Heavy Meson Lifetimes using Gradient Flow

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P H
Particle Physics Phenomenology after the Higgs Discovery

DFG

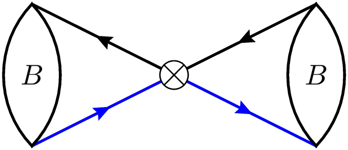
Lifetimes: Theory formulation

- Heavy quark expansion

$$\Gamma(B \to X) = \sum_i \Gamma_i \langle B|O_i|B\rangle$$

Contribution from $\Delta B = 0$ four quark operators

$$\mathcal{Q}_1 = \left(\bar{b}\gamma_\mu P_L q\right)\left(\bar{q}\gamma_\mu P_L b\right)$$
$$\mathcal{Q}_2 = \left(\bar{b} P_L q\right)\left(\bar{q} P_R b\right)$$
$$\mathcal{T}_1 = \left(\bar{b}\gamma_\mu P_L t^a q\right)\left(\bar{q}\gamma_\mu P_L t^a b\right)$$
$$\mathcal{T}_2 = \left(\bar{b} P_L t^a q\right)\left(\bar{q} P_R t^a b\right)$$



All Feynman diagrams were drawn using FeynGame

Lifetimes: Theory formulation

- Heavy quark expansion

$$\Gamma(B \to X) = \sum_i \Gamma_i \langle B|O_i|B\rangle$$

Contribution from $\Delta B = 0$ four quark operators

- Wilson coefficients Γ_i perturbative

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_i^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma_i^{(2)} + \dots$$

- Matrix elements $\langle B|O_i|B\rangle$ non-perturbative

- No up-to-date lattice results

Difficulties on the lattice

- Operators mix under renormalization

$$O_i = \sum_j Z_{ij} O_j^B$$

- Proper combination of different regulators

$$\Gamma(B \to X) = \sum_i \Gamma_i(\mu) \langle B|O_i|B\rangle (\mu)$$

Possible solution: Gradient flow

The QCD gradient flow

- QCD extended by flow time τ
  [Lüscher 2010]

$$B_\mu(\tau = 0, x) = A_\mu(x)$$

- Flow equations describe $\tau > 0$ behavior

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu}$$

  flowed field strength tensor

$$G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + f^{abc} B^b_\mu B^c_\nu$$

- Note that $[\tau] = -2$

Gradient flow renormalization

- Solution of the flow equations leads to **exponential damping factors**

s, ν, b  t, μ, a (momentum p) $\qquad \sim e^{-(t+s)p^2}$

matrix elements of flowed operators are finite

[Lüscher, Weisz 2011; Hieda, Makino, Suzuki 2017]

- QCD limit $\tau \to 0$?

The short-flow-time expansion

- Short-flow-time expansion

$$\tilde{O}_i(\tau) \sim \sum_j \zeta_{ij}(\tau) O_j + \tau \ldots \qquad \Rightarrow \qquad O_i \sim \sum_j \zeta_{ij}^{-1}(\tau) \tilde{O}_j(\tau) + \tau \ldots$$

$$\langle B | O_i | B \rangle (\mu) = \zeta_{ji}^{-1}(\mu, \tau) \langle B | \tilde{O}_i | B \rangle (\tau)$$

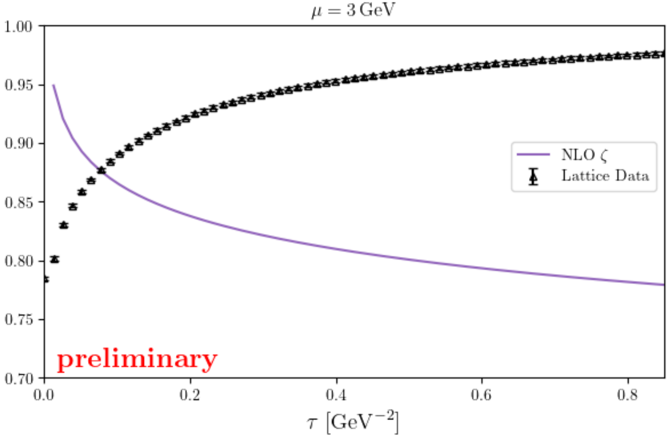
- Mixing shifted to the perturbative calculation

Setup for the calculation

- `qgraf` [Nogueira 1991]
- `tapir` [Gerlach, Herren, Lang 2022]
- `exp` [Harlander, Seidensticker, Steinhauser 1998, Seidensticker 1999]
- `FORM` [Vermaseren 1989]
- `Kira` [Maierhöfer, Usovitsch, Uwer 2017; Klappert, Lange, Maierhöfer, Usovitsch 2020]
- `FireFly` [Klappert, Lange 2019], [Klappert, Klein, Lange 2020]
- `ftint` [Harlander, Nellopoulos, Olsson, Wesle 2024]
  - `pySecDec` [Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke 2017]

Lattice determination

$$\langle B|O_i|B\rangle(\mu) = \zeta_{ji}^{-1}(\mu,\tau)\,\langle B|\tilde{O}_i|B\rangle(\tau)$$



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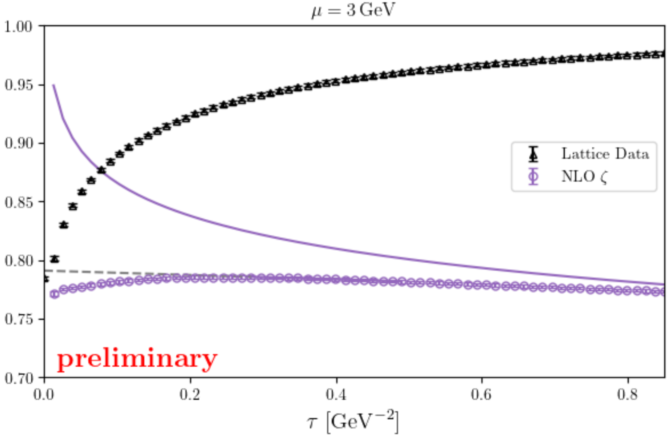

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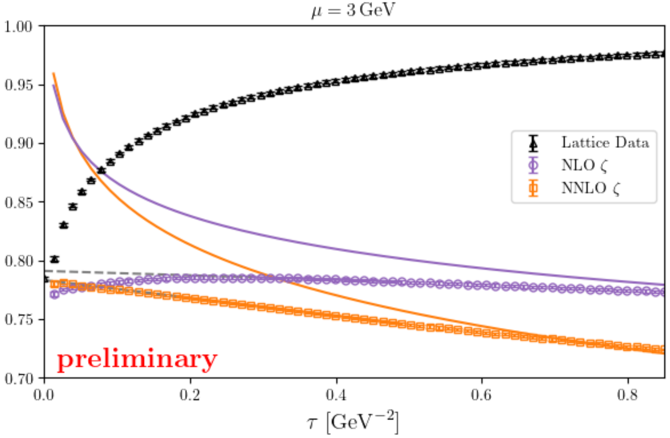
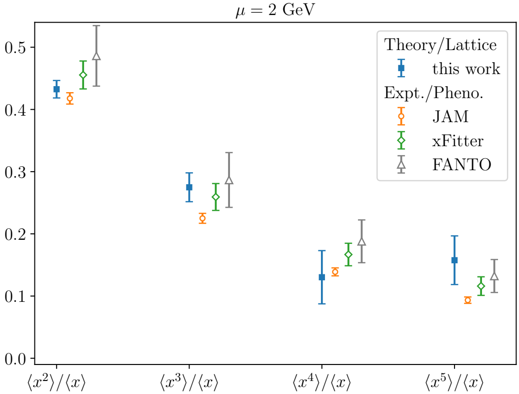


Conclusion and Outlook

- Lifetimes of mesons with an heavy quark calculated with HQE
- Gradient flow allows lattice calculation of non-perturbative $\Delta B = 0$ matrix elements
- Perturbative matching necessary
- First results for D mesons $\rightarrow$ Future extension to B mesons
- Inclusion of Eye Diagrams $\rightarrow$ penguin operators

Other gradient flow applications

- Energy momentum tensor [Makino, Suzuki 2014]
- Parton distribution functions [Shindler 2023]

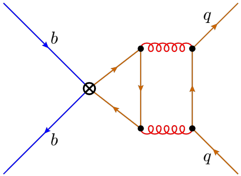


Moments of PDFs using the gradient flow. Figure from [Francis, . . . , JK, . . . 2025]

Penguin operators

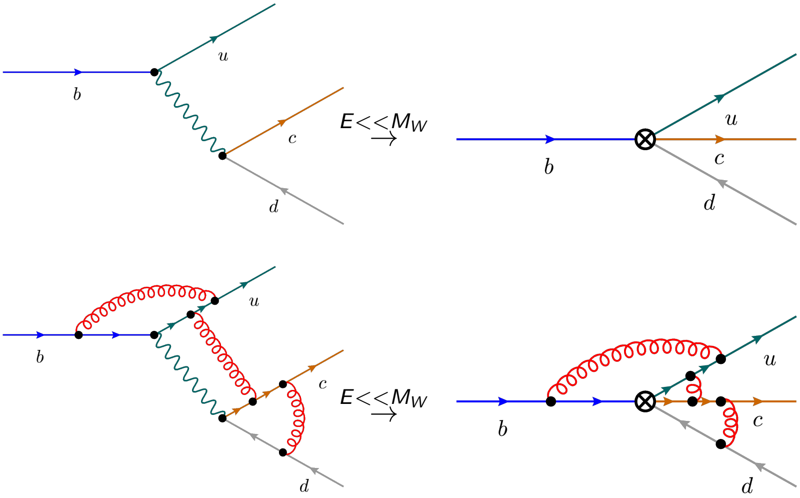
- Closed fermion loops introduce penguin operators

$$\mathcal{Q} = \left(\bar{b}\Gamma q\right)\left(\bar{q}\Gamma b\right) \qquad \mathcal{P}_Q = \left(\bar{b}\Gamma b\right)\sum_q\left(\bar{q}\Gamma q\right)$$



- Lower dimensional operators
- Extends the operator basis

The effective Hamiltonian



$$\mathcal{H}_{eff} = C(\bar{b}\Gamma_\mu u)(\bar{c}\Gamma^\mu d) + \dots$$

Optical theorem

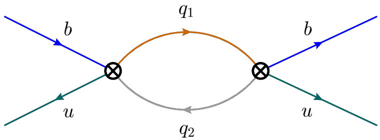
- Optical theorem

$$\Gamma(B \to X) = \frac{1}{2m_B} \langle B|\mathcal{T}|B\rangle$$

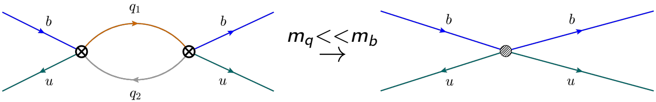
- Transition operator

$$\mathcal{T} = \mathrm{Im}\left\{ i \int d^4x T[\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)] \right\}$$

- Double insertions



The heavy quark expansion



- Operator product expansion

$$\Gamma(B \to X) = \ldots + \Gamma_6 \langle B|(\bar{b}\Gamma u)(\bar{u}\Gamma b)|B\rangle + \ldots$$

- $\Delta B = 0$ Operators

Preliminary results

$$\mathcal{O}_i = \zeta_{ij}^{-1}\tilde{\mathcal{O}}_j + \tau \ldots$$

$$\begin{aligned}
\zeta_{22}^{-1} = 1 &+ \left(\frac{\alpha_s}{\pi}\right)\left(\frac{4}{3} + 4L_{\mu t}\right) \\
&+ \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{2291}{72} + \frac{7}{4}\zeta_2 + \frac{397}{9}\ln(2) - \frac{89}{2}\ln(3) - \frac{26}{9}\ln^2(2)\right. \\
&- \frac{47}{2}\mathrm{Li}_2\left(\frac{1}{4}\right) - n_\mathrm{f}\left(\frac{10}{9} + \frac{1}{3}\zeta_2\right) + L_{\mu t}\left(\frac{425}{12} - \frac{52}{9}\ln(2) - \frac{10}{9}n_\mathrm{f}\right) \\
&\left. + L_{\mu t}^2\left(\frac{27}{2} - \frac{1}{3}n_\mathrm{f}\right)\right] + \mathcal{O}\left(\alpha_s^3\right)
\end{aligned}$$

where $L_{\mu t} = \ln(2\mu^2 t) + \gamma_E$

Evanescent operators 1 loop

$$\mathcal{E}^{(1)}_{Q_1} = \left(\bar{q}_1 \gamma_{\mu\nu\rho} P_L q_2\right) \left(\bar{q}_3 \gamma_{\mu\nu\rho} P_L q_4\right) - 16\mathcal{Q}_1$$
$$\mathcal{E}^{(1)}_{Q_2} = \left(\bar{q}_1 \gamma_{\mu\nu} P_L q_2\right) \left(\bar{q}_3 \gamma_{\mu\nu} P_L q_4\right) - 4\mathcal{Q}_2$$
$$\mathcal{E}^{(1)}_{T_1} = \left(\bar{q}_1 \gamma_{\mu\nu\rho} P_L t^a q_2\right) \left(\bar{q}_3 \gamma_{\mu\nu\rho} P_L t^a q_4\right) - 16\mathcal{T}_1$$
$$\mathcal{E}^{(1)}_{T_2} = \left(\bar{q}_1 \gamma_{\mu\nu} P_L t^a q_2\right) \left(\bar{q}_3 \gamma_{\mu\nu} P_L t^a q_4\right) - 4\mathcal{T}_2$$

Evanescent operators 2 loop

$$\mathcal{E}^{(2)}_{Q_1} = \left(\bar{q}_1\gamma_{\mu\nu\rho\sigma\tau}P_L q_2\right)\left(\bar{q}_3\gamma_{\mu\nu\rho\sigma\tau}P_L q_4\right) - 256\mathcal{Q}_1$$
$$\mathcal{E}^{(1)}_{Q_2} = \left(\bar{q}_1\gamma_{\mu\nu\rho\sigma}P_L q_2\right)\left(\bar{q}_3\gamma_{\mu\nu\rho\sigma}P_L q_4\right) - 16\mathcal{Q}_2$$
$$\mathcal{E}^{(1)}_{T_1} = \left(\bar{q}_1\gamma_{\mu\nu\rho\sigma\tau}P_L t^a q_2\right)\left(\bar{q}_3\gamma_{\mu\nu\rho\sigma\tau}P_L t^a q_4\right) - 256\mathcal{T}_1$$
$$\mathcal{E}^{(1)}_{T_2} = \left(\bar{q}_1\gamma_{\mu\nu\rho\sigma}P_L t^a q_2\right)\left(\bar{q}_3\gamma_{\mu\nu\rho\sigma}P_L t^a q_4\right) - 16\mathcal{T}_2$$

Evanescent Operators scheme

- Evanescent operators not uniquely defined

$$E_i' = E_i + \varepsilon a_i^{(1)} + \varepsilon^2 a_i^{(2)} + \dots$$

- Values of $a_i^{(n)}$ define scheme of evanescent operators
- Our calculation in general scheme
- Scheme of ζ_{ji}^{-1} must be the same as scheme of wilson coefficient Γ_j

$$\tilde{\Gamma}_i = \Gamma_j \zeta_{ji}^{-1}$$

- Scheme dependence cancels

Calculating the mixing matrix

- It is possible to construct projectors $P_n[X]$ so that

  [Gorishny, Larin, Tkachov 1983; Gorishny, Larin 1986]

$$P_n[\mathcal{O}_m] = \delta_{nm}$$

  holds to all orders in perturbation theory
- Using these projectors on the flowed operators

  [Harlander, Kluth, Lange 2019]

$$\tilde{\mathcal{O}}_n(t) \approx \sum_m \xi^B_{nm} \mathcal{O}^B_m + \ldots$$

  leads to

$$\boxed{P_n[\tilde{\mathcal{O}}_m] = \xi^B_{mn}}$$

Form of the Projectors

$$P_n[\mathcal{O}_m] = \delta_{nm}$$

- The projectors have the general form

$$P_n[X] = \sum_k \Pi_k(\partial_p, \partial_m) \langle f_k|X|i_k\rangle \Big|_{p=m=0}$$

- Because all scales are set to zero this only has to hold at tree level

Flowed quark fields

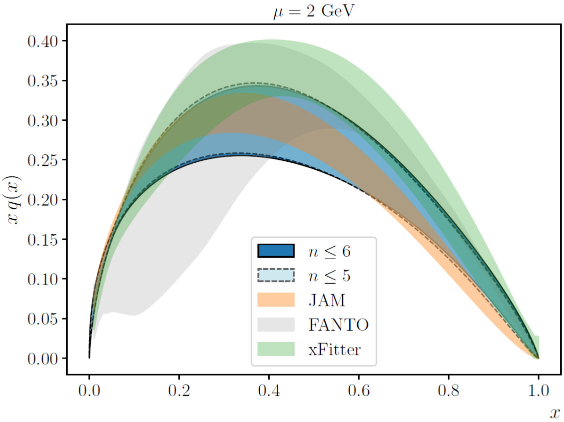
- The flow equations for the flowed quark field χ are

$$\partial_t \chi = \triangle \chi - \kappa \partial_\mu B^a_\mu T^a \chi,$$
$$\partial_t \overline{\chi} = \overline{\chi} \overleftarrow{\triangle} + \kappa \overline{\chi} \partial_\mu B^a_\mu T^a,$$

$$\chi^i(t=0, x) = \psi^i(x)$$

Reconstructed PDF

$$q(x) \sim x^{\alpha}(1-x)^{\beta}$$



For details see [Francis, . . . , JK, . . . 2025]