TECHNISCHE UNIVERSITÄT DRESDEN

INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS

On-shell Renormalization of Vector-like Lepton Models

DESY Theory Workshop - 2025

Kilian Möhling ▪ Dominik Stöckinger ▪ Hyejung Stöckinger-Kim

based on: JHEP 10 (2024) 170 [2407.09421]

TU Dresden, Institut für Kern- und Teilchenphysik

Hamburg, DESY, 24.09.2025

Motivation

Standard Model: *chiral* fermions, e.g. $l_L \equiv (\nu_L, e_L) \in \mathbf{2}_{-1/2}$ and $e_R \in \mathbf{1}_{-1}$

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Special case: coupling to l_L or e_R and Φ $\Longrightarrow$ *vector-like leptons* (VLL)

Name	N	E	L	$L_{3/2}$	N^a	E^a
Rep.	$\mathbf{1}_0$	$\mathbf{1}_{-1}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-3/2}$	$\mathbf{3}_0$	$\mathbf{3}_{-1}$

Phenomenology at leading order

Combinations of VLL lead to additional *breaking* of lepton chiral symmetry, e.g.

$$L \oplus E : \qquad \mathcal{L} \supset -\lambda_E \overline{l_L} \mathsf{P}_R E \Phi - \lambda_L \overline{L} \mathsf{P}_R e_R \Phi - \overline{L} \Big(\lambda \, \mathsf{P}_R + \bar{\lambda} \, \mathsf{P}_L \Big) E \Phi$$

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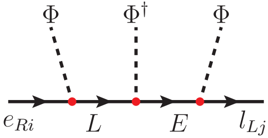

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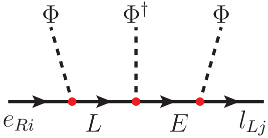

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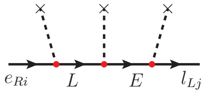
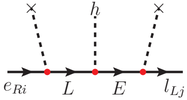
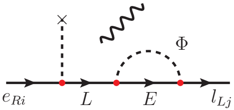
$\xrightarrow{\text{EWSB}}$



$$\Delta m_{ij} \approx -\frac{\lambda_E^i \bar{\lambda} \lambda_L^j v^3}{M_E M_L}$$



$$\Delta \lambda_{ij} \approx -3 \frac{\lambda_E \bar{\lambda} \lambda_L v^2}{M_E M_L}$$



$$\Delta a_i \approx -\frac{\lambda_E^i \bar{\lambda} \lambda_L^i v m_\mu}{16\pi^2 M_E M_L}$$

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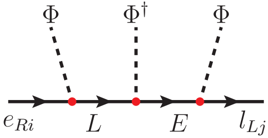

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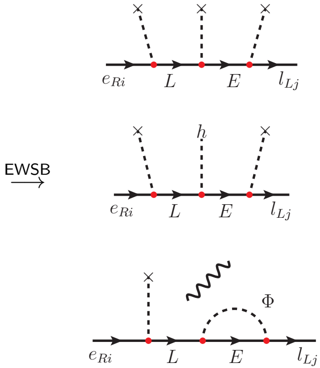

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$\Longrightarrow$ strong correlations between chirality-flipping observables

Kannike et al. [1111.2551], Dermisek et al. [1305.3522]

LHC: Measurement of muon–Higgs coupling

$$R_{\mu\mu} = \frac{\Gamma(h \to \mu\mu)}{\Gamma(h \to \mu\mu)_{\mathsf{SM}}} = 1.21 \pm 0.35$$

and mass-exclusion limits ATLAS [2411.07143]

$$M_L \gtrsim 1270 \text{ GeV}, \; M_E \gtrsim 400 \text{ GeV}$$

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Model prediction $(L \oplus E)$

$$\Delta\lambda_{\mu\mu}, \Delta a_\mu \propto \frac{\lambda_E \bar{\lambda} \lambda_L}{M_E M_L}$$

$$R_{\mu\mu} = \left| 1 + \frac{\Delta\lambda_{\mu\mu}}{\lambda^{\text{SM}}_{\mu\mu}} \right|^2 \approx \left| 1 - 0.87 \frac{\Delta a_\mu}{10^{-9}} \right|^2$$

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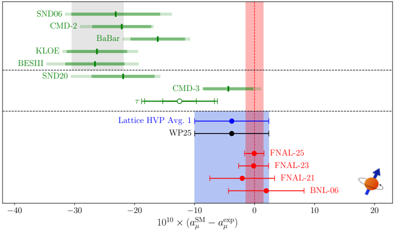

Muon g-2 WP25 Phys.Rept. 1143 (2025) 1-158 [2505.21476]

$\Longrightarrow$ see talk by Gilberto Colangelo tomorrow

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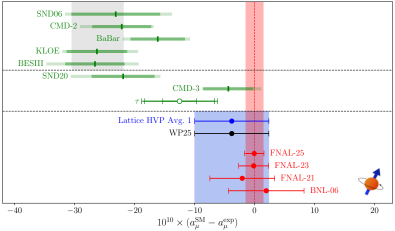

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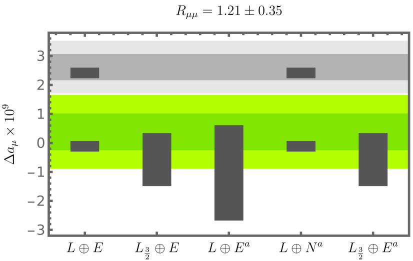

P. Athron, KM, D. Stöckinger, H. Stöckinger-Kim [2507.09289]

Why go beyond leading order?

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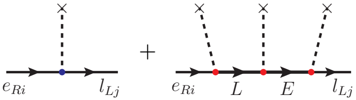
SM Lepton **mass** at tree-level



$$+ \quad \ldots \quad \sim \quad y_{ij} v + \frac{\lambda_L^i \bar{\lambda} \lambda_E^j}{M_E M_L} v^3 + \mathcal{O}(v^5)$$

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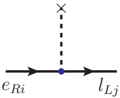
SM Lepton **mass** at tree-level vs one-loop



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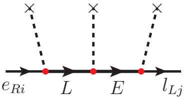

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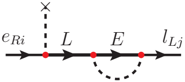

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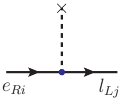

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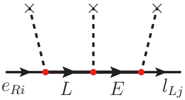

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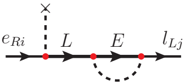

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mass suppression "looses" to loop-suppression when

$$\frac{16\pi^2 v^2}{M^2} \lesssim 1 \qquad \Leftrightarrow \qquad M \gtrsim \mathcal{O}(1\ \text{TeV})$$

$\Rightarrow$ what about other observables?

- e.g. muon-Higgs coupling: LHC at $\mathcal{O}(10\%)$ vs tree-level correction $\mathcal{O}(100\%)$

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⟶ after EWSB mixing between (e_L, L_L^-, E_L) and (e_R, E_R, L_R^-)

$$\mathscr{M}^- = \begin{array}{c} \\ \bar{e}_{Li} \\ \bar{L}_L^- \\ \bar{E}_L \end{array} \begin{array}{c} \begin{matrix} e_{Rj} & E_R & L_R^- \end{matrix} \\ \begin{pmatrix} y^e_{ij} v & \lambda^i_E v & 0 \\ \lambda^j_L v & \lambda v & M_L \\ 0 & M_E & \bar{\lambda} v \end{pmatrix} \end{array} \quad \Longrightarrow \quad U_L^{-\dagger} \mathscr{M}^- U_R^- = \mathrm{diag}(m_i)$$

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Problem: $m_{e,\mu,\tau}$ fixed $\hookrightarrow$ requires (numerical) inversion of SVD to solve for y_e

On-shell scheme: motivation

$\Longrightarrow$ calculation of lepton – Higgs/ Z coupling at one-loop requires *practicable* renormalization scheme

Major issues

- obtaining correct SM lepton masses require solving SVD for y_i^e at tree-level. one-loop mass shift Δm_i makes this tedious / less efficient
- SM-VLL couplings induces off-diagonal mass corrections Δm_{ij} and mixing at external legs $\Longrightarrow$ non-trivial LSZ normalization $\mathcal{Z}_{ij}$
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Solution: $\Longrightarrow$ on-shell scheme KM et al. [JHEP 10 (2024) 170]

$\equiv i\Sigma_{ba}(p)$:

ren. cond. for $p^2 \to m_a^2$

$$\underbrace{\widetilde{\mathrm{Re}}\,\Sigma_{ba}(p) u_a(p) = 0,}_{\text{removes mixing and } \Delta m} \qquad \underbrace{\frac{1}{\not{p} - m_a}\widetilde{\mathrm{Re}}\,\Sigma_{aa} u(p) = 0}_{\text{enforces } \mathcal{Z} = \mathbb{1}}$$

On-shell mass renormalization

Compare to:

- **QED:** 1 fundamental parameter m fixed by OS condition
- **SM quark sector:** fundamental parameters $y_{ij} \leftrightarrow m_i$ and V_{CKM}
- **2HDM:** 4 scalar masses + mixing angle α (+...)

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$\implies$ OS conditions fix 5 of in total 13 parameter ren. consts.

fixed by OS $\rightarrow$

$$\delta m_{aa} = \left[U_L^{-\dagger} \begin{pmatrix} \delta(y^e v) & \delta(\lambda_E^i v) & 0 \\ \delta(\lambda_L^i) & \delta(\lambda v) & \delta M_L \\ 0 & \delta M_E & \delta(\bar{\lambda} v) \end{pmatrix} U_R^- \right]_{aa}$$

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fixed by OS (δm_{aa})

free ren. consts. (δM_E etc.)

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fixed by OS → δm_{aa}

free ren. consts. → δM_E, δM_L, $\delta(y^e v)$, $\delta(\lambda_L^i)$

e.g. $\overline{\text{MS}}$ → $\delta(\bar{\lambda} v)$, $\delta(\lambda v)$, $\delta(\lambda_E^i v)$

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fixed by OS (δm_{aa})

fixed at tree-level ($U_L^{-\dagger}$, U_R^-)

free ren. consts. (δM_E, ...)

e.g. $\overline{\mathsf{MS}}$ ($\delta(\bar{\lambda} v)$, ...)

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  $\hookrightarrow 2 \times 2$ *mixing matrix* $\implies$ reparametrization in terms of α and m_i

for VLL: 5×5 mixing matrix = better avoid explicit parametrization

$\implies$ OS conditions fix 5 of in total 13 parameter ren. consts.

$$\delta m_{aa} = \left[U_L^{-\dagger} \begin{pmatrix} \delta(y^e v) & \delta(\lambda_E^i v) & 0 \\ \delta(\lambda_L^i) & \delta(\lambda v) & \delta M_L \\ 0 & \delta M_E & \delta(\bar{\lambda} v) \end{pmatrix} U_R^- \right]_{aa}$$

fixed by OS → δm_{aa}

fixed at tree-level → $U_L^{-\dagger}$, U_R^-

free ren. consts. → δM_L, δM_E, $\delta(y^e v)$

e.g. $\overline{\text{MS}}$ → $\delta(\lambda_E^i v)$, $\delta(\lambda_L^i)$, $\delta(\lambda v)$, $\delta(\bar{\lambda} v)$

$\implies$ much simpler to disentangle and easy to solve (linear equation...)

but: unlike SM or 2HDM → off-diagonal δm_{ab} remain

Muon–Higgs coupling at one-loop

$$\lambda_{\mu\mu}^{\mathsf{eff}} = \lambda_{\mu\mu}^{\mathsf{tree}}\Big(1 - \alpha B\Big) + \Gamma_{\mu\mu}^{1\ell}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma_{\mu\mu}^{ct}$$

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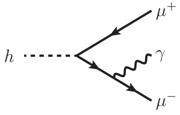
real radiation
(IR div.)



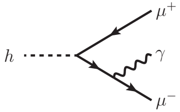
real radiation

Muon–Higgs coupling at one-loop

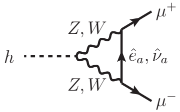
$$\lambda_{\mu\mu}^{\text{eff}} = \lambda_{\mu\mu}^{\text{tree}}\left(1-\alpha B\right) + \Gamma_{\mu\mu}^{1\ell}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma_{\mu\mu}^{ct}$$

real radiation (IR div.) → αB

genuine 1ℓ diagrams (UV + IR div.) → $\Gamma_{\mu\mu}^{1\ell}$



real radiation



genuine one-loop

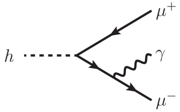
Muon–Higgs coupling at one-loop

$$\lambda_{\mu\mu}^{\text{eff}} = \lambda_{\mu\mu}^{\text{tree}}\left(1-\alpha B\right) + \Gamma_{\mu\mu}^{1\ell}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma_{\mu\mu}^{ct}$$

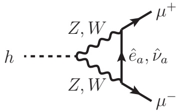
real radiation (IR div.)

genuine 1ℓ diagrams (UV + IR div.)

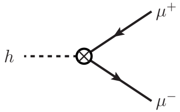
counterterm contribution (UV + IR div.)



real radiation



genuine one-loop



counterterm

$$\lambda_{\mu\mu}^{\text{eff}} = \lambda_{\mu\mu}^{\text{tree}}\Big(1 - \alpha B\Big) + \Gamma_{\mu\mu}^{1\ell}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma_{\mu\mu}^{ct}$$

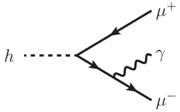
real radiation (IR div.) → αB

genuine 1ℓ diagrams (UV + IR div.) → $\Gamma_{\mu\mu}^{1\ell}$

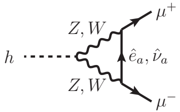
counterterm contribution (UV + IR div.) → $\delta\Gamma_{\mu\mu}^{ct}$

Important checks

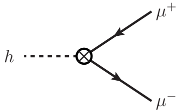
- cancellation of UV and IR divergences ✓
- cancellation of residual renormalization scale dependence ✓
- decoupling behaviour in physical observables ✓



real radiation



genuine one-loop



counterterm

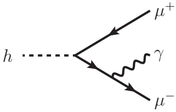
Muon–Higgs coupling at one-loop

$$\lambda_{\mu\mu}^{\text{eff}} = \lambda_{\mu\mu}^{\text{tree}} \left(1 - \alpha B\right) + \Gamma_{\mu\mu}^{1\ell}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma_{\mu\mu}^{ct}$$

real radiation (IR div.)

genuine 1ℓ diagrams (UV + IR div.)

counterterm contribution (UV + IR div.)



real radiation

Important checks

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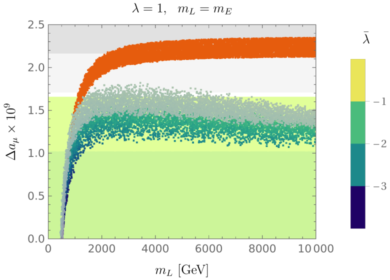
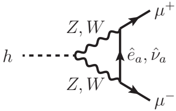
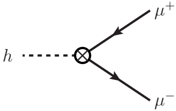




genuine one-loop



counterterm

Summary & Outlook

- VLL have strong impact on Higgs and flavour (violating) physics and upcoming experiments will extensively probe the interesting parameter regions.
- Era of high precision measurements $\rightleftarrows$ era of high precision theory
  $\hookrightarrow$ first step: (on-shell) renormalization scheme ✓
- **Next:** application to EW precision and LFV observables *and*
  complete NLO correlation: calculation of Δa_μ at NLO (2-loop)

Thank you!

Backup

Backup - Current and Future Experimental Bounds

Collider constraints

Higgs	Z
$\text{BR}(h \to \mu e) < 4.4 \times 10^{-5}$	$\text{BR}(Z \to \mu e) < 2.6 \times 10^{-7}$
$\text{BR}(h \to \tau e) < 2.0 \times 10^{-3}$	$\text{BR}(Z \to \tau e) < 5.0 \times 10^{-6}$
$\text{BR}(h \to \tau\mu) < 1.5 \times 10^{-3}$	$\text{BR}(Z \to \tau\mu) < 6.5 \times 10^{-6}$

LFV decays

$\ell \to \ell'\gamma$	$\ell \to 3\ell'$
$\text{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$	$\text{BR}(\mu \to 3e) < 1.0 \times 10^{-12}$
$\text{BR}(\tau \to \ell\gamma) \lesssim 4 \times 10^{-8}$	$\text{BR}(\tau \to 3\ell) \lesssim 2 \times 10^{-8}$

$\mu \to e$ conversion

$$\Gamma(\mu^- \text{Au} \to e^- \text{Au})/\Gamma_{\text{capt}}^{\text{Au}} < 7 \times 10^{-13}$$
$$\Gamma(\mu^- \text{Ti} \to e^- \text{Ti})/\Gamma_{\text{capt}}^{\text{Ti}} < 4.3 \times 10^{-12}$$

Future sensitivities:

$\Gamma(\mu^- \text{Al} \to e^- \text{Al})/\Gamma_{\text{capt}}^{\text{Au}} \sim 6 \times 10^{-16}$

$\text{BR}(\mu \to e\gamma) \sim 6 \times 10^{-14}$

$\text{BR}(\mu \to 3e) \sim 10^{-16}$

On-shell scheme: set-up

Example: $L \oplus E$ multiplets with same QN $\to \varepsilon_{Ra} = (e_{Ri}, E_R)$ and $\ell_{La} = (l_{Li}, L_L)$

$$\mathcal{L} \supset -M_E^a \overline{E}_L \varepsilon_{Ra} - M_L^a \bar{\ell}_{La} L_R - \bar{\ell}_{La} Y_{ab} \varepsilon_{Rb} \Phi - \bar{\lambda} \overline{L}_R E_R \Phi, \qquad Y = \begin{pmatrix} y^e & \lambda_E \\ \lambda_L & \lambda \end{pmatrix}$$

Parameter transformation: $M_i^a \to M_i^a + \delta M_i^a, \quad Y_{ab} \to Y_{ab} + \delta Y_{ab}, \quad \bar{\lambda} \to \lambda + \delta\lambda$

$\Rightarrow$ *Redundancy:* $\ell_L \to V_L \ell_L$ and $\varepsilon_R \to V_R \varepsilon_R$. $V_{L/R}$ can be used to make *gauge-basis* field renormalization hermitian **or** set *bare* $M_i^a = \delta_{a4} M_i$ and $y_{ij}^e = y_i^e \delta_{ij}$

Field transformation: multiplets under broken gauge group

$$\begin{pmatrix} \varepsilon_R \\ L_R^- \end{pmatrix} \to U_R^- Z_R^{\frac{1}{2}} \hat{e}_R, \qquad \begin{pmatrix} \ell_L^- \\ E_L \end{pmatrix} \to U_L^- Z_L^{\frac{1}{2}} \hat{e}_R$$

- unitary 5×5 $U_{L/R}^-$ diagonalize *renormalized* mass matrix

$\Longrightarrow$ off-diagonal mass (Yukawa) renormalization constants

$$\delta m_{ab} = U_L^{-\dagger} \begin{pmatrix} \delta(Yv) & \delta M_L^a \\ \delta M_E^a & \delta(\bar{\lambda} v) \end{pmatrix} U_R^- \neq \delta m_a \delta_{ab}$$

Compared to 2HDM

$$\begin{pmatrix} H \\ h \end{pmatrix} = \mathcal{R}_\alpha \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

2×2 mass diagonalization matrix R_α
$\Longrightarrow$ explicit parametrization in terms α
$\Longrightarrow$ some of the fundamental parameters can traded for pole masses *and* α

$$\underbrace{\widetilde{\mathrm{Re}}\,\Sigma_{ba}(p)u_a(p) = 0,}_{\text{50 equations}} \quad \underbrace{\frac{1}{\not p - m_a}\widetilde{\mathrm{Re}}\,\Sigma_{aa}u(p) = 0,}_{\text{5 equations}}$$

but: 50 Z_{ab} and 2 + 11 parameters

$\Longrightarrow$ OS conditions fix $Z_{L/R}$ and $\delta m_{aa} \equiv \boldsymbol{\delta m}$, but not off-diagonal δm_{ab}

$\Longrightarrow$ similar problem to tree-level m_i vs y_i^e, but now $U_{L/R}$ are already known
$\hookrightarrow$ choose 5 params. (Dirac masses and y_i^e), fix rest by external conditions.

$$\boldsymbol{\delta m} = \boldsymbol{\kappa} \begin{pmatrix} \delta(y^e v) \\ \delta M_L \\ \delta M_E \end{pmatrix} + \boldsymbol{\delta c}$$

fixed by OS ($\boldsymbol{\delta m}$); determined from $U_{L/R}$ ($\boldsymbol{\kappa}$); fundamental ren. const. ($\delta(y^e v)$, δM_L, δM_E); remaining consts. (fixed e.g. in $\overline{\text{MS}}$) ($\boldsymbol{\delta c}$)

$\Longrightarrow$ fundamental ren. consts. are given by $\boldsymbol{\kappa}^{-1}(\boldsymbol{\delta m} - \boldsymbol{\delta c})$ and can be obtained numerically or perturbatively.

Note

- m and δm of leptons with different charge are not independent
  $\hookrightarrow$ OS cond. on all leptons fixes more parameters (e.g. also λ or $\bar{\lambda}$) [JHEP 10 (2024) 170]
- alternatively: leave masses of some leptons (e.g. doubly charged or heavy neutrinos) off-shell and compute one-loop mass shift (preferable e.g. in the triplet models)

On-shell Renormalization Constants: Gauge Sector

$$\widetilde{\mathrm{Re}}\,\Sigma_h(M_h^2) = 0, \qquad \lim_{p^2\to M_h^2} \frac{1}{p^2-M_h^2}\widetilde{\mathrm{Re}}\,\Sigma_h'(M_h^2) = 0, \qquad \text{and}$$

$$\widetilde{\mathrm{Re}}\,\Sigma^{\mu\nu}_{VV'}(q)\epsilon_\nu(q)\Big|_{q^2=M^2_{V'}} = 0, \qquad \lim_{q^2\to M_V^2} \frac{1}{q^2-M_V^2}\Sigma^{\mu\nu}_{VV}(q)\epsilon_\nu(q) = 0$$

The vector two-point function has the general covariant decomposition

$$-i\Sigma^{\mu\nu}_{VV'}(q) = -i\Sigma^T_{VV'}(q^2)\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - i\Sigma^L_{VV'}(q^2)\frac{q^\mu q^\nu}{q^2}$$

Inserting this into the on-shell conditions gives the following renormalization constants at one-loop order

$$\begin{aligned}
\delta Z_h &= -\widetilde{\mathrm{Re}}\,\Sigma_h'(M_h^2) & \delta M_h^2 &= \widetilde{\mathrm{Re}}\,\Sigma_h(M_h^2)\\
\delta Z_{VV} &= -\widetilde{\mathrm{Re}}\,\Sigma^{T\prime}_{VV}(M_V^2), & \delta M_V^2 &= \widetilde{\mathrm{Re}}\,\Sigma^T_{VV}(M_V^2)\\
\delta Z_{AZ} &= -\frac{2}{M_Z^2}\widetilde{\mathrm{Re}}\,\Sigma^T_{AZ}(M_Z^2) & \delta Z_{ZA} &= \frac{2}{M_Z^2}\widetilde{\mathrm{Re}}\,\Sigma^T_{AZ}(0)
\end{aligned}$$

The charge and vev renormalization constants are given by

$$\frac{\delta e}{e} = -\frac{1}{2}\left(\delta Z_{AA} + \frac{s_W}{c_W}\delta Z_{ZA}\right), \qquad \frac{\delta v}{v} = \frac{\delta M_W^2}{2M_W^2} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{2M_Z^2} - \frac{\delta M_W^2}{2M_W^2}\right) - \frac{\delta e}{e}$$

On-shell Renormalization Constants: Lepton Sector

$$\Sigma_{ab}(p) = \Sigma^R_{ab}(p^2)\not{p}\mathsf{P_R} + \Sigma^L_{ab}(p^2)\not{p}\mathsf{P_L} + \Sigma^{SR}_{ab}(p^2)\mathsf{P_R} + \Sigma^{SL}_{ab}(p^2)\mathsf{P_L}.$$

$$\text{for } a = b \begin{cases} (\delta m)_{aa} &= \frac{1}{2}\,\widetilde{\mathrm{Re}}\left[m_a\Sigma^R_{aa} + m_a\Sigma^L_{aa} + \Sigma^{SR}_{aa} + \Sigma^{SL}_{aa}\right]_{p^2=m_a^2} \\ (\delta Z_R)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^R_{aa} + m_a\left(\Sigma^{SR\prime}_{aa} + \Sigma^{SL\prime}_{aa}\right) + m_a^2\left(\Sigma^{R\prime}_{aa} + \Sigma^{L\prime}_{aa}\right)\right]_{p^2=m_a^2} \\ (\delta Z_L)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^L_{aa} + m_a\left(\Sigma^{SR\prime}_{aa} + \Sigma^{SL\prime}_{aa}\right) + m_a^2\left(\Sigma^{R\prime}_{aa} + \Sigma^{L\prime}_{aa}\right)\right]_{p^2=m_a^2} \end{cases}$$

$$\text{for } a \neq b \begin{cases} (\delta Z_R)_{ab} &= \frac{2}{m_a^2-m_b^2}\Big[m_b^2\,\widetilde{\mathrm{Re}}\,\Sigma^R_{ab} + m_a m_b\,\widetilde{\mathrm{Re}}\,\Sigma^L_{ab} + m_a\,\widetilde{\mathrm{Re}}\,\Sigma^{SR}_{ab} \\ &\qquad + m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{SL}_{ab} - m_a(\delta m)_{ab} - m_b(\delta m^\dagger)_{ab}\Big]_{p^2=m_b^2} \\ (\delta Z_L)_{ab} &= \frac{2}{m_a^2-m_b^2}\Big[m_b^2\,\widetilde{\mathrm{Re}}\,\Sigma^L_{ab} + m_a m_b\,\widetilde{\mathrm{Re}}\,\Sigma^R_{ab} + m_a\,\widetilde{\mathrm{Re}}\,\Sigma^{SL}_{ab} \\ &\qquad + m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{SR}_{ab} - m_a(\delta m^\dagger)_{ab} - m_b(\delta m)_{ab}\Big]_{p^2=m_b^2} \end{cases}.$$

Muon–Higgs coupling at one-loop

Impact on correlation: *SMEFT* $\to$ coefficients at one-loop

$$y_\mu^{\text{SMEFT}} = y_\mu + \underbrace{\mathcal{O}\left(\frac{\lambda_i^3}{16\pi^2}\right)}_{1\ell}, \qquad C_{e\Phi}^{\mu\mu} = \frac{\lambda_i^3 v^2}{M^2}\left[\underbrace{\ 1\ }_{\text{tree-level}} + \underbrace{\mathcal{O}\left(\frac{\lambda_i^2}{16\pi^2}\right)}_{1\ell}\right]$$

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$\implies$ (very) large correction to y_μ^{SMEFT} cancels between y_μ^{eff} and m_μ

$$\begin{aligned} \frac{m_\mu}{v} &= y_\mu^{\text{SMEFT}} + C_{e\Phi}^{\mu\mu} v^2 \\ \lambda_{\mu\mu}^{\text{eff}} &= y_\mu^{\text{SMEFT}} + 3 C_{e\Phi}^{\mu\mu} v^2 \end{aligned} \implies \boxed{\frac{\lambda_{\mu\mu}^{\text{eff}}}{\lambda_\mu^{\text{SM}}} = 1 - 0.87\frac{\Delta a_\mu}{10^{-9}}\left(1 + \mathcal{O}(1\ell)\right)}$$

Muon–Higgs coupling at one-loop

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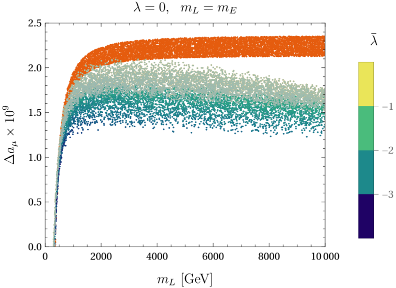
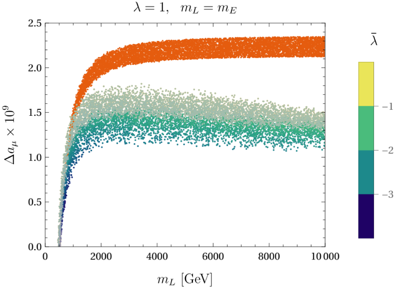
$\Longrightarrow$ (very) large correction to y_μ^{SMEFT} cancels between y_μ^{eff} and m_μ

$$\frac{m_\mu}{v} = y_\mu^{\text{SMEFT}} + C_{e\Phi}^{\mu\mu} v^2$$
$$\lambda_{\mu\mu}^{\text{eff}} = y_\mu^{\text{SMEFT}} + 3C_{e\Phi}^{\mu\mu} v^2$$

$$\Longrightarrow \quad \boxed{\frac{\lambda_{\mu\mu}^{\text{eff}}}{\lambda_\mu^{\text{SM}}} = 1 - 0.87\,\frac{\Delta a_\mu}{10^{-9}}\left(1 + \mathcal{O}(1\ell)\right)}$$

BUT: remaining one-loop corrections are still significant





Muon–Higgs coupling at one-loop

$\Longrightarrow$ remaining large loop effects e.g. from corrections to δv

$$\frac{\delta v}{v}\bigg|_{\mathsf{VLL}} \sim \frac{1}{16\pi^2} \sum_i |\lambda_i|^2 \ln\left(\frac{M}{m_h}\right)$$

$+$ enhancement from mixing

$$y_{\mu\mu}^{\mathsf{eff}}\bigg|_{\delta v} \simeq C_{e\Phi}^{\mu\mu} v^2 \times 6\,\tfrac{\delta v}{v}$$

Leading correction $(L \oplus E)$

$$\frac{y_{\mu\mu}^{\mathsf{eff}}}{y_\mu^{\mathsf{SM}}}\bigg|_{1\ell} \simeq \frac{\lambda_E^\mu \bar{\lambda} \lambda_L^\mu v^3}{16\pi^2 M^2 m_\mu} \left[\sum_i \lambda_i^2 - 12\bar{\lambda}\lambda\right] \ln\left(\frac{M}{m_h}\right)$$