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Determining masses of hadronic bound states using a holographic ansatz

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Motivation

How do we compute observables in strongly coupled theories?

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How do we compute observables in strongly coupled theories?

- Lattice computations $\Rightarrow$ expensive
- AdS/CFT

Advantages of AdS/CFT:	less expensive, straight forward method to calculate e.g bound states
Restrictions of AdS/CFT:	Large N_c, SUSY

AdS/CFT-correspondence

- $AdS_5 \times S_5 \Leftrightarrow \mathcal{N} = 4\ SYM$

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- N_c D3-Branes

$$\begin{aligned} ds^2 &= G_{MN} dx^M dx^N \\ &= \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} \left(dr^2 + r^2 d\Omega_5^2 \right) \end{aligned}$$

$$r^2 = x_m x^m$$

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						

AdS/CFT-correspondence

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- N_c D3-Branes

$$ds^2 = G_{MN} dx^M dx^N$$
$$= \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} \left(d\rho^2 + \rho^2 d\Omega_3^2 + dx_8^2 + dx_9^2 \right)$$

$$r^2 = \rho^2 + x_8^2 + x_9^2$$

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		

AdS/CFT-correspondence

- $AdS_5 \times S_5 \Leftrightarrow \mathcal{N} = 4\ SYM$
- N_c D3-Branes + N_f D7-Branes

$$\begin{aligned} ds^2 &= G_{MN} dx^M dx^N \\ &= \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} \left(d\rho^2 + \rho^2 d\Omega_3^2 + dx_8^2 + dx_9^2 \right) \end{aligned}$$

$$ds_{D7}^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} \left(1 + {x'_8}^2 + {x'_9}^2 \right) d\rho^2 + \frac{R^2 \rho^2}{r^2} d\Omega_3^2$$

$$r^2 = \rho^2 + x_8^2 + x_9^2$$

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		

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Bottom-up model

AdS_5-Model with a modified metric

$$ds^2 = r^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{r^2} d\rho^2$$
$$r^2 = \rho^2 + L(\rho)^2$$
$$\sqrt{-g} = \rho^3$$

Scalar mesons

$$S_\phi = \int d^4x \, d\rho \, \sqrt{-\det g} \left(g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2 \right)$$

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$$0 = \left(\partial_\rho^2 + \mathcal{P}_1 \partial_\rho + \frac{1}{r^4} \partial_\mu \partial^\mu + \mathcal{P}_2 \right) \phi$$

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$$\phi \left(\rho, x^\mu \right) = \phi_\rho(\rho) \exp(\mathrm{i}k \cdot x) \qquad \Rightarrow \partial_\mu \partial^\mu \phi = M^2 \phi$$

Scalar mesons

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$$0 = \left(\partial_\rho^2 + \mathcal{P}_1 \partial_\rho + \frac{1 + L'^2}{r^4} M^2 + \mathcal{P}_2 \right) \phi$$

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Flavor symmetry

Top down:

- N_f D7 branes
- global $\mathrm{SU}(N_f)$

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Top down:

- N_f D7 branes
- global $\mathrm{SU}(N_f)$
- breaking by separating the branes

Bottom up:

$$S_\phi = \int d^4x\, d\rho\, \phi \tilde{\mathcal{L}}(r) \phi$$

$$\Rightarrow \qquad S = \int d^4x d\rho \,\mathrm{Tr} \left\{ \phi_{ij} \tilde{\mathcal{L}}(r) \delta_{jk} \phi_{kl} \right\}$$

Flavor symmetry breaking

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$$\tilde{\mathcal{L}}(r)\delta_{jk} \longrightarrow \tilde{\mathcal{L}}_{jk} := \mathsf{diag} \left(\tilde{\mathcal{L}}(r_1), ..., \tilde{\mathcal{L}}(r_{N_f}) \right)_{jk}$$

$$S = \int d^4x d\rho \, \mathrm{sTr} \left\{ \tilde{\phi}_{ij} \tilde{\mathcal{L}}_{jk} \tilde{\phi}_{kl} \right\}$$

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$$S = \int d^4x d\rho \,\mathrm{sTr}\left\{\tilde{\phi}_{ij}\tilde{\mathcal{L}}_{jk}\tilde{\phi}_{kl}\right\}$$

Parametrization:

$$\phi_{ij} = \phi_a T^a_{ij} \qquad \tilde{\mathcal{L}}_{jk} = p^i_m \tilde{\mathcal{L}}(r_i) T^m_{jk}$$

$$\Rightarrow \quad S = \frac{1}{2}\int d^4x d\rho \,\phi_a \left(p^i_0 \tilde{\mathcal{L}}(r_i)\delta_{ac} + \frac{1}{2} p^i_b \tilde{\mathcal{L}}(r_i) d_{abc}\right)\phi_c$$

Flavor symmetry breaking

$$S = \int d^4x d\rho \, \mathrm{Tr} \left\{ \phi_{ij} \tilde{\mathcal{L}}(r) \delta_{jk} \phi_{kl} \right\}$$

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For Fermions

$$S = \frac{1}{2} \int d^4x d\rho \, \Psi_a \left(p^i_0 \tilde{\mathcal{L}}(r_i) \delta_{ac} + \frac{1}{2} p^i_b \tilde{\mathcal{L}}(r_i) f_{abc} \right) \Psi_c$$

Comparison to SM hadrons ($\mathrm{SU}(3) \to \mathrm{SU}(2) \otimes \mathrm{U}(1)$)

	calculated mass	measured mass	particle
scalar	$449\ MeV\ (+225.3\%)$	$138\ MeV$	π
	$496\ MeV$ (fitted)	$496\ MeV$	K
	$519\ MeV\ (-5.2\%)$	$548\ MeV$	η
vector	$716\ MeV\ (-7.6\%)$	$775\ MeV$	ρ
	$821\ MeV\ (-8.1\%)$	$892\ MeV$	K
	$869\ MeV\ (-14.8\%)$	$1019\ MeV$	ϕ
baryon	$977\ MeV\ (+4.0\%)$	$939\ MeV$	n/p
	$1137\ MeV\ (-3.1\%)$	$1174\ MeV$	Λ/Σ
	$1299\ MeV\ (-1.5\%)$	$1318\ MeV$	Ξ

Input: $m_{u/d} = 3.49\ MeV,\ m_s = 93.5\ MeV,\ M_K = 496\ MeV$

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Conclusion and Outlook

Conclusion:

- use AdS/CFT to predict masses of bound states
- introduction of flavor symmetry and its breaking
- QCD dual model

Outlook:

- Top down approach with arbitrary $\mathrm{SU}(N)$
- Origin of f_{abc} for fermions
- Other symmetries $\mathrm{SO}(N),\ \mathrm{Sp}(2N)$

Thank you for your attention!

The embedding L

$$L \sim m_q + \frac{\langle q\bar{q}\rangle}{\rho^2}$$

$\Rightarrow$ scalar field

$$S = \int d^4x \, d\rho \, \rho^3 \left(\partial_a \phi \partial_b \phi g^{ab} + m^2 \phi^2 \right)$$

$$\Rightarrow \qquad \partial_\rho \left(\rho^3 \partial_\rho L \right) + \rho(3 + m^2) L = 0$$

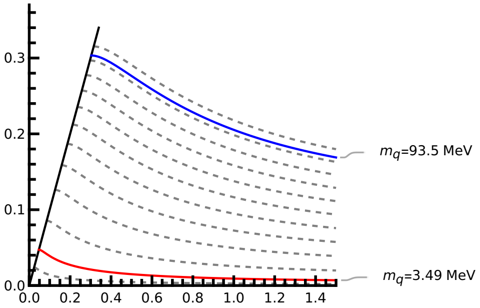
$$\begin{aligned} m^2 =& \Delta(\Delta - 4) \\ \Delta =& 3 + \gamma \\ \gamma =& -\frac{2\alpha_s}{\pi} \end{aligned}$$

$$\Rightarrow \partial_\rho \left(\rho^3 \partial_\rho L \right) + \rho \frac{2\alpha_s}{\pi} \left(2 - \frac{2\alpha_s}{\pi} \right) L = 0$$

$$\partial_\rho\left(\rho^3\partial_\rho L\right)+\rho\frac{2\alpha_s}{\pi}\left(2-\frac{2\alpha_s}{\pi}\right)L=0$$

$$L(\rho_{IR})=\rho_{IR}$$

$$L'(\rho_{IR})=0$$



The embedding L

$$L \sim m_q + \frac{\langle q\bar{q}\rangle}{\rho^2}$$

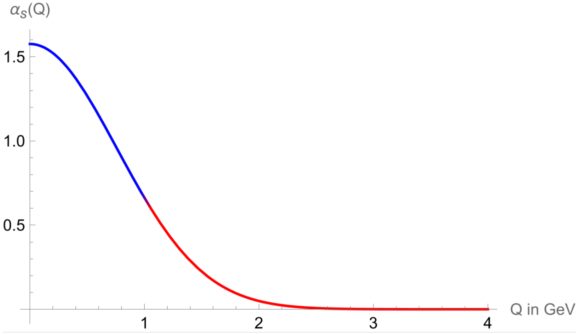
$\Rightarrow$ scalar field

$$\begin{aligned}
S &= \int d^4x\, d\rho\, \rho^3 \left(\partial_a\phi\partial_b\phi g^{ab} + m^2\phi^2\right) \\
&\overset{\tilde{\phi}=\rho\phi}{=} \int d^4x\, d\rho \left(\rho g^{ab}\partial_a\tilde{\phi}\partial_b\tilde{\phi} + \left(\frac{r^2}{\rho} + 2(\rho + LL') + \rho m^2\right)\tilde{\phi}^2\right) \\
&\overset{L=\tilde{\phi}}{=} \int d^4x\, d\rho \left(\rho^3\partial_\rho L\partial_\rho L + \rho\left(3+m^2\right)L^2\right) + \mathcal{O}(L^3)
\end{aligned}$$

$$\Rightarrow \partial_\rho\left(\rho^3\partial_\rho L\right) + \rho(3+m^2)L = 0$$

The strong coupling α_s

- for high energy $\Rightarrow$ perturbation
- in the infrared: bell curve [?]



scalar boundary conditions

$$0 = \left(\partial_\rho^2 + \mathcal{P}_1 \partial_\rho + \frac{1 + L'^2}{r^4} M^2 + \mathcal{P}_2 \right) \tilde{\phi}$$

$$\begin{aligned} \tilde{\phi}_\rho \left(\rho_{IR} \right) =& 1 \\ \partial_\rho \tilde{\phi}_\rho \left(\rho_{IR} \right) =& 0 \end{aligned}$$

normalizable state $\Rightarrow$ ϕ_ρ vanishes in the UV
$\Rightarrow$ vary M until ϕ_ρ vanishes at a chosen UV cutoff

Hadronic bound states

- start with an $AAdS$-action of the desired type of field
- potentially fix their mass dimensions
- match the AdS masses
- calculate the gravitational eoms
- identify the Minkowski part with the field
- solve for the masses of the Minkowski part

potential problems

- for bosonic states $\Rightarrow q\bar{q}$ states
- qqq state not a color singlet for large N_c
- solution: FA_2F state $\Rightarrow$ gaugino
- $N_c = 3$ with $\frac{1}{N_c^2}$ corrections $\Rightarrow$ accuracy of $\sim 10\%$

$\mathrm{SU}(3) \to \mathrm{SU}(2) \otimes \mathrm{U}(1)$

$$S = \sum_{a=1}^{N_f^2-1} \tilde{p}_a^i \int d^4x d\rho\, \tilde{\phi}_a \tilde{\mathcal{L}}_{q_i} \tilde{\phi}_a$$

$$\tilde{\mathcal{L}} = \text{diag}\left(\tilde{\mathcal{L}}_{u/d}, \tilde{\mathcal{L}}_{u/d}, \tilde{\mathcal{L}}_s\right)$$

$\tilde{p}_a^i$ Meson

$a \backslash i$	1	3
1,2,3	1	0
4,5,6,7	$\frac{1}{2}$	$\frac{1}{2}$
8	$\frac{1}{3}$	$\frac{2}{3}$

$\tilde{p}_a^i$ Baryon

$a \backslash i$	1	3
1,2	$\frac{7}{6}$	$-\frac{1}{6}$
3,4,5,6	$\frac{2}{3}$	$\frac{1}{3}$
7,8	$\frac{1}{6}$	$\frac{5}{6}$

Actions

scalar:

$$S_\phi = \int d^4x\, d\rho \left(\rho g^{ab} \partial_a \phi \partial_b \phi + \left(\frac{r^2}{\rho} + 2(\rho + LL') + \rho m^2 \right) \phi^2 \right)$$

fermion:

$$S_\Psi = \int d^4x\, d\rho\, \bar{\Psi} \left(r\gamma^\rho \partial_\rho + \frac{1}{r}\gamma^\mu \partial_\mu + \left(2r' - \frac{3r}{2\rho} + \frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2} \right) \gamma^\rho + m \right) \Psi$$

vector:

$$S_V = \int d^4x\, d\rho\, \rho^3 \left(-\frac{1}{4} g^{ab} g^{cd} F_{ac} F_{bd} + \frac{m^2}{2} g^{ab} V_a V_b \right)$$

Equations of motion

scalar:

$$\left(\partial_\rho^2 + \left(\frac{1}{\rho} + \frac{2(\rho + LL')}{r^2}\right)\partial_\rho + \frac{M^2}{r^4} - \frac{1}{\rho^2} + \frac{2\,(\rho + LL')}{\rho r^2} + \frac{m^2}{r^2}\right)\phi = 0$$

fermion;

$$\left(r^2\partial_\rho^2 + \left(6rr' - 3\frac{r^2}{\rho}\right)\partial_\rho - \left(\frac{3r}{2\rho}m - r'm - rm'\right)\gamma^\rho + \frac{M^2}{r^2} - \frac{9rr'}{2\rho} - m^2 + 2rr'' + \frac{3r^2}{2\rho^2}\right)\Psi = 0$$

vector:

$$\left(\partial_\rho^2 + \frac{3}{\rho}\partial_\rho + \frac{M^2}{r^4} - \frac{m^2}{r^2}\right)V_\mu = 0$$

$\mathrm{SU}(4) \to \mathrm{SU}(2) \otimes \mathrm{SU}(2)$

$\tilde{p}_a^i$ Meson

$a \backslash i$	1	3	multiplet
1-3	1	0	**3**
4-12	$\frac{1}{2}$	$\frac{1}{2}$	**9**
13-15	0	1	**3**

$\tilde{p}_a^i$ Baryon

$a \backslash i$	1	3	multiplet
1-4	1	0	**4**
5-11	$\frac{1}{2}$	$\frac{1}{2}$	**7**
12-15	0	1	**4**

$\mathrm{SU}(4) \to \mathrm{SU}(2) \otimes \mathrm{SU}(2)$ predicted masses

	calculated mass	multiplet	particle
Scalar	$443\ MeV$	**3**	S_1
	$496\ MeV$ (fitted)	**9**	S_2
	$594\ MeV$	**3**	S_3
Vector	$636\ MeV$	**3**	V_1
	$729\ MeV$	**9**	V_2
	$882\ MeV$	**3**	V_3
Baryon	$1042\ MeV$	**4**	B_1
	$1194\ MeV$	**7**	B_2
	$1349\ MeV$	**4**	B_3

UV cutoff

Cut off	$1\ GeV$	$50\ GeV$	$100\ GeV$	$500\ GeV$
π	$444\ MeV$	$449\ MeV$	$449\ MeV$	$449\ MeV$
K_{scalar}	$496\ MeV$	$496\ MeV$	$496\ MeV$	$496\ MeV$
η	$522\ MeV$	$519\ MeV$	$519\ MeV$	$519\ MeV$
ρ	$624\ MeV$	$716\ MeV$	$716\ MeV$	$716\ MeV$
K_{vector}	$721\ MeV$	$821\ MeV$	$821\ MeV$	$821\ MeV$
ϕ	$767\ MeV$	$869\ MeV$	$869\ MeV$	$869\ MeV$
n/p	$834\ MeV$	$976\ MeV$	$977\ MeV$	$977\ MeV$
Σ/Λ	$964\ MeV$	$1137\ MeV$	$1137\ MeV$	$1138\ MeV$
Ξ	$1108\ MeV$	$1298\ MeV$	$1299\ MeV$	$1299\ MeV$

Metric factor

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}\left(\left(1+L'^2\right)d\rho^2 + \rho^2 d\Omega_3^2\right)$$

	with $1+L'^2$	without $1+L'^2$
π	$449\ MeV$	$441\ MeV$
K_{scalar}	$496\ MeV$	$496\ MeV$
η	$519\ MeV$	$523\ MeV$
ρ	$716\ MeV$	$692\ MeV$
K_{vector}	$821\ MeV$	$795\ MeV$
ϕ	$869\ MeV$	$842\ MeV$
n/p	$977\ MeV$	$969\ MeV$
Σ/Λ	$1137\ MeV$	$1130\ MeV$
Ξ	$1299\ MeV$	$1294\ MeV$

Spin connection factor

$$\omega_i^{j\rho} = -\omega_i^{\rho j} = \frac{1}{\sqrt{1+L'^2}}\left(1 - \frac{\rho r'}{r}\right)\varepsilon_i^j$$

$$\not{D}_{AAdS} = \frac{r}{\sqrt{1+L'^2}}\gamma^\rho\partial_\rho + \frac{1}{r}\gamma^\mu\partial_\mu + \frac{1}{\sqrt{1+L'^2}}\left(\frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2} + 2r'\right)\gamma^\rho$$

	with $\frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2}$	without $\frac{\rho}{2r} - \frac{\rho^2 r'}{2r^2}$
n/p	$977\ MeV$	$981\ MeV$
Σ/Λ	$1137\ MeV$	$1133\ MeV$
Ξ	$1299\ MeV$	$1292\ MeV$

DBI-action of a D7-brane:

$$S_{D7}^{b} = -\,\mu_7 \int d^8\xi\, e^{\Phi} \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C^{(4)}] \wedge F \wedge F$$

$$S_{D7}^{f} = \frac{\mu_7}{2} \int d^8\xi\, e^{\Phi} \sqrt{-\det g}\,\bar{\Psi}\mathcal{P}_{-}\Gamma^{A}\left(D_A + \frac{1}{8}\frac{i}{2\cdot 5!}F_{NPQRS}\Gamma^{NPQRS}\Gamma_A\right)\Psi$$

The problem is:

$$\mathrm{sTr}\left\{AX^{-1}XB\right\} \neq \mathrm{sTr}\left\{AB\right\}$$

For $\mathrm{SU}(3)$ there is no difference in the resulting splitting

symmetric Trace of multiple Generators

$$
\begin{aligned}
\mathrm{sTr}\left\{T_a T_i T_j T_b\right\} = \frac{1}{6}\Bigg(& \frac{1}{2N}\delta_{ai}\delta_{bj} + \frac{1}{4}\left(d_{aie} + f_{aie}\right) d_{jbe} \\
& \frac{1}{2N}\delta_{aj}\delta_{bi} + \frac{1}{4}\left(d_{aje} + f_{aje}\right) d_{ibe} \\
& \frac{1}{2N}\delta_{ab}\delta_{ij} + \frac{1}{4}\left(d_{abe} + f_{abe}\right) d_{ije} \Bigg)
\end{aligned}
$$

The terms with f-symbols cancel out