JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Alexander von HUMBOLDT STIFTUNG

A weak effective theory approach to probing ALPs in B-decays

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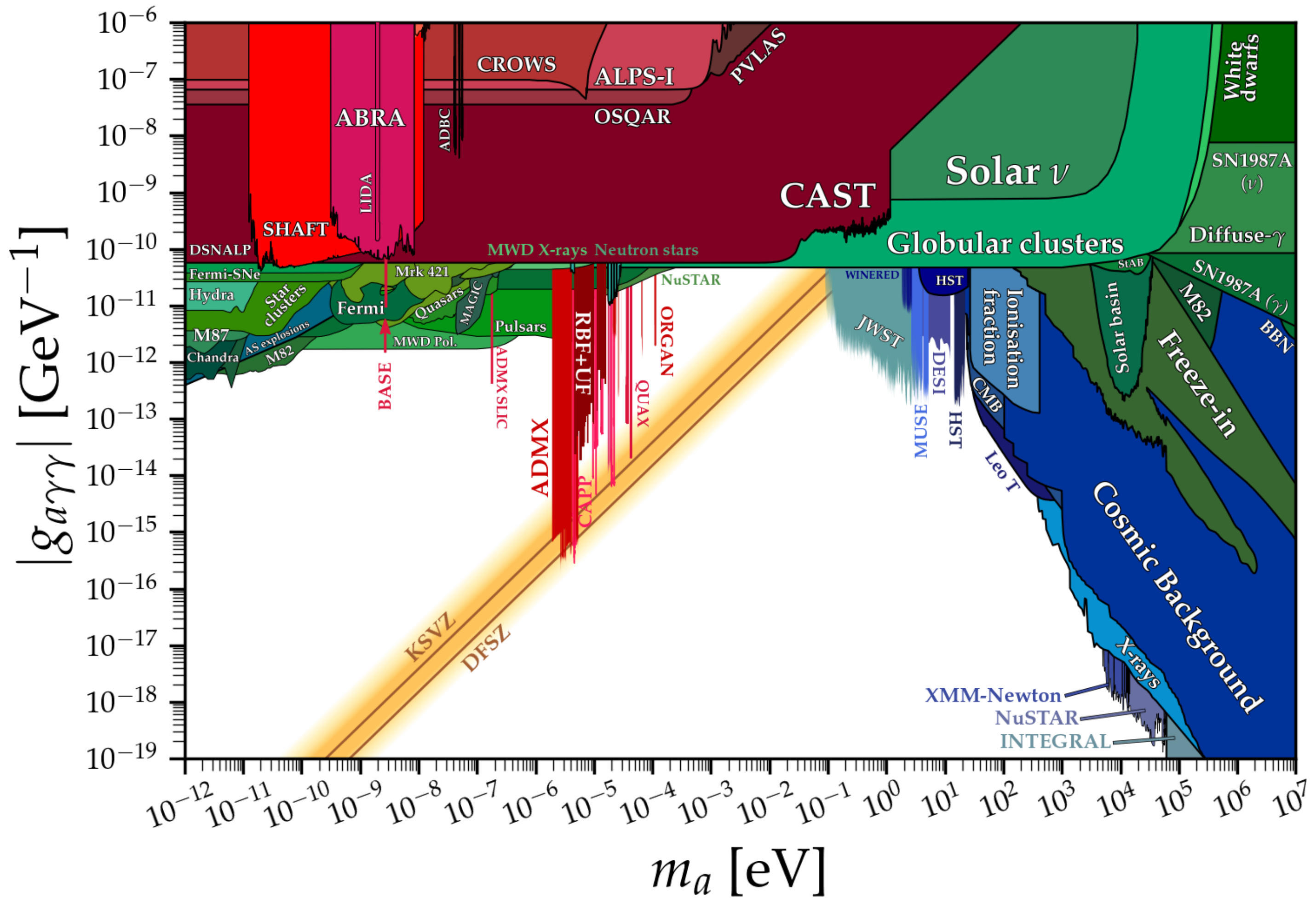
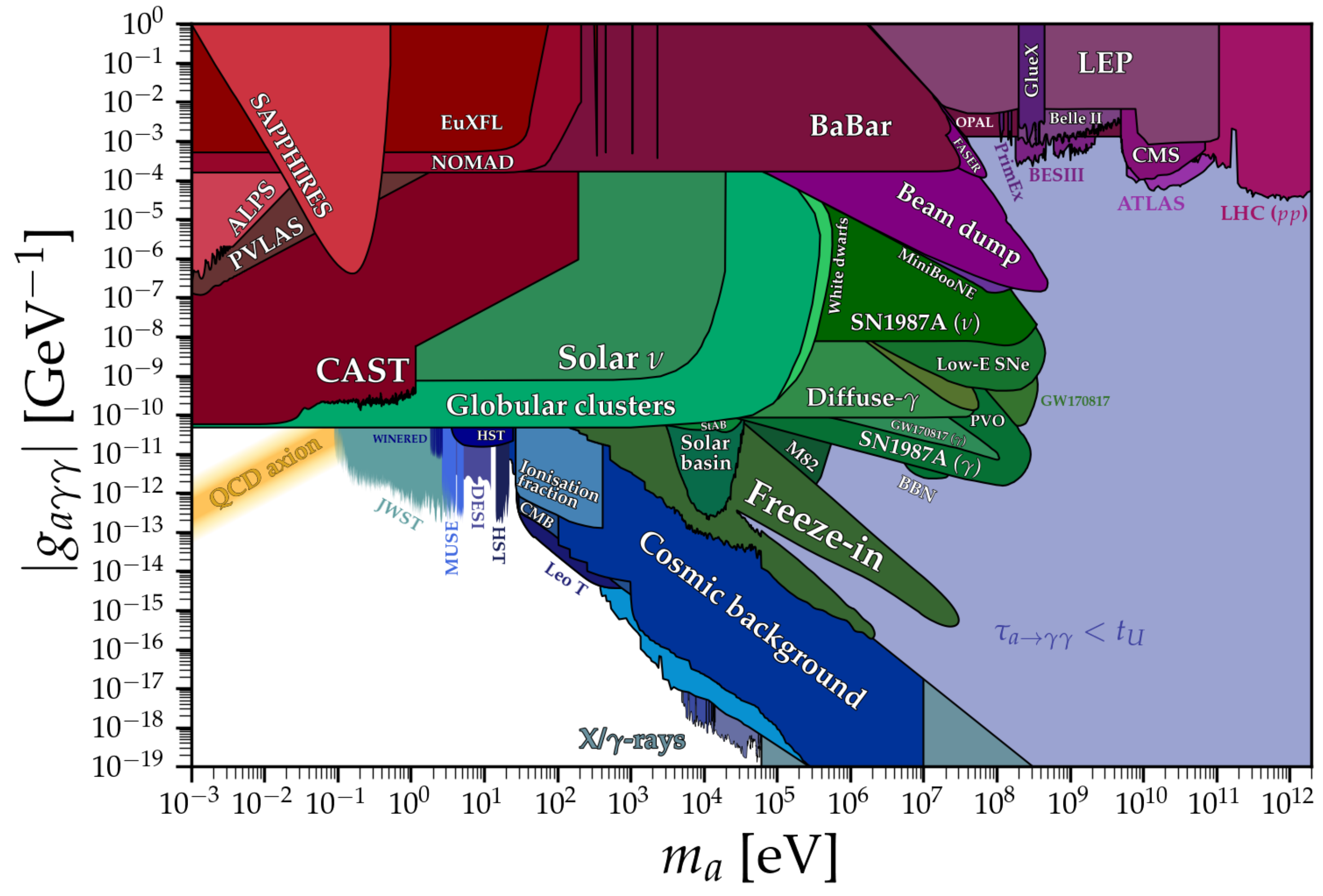
Outline of the talk

- Axion and axion-like particles
- Effective Lagrangian for an ALP coupled to the Standard Model
- Direct probe of ALP couplings through flavour observable
- ALP effective Lagrangian and weak effective theory
- $B^{\pm} \to K^{\pm}a$ with QCD factorization
- Bounds on ALP couplings: at low-energy and at the UV
- Side remark: generation of new operators for ALP Lagrangian in the non-derivative basis

Axions and axion-like particles

- Peccei-Quinn solution to strong CP problem.
- Axion mass and couplings to Standard Models are inversely related to the scale f_a.
- More generally, ALPs arise as pseudo Nambu- Goldstone bosons of spontaneously broken global $U(1)$ symmetry.
- Depending on a particular mass window different categories of searches become important.
- For heavier ALPs, couplings to SM particles can be probed in particle physics experiments.





[https://cajohare.github.io/AxionLimits/]

Effective Lagrangian for an ALP

- Most general effective Lagrangian for a pseudoscalar boson a coupled to SM via classically shift-invariant interactions:

[Georgi, Kaplan, Randall (1986)]

$$\mathcal{L}^{D\leq 5}_{\text{ALP, eff}} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,o}}{2}a^2 + \frac{\partial^\mu a}{f}\sum_F \bar{\Psi}_F \boldsymbol{c}_F \gamma^\mu \Psi_F + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G^A_{\mu\nu}\tilde{G}^{A,\mu\nu} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{f}W^I_{\mu\nu}\tilde{W}^{I,\mu\nu} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

- After the EWSB, the ALP Lagrangian reduces to:

$$\mathcal{L}^{D\leq 5}_{\text{eff}}(\mu_w) = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,0}}{2}a^2 + \mathcal{L}'_{\text{ferm}}(\mu) + c_{GG}\frac{a}{f}G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu} + \cdots$$

With the couplings to Standard Model fermions:

$$\mathcal{L}_{\text{ferm}}(\mu) = \frac{\partial^\mu a}{f}\left[\bar{u}_L\boldsymbol{k}_U(\mu)\gamma_\mu u_L + \bar{u}_R\boldsymbol{k}_u(\mu)\gamma_\mu u_R + \bar{d}_L\boldsymbol{k}_D(\mu)\gamma_\mu d_L + \bar{d}_R\boldsymbol{k}_d(\mu)\gamma_\mu d_R + \bar{\nu}_L\boldsymbol{k}_\nu(\mu)\gamma_\mu \nu_L + \bar{e}_L\boldsymbol{k}_E(\mu)\gamma_\mu e_L + \bar{e}_R\boldsymbol{k}_e(\mu)\gamma_\mu e_R\right]$$

In the up-aligned flavour basis:

$$\boldsymbol{k}_U = \boldsymbol{c}_Q, \quad \boldsymbol{k}_E = \boldsymbol{k}_\nu = \boldsymbol{c}_L, \quad \boldsymbol{k}_{u,d,e} = \boldsymbol{c}_{u,d,e}, \quad \boldsymbol{k}_D = \boldsymbol{V}^\dagger_{CKM}\boldsymbol{c}_Q\boldsymbol{V}_{CKM}$$

Direct probes of ALP couplings:

Observable	Mass range [MeV]	ALP decay mode	Constrained coupling c_{ij}	Limit (95% CL) on $c_{ij} \cdot \left(\frac{\text{TeV}}{f}\right) \cdot \sqrt{\mathcal{B}}$	Limit (95% CL) on $c_{ij}/\lvert V_{ti}^* V_{tj}\rvert \cdot \left(\frac{\text{TeV}}{f}\right) \cdot \sqrt{\mathcal{B}}$
$\text{Br}(K^- \to \pi^- a(\text{inv}))$	$0 < m_a < 261^{(*)}$	long-lived	$\lvert k_D + k_d \rvert_{12}$	1.2×10^{-9}	3.9×10^{-6}
$\text{Br}(K_L \to \pi^0 a(\text{inv}))$	$0 < m_a < 261$	long-lived	$\lvert \text{Im}[[k_D + k_d]_{12}\rvert$	8.1×10^{-9}	7.0×10^{-5}
$\text{Br}(K^- \to \pi^- \gamma\gamma)$	$m_a < 108$	$\gamma\gamma$	$\lvert k_D + k_d \rvert_{12}$	2.1×10^{-8}	6.9×10^{-5}
$\text{Br}(K^- \to \pi^- \gamma\gamma)$	$220 < m_a < 354$	$\gamma\gamma$	$\lvert k_D + k_d \rvert_{12}$	2.0×10^{-7}	6.5×10^{-4}
$\text{Br}(K_L \to \pi^0 \gamma\gamma)$	$m_a < 110$	$\gamma\gamma$	$\lvert \text{Im}[[k_D + k_d]_{12}]\rvert$	1.3×10^{-8}	1.1×10^{-4}
$\text{Br}(K_L \to \pi^0 \gamma\gamma)$	$m_a < 363^{(\maltese\maltese)}$	$\gamma\gamma$	$\lvert \text{Im}[[k_D + k_d]_{12}]\rvert$	1.3×10^{-7}	1.1×10^{-3}
$\text{Br}(K^+ \to \pi^+ a(e^+e^-))$	$1 < m_a < 100$	e^+e^-	$\lvert k_D + k_d \rvert_{12}$	3.4×10^{-7}	1.1×10^{-3}
$\text{Br}(K_L \to \pi^0 e^+e^-)$	$140 < m_a < 362$	e^+e^-	$\lvert \text{Im}[[k_D + k_d]_{12}]\rvert$	3.1×10^{-9}	2.6×10^{-5}
$\text{Br}(K_L \to \pi^0 \mu^+\mu^-)$	$210 < m_a < 350$	$\mu^+\mu^-$	$\lvert \text{Im}[[k_D + k_d]_{12}]\rvert$	4.0×10^{-9}	3.4×10^{-5}
$\text{Br}(B^+ \to \pi^+ e^+e^-)$	$140 < m_a < 5140$	e^+e^-	$\lvert k_D + k_d \rvert_{13}$	7.0×10^{-7}	8.7×10^{-5}
$\text{Br}(B^+ \to \pi^+ \mu^+\mu^-)$	$211 < m_a < 5140^{(\ddagger\ddagger)}$	$\mu^+\mu^-$	$\lvert k_D + k_d \rvert_{13}$	1.2×10^{-7}	1.4×10^{-5}
$\text{Br}(B^- \to K^- \nu\bar{\nu})$	$0 < m_a < 4785$	long-lived	$\lvert k_D + k_d \rvert_{23}$	6.2×10^{-6}	1.6×10^{-4}
$\text{Br}(B \to K^* \nu\bar{\nu})$	$0 < m_a < 4387$	long-lived	$\lvert k_D - k_d \rvert_{23}$	4.1×10^{-6}	1.1×10^{-4}
$d\text{Br}/dq^2(B^0 \to K^{*0} e^+e^-)_{[0.0,0.05]}$	$1 < m_a < 224$	e^+e^-	$\lvert k_D - k_d \rvert_{23}$	6.4×10^{-7}	1.6×10^{-5}
$d\text{Br}/dq^2(B^0 \to K^{*0} e^+e^-)_{[0.05,0.15]}$	$224 < m_a < 387$	e^+e^-	$\lvert k_D - k_d \rvert_{23}$	9.3×10^{-7}	2.4×10^{-5}
$\text{Br}\big(B^- \to K^- a(\mu^+\mu^-)\big)$	$250 < m_a < 4700^{(\dagger)}$	$\mu^+\mu^-$	$\lvert k_D + k_d \rvert_{23}$	4.4×10^{-8}	1.1×10^{-6}
$\text{Br}\big(B^0 \to K^{*0} a(\mu^+\mu^-)\big)$	$214 < m_a < 4350^{(\dagger)}$	$\mu^+\mu^-$	$\lvert k_D - k_d \rvert_{23}$	5.1×10^{-8}	1.3×10^{-6}
$\text{Br}(B^- \to K^- \tau^+\tau^-)$	$3552 < m_a < 4785$	$\tau^+\tau^-$	$\lvert k_D + k_d \rvert_{23}$	8.2×10^{-5}	2.1×10^{-3}
$\text{Br}(D^0 \to \pi^0 e^+e^-)$	$1 < m_a < 1730^{(\ddagger)}$	e^+e^-	$\lvert k_U + k_u \rvert_{12}$	2.8×10^{-5}	–
$\text{Br}(D^+ \to \pi^+ e^+e^-)$	$200 < m_a < 1730^{(\dagger\dagger)}$	e^+e^-	$\lvert k_U + k_u \rvert_{12}$	8.4×10^{-6}	–
$\text{Br}(D_s^+ \to K^+ e^+e^-)$	$200 < m_a < 1475^{(\maltese)}$	e^+e^-	$\lvert k_U + k_u \rvert_{12}$	2.4×10^{-5}	–
$\text{Br}(D^+ \to \pi^+ \mu^+\mu^-)$	$250 < m_a < 1730^{(**)}$	$\mu^+\mu^-$	$\lvert k_U + k_u \rvert_{12}$	2.1×10^{-6}	–
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[Bauer, Neubert, Renner, Schnubel, Thamm (2021)]

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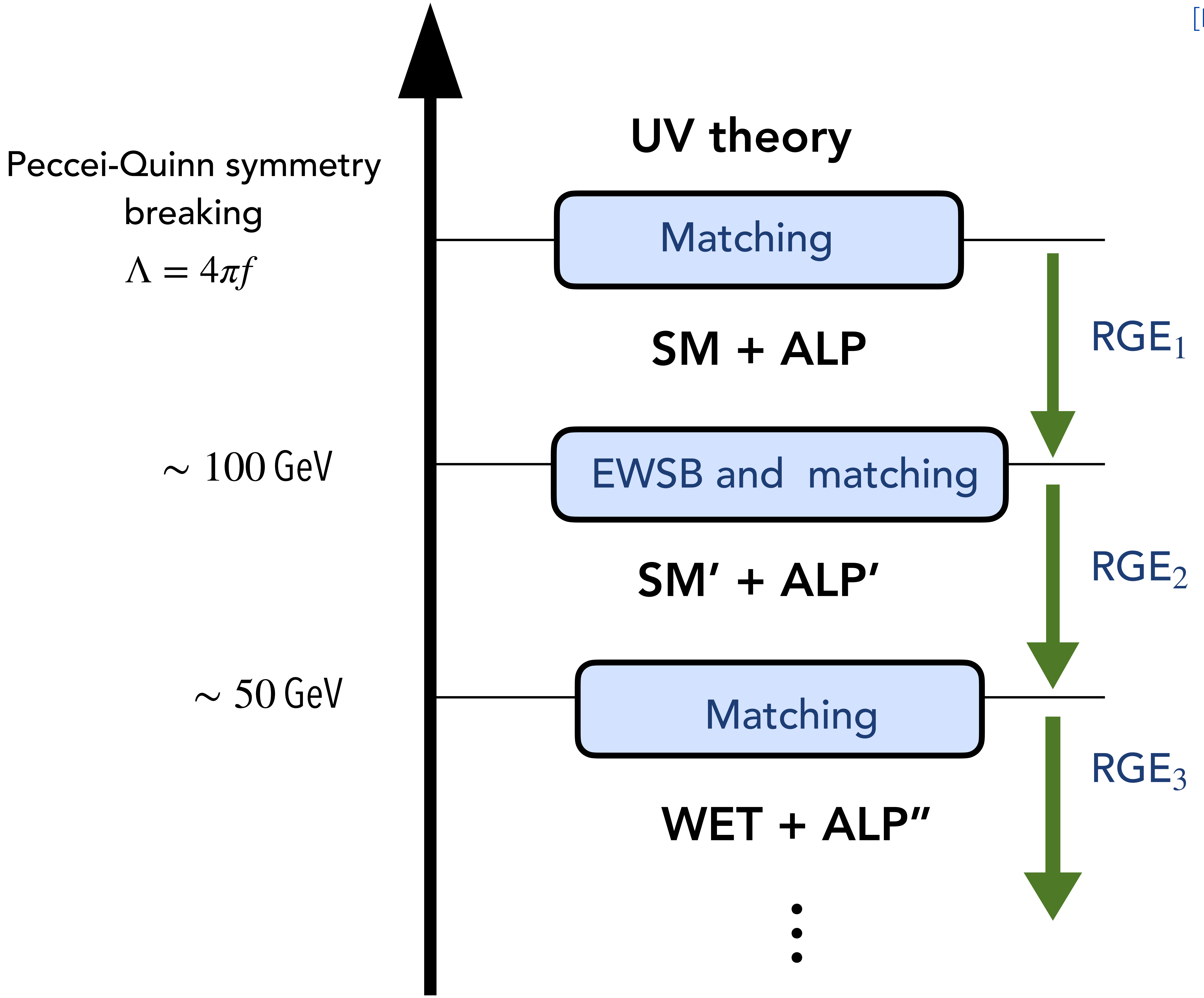
[Bauer, Neubert, Renner, Schnubel, Thamm (2021)]

RG evolution from UV down to low-energy scale

[Bauer, Neubert, Renner, Schnubel, Thamm (2020)]

[Chala, Guedes, Ramos, Santiago (2020)]



Weak effective interaction

- In the case of B-decays involving flavor-diagonal ALP couplings, we assume that the flavor change is induced by effective weak interactions.

- The relevant weak effective operators are:

$$\mathcal{Q}_1 = (\bar{s}_i\, p_i)_{(V-A)}(\bar{p}_j\, b_j)_{(V-A)} \qquad \mathcal{Q}_2 = (\bar{s}_i\, p_j)_{(V-A)}(\bar{p}_j\, b_i)_{(V-A)}$$

$$\mathcal{Q}_3 = (\bar{s}_i\, b_i)_{(V-A)} \sum_q (\bar{q}_j\, q_j)_{(V-A)} \qquad \mathcal{Q}_4 = (\bar{s}_i\, b_j)_{(V-A)} \sum_q (\bar{q}_j\, q_i)_{(V-A)}$$

$$\mathcal{Q}_5 = (\bar{s}_i\, b_i)_{(V-A)} \sum_q (\bar{q}_j\, q_j)_{(V+A)} \qquad \mathcal{Q}_6 = (\bar{s}_i\, b_j)_{(V-A)} \sum_q (\bar{q}_j\, q_i)_{(V+A)}\,.$$

$$\mathcal{Q}_{8g} = \frac{g_s m_b}{8\pi^2}\bar{s}_i\, \sigma^{\mu\nu}(1+\gamma_5)t^a_{ij}\, b_j\, G^a_{\mu\nu}$$

$p = u, c$
$q = u, d, c, s, b$

- Weak effective Hamiltonian:

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 \mathcal{Q}_1^{(p)} + C_2 \mathcal{Q}_2^{(p)} + \sum_{i=3,\cdots,6} C_i \mathcal{Q}_i + C_{8g}\, \mathcal{Q}_{8g} \right) + \text{h.c.}$$

$B^\pm \to K^\pm a$ in QCD Factorization

- For exclusive decays for heavy mesons, at large kaon recoil, the separation of short- and long-distance dynamics is achieved through QCD factorization

$$< K\ a|\mathscr{L}_{tot}|B > = \underbrace{\mathcal{T}^I_\mu(q^2)\ F^\mu_{B\to K}(q^2)}_{\text{Factorizable contribution}} + \iint du\ d\omega\ \underbrace{\Phi^K(u)_{\beta\alpha}\ \mathcal{T}^{II}_{\beta\alpha\rho\eta}(q^2,u,\omega)}_{\text{Non-factorizable contribution}}\ \Phi^B_{\rho\eta}$$

- Hard-scattering kernels: $\mathcal{T}^{I,II}$ are associated with short-distance dynamics.

- Form factor: $F^\mu_{B\to M} \equiv < M(k)|\bar{q}_1\Gamma^\mu q_2|B(p) >$

- Meson-distribution amplitude:

$$\left[\Phi^M\right]_{\alpha\beta} = \left[\frac{if_M}{4}\left\{\not{p}\gamma_5\Phi(u) - \mu_M\gamma_5\left(\Phi_p - i\sigma_{\mu\nu}\frac{p^\mu\bar{n}^\nu}{p\cdot\bar{n}}\frac{\Phi'_\sigma(u)}{6} + i\sigma_{\mu\nu}p^\mu\frac{\Phi_\sigma}{6}\frac{\partial}{\partial k_{\perp\nu}}\right)\right\}\right]_{\alpha\beta}$$

$$\left[\Phi^B\right]_{\alpha\beta} = -\left[\frac{if_Bm_B}{4}\left(\frac{1+\not{v}}{2}\right)\left\{\Phi^B_+(\omega)\not{\bar{n}} + \Phi^B_-(\omega)\left(\not{n} - \omega\gamma^\mu\frac{\partial}{\partial l_{\mu\perp}}\right)\right\}\gamma_5\right]_{\alpha\beta}$$

} are associated with long-distance, non-perturbative dynamics.

$B^\pm \to K^\pm a$: Kinematics

- Four vectors of B-meson, Kaon and the ALP momenta:

$$p_B^\mu = m_B\, v^\mu = m_B \frac{n^\mu}{2} + m_B \frac{\bar{n}^\mu}{2}$$

$$p_K^\mu = \left(E,0,0,\sqrt{E^2 - m_K^2}\right) = \left(E + \sqrt{E^2 - m_K^2}\right)\frac{n^\mu}{2} + \left(E - \sqrt{E^2 - m_K^2}\right)\frac{\bar{n}^\mu}{2}$$

$$\approx\ 2E\frac{n^\mu}{2} + \frac{m_K^2}{2E}\frac{\bar{n}^\mu}{2}$$

$$p_a^\mu = \left(E_a,0,0,-\sqrt{E^2 - m_K^2}\right) \approx \frac{m_a^2}{m_B}\frac{n^\mu}{2} + m_B\frac{\bar{n}^\mu}{2}$$

$$n^\mu = (1,0,0,+1)$$
$$\bar{n}^\mu = (1,0,0,-1)$$

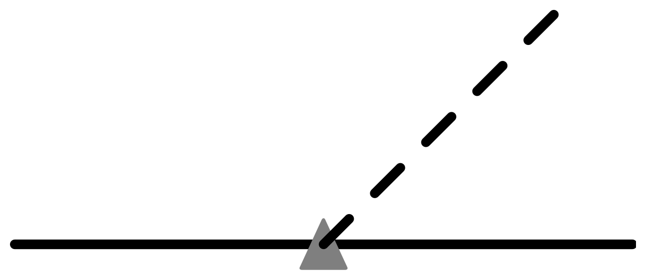
- Kaon is collinear, depending on the mass of the ALP, the ALP becomes either anti-collinear or hard anti-collinear.

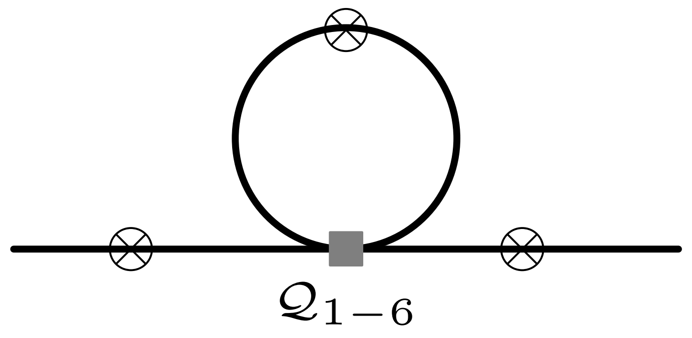
- The momentum carried by the partons inside the mesons:
  - B-meson: $p_b = m_b v^\mu - l^\mu$, $l^\mu = l_+\frac{\bar{n}^\mu}{2} + l_-\frac{n^\mu}{2} + l_\perp^\mu$
  - Kaon: $p_s = uEn^\mu + k_\perp^\mu,$ $p_u = (1-u)En^\mu - k_\perp^\mu$

$B^{\pm} \to K^{\pm}a$: Diagrams

Factorizable

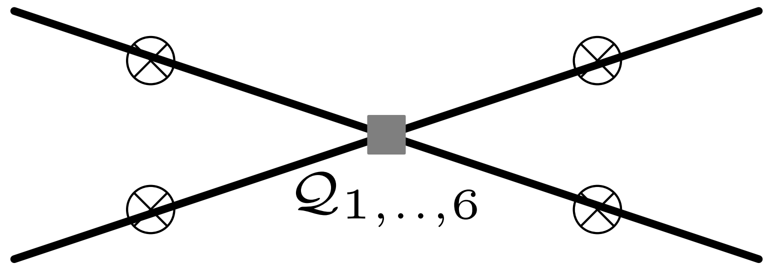
- Form factor correction:





Non- factorizable

- Weak annihilation:



- Hard spectator scattering:

Bounds on ALP couplings

- We use LHCb search for $B^+ \to K^+\chi$ with $\chi \to \mu^+\mu^-$ to get 95% CL bounds on ALP couplings one at a time, considering prompt decay of the ALP.
  [LHCb:1612.07818]

- Bounds on flavour violating coupling is much more stringent than flavour-conserving ones.

- If there is no flavor symmetry at the UV, it pushes the scale to be very high.

$C_i(\mu_b)$	in units of $[\mathrm{GeV}^{-1}]$	
	$m_a = 0.5$ GeV	$m_a = 2$ GeV
$(\kappa_d + \kappa_D)_{23}$	8.71×10^{-11}	5.39×10^{-11}
$(\kappa_b - \kappa_B)$	2.87×10^{-4}	1.90×10^{-4}
$(\kappa_c - \kappa_C)$	1.34×10^{-3}	5.83×10^{-4}
$(\kappa_d - \kappa_D)$	1.09×10^{-2}	2.04×10^{-2}
$(\kappa_s - \kappa_S)$	1.56×10^{-2}	2.14×10^{-2}
$(\kappa_b - \kappa_S)$	1.66×10^{-2}	1.04×10^{-2}
C_{GG}	1.43×10^{-4}	9.10×10^{-5}

Bounds on ALP couplings at the UV

- We consider flavour-universal couplings at the UV scale $\Lambda = 4\pi f$. This way large flavor changing coupling at low-energy can be avoided.
- Through RG running, flavour changing couplings are generated at low-energy.
- The bounds are stronger by at least one order of magnitude over the previous bounds.

[Bauer, Neubert, Renner, Schnubel, Thamm (2021)]

$C_i(\Lambda)$	in units of [GeV^{-1}]	
	$m_a = 0.5$ GeV	$m_a = 2$ GeV
$C_{GG}(\Lambda)$	3.34×10^{-5}	2.06×10^{-5}
$C_{BB}(\Lambda)$	4.78	1.13
$C_{WW}(\Lambda)$	1.17×10^{-3}	2.77×10^{-3}
$C_e(\Lambda)$	3.47×10^{-3}	2.15×10^{-3}
$C_u(\Lambda)$	1.10×10^{-7}	6.82×10^{-8}
$C_d(\Lambda)$	2.28×10^{-5}	1.41×10^{-5}
$C_L(\Lambda)$	4.92×10^{-5}	3.05×10^{-5}
$C_Q(\Lambda)$	1.11×10^{-7}	6.89×10^{-8}

Appearance of new operators in the non-derivative basis

- ALP Lagrangian with classically shift-invariant interactions:

$$\mathcal{L}^{D\leq 5}_{\text{ALP, eff}} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,o}}{2}a^2 + \frac{\partial^\mu a}{f}\sum_F \bar{\Psi}_F\, \boldsymbol{c}_F\, \gamma^\mu\, \Psi_F + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G^A_{\mu\nu}\tilde{G}^{A,\mu\nu} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{f}W^I_{\mu\nu}\tilde{W}^{I,\mu\nu} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

- Alternative form of the Lagrangian:

$$\frac{1}{2}\mathcal{L}^{D\leq 5}_{\text{eff}} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m^2_{a,o}}{2}a^2 - \frac{a}{f}(\bar{Q}\phi\tilde{Y}_d d_R + \bar{Q}\tilde{\phi}\tilde{Y}_u u_R + \bar{L}\phi\tilde{Y}_e e_R + \text{h.c.}) + \tilde{c}_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + \tilde{c}_{WW}\frac{\alpha_s}{4\pi}\frac{a}{f}W^a_{\mu\nu}\tilde{W}^{a,\mu\nu} + \tilde{c}_{BB}\frac{\alpha_s}{4\pi}\frac{a}{f}B_{\mu\nu}\tilde{B}^{\mu\nu}$$

- Relating the couplings of two Lagrangians:

$$\tilde{\boldsymbol{Y}}_d = i(\boldsymbol{Y}_d\boldsymbol{c}_d - \boldsymbol{c}_Q\boldsymbol{Y}_d), \quad \tilde{\boldsymbol{Y}}_u = i(\boldsymbol{Y}_u\boldsymbol{c}_u - \boldsymbol{c}_Q\boldsymbol{Y}_u), \quad \tilde{\boldsymbol{Y}}_e = i(\boldsymbol{Y}_d\boldsymbol{c}_e - \boldsymbol{c}_L\boldsymbol{Y}_e),$$

$$\tilde{c}_{GG} = c_{GG} + \frac{1}{2}\,\text{Tr}(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\boldsymbol{c}_Q), \quad \tilde{c}_{WW} = c_{WW} - \frac{1}{2}\,\text{Tr}(N_c\boldsymbol{c}_Q + \boldsymbol{c}_L), \quad \tilde{c}_{BB} = c_{BB} + \text{Tr}\left[(\frac{4}{3}\boldsymbol{c}_u + \frac{1}{3}\boldsymbol{c}_d - \frac{1}{6}\boldsymbol{c}_Q + \boldsymbol{c}_e - \boldsymbol{c}_L)\right].$$

Appearance of new operators in the non-derivative basis

- The presence of the weak operators modifies the usual SM equations of motion for the fermions, and when we use integration by parts on the derivative-involved ALP–fermion operator together with these modified equations, new higher-dimensional operators appear:

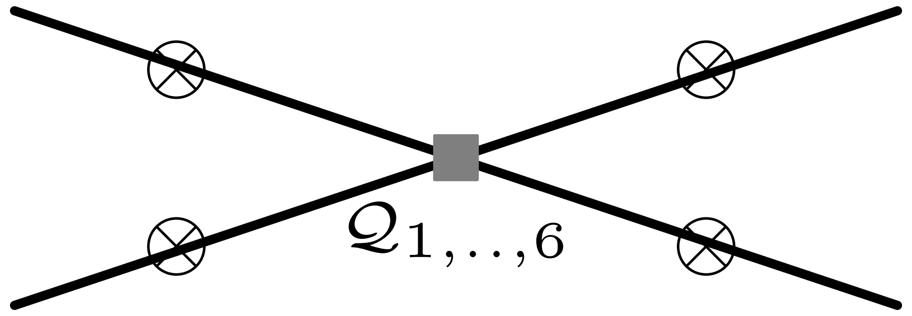
$$\mathcal{Q}_{1,2} = -i\frac{a}{f^2}(k_S - k_B)(\bar{s}_L\gamma_\mu p_L)(\bar{p}_L\gamma_\mu b_L)$$

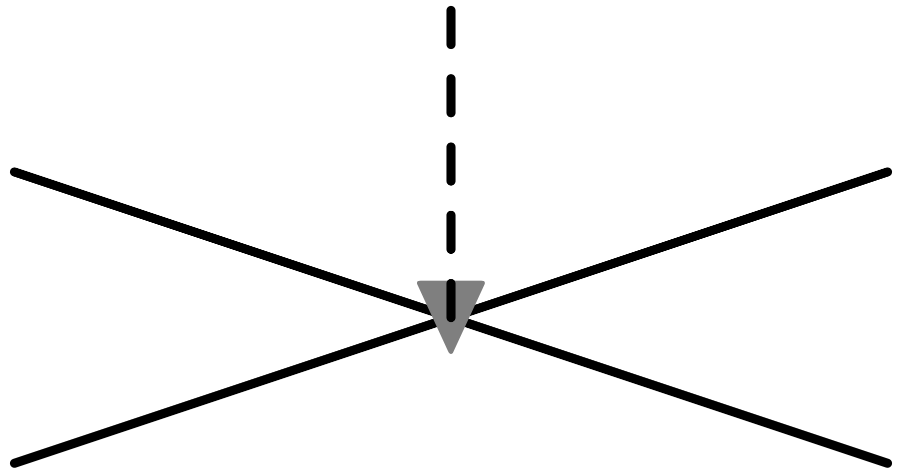
$$\mathcal{Q}_{3,4} = -i\frac{a}{f^2}(k_S - k_B)(\bar{s}_L\gamma_\mu b_L)\sum_q(\bar{q}_L\gamma_\mu q_L)$$

$$\mathcal{Q}_{5,6} = -i\frac{a}{f^2}(k_S - k_B)(\bar{s}_L\gamma_\mu b_L)_{(V-A)}\sum_q(\bar{q}_R\gamma_\mu q_R)\,.$$

$p = u, c$
$q = u, d, c, s, b$

  - Weak annihilation diagrams to consider in the non-derivative basis





Conclusions:

- Axions and ALPs belong to a class of well-motivated light BSM particles with weak couplings to the Standard Model.

- Rare meson decays like $B^+ \to K^+ a$, provide strong bounds for ALPs of few GeV mass range.

- We separate short-distance physics from long-distance, non-perturbative physics through QCD factorization approach.

- At the low energy scale, flavour-violating coupling is more constrained than flavour-conservaing ones.

- If we consider flavour universal coupling at the UV scale, our bounds are stronger up to two orders of magnitude in case of couplings to gluons and up-quarks.

- If we work in the non-derivative basis of the ALP operators along with weak effective operators, we would need to add new higher dimensional operators in our calculation as well, to be able to get the complete result.

Thank you for your attention!