# Associators for string amplitudes and number theory

*based on arXiv:2412.17579 with J. Broedel and F. Zerbini*
*and arXiv:2505.23385*

Konstantin Baune,
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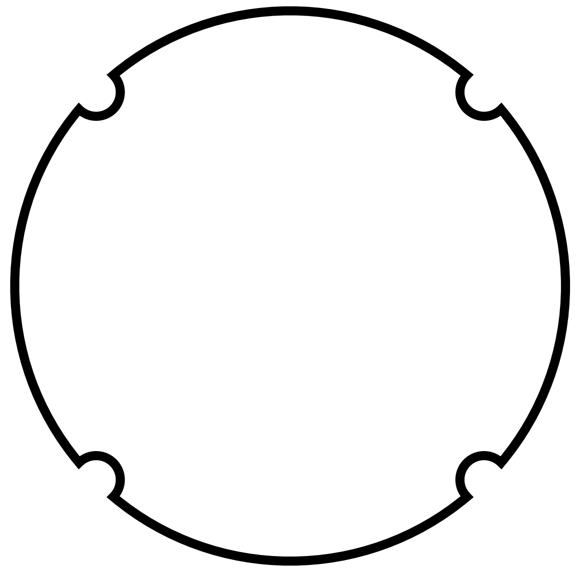
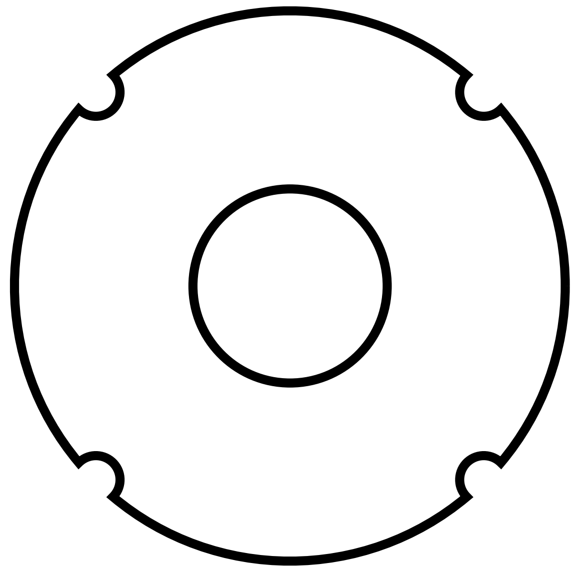
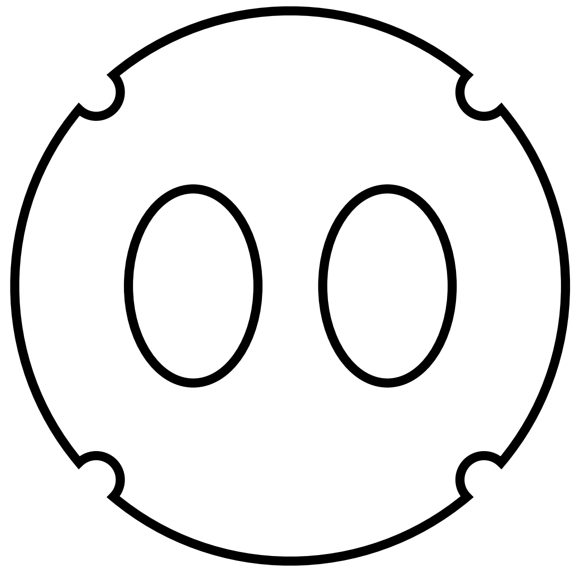
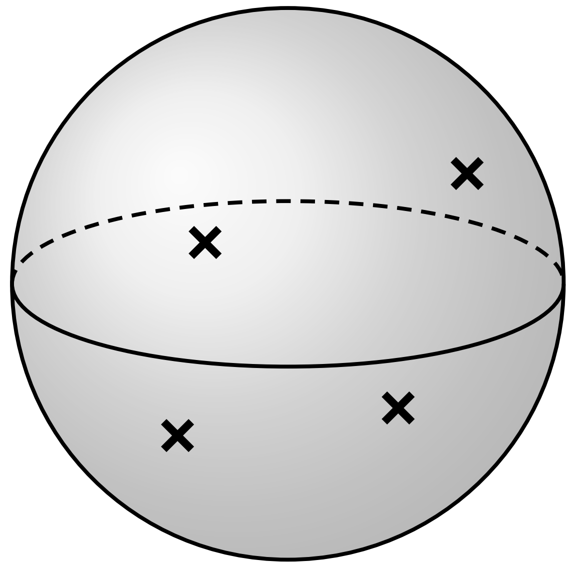
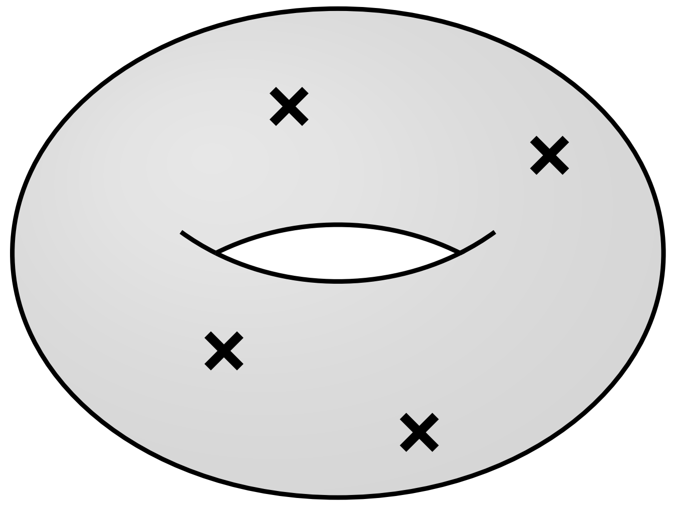
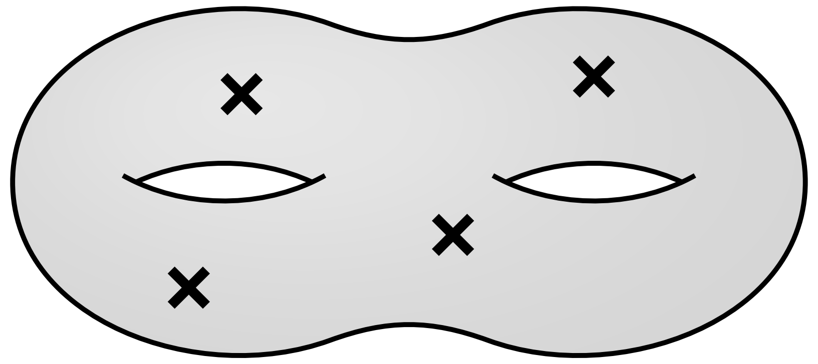
SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

ETH zürich

# String amplitudes

## loop-expansion

| | tree-level | | one-loop | | two-loop | | |
|---|---|---|---|---|---|---|---|
| open string: $\mathcal{A}_4 = g_s^{-2}$ |  | $+$ |  | $+\ g_s^2$ |  | $+$ | $\ldots$ |
| closed string: $\mathcal{M}_4 = g_s^{-2}$ |  | $+$ |  | $+\ g_s^2$ |  | $+$ | $\ldots$ |

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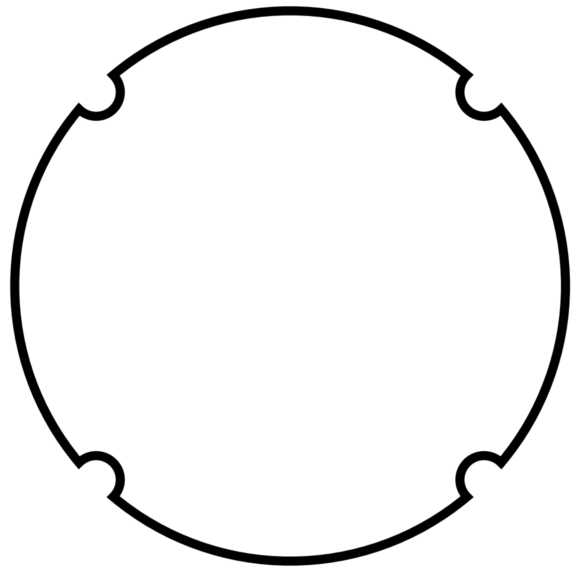
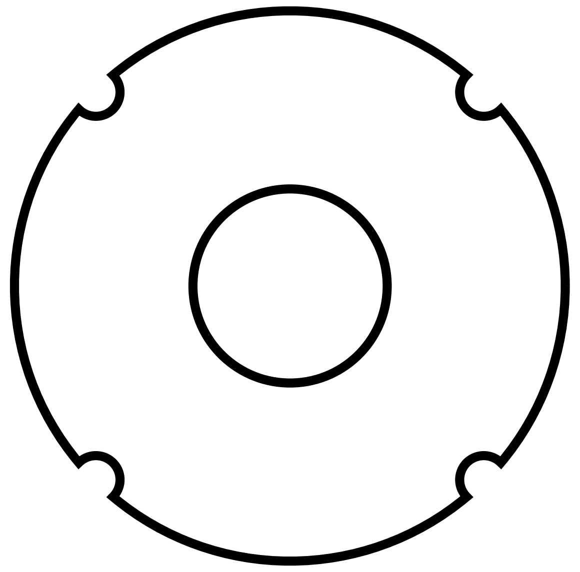
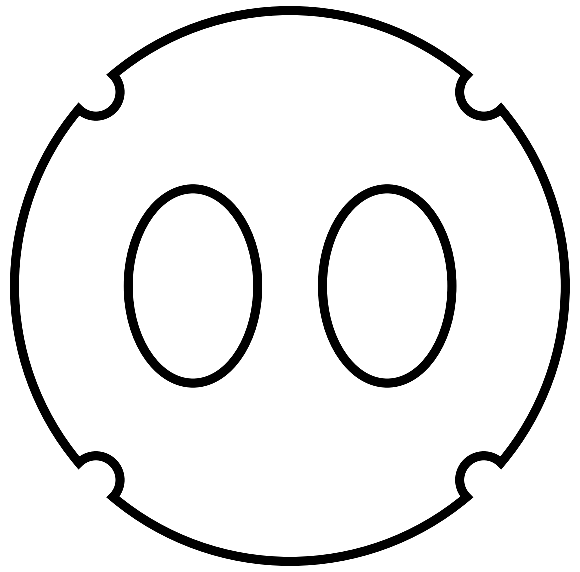
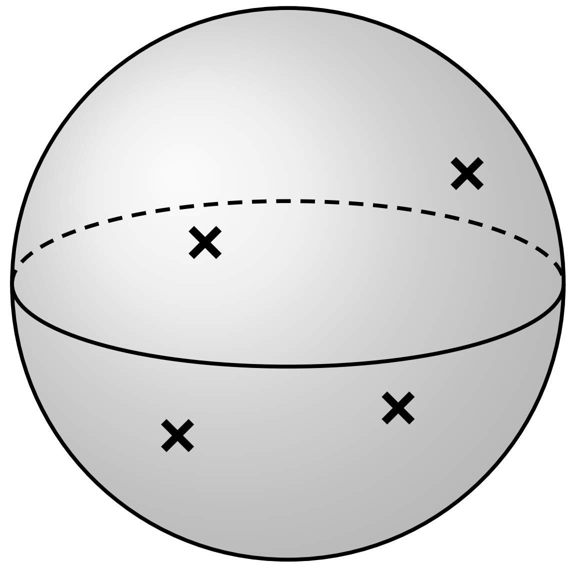
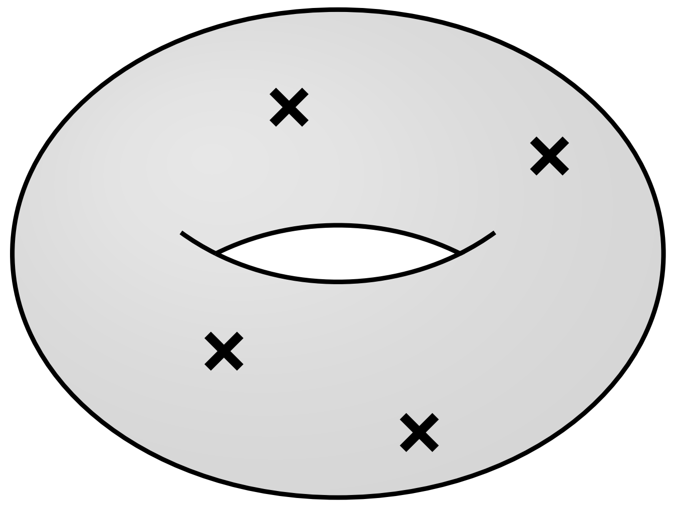
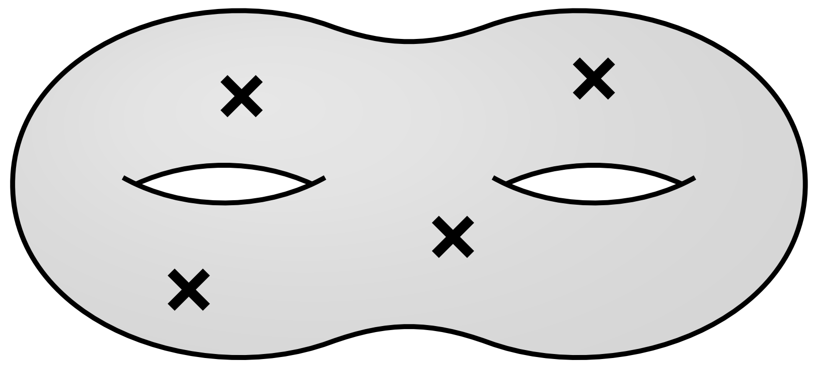
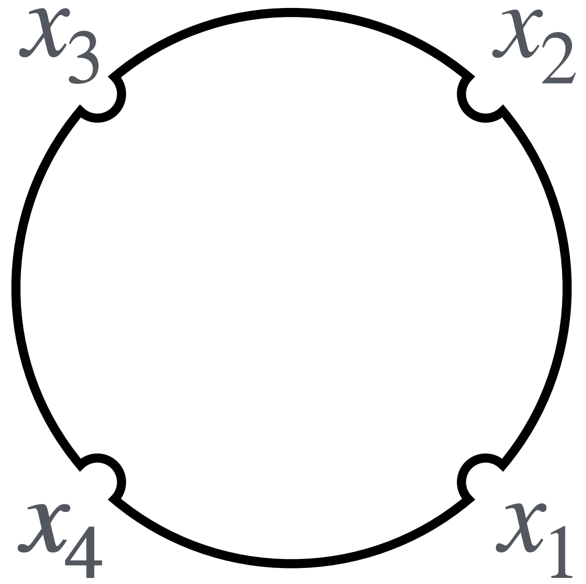
| open string: $\mathscr{A}_4 = g_s^{-2}$ |  | $+$ |  | $+\ g_s^{2}$ |  | $+$ | $\dots$ |
| closed string: $\mathscr{M}_4 = g_s^{-2}$ |  | $+$ |  | $+\ g_s^{2}$ |  | $+$ | $\dots$ |

# String amplitudes

## tree-level 4-point cases

open string: $A_4(1,2,3,4)(s,t) =$




$$= \left(\frac{1}{s} + \frac{1}{t}\right) \exp\left[\sum_{n\geq 2} (-1)^n \frac{\zeta_n}{n} \left(s^n + t^n - (s+t)^n\right)\right]$$

Mandelstam variables
$s_{ij} = \alpha'(p_i + p_j)^2$

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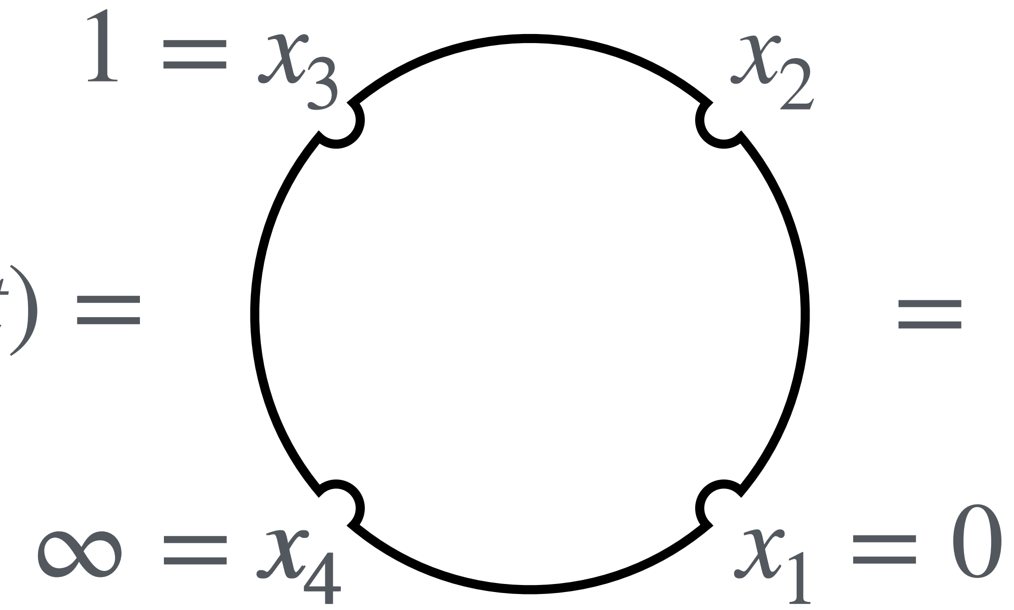
open string: $A_4(1,2,3,4)(s,t) =$ 

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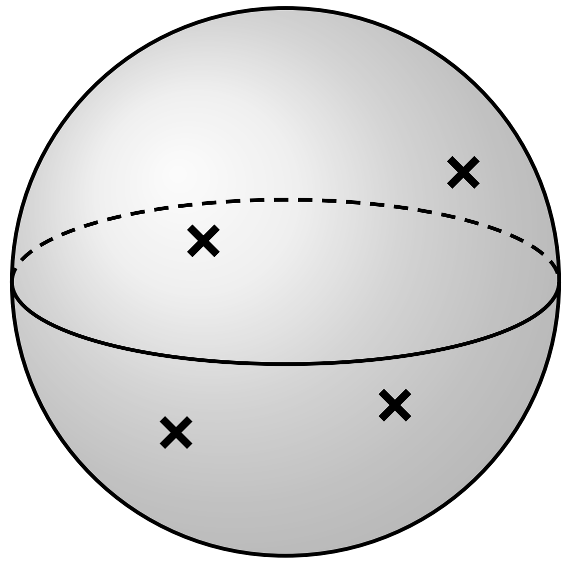
closed string: $M_4(s,t,u) =$  $= \left(\frac{1}{s} + \frac{1}{t}\right) \exp\left[\sum_{\substack{n\geq 2 \\ n \text{ odd}}} (-1)^n \frac{2\zeta_n}{n} \left(s^n + t^n - (s+t)^n\right)\right]$

# String amplitudes

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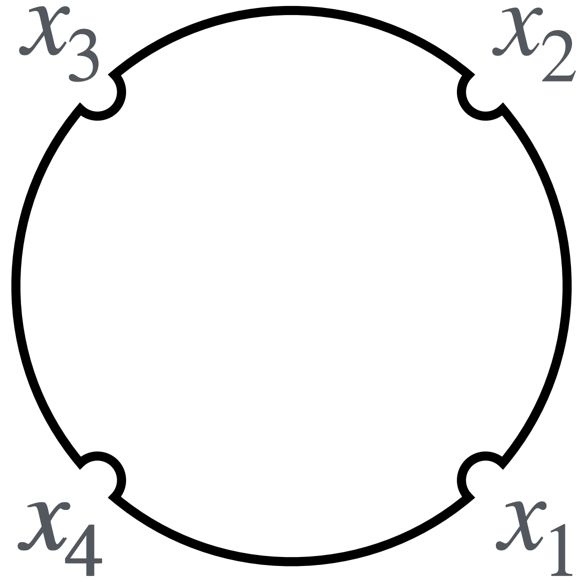


$\infty = x_4$ $\quad x_1 = 0$

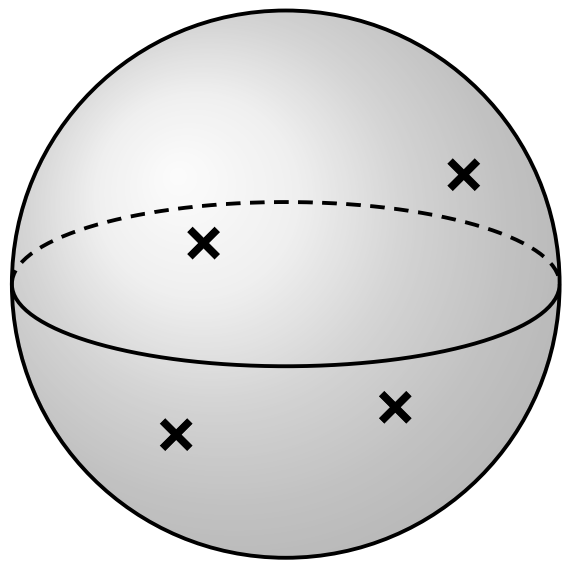
$$= \left(\frac{1}{s}+\frac{1}{t}\right)\exp\left[\sum_{n\geq 2}(-1)^n\frac{\zeta_n}{n}\left(s^n+t^n-(s+t)^n\right)\right]$$

Mandelstam variables
$s_{ij} = \alpha'(p_i+p_j)^2$

**single-valued map**

simplest cases:
$\mathrm{sv}(\zeta_{2n}) = 0 =: \zeta^{\mathrm{sv}}_{2n}$
$\mathrm{sv}(\zeta_{2n+1}) = 2\zeta_{2n+1} =: \zeta^{\mathrm{sv}}_{2n+1}$

closed string: $M_4(s,t,u) =$



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$$\Longrightarrow \quad \mathrm{sv}(A_n) = M_n$$

[Stieberger; Stieberger, Taylor; Brown, Dupont]

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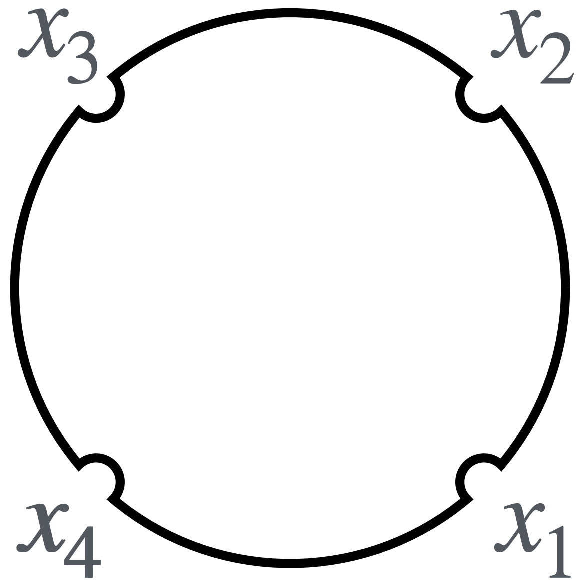


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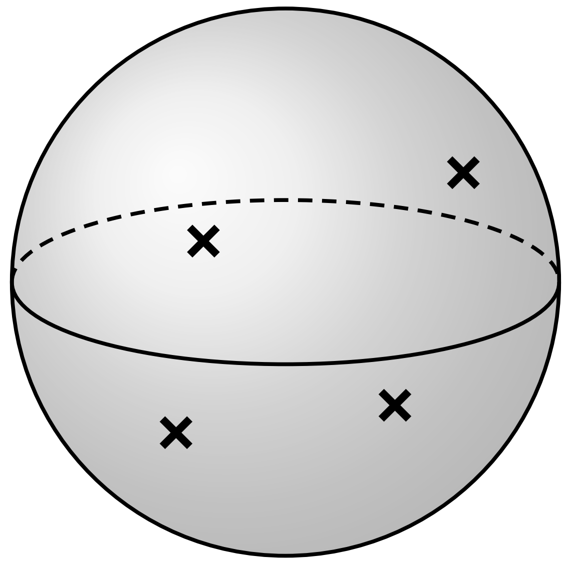

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[Stieberger; Stieberger, Taylor; Brown, Dupont]

# (Single-valued) polylogarithms

**Polylogarithms:** $L_{x_{i_1}\cdots x_{i_s}}(z) := \int_0^z \omega_{i_1}(t)\, L_{x_{i_2}\cdots x_{i_s}}(t), \quad i_k \in \{0,1\}, \qquad \omega_0(z) = \frac{dz}{z},\ \omega_1(z) = \frac{dz}{z-1}$

have branch cuts $\Longrightarrow$ multi-valued functions

special values at $z = 1$: *multiple zeta values* (MZVs): $L_{\underbrace{0\cdots 01}_{n_1}\cdots\underbrace{0\cdots 01}_{n_r}}(1) = (-1)^r\, \zeta_{n_1\cdots n_r}$

e.g. $L_{01}(z) = \int_0^z \frac{dt_1}{t_1} \int_0^{t_1} \frac{dt_2}{t_2 - 1} \longrightarrow L_{01}(1) = -\frac{\pi^2}{6} = -\zeta_2$

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**Single-valued polylogarithms:** specific combinations of polylogs to cancel branch cuts: $\mathscr{L}_w = \mathrm{sv}(L_w)$

[Brown '04] e.g. $\mathscr{L}_{01}(z) = L_{01}(z) + L_{10}(\bar{z}) + L_0(z)\, L_1(\bar{z}), \quad \mathscr{L}_{001}(z) = L_{001}(z) + L_{100}(\bar{z}) + L_{00}(z) L_1(\bar{z}) + L_0(z) L_{10}(\bar{z})$

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special values at $z=1$: *single-valued multiple zeta values* (svMZVs): [Brown '13]

e.g. $\zeta_2^{\mathrm{sv}} = \mathscr{L}_{01}(1) = \zeta_2 - \zeta_2 = 0, \qquad \zeta_3^{\mathrm{sv}} = \mathscr{L}_{001}(1) = \zeta_3 + \zeta_3 = 2\zeta_3$

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special values at $z = 1$: *single-valued multiple zeta values* (svMZVs): [Brown '13]

e.g. $\zeta^{\mathrm{sv}}_{3,3,7} = 2\zeta_{3,3,7} + 12\zeta_5\zeta_{3,5} - 14\zeta_3^2\zeta_7 + 60\zeta_3\zeta_5^2 + 407\zeta_2\zeta_{11} + \frac{112}{5}\zeta_2^2\zeta_9 - \frac{64}{35}\zeta_2^3\zeta_7$ [Stieberger '13]

# Knizhnik—Zamolodchikov equation and associators

Let $F(x)$ be a vector that is solution to the **Knizhnik–Zamolodchikov equation**

$$\frac{d}{dx}F(x) = \left(\frac{e_0}{x} + \frac{e_1}{x-1}\right)F(x),$$

for square matrices $e_0$ and $e_1$.

Then the **regularized limits** $\begin{cases} C_0 = \lim\limits_{x\to 0} x^{-e_0}F(x) \\ C_1 = \lim\limits_{x\to 1}(1-x)^{-e_1}F(x) \end{cases}$ are connected through the

**Drinfeld associator** $\Phi(e_0, e_1) = \sum\limits_{w\in\{e_0,e_1\}^\times} w\,\zeta_w = 1 + \zeta_2[e_0, e_1] + \zeta_3[e_0 + e_1, [e_0, e_1]] + \ldots$

generating series of MZVs

through the equation $\boxed{\Phi(e_0, e_1)\,C_0 = C_1}$

# Open-string amplitude recursion [Broedel, Schlotterer, Stieberger, Terasoma '14]

Goal: formalism translating between $A_{n-1}$ and $A_n$

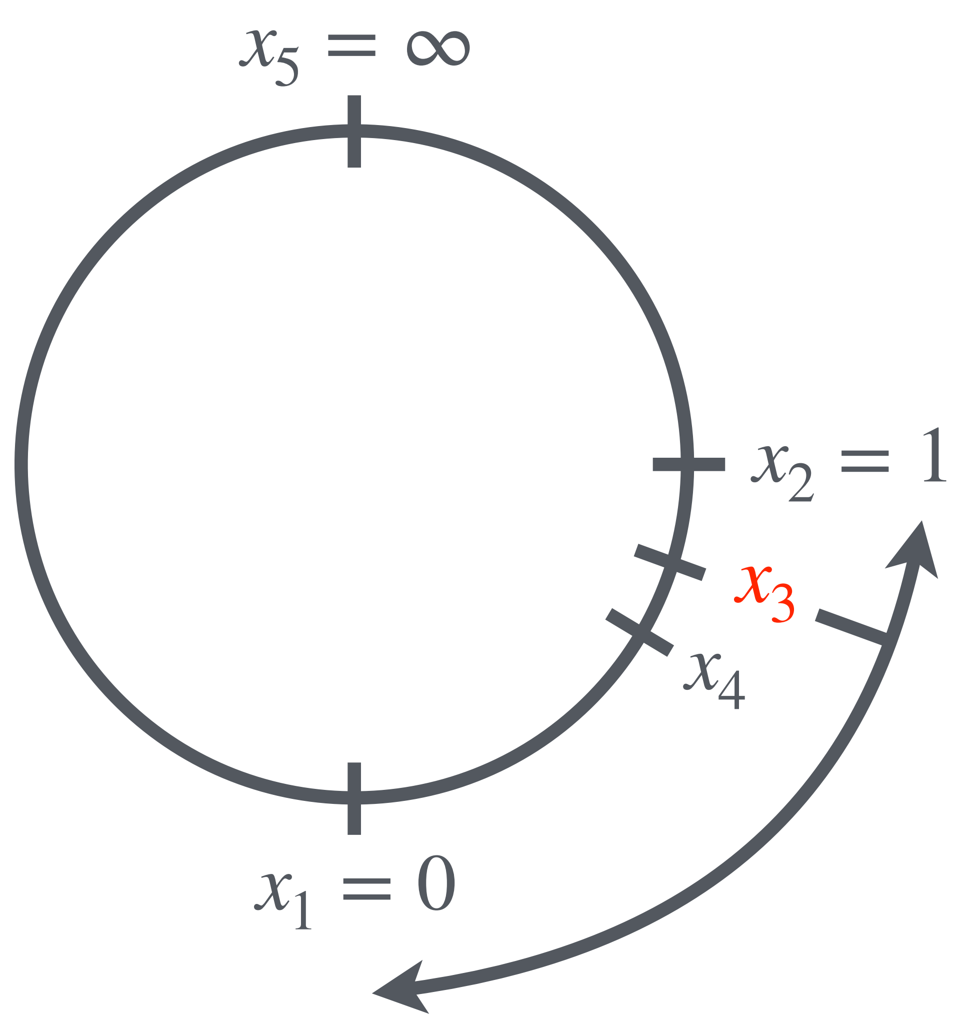
Method: $$\frac{d}{dx}F(x) = \left(\frac{e_0}{x} + \frac{e_1}{x-1}\right)F(x), \qquad \begin{cases} C_0 = \lim\limits_{x\to 0} x^{-e_0}F(x) \\ C_1 = \lim\limits_{x\to 1}(1-x)^{-e_1}F(x) \end{cases} \qquad \Phi(e_0,e_1)\,C_0 = C_1$$

*How to construct $F$?*

Take string amplitude $$s_{14}\int_0^1 \frac{dx_4}{x_4} x_4^{s_{14}}(1-x_4)^{s_{24}}$$

Use unintegrated „vacuum" insertion $x_3$

$$F_4(x_3) = s_{14}\int_0^{x_3} \frac{dx_4}{x_4} x_4^{s_{14}}(1-x_4)^{s_{24}} x_3^{s_{13}}(1-x_3)^{s_{23}}(x_3-x_4)^{s_{34}}$$



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from 3- to 4-points

$$C_0 = \lim_{x_3 \to 0} x_3^{-e_0} F_4(x_3) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$C_1 = \lim_{x_3 \to 1} (1 - x_3)^{-e_1} F_4(x_3) = \begin{pmatrix} \frac{\Gamma(1+s)\,\Gamma(1+t)}{\Gamma(1+s+t)} \\ \frac{\Gamma(1+s)\,\Gamma(1+t)}{\Gamma(1+s+t)} - 1 \end{pmatrix}$$

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3-point amp

4-point amp (Veneziano)

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$$\Phi(e_0, e_1)\, C_0 = C_1$$

from 3- to 4-pts using the Drinfeld associator

$\Longrightarrow$ Veneziano amplitudes only has MZVs in $\alpha'$-expansion

# Open-string amplitude recursion [Broedel, Schlotterer, Stieberger, Terasoma ’14]

from $(n-1)$- to $n$-point amplitude

Drinfeld associator (gen. series of MZVs)

vector containing $n$-pt open-string amp

$$\Phi(e_0, e_1)\, C_0 = C_1$$

$(n-2)! \times (n-2)!$ matrices

vector containing $(n-1)$-pt open-string amp

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What about closed-string amplitudes?

# Closed-string amplitude recursion

[KB, Broedel, Zerbini '24]

<u>Goal:</u> formalism translating between $M_{n-1}$ and $M_n$

<u>Method:</u> use single-valued projection to lift open-string recursion to closed-string recursion

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[KB, Broedel, Zerbini '24]

**Goal:** formalism translating between $M_{n-1}$ and $M_n$

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**Thm. ([KB, Broedel, Zerbini]):** If $\mathscr{F}(z)$ is single-valued in $z$ and satisfies the Kn.–Za. eq., then

we can relate the limits $\begin{cases} \mathscr{C}_0 = \lim\limits_{z\to 0} |z|^{-2e_0}\mathscr{F}(z) \\ \mathscr{C}_1 = \lim\limits_{z\to 1} |1-z|^{-2e_1}\mathscr{F}(z) \end{cases}$ through $\mathscr{C}_1 = \Phi^{\mathrm{sv}}(e_0, e_1)\mathscr{C}_0$,

with $\Phi^{\mathrm{sv}} = \Sigma_w w\,\mathscr{L}_w(1) = \Sigma_w w\,\zeta_w^{\mathrm{sv}} = \mathrm{sv}(\Phi)$ the **Deligne associator**.

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with $\Phi^{\mathrm{sv}} = \Sigma_w w\, \mathscr{L}_w(1) = \Sigma_w w\, \zeta^{\mathrm{sv}}_w = \mathrm{sv}(\Phi)$ the **Deligne associator**.

Use $\mathscr{F} = \mathrm{sv}(F)$:

$\Longrightarrow$ single-valued solution to Kn.–Za. eq. with the same matrices $e_0$ and $e_1$

$\Longrightarrow$ limits contain closed-string amplitudes, related through an associator!

# Closed-string amplitude recursion

[KB, Broedel, Zerbini '24]

from 3- to 4-point amplitude

Virasoro—Shapiro (4-point amp)

3-point amp

$$\Phi^{\mathrm{sv}}(e_0, e_1)\,\mathscr{C}_0 = \Phi^{\mathrm{sv}}\begin{pmatrix}1\\0\end{pmatrix} = \mathscr{C}_1 = \begin{pmatrix}\dfrac{\Gamma(1+s)\,\Gamma(1+t)\,\Gamma(1-s-t)}{\Gamma(1-s)\,\Gamma(1-t)\,\Gamma(1+s+t)}\\ \cdots\end{pmatrix}$$

$$e_0 = \begin{pmatrix}s & -s\\ 0 & 0\end{pmatrix}, \quad e_1\begin{pmatrix}0 & 0\\ -t & t\end{pmatrix}$$

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## from $(n-1)$- to $n$-point amplitude

Deligne associator (gen. series of svMZVs)

vector containing $n$-pt closed-string amp

$$\Phi^{\rm sv}(e_0, e_1)\,\mathscr{C}_0 = \mathscr{C}_1$$

$(n-2)!\times(n-2)!$ matrices

vector containing $(n-1)$-pt closed-string amp

$\Longrightarrow$ closed-string amplitudes only have **svMZVs** in $\alpha'$-expansion

Can this be generalized beyond flat space?

# String amplitudes in AdS

[Alday, Hansen, Silva et al. '22-'25]

## large radius expansion

$$A(s,t) = A^{(0)}(s,t) + \frac{\alpha'}{R^2} A^{(1)}(s,t) + \left(\frac{\alpha'}{R^2}\right)^2 A^{(2)}(s,t) + \ldots$$

flat-space tree-level amp
(Veneziano/Virasoro—Shapiro)

first curvature
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flat-space tree-level amp
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$A^{(k)}(s,t)$ is a linear combination of integrals of the form

Open string:

$$J_w(s,t) = \int_0^1 dx\, x^{s-1}(1-x)^{t-1} L_w(x)$$

Closed string:

$$I_w(s,t) = \int_{\mathbb{C}} d^2z\, |z|^{2(s-1)} |1-z|^{2(t-1)} \mathscr{L}_w(z)$$

flat-space amplitudes
with insertions of
(sv-)polylogarithms

[ KB '25 ]

# AdS amplitude building blocks from associators

Open string: $$J_w(s,t) = \int_0^1 dx\, x^{s-1}(1-x)^{t-1} L_w(x)$$

Closed string: $$I_w(s,t) = \int_{\mathbb{C}} d^2z\, |z|^{2(s-1)} |1-z|^{2(t-1)} \mathscr{L}_w(z)$$

flat-space amplitudes with insertions of (sv-)polylogarithms

Flat-space associator structure directly translates to AdS:

e.g. $$J_{001}(s,t) = \frac{1}{st}(s+t,0,1,0,0,0,0,0) \cdot \Phi(e_0, e_1) \cdot (0,0,0,0,0,0,1,0)^T$$

8x8 matrices

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sv-map

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Flat-space associator structure directly translates to AdS:

In general:

$J_w$ generated through **Drinfeld associator** $\Longrightarrow$ **multiple zeta values**

$I_w$ generated through **Deligne associator** $\Longrightarrow$ **single-valued multiple zeta values**

# Summary

**Tree-level string amplitudes** in flat space and AdS background can be **generated through associators**

- *Open string*: **Drinfeld associator** $\Longrightarrow$ **multiple zeta values** in $\alpha'$-expansion
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- Closed-string **one-loop** recursion? (lifting open-string one-loop recursion of [Broedel, Kaderli '22])
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*Thank you!*