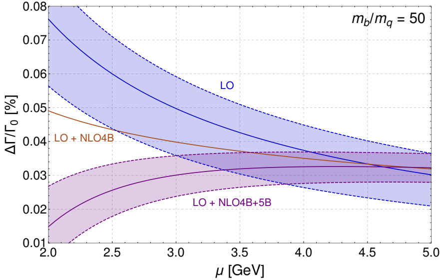
$$\Delta\Gamma/\Gamma_0\Big|_{m_q=m_b/50}[\%] = (0.0600)_{\text{LO}} - (0.0168)_{\text{NLO4B}} - (0.0179)_{\text{NLO4B+5B}}$$

$$\Delta\Gamma/\Gamma_0\Big|_{m_q=m_b/20}[\%] = (0.0274)_{\text{LO}} - (0.0158)_{\text{NLO4B}} - (0.0061)_{\text{NLO4B+5B}}$$

- μ=2.5GeV , $\delta = 1 - 2E_0/m_b = 0.3305$

$\Rightarrow$ 5B contribution become more important at $m_q \to 0$ due to $\ln^2(m_b/m_q)$

μ-dependence



Varying μ_0, m_b, z_c, m_t, $\alpha_s(M_Z)$

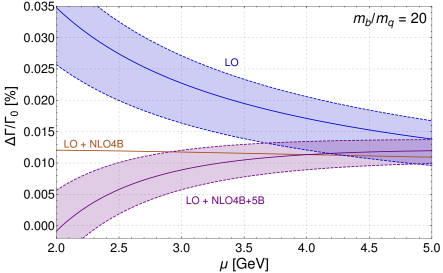
Reduction of uncertainty bands

Curve flatter → RG-Invariance

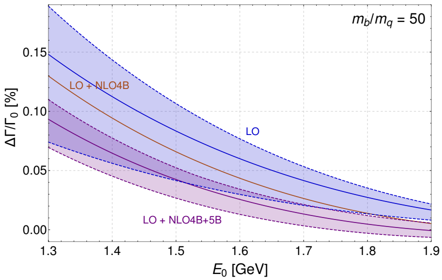
$m_b/m_q = 20$

LO+NLO4B curve extremely flat

→ Accidentally



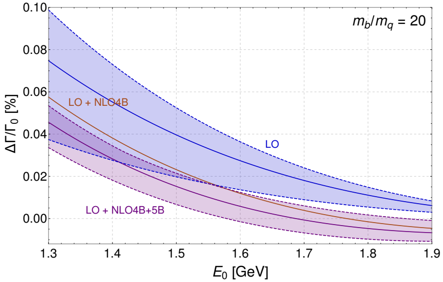
Photon energy cut dependence



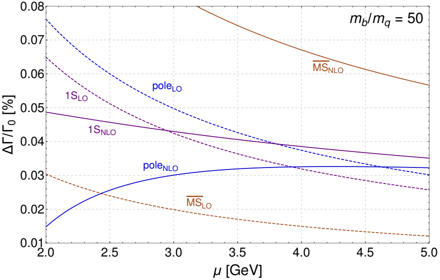
$$E_0 \equiv (1-\delta)\frac{m_b}{2}$$

Varying μ, μ_0, m_b, z_c, m_t, $\alpha_s(M_Z)$

Reduction of uncertainty bands



μ-dependence with different mass schemes



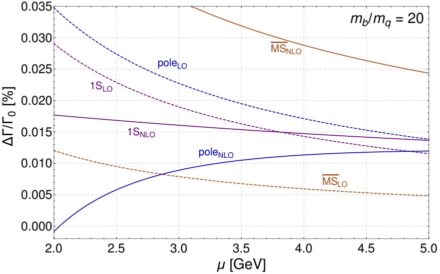
Mass scheme

$$m_b^{\text{pole}} = \overline{m}_b(\overline{m}_b)\left(1 + \frac{16}{3}\frac{\alpha_s(\overline{m}_b)}{4\pi} + \dots\right)$$

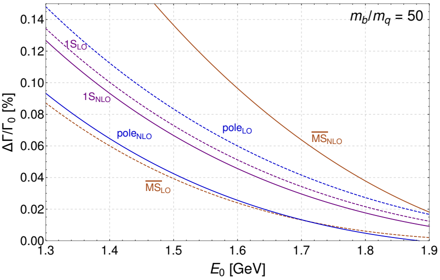
$$m_b^{\text{pole}} = m_b^{\text{1S}}\left(1 + \frac{C_F^2}{8}\,\alpha_s^2(\mu) + \dots\right)$$

$\overline{\text{MS}}$-scheme large scheme dependence

$\rightarrow \overline{m}_c$ dependence not tamed



Photon energy cut dependence with different mass schemes



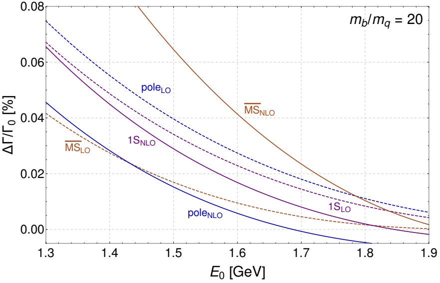
Mass scheme

$$m_b^{\text{pole}} = \overline{m}_b(\overline{m}_b)\left(1 + \frac{16}{3}\frac{\alpha_s(\overline{m}_b)}{4\pi} + \dots\right)$$

$$m_b^{\text{pole}} = m_b^{\text{1S}}\left(1 + \frac{C_F^2}{8}\alpha_s^2(\mu) + \dots\right)$$

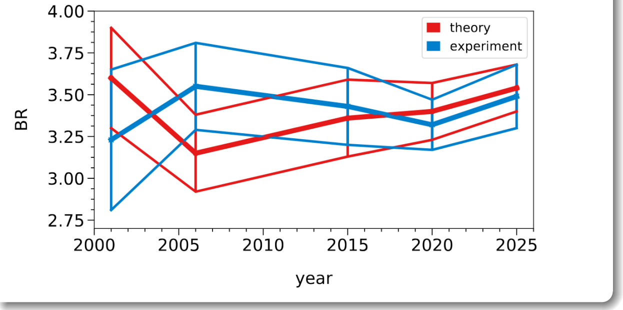
$\overline{\text{MS}}$-scheme large scheme dependence

$\to \overline{m}_c$ dependence not tamed



Conclusion

- $\bar{B} \to X_s\gamma$ represents a standard candle in search for New Physics
- Belle II $\to \pm 2.6\%$ uncertainty
- Theory uncertainty must ideally be comparable size
- Multi-parton formally completes NLO
    - Outlined the computation
    - Shown some numerical results



[Plot taken from CKM talk:Steinhauser;25]

Input parameters for numerical results

$m_t^{\text{pole}} = (172.4 \pm 0.7)$ GeV	$\alpha_s^{(5)}(M_Z) = 0.1180 \pm 0.0009$
$m_b^{\text{pole}} = (4.78 \pm 0.06)$ GeV	$M_Z = 91.1880$ GeV
$m_b^{\text{1S}} = (4.65 \pm 0.03)$ GeV	$M_W = 80.3692$ GeV
$\overline{m}_b(\overline{m}_b) = (4.183 \pm 0.007)$ GeV	$\mu_0 = (160 \pm 80)$ GeV
$m_c^{\text{pole}} = (1.67 \pm 0.07)$ GeV	$\mu = 2.5^{+2.5}_{-0.5}$ GeV
$\overline{m}_c(\overline{m}_c) = (1.2730 \pm 0.0046)$ GeV	$z_c = m_c^2/m_b^2 = 0.12 \pm 0.03$
$\lambda_u = -0.0086 + 0.0186\,i$	$E_0 = 1.6$ GeV
$\lambda_c = -0.9914 - 0.0186\,i$	