Effective Field Theories for Higgs Sector Extensions — when SMEFT is not enough

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Motivation

Why we need BSM physics

The SM cannot explain

- dark Matter
- matter – antimatter asymmetry
- gravity
- neutrino masses
- ...

Why we have not found BSM physics

Maybe BSM physics is

- too heavy to be found at colliders
- very weakly coupled to the SM
- not a particle
- ...

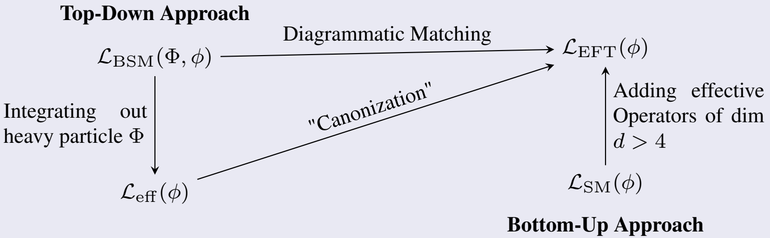
The role of Effective Field Theories (EFTs)

If BSM physics contains a heavy particle,
EFTs can be used to search for traces at collider-accessible energies, providing

- a largely model-agnostic bottom-up approach
- a systematic separation of scales
- a framework to consider classes of theories
- connection to BSM models via top-down matching

Effective Field Theory

The EFT pipeline



$$\mathcal{L}_{\mathrm{EFT}}(\phi) = \mathcal{L}_{\mathrm{SM}}(\phi) + \sum_{d>4}^{n} \frac{C_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}(\phi) + \mathcal{O}(\Lambda^{3-n})$$

What we would like to have

- fully automated matching
- global fit of the $C_i^{(d)}$

Challenges

- e.g. 2499 $C_i^{(6)}$ in SMEFT
- choosing the right BSM $\rightarrow$ EFT limit
- renormalization
- canonization can involve field redefinitions & IBP

Related Work

Functional matching

- Early work [Chan, 1986; Gaillard, 1986; Cheyette, 1988]
- Heavy Higgs in SM (non-linear repr.) [Dittmaier, Grosse-Knetter, 1995; Dittmaier, Grosse-Knetter, 1996]
- Recent revival [Henning, Lu, Murayama, 2016; Boggia, Gomez-Ambrosio, Passarino, 2016; Fuentes-Martin, Portoles, Ruiz-Femenia, 2016; Buchalla et al., 2017]
- Automation [Criado, 2018; Fuentes-Martin et al., 2021; Cohen, Lu, Zhang, 2021]

More general on EFTs

- HEFT vs. SMEFT [Alonso, Jenkins, Manohar, 2016; Cohen et al., 2021]
- Proper choice of EFT limit [Boggia, Gomez-Ambrosio, Passarino, 2016; Dittmaier, SS, Stahlhofen, 2021; Dawson et al., 2023]
- Higgs singlet extension [Buchalla et al., 2017; Ellis et al., 2017; Jiang et al., 2019; Haisch et al., 2020]

Our work [Dittmaier, SS, Stahlhofen, 2021; more in preparation]

- Integrating out fields of mass eingenstates $\rightarrow$ mixing
- Non-linear Higgs representation
- Renormalization of BSM sector
- Validation against full NLO BSM predictions for observables

The EFT Limit

EFT power counting

$$\mathcal{L}_{\text{BSM}}(\Phi, \phi, \mathcal{C}, c) \xrightarrow[\text{EFT limit: } \zeta \ll 1]{\text{Matching}} \mathcal{L}_{\text{SM}}(\phi, c) + \sum_{d>4}^{n} \zeta^{d-4} C_i^{(d)}(\mathcal{C}', c) \mathcal{O}_i^{(d)}(\phi) + \mathcal{O}(\zeta^{n-3})$$

Φ	BSM Fields	ϕ	SM Fields	$\zeta = \frac{v}{\Lambda}$	$\mathcal{C}_i \sim \zeta^{z_i}$	for $\zeta \ll 1$
$\mathcal{C}$	BSM Parameters	c	SM Parameters		$c \sim \zeta^0$	for $\zeta \ll 1$

Defining a reasonable EFT limit

- decoupling $\lim_{\zeta \to 0} \mathcal{L}_{\text{BSM}}(\Phi, \phi, \mathcal{C}, c) = \mathcal{L}_{\text{SM}}(\phi, c)$
- independence of SM input parameters from the heavy scale $c \sim \zeta^0$ for $\zeta \ll 1$
- BSM parameters $\mathcal{C}_i \sim \zeta^{z_i} \Rightarrow$ choice of z_i defines EFT limit
  $\hookrightarrow$ different choices can lead to different EFTs
  $\hookrightarrow$ it is crucial to specify the EFT limit via the z_i for all input parameters
- the renormalization scheme for the $c, \mathcal{C}$ needs to respect the EFT limit
  [Dittmaier, SS, Stahlhofen, 2021]
- $\mathcal{C}_i \sim \Lambda$ (e.g. heavy mass) $\Leftrightarrow z_i = -1$

Functional Matching: Integrating out heavy Degrees of Freedom

Basic idea

$$\int \mathcal{D}\phi \int \mathcal{D}\Phi \exp\left[\mathrm{i}\int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{BSM}}(\Phi, \phi)\right] = \int \mathcal{D}\phi \exp\left[\mathrm{i}\int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{eff}}(\phi)\right]$$

We use

- Background Field Method [DeWitt, 1967; Abbott, 1981], field $\to$ background field + quantum field: $\varphi \to \hat{\varphi} + \varphi$.
- Separation of Modes implements Method of Regions [Beneke, Smirnov, 1998], field $\to$ hard modes + soft modes: $\varphi \to \varphi_h + \varphi_s$.

Eliminating the heavy field(s)

$$\mathcal{L}_{\mathrm{BSM}}(\Phi, \phi) \longrightarrow \mathcal{L}_{\mathrm{BSM}}(\hat{\Phi}_h + \Phi_h + \hat{\Phi}_s + \Phi_s)$$

- $\mathcal{L}_{\mathrm{BSM}}(\Phi, \phi)$ — Integrating out Φ — $\mathcal{L}_{\mathrm{eff}}(\phi)$
- $\hat{\Phi}_h$ — Drop — 0
- Φ_h — Integrating out — $\delta\mathcal{L}_{\mathrm{eff}}^{1-\mathrm{loop}}$
- $\hat{\Phi}_s$, Φ_s — Elimination via EOMs — $\mathcal{L}_{\mathrm{eff}}^{\mathrm{tree}}$

$$\mathcal{L}_{\mathrm{eff}}(\phi) = 0 + \delta\mathcal{L}_{\mathrm{eff}}^{1-\mathrm{loop}} + \mathcal{L}_{\mathrm{eff}}^{\mathrm{tree}}$$

Functional Matching: Renormalization and Canonization

Renormalization

$$\left.\begin{aligned} c_{i,0} &= c_i + \delta c_i \\ \mathcal{C}_{i,0} &= \mathcal{C}_i + \delta \mathcal{C}_i \end{aligned}\right\} \longrightarrow \quad \delta\mathcal{L}^{\mathrm{ct}}_{\mathrm{BSM}} \xrightarrow[\Phi\ \mathrm{EOM}]{\zeta \ll 1} \delta\mathcal{L}^{\mathrm{BSM,ct}}_{\mathrm{eff}},$$

$$\Rightarrow \qquad \mathcal{L}_{\mathrm{eff}} = \underbrace{\mathcal{L}^{\mathrm{tree}}_{\mathrm{eff}} + \delta\mathcal{L}^{1-\mathrm{loop}}_{\mathrm{eff}} + \delta\mathcal{L}^{\mathrm{BSM,ct}}_{\mathrm{eff}} - \delta\mathcal{L}^{\mathrm{ct}}_{\mathrm{eff}}}_{\mathcal{L}^{\mathrm{ren}}_{\mathrm{eff}}} + \delta\mathcal{L}^{\mathrm{ct}}_{\mathrm{eff}}.$$

- Renormalized $c_i, \mathcal{C}_i$ may be enhanced by ζ^{-n} in the EFT power counting w.r.t. $c_{i,0}, \mathcal{C}_{i,0}$
- This is renormalization and tadpole scheme dependent
- $\overline{\mathrm{MS}}$ renormalization for enhanced c_i or $\mathcal{C}_i$ can spoil EFT [Dittmaier, SS, Stahlhofen, 2021]

Canonization (field redefinitions and IBP)

Consider $\mathcal{L}_{\mathrm{eff}}(\phi)$ and $\mathcal{L}'_{\mathrm{eff}}(\phi) = \mathcal{L}_{\mathrm{eff}}(\phi') + \partial\mathcal{L}''$ with $\phi' = \phi + f(\phi, \partial\phi, \partial\partial\phi, \dots)$

If $f(\phi)$ is suppressed by at least one order in a coupling or ζ, $\mathcal{L}_{\mathrm{eff}}$ and $\mathcal{L}'_{\mathrm{eff}}$ generate the same S-matrix.

$\hookrightarrow$ Deciding whether two effective Lagrangians are equivalent is non-trivial.

An Example: The Higgs Singlet Extension of the SM

The Lagrangian of the HSESM Higgs sector [Schabinger, Wells, 2005]

$$\mathcal{L}_{\text{Higgs,HSESM}} = \frac{1}{2}\,\text{tr}\left[(D_\mu \mathbf{\Phi})^\dagger (D^\mu \mathbf{\Phi})\right] + \frac{1}{2}\mu_2^2\,\text{tr}\left[\mathbf{\Phi}^\dagger \mathbf{\Phi}\right] - \frac{1}{16}\lambda_2\,\text{tr}\left[\mathbf{\Phi}^\dagger \mathbf{\Phi}\right]^2$$
$$+ \frac{1}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) + \mu_1^2 \sigma^2 - \lambda_1 \sigma^4 - \frac{1}{2}\lambda_{12}\sigma^2\,\text{tr}\left[\mathbf{\Phi}^\dagger \mathbf{\Phi}\right]$$

non-linear realization: $\mathbf{\Phi} = \frac{1}{\sqrt{2}}(v_2 + h_2)\,\boldsymbol{U}$, $\quad \boldsymbol{U} = \exp\left(\frac{\mathrm{i}\tau_a \varphi_a}{2v_2}\right)$ $\quad \sigma = v_1 + h_1$

Mass eigenfields after mixing:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

New input parameters:

$$\{\mu_1, \lambda_1, \lambda_{12}\} \leftrightarrow \{M_\text{H}, \sin\alpha, \lambda_{12}\}$$

$$M_\text{h} = 125\,\text{GeV} \ll M_\text{H} \sim \zeta^{-1}$$

A Proper EFT limit [Dittmaier, SS, Stahlhofen, 2021; Dawson et al., 2023]

$$M_\text{H} \sim \zeta^{-1},\ \sin\alpha \sim \zeta,\ \lambda_{12} \sim \zeta^0 \quad \Rightarrow \quad c_i \sim \zeta^0 \text{ for all SM parameters } c_i$$

$$\Rightarrow \quad \mu_1, \mu_2, v_1 \sim \zeta^{-1} \qquad \lambda_1, \lambda_2, \lambda_{12}, v_2, c_\text{SM} \sim \zeta^0$$

$\Rightarrow$ decoupling

$\sin(\alpha)^{\overline{\text{MS}}}_{\text{FJTS}} \sim \zeta^{-1}$ $\Rightarrow$ non-decoupling artifacts at NLO $\Rightarrow$ bad renormalization scheme

Results

Final form of the Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^{\text{ren}} + \delta\mathcal{L}_{\text{eff}}^{\text{ct}}$$
$$= \mathcal{L}_{\text{SM}} + \delta\mathcal{L}_{\text{eff}}^{\text{tree}} + \delta\mathcal{L}_{\text{eff}}^{1-\text{loop}} + \delta\mathcal{L}_{\text{eff}}^{\text{ct}}$$

Tree level

$$\delta\mathcal{L}_{\text{eff}}^{\text{tree}} = -\frac{\sin^2\alpha}{2v_2^2}\mathcal{O}_{\Phi\Box} + \mathcal{O}(\zeta^4)$$

1-loop level

$$\delta\mathcal{L}_{\text{eff}}^{1-\text{loop}} = \sum_{n=1}^{6} C_n^{\text{SMEFT}}\mathcal{O}_n^{\text{SMEFT}} + \sum_{n=1}^{8} C_n^{\text{HEFT}}\mathcal{O}_n^{\text{HEFT}} + \mathcal{O}(\zeta^4)$$

- The first six operators are SMEFT operators in the Warsaw basis [Grzadkowski et al., 2010].
- The other eight operators cannot be brought into SMEFT form,
  $\hookrightarrow$ the EFT of the HSESM in the proper limit $M_{\text{H}} \sim \zeta^{-1}$, $\sin\alpha \sim \zeta$, $\zeta \to 0$, is NOT of SMEFT type.
- $\lim_{\alpha\to 0} C_n^{\text{HEFT}} = 0 \quad \Rightarrow \quad \lim_{\alpha\to 0} \mathcal{L}_{\text{eff}} \subset \text{SMEFT}\,.$
- Example operators:

$$\mathcal{O}_2^{\text{SMEFT}} = \mathcal{O}_{\Phi\Box} = (\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi), \qquad \mathcal{O}_5^{\text{HEFT}} = \frac{2}{g_2^2}(v_2+h)^2\Box\,\text{tr}[(D_\mu\boldsymbol{U})^\dagger(D^\mu\boldsymbol{U})].$$

Interpretation

Higgs Effective Field Theory (HEFT) [Feruglio, 1993; Brivio, Trott, 2019]

Similar to SMEFT, but there are some differences

- Both are invariant under the SM gauge group $G_{\mathrm{SM}} = SU(3)_C \times SU(2)_W \times U(1)_Y$
- The Higgs and Goldstone fields do not form a complex doublet, but rather a real singlet and triplet
  $\hookrightarrow$ more operators are allowed
  $\hookrightarrow$ more general than SMEFT
  $\hookrightarrow$ $\mathcal{L}_{\mathrm{HEFT}} \supset \mathcal{L}_{\mathrm{SMEFT}}$.
- No canonical basis exists yet.

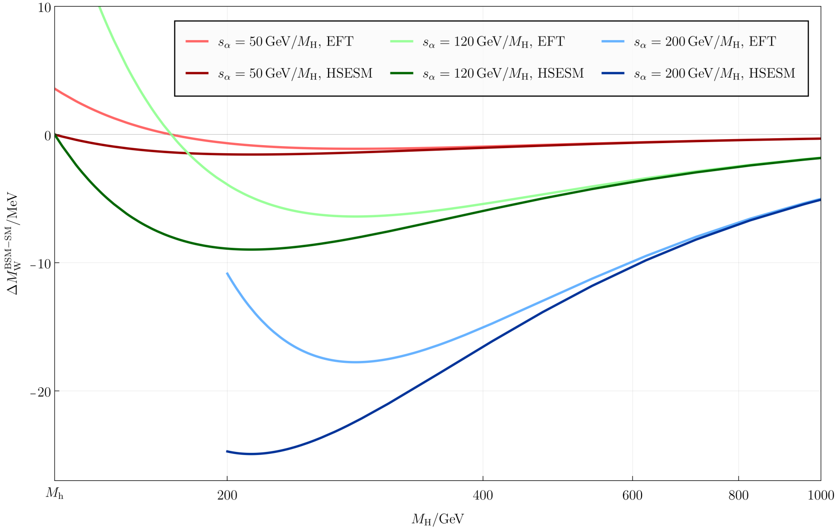
Interpretation

- The resulting effective Lagrangian is of HEFT rather than SMEFT type.
- EFT Higgs h cannot be embedded in an $SU(2)_W$ doublet due to mixing of BSM Higgs fields
  $\hookrightarrow$ we integrated out part of the SM-like Higgs doublet $\mathbf{\Phi}$
  $\hookrightarrow$ the non-SMEFT contributions vanish in the limit $\alpha \to 0$.

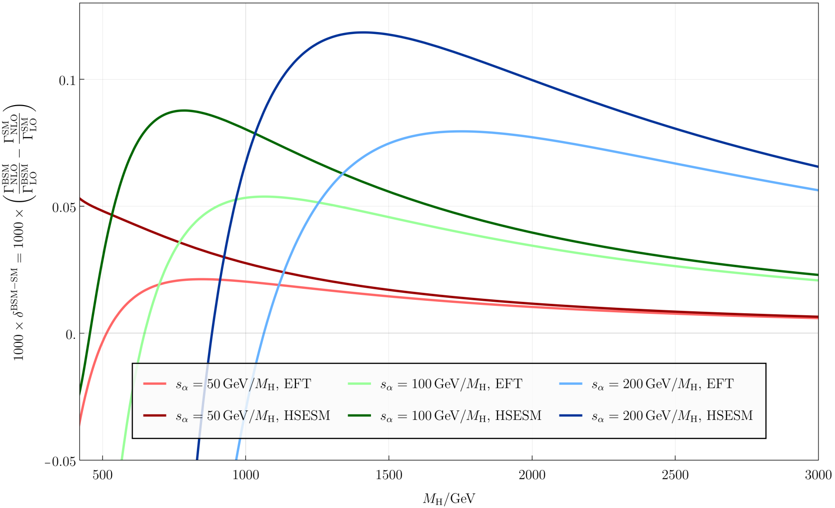
$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\,(v_2 + h_2)\,\boldsymbol{U}, \qquad \sigma = v_1 + h_1$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Phenomenological validation: W-Boson Mass from muon decay



Phenomenological validation: H $\to$ WW $\to \nu_e e^+ \mu^- \bar{\nu}_\mu$



- Prophecy4f [Bredenstein et al., 2006; Altenkamp, Boggia, Dittmaier, 2018] extended to HSESM − EFT
- G_μ scheme used for α_{em}

Why SMEFT cannot be enough

Observable-based proof — breakdown of diagrammatic matching to SMEFT

- Identify all n SMEFT Wilson coefficients C_i that could contribute
- Choose a set of $n+1$ observables X_j
- Compute X_j^{EFT} in the EFT (equivalent to X_j^{HSESM} from the full theory to ζ^2)
- Compute $X_j^{\mathrm{SMEFT}}(C_i)$ in SMEFT
- Solve the system $X_j^{\mathrm{EFT}} = X_j^{\mathrm{SMEFT}}(C_i)$ for the C_i
- If no solution exists, SMEFT cannot be enough to describe the EFT

Nonzero SMEFT Wilson Coefficients

$\{C_i\}$ of the complete Warsaw Basis

↓ massless fermions, no CP violation

$$\{C_\Phi, C_{\Phi\Box}, C_{\Phi D}, C_W, C_{\Phi W}, C_{\Phi B}, C_{\Phi WB}\}$$

↓ fix C_Φ via e.g. $hh \to hhhh$; fix C_W via e.g. $ZZ \to WWWW$

$$\{C_{\Phi\Box}, C_{\Phi D}, C_{\Phi W}, C_{\Phi B}, C_{\Phi WB}\}$$

Observables

- W-boson mass M_{W}
- Partial width $\Gamma_{\mathrm{Z}\nu_i\bar{\nu}_i}$ of the Z-boson
- Form factors $F_1^{\mathrm{h}\gamma\gamma}$, $F_1^{\mathrm{h}\gamma\mathrm{Z}}$, F_1^{hWW}, F_2^{hWW}

$$\Gamma_{\mu\nu}^{\mathrm{h}V_1V_2} = g_{\mu\nu} F_1^{\mathrm{h}V_1V_2} + k_{1\nu}k_{2\mu} F_2^{\mathrm{h}V_1V_2} + \ldots,$$

Result

No Solution exists

$\Rightarrow$ SMEFT is not enough

Conclusion

Summary

- Heavy Higgs mass eigenstate of the HSESM integrated out.
- Carefully defined a decoupling EFT limit such that SM parameters scale as Λ^0 for $\Lambda \to \infty$.
- Higgs mixing and HSESM renormalization fully taken into account.
- Effective Lagrangian is of to HEFT rather than SMEFT type.
- The non-SMEFT nature of the EFT is due to the mixing of the Higgs fields
  $\hookrightarrow$ part of the SM-like Higgs doublet integrated out
  $\hookrightarrow$ non-SMEFT contributions vanish in the limit $\alpha \to 0$.
- Validated predictions for electroweak precision observables in the EFT
  $\hookrightarrow$ proper asymptotic convergence to the full result in the EFT limit.
- Proof via observables/diagrammatic matching that SMEFT is not enough
- Many BSM models have similar mixing, e.g. THDM,
  $\hookrightarrow$ Be careful when integrating out heavy fields with mixing,
  $\hookrightarrow$ EFTs beyond SMEFT might be required to fully capture low energy physics.

Backup Slides

The Background-Field Method [DeWitt, 1967; Abbott, 1981; Dittmaier, Grosse-Knetter, 1995]

Idea: Split all fields into background and quantum fields $\varphi \to \tilde{\varphi} = \hat{\varphi} + \varphi$, to separate loop effects

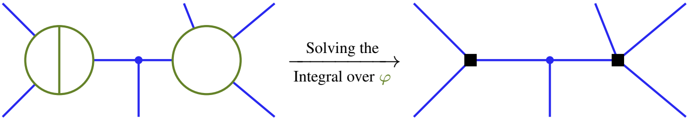
Formally

$$\mathrm{i}\mathcal{W}[J] = \int_{\text{connected}} \mathcal{D}\varphi \, \exp\left(\mathrm{i}\int \mathrm{d}^4x \, \mathcal{L}(\varphi) + J\varphi\right)$$

$$= \int_{\substack{\text{connected}\\\text{tree}}} \mathcal{D}\hat{\varphi} \, \exp\left(\mathrm{i}\int \mathrm{d}^4x \, J\hat{\varphi}\right) \underbrace{\int_{\text{1PI}} \mathcal{D}\varphi \, \exp\left(\mathrm{i}\int \mathrm{d}^4x \, \mathcal{L}(\hat{\varphi}+\varphi)\right)}_{\exp\left(\mathrm{i}\hat{\Gamma}[\hat{\varphi}]\right)}$$

We gain

- We can choose the gauges for background and quantum fields independently
- Integral over quantum fields is bilinear in φ at 1-loop, can be reduced to solvable Gaussian
- An intuitive picture for "Integrating out" quantum fields:



However, we only want to integrate out the heavy quantum fields

The Separation of Modes

Method of Regions [Beneke, Smirnov, 1998]

If a Feynman Integral depends on two scales, $m \ll \Lambda$ then

$$\int \mathrm{d}^D p\, f(m,\Lambda,p) = \left[\int \mathrm{d}^D p\, f_m(m,\Lambda,p) + \int \mathrm{d}^D p\, f_\Lambda(m,\Lambda,p)\right],$$

where $f_i(m,\Lambda,p)$ is the Taylor-Expansion of f in p about the point $p = i$.
This works only in Dimensional Regularization, thanks to $\int \mathrm{d}^D p\, \left(p^2\right)^n = 0,\ n \in \mathbb{Z}$!

We use this to split the fields φ further into light and heavy components

$$\varphi(m,\Lambda,p) = \varphi_s(m,\Lambda,p\sim m) + \varphi_h(m,\Lambda,p\sim\Lambda),$$

that carry only soft and hard momentum modes respectively via Taylor expansion.

Separation of Modes + Background Field Method [Dittmaier, SS, Stahlhofen, 2021]

$$\varphi \to \tilde{\varphi} = \hat{\varphi}_s + \hat{\varphi}_h + \varphi_s + \varphi_h$$

$$\mathrm{i}\mathcal{W}[J] = \int_{\substack{\text{conected}\\\text{tree}}} \mathcal{D}\hat{\varphi}_s \exp\left(\mathrm{i}\int \mathrm{d}^4x\, J\hat{\varphi}_s\right) \int_{\text{1PI}} \mathcal{D}\varphi_s \int_{\text{1PI}} \mathcal{D}\varphi_h \exp\left(\mathrm{i}\int \mathrm{d}^4x\, \mathcal{L}(\hat{\varphi}_s + \varphi_s + \varphi_h)\right)$$

Note: In the low-energy region of the EFT, $\hat{\varphi}_h = 0$.

Equations of Motion

What to do with the soft modes of heavy fields $\hat{\Phi}_s$, Φ_s?

Notation: Φ for heavy fields of mass $M_\Phi \sim \Lambda$, ϕ for light (SM) fields of mass $m_\phi \ll \Lambda$

Iterative Solution of EOM for a Scalar Particle

$$(\Box + \Lambda^2)\Phi_s = \sum_{i=0}^{\infty} f_i[\phi_s]\Phi_s^i$$

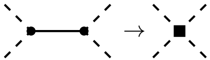
$$\Phi_{l,0}[\phi_l] = \frac{f_0[\phi_s]}{\Lambda^2} + \mathcal{O}\left(\Lambda^{\kappa-3}\right)$$

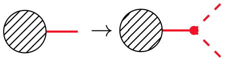
$$\Phi_{l,n}[\phi_l] = \frac{1}{\Lambda^2}\left(\sum_{i=0}^{\infty} f_i[\phi_s]\Phi_{l,n-1}^i - \Box\Phi_{l,n-1}\right) + \mathcal{O}\left(\Lambda^{(n+1)(\kappa-2)-1}\right)$$

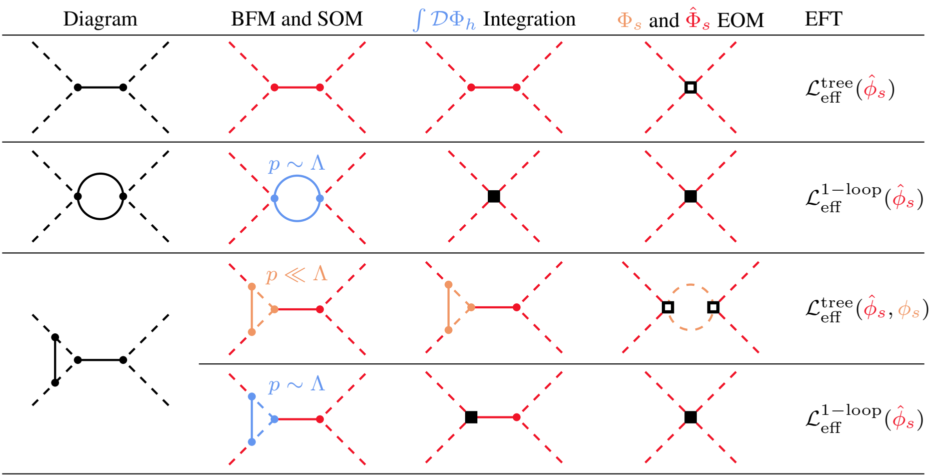
This only works if $f_0[\phi_l] \neq 0$ is leading order in Λ on the RHS, and $f_0[\varphi_s] = \mathcal{O}(\Lambda^\kappa)$ with $\kappa < 2$

Notes:

- We can treat $\hat{\Phi}_s, \Phi_s = \mathcal{O}\left(\Lambda^{\kappa-2}\right)$ in calculations
- This effectively expands all $\hat{\Phi}_s$, Φ_s Propagators in $\frac{p^2}{\Lambda^2} \to 0$, $\frac{\mathrm{i}}{p^2-\Lambda^2} \to -\frac{\mathrm{i}}{\Lambda^2}\sum_{n=0}^{N}\left(\frac{p^2}{\Lambda^2}\right)^n + \mathcal{O}\left(\Lambda^{-(N+4)}\right)$
- This also replaces external heavy fields $\hat{\Phi}_s$ lines $\hat{\Phi}_s \to \hat{\Phi}_{l,N-1}[\hat{\phi}_l] + \mathcal{O}\left(\Lambda^{N(\kappa-2)-1}\right)$







— heavy field Φ

- - - light field ϕ

$$\mathcal{L}_{\rm BSM}(\Phi, \phi) \longrightarrow \mathcal{L}_{\rm BSM}(\hat{\Phi}_h + \Phi_h + \hat{\Phi}_s + \Phi_s)$$

$$\mathcal{L}_{\rm eff}(\phi) = 0 + \mathcal{L}_{\rm eff}^{1-\rm loop} + \mathcal{L}_{\rm eff}^{\rm tree}$$

Full Set of Non-SMEFT Operators

$$\mathcal{O}_1^{\text{HEFT}} = -\frac{2}{g_2^2}(v_2+h)^2 \,\text{tr}\left\{[D_B^\mu(U^\dagger(D_\mu U))][D_B^\nu(U^\dagger(D_\nu U))]\right\}$$

$$\mathcal{O}_2^{\text{HEFT}} = -\frac{2}{g_2^2}(v_2+h)^2 \,\text{tr}\left\{[D_B^\mu(U^\dagger(D^\nu U))][D_{B\mu}(U^\dagger(D_\nu U))]\right\}$$

$$\mathcal{O}_3^{\text{HEFT}} = -\frac{2}{g_2^2}(v_2+h)^2 \,\text{tr}\left\{[D_B^\mu(U^\dagger(D_\nu U))][D_B^\nu(U^\dagger(D_\mu U))]\right\}$$

$$\mathcal{O}_4^{\text{HEFT}} = \frac{\mathrm{i}}{g_2^2}(v_2+h)^2 B^{\mu\nu} \,\text{tr}\left(\tau^3[(D_\mu U)^\dagger,(D_\nu U)]\right)$$

$$\mathcal{O}_5^{\text{HEFT}} = \frac{2}{g_2^2}(v_2+h)^2 \partial^\mu\partial^\nu \,\text{tr}\left[(D_\mu U)^\dagger(D_\nu U)\right]$$

$$\mathcal{O}_6^{\text{HEFT}} = \frac{2}{g_2^2}(v_2+h)^2 \Box \,\text{tr}\left[(D_\mu U)^\dagger(D^\mu U)\right],$$

$$\mathcal{O}_7^{\text{HEFT}} = \frac{2}{g_2^2}(\partial^\mu h)(\partial^\nu h) \,\text{tr}\left[(D_\mu U)^\dagger(D_\nu U)\right]$$

$$\mathcal{O}_8^{\text{HEFT}} = \frac{4}{g_2^4}(v_2+h)^2 \,\text{tr}\left[(D_\mu U)^\dagger[(D^\mu U)^\dagger,(D^\nu U)](D_\nu U)\right]$$

with

$$D_B^\mu\phi = \partial^\mu\phi + \mathrm{i}\frac{e}{2c_{\text{w}}}B^\mu[\tau^3,\phi] \qquad \Phi = \frac{1}{\sqrt{2}}(v_2+h_2)\,U \qquad U = \exp\left(2\mathrm{i}\frac{\varphi}{v_2}\right)$$

Phenomenology

What can we do with our EFT?

- Pheno predictions at 1-loop, $\mathcal{O}(\zeta^2)$,
- Compare to the full theory prediction.

Final form of the Effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \mathcal{L}^{\rm tree}_{\rm BSM,eff} + \delta\mathcal{L}^{\rm ct}_{\rm BSM,soft} + \mathcal{L}^{1-\rm loop,ren}_{\rm eff}$$

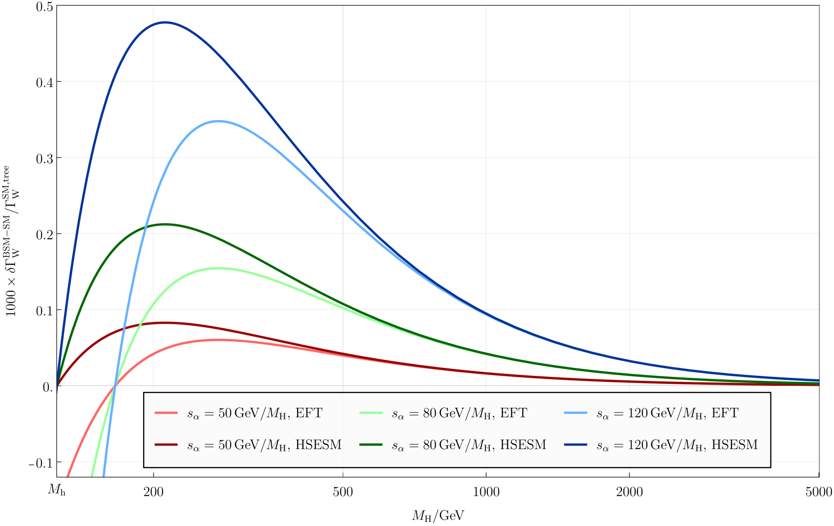
Ingredients

- The SM contribution up to 1-loop,
- $\mathcal{L}^{1-\rm loop,ren}_{\rm eff}$ in tree-level diagrams,
- $\mathcal{L}^{\rm tree}_{\rm BSM,eff}$ in up to 1-loop diagrams,
- Counterterms from $\delta\mathcal{L}^{\rm ct}_{\rm BSM,soft}$.

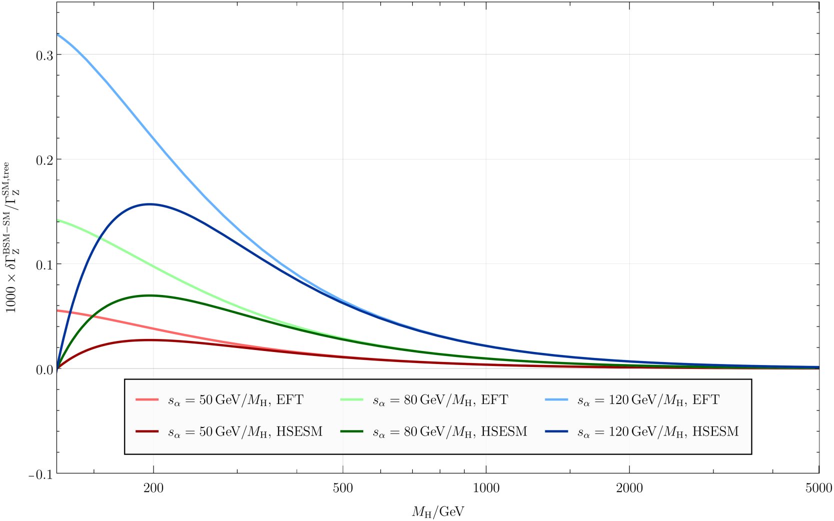
Notes

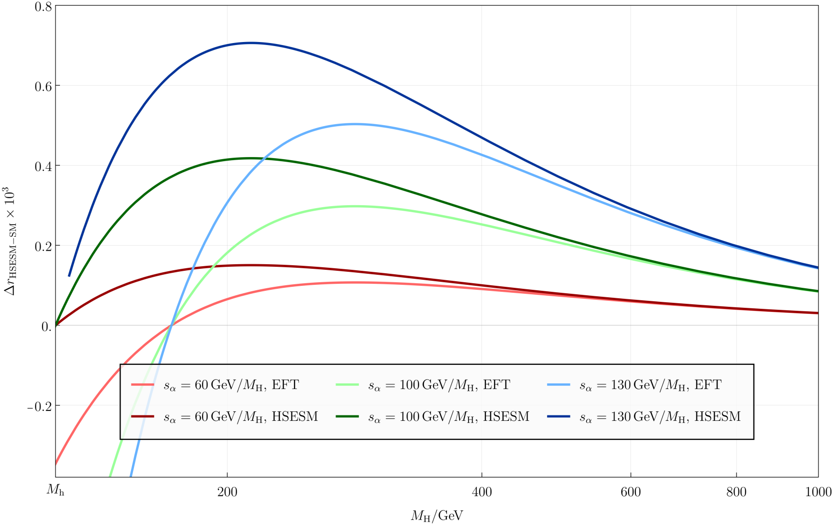
- Electroweak precision observables will be sensitive to BSM effects.
- We neglect fermion masses.
- We use on-shell renormalization for SM parameters, $M_{\rm H},\ s_\alpha$ and $\overline{\rm MS}$ for λ_{12}.
- We choose trajectories in BSM parameter space $M_{\rm H} = v/\zeta,\ s_\alpha = \text{const.}/M_{\rm H} \sim \zeta,\ M_{\rm H} \to \infty$.
- We calculate to order $\mathcal{O}(\zeta^2)$.
- EFT and full theory should converge in the sense $\text{HSESM} \sim \text{EFT} + \mathcal{O}(\zeta^3) \sim \text{SM} + \zeta^2\delta\text{EFT} + \mathcal{O}(\zeta^3)$ for $M_{\rm H} \to \infty$.

W-Boson Width $W \to f\bar{f}'$



Z-Boson Width $Z \to f\bar{f}$

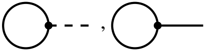




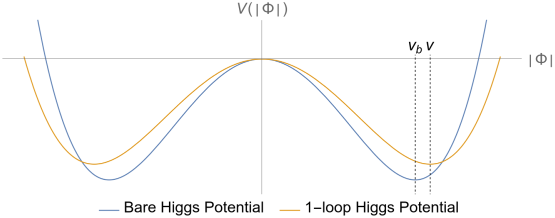
0.8
0.6
0.4
0.2
0.
-0.2
$\Delta r_{\mathrm{HSESM-SM}} \times 10^3$
M_h
200
400
600
800
1000
M_H/GeV
$s_\alpha = 60\,\mathrm{GeV}/M_H$, EFT
$s_\alpha = 100\,\mathrm{GeV}/M_H$, EFT
$s_\alpha = 130\,\mathrm{GeV}/M_H$, EFT
$s_\alpha = 60\,\mathrm{GeV}/M_H$, HSESM
$s_\alpha = 100\,\mathrm{GeV}/M_H$, HSESM
$s_\alpha = 130\,\mathrm{GeV}/M_H$, HSESM

Renormalization of the Vacuum

Remember: The SM is Taylor-expanded around the vacuum expectation value.
Which one? → Tadpole renormalization



$\sim \Lambda^2$



Important Tadpole Renormalization Schemes [Dittmaier, Rzehak, 2022]

Parameter Renormalized Tadpole Scheme
[Bohm, Spiesberger, Hollik, 1986]

- Sets the Renormalized Tadpole to zero
  ↪ Expands around the renormalized vacuum
  ↪ Perturbatively stable
- In general not gauge invariant
- No tadpoles in renormalization constants

Fleischer Jegerlehner Tadpole Scheme
[Fleischer, Jegerlehner, 1981]

- Sets the Bare Tadpole to zero
  ↪ Expands around the bare vacuum
  ↪ Perturbatively unstable
- Gauge invariant
- Tadpoles appear in renormalization constants

Expressions for Matching Observables

$$\Gamma^{\mathrm{h}V_1V_2}_{\mu\nu} = g_{\mu\nu} F_1^{\mathrm{h}V_1V_2} + k_{1\nu} k_{2\mu} F_2^{\mathrm{h}V_1V_2} + \ldots,$$

$$\Gamma_{\mathrm{W}}|_{\mathrm{hard}} = \frac{v_2^2}{2s_{\mathrm{w}}^2}(4c_{\mathrm{w}}s_{\mathrm{w}}C_{\Phi WB} + c_{\mathrm{w}}^2 C_{\Phi D} + 8s_{\mathrm{w}}^2 C_2^{\mathrm{HEFT}}),$$

$$\Gamma_{\mathrm{Z}\nu_i\bar{\nu}_i}|_{\mathrm{hard}} = \frac{v_2^2}{2s_{\mathrm{w}}^2}(4c_{\mathrm{w}}s_{\mathrm{w}}C_{\Phi WB} + \left(1-2s_{\mathrm{w}}^2\right) C_{\Phi D} + 8s_{\mathrm{w}}^2 C_2^{\mathrm{HEFT}}),$$

$$F_1^{\mathrm{h}\gamma\gamma}|_{\mathrm{hard}} = 2v_2(k_1^2+k_2^2-k_{\mathrm{h}}^2)(c_{\mathrm{w}}^2 C_{\Phi B} + s_{\mathrm{w}}^2 C_{\Phi W} - c_{\mathrm{w}}s_{\mathrm{w}}C_{\Phi WB}),$$

$$F_1^{\mathrm{h}\gamma\mathrm{Z}}|_{\mathrm{hard}} = v_2(k_1^2+k_2^2-k_{\mathrm{h}}^2)(2c_{\mathrm{w}}s_{\mathrm{w}}C_{\Phi B} - 2s_{\mathrm{w}}c_{\mathrm{w}}C_{\Phi W} + (1-2s_{\mathrm{w}}^2)C_{\Phi WB}),$$

$$F_1^{\mathrm{hWW}}|_{\mathrm{hard}} = 2v_2(k_1^2+k_2^2-k_{\mathrm{h}}^2)(C_{\Phi W} + C_2^{\mathrm{HEFT}}) - 4v_2 k_{\mathrm{h}}^2 C_6^{\mathrm{HEFT}} + \frac{e^2 v_2^3}{8s_{\mathrm{w}}^4}\left(4c_{\mathrm{w}}s_{\mathrm{w}}C_{\Phi WB} + 4s_{\mathrm{w}}^2 C_{\Phi\Box} + (1-2s_{\mathrm{w}}^2)C_{\Phi D} + 8s_{\mathrm{w}}^2 C_2^{\mathrm{HEFT}}\right),$$

$$F_2^{\mathrm{hWW}}|_{\mathrm{hard}} = 4v_2(C_{\Phi W} - C_3^{\mathrm{HEFT}} - C_5^{\mathrm{HEFT}}),$$

Result of solving the system of equations

$$0 \overset{!}{=} C_2^{\mathrm{HEFT}} + C_3^{\mathrm{HEFT}} + C_5^{\mathrm{HEFT}} = \frac{(D-6)e^2 s_\alpha^2 I_{20}}{16\pi^2 D\,(D^2-4)\,s_{\mathrm{w}}^2 v_2^2} \neq 0,$$

contains a logarithm of the heavy scale M_{H} $\rightarrow$ cannot be canceled by contributions of the soft modes
$\Rightarrow$ SMEFT cannot fully describe the HSESM in the limit $M_{\mathrm{H}} \rightarrow \infty$, $s_\alpha \sim M_{\mathrm{H}}{}^{-1}$.