Dark Matter Aspects of Composite Higgs Models

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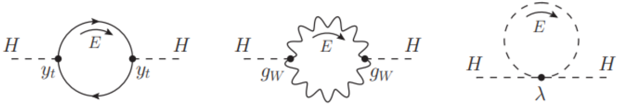
Overview

The Hierarchy Problem



$$\delta m_H^2 \approx \frac{3y_t^2}{8\pi^2}\Lambda_{SM}^2$$

- $\Lambda_{SM} \sim \mathrm{TeV}$ or excessive fine-tune
- Why $v_{\mathrm{EW}} \ll M_{\mathrm{pl}}$?

Composite Higgs (CH)

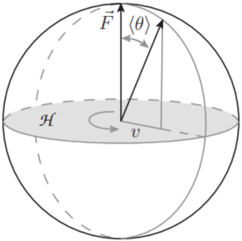


Figure: EWSB by vacuum misalignment [Panico, 1506.01961]

EWSB in CH

$$\begin{array}{ccc} & & \mathcal{G} \\ & & \vec{F}\big\downarrow \Rightarrow H \in \text{pNGBs} \\ SU(2)_L \times SU(2)_R & \in & \mathcal{H} \\ \big\downarrow \text{Higgs vev} & & \\ SU(2)_D & & W^{\pm}_{\mu}, Z_{\mu} \end{array}$$

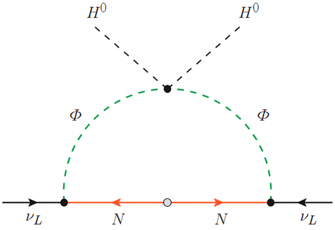
- ! Higgs: a pseudo Nambu-Goldstone-Boson (pNGB) $\in \mathcal{G}/\mathcal{H}$
- ! Custodial symmetry $SU(2)_L \times SU(2)_R \in \mathcal{H}$
- ➤ Minimal coset SU(4)/Sp(4)

pNGBs		
$SU(2)^2$	$SU(2)_D$	name
(2,2)	3	φ
	1	H
(1,1)	1	η

Dark Matter (DM) and Neutrino Mass

- DM: $\sim 85\%$ of total matter content
- $m_\nu < 0.12\mathrm{eV}$

Scotogenic Models:



- A discrete $\mathbb{Z}_2$ symmetry
- Majorana fermion N $(1,1)_-$
- Scalar bi-doublet Φ $(2,2)_-$
- Loop suppression $\rightarrow$ reduced seesaw scale $M_N \sim$ TeV
- DM candidate: lightest $\mathbb{Z}_2$-odd particle

➥ **DM and natrually small neutrino mass simutaneously!**

pNGBs in the SU(6)/Sp(6) coset

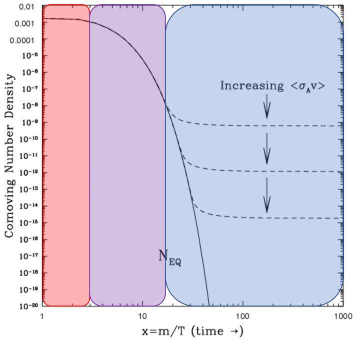
14 pNGBs in the SU(6)/Sp(6) coset:

	EW vacuum ($\theta = 0$) $SU(2)_L \times SU(2)_R$	CH vacuum ($\theta \neq 0$) $SU(2)_D$
$\mathbb{Z}_2$–even	$H = (2,2)_+$ $\eta_1 = (1,1)_+$ $\eta_2 = (1,1)_+$	$h = 1_+$ $\eta_1 = 1_+$ $\eta_2 = 1_+$
$\mathbb{Z}_2$–odd	$\Delta = (3,1)_-$ $\eta_3 = (1,1)_-$ $\Phi = (2,2)_-$	$\Delta^\pm, \Delta^0 = 3_-$ $\eta_3 = 1_-$ $\phi^\pm, \phi^0 = 3_-$ $\eta_4 = 1_-$

Relic Density: Freeze–Out

DM creation and annihilation: $\mathrm{SM} + \mathrm{SM} \leftrightarrow \mathrm{DM} + \mathrm{DM}$

Boltzmann Eq.: $\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{\mathrm{eq}}^2)$



- $n_{\mathrm{eq}} \propto (\frac{mT}{2\pi})^{3/2} e^{-m/T}$
- $Hn \gg \langle \sigma v \rangle n^2 \rightarrow n \propto a^{-3}$: freeze-out
- $\Omega_c h^2 = 0.1200$ in ΛCDM

Image credit: A. Green

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$$\lambda' s = f(\tilde{g}, g_{V\pi\pi}, \theta, M_V, M_A)$$

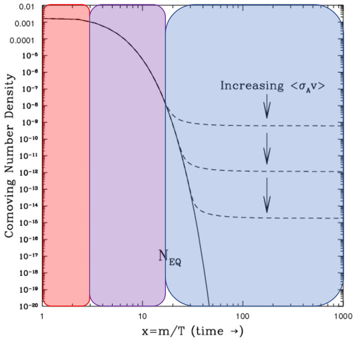

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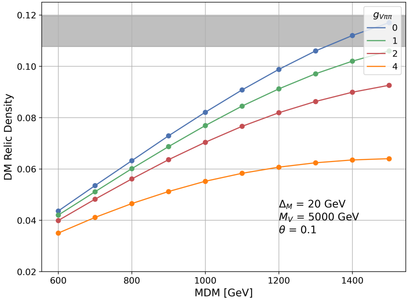
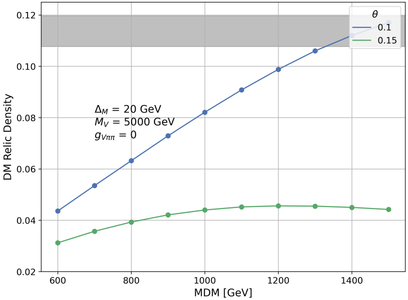
Image credit: A. Green

$\eta_3((1,1)_- \to 1_-)$ Freeze-Out Relic Density

M_{DM}: DM mass, here mass of η_3
$g_{V\pi\pi}$: Effective Vector-pNGB-pNGB coupling
θ: Misalignment angle



(a) Ω vs. M_{DM} for different $g_{V\pi\pi}$



(b) Ω vs. M_{DM} for different θ

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η_3 Freeze–Out Relic Density

Δ_M: Mass difference between other pNGBs and DM candidate
M_V: Heavy vector mass

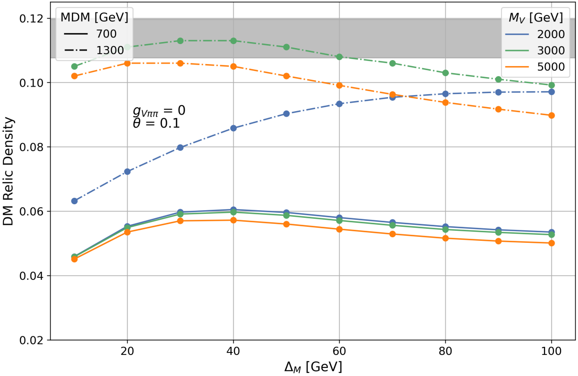

Figure: Ω vs. Δ_M for different M_V and M_{DM}

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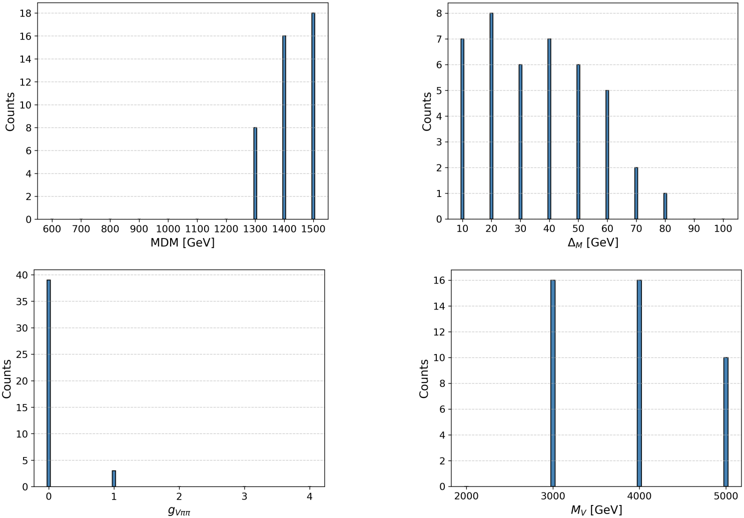
What the RD Calculation tells us

For the case of η_3 being the DM candidate:

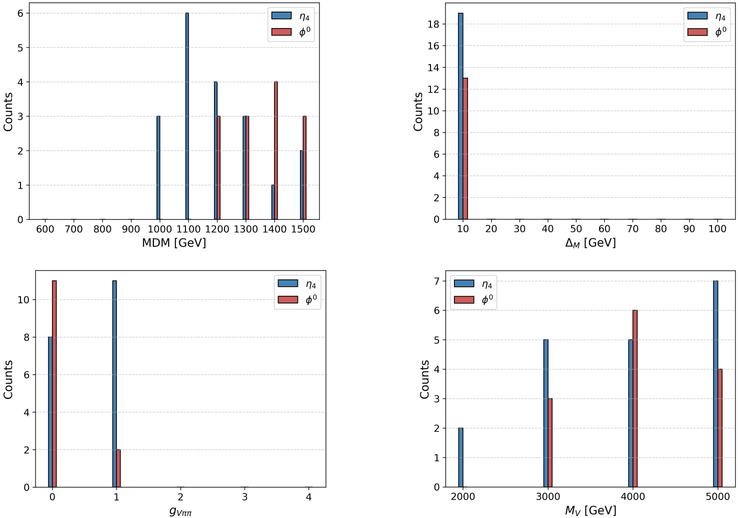


What the RD Calculation tells us

For $\Delta^0((3,1)_- \to 3_-)$: No viable points in parameter space found

What the RD Calculation tells us

For $\phi^0((2,2)_- \to 3_-)$ and $\eta_4((2,2)_- \to 1_-)$:



Conclusion

Observations:

- The unbroken Sp(6) subgroup contains a discrete $\mathbb{Z}_2$ symmetry
- Four potential DM candidates among pNGBs in the SU(6)/Sp(6) coset
- Δ^0 as a DM candidate does not work in the scaned region of parameter space
- Relic Density calculation favors large pNGB masses and the decoupling limit

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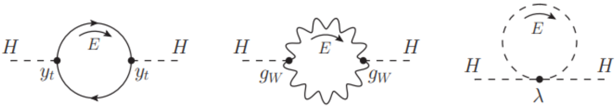
Next steps:

- Relevant signatures at LHC
- Identifying ingredients for a scotogenic setup
- Constraints from neutrino masses and EW precision tests

Thank you for your attention!

The Hierarchy Problem

An open question in particle physics:



$$\delta m_H^2 = \int_0^{\lesssim \Lambda_{\rm SM}} dE \, \frac{dm_H^2}{dE}(E; p_{\rm true}) + \int_{\lesssim \Lambda_{\rm SM}}^{\infty} dE \, \frac{dm_H^2}{dE}(E; p_{\rm true})$$
$$= \delta_{\rm SM} m_H^2 + \delta_{\rm BSM} m_H^2$$
$$\approx \frac{3 y_t^2}{8\pi^2} \Lambda_{\rm SM}^2 + ?$$

Composite Higgs (CH)

A lesson from QCD:

- **Dimensional transmutation**: QCD coupling runs logrithmically between two vastly separated scales $\ln\left(M_{pl}/\Lambda_{\mathrm{QCD}}\right) \sim 100$
- Below Λ_{QCD}: condensation and pions as **pNGB** from chiral symmetry breaking

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A lesson from QCD:

- **Dimensional transmutation**: QCD coupling runs logrithmically between two vastly separated scales $\ln\left(M_{pl}/\Lambda_{\rm QCD}\right) \sim 100$
- Below $\Lambda_{\rm QCD}$: condensation and pions as **pNGB** from chiral symmetry breaking

Composite Sector

$\mathcal{G} \xrightarrow{\text{SSB}} \mathcal{H}$	Eg. $\mathrm{SU}(4) \xrightarrow[m_*]{\langle\bar{\psi}\psi\rangle \neq 0} \mathrm{Sp}(4)$
$H \in \mathcal{G}/\mathcal{H}$	

$\mathcal{L}_{\rm int}$, $\not{\mathcal{G}}$

Elementary Sector

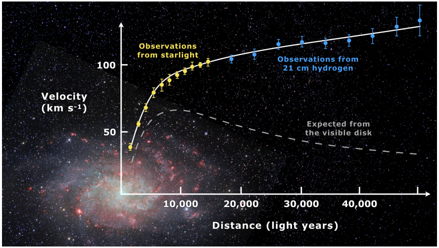
W^μ, B^μ

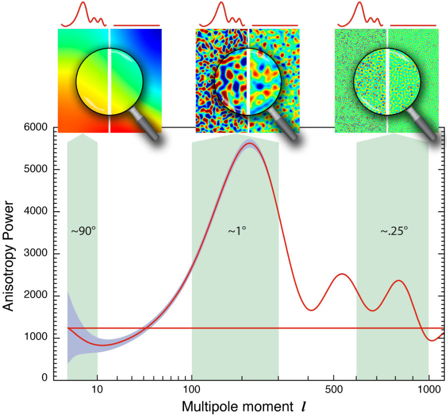
$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$

- $m_H <$ confinement scale m_*
- m_H insensitive to physics above $m_* \sim \mathrm{TeV}$
- Natural hierarchy between m_* and $\Lambda_{UV} \gg \mathrm{TeV}$

Appendix

Dark Matter (DM)





What we know about DM:

- ✓ interacting gravitationally
- ✓ $\sim$ 85% of total matter content
- ✓ cold: non-relativistic in early universe

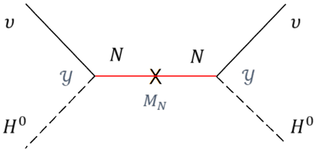
Image credit: NASA

Scotogenic Models

Weinberg operator: $\Lambda^{-1} H^0 H^0 \nu_i \nu_j$
Three realizations of seesaw mechanism by adding:

1. Fermion singlets (Majorana neutrinos) $\times (LH)_s$
2. Scalar triplet $\times (LL)_t$
3. Fermion triplets $\times (LH)_t$

Type I Seesaw:



- Majorana neutrino N with mass M_N
- $m_\nu \sim \kappa \frac{\langle H^0 \rangle^2}{M_N}$
- Typical seesaw scale $M_N \sim \mathcal{O}(10^9)\mathrm{GeV}$

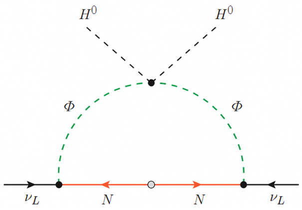
Appendix

Scotogenic Models

Weinberg operator: $\Lambda^{-1} H^0 H^0 \nu_i \nu_j$
Three realizations of seesaw mechanism by adding:

1. Fermion singlets (Majorana neutrinos) $\times (LH)_s$
2. Scalar triplet $\times (LL)_t$
3. Fermion triplets $\times (LH)_t$

$\Rightarrow$ Three types of scotogenic setups



$$\mathcal{L} \supset y_{na\alpha} \overline{N}_n \eta_a l_L^\alpha + \frac{1}{2} M_{N_n} \overline{N^c}_n N_n + \lambda \eta^2 H^2 + \text{h.c.}$$

$$m_\nu = \frac{\lambda v^2}{32\pi^2 M_N^2} y^T f_{\text{loop}} y$$

Embedding EW Symmetry in SU(6)

hyper-fermions	$SU(2)_L \times SU(2)_R$	$\mathbb{Z}_2$
$\Psi_1 \equiv (\psi_1\ \psi_2)^T$	(2,1)	+
$\Psi_2 \equiv (\psi_3\ \psi_4)^T$	(1,2)	+
$\Psi_3 \equiv (\psi_5\ \psi_6)^T$	(2,1)	-

- Three commuting SU(2) subgroups $\in$ Sp(6)
- $SU(2)_L = SU(2)_1 + SU(2)_3$
- $SU(2)_R = SU(2)_2$
- $\mathbb{Z}_2$ symmetry present

$$SU(2)_L : T_L^i = 1/2 \begin{pmatrix} \sigma^i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^i \end{pmatrix} ; \qquad SU(2)_R : T_R^i = 1/2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sigma_i^T & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Vacuum Misalignment

EW Vacuum:

$$\langle\psi\psi\rangle \sim \Sigma_0 = \begin{pmatrix} i\sigma_2 & 0 & 0 \\ 0 & -i\sigma_2 & 0 \\ 0 & 0 & i\sigma_2 \end{pmatrix}$$

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \in \mathrm{Sp}(6)$$
$$14 \text{ pNGBs} \in \mathrm{SU}(6)/\mathrm{Sp}(6)$$

pause

$$\xrightarrow[\text{Higgs vev}]{\text{explicit breakings}}$$

CH Vacuum:

$$\Sigma_\theta = \Omega(\theta)\cdot\Sigma_0\cdot\Omega^T(\theta) = \begin{pmatrix} i\sigma_2\cos\theta & \mathbb{I}_2\sin\theta & 0 \\ -\mathbb{I}_2\sin\theta & -i\sigma_2\cos\theta & 0 \\ 0 & 0 & i\sigma_2 \end{pmatrix}$$

$\Omega(\theta) = \exp\left(\sqrt{2}i\theta X^H\right)$: rotation induced by Higgs vev

$$\mathrm{SU}(2)_V,\ W_\mu^\pm,\ Z_\mu$$

Heavy Spin-1 Resonances

Hidden Symmetry Formulism

$$
\begin{tikzcd}
\mathcal{G}_0 \arrow[d, "\pi_0"] & & \mathcal{G}_1 \arrow[d, "\pi_1"] \\
\mathcal{H}_0 \arrow[dr] & & \mathcal{H}_1 \arrow[dl] \\
& \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \in \mathcal{H}_\mathrm{v} \arrow[d, "\mathrm{EWSB}"] & \\
& \mathrm{SU}(2)_V &
\end{tikzcd}
$$

$\pi_\mathrm{p}, \cancel{\pi_\mathrm{u}} \to \mathcal{A}_\mu$

$\pi_\mathrm{p}, \mathcal{A}_\mu$

$\cancel{K} \to \mathcal{V}_\mu$

$\pi_\mathrm{p}, \mathcal{A}_\mu, \mathcal{V}_\mu$

$\pi_\mathrm{p} - 3, W^\pm_\mu, Z_\mu, \mathcal{A}_\mu, \mathcal{V}_\mu$

$$n_\mathrm{Axi} = \dim[\mathcal{G}] - \dim[\mathcal{H}]$$
$$n_\mathrm{Vec} = \dim[\mathcal{G}] - n_\mathrm{Axi}$$

Heavy Spin–1 Resonances

$$\psi\bar{\psi}: \quad \mathbf{N} \otimes \bar{\mathbf{N}} = \mathrm{Adj.} \oplus \mathbf{1}$$
$$\mathbf{35}_{SU(6)} = \mathbf{14}_{Sp(6)} + \mathbf{21}_{Sp(6)}$$

	$\mathcal{V}_\mu$ name	$\mathcal{V}_\mu$ $SU(2)^2$	$\mathcal{V}_\mu$ $SU(2)_V$	$\mathcal{A}_\mu$ name	$\mathcal{A}_\mu$ $SU(2)^2$	$\mathcal{A}_\mu$ $SU(2)_V$
$\mathbb{G}_0/\mathbb{H}_0$	$v_{1\mu}$	$(3,1)\oplus(1,3)$	3	$a_{1\mu}$	(2,2)	3
	$v_{2\mu}$	$(3,1)\oplus(1,3)$	3	$y_{1\mu}$		1
	$r_{1\mu}$	(2,2)	3	$y_{2\mu}$	(1,1)	1
	$x_{1\mu}$		1			
$\mathbb{Z}_2$–odd	$r_{2\mu}$	(3,1)	3	$a_{2\mu}$	(3,1)	3
	$x_{2\mu}$	(1,1)	1	$y_{4\mu}$	(1,1)	1
	$r_{3\mu}$	(2,2)	3	$a_{3\mu}$	(2,2)	3
	$x_{3\mu}$		1	$y_{5\mu}$		1
$\mathbb{Z}_2$–even	$v_{3\mu}$	(3,1)	3	$y_{3\mu}$	(1,1)	1

Mixing with the EW Bosons

$$\mathcal{M}_N^2 = \begin{pmatrix} \frac{g'^2 M_V^2(1+ws_\theta^2)}{\tilde{g}^2} & -\frac{gg' M_V^2 ws_\theta^2}{\sqrt{2}\tilde{g}} & -g'\frac{rM_A^2 s_\theta}{\sqrt{2}\tilde{g}} & -g'\frac{M_V^2}{\sqrt{2}\tilde{g}} & -g'\frac{M_V^2 c_\theta}{\sqrt{2}\tilde{g}} & 0 \\ -\frac{gg' M_V^2 ws_\theta^2}{\sqrt{2}\tilde{g}} & \frac{g^2 M_V^2(2+ws_\theta^2)}{\tilde{g}^2} & g\frac{rM_A^2 s_\theta}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2 c_\theta}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2}{\tilde{g}} \\ -g'\frac{rM_A^2 s_\theta}{\sqrt{2}\tilde{g}} & g\frac{rM_A^2 s_\theta}{\sqrt{2}\tilde{g}} & M_A^2 & 0 & 0 & 0 \\ -g'\frac{M_V^2}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2}{\sqrt{2}\tilde{g}} & 0 & M_V^2 & 0 & 0 \\ -g'\frac{M_V^2 c_\theta}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2 c_\theta}{\sqrt{2}\tilde{g}} & 0 & 0 & M_V^2 & 0 \\ 0 & -g\frac{M_V^2}{\tilde{g}} & 0 & 0 & 0 & M_V^2 \end{pmatrix} \begin{pmatrix} B_\mu \\ \widetilde{W}_\mu^3 \\ a_{1\mu}^0 \\ v_{1\mu}^0 \\ v_{2\mu}^0 \\ v_{3\mu}^0 \end{pmatrix}$$

Mixing with the EW Bosons

$$\mathcal{M}_C^2 = \begin{pmatrix} \frac{g^2 M_V^2 (2+ws_\theta^2)}{\tilde{g}^2} & -g\frac{rM_A^2 s_\theta}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2 c_\theta}{\sqrt{2}\tilde{g}} & -g\frac{M_V^2}{\tilde{g}} \\ -g\frac{rM_A^2 s_\theta}{\sqrt{2}\tilde{g}} & M_A^2 & 0 & 0 & 0 \\ -g\frac{M_V^2}{\sqrt{2}\tilde{g}} & 0 & M_V^2 & 0 & 0 \\ -g\frac{M_V^2 c_\theta}{\sqrt{2}\tilde{g}} & 0 & 0 & M_V^2 & 0 \\ -g\frac{M_V^2}{\tilde{g}} & 0 & 0 & 0 & M_V^2 \end{pmatrix}, \begin{pmatrix} \widetilde{W}_\mu^+ \\ a_{1\mu}^+ \\ v_{1\mu}^+ \\ v_{2\mu}^+ \\ v_{3\mu}^+ \end{pmatrix} = \mathcal{C} \begin{pmatrix} W_\mu^+ \\ A_{1\mu}^+ \\ V_{1\mu}^+ \\ V_{2\mu}^+ \\ V_{3\mu}^+ \end{pmatrix}$$

INTERACTIONS

Maurer-Cartan Forms:

$$
\begin{aligned}
iU_1^\dagger D_\mu U_1 &= iU_1^\dagger(\partial_\mu - i\hat{g}\tilde{W}^i_\mu T^i_L - i\hat{g}' B_\mu T^3_R)U_1 \\
iU_2^\dagger D_\mu U_2 &= iU_2^\dagger(\partial_\mu - i\tilde{g}\mathcal{V}^i_\mu \tilde{S}^i - i\tilde{g}' \mathcal{A}^I_\mu \tilde{X}^I)U_2
\end{aligned}
$$

Building blocks:

$$
d_{i,\mu} \to d'_{i,\mu} = h(g_i, \pi_i) d_{i,\mu} h^\dagger(g_i, \pi_i), \tag{1}
$$

$$
e_{i,\mu} \to e'_{i,\mu} = h(g_i, \pi_i)(e_{i,\mu} + i\partial_\mu) h^\dagger(g_i, \pi_i) \tag{2}
$$

CCWZ Lagrangian:

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{4}\mathrm{Tr}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{1}{4}\mathrm{Tr}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{1}{4}\mathrm{Tr}\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu} \\
&+ \frac{f_0^2}{4}\mathrm{Tr}d_{0\mu}d_0^\mu + \frac{f_1^2}{4}\mathrm{Tr}d_{1\mu}d_1^\mu + \frac{rf_1^2}{2}\mathrm{Tr}d_{0\mu}Kd_1^\mu K^\dagger + \frac{f_K^2}{4}\mathrm{Tr}D^\mu K(D_\mu K)^\dagger.
\end{aligned}
$$

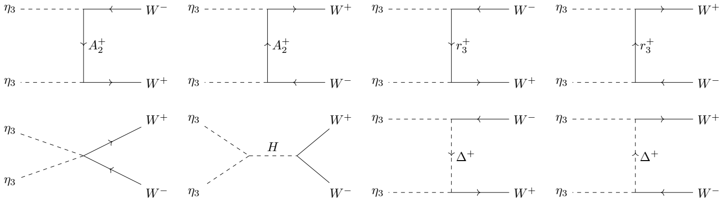
INTERACTIONS

$$
\begin{aligned}
\mathrm{i}U_i^\dagger D_\mu U_i \approx & i(\mathbb{I}_6 - \frac{i\sqrt{2}}{f_i}\Pi - \frac{1}{f_i^2}\Pi^2)(\partial_\mu - i\mathcal{F}_\mu)(\mathbb{I}_6 + \frac{i\sqrt{2}}{f_i}\Pi - \frac{1}{f_i^2}\Pi^2) \\
= & - i\mathcal{F}_\mu \\
& + \frac{1}{f_i}(i\sqrt{2}\partial_\mu\Pi + \sqrt{2}\mathcal{F}_\mu\Pi - \sqrt{2}\Pi\mathcal{F}_\mu) \\
& + \frac{1}{f_i^2}(i\mathcal{F}_\mu\Pi^2 + i\Pi^2\mathcal{F}_\mu - 2i\Pi\mathcal{F}_\mu\Pi) \\
& + \mathcal{O}(\frac{1}{f_i^3})
\end{aligned}
$$

$$\Rightarrow V_\mu V^\mu (\rightarrow \text{ vector mixing}) + \pi V_\mu V^\mu + \pi\partial_\mu\pi V^\mu + \pi\pi V_\mu V^\mu$$

An Example: $\eta_3 + \eta_3 \to W^+ + W^-$

η_3 :
$(1,1)_- \to 1_-$



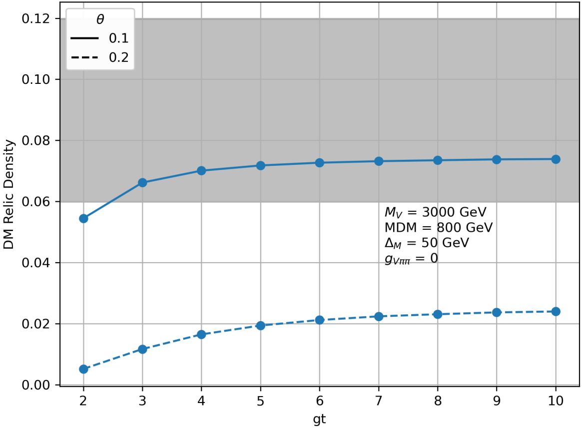
From the kinematic part:

$$\lambda_1 \cdot \eta_3 A_2^{\pm,\mu} W_\mu^\mp + \lambda_2 \cdot \eta_3 r_3^{\pm,\mu} W_\mu^\mp$$
$$+\lambda_3 \cdot (\partial_\mu \eta_3 \Delta^\pm - \partial_\mu \Delta^\pm \eta_3) W_\mu^\mp$$
$$+\lambda_4 \cdot \eta_3 \eta_3 W_\mu^+ W^{-,\mu}$$

$$\lambda\prime s = f(\tilde{g}, g_{V\pi\pi}, \theta, M_V, M_A)$$

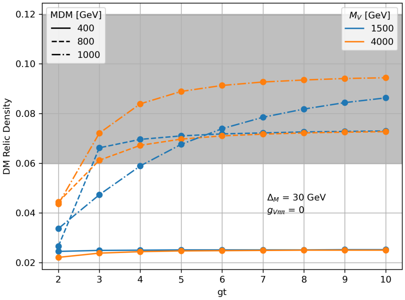
From the scalar potential: $c_1 v_{\mathrm{EW}} \cdot \eta_3 \eta_3 H + c_1 \cdot \eta_3 \eta_3 HH$

η_3 Freeze–Out Relic Density

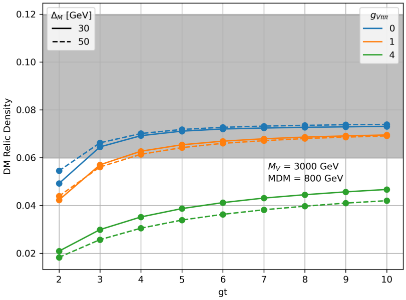


η_3 Freeze-Out Relic Density

$\tilde{g}$: "gauge coupling" of the composite sector



(a) Ω vs. $\tilde{g}$ for different m_{DM} and M_V



(b) Ω vs. $\tilde{g}$ for different Δ_M and $g_{V\pi\pi}$

η_4 Freeze–Out Abundance