NLO corrections and Decoherence effects

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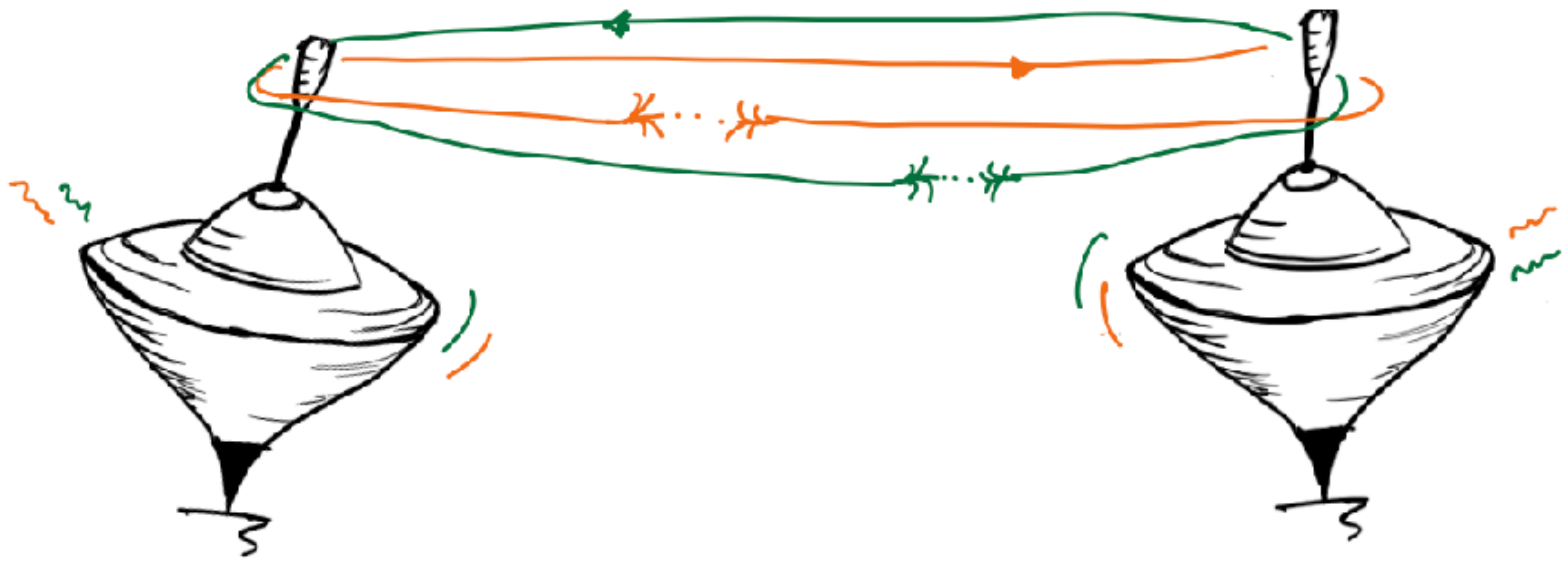
[quant-ph/2504.07030]



with Fabio Maltoni, Alan Barr and Leonardo Satrioni

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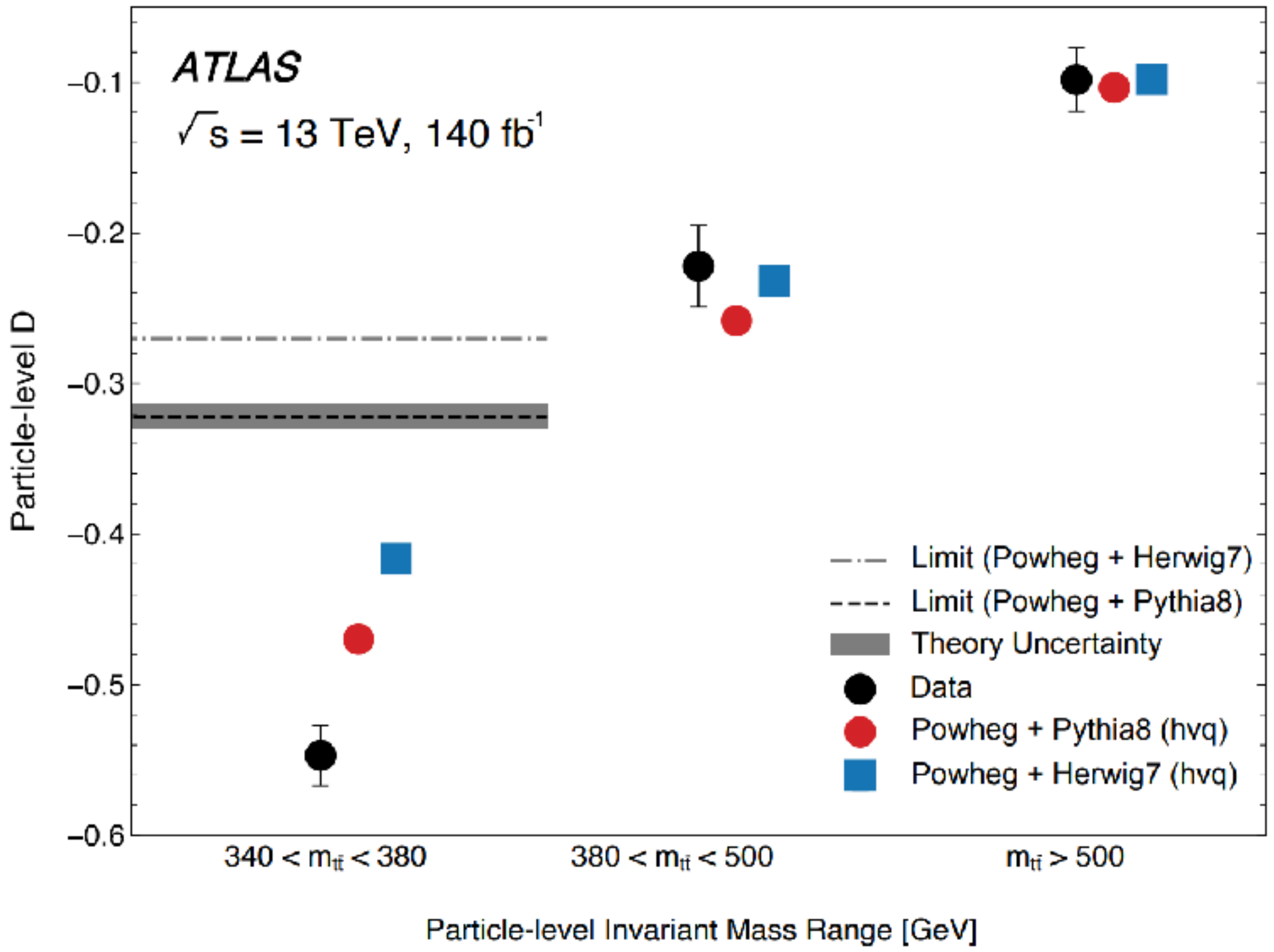
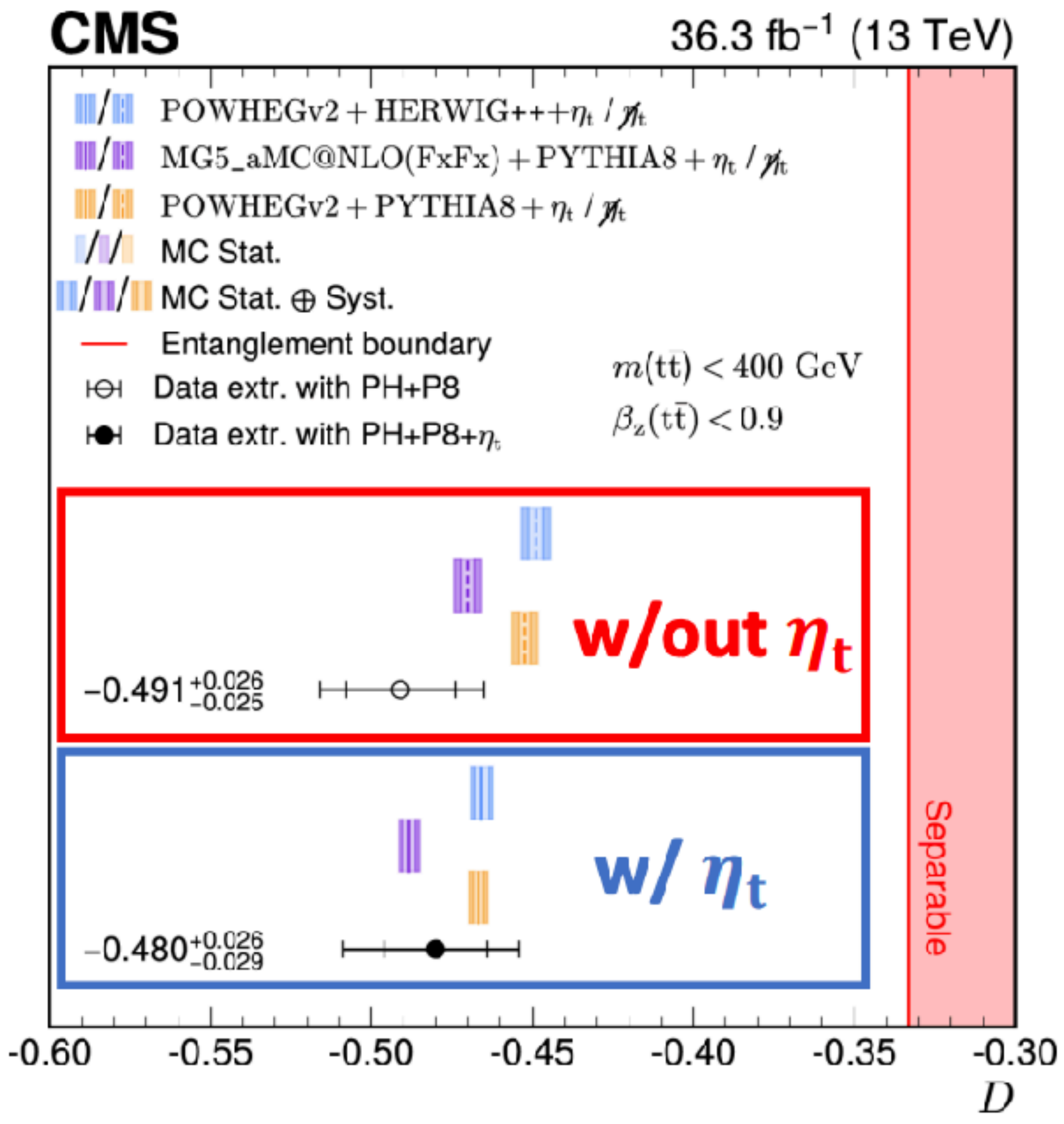
DESY - 2025

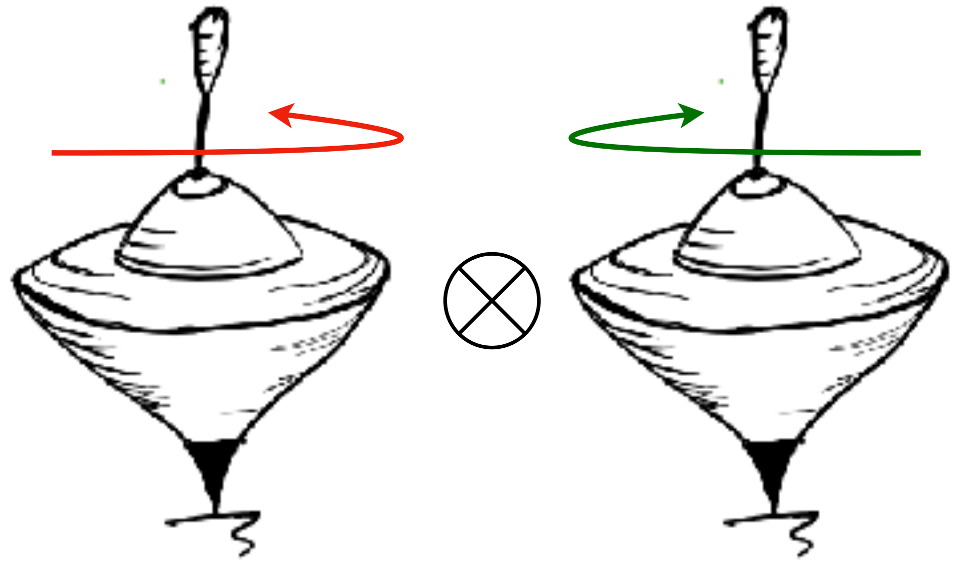
Entanglement at LHC





D < -1/3 ⟷ entangled

Top pair as a bipartite qubit system

Two qubit 

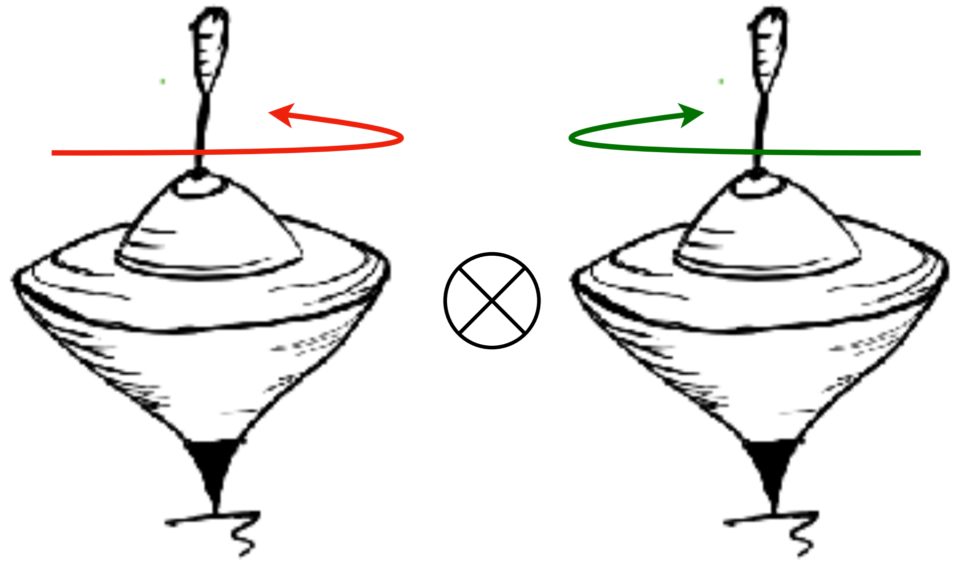
$$|\psi_{\rm ab}\rangle = c_1|\uparrow\uparrow\rangle + c_2|\uparrow\downarrow\rangle + c_3|\downarrow\uparrow\rangle + c_4|\downarrow\downarrow\rangle \;= \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

Density matrix is 4x4

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_i^- \mathbb{1}_2 \otimes \sigma^i + C_{ij}\, \sigma^i \otimes \sigma^j}{4}.$$

Top pair as a bipartite qubit system

Two qubit



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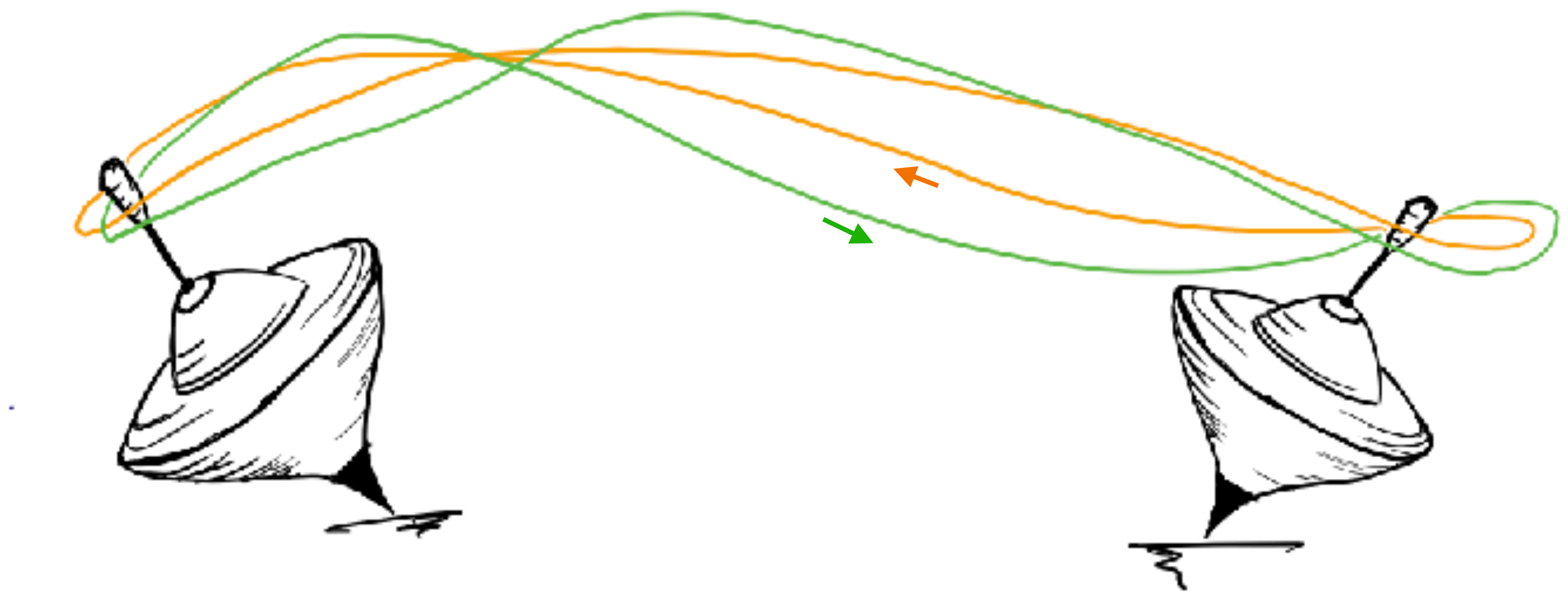
Entanglement

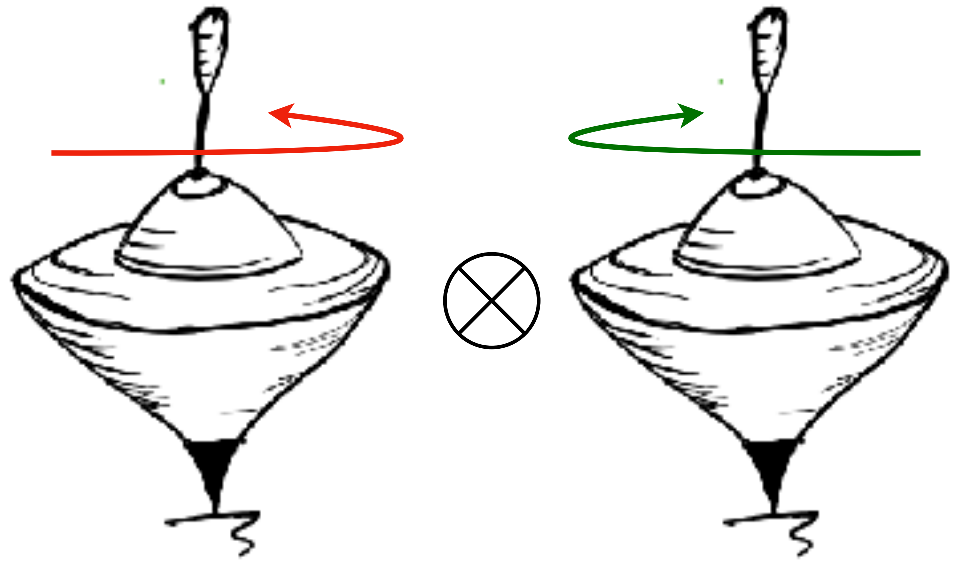


Given a bipartite system $\mathcal{H}_{\rm ab} = \mathcal{H}_{\rm a} \otimes \mathcal{H}_{\rm b}$

Can you write $|\psi_{\rm ab}\rangle = |\psi_{\rm a}\rangle \otimes |\psi_{\rm b}\rangle$

No? Then it is entangled.

Top pair as a bipartite qubit system

Two qubit 

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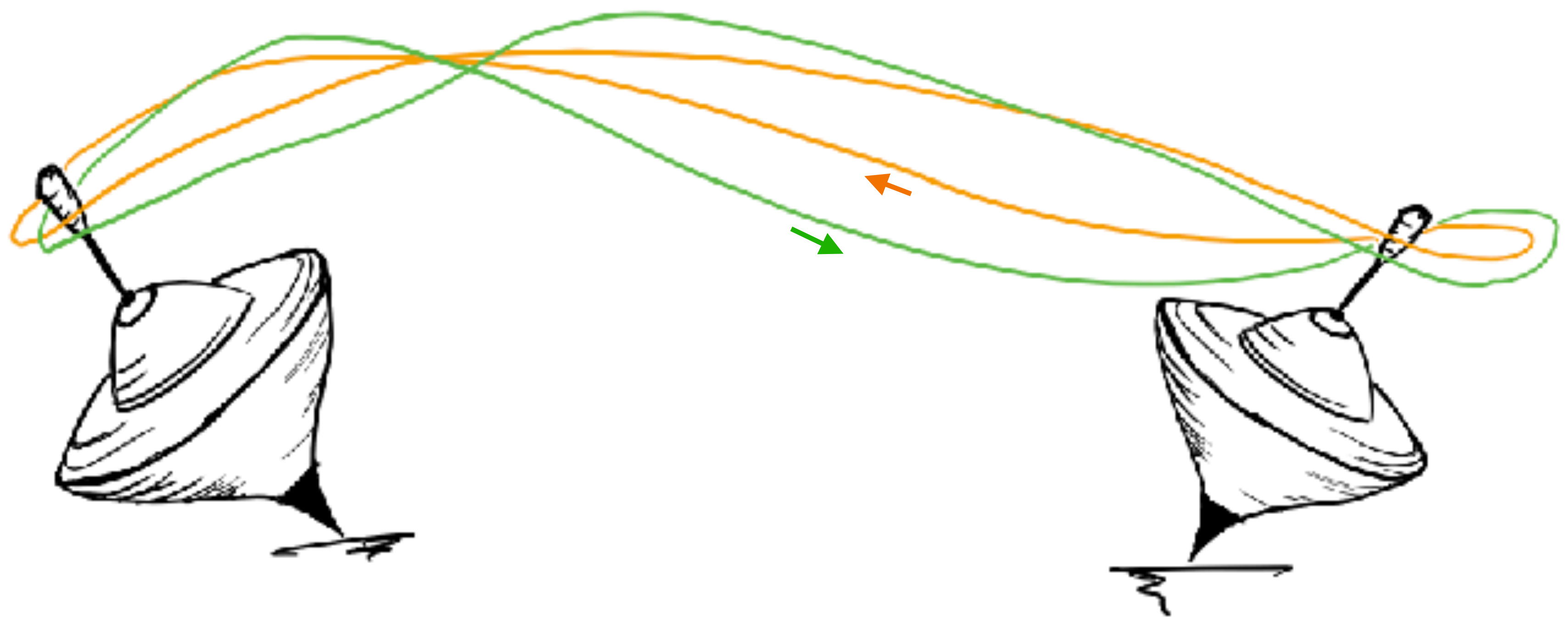
Entanglement



Peres-Horodecki Criterion:

$$\Delta \equiv -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$

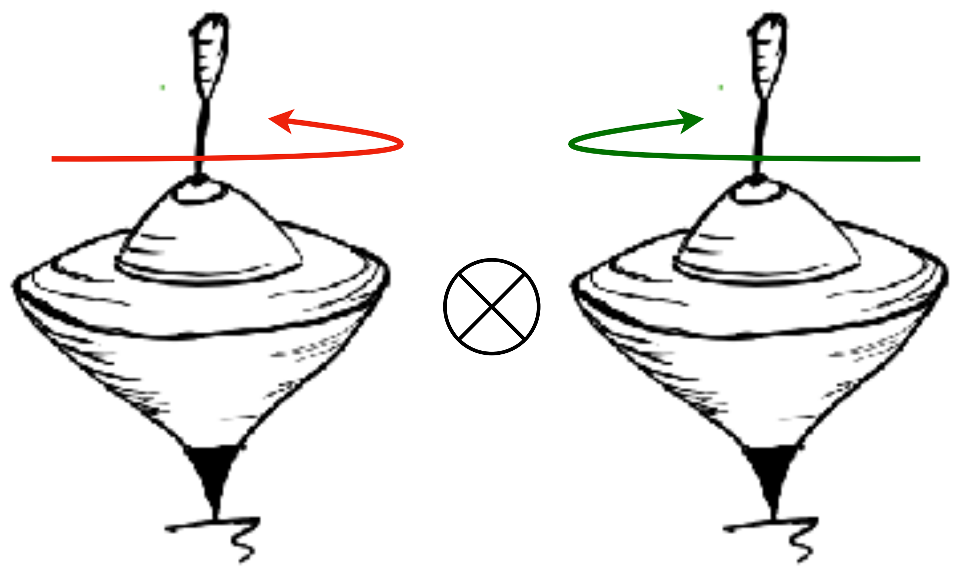
Concurrence (max =1):

$$C[\rho] = \max(\Delta/2, 0)$$

Top pair as a bipartite qubit system

[Afik and de Nova, 21']

Two qubit



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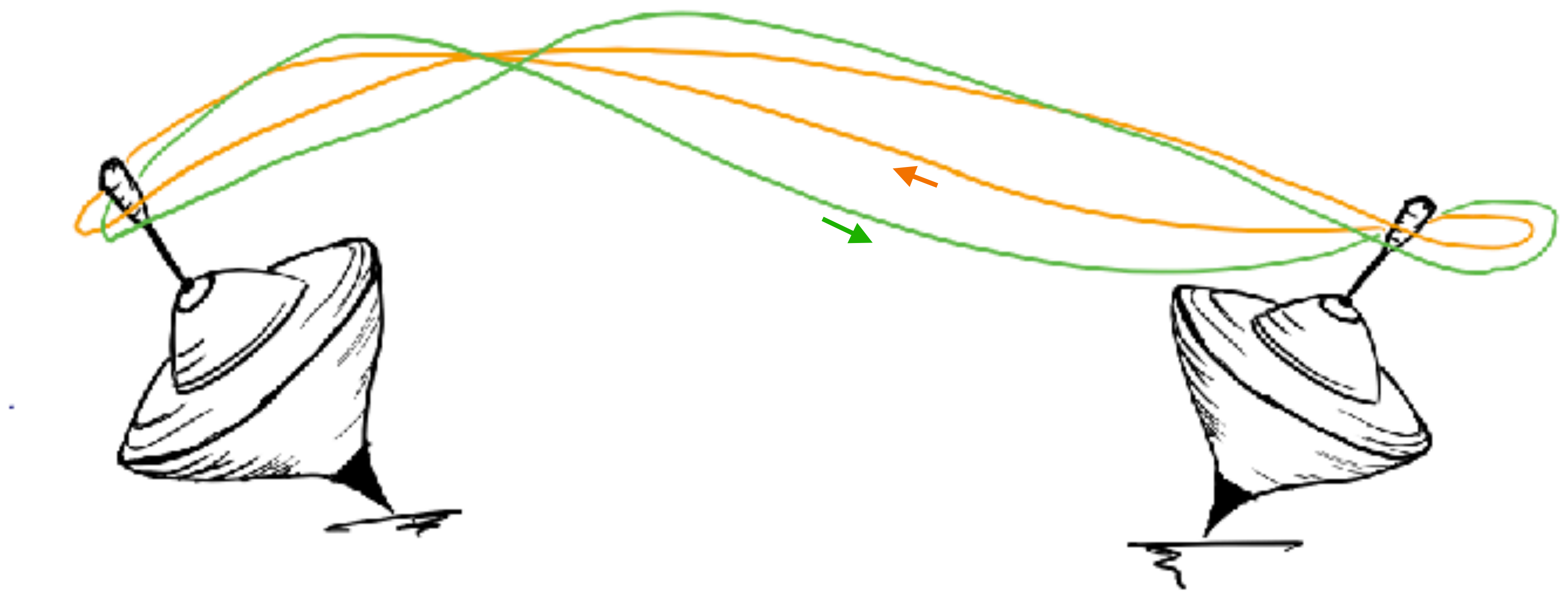
NB: Spin-Correlations $\supseteq$ Entanglement



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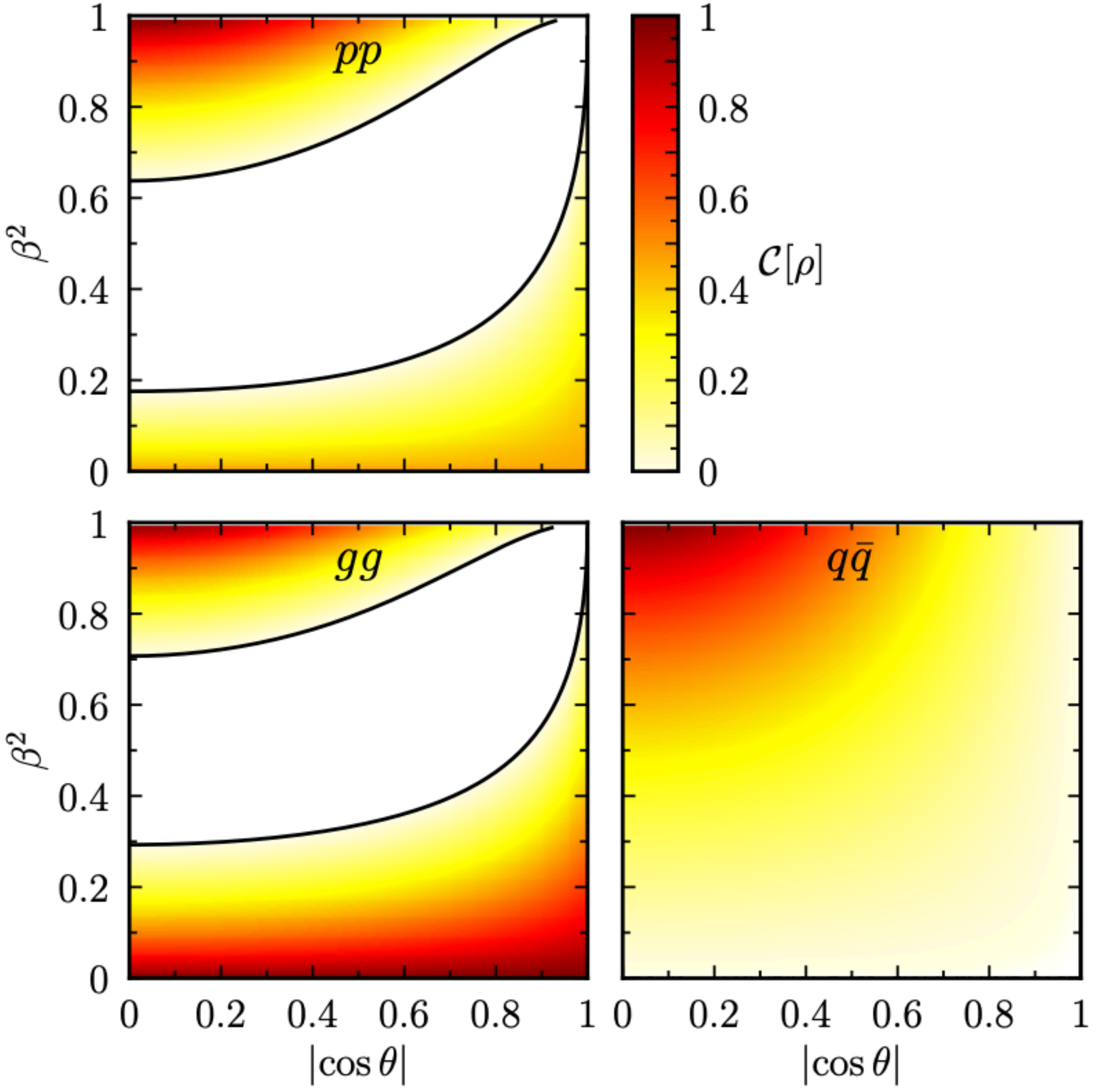
What's the story for the SM?



[Aoude, Madge, Maltoni, Mantani, '22]

Two-to-two scattering: 2 d.o.f

velocity and angle of the top

$$\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \qquad \cos\theta$$

White regions: zero-entanglement

Maximal entanglement points/regions

- At threshold: $\beta^2 = 0, \forall\theta$
- high-E: $\beta^2 \to 1, \cos\theta = 0$

[Afik and de Nova, 21']

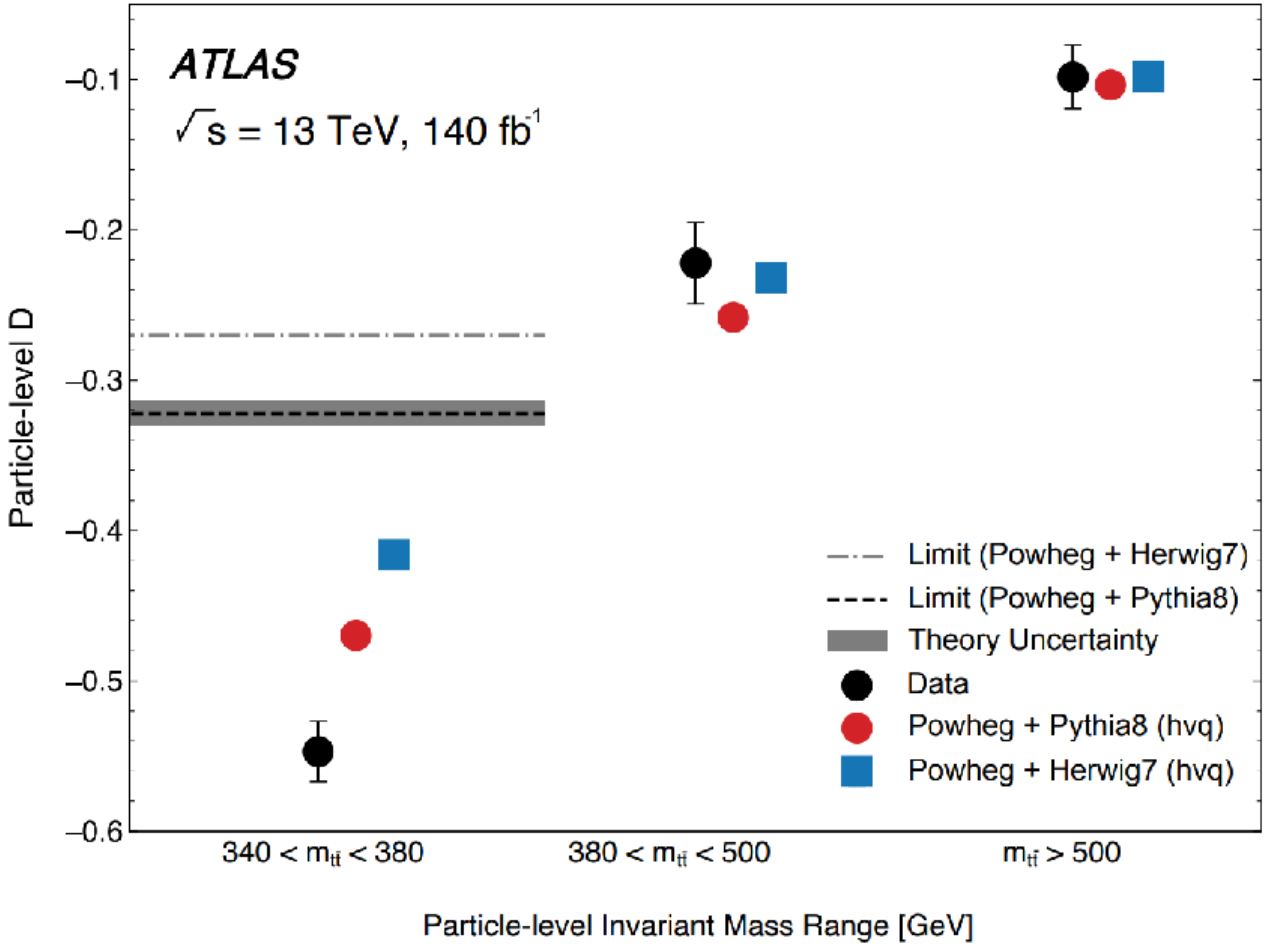
Review by [Barr, Fabbrichesi, Floreanini, Gabrielli, Mazola, '24]

See Also [Abel, Dittmar, Dreiner '92] [Dittmar, Dreiner '96]

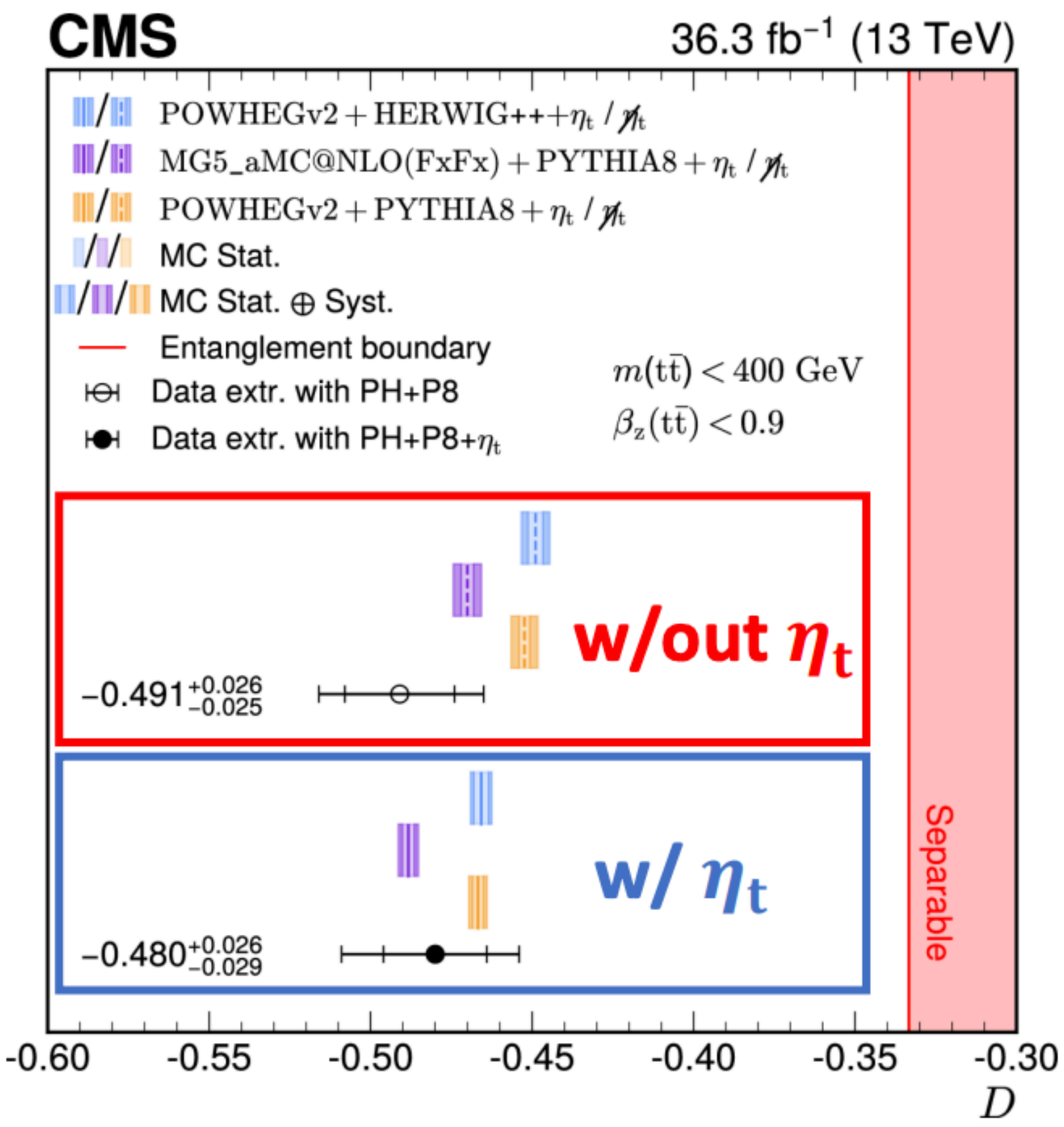
ATLAS and CMS measurement

$^{*}\eta_t$ topponium modelling

At threshold $\beta^2 \sim 0$



[Nature 633, 542-547 (2024)]



[Rep. Prog. Phys. 86 (2024) 117801]

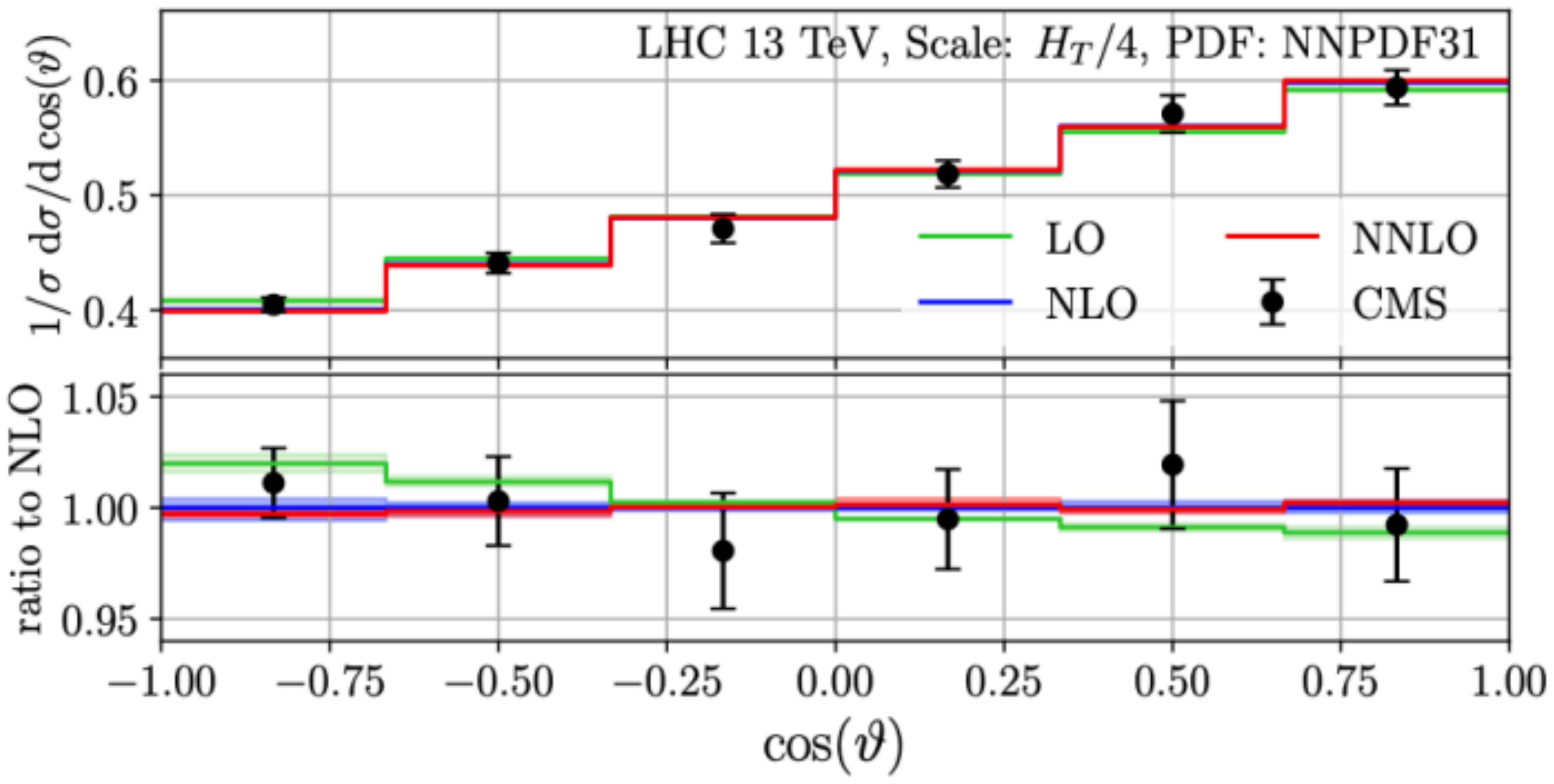
$$\mathrm{tr}[C_{ij}]/3 \equiv D < -1/3 \qquad \Leftrightarrow \qquad \text{entangled}$$

Going beyond LO?

We assume that NLO corrections in the ttbar entanglement are small

Knowledge from many spin-correlations studies [Czakon, Mitov, Poncelet '08]

Coefficient	LO ($\times 10^3$)	NLO ($\times 10^3$)	NNLO ($\times 10^3$)	CMS ($\times 10^3$)
B_1^k	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+0}_{-1} [sc] ± 2 [mc]	-1^{+0}_{-1} [sc] ± 4 [mc]	5 ± 23
B_1^r	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+1}_{-0} [sc] ± 2 [mc]	0^{+1}_{-2} [sc] ± 2 [mc]	-23 ± 17
B_1^n	0^{+0}_{-0} [sc] ± 1 [mc]	3^{+1}_{-1} [sc] ± 1 [mc]	4^{+1}_{-0} [sc] ± 3 [mc]	6 ± 13
B_2^k	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+0}_{-1} [sc] ± 1 [mc]	-5^{+2}_{-3} [sc] ± 3 [mc]	7 ± 23
B_2^r	0^{+0}_{-0} [sc] ± 1 [mc]	0^{+2}_{-0} [sc] ± 1 [mc]	-2^{+0}_{-1} [sc] ± 2 [mc]	-10 ± 20
B_2^n	0^{+0}_{-0} [sc] ± 1 [mc]	-2^{+0}_{-1} [sc] ± 1 [mc]	-3^{+1}_{-0} [sc] ± 3 [mc]	17 ± 13
C_{kk}	324^{+7}_{-7} [sc] ± 1 [mc]	330^{+2}_{-2} [sc] ± 3 [mc]	323^{+2}_{-5} [sc] ± 6 [mc]	300 ± 38
C_{rr}	6^{+5}_{-5} [sc] ± 1 [mc]	58^{+18}_{-12} [sc] ± 2 [mc]	69^{+8}_{-7} [sc] ± 3 [mc]	81 ± 32
C_{nn}	332^{+1}_{-0} [sc] ± 1 [mc]	330^{+1}_{-1} [sc] ± 2 [mc]	326^{+1}_{-1} [sc] ± 4 [mc]	329 ± 20
$C_{nr} + C_{rn}$	1^{+0}_{-0} [sc] ± 1 [mc]	-1^{+1}_{-0} [sc] ± 3 [mc]	-4^{+4}_{-0} [sc] ± 6 [mc]	-4 ± 37
$C_{nr} - C_{rn}$	0^{+0}_{-1} [sc] ± 1 [mc]	-1^{+1}_{-0} [sc] ± 2 [mc]	2^{+4}_{-2} [sc] ± 8 [mc]	-1 ± 38
$C_{nk} + C_{kn}$	0^{+0}_{-0} [sc] ± 1 [mc]	2^{+1}_{-0} [sc] ± 1 [mc]	3^{+4}_{-1} [sc] ± 3 [mc]	-43 ± 41
$C_{nk} - C_{kn}$	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+1}_{-1} [sc] ± 2 [mc]	6^{+0}_{-2} [sc] ± 7 [mc]	40 ± 29
$C_{rk} + C_{kr}$	-229^{+4}_{-4} [sc] ± 1 [mc]	-203^{+9}_{-7} [sc] ± 2 [mc]	-194^{+8}_{-6} [sc] ± 7 [mc]	-193 ± 64
$C_{rk} - C_{kr}$	1^{+0}_{-0} [sc] ± 1 [mc]	1^{+0}_{-1} [sc] ± 4 [mc]	-1^{+1}_{-3} [sc] ± 5 [mc]	57 ± 46



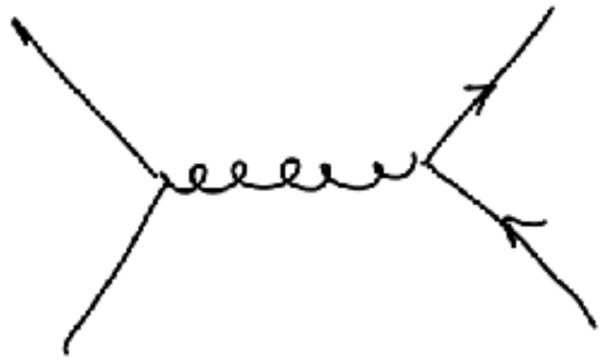
If we plan to measure spin entanglement at LHC with more and more precision,

revisit this assumption with a more quantum info perspective

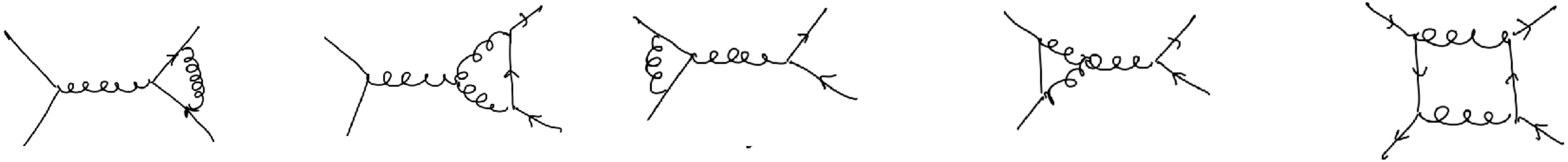
NLO corrections

For ttbar production at NLO (in QCD) we need…

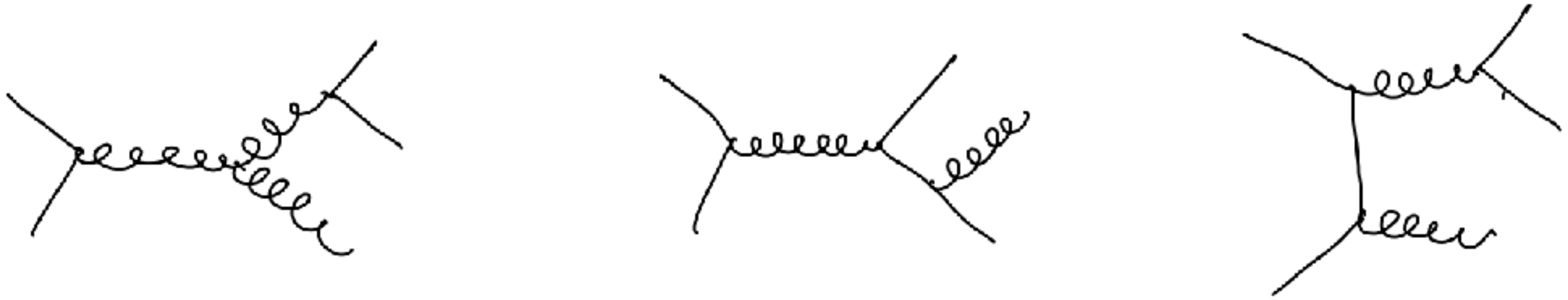
tree



virtual



real



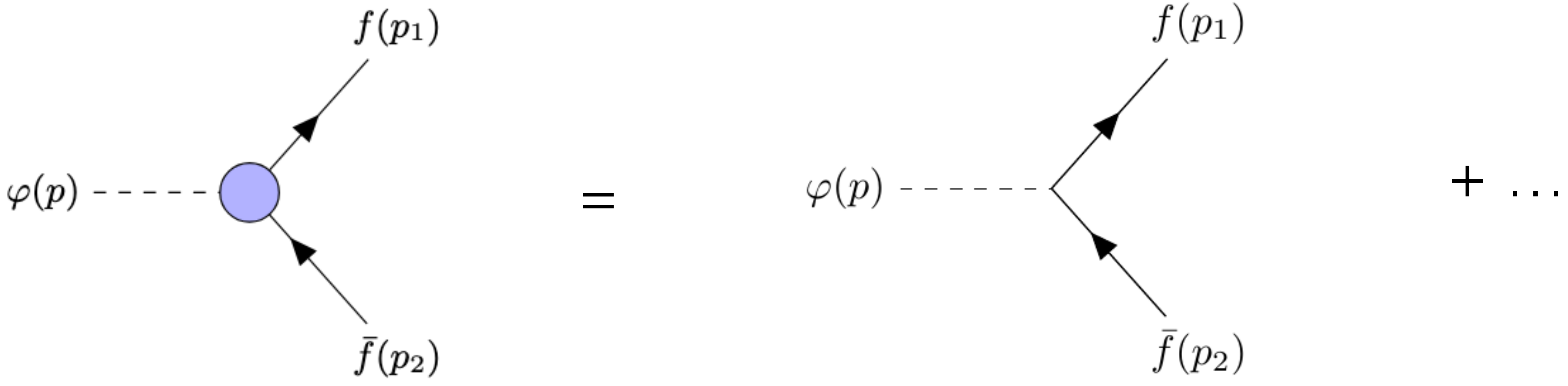
Extra radiation?
Larger Hilbert space.

If the radiation is unresolvable, can we see this as a quantum map?

Decoherence!

Let’s do an easier example…

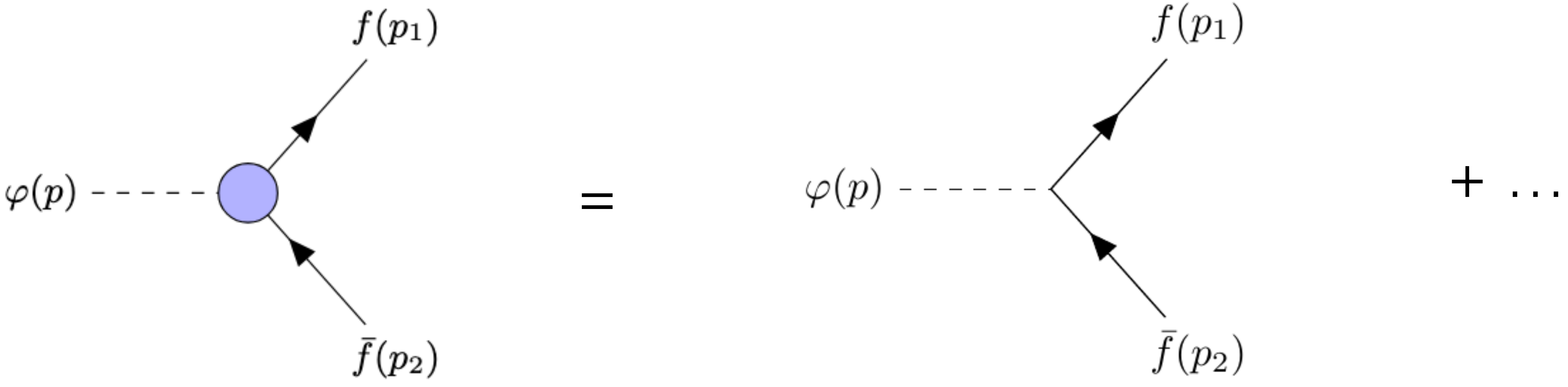
Fermion pair from a scalar decay



Let's do an easier example…

$$R_{\rm LO} \sim \mathcal{A} \otimes \mathcal{A}^\dagger$$

Fermion pair from a scalar decay



At tree-level: $R_{\rm LO} = \frac{4N_C y_f^2 m_f^2 \beta^2}{1-\beta^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad \rho_{\rm LO} = \frac{1}{{\rm tr}[R_{\rm LO}]} R_{\rm LO} = |\Psi^+\rangle\langle\Psi^+|$

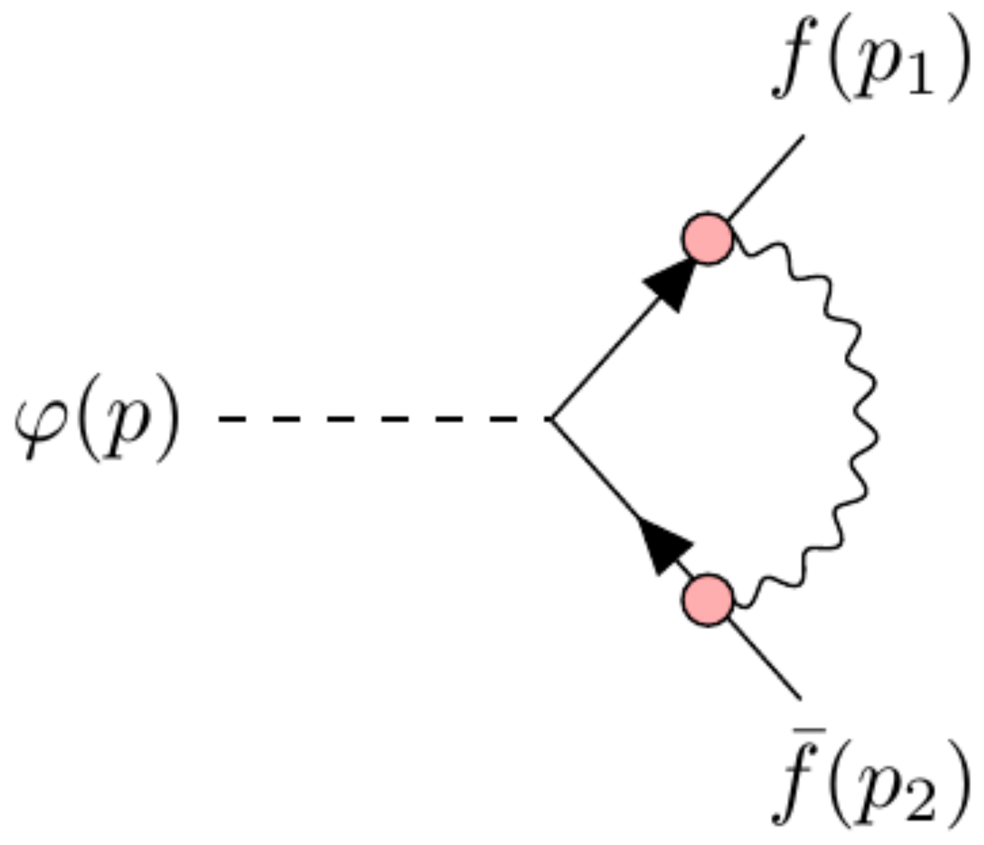
Maximally entangled: controlled place to study entanglement decrease

NLO corrections

General interaction: Scalar, pseudo scalar, vector and axial

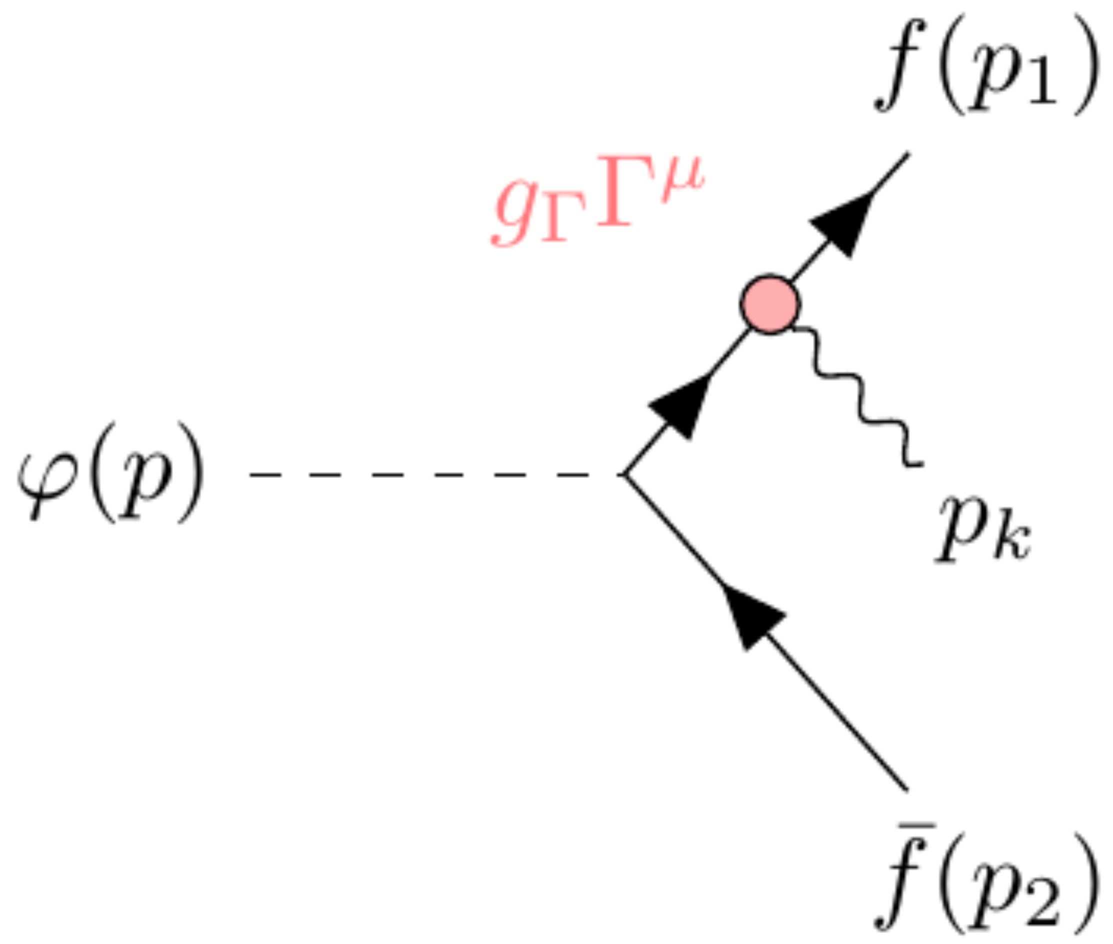
$$g_\Gamma \Gamma^\mu = \{g_S 1, g_P \gamma^5, g_V \gamma^\mu, g_A \gamma^\mu \gamma^5\}$$

Virtual correction: one-loop



+

Real emission



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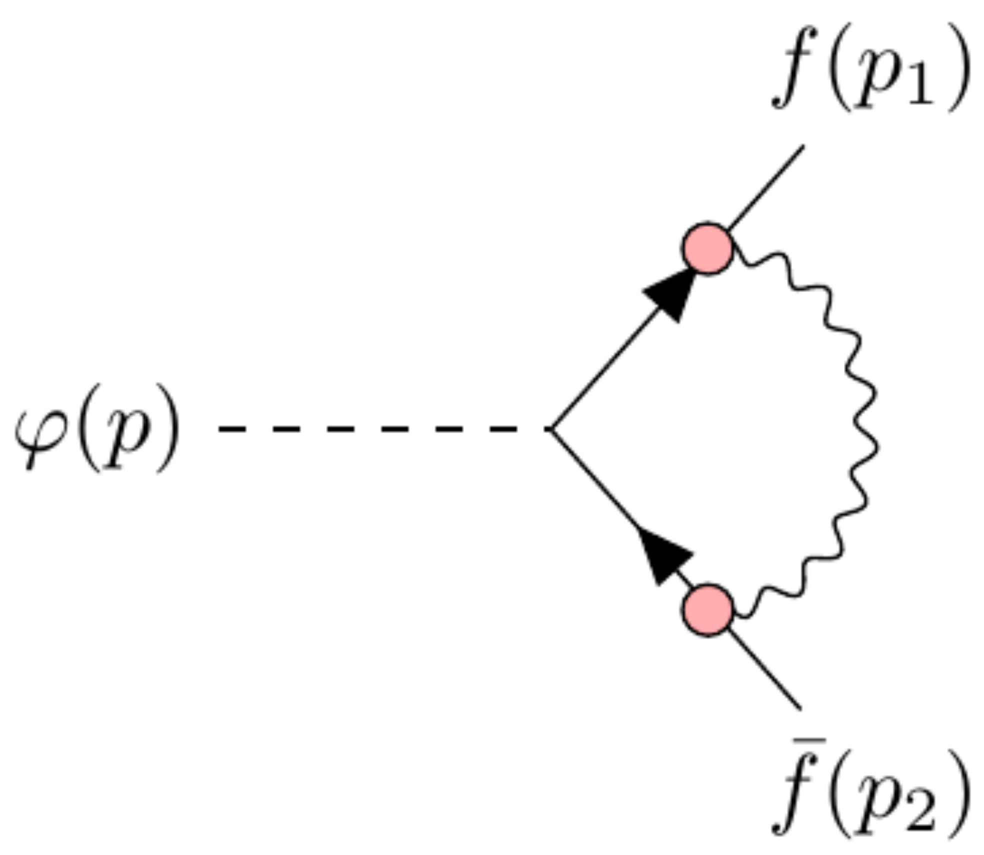

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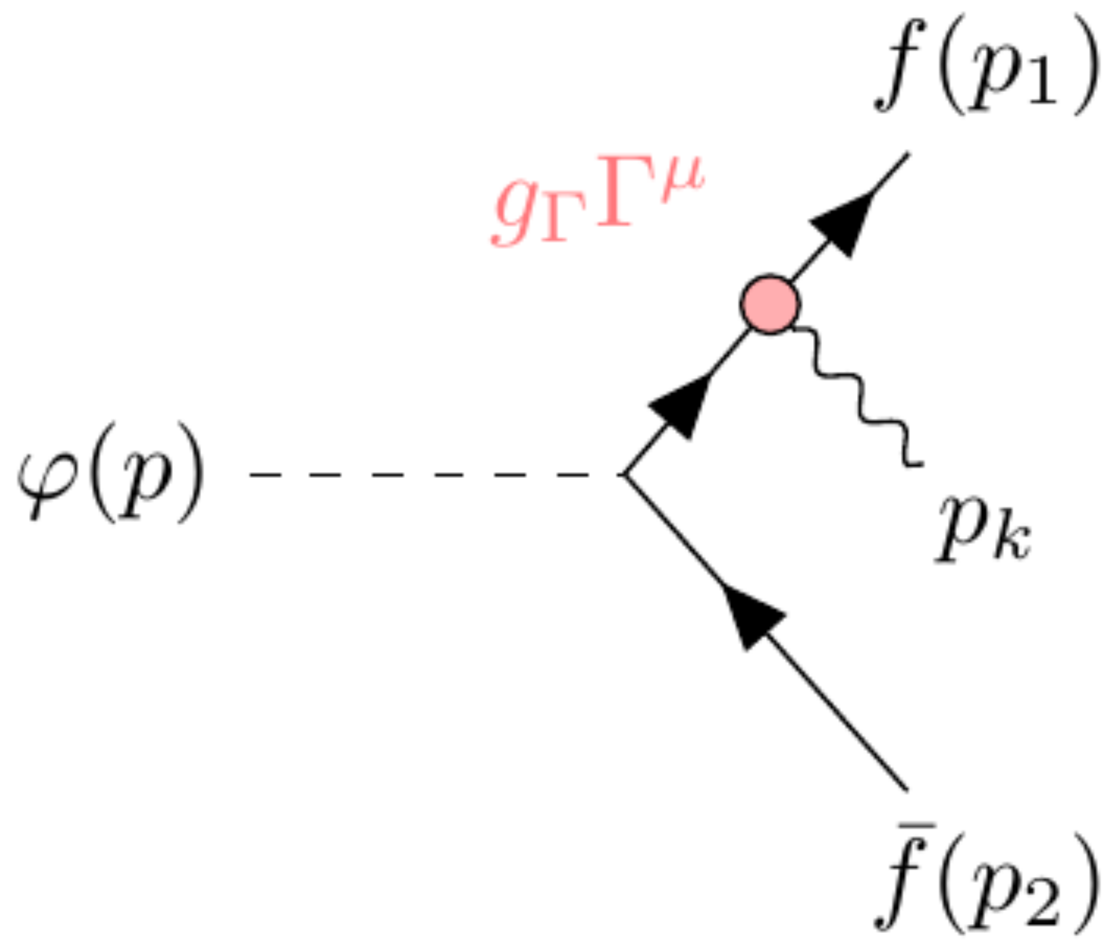


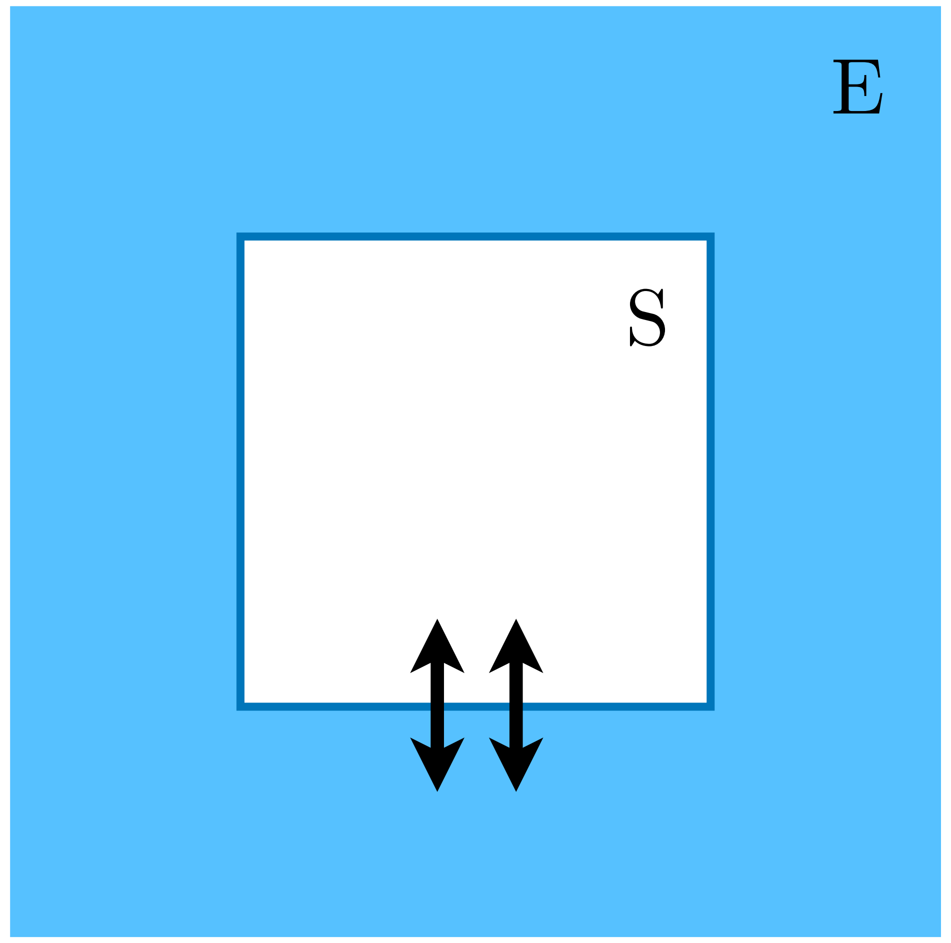
Trace over the extra d.o.f (environment)

↓

Same Hilbert space

Quantum Map.
Open Quantum system

Open Quantum System

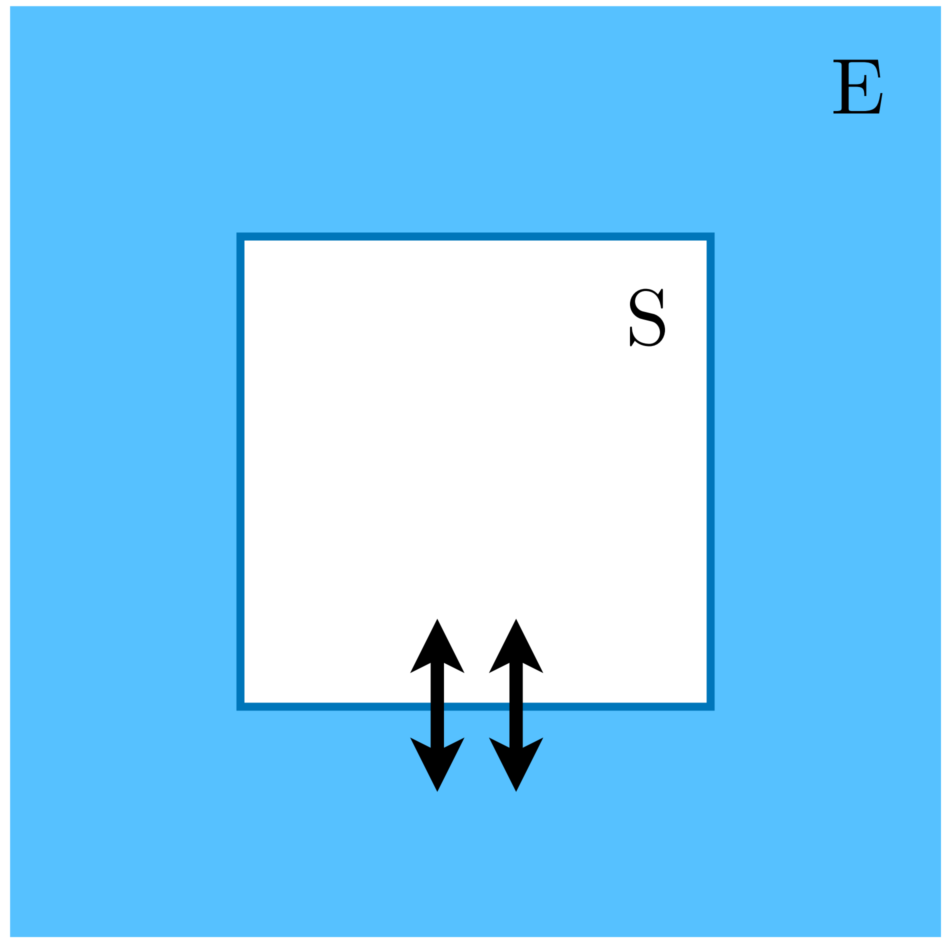


Full system evolution: $\rho'(t) = U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)$

Reduced system: $\rho_{\mathrm{S}}(t) = \mathrm{tr}_E\left[U(t)\rho_{\mathrm{S}}(0) \otimes \rho_E(0)U^\dagger(t)\right]$

See Clara's Murgi Talk tomorrow!

Open Quantum System



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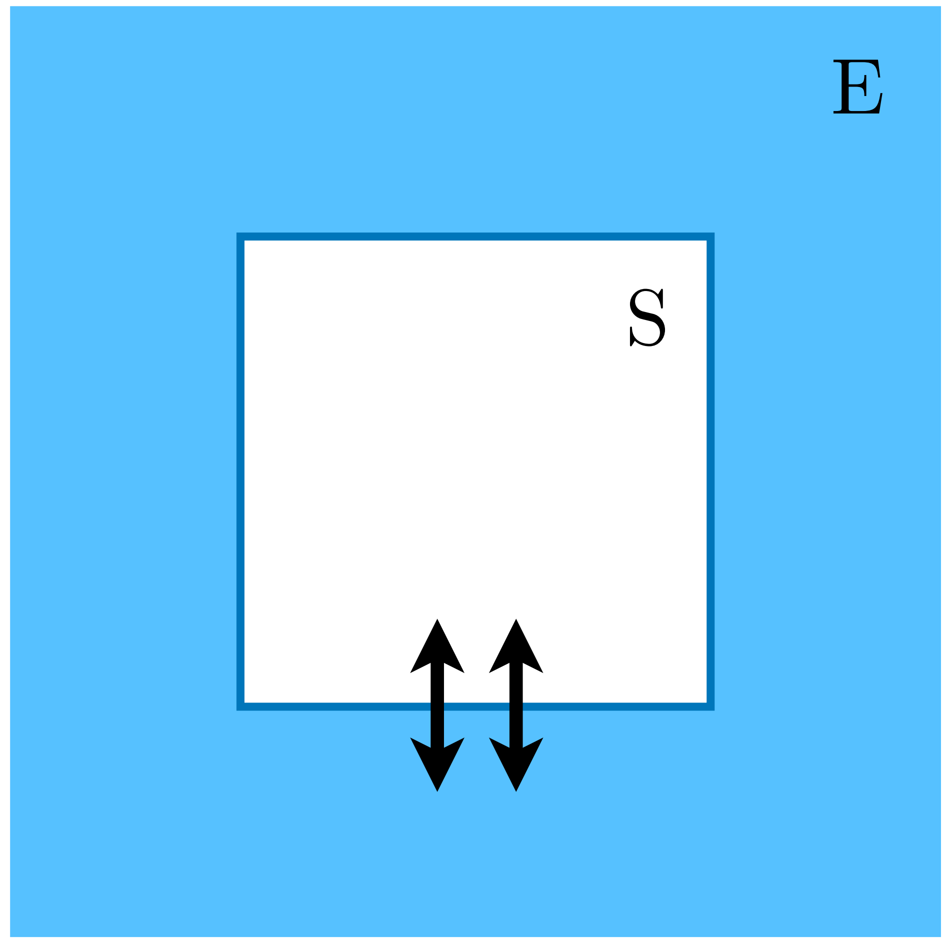
Operator-sum representation (Kraus operators) $\rho_{\mathrm{S}}(t) = \sum_j K_j \rho_{\mathrm{S}}(0) K_j^\dagger$

For bipartite qubits: K_j Tensor product of Pauli

s.t $\sum_j K_j K_j^\dagger = 1$

Environment as unresolved radiation



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Operator-sum representation
(Kraus operators)

$$\rho_{\mathrm{S}}(t) = \mathcal{E}[\rho_S(0)] = \sum_j K_j \rho_S(0) K_j^\dagger$$

We trace out the **unresolved** interaction: soft or collinear

$$\mathrm{tr}_{\mathcal{H}_k}[\cdot] = \int \mathrm{d}\Phi(k) \sum_{\sigma=\pm} \langle k, \sigma| \cdot |k, \sigma\rangle$$

If it's resolved: three-body decay

NLO reduced density matrix

After the trace, evolution in terms of Kraus operators = Quantum Map $\mathcal{E}[\rho]$

$$\rho^{\text{red}}_{\text{LO+NLO}} = \sum_j K_j \rho_{\text{LO}} K_j^\dagger \qquad \text{s.t} \qquad \sum_j K_j K_j^\dagger = 1$$

$$= \mathfrak{p}_{\text{LO}}\, \mathbb{1} \rho_{\text{LO}} \mathbb{1} + \bar{\mathcal{E}}_{\text{V}}[\rho_{\text{LO}}] + \bar{\mathcal{E}}_{\text{R}}[\rho_{\text{LO}}]$$

LO contribution

"Map" of virtual emission — UV and IR divergent

"Map" of real emission — IR divergent

Virtual radiation map

Scalar current at tree-level $\bar{u}(p_1, h_1)v(p_2, h_2)$

Virtual $\bar{u}(p_1, h_1)\mathbb{T}_{\text{virt.}}v(p_2, h_2) = \tilde{T}_{\text{virt.}}\bar{u}(p_1, h_1)v(p_2, h_2)$

↓ One-loop "tensor" integral (w/.gamma's)

↘ Scalar integral (w/o gamma 's) (Passarino Veltman B's and C's)

Virtual radiation map

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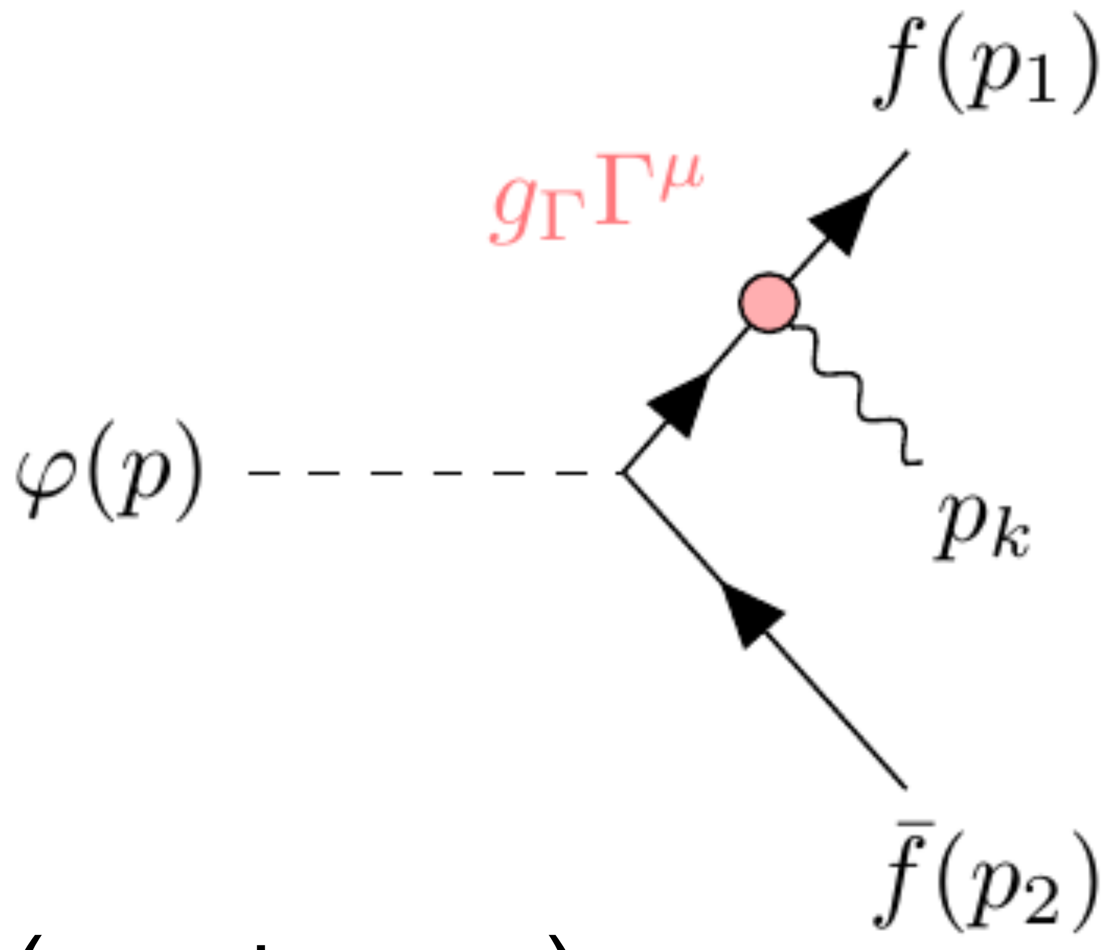
One-loop
"tensor" integral (w/.gamma's)

Scalar integral (w/o.gamma 's)
(Passarino Veltman B's and C's)

The map is just an identity $\bar{\mathcal{E}}_{\text{V}}[\rho_{\text{LO}}] = \mathsf{p}_{\text{V}}\mathbb{1}\rho_{\text{LO}}\mathbb{1}$

This is special for scalar decay.
It would generalise for a vector current

Real radiation map



Real emissions are different

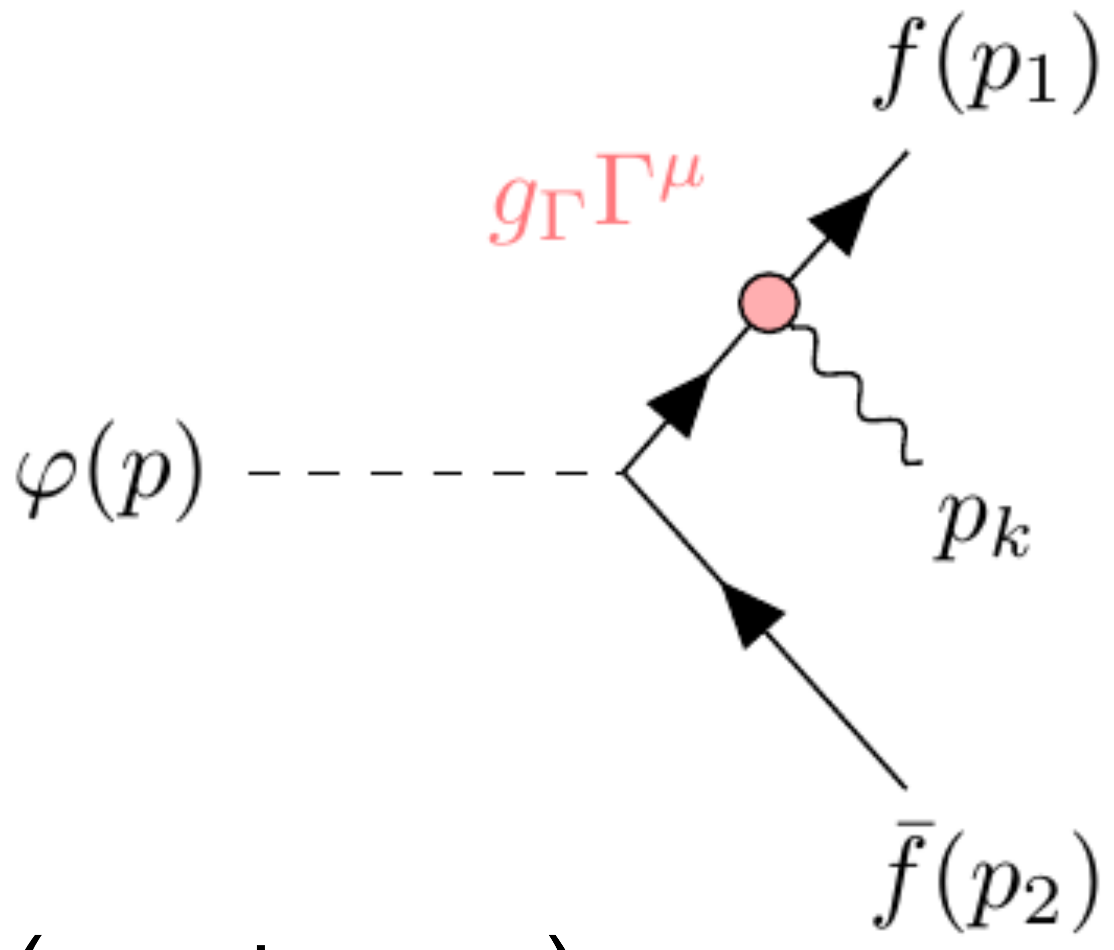
→ change the LO spin-structure

Let's split into a soft and hard emission (w.r.t. ω_0)

$$\bar{\mathcal{E}}_{\mathrm{R}}[\rho_{\mathrm{LO}}] = \bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{soft}}[\rho_{\mathrm{LO}}] + \bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{hard}}[\rho_{\mathrm{LO}}]$$

Unresolved radiation

Real radiation map



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Unresolved radiation

In principle, both are written as operator-sum representation (Kraus)

$$\bar{\mathcal{E}}_{\mathrm{R}}[\rho_{\mathrm{LO}}] = \sum_j K_j \rho_{\mathrm{LO}} K_j^\dagger$$

↓ Built of Pauli matrices

Soft part

We can use the soft theorem

$$\mathcal{M}_{n+1} = \sum_{i=1}^{n} \left[\frac{p_i \cdot \varepsilon_h(k)}{p_i \cdot k} + \cdots \right] \mathcal{M}_n$$

Scalar function

Next-to-leading soft (change structures)

Leading-soft map

$$\bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{soft}}[\rho_{\mathrm{LO}}] = \overbrace{\underbrace{\mathsf{p}_{\mathrm{R}}^{\mathrm{soft}} \mathbb{1} \rho_{\mathrm{LO}} \mathbb{1}}_{\mathrm{scalar,vector}} + \mathsf{q}_5^{\mathrm{soft}} \sum_{j \neq \mathrm{id}} K_j \rho_{\mathrm{LO}} K_j^{\dagger}}^{\mathrm{pseudoscalar,axial}}$$

p's cancel the IR divergence of virtual: KLN theorem

Hard (collinear) emission part

- Now, this has a non-trivial Kraus operator part

$$\bar{\mathcal{E}}_{\mathrm{R}}^{\mathrm{hard}}[\rho_{\mathrm{LO}}] = \mathsf{p}_{\mathrm{R}}^{\mathrm{hard}} \mathbb{1} \rho_{\mathrm{LO}} \mathbb{1} + \mathsf{q}^{\mathrm{hard}} \sum_{j \neq \mathrm{id}} K_j \rho_{\mathrm{LO}} K_j^{\dagger}$$

without the identity

non-zero q = decoherence

Change in spin-structure: dipole-like interaction (IR finite)

Taking the collinear limit for the emission …

Full NLO map

$$\mathcal{E}_{\text{full}}[\rho_{\text{LO}}] = \mathsf{p}_{\text{id}}\, \mathbb{1}\rho_{\text{LO}}\mathbb{1} + \mathsf{q}\sum_{j\neq\text{id}} K_j\rho_{\text{LO}}K_j^\dagger$$

$$\mathsf{p}_{\text{id}} = \left(\mathsf{p}_{\text{LO}} + \mathsf{p}_{\text{V}} + \mathsf{p}_{\text{R}}^{\text{soft}} + \mathsf{p}_{\text{R}}^{\text{hard}}\right)$$

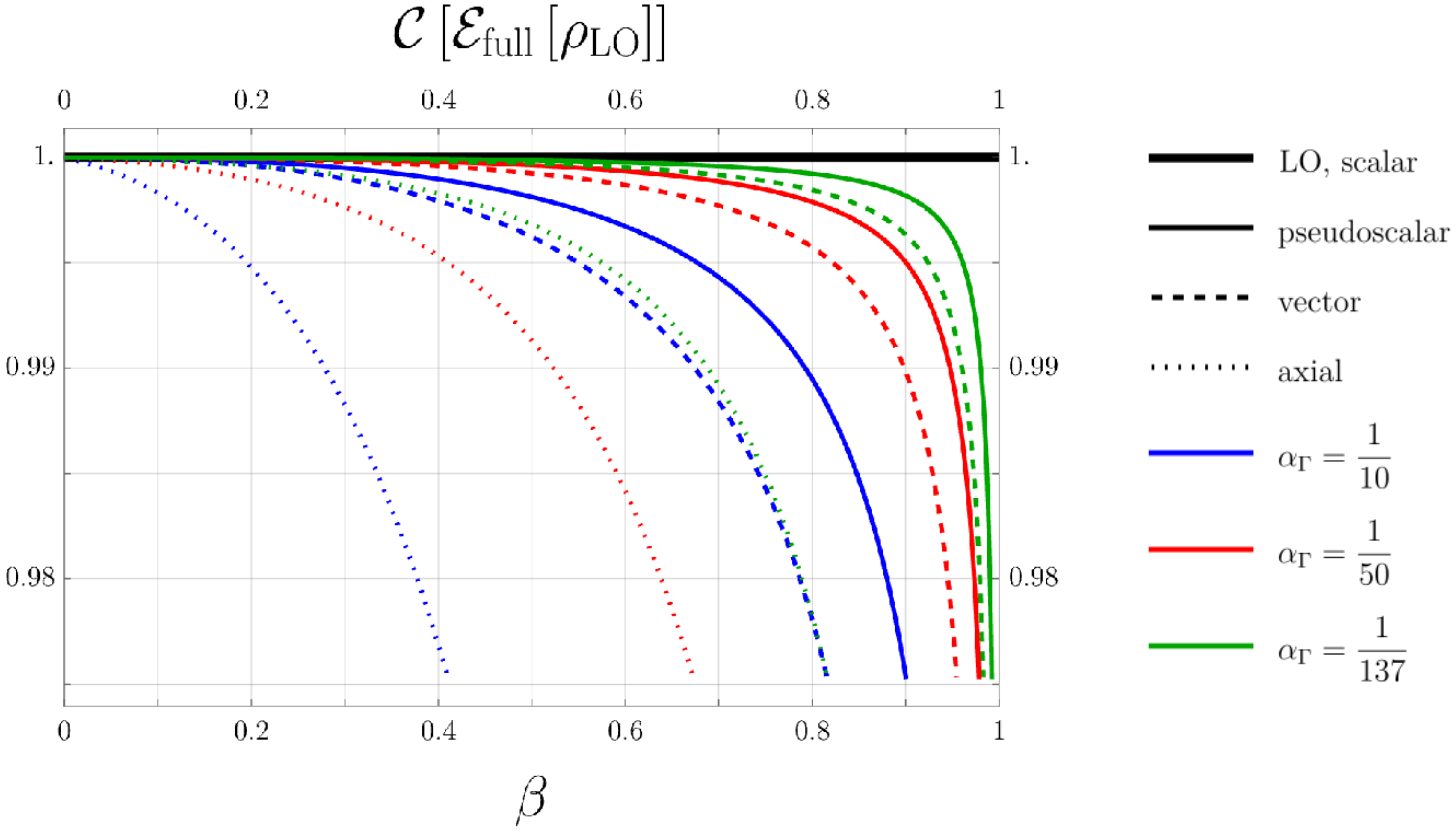
Identity part:
does not change entanglement

$$\mathsf{q} = \mathsf{q}^{\text{hard}} + \mathsf{q}_5^{\text{soft}}$$

Non-trivial Kraus part:
Decoherence!

*in the leading soft/collinear limit

Decoherence



$\beta = \sqrt{1 - 4m_f^2/m_\varphi^2}$

Entanglement is lost mainly due to collinear emission ⟶ Small effects ~ 1%

Conclusions

- NLO corrections are still expected to be small
- Full pheno study required to know the exact impact (WIP)
- Three main effects drive the smallness of decoherence in ttbar

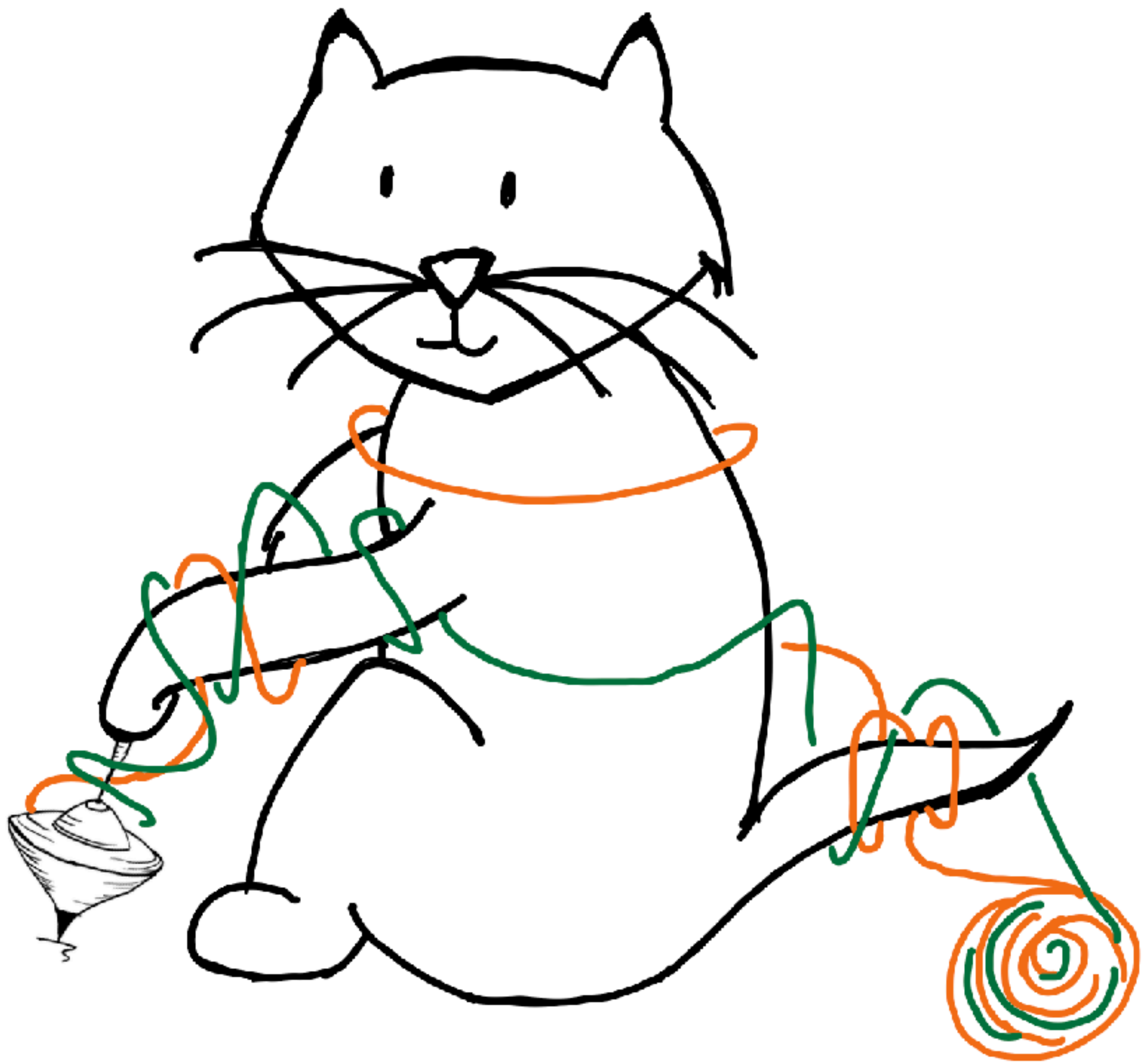
NLO: α is small

Leading soft radiation is a scalar function

Collinear emission: $1/m_t$

- Many new directions
    - Next-to-leading soft/collinear
    - Gravity?
    - Pheno study
    - Qutrits
    - …

Thank you!



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UKRI

IR Safe?

- IR cancellations are in the “identity part” of the map

  Cancel as in the cross-section/decay: KLN theorem

- By power counting, one can see that the identity part contains integrals of

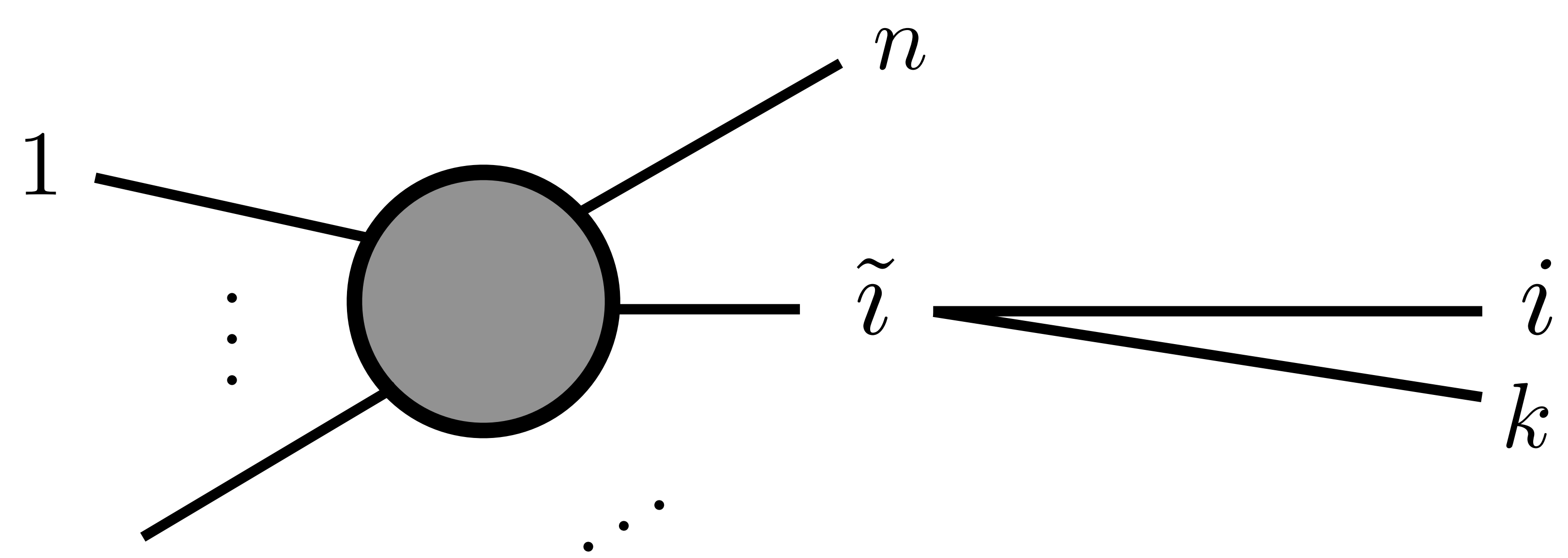
$$\int \mathrm{d}\Phi(k) \frac{1}{(p_i \cdot k)(p_j \cdot k)} \qquad i, j = 1, 2 \longrightarrow \text{IR divergent}$$

- While the non-trivial Kraus has (rank-n tensor integral)

$$\int \mathrm{d}\Phi(k) \frac{k^{\mu_1} \dots k^{\mu_n}}{(p_i \cdot k)(p_j \cdot k)} \qquad \text{IR finite for n > 0}$$

Collinear again…

In the collinear limit, when a n-parton system undergoes a splitting $\tilde{\imath} \to ik$



The amplitude factorize

$$\mathcal{M}_{n+1}^{\lambda_i \lambda_k}(\cdots, p_i, p_k, \cdots) = \mathcal{S}_{\tilde{\imath}\to ik}^{\lambda_{\tilde{\imath}}\lambda_i\lambda_k} \; \mathcal{M}_n^{\lambda_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)$$

→ Helicity dependent AP splitting function

used for spin correlations in Parton showers [Richardson, Webster ’18] [Hamilton, Karlberg, Salam, Scyboz, Verheyen ’21]

Splitting functions as Kraus operators

Density matrix before the splitting $\rho^{\lambda_{\tilde{\imath}}\lambda'_{\tilde{\imath}}} = \frac{1}{\mathcal{N}_i}\,\mathcal{M}^{\lambda_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)\;\overline{\mathcal{M}}^{\lambda'_{\tilde{\imath}}}(\cdots, p_{\tilde{\imath}}, \cdots)\,.$

After the splitting in the col. limit $\rho^{(\lambda_i\lambda_k)(\lambda'_i\lambda'_k)} = \left[\mathcal{S}^{\lambda_{\tilde{\imath}}\lambda_i\lambda_k}_{\tilde{\imath}\to ik}\right]\,\rho^{\lambda_{\tilde{\imath}}\lambda'_{\tilde{\imath}}}\,\left[\mathcal{S}^{\lambda'_{\tilde{\imath}}\lambda'_i\lambda'_k}_{\tilde{\imath}\to ik}\right]^{\dagger}\left(\frac{\mathcal{N}_i}{\mathcal{N}_{\tilde{\imath}k}}\right)$

Tracing over the unresolved d.o.f

$$\bar{\mathcal{E}}_{\mathrm{col}}[\rho] = \rho^{\lambda_i\lambda'_i}_{\mathrm{red}} = \sum_{\sigma=\pm}\int_{p_k}\mathcal{S}^{\lambda_{\tilde{\imath}}\lambda_i\sigma}_{\tilde{\imath}\to ik}\cdot\rho^{\lambda_{\tilde{\imath}}\lambda'_{\tilde{\imath}}}\cdot\mathcal{S}^{\lambda'_{\tilde{\imath}}\lambda'_i\sigma}_{\tilde{\imath}\to ik} = \mathsf{q}^{\mathrm{hard}}\sum_{j\neq\mathrm{id}}K_j\rho_{\mathrm{LO}}K_j^{\dagger}$$

Splitting functions as Kraus (here: one emission)