Renormalisation scheme dependence of the trilinear Higgs coupling in extended scalar sectors

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DESY.

Motivation: Trilinear Coupling

[Bahl, Braathen, Gabelmann, Weiglein 2305.03015]

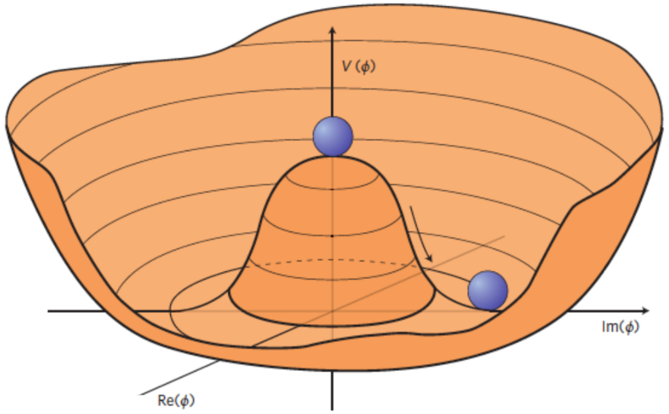
- Trilinear Higgs coupling determines the shape of the Higgs potential:
  - crucial for understanding dynamics of electroweak phase transitions

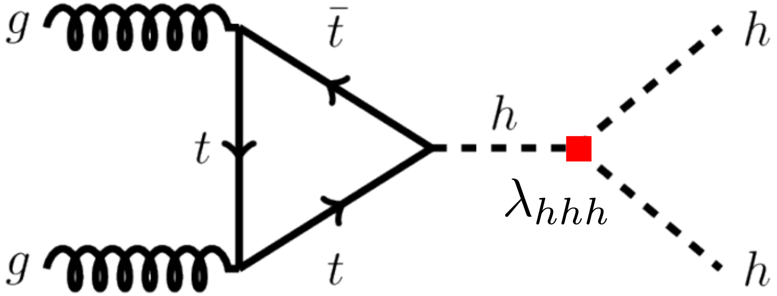
$$V_{\mathrm{SM}} \supset \frac{m_h^2}{2} h^2 + \underbrace{\frac{3m_h^2}{v}}_{\lambda_{hhh}^{\mathrm{SM},0}/6} h^3 + \frac{3m_h^2}{v^2} h^4$$

  - sensitive to BSM physics
  - exp. limit: $-1.2 < \frac{\lambda_{hhh}}{\lambda_{hhh}^{\mathrm{SM},0}} < 7.2$ at 95% C.L.

[ATLAS, 2406.09971]

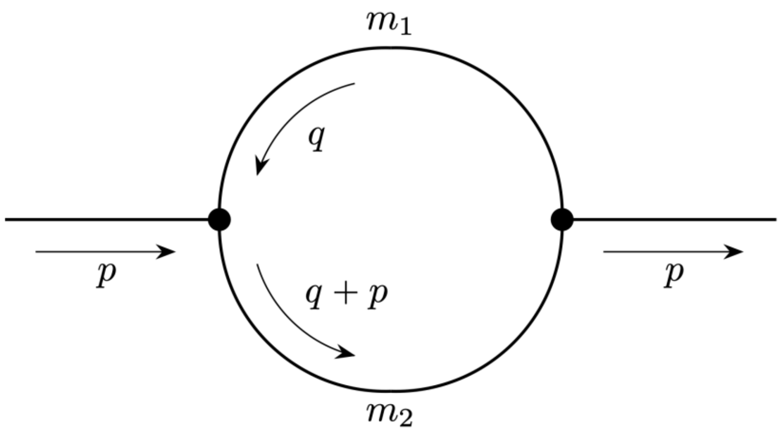
[CMS, 2407.13554]





Motivation: Precision Calculations

- reliable and numerically stable theoretical predictions of physical observables are imperative
  - important for comparison with experimental data
  - needed for consistent constraints on parameter space of BSM models



understanding of renormalisation scheme dependence needed

Singlet Extension of the Standard Model (SSM)

[Bahl, Braathen, Gabelmann, Weiglein 2305.03015]

- we consider the real singlet extension of the SM

$$V(\Phi, S) = \mu^2 |\Phi|^2 + \frac{\lambda}{2} |\Phi|^4 + \kappa_{SH} |\Phi|^2 S + \frac{\lambda_{SH}}{2} |\Phi|^2 S^2 + \frac{m_S^2}{2} S^2 + \frac{\kappa_S}{3} S^3 + \frac{\lambda_S}{2} S^4$$

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- after symmetry breaking: $\Phi = \frac{1}{\sqrt{2}} \binom{\sqrt{2} G^+}{v+h+iG}$ $\quad S = s + v_S$
- the fields mix to 2 CP-even eigenstates $h_{1,2}$
- the Lagrangian parameters are traded for $\boxed{m_{h_1}, m_{h_2}, \alpha, v, v_S, \kappa_S, \kappa_{SH}}$

5 new parameters

- trilinear coupling at tree level: $\lambda_{h_1 h_1 h_1} = 3 m_{h_1}^2 \left(\frac{\cos^3 \alpha}{v} + \frac{\sin^3 \alpha}{v_S} \right)$

Choice of Renormalization scheme

OS renormalisation

- renormalised parameters correspond to measurable quantities
- OS condition: $m_0^2 = m_{\mathrm{OS}}^2 + \delta m_{\mathrm{OS}}^2$

$$\hat{\Sigma}\left(p^2 = m^2\right) = 0 \Rightarrow \delta m_{\mathrm{OS}}^2 = \Sigma(p)|_{p^2=m^2}$$

renormalised self energy

self energy

Choice of Renormalization scheme

OS renormalisation	**$\overline{\text{MS}}$ renormalisation**
• renormalised parameters correspond to measurable quantities • OS condition: $m_0^2 = m_{\text{OS}}^2 + \delta m_{\text{OS}}^2$ $\hat{\Sigma}\left(p^2 = m^2\right) = 0 \Rightarrow \delta m_{\text{OS}}^2 = \Sigma(p)\big\rvert_{p^2=m^2}$ renormalised self energy self energy	• no conditions $\longrightarrow$ only UV divergent part and constants are subtracted $\Delta = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi$ • μ dependence $\longrightarrow$ running of parameters

$$m_B = m^{\overline{\text{MS}}}(\mu) + \delta m^{\overline{\text{MS}}}(\mu) = m^{\text{OS}} + \delta m^{\text{OS}}$$

Constructed renormalisation schemes

<table>
<tr><th></th><th>m_{h_1}</th><th>m_{h_2}</th><th>α</th><th>κ_S</th><th>κ_{SH}</th><th>v_S</th><th>Tads</th></tr>
<tr><td>OS</td><td colspan="3">OS</td><td colspan="2">MS</td><td>-</td><td>OS</td></tr>
<tr><td>MS</td><td colspan="5">MS</td><td>-</td><td>OS</td></tr>
<tr><td>OSMSalpha</td><td colspan="2">OS</td><td colspan="3">MS</td><td>-</td><td>OS</td></tr>
<tr><td>OSkappas</td><td colspan="5">OS</td><td>-</td><td>OS</td></tr>
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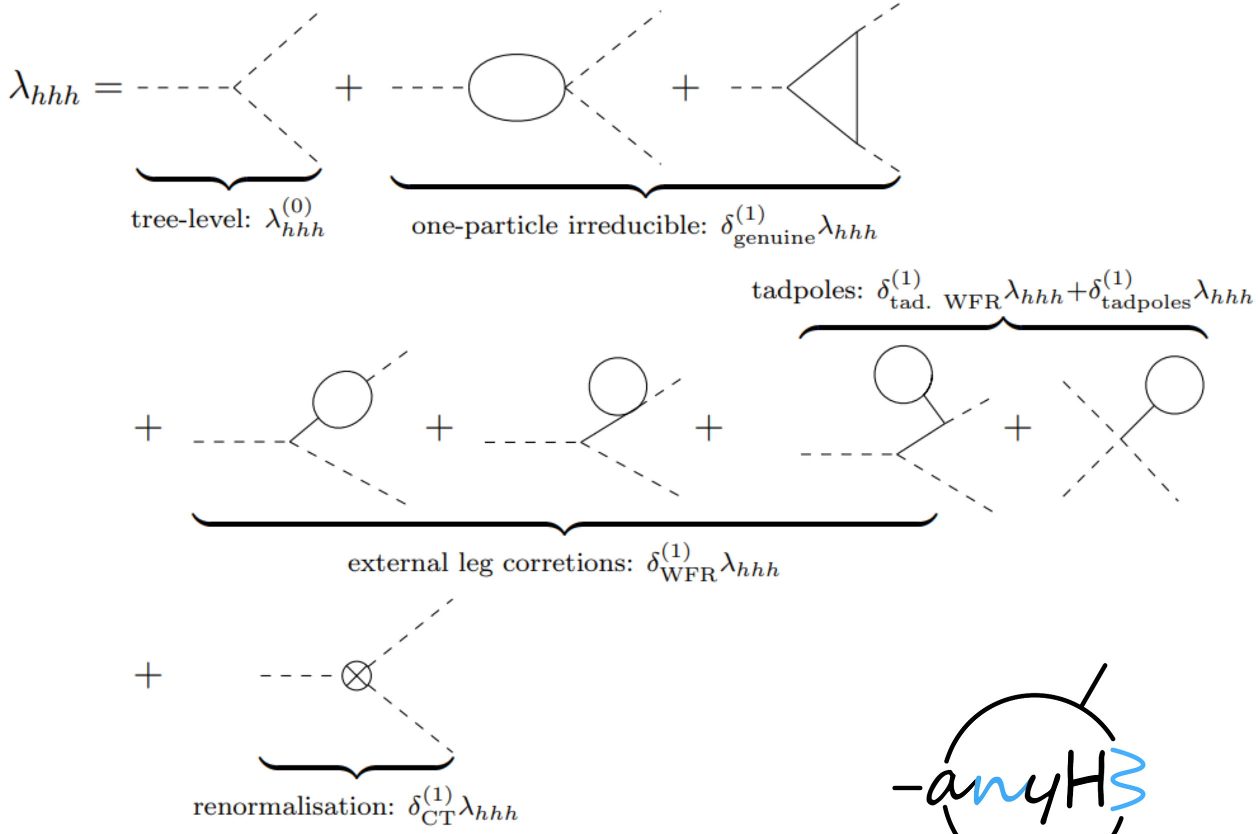
Computing the Trilinear Coupling: anyH3

[Bahl, Braathen, Gabelmann, Weiglein; 2305.03015]

- Python code anyH3 allows fast computation of λ_{hhh} at full one-loop level
- customize:
  - parameters/couplings
  - renormalisation
  - tadpole treatment
- applicable to all renormalisable models via UFO files

  [Degrande et al.; 1108.2040]

  [Darmé et al.; 2304.09883]



Analytic approximations

Applied simplifications:

1. set of particles: $h_1, h_2, G^\pm, G^0$
2. massless Goldstones
3. gaugeless limit: $m_W, m_Z \to 0$
4. α expansion
5. $m_{h_1} = 0 \to$ BSM contribution

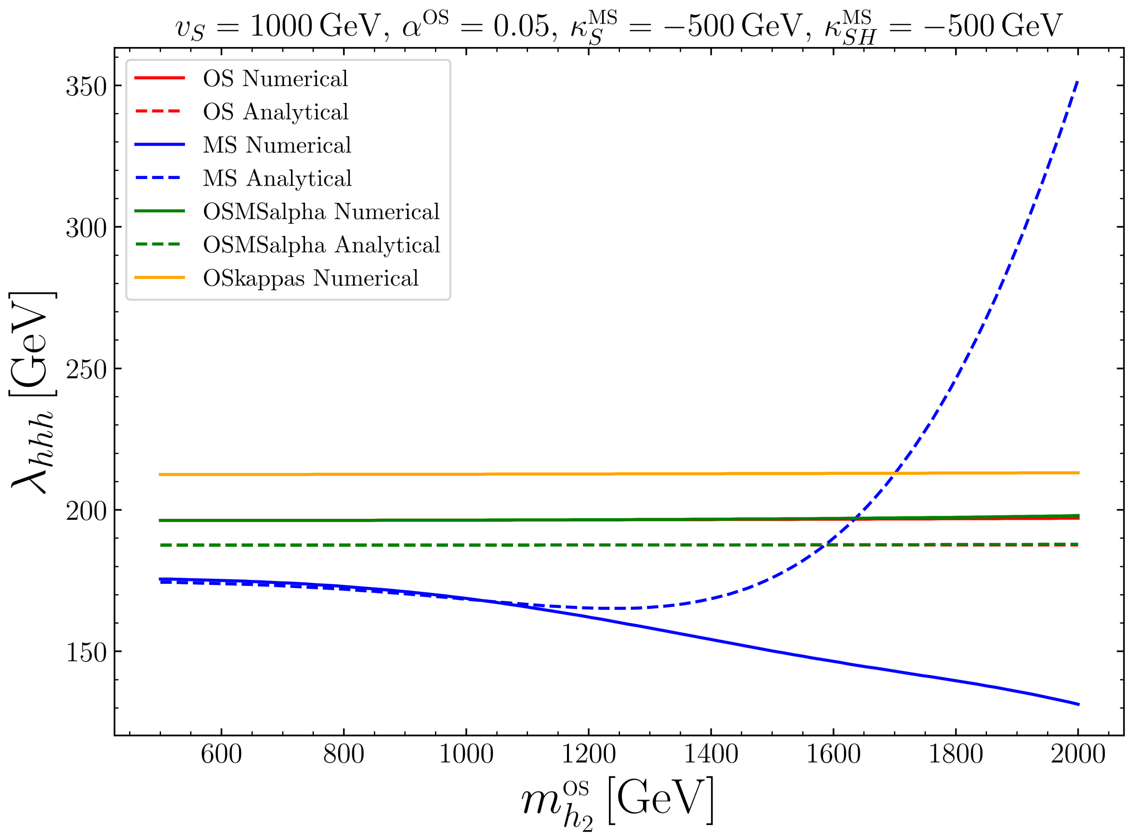
- Additional treelevel expression derived

$$\lambda_{hhh}^{OS,(1)} = \frac{\kappa_{SH}^2 v^2}{64 m_{h_2}^2 \pi^2 v_S^4}\Big[6\alpha m_{h_2}^2(-4+\sqrt{3}\pi)v_S - 2\kappa_{SH} v_S v - \alpha(-3+\sqrt{3}\pi)\big(2\kappa_S v_S^2 - 3\kappa_{SH} v^2\big)\Big]$$

$$\begin{aligned}\lambda_{hhh}^{MS,(1)} = \frac{1}{64 m_{h_2}^2 \pi^2 v_S^4 v^2}\Big\{ & 6\alpha m_{h_2}^6 v_S^3 + 6\kappa_{SH} m_{h_2}^4 v_S^3 v \\ & - 6\alpha\kappa_{SH}^2 v_S(-2m_{h_2}^2 + \kappa_S v_S)v^4 - 2\kappa_{SH}^3 v_S v^5 + 9\alpha\kappa_{SH}^3 v^6 \\ & - 6\Big[\kappa_{SH} m_{h_2}^2 v_S^2 v(m_{h_2}^2 v_S - \kappa_{SH} v^2) \\ & + \alpha\big(m_{h_2}^6 v_S^3 + 2\kappa_{SH} m_{h_2}^2 v_S^2(2m_{h_2}^2 - \kappa_S v_S)v^2 \\ & + \kappa_{SH}^2 v_S(9m_{h_2}^2 - 2\kappa_S v_S)v^4 + 3\kappa_{SH}^3 v^6\big)\Big] \cdot \ln\left(\frac{m_{h_2}^2}{\mu^2}\right)\Big\}\end{aligned}$$

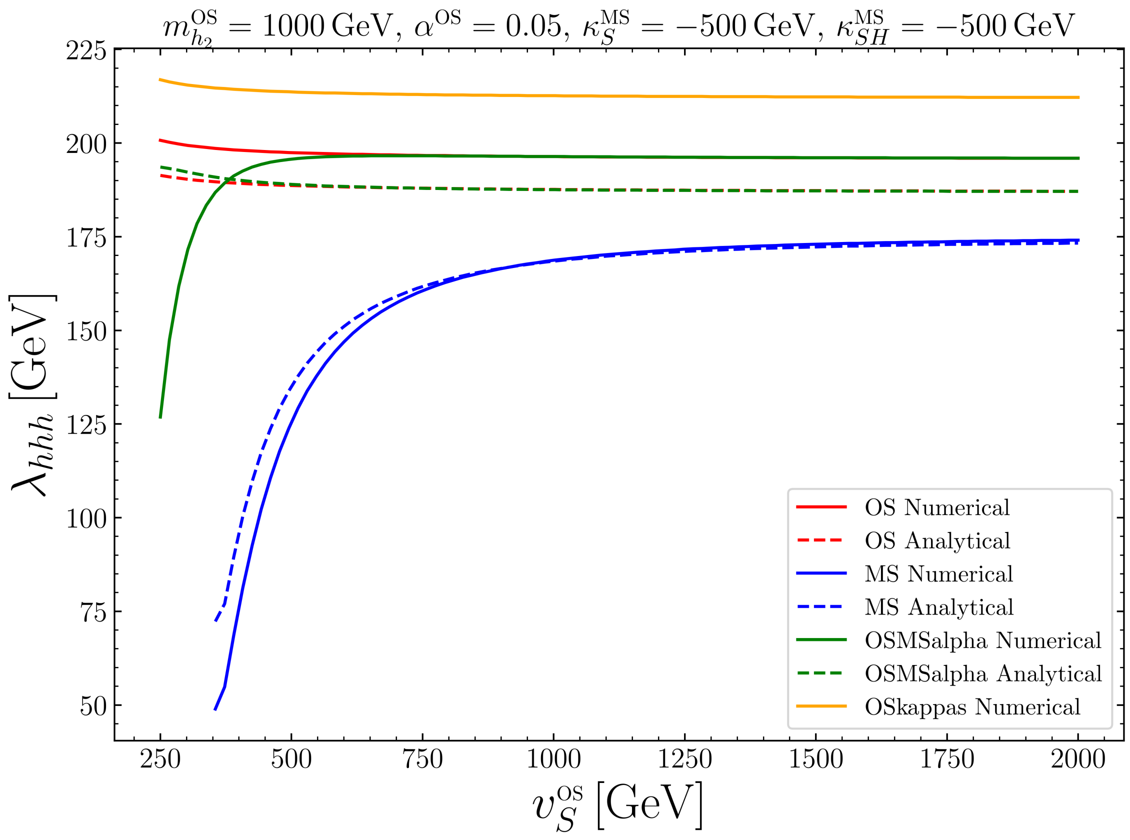
Scheme Dependence of the Trilinear Coupling

- different OS schemes show good
- constant shift between numerical and analytical calculation
- MS result differs for large BSM masses



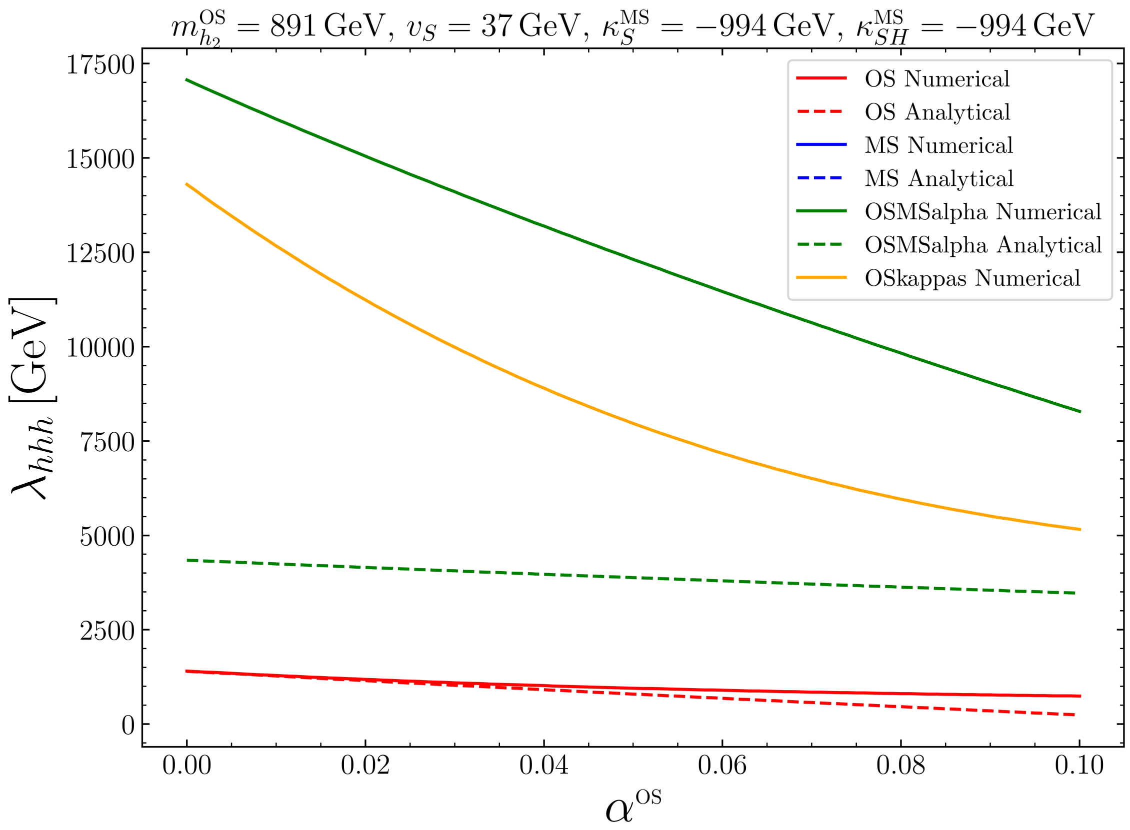
Scheme Dependence of the Trilinear Coupling

- high sensitivity to small singlets vevs
- MS conversion breaks down
- in general small deviations



Scheme Dependence of the Trilinear Coupling

- benchmark points with large corrections show a much greater uncertainty
- especially for small singlet vevs



Outlook

- Investigations of the renormalisation scheme dependence of the trilinear coupling in the SSM, derivation of analytic formulas
- Generally, larger differences between the schemes for large corrections to the coupling
- Extension to other BSM models e.g. 2HDM

Goal: derive a quantitative description of the uncertainty from the renormalisation scheme dependence $\longrightarrow$ deliver a robust, consistent prediction for λ_{hhh}

Backup Slides

Tadpole Treatment

- Tadpole-free $\overline{\text{MS}}$ scheme:

$$T_h = t_h + \delta^{(1)} t_h = 0 \Rightarrow t_h = -\delta^{(1)} t_h = -\left.\frac{\partial V^{(1)}}{\partial h}\right|_{\text{min}}$$

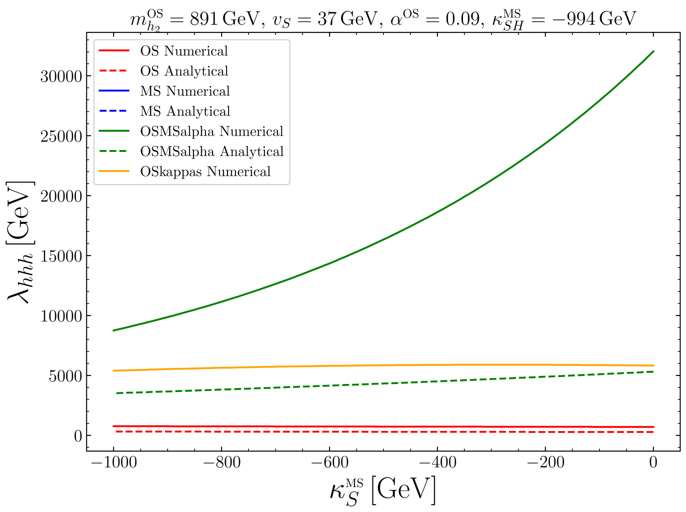
- OS tadpole renormalisation:

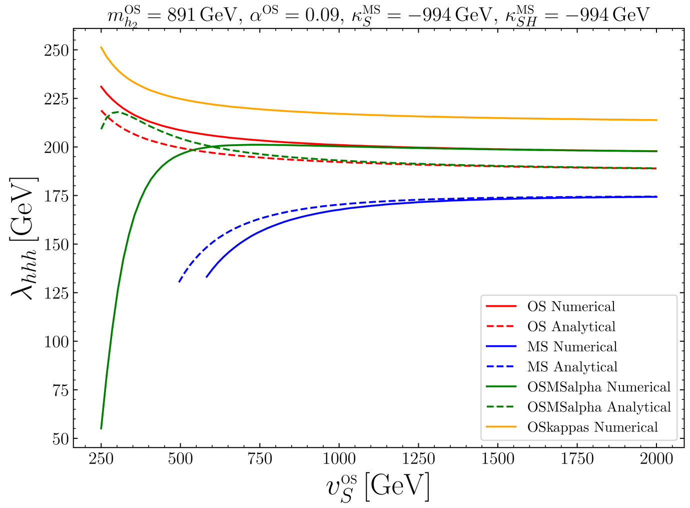
$$t_h = 0 \qquad \delta_{\text{CT}}^{(1)} t_h = -\delta^{(1)} t_h$$

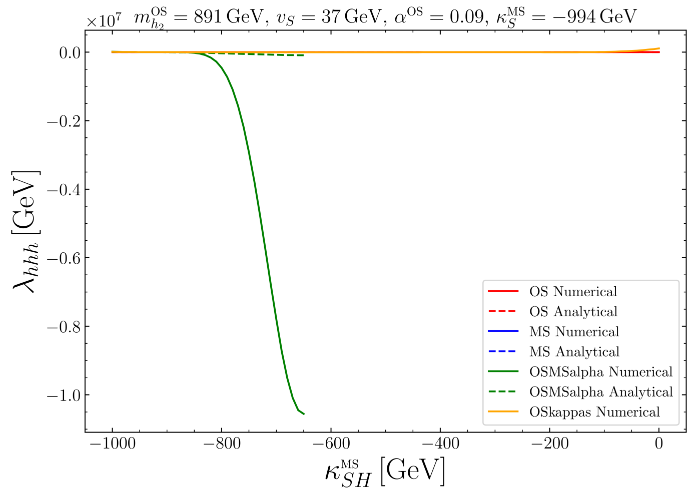
- MS tadpole renormalisation at the tree-level minimum (FJ):

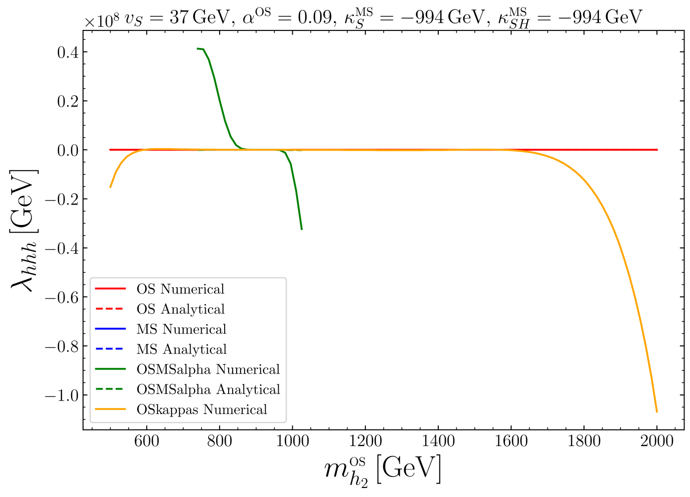
$t_h = 0$ and $\overline{\text{MS}}$ renormalised tadpoles

Benchmark Point

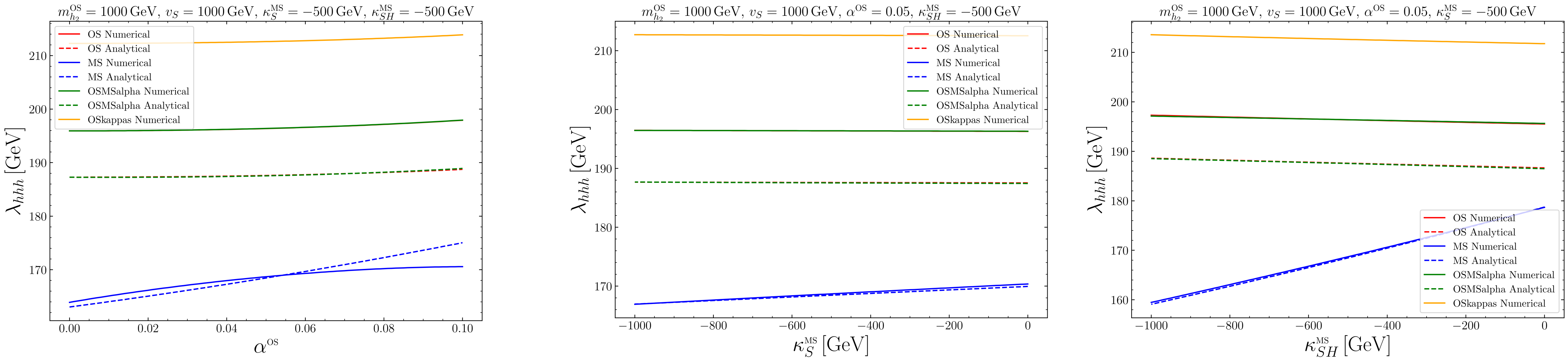








Test Point



$m_{h_2}^{\mathrm{OS}} = 1000\,\mathrm{GeV},\ v_S = 1000\,\mathrm{GeV},\ \kappa_S^{\mathrm{MS}} = -500\,\mathrm{GeV},\ \kappa_{SH}^{\mathrm{MS}} = -500\,\mathrm{GeV}$
$m_{h_2}^{\mathrm{OS}} = 1000\,\mathrm{GeV},\ v_S = 1000\,\mathrm{GeV},\ \alpha^{\mathrm{OS}} = 0.05,\ \kappa_{SH}^{\mathrm{MS}} = -500\,\mathrm{GeV}$
$m_{h_2}^{\mathrm{OS}} = 1000\,\mathrm{GeV},\ v_S = 1000\,\mathrm{GeV},\ \alpha^{\mathrm{OS}} = 0.05,\ \kappa_S^{\mathrm{MS}} = -500\,\mathrm{GeV}$
OS Numerical
OS Analytical
MS Numerical
MS Analytical
OSMSalpha Numerical
OSMSalpha Analytical
OSkappas Numerical
λ_{hhh} [GeV]
α^{OS}
κ_S^{MS} [GeV]
$\kappa_{SH}^{\mathrm{MS}}$ [GeV]

Counter Terms

$$\delta\alpha = \frac{1}{2}\frac{\Sigma_{h_1h_2(p^2=m_{h1}^2)} + \Sigma_{h_1h_2(p^2=m_{h2}^2)}}{m_{h_2}^2 - m_{h_1}^2}$$

$$\delta\kappa_S = \frac{\frac{\partial\lambda_{HHH}^{(0)}}{\partial\kappa_{SH}}\left(\delta_{\text{gen+wfr}}^{(1)}\lambda_{hHH} + \sum_x \delta_x^{\text{CT}}\lambda_{hHH}\right) - \frac{\partial\lambda_{hHH}^{(0)}}{\partial\kappa_{SH}}\left(\delta_{\text{gen+wfr}}^{(1)}\lambda_{HHH} + \sum_x \delta_x^{\text{CT}}\lambda_{HHH}\right)}{\frac{\partial\lambda_{hHH}^{(0)}}{\partial\kappa_S}\frac{\partial\lambda_{HHH}^{(0)}}{\partial\kappa_{SH}} - \frac{\partial\lambda_{hHH}^{(0)}}{\partial\kappa_{SH}}\frac{\partial\lambda_{HHH}^{(0)}}{\partial\kappa_S}}$$

$$\delta\kappa_{SH} = \frac{\frac{\partial\lambda_{hHH}^{(0)}}{\partial\kappa_S}\left(\delta_{\text{gen+wfr}}^{(1)}\lambda_{HHH} + \sum_x \delta_x^{\text{CT}}\lambda_{HHH}\right) - \frac{\partial\lambda_{HHH}^{(0)}}{\partial\kappa_S}\left(\delta_{\text{gen+wfr}}^{(1)}\lambda_{hHH} + \sum_x \delta_x^{\text{CT}}\lambda_{hHH}\right)}{\frac{\partial\lambda_{hHH}^{(0)}}{\partial\kappa_S}\frac{\partial\lambda_{HHH}^{(0)}}{\partial\kappa_{SH}} - \frac{\partial\lambda_{hHH}^{(0)}}{\partial\kappa_{SH}}\frac{\partial\lambda_{HHH}^{(0)}}{\partial\kappa_S}}$$